Double resonance Raman modes in monolayer and few-layer MoTe$_2$
Double resonance Raman modes in monolayer and few-layer MoTe$_2$

Huaihong Guo,$^{1,*}$ Teng Yang,$^{2,1}$ Mahito Yamamoto,$^3$ Lin Zhou,$^4$ Ryo Ishikawa,$^5$ Keiji Ueno,$^6$
Kazuhiro Tsukagoshi,$^3$ Zhidong Zhang,$^2$ Mildred S. Dresselhaus,$^{4,7}$ and Riiichiro Saito$^1$

$^1$Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan

$^2$International Center for Materials Nanoarchitectonics (WPI-MANA), National Institute for Materials Science (NIMS), Tsukuba, Ibaraki 305-0044, Japan

$^3$Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4037, USA

$^4$Department of Chemistry, Graduate School of Science and Engineering, Saitama University, Saitama 338-8570, Japan

$^5$Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan

$^6$Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307, USA

$^7$Department of Functional Materials Science, Graduate School of Science and Engineering, Saitama University, Saitama 338-8570, Japan

(Received 13 January 2015; revised manuscript received 11 March 2015; published 12 May 2015)

We study the second-order Raman process of mono- and few-layer MoTe$_2$, by combining $ab$ initio density functional perturbation calculations with experimental Raman spectroscopy using 532, 633, and 785 nm excitation lasers. The calculated electronic band structure and the density of states show that the resonance Raman process occurs at the $M$ point in the Brillouin zone, where a strong optical absorption occurs due to a logarithmic Van Hove singularity of the electronic density of states. The double resonance Raman process with intervalley electron-phonon coupling connects two of the three inequivalent $M$ points in the Brillouin zone, giving rise to second-order Raman peaks due to the $M$-point phonons. The calculated vibrational frequencies of the second-order Raman spectra agree with the observed laser-energy-dependent Raman shifts in the experiment.

DOI: 10.1103/PhysRevB.91.205415 PACS number(s): 33.40.+f, 78.30.Hv, 63.20.kd, 31.15.A–

I. INTRODUCTION

Transition-metal dichalcogenides (TMDs) have been intensively studied over the past decades, and have recently attracted tremendous attention for a number of applications [1–5], since TMDs have a finite energy band gap which is more advantageous than graphene. Among the TMDs, MoTe$_2$ has a stronger spin-orbit coupling than other Mo-related TMD materials (MoX$_2$, $X = $ S, Se), which leads to a longer decoherence time both for valley and spin freedom [5], and thus MoTe$_2$ is considered to be used for ideal valleytron devices [6,7]. Unlike other TMDs, MoTe$_2$ has unique structural properties, such as a thermally induced structural phase transition at around 900 °C from the $\alpha$-phase diamagnetic semiconductor to the $\beta$-phase paramagnetic metal [8,9]. MoTe$_2$ has the smallest direct energy band gap (∼1.1 eV) at the $K$ point (hexagonal corner) in the Brillouin zone (BZ) among all semiconducting TMDs, and the band gap is similar to the indirect gap (∼1.1 eV) of silicon [10,11]. MoTe$_2$ is thus suitable for nanoelectronics devices [4,5] and for studying exciton effects at the $K$ points [12]. Raman spectroscopy is a useful tool for characterizing TMDs since we thus get much information [13–15], such as the number of layers, and the electronic and phonon properties of TMDs. Recent Raman spectroscopy studies of monolayer and few-layer MoTe$_2$ using visible light (∼1.9–2.5 eV) show that there exists some unassigned Raman spectral peaks which cannot be assigned to a first-order Raman process [5,16], and the assignment of these Raman spectra is the subject of the present paper.

