3D weak-dispersion reverse time migration using a stereo-modeling operator

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3D weak-dispersion reverse time migration using a stereo-modeling operator

Jingshuang Li¹, Michael Fehler², Dinghui Yang³, and Xueyuan Huang³

ABSTRACT

Reliable 3D imaging is a required tool for developing models of complex geologic structures. Reverse time migration (RTM), as the most powerful depth imaging method, has become the preferred imaging tool because of its ability to handle complex velocity models including steeply dipping interfaces and large velocity contrasts. Finite-difference methods are among the most popular numerical approaches used for RTM. However, these methods often encounter a serious issue of numerical dispersion, which is typically suppressed by reducing the grid interval of the propagation model, resulting in large computation and memory requirements. In addition, even with small grid spacing, numerical anisotropy may degrade images or, worse, provide images that appear to be focused but position events incorrectly. Recently, stereo-operators have been developed to approximate the partial differential operator in space. These operators have been used to develop several weak-dispersion and efficient stereo-modeling methods that have been found to be superior to conventional algorithms in suppressing numerical dispersion and numerical anisotropy. We generalized one stereo-modeling method, fourth-order nearly analytic central difference (NACD), from 2D to 3D and applied it to 3D RTM. The RTM results for the 3D SEG/EAGE phase A classic data set 1 and the SEG Advanced Modeling project model demonstrated that, even when using a large grid size, the NACD method can handle very complex velocity models and produced better images than can be obtained using the fourth-order and eighth-order Lax-Wendroff correction (LWC) schemes. We also applied 3D NACD and fourth-order LWC to a field data set and illustrated significant improvements in terms of structure imaging, horizon/layer continuity and positioning. We also investigated numerical dispersion and found that not only does the NACD method have superior dispersion characteristics but also that the angular variation of dispersion is significantly less than for LWC.

INTRODUCTION

Two steps are needed to obtain subsurface seismic images that are of sufficient quality to be used for resource identification and reservoir characterization in complex geologic environments. First, we need to reliably estimate parameters such as velocity and density to form a basic model for conducting imaging. Second, we need migration algorithms that reliably account for wave phenomena in heterogeneous models. Due to the 3D nature of geologic structures, 2D migrated images may not be accurate due to out-of-plane 3D effects. Producing 3D seismic images is desirable to make seismic interpretation easier and help facilitate sound business decision making.

Reverse time migration (RTM) (Baysal et al., 1983; Whitmore, 1983) has drawn considerable attention as the most powerful depth imaging method because of its ability to handle complex velocity models without dip limitations. Even though it was first introduced about three decades ago, RTM has only recently become widely used in the industry because computational capability has finally caught up with the large memory requirement and computational cost. RTM uses the two-way wave equation and is relevant for challenging geologic environments because it does not suffer from dip limitation. Three-dimensional RTM is a suitable approach for seismic imaging in the case of strong lateral variations of velocity. However, 3D RTM still remains expensive, in spite of the incredible increase in computing power.
The kernel of RTM is a method for modeling the two-way wavefield by solving the full wave equation. The finite-difference (FD) method is a popular and easy way to implement RTM due to its efficiency and flexibility. A large number of FD schemes have been developed to solve acoustic and elastic wave equations (Alford et al., 1974; Kelly et al., 1976; Dablain, 1986; Takeuchi and Geller, 2000) and also applied to anisotropic and viscoelastic problems (Robertsson et al., 1994; Blanch and Robertson, 1997; Zhang et al., 1999; Takeuchi and Geller, 2000). Classical FD methods suffer from serious numerical dispersion when too few samples per wavelength are used (Yang et al., 2006). To minimize or eliminate numerical dispersion, many researchers have attempted to develop new algorithms by redefining the operators for spatial and temporal differentiation or making some special treatments (Kosloff and Baysal, 1982; Dablain, 1986; Fei and Larner, 1995; Zhang et al., 1999; Mizutani et al., 2000; Zheng and Zhang, 2005). However, as discussed in our previous work (Yang et al., 2007), the theoretical properties of these methods or techniques have some disadvantages. For example, although the so-called flux-corrected transport technique can suppress the numerical dispersion (Fei and Larner, 1995; Zhang et al., 1999; Yang et al., 2002; Zheng et al., 2006), it is unable to fully recover the resolution lost by the numerical dispersion when the grid size is too coarse (Yang et al., 2002). The spatial operator for the pseudospectral method (Kosloff and Baysal, 1982; Fornberg, 1987) can be exact up to the Nyquist frequency, but it requires the fast Fourier transform of the wavefield, which needs additional computation and introduces difficulties of how to handle nonperiodic boundary conditions and implementation in parallel computing environments (Mizutani et al., 2000). The performance of high-order FD methods such as the high-order FD scheme (e.g., Fornberg, 1990; Igel et al., 1995), high-order compact FD scheme (Lele, 1992), or the so-called Lax-Wendroff correction (LWC) scheme (Lax and Wendroff, 1964; Dablain, 1986; Robertson et al., 1994; Blanch and Robertson, 1997) are also affected by numerical dispersion. Using higher order schemes or finer spatial grids are two ways to suppress numerical dispersion. Unfortunately, the suppression of numerical dispersion is a trade-off with efficiency because finer spatial grids bring shorter temporal step sizes, resulting in large storage space requirements and increased computational cost. Staggered-grid FD methods (Virieux, 1986; Fornberg, 1990; Igel et al., 1995) can further reduce numerical dispersion, but they still suffer from it when too few samples per wavelength are used (Sei and Symes, 1995). Furthermore, staggered-grid FD methods may suffer from numerical anisotropy and have additional errors for the anisotropic case (Igel et al., 1995).

