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Robust Power Allocation for Energy-Efficient Location-Aware Networks

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Abstract—In wireless location-aware networks, mobile nodes (agents) typically obtain their positions through ranging with respect to nodes with known positions (anchors). Transmit power allocation not only affects network lifetime, throughput, and interference, but also determines localization accuracy. In this paper, we present an optimization framework for robust power allocation in network localization to tackle imperfect knowledge of network parameters. In particular, we formulate power allocation problems to minimize the squared position error bound (SPEB) and the maximum directional position error bound (mDPEB), respectively, for a given power budget. We show that such formulations can be efficiently solved via conic programming. Moreover, we design an efficient power allocation scheme that allows distributed computations among agents. The simulation results show that the proposed schemes significantly outperform uniform power allocation, and the robust schemes outperform their non-robust counterparts when the network parameters are subject to uncertainty.

Index Terms—Localization, wireless networks, resource allocation, semidefinite programming (SDP), second-order conic programming (SOCP), robust optimization.

I. INTRODUCTION

Positional information is of critical importance for future wireless networks, which will support an increasing number of location-based applications and services [1]–[9]. Example applications include cellular positioning, search and rescue work, blue-force tracking, etc., covering civilian life to military operations. In GPS-challenged environments, wireless network localization typically refers to a process that determines the positions of mobile nodes (agents) based on the measurements with respect to mobile/static nodes with known positions (anchors), as illustrated in Fig. 1. With the rapid development of advanced wireless techniques, wireless network localization has attracted numerous research interests in the past decades [10]–[20].

Localization accuracy is a critical performance measure of wireless location-aware networks. In recent work [5], [6], the fundamental limits of wideband localization have been derived in terms of the squared position error bound (SPEB) and directional position error bound (DPEB). It shows that localization accuracy is related to several aspects of design, including network topology, signal waveforms, and transmit power. Power allocation for wireless network localization plays a critical role in reducing localization errors or energy consumption, when the nodes are subject to limited power resources or quality-of-service (QoS) requirements [21]–[23]. Optimal or near-optimal trade-off between localization errors and energy consumption can be obtained by optimization methods, which have played an important role in maximizing communication and networking performance under limited resources [24]–[31]. The authors in [32] formulated several optimization problems for anchor power allocation in wideband localization systems, and derived the optimal solution for single-agent networks. In [33], it exploited the geometrical interpretation of localization information to minimize the maximum DPEB (mDPEB).

In [34], it investigated the localization using MIMO radar systems, and adopted the constraint relaxation and domain decomposition methods to obtain sub-optimal solutions for power allocation. In general, how to optimally allocate the transmit power in location-aware networks still remains as an open problem.

Power allocation schemes should be adapted to the instan-

1The mDPEB characterizes the maximum position error of an agent over all directions.
taneous network conditions, such as network topology and channel qualities, for optimizing the localization performance. Previous work on power allocation in location-aware networks assumes that the network parameters such as nodes’ positions and channel conditions are perfectly known [32]–[34]. However, these parameters are obtained through estimation and hence subject to uncertainty. The power allocation based on imperfect knowledge of network parameters often leads to suboptimal or even infeasible solutions in realistic networks [35]–[37]. Therefore, it is essential to design a robust scheme to combat the uncertainty in network parameters.

In this paper, we present an optimization framework for robust power allocation in network localization to tackle imperfect knowledge of network parameters. Specifically, we treat the fundamental limits of localization accuracy, i.e., SPEB and mDPEB, as the performance metrics. The main contributions are summarized as follows.

- We formulate optimization problems for power allocation to minimize SPEB/mDPEB subject to limited power resources, and prove that these formulations can be transformed into conic programs.\(^2\)
- We propose a robust optimization method for the worst-case SPEB/mDPEB minimization in the presence of parameter uncertainty. The proposed robust formulations retain the same form of conic programs as their non-robust counterparts.
- We develop a distributed algorithm for robust power allocation, which decomposes the original problem into several subproblems enabling parallel computations among all the agents without loss of optimality.

The rest of the paper is organized as follows. In Section II, we describe the system model and introduce the performance metrics. In Section III, we formulate the power allocation problems into conic programs. In Section IV, robust power allocation schemes are proposed to combat the uncertainty in network parameters. In Section V, we further decompose our robust formulation into several subproblems that can be independently solved by each agent. In Section VI, the performance of the proposed schemes is investigated through simulations. Finally, the paper is concluded in Section VII.

**Notations:** We use lowercase and uppercase bold symbols to denote vectors and matrices, respectively; \(\text{det}(A)\) and \(\text{tr}(A)\) denote the determinant and trace of matrix \(A\), respectively; the superscript \((\cdot)^T\) and \(\|\cdot\|\) denote the transpose and Euclidean norm of its argument, respectively; matrices \(A \succeq B\) denotes that \(A - B\) is positive semidefinite. We define the unit vector \(\mathbf{u}(\phi) = [\cos \phi \ \sin \phi]^T\). We use calligraphic symbols, e.g., \(\mathcal{N}\), to denote sets, and \(\mathbb{E}\{\cdot\}\) and \(\Pr\{\cdot\}\) to denote the expectation and probability operators, respectively.

## II. System Model

In this section, we describe the system model, and introduce two performance metrics of location-aware networks.

2Conic programs can be efficiently solved by off-the-shelf optimization tools [27], [38]

### A. Network Settings

Consider a 2-D location-aware network consisting of \(N_a\) agents and \(N_b\) anchors, where the sets of agents and anchor are denoted by \(N_a = \{1, 2, \ldots, N_a\}\) and \(N_b = \{N_a + 1, N_a + 2, \ldots, N_a + N_b\}\), respectively. The 2-D position of node \(k\) is denoted by \(p_k\). The angle and distance between nodes \(k\) and \(j\) are given by \(\phi_{kj}\) and \(d_{kj}\), respectively. The anchors are mobile/static nodes with known positions, and subject to limited power resources. The agents aim to determine their positions based on the radio signals transmitted from the anchors. For instance, agents can obtain the signal metrics such as time-of-arrival (TOA) from the received signals, and then calculate their positions via triangulation [5].

The multipath received waveform at agent \(k\) from anchor \(j\) is modeled as [5]

\[
r_{kj}(t) = \sum_{l=1}^{L_{kj}} \sqrt{P_{kj}} \cdot \alpha_{l(kj)} \cdot s(t - \tau_{l(kj)}) + z_{kj}(t), \quad t \in [0, T_{ob})
\]

where \(x_{kj}\) is the power of the transmit waveform from anchor \(j\) to agent \(k\), \(s(t)\) is a known transmit waveform, \(\alpha_{l(kj)}\) and \(\tau_{l(kj)}\) are the amplitude and delay, respectively, of the \(l\)th path, \(L_{kj}\) is the number of multipath components, \(z_{kj}(t)\) represents additive white Gaussian noise (AWGN) with two-side power spectral density \(N_0/2\), and \([0, T_{ob})\) is the observation interval.