Raman spectra of TMDs have been reported for several decades [17–29]. The first-order Raman process in TMDs has been widely investigated and is now well understood [16,21,22,24,30–34]. However, the study of second-order Raman spectra did not yet give consistent assignments for different TMDs [18,25,26,35–37]. For example, Stacy et al. [26] and many other groups [15,18,29,38–40] assigned some of the Raman peaks of 2H-MoS$_2$ to either combination or overtone second-order phonon modes occurring at the $M$ point (center of hexagonal edge) in the Brillouin zone. Terrones et al. [37] also assigned phonon modes in few-layer WS$_2$ according to the $M$-point phonon. On the other hand, Sourisseau et al. [27,28] ascribed the second-order Raman process of 2H-WS$_2$ to the $K$-point phonons. Berkdemir et al. [41] considered that electrons are scattered by the $M$-point phonons between the $K$ point and $I$ point (one point along the $\Gamma K$ direction), and that this process is responsible for the double resonant Raman process in few-layer WS$_2$. Furthermore, Sekine et al. [25] found a dispersive Raman peak in 2H-MoS$_2$ and interpreted this peak in terms of a two-phonon Raman process at the $K$ point. The different assignments for both few-layer or bulk MoS$_2$ and WS$_2$, either at the $M$ or $K$ points, have not been well investigated, since most of these assignments to second-order Raman peaks have been made merely by comparing phonon frequencies with reference to some inelastic neutron-scattering results [19,28]. Thus a detailed analysis with use of double resonance Raman theory is needed, especially for understanding both mono- and few-layer MoTe$_2$, which are discussed in this paper.

In the present paper, we mainly study the second-order Raman spectra of monolayer MoTe$_2$, based on $ab$ initio electronic band calculations, density functional perturbation theory, and associated experimental Raman spectroscopy.

$^1$hhguo@alum.imr.ac.cn
$^1$yangteng@imr.ac.cn
From our electronic band structure calculation, we have found a saddle point of the energy dispersion at the $M$ points in the BZ, which gives rise to a two-dimensional Van Hove singularity (VHS) in the electronic density of states. The energy separation between two VHSs (2.30 and 2.07 eV) are matched to the 2.33 eV (532 nm) and 1.96 eV (633 nm) laser excitation energies in our experiment in which resonant optical absorption occurs at the $M$ points. Consequently, an intervalley electron-phonon scattering process occurs between two of the three inequivalent $M (M', M'')$ points, giving rise to a double resonance Raman process. We propose that this double resonance process occurring at the $M$ points is essential for the interpretation of the observed second-order Raman peaks of MoTe$_2$. The experimental second-order Raman frequencies as a function of laser excitation energy confirm our interpretation of the double resonance Raman spectra.

The organization of the paper is as follows. In Secs. II and III, we briefly show computational details and experimental methods, respectively. In Sec. IV, both experimental and theoretical results for mono- and few-layer MoTe$_2$ Raman spectra are given. We assign second-order Raman peaks by comparing the calculated results with the observed Raman spectra. In Sec. V, a summary of the paper is given. Further information such as layer-number dependence is discussed in the Supplemental Material [42].

II. COMPUTATIONAL DETAILS

We performed the electronic and phonon energy dispersion calculations on monolayer and few-layer MoTe$_2$ by using first-principles density functional theory within the local density approximation (LDA) as implemented in the QUANTUM-ESPRESSO code [43]. The structures of monolayer and few-layer MoTe$_2$ that are used in the present experiment are $\alpha$-MoTe$_2$ [16]. The $AA'$ and $AA''A$ stacking geometries are used in the computational study for bilayer and trilayer MoTe$_2$, respectively, since these stacking geometries are more stable than the other possibilities [44]. $AA'$ stacking refers to the geometry which has a Mo (Te) atom of one layer on top of a Te (Mo) atom of the other layer. The few-layer MoTe$_2$ separation from one unit cell to another unit cell was taken as 25 Å in the calculation to eliminate the inter-few-layer interaction. We used projector augmented-wave (PAW) pseudopotentials [45] with a plane-wave cutoff energy of 65 Ry to describe the interaction between electrons and ions. The spin-orbit split electronic band structures were calculated by relativistic pseudopotentials derived from an atomic Dirac-like equation. The atomic coordinates are relaxed until the atomic force is less than $10^{-5}$ Ry/Bohr. The lattice parameters used in this paper are listed in Table S1 of the Supplemental Material [42] and agree well with the experiment and other calculations [32,46,47]. The Monkhorst-Pack scheme [48] is used to sample the Brillouin zone (BZ) over a $17 \times 17 \times 1$ $k$ mesh for MoTe$_2$. The phonon energy dispersion relations of MoTe$_2$ are calculated based on density functional perturbation theory [49]. In order to visualize the vibrational modes at the $M$ point, we use a $2 \times 2 \times 1$ supercell in which the $M$ point is zone folded to the $\Gamma$ point in the folded BZ.