To effectively suppress the numerical dispersion caused by the discretization of the wave equation, a so-called nearly analytic discrete operator was introduced to approximate the partial differential operators in space (Yang et al., 2003, 2004, 2006). As described by Yang et al. (2003), it simultaneously uses the wave displacement and its gradients to determine the high-order spatial derivatives. The expression of wave displacements is in a nearly analytical form, which makes the method different from conventional methods. Combined with different time schemes, several methods have been developed (Wang et al., 2009; Chen et al., 2010; Yang and Wang, 2010; Ma et al., 2011) based on this operator, and each has been found to be better than conventional algorithms at suppressing numerical dispersion and saving computation and memory cost. The approach is initially proposed by Konndoh (1991) and applied to solve parabolic equations (Konndoh et al., 1994). We refer to this class of modeling methods, stereo-modeling methods, for reasons that will be described below. In this paper, we extend one of the stereo-modeling methods from 2D to 3D, discuss issues of numerical dispersion, numerical anisotropy, and computational efficiency, and, finally, demonstrate its applicability to RTM by investigating several 3D data sets. Finally, we provide specific examples of how numerical dispersion varies with grid spacing, frequency, and propagation angle to show the advantages of the stereo-modeling method compared with conventional FD methods.

**METHOD**

In this section, we briefly introduce the concept of stereo-modeling method and illustrate its basic formulation. For simplicity, we limit our discussion to the scalar isotropic case, which is described by the following wave equation:

$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \Delta u(x, t), \quad (1)$$

where $u(x, t)$ is the wavefield, $c$ is the propagation velocity in the medium, and the position vector is indicated by $x = (x, y, z)$. If a temporal differencing scheme is applied, we can get a half discretized equation. For example, with the truncated Taylor series expansion in the time domain and equation 1, we can get the following equation to compute the wavefield at time $t_{n+1}$:

$$u_{i,j,k}^{n+1} = 2u_{i,j,k}^n - u_{i,j,k}^{n-1} + \frac{\Delta t^2}{12} \left( \frac{\partial^4 u}{\partial t^4} \right)_{i,j,k} + \frac{\Delta t^4}{12} \left( \frac{\partial^2 u}{\partial r^2} \right)_{i,j,k}, \quad (2)$$

where $\Delta t$ denotes the time increment, $n$ is time index, and $i$, $j$, and $k$ are space indexes for $x$, $y$, and $z$ directions, respectively. The main difference between the stereo-modeling and conventional FD methods is the evaluation of the Laplace operator and the high-order spatial derivatives.

The stereo-modeling method borrows its name from an analogy to stereotomography (Billette and Lambaré, 1998) for ray-based seismic velocity. As is well known, traveltome tomography uses only picked travel times in velocity inversion. Stereotomography uses not only picked travel times but also picked gradients of traveltime versus distance, which include local coherence information. Joint inversion of traveltimes and gradients greatly increases the resolution of inverted model parameters. Analogously, conventional FD modeling uses only the wavefield for computing high-order spatial derivatives. Correspondingly, stereo-modeling uses not only the wavefield but also its gradient or first-order spatial derivative when constructing high-order spatial derivatives. During wave propagation, the wavefield and its gradient are propagated simultaneously. Including wave gradient information greatly increases the accuracy of the representation of the high-order spatial derivatives, thus improving the wave simulation quality by reducing numerical dispersion and numerical anisotropy while leading to improved efficiency.

