Hybrid experimental–numerical analysis of basic ductile fracture experiments for sheet metals

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1016/j.ijsolstr.2009.12.011">http://dx.doi.org/10.1016/j.ijsolstr.2009.12.011</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>Elsevier</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Fri Dec 07 22:40:04 EST 2018</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/96412">http://hdl.handle.net/1721.1/96412</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
Hybrid experimental–numerical analysis of basic ductile fracture experiments for sheet metals

Matthieu Dunand, Dirk Mohr

Abstract

A basic ductile fracture testing program is carried out on specimens extracted from TRIP780 steel sheets including tensile specimens with a central hole and circular notches. In addition, equi-biaxial punch tests are performed. The surface strain fields are measured using two- and three-dimensional digital image correlation. Due to the localization of plastic deformation during the testing of the tensile specimens, finite element simulations are performed of each test to obtain the stress and strain histories at the material point where fracture initiates. Error estimates are made based on the differences between the predicted and measured local strains. The results from the testing of tensile specimens with a central hole as well as from punch tests show that equivalent strains of more than 0.8 can be achieved at approximately constant stress triaxialities to fracture of about 0.3 and 0.66, respectively. The error analysis demonstrates that both the equivalent plastic strain and the stress triaxiality are very sensitive to uncertainties in the experimental measurements and the numerical model assumptions. The results from computations with very fine solid element meshes agree well with the experiments when the strain hardening is identified from experiments up to very large strains.

1. Introduction

The use of sheet materials of high strength-to-weight ratio is essential in modern transportation vehicle engineering. This choice is driven by the constant quest for lower structural weight in an attempt to improve fuel efficiency, vehicle dynamics and cost efficiency. Candidate materials are advanced high strength steels which feature a tensile strength that is more than twice as high as that of conventional low carbon or HSLA steels. Special aluminum and magnesium alloys are also being considered, mostly for castings and extrusions. The common feature of light weight metal structures is that ductile fracture frequently limits their formability and crashworthiness.

Ductile fracture is generally described by the nucleation and growth of voids that ultimately link to form cracks. The early studies of McClintock (1968) and Rice and Tracey (1969) analyzed the evolution of cylindrical and spherical holes in a ductile matrix. Their results show that void growth is governed by the stress triaxiality. Gurson (1977) proposed a porous plasticity model which includes the void volume fraction as internal variable. Some phenomenological fracture models make use of Gurson’s evolution equations assuming that ductile fracture occurs as the void volume fraction reaches a critical threshold value. The original Gurson model has been repeatedly improved by accounting for the loss of stress-carrying capacity associated with void coalescence (e.g. Tvergaard and Needleman, 1984), by incorporating enhanced strain hardening models (e.g. Leblond et al., 1995), by describing void shape effects (e.g. Pardoen and Hutchinson, 2000) and by incorporating plastic anisotropy (e.g. Benzarga et al., 2004) and shear (e.g. Nahshon and Hutchinson, 2008). A comprehensive review of modified Gurson models can be found in Lassance et al. (2007).

As an alternative to micromechanics inspired fracture models, phenomenological models have been developed to predict ductile fracture without modeling void nucleation and growth. It is assumed that fracture occurs at a point of the body where a weighted measure of the accumulated plastic strain reaches a critical value. A comparative study of various weighting functions (including models based on the work of McClintock (1968), Rice and Tracey (1969), LeRoy et al. (1981), Cockcroft and Latham (1968), Oh et al. (1979), Brozzo et al. (1972) and Clift et al. (1990)) showed that none of them can accurately describe the fracture behavior of a given material over a large range of stress triaxialities (Bao and Wierzbicki, 2004). Attempts to define a more general fracture criterion have lead to the introduction of the third invariant of the stress tensor in the weighting function (e.g. Wierzbicki and Xue,
Numerous experimental investigations have been carried out to characterize ductile fracture. Clausing (1970) performed an experimental study on axisymmetric and plane strain tensile fracture specimens of several materials and found a lower ductility for plane strain loading. Hancock and Mackenzie (1976) investigated the relationship between the ductility and the stress triaxiality for three different steels. They used smooth and U-notched axisymmetric tensile specimens and concluded that for all studied materials, the ductility is decreasing with stress triaxiality; the same authors also found good agreement between their experimental results and the predictions by Rice and Tracey’s (1969) fracture model. Hancock and Brown (1983) compared experimental results from notched axisymmetric specimens and flat grooved plane strain specimens and concluded that the ductility was determined by the stress state, and not the strain state. Using split Hopkinson bars, Johnson and Cook (1985) performed a dynamic torsion test on axisymmetric and plane strain tensile fracture specimens. Hancock and Brown (1983) compared experimental results and the predictions by Rice and Tracey’s (1969) fracture model. Hancock and Brown (1983) compared experimental results from notched axisymmetric specimens and flat grooved plane strain specimens and concluded that the ductility was determined by the stress state, and not the strain state. Using split Hopkinson bars, Johnson and Cook (1985) performed a dynamic torsion test on axisymmetric and plane strain tensile fracture specimens. Hancock and Brown (1983) compared experimental results and the predictions by Rice and Tracey’s (1969) fracture model. Hancock and Brown (1983) compared experimental results from notched axisymmetric specimens and flat grooved plane strain specimens and concluded that the ductility was determined by the stress state, and not the strain state. Using split Hopkinson bars, Johnson and Cook (1985) performed a dynamic torsion test on axisymmetric and plane strain tensile fracture specimens.

For each type of specimen, the accuracy of the hybrid experimental–numerical approach. In other words, a detailed finite element analysis of each experiment is required to identify the stress and strain histories prior to fracture need to be determined in a hybrid experimental–numerical approach. In other words, a detailed finite element analysis of each experiment is required to identify the stress and strain histories. Using these coefficients, the corresponding yield stress ratios (Table 2) are calculated to define the Hill’48 yield function in the commercial finite element code Abaqus (2007). We assume $R_{13} = R_{23} = 1$ (value for isotropic materials) due to the lack of experimental data for out-of-plane shear. Mohr et al. (2010) have shown that the Swift law provides a good approximation of the deformation resistance before the onset of necking (see Table 2 for the model parameters $A$, $r_0$ and $n$). The stress–strain curve for strains greater than 0.2 (beyond the point of necking) is obtained from inverse analysis which is discussed in Sections 4 and 5.