We consider that the measurements between anchors and agents do not interfere each other by using medium access control, and the network is synchronized such that the inter-node distance is estimated using one-way time-of-flight (TOF).\(^3\) Our work can be extended to asynchronous networks where round-trip TOF is employed for distance estimation, and it will be discussed in Section III.

### B. Position Error Bound

The SPEB introduced in [5] is a performance metric that characterizes the localization accuracy, defined as

\[
\mathcal{P}(p_k) \triangleq \text{tr}\{J_e^{-1}(p_k; \{x_{kj}\})\}
\]

where \(J_e(p_k; \{x_{kj}\})\) is the equivalent Fisher information matrix (EFIM) for agent \(k\’s\) position \(p_k\). Using the information inequality [39], we can show that the squared position error is bounded below as

\[
\mathbb{E}\{||\hat{p}_k - p_k||^2\} \geq \mathcal{P}(p_k)
\]

where \(\hat{p}_k\) is an unbiased estimate of the position \(p_k\). The EFIM in (2) can be derived based on the received waveform in (1) as a \(2 \times 2\) matrix [5]

\[
J_e(p_k; \{x_{kj}\}) = \sum_{j \in N_b} \xi_{kj} x_{kj} \mathbf{J}_e(\phi_{kj})
\]

where \(\mathbf{J}_e(\phi_{kj}) = \mathbf{u}(\phi_{kj})\mathbf{u}(\phi_{kj})^T\) is a \(2 \times 2\) matrix, and \(\xi_{kj}\) is a positive coefficient determined by the channel properties.

\(^3\)There are two common ways for inter-node distance estimation based on TOA: one-way TOF (only anchor transmits) or round-trip TOF (both anchor and agent transmit). The former requires anchors and agents to be synchronized for distance estimation.
given by,
\[ \xi_{kj} = \frac{8\pi^2W^2}{c^2}(1 - \chi_{kj})\frac{(\alpha_{kj})^2}{N_0} \] (4)
with \( W \) as the effective bandwidth, \( c \) as the light speed, \( \chi_{kj} \) as path-overlap coefficient characterizing the effect of multi-path propagation for localization, \( N_0 \) as the noise spectrum density.\footnote{The derivation of \( \xi_{kj} \) is given in \cite{5}, and this parameter can be obtained through channel estimation.}

Since the SPEB characterizes the fundamental limit of localization accuracy and is achievable in high SNR regimes, we will use it as a performance metric for location-aware networks, and allocate the transmit power to optimize the system performance by minimizing the SPEB.

C. Directional Decoupling of SPEB

We then introduce the notations of DPEB and mDPEB \footnote{Although the structure of SPEB is derived based on the received waveforms for wideband systems in \cite{5}, it is also observed in other TOA- or RSS-based localization systems, e.g., \cite{16, 40, 41, 42}.}. The EFIM (3) can be written, by eigen decomposition, as
\[ \mathbf{J}_e(\mathbf{p}_k; \{x_{kj}\}) = \mathbf{U}_{\theta_k} \begin{bmatrix} \mu_{1,k} & 0 \\ 0 & \mu_{2,k} \end{bmatrix} \mathbf{U}_{\theta_k}^T \]
where \( \mu_{1,k} \) and \( \mu_{2,k} \) are the ordered eigenvalues of EFIM (\( \mu_{1,k} \geq \mu_{2,k} \)), given by
\[ \mu_{1,k}, \mu_{2,k} = \frac{1}{2} \left( \sum_{j \in N_h} \xi_{kj} x_{kj} \pm \left\| \sum_{j \in N_h} \xi_{kj} x_{kj} \mathbf{u}(2\phi_{kj}) \right\| \right) \]
and \( \mathbf{U}_{\theta_k} \) is a rotation matrix with angle \( \theta_k \), given by
\[ \mathbf{U}_{\theta_k} = \begin{bmatrix} \cos \theta_k & -\sin \theta_k \\ \sin \theta_k & \cos \theta_k \end{bmatrix}. \]

Geometrically, the EFIM for agent \( k \) can be viewed as an information ellipse given by \( \mathbf{z} \in \mathbb{R}^2 : \mathbf{z}^T \mathbf{J}_e^{-1}(\mathbf{p}_k; \{x_{kj}\}) \mathbf{z} = 1 \) (see Fig. 2), where \( 2\sqrt{\mu_{1,k}} \) and \( 2\sqrt{\mu_{2,k}} \) give the major axis and minor axis, respectively.

**Definition 1:** The directional position error bound (DPEB) of agent \( k \) along the direction \( \varphi \) is defined as
\[ \mathcal{P}(\mathbf{p}_k; \varphi) \triangleq \mathbf{u}(\varphi)^T \mathbf{J}_e^{-1}(\mathbf{p}_k, \{x_{kj}\}) \mathbf{u}(\varphi). \]

**Proposition 1:** The mDPEB of agent \( k \) is
\[ \max_{\varphi \in [0, 2\pi)} \{ \mathcal{P}(\mathbf{p}_k; \varphi) \} = \frac{1}{\mu_{2,k}} \] (5)

**Proof:** See Appendix A.

Proposition 1 can also be understood via the information ellipse of EFIM. The information for localization achieves the maximum along the major axis and the minimum along the minor axis. Due to the reciprocal, the SPEB is dominated by the mDPEB, which is the inverse of the smaller eigenvalue of the EFIM. Therefore, in order to improve the localization performance, it is more helpful to maximize the smaller eigenvalue of EFIM, equivalently to minimize the mDPEB that characterizes the maximum position error of an agent over all directions. We will use mDPEB as another performance metric of localization accuracy.

III. OPTIMAL POWER ALLOCATION VIA CONIC PROGRAMMING

In this section, we formulate the power allocation problem using SPEB and mDPEB as the objective functions, respectively. We show that the SPEB minimization is a semidefinite program (SDP) and the mDPEB minimization is a second-order conic program (SOCP).

A. Problem Formulation Based on SPEB

We first consider the problem of optimal power allocation that minimizes the total SPEB while the network is subject to a budget of power consumption. The problem can be formulated as \footnote{The structure of the problem retains with additional linear constraints, such as the maximum transmit power from anchor \( j \) to agent \( k \), and the maximum total transmit power from anchor \( j \). See Remark 2 for more discussion.}

\[ \mathcal{P}_1 : \min_{\{x_{kj}\}} \sum_{k \in N_i} \text{tr}\{\mathbf{J}_e^{-1}(\mathbf{p}_k; \{x_{kj}\})\} \quad (6) \]
\[ \text{s.t.} \quad \sum_{k \in N_i} \sum_{j \in N_h} x_{kj} \leq P_{\text{tot}} \quad (7) \]
\[ x_{kj} \geq 0, \quad \forall k \in N_{a}, \quad \forall j \in N_{b} \quad (8) \]
where (7) gives the total transmit power budget \( P_{\text{tot}} \) for all the anchors. We first show the convexity of the above problem in the following proposition.