To illustrate the main difference between conventional FD and stereo-modeling schemes, we take the 1D case as an example.
The conventional representation of spatial derivatives is given by (Fornberg, 1988)
\[ \frac{d^m f(x)}{dx^m} \bigg|_{x=x_j} = \sum_{j=0}^{n} a_{n,j}^m f(x_j) \quad m = 1, \ldots, M; n = m, m+1, \ldots, N, \]
where \( M \) is the order of the highest order derivatives to be approximated and \( a_{n,j}^m \) is the coefficient for the \( m \)th-order derivative of function \( f(x) \) at grid point \( j \) when \( n + 1 \) points are used. Note that the number of terms in the sum \( n \) depends on the order of the derivative. Inspection of this formula leads us to conclude that conventional FD has an expanded computational stencil, which means the number of grid points used is no less than the order of the derivative to be calculated. For example, for a fourth-order derivative (\( m = 4 \)), at least four grid points (\( n \geq 4 \)) are needed, and the higher the order of the derivative, the more grid points are needed. Furthermore, when used in a wave equation solver, only grid points along axis directions are used, which makes a linear computational stencil structure. Large numerical dispersion will appear unless one makes a careful selection of grid size, which means that the number of grid points per wavelength should be large. In contrast to conventional FD approaches, the representation of high-order spatial derivatives for the stereo-modeling approximation is formulated as follows (Yang et al., 2003, 2004, 2006; Tong and Yang, 2013):
\[ \frac{d^m f(x)}{dx^m} \bigg|_{x=x_j} \approx \sum_{j=-L}^{L} \left( a_{n,j}^m f(x_j) + \beta_{j}^m \frac{df(x_j)}{dx} \right) \quad m = 1, \ldots, 4L + 1. \]

In this formula, function values and their first-order derivatives are used in the construction of higher order derivatives. The value of \( L \) is independent of \( m \) and only depends on the accuracy of the method.

Larger values of \( L \) lead to higher spatial accuracy. We can achieve \( 4L \)th order accuracy in space when we use \( 2L + 1 \) grid points for the 1D case. For example, we can achieve fourth-order accuracy in space when \( L \) is 1. Thus, to achieve fourth-order accuracy in space for a 1D case, only three grid points are required for all high-order derivatives. To achieve higher order accuracy, the number of grid points needed increases, but it will remain the same for all derivatives. The number of grid points used for different order derivatives to achieve different orders of accuracy for conventional FD and stereo-modeling is listed in Table 1. Stereo-modeling thus has a compact computational stencil. When used in a wave equation solver, the computational stencil has an areal structure for 2D case and cubic structure for 3D case as shown in Figure 1. The 2D 9-point (\( 3 \times 3 \)) and 3D 27-point (\( 3 \times 3 \times 3 \)) stencils for stereo-modeling (nearly analytic central difference [NACD]) use off-axis points, which, we will show later, leads to much less numerical anisotropy and numerical dispersion than when using a conventional linear stencil. An added advantage is that the spatial gradient information available from stereo-modeling would be useful for angle gather computation in RTM, filtering, some new imaging conditions (Fleury and Vasconcelos, 2010), and some new acquisition approaches including the direct use of measured wavefield gradients.

We will now briefly illustrate the theory of 3D NACD. NACD gets its name because we use a central difference operator and save fourth-order terms in the time and space domains. The NACD method is relatively straight forward, and the time discretization is simpler than that of the other stereo-modeling methods, which use more complex time discretization, for example, the symplectic partitioned Runge-Kutta (Ma et al., 2011). The discretization in space is also different from other stereo-modeling methods, which are based on interpolation extrapolation. The differences can be seen, for example, by comparing the formulas in Appendix A with those by Yang et al. (2007). Because we need to update the gradients as well as the displacements at each time step in stereo-modeling, we introduce the definition \( U = (u, \partial u/\partial x, \partial u/\partial y, \partial u/\partial z) \) and replace \( u \) with \( U \) in equation 2 to get

![Figure 1. Schematic diagram of the 3D computational stencil for (a) NACD and (b) fourth-order LWC.](image-url)

| Table 1. The number of grid points used for different order derivatives to achieve different orders of accuracy for conventional 1D FD and stereo-modeling. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Order of derivative | Secnd | Fourth | Sixth | 2mth | Fourth | Eighth | Twelfth | 4mth |
| Second | 3 | 5 | 7 | 2m + 1 | 3 | 5 | 7 | 2m + 1 |
| Third | 4 | 6 | 8 | 2m + 2 | 3 | 5 | 7 | 2m + 1 |
| Fourth | 5 | 7 | 9 | 2m + 3 | 3 | 5 | 7 | 2m + 1 |
\[ U_{i,j,k}^{n+1} = 2U_{i,j,k}^n - U_{i,j,k}^{n-1} + \left( \frac{c \Delta t}{\Delta x^2} \right) \left( \frac{\partial U}{\partial x} \right)_{i,j,k}^n + \frac{(c \Delta t)^2}{12} \left( \Delta^2 U \right)_{i,j,k}^n. \]  