### 2. Material and Constitutive Modeling

#### 2.1. Material

A TRIP780 steel is chosen for the present study. It is a representative of the class of advanced high strength materials that are of growing importance in lightweight automotive engineering. All specimens are extracted from 1.5 mm thick sheets provided by POSCO (Korea). This TRIP-assisted steel features a complex multiphase microstructure consisting of ferrite, bainite, martensite and metastable retained austenite. The exact chemical composition of the present TRIP780 material as measured by energy-dispersive X-ray analysis is given in Table 1. Micrographs reveal a fine grain structure with a maximum grain size of about 10 μm and an austenite content of about 6%.

#### 2.2. Plasticity Model

The plastic behavior of this TRIP steel under monotonic loading conditions has been investigated by Mohr et al. (2010) over a wide range of stress states using a combined tension and shear testing technique for sheet materials. For monotonic loading conditions, the plastic behavior of the TRIP780 material is modeled using a standard plasticity theory with: (1) an anisotropic quadratic Hill (1948) yield function, (2) associated flow rule and (3) an isotropic hardening model. Formally, the yield function may be written as

$$f(\sigma, \dot{\varepsilon}) = \sigma_{\text{Hill}} - k = 0.$$  

where $\sigma_{\text{Hill}}$ is the Hill’48 equivalent stress and $\dot{\varepsilon}$ is the corresponding work-conjugate Hill’48 equivalent plastic strain. The deformation resistance $k$ (yield stress) is defined through the hardening law

$$dk = H(\dot{\varepsilon})d\dot{\varepsilon}.$$  

The coefficients of the yield function are calibrated based on the measured Lankford coefficients $r_{0} = 0.89$, $r_{45} = 0.82$ and $r_{90} = 1.01$. Using these coefficients, the corresponding yield stress ratios (Table 2) are calculated to define the Hill’48 yield function in the commercial finite element code Abaqus (2007). We assume $R_{13} = R_{23} = 1$ (value for isotropic materials) due to the lack of experimental data for out-of-plane shear. Mohr et al. (2010) have shown that the Swift law

$$k = A(\dot{\varepsilon}^{p} + r_{0})^{n}$$  

provides a good approximation of the deformation resistance before the onset of necking (see Table 2 for the model parameters $A$, $r_0$ and $n$). The stress–strain curve for strains greater than 0.2 (beyond the point of necking) is obtained from inverse analysis which is discussed in Sections 4 and 5.

### Table 1

Chemical composition of the TRIP780 material in wt.%

<table>
<thead>
<tr>
<th>Element</th>
<th>C</th>
<th>Al</th>
<th>Mn</th>
<th>Si</th>
<th>Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.70</td>
<td>0.47</td>
<td>2.50</td>
<td>0.59</td>
<td>0.08</td>
</tr>
</tbody>
</table>
3. Methodology

The goal of this work is to obtain reliable time histories of the stress triaxiality and the equivalent plastic strain at the point of the onset of fracture. In particular, we focus on three fracture tests that may be easily performed on full-thickness specimens: (1) notched tensile specimens, (2) a tensile specimen with central hole and (3) a disc specimen for punch testing. It is noted that the first two types of specimens can develop diffuse necking (localization through the width of the gage section) prior to the onset of localized necking (through the thickness localization) while the punch test cannot develop diffuse necking because of its geometry. Here, we make use of a hybrid experimental–numerical approach to determine the loading histories. As a result, the determination of the strain to fracture is not affected by these different necking behaviors. In this section, we discuss the methodology to assess the error of the hybrid experimental–numerical approach.

3.1. Determination of the onset of fracture

The displacement fields on the specimen surface are measured using either two- or three-dimensional digital image correlation (DIC). Based on the DIC measurements, we define the instant of onset of fracture (not the location) by the first detectable discontinuity in the measured displacement field at the specimen surface. Subsequently, a finite element simulation is performed of each experiment. Post-processing of those simulations gives then access to the evolution of the stress triaxiality and the equivalent plastic strain. For the three types of experiments performed within this study, it is assumed that the location of the onset of fracture coincides with the location of the highest equivalent plastic strain within the specimen at the instant of onset of fracture. The corresponding equivalent plastic strain is referred to as fracture strain $\gamma_f$.

3.2. Sources of error

3.2.1. Experimental error

Among all experimental uncertainties, we consider the error in the optical displacement field measurement as critically important for the determination of the fracture strain and triaxiality. Possible errors in the initial specimen geometry including thickness can be easily eliminated through dimensional verification prior to each experiment. The accuracy of the DIC procedure used to measure the fracture displacement depends mainly on the quality of the speckle pattern applied on the specimen and on the interpolation function used during the correlation. In order to get the least error, the pattern has been applied following the recommendations of Sutton et al. (2009) in terms of speckle size and density. Based on correlations performed on computer-generated sinusoidal waves, Sutton et al. (2009) concluded that for an appropriate speckle pattern, cubic B-spline displacement field interpolation functions produce an interpolation bias of about 1/40 pixel.

3.2.2. Computational error

The solution obtained from finite element analysis usually differs from the exact solution of the corresponding physical problem. In addition to shortcomings of the material model, the FEA is affected by different sources of errors (e.g. Bathe, 1996). In particular, errors associated with the spatial and time discretization as well as the constitutive model are monitored in this study:

- Spatial discretization errors are minimized by increasing the number of elements. To find a suitable mesh, we start with a coarse mesh that is successively refined by dividing the elements’ characteristic dimensions by two until the result converges. It is considered that convergence is achieved when an additional element division does not change the final plastic strain by more than 0.5%.
- Time discretization errors are minimized by increasing the number of implicit time steps. It is considered that convergence is achieved when adding 50% more time increments does not change the final plastic strain by more than 0.5%.

Furthermore, round-off errors are minimized by using the double precision floating point format in our computations with explicit time integration.