The optical absorption spectrum is calculated by the real ($\epsilon'$) and imaginary ($\epsilon''$) parts of the dielectric function as a function of photon energy, respectively, based on the PAW methodology [50] and the conventional Kramers-Kronig transformation. The absorption coefficient $\alpha$ is described by $\alpha = 4\pi \frac{\kappa}{h\omega} E_{\nu}/(\epsilon' + \epsilon''^2 - \epsilon'^2)\kappa$ is the Planck constant, $c$ is the speed of light, and $\kappa$ is the extinction coefficient [51], that is, $\kappa = \sqrt{(\epsilon'^2 + \epsilon''^2 - \epsilon')/2}$.

III. EXPERIMENTAL METHOD

Bulk crystals of $\alpha$-MoTe$_2$ were prepared through a chemical-vapor transport method [52]. Atomically thin crystals of MoTe$_2$ were mechanically exfoliated from the bulk crystals onto silicon substrates with a 90-nm-thick oxide layer on top. The thicknesses of the atomic crystals were identified using optical microscopy and first-order Raman spectroscopy [16]. Raman spectroscopy measurements for monolayer MoTe$_2$ were performed using 532, 633, and 785 nm excitation lasers for discussing the observed phonon dispersion due to double resonance Raman theory [53]. Raman spectroscopy studies of few-layer MoTe$_2$ under 532 nm laser excitation were also performed and the results are discussed in the Supplemental Material [42]. The grating sizes were 1200, 1800, and 2400 lines/mm for the 785, 633, and 532 nm laser excitation measurements, respectively. The magnification of the objective lens was 100×. The accumulation times were 150–200 seconds. The laser power was kept below 0.1, 0.2, and 3.0 mW for the 532, 633, and 785 nm excitation lasers, respectively, in order to avoid damage to the samples. All measurements were performed at room temperature in the backscattering configuration. Since this paper gives greater emphasis to a theoretical understanding of the second-order Raman scattering processes, we do not present many experimental details. However, more details about the characterization of the number of layers in our MoTe$_2$ samples can be obtained in one of the previous works on MoTe$_2$ synthesis and characterization, based on the first-order Raman spectra [16].

IV. RESULTS AND DISCUSSION

Figure 1(a) shows Raman spectra of mono- to trilayer MoTe$_2$ under the 532 nm (2.33 eV) laser excitation. The Raman spectra show three strong peaks, including the in-plane $E_{2g}^1$, mode and the out-of-plane $A_{1g}$ mode for monolayer and few-layer MoTe$_2$, as well as the $E_{1g}$ mode and the $B_{1g}$ mode for bi- and trilayer MoTe$_2$, which were observed previously [5,16,54]. These spectra are assigned to the first-order Raman spectra of the $\Gamma$-point phonons [16]. However, we observe additional peaks with relatively small intensities, which cannot be assigned as first-order Raman spectra [16]. In Fig. 1(a), we denote the weak Raman spectra as $\omega_i$ ($i = 1, 2, \ldots, 6$).

Since the second-order Raman spectra are dispersive as a function of excitation energy [53], we performed Raman spectroscopy measurements using different excitation laser energies. In Fig. 1(b), we show the Raman spectra of monolayer MoTe$_2$ under 2.33 eV (532 nm), 1.96 eV (633 nm), and 1.58 eV (785 nm) laser excitation energies. Here the Raman intensities are normalized using the nonresonant Si peak intensity at 520 cm$^{-1}$ [not shown in Fig. 1(b)]. We find that the peak positions of each $\omega_i$ either upshift or downshift
by up to several cm\(^{-1}\) for different laser excitation energies, suggesting that these peaks are associated with a second-order Raman process \[53\].

Considering that the second-order process is usually connected to the electronic properties by double resonance Raman theory \[53\], we calculate the electronic band structure, the electronic density of states (DOS), and the optical absorption spectra of monolayer MoTe\(_2\), as shown in Fig. 2. Our calculated energy dispersion reproduces the main experimentally observed features \[11,32,56\] such as the direct electronic band

![Diagram](image-url)

**FIG. 1.** (Color online) (a) Raman spectra of mono-, bi-, and trilayer MoTe\(_2\) under 2.33 eV (532 nm) laser excitation energy, showing the small intensity peaks \(\omega_i\) \((i = 1, 2, \ldots, 6)\) and an additional calibration peak from Si besides the typical Raman peaks of \(A_{1g}, E_{1g},\) and \(B_{1g}\) that are frequently observed in MoTe\(_2\) \[5,16\]. (b) Raman spectra of monolayer MoTe\(_2\) under 2.33 eV (532 nm), 1.96 eV (633 nm), and 1.58 eV (785 nm) laser excitation energies.