(5)

Then, following the methodology that has been previously described for the 2D case of NACD (Yang et al., 2012), we apply the central difference discretization to the second-order spatial derivatives and keep the high-order terms required to achieve fourth-order accuracy in space. This can be seen as doing Taylor expansions in space. Then, equation 5 becomes

\[ \frac{U_{i,j,k}^{n+1} - 2U_{i,j,k}^n + U_{i,j,k}^{n-1}}{\Delta t^2} = c^2 \frac{U_{i+1,j,k}^n - 2U_{i,j,k}^n + U_{i-1,j,k}^n}{\Delta x^2} + c^2 \frac{U_{i,j+1,k}^n - 2U_{i,j,k}^n + U_{i,j-1,k}^n}{\Delta y^2} + c^2 \frac{U_{i,j,k+1}^n - 2U_{i,j,k}^n + U_{i,j,k-1}^n}{\Delta z^2} \]

\[ + \left( \frac{c^2 \Delta t^2}{12} - \frac{c^4 \Delta t^4}{12} \right) \left( \frac{\partial^4 U}{\partial x^4} \right)_{i,j,k}^n + \left( \frac{c^2 \Delta t^2}{12} - \frac{c^4 \Delta t^4}{12} \right) \left( \frac{\partial^4 U}{\partial y^4} \right)_{i,j,k}^n + \left( \frac{c^2 \Delta t^2}{12} - \frac{c^4 \Delta t^4}{12} \right) \left( \frac{\partial^4 U}{\partial z^4} \right)_{i,j,k}^n. \]  

(6)

The key now is how to solve the high-order (≥ 4) spatial derivatives; this is the main difference from the conventional FD method. Note that although equation 6 appears to have maximum fourth-order derivatives, fifth-order derivatives are required to get accurate solutions for high-order problems. This can be seen as doing Taylor expansions in space. Then, equation 5 becomes

\[ \frac{U_{i,j,k}^{n+1} - 2U_{i,j,k}^n + U_{i,j,k}^{n-1}}{\Delta t^2} = c^2 \frac{U_{i+1,j,k}^n - 2U_{i,j,k}^n + U_{i-1,j,k}^n}{\Delta x^2} + c^2 \frac{U_{i,j+1,k}^n - 2U_{i,j,k}^n + U_{i,j-1,k}^n}{\Delta y^2} + c^2 \frac{U_{i,j,k+1}^n - 2U_{i,j,k}^n + U_{i,j,k-1}^n}{\Delta z^2} \]

\[ + \left( \frac{c^2 \Delta t^2}{12} - \frac{c^4 \Delta t^4}{12} \right) \left( \frac{\partial^4 U}{\partial x^4} \right)_{i,j,k}^n + \left( \frac{c^2 \Delta t^2}{12} - \frac{c^4 \Delta t^4}{12} \right) \left( \frac{\partial^4 U}{\partial y^4} \right)_{i,j,k}^n + \left( \frac{c^2 \Delta t^2}{12} - \frac{c^4 \Delta t^4}{12} \right) \left( \frac{\partial^4 U}{\partial z^4} \right)_{i,j,k}^n. \]  

(7)

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### Numerical Results

We will first show forward wavefield propagation simulations to demonstrate the advantage of our method with regard to numerical dispersion. We will then show several 3D RTM examples using our modeling method on synthetic and field data.

To demonstrate the numerical dispersion of the method in the 3D case, we consider the following 3D scalar wave equation:

\[ \frac{\partial^2 u}{\partial t^2} - c_0^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = f(t) \delta(x_s, y_s, z_s). \]  

(7)

We first test a simple homogeneous case, choosing the acoustic velocity to be 4 km/s and the Courant number (\(c \Delta t / h\), 

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**Table 2. Comparison of the computational cost for NACD, fourth-order LWC, and eighth-order LWC.**