3.3. Error estimation

Due to the redundancy of measurements, we compare the logarithmic strain history obtained from DIC with that computed by FEA at the point on the surface where the first displacement field discontinuity appears. For every time step in the finite element simulation, there is a difference between the computed strain increment $\Delta e_{\text{FEA}}$ at the surface of the specimen and the actual strain increment $\Delta e_{\text{DIC}}$ measured by DIC. The error affecting the determined plastic strain increment at the location of the onset of fracture, $\delta (\Delta e^p)$, is then estimated based on the strain increment difference on the specimen surface,

$$\delta (\Delta e^p) \approx \frac{\partial e^p}{\partial e_{\text{DIC}}} |\Delta e_{\text{DIC}} - \Delta e_{\text{FEA}}|.$$  (4)

Furthermore, we estimate the error in the accumulated equivalent plastic strain as

$$\delta \bar{\varepsilon}^p \approx \int_0^{t^f} \left( \frac{|\Delta e_{\text{DIC}} - \Delta e_{\text{FEA}}|}{|\Delta e_{\text{DIC}}|} \right) d\bar{\varepsilon}.$$  (5)

It is noted that the above error estimate represents both computational and experimental uncertainties. Using the definition of the stress triaxiality,

$$\eta = \frac{\sigma_m}{\sigma_{\text{VM}}} \quad \text{with} \quad \sigma_m = \frac{tr(\sigma)}{3},$$  (6)

the error in the stress triaxiality is related to the error in the hydrostatic stress $\sigma_m$ and that in the von Mises stress,

$$\delta \eta = \frac{1}{\sigma_{\text{VM}}} [\delta \sigma_m + \eta \delta \sigma_{\text{VM}}].$$  (7)

It is assumed that all the components of the stress tensor are computed with the same relative error,

$$\frac{\delta \sigma_m}{\sigma_m} = \frac{\delta \sigma_{\text{VM}}}{\sigma_{\text{VM}}} = \frac{\delta \sigma_{\text{Hill}}}{\sigma_{\text{Hill}}},$$  (8)

and thus
Using the hardening law, the first-order estimate of the error in stress triaxiality may be written as

$$\delta \eta = 2 \frac{\eta}{\sigma_{VM}} \delta \sigma_{hill}.$$  \hspace{1cm} (9)

4. Notched tensile tests

The first type of specimen considered in this study is a flat tensile specimen with circular cutouts (Fig. 1a–c). The stress triaxiality within the specimen is a function of the notch radius. For very large notch radii the stress state near the specimen center (prior to necking) corresponds to uniaxial tension, while the plane strain condition (along the width direction) is achieved for very small notch radii. In the case of isotropic materials, this variation of stress state corresponds to a range of triaxialities from 0.33 to 0.58.

4.1. Experimental procedure

Specimens are extracted from the sheet material using waterjet cutting. The specimen loading axis is always oriented along the rolling direction. All specimens are 20 mm wide and feature a 10 mm wide notched gage section. Three different notch radii are considered: \(R = 20 \text{ mm}, R = 10 \text{ mm} \text{ and } R = 6.67 \text{ mm}\). The specimens are tested on a hydraulic testing machine (Instron Model 8080) with custom-made high pressure clamps. All experiments are carried out under displacement control at a constant crosshead velocity of 0.5 mm/min.

During the tests, two digital cameras (QImaging Retiga 1300i with 105 mm Nikon Nikkor lenses) take about 300 pictures (resolution 1300 × 1300 pixels) of the speckle-painted front and back surface of the specimens. The pictures of the front surface are used to determine the displacements of the specimen boundaries. The front camera is positioned at a distance of 1.25 m to take pictures of the entire specimen (square pixel edge length of 51 \(\mu\text{m}\)). The photographs of the back face are used to perform accurate DIC measurements of the displacement field at the center of the specimen gage section. For that purpose, the second camera is positioned at a distance of 0.25 m which reduces the square pixel edge length to 9.5 \(\mu\text{m}\). The average speckle size is about 70 \(\mu\text{m}\) on both faces. The displacement field is calculated by DIC (VIC2D, Correlated Solutions) based on the assumption of an affine transformation of the 21 × 21 pixel neighborhood of each point of interest. The logarithmic axial strain at the center of the specimen is computed from the relative vertical displacement \(\Delta v\) of two points located at the center of the specimen.

$$\varepsilon = \ln \left(1 + \frac{\Delta v}{\Delta y}\right).$$  \hspace{1cm} (11)

Both points are located on the vertical axis of symmetry at an initial distance of \(\Delta y = 20 \text{ pixels} (190 \mu\text{m})\). For each specimen geometry, we also performed an interrupted test: the monotonic displacement loading has been interrupted at a crosshead displacement of approximately 98% of the measured displacement to fracture. Subsequently, two 12 mm long samples have been extracted from the deformed specimen gage section; the small samples are embedded in an epoxy matrix for polishing; low magnification pictures are then taken to determine the thickness profile along the specimen’s planes of symmetry.

4.2. Experimental results

The force–displacement curves for the three different notched geometries (black solid dots in Fig. 2) are shown all the way to fracture. All feature a force maximum before fracture occurs. The displacement to fracture presents small variations for different tests carried out on a given geometry (less than 3%). The measured fracture displacements and the corresponding experimental uncertainty are summarized in Table 3a. DIC analysis of the strain fields shows significant strain localization near the center of the specimens. The evolution of the logarithmic axial strain with respect to displacement is shown as solid blue dots in Fig. 2. Irrespective of the notch radius, two consecutive increases of the local strain rate become apparent. The first corresponds to the development of diffuse necking, while the second indicates the onset of localized necking. The localized necking provokes a severe thickening of the specimen. For that purpose, the second camera is positioned at a distance of 0.25 m which reduces the square pixel edge length to 9.5 \(\mu\text{m}\). The average speckle size is about 70 \(\mu\text{m}\) on both faces. The displacement field is calculated by DIC (VIC2D, Correlated Solutions) based on the assumption of an affine transformation of the 21 × 21 pixel neighborhood of each point of interest. The logarithmic axial strain at the center of the specimen is computed from the relative vertical displacement \(\Delta v\) of two points located at the center of the specimen.
ness reduction at the center of the specimens. The measured thickness variations along the axial plane of symmetry of the samples obtained from interrupted tests are depicted in Fig. 3. Observe the severe thickness reduction for all three geometries.

4.3. Finite element model

Implicit finite element simulations are performed of each experiment using Abaqus/standard. Reduced-integration eight-node 3D solid elements (type C3D8R of the Abaqus element library) are used to mesh the specimens. Exploiting the symmetry of the specimen geometry, material properties and loading conditions, only one eighth of the specimen is modeled: the mesh represents the upper right quarter of the specimen, with half its thickness (Fig. 4). A constant velocity is uniformly imposed to the upper boundary. A zero-normal displacement condition is imposed to the three boundaries that correspond to symmetry planes.

The effect of mesh density and time discretization on the computational predictions is studied for the \( R = 10 \text{ mm} \) notch specimen geometry. Four meshes are considered (Fig. 4):

(i) coarse mesh with an element edge length of \( l_e = 400 \mu\text{m} \) at the specimen center and \( n_t = 2 \) elements in thickness direction (half-thickness),
(ii) medium mesh with \( l_e = 200 \mu\text{m} \) and \( n_t = 4 \),
(iii) fine mesh with \( l_e = 100 \mu\text{m} \) and \( n_t = 8 \),
(iv) very fine mesh with \( l_e = 50 \mu\text{m} \) and \( n_t = 16 \).