![Diagram](image-url)

**FIG. 2.** (Color online) (a) Electronic energy band structure \(E(k)\) and the density of states (DOS), (b) the real (\(\epsilon'\)) and imaginary (\(\epsilon''\)) parts of the dielectric function and the optical absorption coefficient \(\alpha\) as a function of photon energy \(E_{\gamma}\), and (c) an equienergy contour plot of the two conduction bands [(c1) and (c2)] with the same spin for monolayer MoTe\(_2\). Energy bands are split by spin-orbit interaction; the conduction bands (c1) and (c2) and the valence bands which have the same spin \[6,55\] are highlighted in red solid lines in (a). When we consider the energy separation between two Van Hove singularities (VHSs) of the DOS, the peak at 1.80 eV in the absorption spectrum corresponds to the optical transition at the \(M\) point [black double-headed arrow in (a)]. (c) The equienergy contour plot for VHS \(E_{\text{band}} = 1.52\) and 1.57 eV (left) and \(E_{\text{band}} = 1.72\) eV (right). All equienergy contours appear around the \(M\) point. The \(E_{\text{band}}\) energies of 1.52, 1.57, and 1.72 eV [horizontal lines in (a)] correspond, respectively, to the experimental laser energies (1.58, 2.33, and 1.96 eV) in the blue, pink, and orange colors.
gap (1.00 eV) and the spin-orbit splitting of the valence band (∼230 meV) at the K point which are close to 1.10 eV and 250 meV, respectively, obtained from the optical absorption measurements [11]. It is noted that the energy band gap is underestimated by the local density functional method. In order to fit the experimental data, the conduction bands in the calculation are upshifted by 0.10 eV, as are the DOS and optical absorption spectrum. In Fig. 2(a), we show the calculated electronic energy band and DOS. From the parabolic bands around the K point, the two-dimensional (2D) DOS gives constant values, while at the M point, we have found saddle points of the energy dispersion in the electronic band; the corresponding DOS gives a logarithmic ln |E − E₀| divergence, which is known as a Van Hove singularity (VHS) in 2D systems.

In Fig. 2(b), we show the calculated real (ε⁺) and imaginary (ε'') parts of the dielectric constant and the optical absorption coefficient α as a function of photon energy. Here we see that ε'' has a nonzero value above the photon energy Eγ = 1.10 eV, which corresponds to the energy gap, and a peak at Eγ ∼ 1.80 eV. A strong optical absorption at 1.80 eV from our calculation agrees well with the large peak at around 1.83 eV from the optical absorption measurement by Ruppert et al. [11] on monolayer MoTe₂. Moreover, the electronic excitation energy of 2.30 eV (2.07 eV) from the third (top) valence band to the lowest (third-lowest) conduction band at the M point matches very well with the laser energy 2.33 eV (1.96 eV) used in our experiment, as indicated by pink (orange) arrows in the band structure of Fig. 2(a). Thus resonant optical absorption at these laser lines is expected.

In Fig. 2(c), we plot the equienergy contour for the conduction band energies Eband which correspond to the laser excitation energies Eγ. More specifically, Eband = 1.52 and 1.57 eV for (c1) and 1.72 eV for (c2) correspond to the Eγ = 1.58, 2.33, and 1.96 eV, respectively, as indicated by blue, pink, and orange arrows in Fig. 2(a). The conduction bands (c1) and (c2) and the valence bands which have the same spin [6,55] are highlighted in red solid lines in the band structure. The equienergy contour lines Ec1−→1−→M (c2−→M) starting from the M point monotonically decreases (increases) along the M-K direction, whereas this band energy increases (decreases) along the M-Γ direction. When we decrease the laser excitation energies from 2.33 to 1.58 eV, the contour lines in Fig. 2(c), left plot, change their shapes around the M point, suggesting that the area inside the contour line for each M point decreases with increasing laser excitation energy. This situation is opposite to the case of graphene for which the equienergy contour is almost circular and the area increases with increasing energy.