**Proposition 2:** The problem \( \mathcal{P}_1 \) is convex in \( x_{kj} \).

**Proof:** See Appendix B.

Since \( \mathcal{P}_1 \) is a convex problem, the optimal solution can be achieved by the standard convex optimization algorithms, e.g., interior point method. We next show that such problem can be converted to a SDP problem, which is a more favorable formulation since many fast real-time optimization solvers are available for SDP \cite{43, 44}.

To obtain an equivalent formulation to \( \mathcal{P}_1 \), we replace the EFIMs in (6) with auxiliary matrices \( \mathbf{M}_{k} \), and add another constraint
\[ \mathbf{M}_{k} \succeq \mathbf{J}_e^{-1}(\mathbf{p}_k; \{x_{kj}\}) \]
Since \( \mathbf{J}_e(\mathbf{p}_k) \) is a positive semidefinite matrix, due to the property of Schur complement, the above inequality is equivalent to
\[ \begin{bmatrix} \mathbf{M}_{k} & \mathbf{I} \\ \mathbf{I} & \mathbf{J}_e(\mathbf{p}_k; \{x_{kj}\}) \end{bmatrix} \succeq 0. \]

Fig. 2: Geometrical interpretation of the EFIM for agent \( k \).
Then, we can obtain a SDP formulation $\mathcal{P}_{1\text{SDP}}$ equivalent to $\mathcal{P}_1$:

$$
\mathcal{P}_{1\text{SDP}}: \min_{\{x_{kj}\}, \{\mu_{2,k}\}} \sum_{k \in n}\sum_{j \in j} \text{tr}\{M_k\}
\text{s.t.}\ [M_k \quad \mathbf{I}] [x_{kj} \quad \mathbf{I}] \succeq 0, \forall k \in n_s,
(7) - (8).
$$

Hence, the optimal solution of $\mathcal{P}_1$ can be efficiently obtained by solving the SDP formulation $\mathcal{P}_{1\text{SDP}}$.

B. Problem Formulation Based on mDPEB

We now consider the minimization of total mDPEB as our objective. The problem can be formulated as

$$\mathcal{P}_2: \min_{\{x_{kj}\}, \{r_k\}} \sum_{k \in n_s} \frac{1}{\mu_{2,k}}
\text{s.t.}\ (7) - (8)
$$

which can be equivalently converted to

$$\mathcal{P}_{2\text{SOCP}}: \min_{\{x_{kj}, r_k\}} \sum_{k \in n_s} \frac{1}{\mu_{2,k}} \sum_{j \in n_s} \xi_{kj} x_{kj} - r_k
\text{s.t.}\ r_k \geq \left[ \sum_{j \in n_s} \xi_{kj} x_{kj} \right] / \left[ 2(\phi_{kj}) \right], \forall k \in n_a (9)
\text{s.t.}\ (7) - (8).
$$

The constraints (9) define $n_a$ second-order cones given by

$$Q_k = \{(r_k, z_k) \in \mathbb{R} \times \mathbb{R}^2 : r_k \geq \|z_k\|\}, \forall k \in n_a
$$

where $z_k = \sum_{j \in n_s} \xi_{kj} x_{kj} \mathbf{u}(\phi_{kj})$. Moreover, the objective is convex in $\{x_{kj}, r_k\}$, since the denominator is a positive linear combination of $\{x_{kj}, r_k\}$, and the reciprocal is a convex and decreasing function which preserves convexity [45]. Thus, we obtain a nonlinear SOCP problem which is convex in $x_{kj}$.

Remark 1: We consider a general model where each anchor can use different transmit power, and our work can be applied to the anchor broadcasting scenario by simply adding constraint $x_{kj} = x_j, \forall k \in n_a$.

Remark 2: Additional linear constraints on transmit power can be imposed depending on the realistic requirements of location-aware networks. For example, we can consider $p_{\text{min}} \leq x_{kj} \leq p_{\text{max}}$ where $p_{\text{min}}$ and $p_{\text{max}}$ are the lower and upper limit of the transmit power from anchor $j$ to agent $k$, respectively; or $\sum_{k \in n_a} x_{kj} \leq p_{\text{tot}}^j$ where $p_{\text{tot}}^j$ is the upper limit of the total transmit power from anchor $j$. Due to the linearity of these constraints, the convexity of the problem is retained, and the optimal solution can be obtained via conic programming.

Remark 3: For the asynchronous networks where round-trip TOF is employed for distance estimation, we need to allocate the transmit power for both anchors and agents. Let $x_{kj}'$ denote the power of the transmit waveform from agent $k$ to anchor $j$. In addition to the total anchor power constraint in (7), we also impose a total power constraint on agents, i.e.,

$$\sum_{k \in n_a} \sum_{j \in n_n} x_{kj}' \leq P_{\text{tot}}^k
$$

where

$$x_{kj}' \geq 0, \forall k \in n_a, \forall j \in n_n.
$$

It can be shown that the EFIM of agent $k$ is given by

$$\mathbf{J}_e(p_k; \{x_{kj}\}) = \sum_{j \in n_s} \xi_{kj} g(x_{kj}, x_{kj}') J_e(\phi_{kj})
$$

where the equivalent power $g(x_{kj}, x_{kj}') = 4(x_{kj}^{-1} + x_{kj}'^{-1})^{-1}$. To derive the maximum total equivalent power, we consider the following problem

$$\max_{\{x_{kj}, x_{kj}'\}} \sum_{k \in n_a} \sum_{j \in n_n} g(x_{kj}, x_{kj}')
\text{s.t.}\ (7) - (8)
(10) - (11).
$$

Using the Karush-Kuhn-Tucker conditions [46], it can be proved that the optimal value is reached as a constant $g(P_{\text{tot}}, P_{\text{tot}}^a)$ if and only if

$$x_{kj}' = \frac{P_{\text{tot}}^k}{P_{\text{tot}}^a} x_{kj}.
$$

Hence, in order to achieve the maximum total equivalent power, the power allocated on anchors and agents should be proportional and consequently, the EFIM for asynchronous network is

$$\mathbf{J}_e(p_k; \{x_{kj}\}) = \sum_{j \in n_s} \xi_{kj} 4P_{\text{tot}}^a / P_{\text{tot}}^a x_{kj} J_e(\phi_{kj})
$$

which is with the same structure as the EFIM of synchronous network in (3). Therefore, the power allocation on both anchors and agents in asynchronous networks can be equivalently converted into anchor power allocation in synchronous networks.

C. Formulations with QoS Guarantee

We next briefly show that the proposed framework also applies to another two types of problem formulations based on different QoS requirements.