<table>
<thead>
<tr>
<th>Method</th>
<th>NACD</th>
<th>Fourth-order LWC</th>
<th>Eighth-order LWC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid size (m)</td>
<td>40</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>CPU time (s)</td>
<td>43.88</td>
<td>568.76</td>
<td>80.06</td>
</tr>
<tr>
<td>Memory usage (array)</td>
<td>120 × 120 × 120 × 12</td>
<td>240 × 240 × 240 × 3</td>
<td>150 × 150 × 150 × 3</td>
</tr>
</tbody>
</table>
$h = \Delta x = \Delta y = \Delta z$ to be 0.2. The computational domain is $0 \leq x \leq 4.8$ km, $0 \leq y \leq 4.8$ km, and $0 \leq z \leq 4.8$ km. A 15-Hz peak frequency Ricker wavelet explosive source is located at the center of the computational domain. Figure 2a, 2b, and 2c shows wavefield snapshots at $t = 0.54$ s on the vertical plane containing the source when using a coarse computational grid ($\Delta x = \Delta y = \Delta z = 40$ m; equivalent to approximately six grid points per dominant wavelength) generated by the (a) NACD, (b) fourth-order LWC (fourth-order in time and space), and (c) eighth-order LWC (fourth order in time and eighth order in space), respectively. To compare the ability of the three methods for producing a comparable wavefield quality, we compute the same wavefield using LWC with finer spatial increments. Figure 2d and 2e shows the wavefield snapshot at $t = 0.54$ s on finer grids, generated by the fourth-order LWC ($\Delta x = \Delta y = \Delta z = 20$ m) and eighth-order LWC ($\Delta x = \Delta y = \Delta z = 32$ m) using the same Courant number. We can see that the wavefronts of the seismic waves shown in Figure 2a, 2d, and 2e, simulated by the NACD, the fourth order LWC and the eighth order LWC, respectively, are similar. Comparing Figure 2a, 2b, and 2c, we can see that the NACD has no available numerical dispersion even though the space increment is 40 m. However, when the LWC and the NACD have similar accuracy, their computational costs are quite different. It took the NACD about 43 s to generate Figure 2a on a workstation, whereas the fourth-order and eighth-order LWC method took about 568 and 80 s, respectively, to generate Figure 2d and 2e under the same condition. For this simple simulation, the computational speed of the NACD is roughly 14 times and 2 times faster than that of the fourth-order and eighth-order LWC, respectively. Furthermore, their memory requirements are different. The...
NACD needs 12 arrays to store the wave displacement and gradient at each spatial grid point, and the number of grid points is $121 \times 121 \times 121$ on the coarse grid (40 m). The fourth-order LWC needs only three arrays to store the wave displacement at each grid point, but the number of grid points on the fine grid (20 m) goes up to $241 \times 241 \times 241$. Thus, the NACD requires only about half of the storage space for the fourth-order LWC for comparable quality. A comparison of the computational requirements for each method is given in Table 2.

3D SEG/EAGE PHASE A CLASSIC DATA SET 1

We now present RTM results using the 3D SEG/EAGE phase A classic data set 1 (Biondi, 2006) using the NACD method and compare with results obtained when using the fourth-order LWC method. In phase A, two 138-shot 3D shot lines that are oriented perpendicular to each other with their intersection at the crest of the salt were acquired. The phase A classic data set 1 was extracted from the acquisition along line 1. For this data set, each shot has six streamers with a maximum of 65 groups per streamer.

Figure 7. Two different 3D views of the subset of SEAM velocity model with (a) crossline section at $x = 4.8$ km, inline section at $y = 4.8$ km, and horizontal plane at $z = 2.4$ km and (b) crossline section at $x = 3$ km, inline section at $y = 3$ km, and horizontal plane at $z = 2.36$ km.

Figure 8. Three-dimensional views of the RTM slices obtained by (a) NACD, (b) fourth-order LWC, and (c) eighth-order LWC for the SEAM model with crossline section at $x = 4.8$ km, inline section at $y = 4.8$ km, and horizontal plane at $z = 2.4$ km that is displayed in Figure 7a.
The group interval is 40 m, the near offset is 160 m, and the far offset is 2720 m. The sample interval is 8 ms, and the recording time is 5 s. The shot and receiver layouts are shown in Figure 3. We choose a Ricker wavelet with a peak frequency of 18 Hz as the source wavelet to provide a close match to the spectra measured from example data set waveforms. For migration, we choose grid sizes of 40 and 20 m in the horizontal and vertical directions, respectively. The vertical plane of the velocity model, directly below the source line, is shown in Figure 4. Figure 5 shows the results of applying RTM to this data set computed using NACD and fourth-order LWC, respectively. The two images in Figure 5 demonstrate that the NACD method produces a better image than the fourth-order LWC method when the same grid spacing is used, especially in the regions above the left and right salt flanks. The white ellipses superimposed on the images highlight regions in which the images have significant differences. In Figure 5a, the whole image is very clear and each interface above the salt is well imaged, but in the corresponding regions in Figure 5b, the interfaces near the salt can hardly be distinguished. Moreover, the salt is correctly positioned in Figure 5a, but in Figure 5b, the salt is a little shifted. Comparison between Figure 5a and 5b shows that most structure of the model can be well imaged by the NACD method even using a coarse grid size, consistent with the results found for the 2D case (Li et al., 2013).