If the laser excitation energy matches the optical transition energy at the M point, we expect that many photoexcited electrons are scattered inelastically by emitting phonons and make intervalley (i.e., M ←→ M′, M′ ←→ M′′, and M ←→ M′′′) transitions, as shown by the solid double-end arrows in the first BZ. The related phonon vectors q measured from the Γ point are also given by the six dashed arrows. (b) The schematics of the double resonance process (electron-photon and electron-phonon processes). This process generates two phonons, (ℏωp,−→q) and (ℏωp,2,−→q).

FIG. 3. (Color online) (a) In the double resonance process, photon-excited electrons are scattered inelastically by emitting phonons and make intervalley (i.e., M ←→ M′, M′ ←→ M′′, and M ←→ M′′′) transitions, as shown by the solid double-end arrows in the first BZ. The related phonon vectors q measured from the Γ point are also given by the six dashed arrows. (b) The schematics of the double resonance process (electron-photon and electron-phonon processes).
In Table I, we list phonon frequencies of the first-order (upper) and the second-order (lower) Raman spectra for both the experiment and the calculation and also the phonon assignments. The first-order experimental Raman spectra \((A_{1g} \text{ and } E_{2g})\) do not change the frequency for the laser energies \((2.33 \text{ and } 1.96 \text{ eV})\) within the experimental error \((1 \text{ cm}^{-1})\) and agree with the calculations. The small frequency downshift of the first-order spectra \((1.7 \text{ cm}^{-1} \text{ at most})\) from 2.33 to 1.58 \text{ eV} laser excitation is checked to be not from the laser-induced thermal effect, as discussed in the Supplemental Material [42]. Although we did not specify the origin of the downshift precisely, the origin of the change for the first order can be neglected for simplicity. The experimental second-order peak positions of \(\omega_i\) are either upshifted or downshifted with the laser energy \(E_\gamma\) by a small magnitude of about \(1-3 \text{ cm}^{-1}\). It is pointed out that the frequency of \(\omega_1\) is upshifted from 2.33 to 1.58 \text{ eV} even though both of the first-order spectra are downshifted. This can be understood by double resonance Raman theory in which the resonance phonon frequency changes along the phonon dispersion near the \(M\) point. In the right of Table I, the second-order phonon frequencies for 2.33 and 1.58 \text{ eV} laser excitations are obtained at \(\bar{q}_{MM'} = \bar{G}M\) (at \(M\)) and \(~10\%M'M'\) (denoted by “off \(M\)”), respectively. For \(E_\gamma = 1.96 \text{ eV}\), the phonon assignments are obtained either at \(\bar{q}_{MM'} = \bar{G}M\) (at \(M\)) or \(~1\%M'M'\) (off \(M\)).

These assignments can be further confirmed by the observed relative Raman intensity and the phonon dispersion relation. First, in Fig. 1(b), a larger relative intensity of Raman spectra \(\omega_i\) can be found at \(E_\gamma = 2.33 \text{ eV}\) than at 1.58 \text{ eV}. This supports the assignments at the \(M\) point and at \(\bar{q}_{MM'} \sim 10\%M'M'\) for \(E_\gamma = 2.33 \text{ and } 1.58 \text{ eV}\), respectively, since the phonon density of states (PhDOS) at the \(M\) point is larger than that at \(\bar{q}_{MM'} \sim 10\%M'M'\). Similarly, for \(E_\gamma = 1.96 \text{ eV}\), a relatively large (small) intensity of the Raman spectra \(\omega_2, \omega_3\), and \(\omega_6\) \((\omega_1, \omega_4, \text{and } \omega_5)\) in Fig. 1(b) may arise from the large (small) PhDOS at the \(M\) point (off the \(M\) point). It is therefore reasonable to assign the \(\omega_2, \omega_3, \text{and } \omega_6\) \((\omega_1, \omega_4, \text{and } \omega_5)\) at the \(M\) point (off the \(M\) point), as underlined in Table I. Second, it is noted that the experimental spectra of \(\omega_4\) (at the three laser energies) are too weak to determine the frequency values. The \(\omega_4\) is assigned as a combination mode related to the \(A_{1g}(M)\) as listed in Table I. Then it is interesting to discuss why the \(\omega_4\) becomes weaker and broader when...
TABLE I. Phonon frequencies of the first- and second-order Raman spectra of monolayer MoTe$_2$ are listed for three different laser energies $E_g$. A comparison for first- and second-order Raman spectra is given between the experimentally unassigned modes $\omega_i$ ($i = 1, 2, \ldots, 6$) and the calculated phonon frequencies both at the $M$ point and near the $M$ point (denoted by “off $M$”) whose $\mathbf{q}$ lies around $\frac{1}{2}[\mathbf{M}K]$ and $10\%[\mathbf{M}M]$ for $E_g = 1.96$ eV and 1.58 eV, respectively. The consistent dispersion behavior for the second-order Raman spectra between the experiment and the theory is underlined. The frequency is given in units of cm$^{-1}$. NA: not available.