1) Energy-efficient Formulation: The objective is to minimize the total transmit power subject to the requirements for agents’ SPEBs, i.e.,

$$\min_{\{x_{kj}\}} \sum_{k \in n_a} \sum_{j \in n_n} x_{kj}
\text{s.t.}\ \text{tr}\{\mathbf{J}_e^{-1}(p_k; \{x_{kj}\})\} \leq \gamma, \forall k \in n_a
\text{s.t.}\ (8)
(13)
$$

Similarly, a formulation for the mDPEB case can be obtained by replacing (13) with

$$\frac{1}{\mu_{2,k}} \leq \gamma, \forall k \in n_a.
\text{(14)}$$
2) Min-max SPEB Formulation: The objective is to minimize the maximum SPEB among all the agents, i.e.,
\[
\min_{\{x_{kj}\}, \gamma} \max_k \left\{ J^{-1}_e(p_k; x_{kj}) \right\} \\
\text{s.t.} \quad (7) - (8).
\]
It can be equivalently transformed into
\[
\min_{\{x_{kj}\}, \gamma} \gamma \\
\text{s.t.} \quad \sum_k \sum_{\epsilon_k \in S_{kj}^\epsilon} \sum_{\delta_k \in S_{kj}^\delta} \left( J^{-1}_e(p_k; x_{kj}) + \epsilon_k + \delta_k \right) \leq \gamma, \quad \forall k \in N_k
\]
which turns out to be with the same structure as the energy-efficient formulation. Similarly, a min-max formulation for the mDPEB case can be obtained by replacing the SPEB with the mDPEB in the constraint.

Note that since the above formulations with QoS guarantee have the same structure as \(\mathcal{P}_1\) or \(\mathcal{P}_2\), which can be solved efficiently by conic programming, we will focus on \(\mathcal{P}_1\) and \(\mathcal{P}_2\) in the following.

To obtain the optimal solutions of \(\mathcal{P}_1\) and \(\mathcal{P}_2\), it requires the network parameters, i.e., the channel parameter \(\xi_{kj}\) and the angle \(\phi_{kj}\). However, \(\xi_{kj}\)’s and \(\phi_{kj}\)’s are usually not perfectly known in realistic networks, and only estimated values are available. When \(\xi_{kj}\)’s and \(\phi_{kj}\)’s are subject to uncertainty, the formulation \(\mathcal{P}_1\) or \(\mathcal{P}_2\) may fail to provide reliable solutions, since the actual SPEB/mDPEB is not necessarily minimized. Therefore, it is essential to design a power allocation scheme which is robust to the uncertainty in network parameters.

IV. ROBUST POWER ALLOCATION UNDER IMPERFECT KNOWLEDGE OF NETWORK PARAMETERS

In this section, we consider the location-aware networks with imperfect knowledge of network parameters, and propose robust optimization methods to minimize the worst-case SPEB/mDPEB under parameter uncertainty.

A. Robust Counterpart of SPEB Minimization

In realistic location-aware networks, the network parameters, i.e., \(\xi_{kj}\) and \(\phi_{kj}\), can be obtained through channel estimation or inferred based on the prior information of agents’ positions, and hence are both subject to uncertainty. We adopt robust optimization methodology, which is developed in recent years to handle the optimization problems with data uncertainty [36]. Typically, the data defining the optimization problem is assumed to lie in a certain bounded set, referred to as uncertainty set. Here we consider the actual channel parameters and angles lie in linear uncertainty sets, i.e.,
\[
\begin{align*}
\xi_{kj} \in S_{kj}^\xi &\triangleq \left[ \xi_{kj} - \epsilon_{kj}, \xi_{kj} + \epsilon_{kj} \right] \\
\phi_{kj} \in S_{kj}^\phi &\triangleq \left[ \phi_{kj} - \phi_{kj}, \phi_{kj} + \phi_{kj} \right]
\end{align*}
\]

where \(\hat{\xi}_{kj}\) and \(\hat{\phi}_{kj}\) denote channel parameter and angle with uncertainty, respectively, and \(\epsilon_{kj}\) and \(\phi_{kj}\) are both small positive numbers denoting the maximum uncertainty in the channel parameter and angle, respectively.\(^9\)

To deal with the network parameter uncertainty, we adopt robust optimization techniques to guarantee the worst-case performance. Instead of using the estimated values, we consider minimizing the largest SPEB over the possible set of actual network parameters, i.e.,
\[
\mathcal{P}_{\mathcal{R}_0}: \min_{\{x_{kj}\}, \gamma} \max_k \left\{ J^{-1}_e(p_k; x_{kj}) \right\} \\
\text{s.t.} \quad (7) - (8).
\]

Since \(\sum_k \sum_{\epsilon_k \in S_{kj}^\epsilon} \sum_{\delta_k \in S_{kj}^\delta} \left( J^{-1}_e(p_k; x_{kj}) + \epsilon_k + \delta_k \right) \leq \gamma, \quad \forall k \in N_k\)

on the other hand, however, the maximization over \(\phi_{kj}\) is not trivial, because
\[
\begin{align*}
\hat{\xi}_{kj} &\triangleq \arg \max_{\xi_{kj} \in S_{kj}^\xi} \sum_k \sum_{\epsilon_k \in S_{kj}^\epsilon} \sum_{\delta_k \in S_{kj}^\delta} \left( J^{-1}_e(p_k; x_{kj}) + \epsilon_k + \delta_k \right) \\
\hat{\xi}_{kj} &\triangleq \arg \max_{\xi_{kj} \in S_{kj}^\xi} \sum_k \sum_{\epsilon_k \in S_{kj}^\epsilon} \sum_{\delta_k \in S_{kj}^\delta} \left( J^{-1}_e(p_k; x_{kj}) + \epsilon_k + \delta_k \right)
\end{align*}
\]

On the right-hand side of (15) is not a convex problem. Hence, it is difficult to obtain a close-form solution of \(\hat{\phi}_{kj}\) since it depends on \(x_{kj}\).

We next consider a relaxation for the robust optimization with respect to \(\phi_{kj}\) and introduce a new matrix
\[
Q_i(\hat{\phi}_{kj}, \hat{\delta}_{kj}) = J_i(\hat{\phi}_{kj}) - \hat{\delta}_{kj} \cdot I
\]

9If uncertainty exists in anchor positions, it can be equivalently converted into the uncertainty in channel qualities [6].
by omitting the variable $\delta_{kj}$ in (16) for simplicity. Then, we replace the matrix $J_r(\phi_{kj})$ with $Q_r(\hat{\phi}_{kj})$ in the previous formulation, and propose a robust counterpart of $\mathcal{P}_1$ given by

$$
\mathcal{P}_{R-1}: \min_{\{x_{kj}\}} \sum_{k \in N_a} \text{tr}\left\{ \left( \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj}) \right)^{-1} \right\}
$$

s.t. \hspace{1cm} \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj}) \geq 0, \forall k \in N_a \hspace{1cm} (18)

(7) – (8).

Again by the property of Schur complement as in $\mathcal{P}_{SDP}^{1}$, the problem $\mathcal{P}_{R-1}$ is equivalent to a SDP formulation, given by

$$
\mathcal{P}_{SDP}^{R-1}: \min_{\{x_{kj}\}, M_k} \sum_{k \in N_a} \text{tr}\{M_k\}
$$

s.t. \hspace{1cm} \left[ M_k \hspace{1cm} \tilde{I} \right] \left[ \tilde{I} \min_{\{x_{kj}\}} \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj}) \right] \geq 0, \forall k \in N_a \hspace{1cm} (19)

(7) – (8).