**SEAM MODEL**

Next, we apply the 3D RTM using the NACD and LWC methods to the SEG Advanced Modeling project (SEAM) model (Fehler, 2008). We use NACD to generate a synthetic data set on a subset of the SEAM model using a grid spacing of 40 m in the x- and y-directions and 20 m in depth. For convenience, we relabel the axes of our selected study region located within the SEAM model to make a new model that has dimensions of $0 \leq x \leq 6$ km, $0 \leq y \leq 6$ km, and $0 \leq z \leq 4.36$ km. The southwest corner of our model is at (12 km, 23.96 km) in the original SEAM model. We use only one shot that is located at the center of an array of receivers. The shot and receiver layout is shown in Figure 6, in which the black star represents the source and the receivers are distributed on a $40 \times 40$ m grid within the box outlined by a solid black line. The sample interval is 8 ms, and the recording time is 3 s. The migrated volume is the entire subset of the SEAM model. In this experiment, we choose a Ricker wavelet with a peak frequency of 20 Hz as the source wavelet. Two different 3D views of the velocity model are shown in Figure 7. Figures 8a and 9a show 3D views of image slices obtained by NACD for the slices that are shown in Figure 7a and 7b, respectively. Corresponding slices obtained from using fourth-order and eighth-order LWC are shown in Figures 8b and 9b and Figures 8c and 9c. The white ellipses in Figures 8 and 9 indicate regions in which the salt and sediments are better imaged by NACD than by LWC, and where we see a lot of dispersion and noise in Figures 8b, 8c, 9b, and 9c, especially in regions in which the velocity is complicated. To look into the details in the images, we extract representative 2D crossline and inline sections from the image volume and show them in Figures 10 and 11, respectively. Comparing Figures 10 and 11, the differences among images migrated by NACD and LWC are very obvious, particularly in areas within the white ellipses. We can see significant numerical dispersion in the images generated by LWC in the complex velocity area, for example, the sharp salt body and the rough salt boundary.

**Figure 9.** Three-dimensional views of the RTM slices obtained by (a) NACD, (b) fourth-order LWC, and (c) eighth-order LWC for the SEAM model with crossline section at $x = 3$ km, inline section at $y = 3$ km, and horizontal plane at $z = 2.36$ km, which is displayed in Figure 7b.
Figure 10. Vertical 3D migrated crossline sections at (a-c) $x = 3$ km and (d-f) $x = 5$ km obtained by (a and d) NACD, (b and e) fourth-order LWC, and (c and f) eighth-order LWC for the subset of the SEAM model.

Figure 11. Vertical 3D migrated inline sections at (a-c) $y = 3$ km and (d-f) $y = 5$ km obtained by (a and d) NACD, (b and e) fourth-order LWC, and (c and f) eighth-order LWC for the subset of the SEAM model.
Once again, our results illustrate that the NACD can give us very clean images with great quality, even using a coarse grid size and high frequency.

3D FIELD DATA

In this section, we use NACD and fourth-order LWC to perform 3D RTM on a 3D marine field data set. This data set has been sorted so that the receivers for one shot are positioned only along the line belonging to the shot. We choose two neighboring shot lines, each consisting of 234 shots, for the 3D migration study. The shot spacing and receiver interval are both 25 m. Correspondingly, the migration grid spacing is 25 and 20 m in horizontal and vertical directions, respectively. We choose a 40-Hz Ricker source to attempt to match the high frequency content of the data. The velocity model that we were provided is relatively smooth. We show the vertical plane directly below one shot line in Figure 12. The corresponding images directly below the same shot line are shown in Figure 13. Images shown are the results obtained after stacking the 3D migrations using the two shot lines giving a total of 468 shots used to construct the image. Comparison between the two images in Figure 13 indicates many improvements to the structure; better continuity and better event positioning are seen in Figure 13a. As highlighted by the black arrows, faults are more clearly imaged and cross-dipping events are placed more properly in the image obtained by NACD, than those in Figure 13b. There is a slight phase shift in the image of Figure 13b compared with Figure 13a, which means there is an error in the numerical velocity of fourth-order LWC. The phase shift is particularly clear in the lower left side of the images in the region inside the black boxes. These images also tell us that NACD works well even using field data with such a high frequency.