<table>
<thead>
<tr>
<th>$E_g$ (eV)</th>
<th>Experiment</th>
<th>Calculation</th>
<th>Phonon assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{1g}$</td>
<td>NA</td>
<td>NA</td>
<td>116.9</td>
</tr>
<tr>
<td>$A_{1g}$</td>
<td>171.1</td>
<td>172.2</td>
<td>first-order</td>
</tr>
<tr>
<td>$E_{1g}^1$</td>
<td>235.8</td>
<td>236.9</td>
<td>first-order</td>
</tr>
</tbody>
</table>

| $\omega_1$ | 138.2 | 140.7 | 139.0 | 137.4 | 137.4/146.6 | 140.1 | $2\text{TA}(M)$ |
| $\omega_2$ | 183.7 | 183.6 | 181.5 | 178.3 | 178.3/177.2 | 177.8 | $E_{1g}^2(M) - \text{LA}(M)$ |
| $\omega_3$ | 205.5 | 205.3 | 205.9 | 197.9 | 197.9/208.2 | 200.5 | $E_{1g}(M) + \text{TA}(M)$ |
| $\omega_4$ | 263.0 | 263.0/256.6 | 261.0 | $A_{1g}(M) + \text{LA}(M)$ |
| $\omega_5$ | 323.3 | 323.3/317.2 | 321.6 | $2A_{1g}(M)$ |
| $\omega_6$ | 348.3 | 348.3/348.5 | 348.0 | $E_{1g}^2(M) + \text{TA}(M)$ |

the first-order $A_{1g}(\Gamma')$ mode becomes stronger by changing the laser excitation energy from 2.33 eV to 1.96 eV/1.58 eV. We expect that the photoexcited electron may choose either an intravalley scattering process $[A_{1g}(\Gamma')]$ or an intervalley scattering process $[A_{1g}(M)]$, so that $A_{1g}(M)$ competes with $A_{1g}(\Gamma')$, resulting in the alternative Raman intensity of $A_{1g}(\Gamma')$ and $\omega_4$ in the experiment. The $\omega_4$ $[A_{1g}(M)]$ has no VHS of PhDOS compared with all the other $\omega$’s at the $M$ point, as seen from Fig. 4(b), which is relevant to the weak intensity of $\omega_4$, too.

In Fig. 5, we show the normal modes of the phonons at the $M$ point in which the phonon vibration amplitude is visualized in the $2 \times 2$ supercell. As seen from Fig. 4(b), the two groups of optical phonon modes ($E_{2g}$ and ($E_{1g}$ or $A_{1g}$)) are well separated by about 50 cm$^{-1}$. The visualization of these $M$-point phonon modes shows that the longitudinal and transverse optical (LO and TO) $E_{2g}(M)$ modes ($\sim$230–300 cm$^{-1}$) and the LA($M$) and TA($M$) modes ($\sim$60–100 cm$^{-1}$) are bond-stretching modes of the Mo-Te bond. The LO $A_{1g}(M)$ and TO $E_{1g}(M)$ modes ($\sim$120–180 cm$^{-1}$) arise from the relative motion of two Te atoms occupying the same sublattice. The six vibrational modes in Fig. 5 are Raman active, since the volume or the area of the supercell under vibration gives a nonzero deformation potential. Thus all the modes can contribute to the second-order overtone, combination, and difference modes. Furthermore, the vibrational magnitude of the TA modes is no less than that of the LA modes, suggesting that the TA modes may induce as large a deformation potential as the LA modes in the electron-phonon interaction. Usually, the TA modes around the $\Gamma$ point do not contribute to the electron-phonon interaction. Nevertheless, this seems not to be the case at the $M$ point.