The meshes are designed such that the elements near the specimen center feature the same dimension in the in-plane directions. In addition to solid element simulations, we make use of first-order plane stress shell elements (S4R) with five integration points in the thickness direction. The simulations are run up to the fracture displacement. Forty equally spaced time increments are used. The corresponding force–displacement curves as well the evolution of the equivalent plastic strain at the center of the specimen with respect to displacement are plotted in Fig. 5a. The force–displacement curves lie on top of each other for all solid element meshes. However, the comparison with the results from shell element simulations shows that solid elements are required in order to provide meaningful predictions after the force maximum has been reached (after the onset of necking). Therefore, we limit our attention to the solid element simulations.

The predictions of force–displacement relationship are approximately mesh size independent, but the mesh density has a noticeable effect on the predicted strains at the specimen center. The final plastic strain computed with the coarse mesh is 7.3% lower than that for the very fine mesh. The relative error between the fine and very fine meshes being only 0.2%. Errors due to time discretization are evaluated by running a simulation on the fine mesh with 40, 60 and 90 implicit time steps. The difference in the final plastic strain between 40 and 90 steps is 0.7%, while a difference of less than 0.2% is observed for 60 time steps. In the following, all simulations of notched tensile tests are performed using at least eight solid elements through the half-thickness and at least 60 equally spaced implicit time increments.

4.4. Extrapolation of the stress–strain curve

The Swift strain hardening curve has been validated for equivalent plastic strains of up to 0.2 (point of necking under uniaxial tension). However, in notched tensile specimens, the plastic strains at the specimen center are much higher than 0.2. The comparison of the experimentally-measured force–displacement curve for \( R = 20 \text{ mm} \) (black solid dots in Fig. 5b) with the simulation results shows that the Swift model assumption overestimates the force level (blue solid line). The assumption of a tangent modulus of

\[ \text{Fig. 2. Experimental (points) and simulation (solid curves) results for tensile specimen with circular cutouts. Force–displacement curves are in black and central logarithmic axial strain versus displacement curves in blue. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)} \]
H_{\text{min}} = 0 for strains greater than 0.2 yields an underestimation of the force level. To obtain a better extrapolation of the measured stress–strain curve, we defined two segments of constant slope $H_1$ and $H_2$; here, $H_1$ corresponds to the range of intermediate plastic strains (from 0.2 to 0.35), $H_2$ to the range of high plastic strains ($>0.35$). The two strain hardening rate moduli (Table 4) are calibrated such that the simulation provides a good prediction of the experimentally-measured force–displacement curve (Fig. 5b).

### 4.5. Comparison of simulations and experiments

In Fig. 2, we show the simulated force–displacement curves (solid black lines) all the way to fracture. The agreement with the experimental results (depicted with black dots) is very good for the 20 mm (Fig. 2a) and 10 mm (Fig. 2b) notch geometries. The force difference between simulation and experimental results is less than 1% in both cases. For the 6.67 mm notch geometry (Fig. 2c), the peak of force, corresponding to the onset of localized necking, is delayed in the simulation by relative displacement of about 5%. As a result, the FEA predicted force drop is too small: the axial force at the onset of fracture is 3.7% higher in the simulation than in the experiment. The comparison of the evolution of the logarithmic axial strain at the center of the gage section with respect to the displacement (depicted in blue in Fig. 2) also shows a good agreement. Irrespective of the notch radius, the simulations are able to describe the characteristic increases in strain rates that have been observed in the experiments. Relative differences between simulation and DIC strains in case of the 20 mm notch geometry are about 3% (Fig. 2a). For the 10 mm notch geometry (Fig. 2b), the computed strain is of up to 10% higher than the DIC measurement. As far as the 6.67 mm notch geometry is concerned (Fig. 2c), the first increase of strain rate is too large in the simulation, while the predicted strain rate increase after the onset of localized necking appears to be smaller than that given by DIC. As a result, differences between simulated and measured strains tend to decrease at the end of the simulation, to be almost zero at fracture. Fig. 3 depicts the thickness profile along the axial plane of symmetry of the specimens (20 mm in red, 10 mm in blue, 6.67 mm in black). Note that both the amplitude of thickness variation and the size of the area of localization are very well predicted by the simulations.

### 4.6. Stress triaxiality and equivalent plastic strain evolution

Fig. 6a–c shows the evolution of the equivalent plastic strain as a function of the stress triaxiality at the center of the gage area. The red solid lines depict the evolution on the specimen surface, while the black solid lines show the evolution at the very center of the specimen (intersection point of all three symmetry planes). The large solid dot marks the onset of fracture that is obtained when using the average fracture displacement from three experiments. The crosses indicate the corresponding simulation results for the measured minimum and maximum displacement to fracture (see also Table 3a). The comparison of the red and black curves clearly

### Table 3a

Experimental results and fracture predictions for the tensile specimens with circular cutouts.

<table>
<thead>
<tr>
<th>Notch radius [mm]</th>
<th>Fracture displacement</th>
<th>Fracture plastic strain</th>
<th>Stress triaxiality at fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value [mm]</td>
<td>Variation [%]</td>
<td>Value [–]</td>
</tr>
<tr>
<td>6.67</td>
<td>2.048</td>
<td>1.28</td>
<td>0.422</td>
</tr>
<tr>
<td>10</td>
<td>2.336</td>
<td>1.21</td>
<td>0.552</td>
</tr>
<tr>
<td>20</td>
<td>2.887</td>
<td>1.22</td>
<td>0.585</td>
</tr>
</tbody>
</table>

Fig. 3. Thickness profile along the axial plane of symmetry for the three geometries. The curves corresponding to the 10 and 6.67 mm notched geometries are shifted by –0.3 mm and –0.6 mm, respectively.