NUMERICAL DISPERSION

The results above indicate that NACD has superior performance compared with LWC for 3D RTM on synthetic and field data sets. To better understand the reason why the NACD method performs better than the LWC method when using a coarse grid size and why it provides a better image under the same condition as the LWC method, we provide a simple numerical dispersion analysis. Numerical dispersion causes the phase velocity to vary with the spatial and temporal frequencies. The computational merit of most numerical schemes always hinges on their ability to minimize this effect. Following the analysis approach presented by Moczo et al. (2000), we investigate the numerical dispersion of the 3D NACD and fourth-order LWC methods. The dispersion relation as a function of the sampling rate (grid spacing per wavelength) is shown in Figure 14. The dispersion curves corresponding to different propagation directions tell us that numerical velocity for fourth-order LWC has bigger error and more numerical anisotropy than that of NACD. This observation is in agreement with our previous discussion about the advantages of stereo-modeling methods. A simple spectral analysis shows that the spectrum of the field data set mainly ranges from about 20 to 80 Hz. To obtain an estimate of the dispersion for RTM of the field data set, we choose a Courant number of $\alpha = 0.2$, the spatial grid size to be 20 m, and the velocity to be 3 km/s. The dispersion relation shown in Figure 14 is then transformed from being a function of the grid sampling rate into a function of frequency. Figure 15 shows representative dispersion...
relation curves, as a function of frequency, corresponding to different propagation directions. The curves would not change if we choose the grid size to be 40 m, but the frequency scale would be divided by 2, so the range would be from 0 to 40 Hz, which fits the case of the SEG/EAGE data set. These curves show that the maximum phase-velocity error of NACD is less than 8%, whereas the maximum error of the fourth-order LWC is as high as 28% over the frequency range of the data sets, which explains the dispersion and the phase shift in those images generated by fourth-order LWC. Comparing the curves for various propagation angles shows that there is significantly more variation in numerical dispersion with propagation angle for fourth-order LWC than for NACD. The numerical velocities for both methods are slower than the actual velocities when the frequency is high, but the fourth-order LWC is worse. That error will cause the events in the image to shift to shallower depths, which explains the results obtained by the fourth-order LWC method in Figures 5b and 13b compared with those for NACD. The differences in depth of events are clearly shown in the regions highlighted by the two boxes in Figure 13b compared with those in Figure 13a. This means that, in practical cases, conventional RTM may yield an image that appears to be in focus, but interfaces will be at incorrect locations. This may lead to incorrect decisions about well positioning that could lead to large economic losses.

CONCLUSIONS

We have proposed and applied a fourth-order 3D stereo-modeling method to RTM and obtained a weak-dispersion prestack depth migration method that allows large extrapolation grid size to be used. Numerical results illustrate that the stereo-modeling method, which uses the wave displacement and its gradient, can greatly increase the computational efficiency and save computer memory through the use of the large spatial increments and the resulting large time steps. Tests on synthetic and field data sets have demonstrated that the stereo-modeling method is effective in imaging even using coarse grids, compared with conventional methods such as the fourth-order and the eighth-order LWC. The stereo-modeling method has significantly less numerical anisotropy than the conventional methods. These results imply that stereo-modeling methods have a promising future in 3D imaging.

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APPENDIX A

APPROXIMATION TO HIGH-ORDER DERIVATIVES

To aid in implementing the 3D NACD method, we provide the approximations of the high-order derivatives related to the wave displacement and its gradient. For convenience, here we present the expressions used in the computation. First, we define $E^{1/2}_x$, $\delta^2_x$, and the other operators as follows:
\( E^{-1} f_{i,j,k} = f_{i-1,j,k} \), \( E^{+1} f_{i,j,k} = f_{i+1,j,k} \),
\( E^{-1} f_{i,j,k} = f_{i,j-1,k} \), \( E^{+1} f_{i,j,k} = f_{i,j+1,k} \),
\( E^{-1} f_{i,j,k} = f_{i,j-1,k} \), \( E^{+1} f_{i,j,k} = f_{i,j+1,k} \),
\( \delta g_{i,j,k} = f_{i+1,j,k} - 2f_{i,j,k} + f_{i-1,j,k} \),
\( \delta g_{i,j,k} = f_{i,j+1,k} - 2f_{i,j,k} + f_{i,j-1,k} \),
\( \delta g_{i,j,k} = f_{i,j+1,k} - 2f_{i,j,k} + f_{i,j-1,k} \),
\( \delta g_{i,j,k} = f_{i,j+1,k} - 2f_{i,j,k} + f_{i,j-1,k} \).