Remark 4: The formulation with QoS guarantee proposed in Section III-C can also be extended to its robust formulation using the above method. By such, the SPEB of each agent is always guaranteed to satisfy its position error requirement. However, if using the non-robust formulation, the requirements for agents’ SPEBs, e.g., (13) or (14), can easily be violated due to imperfect knowledge of network parameters.

Note that from Proposition 3, the new formulation $\mathcal{P}_{R-1}$ is a valid relaxation for $\mathcal{P}_{R-0}$ when the condition (18) holds. Since $Q_r(\hat{\phi}_{kj})$ is not positive definite due to det $(Q_r(\hat{\phi}_{kj})) = \sin \epsilon^\phi_k \sin \epsilon^\phi_k - 1 \leq 0$, such a condition does not necessarily hold for all power allocation $\{x_{kj}\}$. However, we will show that it holds for the optimal power allocation of $\mathcal{P}_{R-0}$ with high probability (w.h.p.) when the number of anchors is large or the uncertainty in angle is small.

Before giving the proposition, we introduce an equivalent expression for the channel parameter $\xi_{kj}$ in (4) as $\xi_{kj} = \zeta_{kj}/d_{kj}^\beta$, where $\zeta_{kj}$ is a positive coefficient characterizing shadowing effect and small-scale fading process, and $\beta$ is the amplitude loss exponent.\textsuperscript{10}

**Proposition 4:** Consider a network where all the nodes are uniformly located in a $R \times R$ square region, the minimum distance between two nodes is $r_0$, and the coefficient $\zeta_{kj}$ has a support on $[\zeta_{min}, \zeta_{max}]$ where $0 < \zeta_{min} \leq \zeta_{max}$. Let $\{x_{kj}\}$ be the optimal solution of $\mathcal{P}_{R-0}$, and $\delta = \sin \epsilon^\phi_k$ where $\epsilon^\phi_k = \max\{\epsilon^\phi_k\}$, then

(a) when $N_b \to \infty$ and $\delta \leq \delta_{max}$, where $\delta_{max}$ is the smallest positive root of equation $4d^4 - 4d^2 - 2\zeta_{max}/\zeta_{min} + \delta_0 = 0$, we have

$$
\Pr\left\{ \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj}) \geq 0 \right\} = 1 - O(\exp(-\eta R_N)), \hspace{1cm} \forall k \in N_a
$$

(b) when $\epsilon^\phi_k \to 0$, we have

$$
\Pr\left\{ \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj}) \geq 0 \right\} = 1 - O((\epsilon^\phi_k)^{N_b/2}), \hspace{1cm} \forall k \in N_a.
$$

**Proof:** See Appendix D. \hfill $\Box$

Remark 5: Proposition 4 implies that the condition (18) holds w.h.p. at the rate indicated by the $O$ notation, where $O(f(n))$ means that the function value is on the order of $f(n)$ [47].

**Remark 6:** Note that Proposition 4 holds for $\{x_{kj}\}$, which implies that the optimal solution of the original robust formulation $\mathcal{P}_{R-0}$ is included in the feasible set of the proposed formulation $\mathcal{P}_{R-1}$ (or $\mathcal{P}_{SDP}^{R-1}$) w.h.p.

B. Robust Counterpart of mDPEB Minimization

We investigate the robust power allocation based on mDPEB formulation $\mathcal{P}_2$. To circumvent the intractable maximization in (15), we consider the robust SPEB formulation $\mathcal{P}_{R-1}$. Specifically, the objective of $\mathcal{P}_{R-1}$ can be written as

$$
\text{tr}\left\{ \left( \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj}) \right)^{-1} \right\} = \frac{1}{\mu_{1,k}} + \frac{1}{\mu_{2,k}} \hspace{1cm} (20)
$$

where $\mu_{1,k}$ and $\mu_{2,k}$ are the two eigenvalues of the matrix $\sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} Q_r(\hat{\phi}_{kj})$, given by

$$
\tilde{\mu}_{1,k}, \tilde{\mu}_{2,k} = \frac{1}{2} \left( \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} (1 - 2 \sin \epsilon^\phi_k) \right) \pm \left\| \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} \mathbf{u}(\hat{\phi}_{kj}) \right\| \hspace{1cm} (21)
$$

Geometrically, $\tilde{\mu}_{1,k}$ and $\tilde{\mu}_{2,k}$ are similar to the DPEB’s in two orthogonal directions. Using Proposition 4, we can show that $\tilde{\mu}_{2,k} \geq 0$ w.h.p. when $N_b$ is large or $\epsilon^\phi_k$ is small. Since $\tilde{\mu}_{1,k} \geq \tilde{\mu}_{2,k}$, the smaller eigenvalue $\tilde{\mu}_{2,k}$ dominates the function in (20). Hence, we formulate a robust counterpart of $\mathcal{P}_2$ based on $\tilde{\mu}_{2,k}$, given by

$$
\mathcal{P}_{R-2}: \min_{\{x_{kj}\}} \sum_{k \in N_a} \tilde{\mu}_{2,k}
$$

s.t. \hspace{1cm} $\tilde{\mu}_{2,k} \geq 0$, $\forall k \in N_a \hspace{1cm} (22)$

(7) – (8).

Given that $\tilde{\mu}_{2,k} \geq 0$, the problem $\mathcal{P}_{R-2}$ is equivalent to the following SOCP problem:

$$
\mathcal{P}_{SOCP}^{R-2}: \min_{\{x_{kj}, r_k\}} \sum_{k \in N_a} \left\{ \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} (1 - 2 \sin \epsilon^\phi_k) \right\} \hspace{1cm} (23)
$$

s.t. \hspace{1cm} $r_k \geq \left\| \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} \mathbf{u}(\hat{\phi}_{kj}) \right\|$, $\forall k \in N_a \hspace{1cm} (24)$

$$
\left\| r_k \right\| \leq \sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj} (1 - 2 \sin \epsilon^\phi_k), \forall k \in N_a \hspace{1cm} (7) – (8).
$$

\textsuperscript{10}We introduce the path loss model here to facilitate the proof of the Proposition 4. However, the robust power allocation schemes do not require $\beta$, since the channel parameter $\xi_{kj}$ can be obtained directly through channel estimation.
Note that the uncertainty in angle $\varepsilon^o_{kj}$ only exists in the objective, and does not affect the second-order conic constraint (24). Hence, the problem $\mathcal{P}_{R-2}^{\text{SOCP}}$ retains the same structure of $\mathcal{P}_2^{\text{SOCP}}$, and its optimal solution can be efficiently obtained.

V. EFFICIENT ROBUST ALGORITHM USING DISTRIBUTED COMPUTATIONS

In this section, we designed a distributed robust algorithm for both SPEB and mDPEB minimization, which decomposes the original formulation into two-stage optimization problems and enables parallel computations among all the agents. The proposed algorithms achieve the global optimal solution with improved computational efficiency.

A. Algorithm for SPEB Minimization

Despite the convexity of the robust SDP formulation $\mathcal{P}_{R-1}^{\text{SDP}}$, there are multiple positive semidefinite constraints imposed for multiple agents, and the computational complexity depends on the number of SDP constraints. To efficiently obtain the power allocation decision for multi-agent networks, we design a distributed implementation for $\mathcal{P}_{R-1}^{\text{SDP}}$, which can be solved using parallel computations among the agents.