Fig. 4. Meshes of the 10 mm notched radius specimen. (a) Coarse mesh with two elements through half the thickness, (b) medium mesh with four elements through half the thickness, (c) fine mesh with eight elements through half the thickness, (d) very fine mesh, with 16 elements through half the thickness.
shows that the stress and strain state at the specimen surface is significantly different from that at the specimen mid-plane. In other words, there is a strong gradient along the thickness direction within the central zone of strain localization. The equivalent plastic strains to fracture inside the specimen are 11.5% (20 mm notch) and 15.8% (10 mm notch) higher inside the specimen than on the surface (see Fig. 7). Localized necking also leads to the development of out-of-plane stress components in the middle of the specimen (while the surface deforms under plane stress), which increases the stress triaxiality. Furthermore, the strains measured at the specimen surface are not representative for the strain to fracture of the material (red line in Fig. 6). It is also noted that the stress triaxiality exhibits very strong variations during loading. For instance, for $R = 20$ mm it increases from $\eta = 0.40$ before the onset of localized necking to $\eta = 0.61$ at the onset of fracture.

4.7. Uncertainty analysis

Three different types of errors affecting both the plastic strain and the stress triaxiality at the onset of fracture are summarized in Table 3b. Considering DIC accuracy and camera resolution, the relative displacement of the specimen boundaries is measured in all experiments with an accuracy of 2.5 $\mu$m. Due to the strain localization at the center of the specimen, the errors in the fracture displacement translate to even larger errors in the fracture strain. This small uncertainty in the measured fracture displacement (relative error of about 0.1%) leads to an error of about 0.004 on the fracture strain and 0.002 on the stress triaxiality at the onset of fracture.
Errors due to the inaccuracy of the constitutive model are computed according to Eq. (5). For all three specimen geometries, the estimated error $\delta_ip^p$ on the plastic strain is less than 0.03 at the onset of localized necking and reaches 0.063 at the point of fracture for the 20 mm notch geometry (0.083 and 0.073 for the 10 mm and 6.67 mm geometries, respectively). This emphasizes the difficulty of modeling the post-necking behavior of the specimen with great accuracy. According to Eq. (10), this corresponds to errors on the stress triaxiality at the onset of fracture of 0.025 for the 20 mm geometry, 0.036 for $R = 10$ mm and 0.032 for $R = 6.67$ mm. The errors affecting the plastic strain versus stress triaxiality curves are depicted in Fig. 6a–c by dashed blue lines. Those lines can be seen as the upper and lower boundaries on the evolution of the actual material state in the stress triaxiality/plastic strain space at the fracture locus. Colored areas surrounding fraction of the actual material state in the stress triaxiality/plastic strain space allows the visualization of the regions where the error on the stress triaxiality is significant.

To illustrate the large error associated with the use of shell elements, we added a red dashed line to Fig. 6b which shows the predicted loading path evolution from a shell element simulation prior to necking, whereas the predicted strain increases to unrealistically high values after the onset of necking.

5. Circular punch test

The circular punch test is a standard sheet metal forming test that characterizes the formability of sheet materials under stress states that are close to equi-biaxial tension. Analogously to our analysis of the notched tensile test, we assess the accuracy of the circular punch test.

5.1. Experimental procedure

The circular sheet specimen is clamped on a circular die and subsequently loaded through a hemispherical punch. The punch and die have a diameter of 44.5 mm and 100 mm, respectively. The clamping pressure is applied through eight M10-12.9 screws. The experiment is carried out in a universal testing machine (MTS, Model G45) at a constant punch velocity of 5 mm/min. In order to limit the effects of friction, a stack of six oil-lubricated 90 µm thick Teflon layers is put between the specimen and the punch during each test.

Three-dimensional digital image correlation (Vic3D, Correlated Solutions) is used to measure the out-of-plane deformation of the specimen. In our vertical experimental set-up, the clamping die is fixed on a special metal frame (Walters, 2009). A leaning mirror is integrated into that frame to record pictures of the speckle-painted bottom surface of the specimen with two digital cameras. The cameras see the specimen at a distance of 2.5 m at an angle of 20° from the punching direction. Each camera records about 300 pictures during the test; the edge length of a square pixel is about 100 µm. The displacement field is calculated by DIC for the entire free surface of the specimen assuming an affine transformation of the 21 × 21 pixels neighborhood of each point. The interpolation of the gray values is performed with a 6-tap filter. The logarithmic strain field is then calculated by averaging the displacement gradient over an area of 11 × 11 pixels.

5.2. Experimental results

The measured force–displacement curve increases monotonically until a sharp drop in force level is observed at the instant of onset of fracture. The recorded crosshead displacement includes the deformation of the clamping fixture as well as the deformation of the punch and the testing frame in addition to the effective punch displacement. Since the punch behaves like a non-linear spring (because of the increasing contact area between the punch and the specimen) it is difficult to extract the displacement associated with permanent deformation of the specimen from these measurements. Moreover, we observed that most of the Teflon layers are torn apart during the punching which may be considered as permanent deformation of the testing device. The initial thickness of the Teflon stack is 0.55 mm, but we measure a final thickness of 0.12 mm after the experiment. In order to eliminate these experimental uncertainties in the punch displacement measurements, we present most experimental results as a function of the punch force instead of the punch displacement.

Fig. 8c depicts the evolution of the maximum principal true strain on top of the dome measured by DIC. Observe that the applied force reaches a plateau in this displacement controlled experiment. Fracture initiates on top of the dome which indicates that friction was close to zero in this experiment. After fracture initiation, cracks propagate along the rolling direction of the sheet. Both

<table>
<thead>
<tr>
<th>Notch radius [mm]</th>
<th>Fracture plastic strain</th>
<th>Stress triaxiality at fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement error [-]</td>
<td>Modeling error [-]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.67</td>
<td>$0.34 \times 10^{-2}$</td>
<td>$7.31 \times 10^{-2}$</td>
</tr>
<tr>
<td>10</td>
<td>$0.43 \times 10^{-2}$</td>
<td>$8.34 \times 10^{-2}$</td>
</tr>
<tr>
<td>20</td>
<td>$0.46 \times 10^{-2}$</td>
<td>$6.34 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 3b. Errors in the evaluation of the plastic strain and stress triaxiality at the onset of fracture.
measured principal strains at the apex of the deformed specimen exceed 0.4 at the onset of fracture. Post-mortem analysis revealed that the sheet thickness is reduced by almost 60%, from 1.43 mm (initial) to 0.58 mm (final).

In addition to measuring the strain at the specimen apex, the DIC measurements are used to verify two important features of this experiment. First, the DIC measurements demonstrate that the radial displacements are negligibly small along the interface between the specimen and the clamping ring (less than 0.05 mm). Second, the DIC measurements demonstrate that the strain maximum prior to fracture is located at the specimen center which re-confirms that friction effects have been successfully eliminated by the lubricated Teflon layers (Burford et al., 1991).