\[ (\partial^2 u / \partial g^2)_{i,j,k} = \frac{6}{\Delta g^2}(E_g - E_{g-1}) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k} - \frac{12}{\Delta g^2} \delta g_{i,j,k}, \]
\[ g = x, y, z, \]  
(A-1)

\[ (\partial^2 u / \partial g^2 \partial e)_{i,j,k} = \frac{3}{\Delta g^2 \Delta e} \delta g_{i,j,k} \left( E_{e} - E_{e+1} \right) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k} 
- \frac{1}{\Delta g^2 \Delta e} \delta g_{i,j,k} \left( E_{e} - E_{e-1} \right) \left( \frac{\partial u}{\partial e} \right)^n_{i,j,k}, \quad e, g = x, y, z, \]  
(A-2)

\[ (\partial^2 u / \partial g^2 \partial d)_{i,j,k} = -\frac{90}{\Delta g^2} (E_g - E_{g-1}) u^{n}_{i,j,k} \]
\[ + \frac{30}{\Delta g^2} (E_{g} + E_{g+1} + 4I) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k}, \quad g = x, y, z, \]  
(A-3)

\[ (\partial^2 u / \partial d^2 \partial e)_{i,j,k} = -\frac{6}{\Delta g^2 \Delta e} \delta g_{i,j,k} \left( E_{e} - E_{e-1} \right) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k} 
+ \frac{3}{\Delta g^2 \Delta e} \left( E_{1} E_{e} + E_{g} E_{e-1} - E_{1} E_{g} E_{e-1} \right) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k}, \quad e, g = x, y, z, \]  
(A-4)

\[ (\partial^2 u / \partial d^2 \partial d)_{i,j,k} = -\frac{6}{\Delta g^2 \Delta e} \delta g_{i,j,k} \left( E_{e} - E_{e-1} \right) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k} 
+ \frac{3}{\Delta g^2 \Delta e} \left( E_{1} E_{e} + E_{g} E_{e-1} - E_{1} E_{g} E_{e-1} \right) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k}, \quad e, g = x, y, z, \]  
(A-5)

With the notation above and following the directional approach, we can get expressions of the high-order mixed spatial derivatives for 3D NACD. The expressions of the other derivatives that are the same as the previous work are also given (Yang et al., 2007).

\[ \left( \frac{\partial^2 u}{\partial g^2} \right)^n_{i,j,k} = -\frac{3}{\Delta g^2 \Delta e} \delta g_{i,j,k} \left( E_g - E_{g-1} \right) u^{n}_{i,j,k} \]
\[ + \frac{3}{\Delta g^2 \Delta e} \delta g_{i,j,k} \left( E_g + E_{g+1} \right) \left( \frac{\partial u}{\partial g} \right)^n_{i,j,k}, \]
\[ \theta, g = x, y, z, \]  
(A-6)

\[ \left( \frac{\partial^2 u}{\partial g^2 \partial d} \right)^n_{i,j,k} = \alpha_1 \cdot \delta g_{i,j,k} \left( 3E_g E_d - E_{g-1} E_d - E_{g+1} E_d + E_{g+1} E_d \right) 
- 6 \left( E_g - E_{d} \right) u^{n}_{i,j,k}, \]
\[ + \alpha_2 \cdot \delta g_{i,j,k} \left( E_g E_d + E_{g+1} E_d - E_{g-1} E_d - E_{g+1} E_d \right) \left( \frac{\partial u}{\partial d} \right)^n_{i,j,k} \]
\[ + \alpha_3 \cdot \delta g_{i,j,k} \left( E_g E_d - E_{g-1} E_d - E_{g+1} E_d + E_{g+1} E_d \right) \left( \frac{\partial u}{\partial d} \right)^n_{i,j,k} 
+ \alpha_4 \cdot \delta g_{i,j,k} \left( E_g E_d + E_{g+1} E_d - E_{g-1} E_d + E_{g+1} E_d \right) \left( \frac{\partial u}{\partial d} \right)^n_{i,j,k}, \]
\[ \text{when } g = x, \quad e = y, \quad d = z, \quad \alpha_1 = \frac{1}{\Delta g^2 \Delta d} \quad \alpha_2 = \frac{1}{\Delta g^2 \Delta d} \quad \alpha_3 = \frac{1}{\Delta g^2 \Delta d} \quad \alpha_4 = \frac{1}{\Delta g^2 \Delta d} \]  
(A-7)

\[ \text{REFERENCES} \]