Specifically, we let $x_{kj} = \rho_{kj} x_k$ where $x_k$ is the total power assigned for locating agent $k$, and $\rho_{kj} \in [0, 1]$ is a fractional number denoting the percentage of $x_k$ allocated to anchor $j$. By introducing the two variables $\rho_{kj}$ and $x_k$, the robust formulation for power allocation can be written as

$$\min_{\{\rho_{kj}, x_k\}} \sum_{k \in N_a} \frac{1}{x_k} \text{tr} \left( \left( \sum_{j \in N_b} \tilde{\xi}_{kj} \rho_{kj} Q_j(\hat{\phi}_{kj}) \right)^{-1} \right)$$

s.t. \hspace{1em} (25)

$$\sum_{j \in N_b} \rho_{kj} \leq 1$$

$$\rho_{kj} \geq 0, \forall k \in N_a, \forall j \in N_b$$

$$\sum_{k \in N_a} x_k \leq P_{\text{tot}}$$

$$x_k \geq 0, \forall k \in N_a.$$  \hspace{1em} (28)

Since the constraints on $\rho_{kj}$ and $x_k$ are separable, and $x_k$ and $\rho_{kj}$ are only related to the SPEB of agent $k$, we can decompose the above problem into two stages. In Stage I, given the total power budget $x_k$ for agent $k$, we consider the optimal allocation of $x_k$ among all the anchors, i.e.,

$$\mathcal{P}_{R-1,k}^{(I)} : \min_{\{\rho_{kj}, M_k\}} \frac{\text{tr}[M_k]}{x_k}$$

s.t. \hspace{1em} (25) – (26).

The optimal solution of $\mathcal{P}_{R-1,k}^{(I)}$ is denoted by $\rho_{kj}^*$, and it is independent of the total power for agent $k$ since $x_k$ only appears as a scalar in the objective and can be removed. Since the problem $\mathcal{P}_{R-1,k}^{(I)}$ is formulated for agent $k$, there are totally $N_a$ problems to be solved in Stage I.

In Stage II, we allocate the total $x_k$ for localizing agent $k$. The objective is the total SPEB of the agents, where the parameter $\rho_{kj}^*$'s are from Stage I $\mathcal{P}_{R-1,k}^{(I)}$. In particular, we let $T_k = \text{tr} \left( \sum_{j \in N_b} \tilde{\xi}_{kj} \rho_{kj}^* Q_j(\hat{\phi}_{kj}) \right)^{-1}$ and formulate the problem as:

$$\mathcal{P}_{R-1,k}^{(II)} : \min_{\{x_k\}} \sum_{k \in N_a} T_k$$

s.t. \hspace{1em} (27) – (28).

The problem $\mathcal{P}_{R-1,k}^{(II)}$ is convex in $x_k$, and the optimal solution is given in a closed form as follows.

**Proposition 5:** Given that $\rho_{kj}^*$ is the optimal solution of $\mathcal{P}_{R-1,k}^{(I)}$, the optimal solution of $\mathcal{P}_{R-1,k}^{(II)}$ is given by

$$x_k^* = \frac{P_{\text{tot}} \sqrt{T_k}}{\sum_{k \in N_a} \sqrt{T_k}}.$$  \hspace{1em} (29)

**Proof:** See Appendix E. $\blacksquare$

The optimal power allocation for the location-aware network is

$$x_k^* = \rho_{kj}^* x_k^*$$  \hspace{1em} (30)

where $x_k^*$ is given in (29). The detailed algorithm is described in the Algorithm 1.

**Algorithm 1** Robust power allocation algorithm for multi-agent networks

**Require:** the angle $\hat{\phi}_{kj}$ and the distance $\hat{d}_{kj}$ between anchor $j$ ($j \in N_b$) and agent $k$ ($k \in N_a$)

1. Set $x_k \leftarrow 1, \forall k \in N_a$
2. Solve the Stage I problems $\mathcal{P}_{R-1,k}^{(I)}$ which gives the optimal solution $\rho_{kj}^*$
3. Set $\rho_{kj} \leftarrow \rho_{kj}^*, \forall k \in N_a, \forall j \in N_b$
4. Solve the Stage II problem $\mathcal{P}_{R-1,k}^{(II)}$ by using (29) to compute the optimal solution $x_k^*$
5. Set $x_{kj}^* \leftarrow \rho_{kj}^* x_k^*, \forall k \in N_a, \forall j \in N_b$

**Remark 7:** Since each Stage I problem $\mathcal{P}_{R-1,k}^{(I)}$ in Algorithm 1 is with a single SDP constraint, its complexity is much lower than the original problem $\mathcal{P}_{R-1}^{\text{SDP}}$ which contains $N_a$ SDP constraints. Moreover, the $N_a$ Stage I problems $\mathcal{P}_{R-1,k}^{(I)}$ can be separately solved by the $N_a$ agents, since each agent itself does not require any information from other agents. Thus, the computation efficiency can be improved by $N_a$ times using the parallel computations among the agents.

**Remark 8:** The proposed distributed algorithm can also be applied to the robust power allocation with individual power constraint, e.g., $\sum_{k \in N_a} x_k \leq P_{\text{tot}}$. In particular, we replace such constraint with $\sum_{k \in N_a} \rho_{kj}^* x_k \leq P_{\text{tot}}$ in the Stage II formulation $\mathcal{P}_{R-1,k}^{(II)}$, while the Stage I formulation $\mathcal{P}_{R-1,k}^{(I)}$ remains the same. In such case, the close-form solution in (30) is not available, however, the optimal solution of the Stage II problem can still be efficiently obtained since the problem is convex. Consequently, we can obtain a sub-optimal solution for the overall problem.

B. Algorithm for mDPEB Minimization

A similar decomposition method can be applied to the mDPEB minimization $\mathcal{P}_{R-2}$, i.e., by introducing two variables
respectively. The optimal power allocation is the product of the optimal solutions of the two-stage problems, given by (30). The algorithm for mDPEB minimization is similar to that of the optimal solutions of the two-stage problems, given by (30).

\[ \sum_{k \in N_n} \xi_{kj} \rho_{kj} u(2 \phi_{kj}) \]

Then, the two-stage formulations are given by

\[ \mathcal{P}_{R_2}^{(I)} : \max_{\mu_{2,k}} \mu_{2,k} \quad \text{s.t.} \quad \mu_{2,k} \geq 0 \]

\[ (25) - (26) \]

and

\[ \mathcal{P}_{R_2}^{(II)} : \min_{\sigma_k} \sum_{k \in N_n} \frac{1}{\mu_{2,k}} \quad \text{s.t.} \quad (27) - (28) \]

respectively. The optimal power allocation is the product of the optimal solutions of the two-stage problems, given by (30). The algorithm for mDPEB minimization is similar to that of Algorithm 1, and hence, we omit the details here.