### Table 4
Stress strain curve beyond the onset of necking.

<table>
<thead>
<tr>
<th>Equivalent plastic strain [-]</th>
<th>0.2</th>
<th>0.35</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent stress [MPa]</td>
<td>1050</td>
<td>1140</td>
<td>1240</td>
</tr>
<tr>
<td>Hardening modulus [MPa]</td>
<td>600</td>
<td>400</td>
<td>100</td>
</tr>
</tbody>
</table>

#### 5.3. Numerical modeling

A quarter of the mechanical system is modeled because of the symmetry of the punch experiment and the orthotropic material behavior. Eight-node reduced-integration solid element meshes are employed in conjunction with the implicit solver of Abaqus. In addition, we make use of a shell element mesh along with the explicit solver of Abaqus because of the high computational efficiency of the shell contact formulation. In all FE models, the punch and the die are modeled as rigid bodies. The portion of the specimen that is clamped in the die is limited to 5 mm in the simulation (i.e. the diameter of the circular specimen in the FE model is 110 mm). The displacements of all nodes located on the outer edge of the specimen are set to zero. A condition of zero-normal displacement is imposed along the two edges that correspond to planes of symmetry. For shell meshes, no rotation around the edge direction is allowed on those two boundaries. A frictionless node-to-surface contact is defined between the punch and the specimen. In the case of shell elements, contact is defined for the sheet surface while thickness variations are taken into account. A constant velocity is applied to the punch, while the die is fixed in space.
The predicted force–displacement curves from an implicit simulation with 100 time steps using a coarse mesh (60/3 reduced-integration solid elements along the radial/thickness direction), an intermediate mesh (120/6) and a fine mesh (240/12) all lie on top of each other (Fig. 8a). Similarly, the results from quasi-static explicit simulations with reduced-integration shell elements (using the same number of integration points through-the-thickness as solid elements along that direction) are all identical. However, there is a noticeable difference between shell and solid element simulations for large punch displacements. This difference may be attributed to the errors associated with the assumption of plane stress and zero plastic out-of-plane shear strains in the shell element formulation. Analysis of the solid element simulations reveals that the out-of-plane compression stress reaches 90 MPa on the contact surface with the punch; the maximal out-of-plane logarithmic shear strain is about 0.035. Unlike the results for the force–displacement curves, the solid element model predictions of the equivalent plastic strain feature a weak mesh size effect. At the center, the final equivalent plastic strain reaches 0.92 for a coarse mesh, whereas it is 0.90 for a fine mesh. Between a medium and fine mesh, the relative difference is almost zero. The results from implicit simulations with different numbers of time steps (65, 100, 150 and 200) revealed only small differences. The final maximal equivalent plastic strain reaches 0.92 when using 65 time steps and 0.90 for 200 steps.

Based on our brief analysis, the punch experiment will be analyzed using a finite element model with (i) 120 solid elements along the radial direction, (ii) 6 solid elements in thickness direction, (iii) 100 implicit time steps and (iv) frictionless kinematic node-to-surface contact.

5.4. Identification of strain hardening response

When plotting the evolution of the major principal strain at the center of the specimen as a function of the punching force (Fig. 8c), it becomes apparent that the simulation model (blue curve) underestimates the strain in comparison with the experiment (solid black dots). Recall that the hardening curve used in the simulation has been calibrated based on the experimental results from uniaxial and notched tensile tests. However, since the maximum equivalent plastic strain reached in a punch test (about 0.9) is still much larger than that reached in a notched tensile test (about 0.6), we may improve the extrapolation of the stress–strain curve for large strains. Here, a third linear hardening segment is introduced for \( \varepsilon > 0.6 \). The best correlation between simulation and the punch experiment is achieved when using a hardening modulus of \( H_s = 100 \text{ MPa} \) in this third segment. The corresponding simulation result is depicted as a black solid line in Fig. 8c. It is emphasized that this modification of the hardening curve does not affect the results from the previous section on notched tensile tests. Note that the hardening curve identification based on the biaxial experiments depends on the choice of the yield function (e.g. Banabic et al., 2000). Up to the strain of necking under uniaxial tension (about 0.2), we may interpret the good agreement of the simulations and the biaxial experiments as a partial validation of the Hill’48 model. Furthermore, the Hill’48 yield surface has been validated for the present material for a similar range of modest strains through multi-axial experiments (see Mohr et al., 2010). However, the assumption of a Hill’48 yield function may become inadequate for very large strains in the case of texture evolution.

5.5. Simulation results and uncertainty analysis

The numerical simulation is performed up to the instance where the computed surface strain equals the measured surface strain at the onset of fracture (\( \varepsilon = 0.461 \)). The simulated curve shows very good agreement with the experimental results (depicted with black dots). Furthermore, the predicted thickness reduction is in excellent agreement with the experiment. Fig. 8d depicts the evolution of the equivalent plastic strain as a function of the stress triaxiality (black curve); here, it is assumed that fracture initiates on the free specimen surface. The loading state at the onset of fracture is depicted as a black dot. Fracture occurs at a computed stress triaxiality of \( \eta = 0.66 \) (equi-biaxial tension).

The modeling error affecting the computation of the equivalent plastic strain and the stress triaxiality is evaluated according to Eqs. (5) and (10); it is shown by blue dashed lines in Fig. 8d. Modeling errors at the onset of fracture are summarized in Table 5. A complete evaluation of the errors affecting the hybrid experimental–numerical result would require evaluating the precision of the 3D DIC method. However, the authors could not identify such an evaluation in the open literature. For the case of one-dimensional DIC, Sutton et al. (2009) reported that the 6-tap optimized filter interpolation function does not produce any significant error. Thus, we neglect this source of error in our analysis.

6. Tensile specimen with central hole

Conventional uniaxial tensile specimens develop a pronounced neck at large strains which yields a change in stress state throughout the experiment from uniaxial tension to transverse plane strain. In an attempt to keep the stress triaxiality more constant throughout the experiment, we make use of tensile specimens with a central hole. The presence of a central hole creates a strain concentration which favors the fracture initiation at the intersection of the hole and the transverse axis of symmetry of the specimen.

6.1. Experimental procedure

The tensile specimens are 20 mm wide and feature an 8 mm diameter circular hole at the center (Fig. 1d). For the first set of specimens, the central hole is cut using a water-jet. In order to obtain a better edge finish, we prepared a second set of specimens with a 7 mm diameter water-jet cut hole that is subsequently enlarged to 8 mm using CNC milling (with a 0.125° diameter end mill). The experimental procedure follows closely the program outlined for the notched specimens. To evaluate the error in the computed strains, we determined the axial logarithmic strain on the transverse symmetry axis at a distance of 40 pixels (380 \( \mu \text{m} \)) from the hole. A measurement right at the edge of the hole is not possible as the DIC algorithm needs a continuous displacement field in the vicinity of the point of interest.