So far we have only considered the electron-involved double resonance process for MoTe$_2$ and have not discussed...
the hole-involved double resonance process. As Venezuela et al. [57] pointed out, the enhancement by the double resonance is strong in graphene for one-electron and one-hole scattering, since the energy denominators of the Raman intensity formula become zero for the symmetric \( \pi \) and \( \pi^* \) bands of graphene (triple resonance). In the case of TMDs, however, a similar effect is not expected since the conduction band and the valence band are not symmetric around the Fermi energy [55]. Since we do not have a program to calculate the electron-phonon matrix element by first-principles calculations, we cannot calculate the Raman intensity for each double resonance process [57], which will be a future work.

Finally, we briefly comment on the exciton effect on the double resonance Raman spectra of MoTe\(_2\). Since the excitonic states appear at the direct energy gap at the \( K \) point of the BZ [58,59], we expect that the excitonic energy is around 1.1 eV for MoTe\(_2\), which is much smaller than the three laser energies we used (1.58–2.33 eV). Since we consider only MoTe\(_2\), we can safely neglect the exciton effect at the \( K \) point. However, in the case of other TMDs (MoS\(_2\) [26], WS\(_2\) [27,28,41,60], and WSe\(_2\) [36,37]), energy gaps at the \( K \) point \( [E_g(K)] \) lie in the range of 1.7–1.9 eV, as shown in Table II. The laser energies used in the experiments lie in a range from 1.83 to 2.41 eV. Thus the exciton effect needs to be considered for some laser energies. Nevertheless, when we discuss the \( M \)-point phonon of the other TMDs, we expect that the second-order Raman spectra do not appear, since the optical transition energies at the \( M \) point are larger than 3.0 eV.

It would be interesting if an ultraviolet laser could be used for detecting the \( M \)-point phonon modes for other TMDs, which will be a future work. In the case of MoTe\(_2\), on the other hand, since \( E_g(M) \sim 1.8 \) eV, we expect the double resonance Raman process for conventional laser energies. Although we did not consider the exciton effect at the \( M \) point explicitly as Mak et al. [61] did in graphene, we believe that the present discussion of the double resonance Raman effect should work well because only optical transition matrix elements that are common in the resonant Raman process are enhanced by the exciton effect, and the relative Raman intensity thus obtained does not change much for the \( M \) point [62,63].

### V. CONCLUSION

In summary, we have investigated the second-order Raman spectra of MoTe\(_2\) systems, based on an \textit{ab initio} density functional calculation and compared these results with the available experimental Raman spectroscopy results. The calculated electronic bands, the Van Hove singularities in the density of states, and the absorption spectra indicate a strong optical absorption at the \( M \) point in the BZ of MoTe\(_2\). Using double resonance Raman theory, a combination/difference or overtone mode of phonons at the \( M \) point is assigned to identify the experimental Raman modes of \( \omega_i \) \( (i = 1, 2, \ldots, 6) \). This study facilitates a better understanding of the electron-phonon interaction and the second-order Raman processes in TMD systems.

### ACKNOWLEDGMENTS

M.Y. and K.T. sincerely acknowledge Professor Meiyan Ni and Dr. Katsunori Wakabayashi for critical suggestions on the experiments. T.Y. and Z.D.Z. acknowledge the NSFC under Grant No. 51331006 and the Key Research Program of Chinese Academy of Sciences (Grant No. KGZD-EW-T06) for financial support. R.S. acknowledges MEXT Grants No. 25286005 and No. 25107005. M.S.D. acknowledges NSF-DMR Grant No. 1004147. K.U. and K.T. acknowledge MEXT Grant No. 25107004. Calculations were performed in the HPC cluster at the Institute of Metal Research in Shenyang, China.

---


The phonon modes $E'$, $E''$, and $A_1$ of monolayer and related Raman-active modes of few-layer MoTe$_2$ are labeled as $E_{1g}$, $E_{2g}$, and $A_{1g}$ of bulk MoTe$_2$ for convenience in discussion.


It is worth pointing out that our assignment of the $M$-point phonon for MoTe$_2$ is similar to that for WS$_2$, but in our case is based on a different intervalley process. Berkdemir et al. [41] used a laser line (514.5 nm) which matches the $K$-point gap of WS$_2$ and therefore may trigger an intervalley electron-phonon process between the $K$ and $I$ points, both of which have almost the same band energy. Thus Berkdemir’s model should be used for WS$_2$ and our model for MoTe$_2$.