VI. SIMULATION RESULTS

In this section, we investigate the localization performance by the proposed power allocation schemes. The total power for localization is normalized to \( P_{\text{tot}} = 1 \), and the channel parameter is given by \( \xi_{kj} = 10 \mu^2 / d_{kj}^2 \). The proposed optimization of power allocation, i.e., SDP and SOCP, are solved by the standard optimization solver CVX [49].

\[ \xi_{kj} = 10 \mu^2 / d_{kj}^2 \]

We choose the free-space propagation model where the path loss exponent is 2 [48].
and outperforms uniform allocation by 70%. We observe that the actual SPEB of robust mDPEB minimization forms the non-robust mDPEB minimization (\(P_3\)) by 35% and set the normalized uncertainty set size \(\varepsilon\) to 2. Therefore, \(\mathcal{P}_{\text{SOCP}}^{R-1}\) outperforms \(\mathcal{P}_{\text{SDP}}^{R-1}\) when the uncertainty in network parameters is not negligible (e.g., \(\varepsilon = 0.2\)).

In Fig. 8, we investigate the actual SPEB with respect to the normalized uncertainty set size \(\varepsilon\). We consider a single-agent network with ten anchors deployed on a circle (similar to Fig. 5). As we observe, the actual SPEB of non-robust schemes quickly increases as the normalized uncertainty set size goes large. When the normalized uncertainty set size is larger than 0.22 and 0.27, respectively, the non-robust SPEB minimization and non-robust mDPEB minimization even perform worse than the uniform allocation, while the robust schemes always achieves better SPEB than all the other schemes. Moreover, the robust mDPEB minimization outperforms the non-robust mDPEB minimization and robust SPEB minimization by 30% and 23%, respectively, when \(\varepsilon = 0.15\). Both Figs. 7 and 8 have demonstrated the advantage of the proposed robust power allocation schemes, especially the mDPEB minimization, in the practical location-aware networks with imperfect knowledge of network parameters.

VII. Conclusion

In this paper, we presented an optimization framework for robust power allocation in network localization based on the performance metrics SPEB and mDPEB. We first showed that the optimal power allocation with perfect network parameters can be efficiently obtained via conic programming, and then proposed robust power allocation schemes to combat uncertainty in network parameters for practical systems. Moreover, we designed an efficient algorithm for robust power allocation.
that allows distributed computations among agents. The simulation results demonstrated that the robust power allocation remarkably outperforms the non-robust power allocation and uniform allocation. Furthermore, we showed that, compared with the SPEB minimization, the mDPEB minimization is more robust to network parameter uncertainty for power allocation.

**APPENDIX A**

**PROOF OF PROPOSITION 1**

The maximization on DPEB in (5) follows that:

$$\max_{\varphi \in [0, 2\pi]} \{ \mathcal{P}(\mathbf{p}_k; \varphi) \} = \max_{\varphi \in [0, 2\pi]} \mathbf{u}(\varphi)^T \mathbf{J}_p^{-1}(\mathbf{p}_k; \{x_{kj}\}) \mathbf{u}(\varphi)$$

$$= \max_{\varphi \in [0, 2\pi]} \mathbf{u}(\varphi)^T \mathbf{U}_{\theta_k}^{-1} \mathbf{J}_p^{-1}(\mathbf{p}_k; \{x_{kj}\}) \mathbf{U}_{\theta_k} \mathbf{u}(\varphi)$$

$$= \max_{\varphi \in [0, 2\pi]} \mathbf{u}(\varphi)^T \mathbf{J}_p^{-1}(\mathbf{p}_k; \{x_{kj}\}) \mathbf{u}(\varphi)$$  \(\text{(31)}\)

where the last equality is due to the fact that the product of a unit vector and a rotation matrix \(\mathbf{U}_{\theta_k}\) is still a unit vector. Now, let \(\varphi' = \theta_k\) in (31), then we have

$$\max_{\varphi \in [0, 2\pi]} \{ \mathcal{P}(\mathbf{p}_k; \varphi) \} = \max_{\theta_k} \left\{ \mu_{1,k} \cos^2 \theta_k + \mu_{2,k} \sin^2 \theta_k \right\}$$

$$= \mu_{2,k}$$

where the last equation is due to \(\mu_{1,k} \geq \mu_{2,k}\).

**APPENDIX B**

**PROOF OF PROPOSITION 2**

Since (7)–(8) are all linear constraints, we only need to show the objective in (6), i.e., the SPEB, is a convex function in \(x_{kj}\). We write the transmit power of agent \(k\) as a vector \(x_k = [x_{k1} \ x_{k2} \ \cdots \ x_{kN_b}]^T\), and the SPEB is a function of \(x_k\), given by

$$f(x_k) = \max_{\varphi \in [0, 2\pi]} \mathbf{u}(\varphi)^T \mathbf{J}_p^{-1}(\mathbf{p}_k; \{x_{kj}\}) \mathbf{u}(\varphi)$$

We choose two arbitrary \(x_k, x'_k \in \mathbb{R}^{N_b}\). Given any \(\alpha \in [0, 1]\), we have

$$f(\alpha x_k + (1 - \alpha)x'_k) \geq \alpha f(x_k) + (1 - \alpha)f(x'_k).$$

The inequality (32) holds since the function \(\text{tr} \{ \mathbf{X}^{-1} \}\) is convex in \(\mathbf{X} > 0\) [45]. If the matrix \(\mathbf{X}\) is singular, the inequality (32) still holds. Since \(\xi_{kj}\) is a positive scaler, \(f(x_k)\) is convex in \(x_k\).

**APPENDIX C**

**PROOF OF PROPOSITION 3**

Let \(\phi_{kj} = \phi_{kj} + \hat{\phi}_{kj}\) and \(\phi_{kj} = \phi_{kj} - \hat{\phi}_{kj}\), we have

$$\mathbf{J}_i(\phi_{kj}) - \mathbf{Q}_i(\hat{\phi}_{kj}, \delta_{kj}) = \begin{bmatrix} \delta_{kj} - \sin \phi_{kj}^+ \sin \phi_{kj}^- & \cos \phi_{kj}^+ \sin \phi_{kj}^- \\ \cos \phi_{kj}^+ \sin \phi_{kj}^- & \delta_{kj} + \sin \phi_{kj}^+ \sin \phi_{kj}^- \end{bmatrix}. $$

We can show that \(\mathbf{J}_i(\phi_{kj}) - \mathbf{Q}_i(\hat{\phi}_{kj}, \delta_{kj})\) is positive semidefinite if

$$\begin{cases} \delta_{kj} \geq \sin \phi_{kj}^+ \sin \phi_{kj}^-; \\ \delta_{kj} \geq |\sin \phi_{kj}|. \end{cases}$$
Note that if the agent is at the corner or on the boundary of the square area, we can rotate the angles accordingly to find such a region.

It can be shown that there exists at least one such pair of anchors with probability $1 + (1 - 2p_0)N_b - 2(1 - p_0)N_b$, where $p_0 = (\varphi^2 - 1)\varphi/2R^2$. Since the probability goes to 1 exponentially with $N_b$, such a pair of anchors can be found w.h.p.