6.2. Experimental results

The measured force–displacement curves for the two sets of specimens are shown in Fig. 9a as crosses for the water-jet cut specimens and as solid dots for the CNC-milled specimens. The water-jet-prepared specimens are extracted from a slightly thinner part of the sheet (1.46 mm instead of 1.5 mm), resulting in a lower force–displacement curve. The measurements demonstrate that

<table>
<thead>
<tr>
<th>Fracture point and error estimation for the circular punched specimen.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture plastic strain</td>
</tr>
<tr>
<td>Value [-]</td>
</tr>
<tr>
<td>0.896</td>
</tr>
</tbody>
</table>
the machining technique has a strong influence on the fracture displacement. It is about 2.1 mm for CNC-milled specimens and only 1.7 mm for the water-jet cut specimens. The water-jet cuts the sheet by abrasion (abrasive jet), which leaves a non-smooth edge with numerous geometric defects. It is speculated that those defects along with some residual plastic strains are responsible for the premature failure of the water-jet cut specimens. Consequently, the results for water-jet cut specimens are discarded in the following analysis.

The force–displacement curve (Fig. 9a) exhibits a peak before fracture occurs. An important width reduction (diffuse necking) is observed within both specimen ligaments (Fig. 10a), which appears to intensify as the force reaches its maximum. The displacement to fracture varies among the CNC-milled specimens (Table 6a). Observe from Fig. 10c that axial strain field features steep gradients around the transverse axis of symmetry of the specimen. The evolution of the surface strain (blue curve in Fig. 9a) shows that the surface strain reaches values of up to 0.7 prior to fracture.

6.3. Numerical modeling

Based on the results from Section 4, eight-node solid elements (with reduced integration) are used to mesh one eighth of the specimen (Fig. 10b). A constant velocity is applied to the upper boundary. A zero-normal displacement condition is imposed to the three boundaries corresponding to symmetry planes. Since the experimental results indicate that through-the-thickness localization is less important with this specimen design than for notched tensile specimens, we assume that eight elements through the half-thickness are enough to describe the stress and strain variations along the thickness direction. However, we use a biased mesh with the smallest elements at the intersection of the hole with the transverse plane of symmetry (Fig. 10b). In this vicinity, the elements have also the same length in the axial and transverse directions. Implicit simulations are performed using a coarse mesh (smallest in-plane element edge length is 120 μm), a medium mesh (60 μm) and a fine mesh (30 μm). As for the notched tensile tests, we find the same force–displacement curves for all mesh sizes. The effect of mesh size on the equivalent plastic strain is also weak for the element located at the hole boundary (on the specimen mid-plane). The final equivalent plastic strain computed with the coarse mesh is 0.83 compared to 0.86 when using the fine mesh.
Table 6a
Experimental results and fracture predictions for the tensile specimen with central hole.

<table>
<thead>
<tr>
<th>Fracture plastic strain</th>
<th>Fracture strain at fracture</th>
<th>Stress triaxiality at fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value [mm]</td>
<td>Variation [%]</td>
<td>Value [-]</td>
</tr>
<tr>
<td>2.089</td>
<td>2.24</td>
<td>0.834</td>
</tr>
</tbody>
</table>

mesh. Errors due to time discretization are also evaluated by running simulations with 50, 75, 100 and 150 time increments. The difference in final plastic strain is already negligible (0.004) when comparing the results for 75 and 100 time increments. Thus, we make use of the implicit analysis with 75 time steps and a medium mesh to determine the loading history to fracture.

6.4. Numerical results and error estimation

The simulated force-displacement curve is depicted as a black solid line in Fig. 9a. It is in excellent agreement with the experimental data (solid dots). The maximum difference between the experimentally-measured and numerically-predicted force level is smaller than 2%. This good correlation is seen as a validation of the strain hardening curve that has been determined from the notched tension and punch tests. The FEA predicted evolution of the surface strain (blue line in Fig. 9a) is also close to the DIC measurements (blue dots). Similar to the results for notched tensile tests, the strains computed by FEA are higher than the DIC measurements. Here, the final computed strain is overestimated by 40%.

The evolution of the equivalent plastic strain as a function of the stress triaxiality is shown in Fig. 9b. The black solid dot highlights the instant of onset of fracture. The differences due to scatter in the measured fracture displacement is represented with solid crosses. The identified fracture strain as well as the stress triaxiality at the onset of fracture are summarized in Table 6b along with the corresponding error estimates.

As compared to the results for notched tensile specimens, the stress triaxiality variations in the tensile specimen with a central hole are small. It varies between 0.277 and 0.338. At the onset of fracture, the stress triaxiality is \( \eta \approx 0.282 \) which is close to uniaxial tension (\( \eta \approx 0.33 \)). The estimated equivalent plastic strain to fracture is 0.83. The relative displacements of the specimen boundaries are computed with a precision of 2.5 \( \mu \)m which translates into an uncertainty of 0.002 for the fracture strain and of less than is 0.001 for the stress triaxiality. The modeling errors according to Eqs. (5) and (10) are 0.095 for the fracture strain and 0.004 for the stress triaxiality.

7. Discussion and recommendations

7.1. Identification of the strain hardening response

The proper identification of the strain hardening model for very large strains is critically important for the reliable determination of the fracture strains. It is emphasized that conventional extrapolation formulas such as the modified Swift model seem to provide a poor approximation of the strain hardening behavior of advanced high strength steels at large strains. The present study shows that the Swift assumption leads to substantial errors in the simulation results after the onset of necking which is consistent with earlier results on martensitic steel (Mohr and Ebnoether, 2009). When hydraulic bulge testing devices are not available or a bulge test is impossible to realize (because of the very large specimen size), we propose the following procedure to identify the strain hardening function \( H(\beta) \):

(i) Uniaxial tensile testing of dogbone specimens up to the strain of necking (ASTM E8M-04, 2004).

(ii) Uniaxial testing of a tensile specimen with a central hole; the stress–strain curve can then be identified through inverse calibration.