Consider a power allocation scheme $\{\tilde{P}_{ki} = \tilde{P}_{ki'} \}$, and we show this scheme satisfies the condition (33) for a sufficiently small $\delta$. Based on the definition of the optimal power allocation, we have

$$\text{tr}\left\{\left(\sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj}^* J_r(\tilde{\phi}_{kj})^{-1}\right)^{-1}\right\} \leq \max_{\{\phi_{kj}\}} \text{tr}\left\{\left(\sum_{j \in N_b} \tilde{\xi}_{kj} x_{kj}^* J_r(\phi_{kj})^{-1}\right)^{-1}\right\}$$

$$\leq \max_{\{\phi_{kj}\}} \text{tr}\left\{\left(\sum_{j \in N_b} \xi_{kj} x_{kj}^* J_r(\phi_{kj})^{-1}\right)^{-1}\right\} = \frac{2^{2\beta}r_0^{2\beta}}{P^{\beta} \sin^2(\pi/2 - 2\Delta^\phi - 2\varphi^\phi)} \cdot$$

Therefore, a sufficient condition for (33) is

$$\frac{2^{2\beta} \sin \varphi^\phi}{\cos^2(2\Delta^\phi + 2\varphi^\phi)} \leq \frac{\zeta_{\min}}{\zeta_{\max}} \quad (34)$$

where $\delta = \sin \varphi^\phi$. Note that the left-hand side of (34) is an increasing function in $\varphi^\phi$, $\Delta^\phi$ and $\varphi^\phi$, when $\Delta^\phi$ and $\varphi^\phi$ are both small positive numbers. Thus, the maximum $\varphi^\phi$ (or equivalently, maximum $\delta$) to satisfy (34) can be obtained by taking the limit $\varphi^\phi \to 1$ and $\Delta^\phi \to 0$. It follows that

$$\frac{2 \sin \varphi^\phi}{\cos^2(2\varphi^\phi)} \leq \frac{\zeta_{\min}}{\zeta_{\max}} \quad (34)$$

and the inequality holds when $0 < \delta = \sin \varphi^\phi \leq \delta_{\max}$, where $\delta_{\max}$ is the smallest positive root of the equation

$$4\delta^4 - 4\delta^2 - 2 \frac{2^{2\beta} \sin \varphi^\phi}{\cos^2(2\varphi^\phi)} - 1 = 0.$$ 

We give some numerical examples: $\delta_{\max} = 0.318$ when $\zeta_{\min}/\zeta_{\max} = 1$; $\delta_{\max} = 0.096$ when $\zeta_{\min}/\zeta_{\max} = 5$. 

For (b): Consider a small angle $\sqrt{2a\varphi^\phi} \to 0$, where $a = (2^{2\beta + 1}P^{2\beta}/\zeta_{\max})(\zeta_{\min}^{2\beta})$. The probability that all $N_b$ anchors locate in such a small angle of the $R \times R$ region is at most $(\sqrt{2a\varphi^\phi})^{N_b}$, which goes to 0 at the rate of polynomial power $N_b/2$ as $\varphi^\phi \to 0$. Hence, we can find two anchors, $i$ and $i'$, whose angle separation is larger than $\sqrt{2a\varphi^\phi}$ and smaller than $\pi - \sqrt{2a\varphi^\phi}$ w.h.p.
We allocate the power equally on these two anchors, and it follows

$$\text{tr}\left\{ \sum_{j \in N_k} \xi_{kj} x_{kj}^* J_f (\phi_{kj})^{-1} \right\}$$

$$\leq \max_{(\phi_{kj})} \text{tr}\left\{ \sum_{j \in N_k} \xi_{kj} \tilde{P}_{kj} J_f (\phi_{kj})^{-1} \right\}$$

$$\leq \max_{(\phi_{kj})} \text{tr}\left\{ \left( \frac{\xi_{\min}}{\sqrt{2R}} \right)^{2\beta} \left( J_f (\phi_{ki}) + J_f (\phi_{kv}) \right) \right\}^{-1}$$

$$= \frac{2\beta R^{2\beta}}{\xi_{\min}} \frac{2}{P_{\text{tot}}} \sin^2 \left( \frac{\phi}{\sqrt{2\alpha \varepsilon - 2\varepsilon}} \right).$$

Finally, we need to show that

$$\frac{2\beta R^{2\beta}}{\xi_{\min}} \frac{2}{P_{\text{tot}}} \sin^2 \left( \frac{\phi}{\sqrt{2\alpha \varepsilon - 2\varepsilon}} \right) \leq \frac{t_0^{2\beta}}{\xi_{\max}} \frac{2}{P_{\text{tot}}} \sin \varepsilon \phi$$

or equivalently,

$$a \leq \frac{\sin^2 \left( \phi / \sqrt{2\sin \varepsilon \phi} \right)}{\sin \varepsilon \phi}.$$

The above inequality holds as $\varepsilon \phi \to 0$, since the limit of its right-hand side is $2a$.

Now, we extend the above proof to the multiple-agent case. In Section V, we decomposed the one-stage problem $\mathcal{R}^{1}_{\text{R-1}}$ into two-stage optimizations. Let $\tilde{P}_{kj}$ and $x_k^*$ denote the optimal solution of $\mathcal{R}^1_{\text{R-1},k}$ and $\mathcal{R}^1_{\text{R-1}}$, respectively. Since the Stage I problem $\mathcal{R}^1_{\text{R-1},k}$ is formulated for each single agent, we can show by the above proof that

$$\sum_{j \in N_k} \xi_{kj} \tilde{P}_{kj} Q_f (\phi_{kj}) \geq 0$$

holds w.h.p. for agent $k$. Moreover, the optimal power allocation is given in (30) as $x_{kj}^* = \rho_{kj} x_k^*$, where $x_k^*$ obtained in Stage II does not affect $\rho_{kj}$. Hence, we can show that the condition (18) holds w.h.p. for multiple-agent networks.

**APPENDIX E**

**PROOF OF PROPOSITION 5**

The Lagrangian function is given by

$$\mathcal{L}(x_k, u_k, v) = \sum_{k \in N_k} \frac{T_k}{x_k} - \sum_k u_k x_k + v \left( \sum_{k \in N_k} x_k - P_{\text{tot}} \right)$$

where $u_k$, $v \geq 0$. The KKT conditions [46] can be derived as

$$\frac{\partial \mathcal{L}}{\partial x_k} = -\frac{T_k}{x_k^2} - u_k + v = 0 \quad (35)$$

$$u_k x_k = 0$$

$$v \left( \sum_{k \in N_k} x_k - P_{\text{tot}} \right) = 0.$$

Since $x_k$ is always positive, we have $u_k = 0$, which leads to $x_k = \sqrt{T_k / v}$ in (35). Moreover, the objective is monotonically decreasing in $x_k$, which implies the optimal allocation must use all the power resource, i.e., $\sum_{k \in N_k} x_k = P_{\text{tot}}$. Hence, the optimal solution is given by (29).

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