One may also model the post-necking behavior of uniaxial tensile test and determine the stress–strain curve from inverse analysis (e.g. Mohr and Ebnoether, 2009). However, since the stress state in the neck of a uniaxial tension specimen is close to transverse plane strain, the maximum equivalent plastic strain achieved using an uniaxial specimen with a central hole is expected to be larger. Furthermore, the stress gradients through the sheet thickness are smaller for the later type of specimen. From an experimental point of view, we note that the location of the zone of localization is a priori known when using a specimen with a central hole. This allows for the proper positioning of the DIC system before the experiment. Note that in a uniaxial tensile test, the position of the emerging neck is unknown before the experiment and may thus occur outside the field of vision of the camera system.

7.2. Numerical modeling

Shell element simulations provide accurate predictions of the large deformation behavior of sheet metal structures before the onset of through-the-thickness necking. However, the strain and stress state predictions of shell element simulations after the onset of through-the-thickness necking are not reliable as out-of-plane stresses become important. The same limitation becomes apparent under the presence of high surface pressures (e.g. final phase of the punch test). Thus, we strongly recommend using solid element meshes to determine the stress and strain histories all the way to fracture. When evaluating the effect of mesh density on the simulation results, it is important to monitor the strain evolution within the zone of localization. The global force–displacement curves are usually not mesh size sensitive since the material within the zone of localization contributes only little to the “internal energy” (elastic strain energy plus plastic dissipation) of the entire structure. As a rule of thumb, we recommend 16 first-order solid elements through the full thickness of the sheet.

7.3. Summary of the loading paths to fracture

Table 6b
Errors in the evaluation of the plastic strain and stress triaxiality at the onset of fracture.

| Fracture plastic strain Displacement error [-] | Modeling error [-] | Total error [-] | Stress triaxiality at fracture Displacement error [-] | Modeling error [-] | Total error [-] |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.22 \times 10^{-2} | 9.49 \times 10^{-2} | 9.71 \times 10^{-2} | 0.03 \times 10^{-2} | 0.43 \times 10^{-2} | 0.46 \times 10^{-2} |
men with a central hole, while it is located on the specimen surface for the punch experiment. We observe the high ductility for stress states close to uniaxial tension and those close to equi-biaxial tension. The strain path for the notched tensile specimen features stress states close to transverse plane strain which exhibit the lowest ductility. As an alternative to showing the results in the $(\eta, \theta)$-plane, we also computed the loading paths to fracture in terms of the principal plastic strains in the plane of the sheet. The minor and major strains shown in Fig. 11b are calculated from the in-plane components of the plastic strain tensor. Note that these strains are different from the eigenvalues of the plastic strain tensor (unless the out-of-plane shear strain components are zero).

7.4. Effect of porosity

All numerical results presented in this paper are obtained under the simplifying assumption that the effect of (micro-)porosity evolution on the effective plastic behavior can be neglected. There is strong theoretical evidence that the evolution of porosity changes the predicted stress triaxialities (e.g. Danas and Ponte Castaneda, 2009a). However, it is difficult to quantify the effect of porosity on the plastic behavior of the TRIP780 steel based on our macroscopic measurements (surface displacement fields and total force). At the same time, the numerical predictions of the non-porous plasticity model employed in this study agree well with all macroscopic measurements for various loading conditions.

The evolution of porosity clearly plays an important role as far as the onset of fracture is concerned. The initial microstructure is void free, but micrographs of highly deformed specimens indicate that voids initiate and grow throughout loading. Fig. 12 shows micrographs of the axial plane of symmetry at the center of the notched specimens prior to fracture (after applying about 97% of the displacement to fracture). Voids and microcracks are clearly visible at this stage of deformation. Observe that the microcracks are aligned with the loading direction. This observation may be explained using the anisotropic porous plasticity model of Danas and Ponte Castaneda (2009b). Their homogenization-based computations show that the severe elongation of initially spherical voids under transverse plane strain loading causes the loss of ellipticity of the effective porous medium. In other words, the axial microcracks may be considered as the result of the coalescence of highly elongated voids. However, in the present case, the onset of fracture is also affected by material heterogeneities at the microstructural level. Energy dispersive X-ray analysis revealed that the locations

![Fig. 11. Stress and strain histories for the five geometries in the stress triaxiality versus equivalent plastic strain space (a) and in the in-plane major strain versus in-plane minor strain space (b).](image1)

![Fig. 12. Micrographs of the axial plane of symmetry of deformed notched tensile specimens: (a) R = 20 mm notch specimen strained to 98.3% of the fracture displacement, (b) R = 6.67 mm notch specimen strained to 96.8% of the fracture displacement. The vertical and horizontal directions of the pictures correspond to the axial and thickness directions of specimen, respectively.](image2)
of the microcracks seen in Fig. 12 coincide with the position of Mn and Mo segregation bands.

8. Conclusions

Five different fracture tests have been performed on specimens extracted from TRIP780 sheets and analyzed in great detail to obtain reliable estimates of the loading path to fracture for stress triaxialities ranging from uniaxial tension to equi-biaxial tension. The main conclusions are:

(1) Shell element simulations are not suitable for the evaluation of the local loading path after the onset of through-the-thickness necking.

(2) Solid element simulations can provide accurate predictions. Both coarse and fine meshes predict usually the same overall force–displacement response, but it is important to evaluate the accuracy of an FE simulation through the comparison of the predicted strains within the neck with DIC surface strain measurements. For the present material and loading conditions, 16 first-order solid elements along the thickness direction provided sufficiently accurate results for the local fields.

(3) It is important to identify the strain hardening curve for large strains from experiments. The analytical extrapolation (e.g. Swift law) based on data for uniaxial tension prior to necking is not sufficiently accurate. When data from hydraulic bulge tests is not available, we recommend the inverse identification of the stress–strain curve using the results from the testing of uniaxial tensile specimen with a large central hole.

(4) The stress triaxiality is approximately constant all the way to fracture for a tensile specimen with a central hole and during a punch test; it increases monotonically throughout notched tensile tests.

Acknowledgements

The partial support of the Joint MIT/Industry AHSS Fracture Consortium is gratefully acknowledged. POSCO Steel is thanked for providing the material. Thanks are due to Dr. L. Greve from Volkswagen for suggesting the tensile specimen with a central hole for fracture testing. Professor T. Wierzbicki from MIT is thanked for valuable discussions. The help of Dr. C. Walters on performing the punch experiments is gratefully acknowledged.

References

Pardoen, T., Hutchinson, J.W., 2000. An extended model for void growth and necking is not sufficiently accurate. When data from hydraulic bulge tests is not available, we recommend the inverse identification of the stress–strain curve using the results from the testing of uniaxial tensile specimen with a large central hole.