A Generalized Multi-Commodity Network Flow Model for the Earth-Moon-Mars Logistics System

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Simple logistics strategies such as "carry-along" and Earth-based "resupply" were sufficient for past human space programs. Next-generation space logistics paradigms are expected to be more complex, involving multiple exploration destinations and in-situ resource utilization (ISRU). Optional ISRU brings additional complexity to the interplanetary supply chain network design problem. This paper presents an interdependent network flow modeling method for determining optimal logistics strategies for space exploration and its application to the human exploration of Mars. It is found that a strategy utilizing lunar resources in the cis-lunar network may improve overall launch mass to low Earth orbit for recurring missions to Mars compared to NASA’s Mars Design Reference Architecture 5.0, even when including the mass of the ISRU infrastructures that need to be pre-deployed. Other findings suggest that chemical propulsion using LOX/LH$_2$, lunar ISRU water production, and the use of aerocapture significantly contribute to reducing launch mass from Earth. A sensitivity analysis
of ISRU reveals that under the given assumptions, local lunar resources become attractive at productivity levels above 1.8 kg/year/kg in the context of future human exploration of Mars.
Nomenclature

\(A\) = flow equilibrium matrix
\(\mathcal{A}\) = set of directed arcs
\(B\) = flow transformation matrix
\(\mathbf{b}, \mathbf{b}\) = net supply/demand
\(C\) = flow concurrency matrix
\(c, c\) = cost per unit flow
\(d, d\) = constant for flow concurrency constraints
\(\mathcal{G}\) = directed network graph
\(g_0\) = standard gravity, 9.80665 m/s\(^2\)
\(I_{\text{sp}}\) = specific impulse, s
\(i, j\) = node index
\(\mathcal{J}\) = objective function
\(k\) = number of different commodities
\(l, l\) = lower bound
\(m\) = mass
\(\mathcal{N}\) = set of nodes
\(u, u\) = arc capacity
\(x, x\) = flow variable
\(\alpha\) = proportional constant for ISRU maintenance requirement
\(\beta\) = proportional constant for ISRU productivity
\(\Delta t\) = duration, days
\(\Delta V\) = change in velocity, km/s
\(\theta\) = aeroshell mass fraction
\(\mu\) = positive multiplier in generalized flows
\(\phi\) = propellant mass fraction
Nomenclature

Subscripts
(·)_i = node i
(·)_{ii} = loop (i,i)
(·)_{ij} = arc (i,j)

Superscripts
(·)^+ = outflow from tail node
(·)^- = inflow into head node
(·)^± = both outflow and inflow
## Nomenclature

### Acronyms

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<th>Acronym</th>
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<tr>
<td>DCO</td>
<td>Deimos capture orbit</td>
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<td>DEIM</td>
<td>Deimos</td>
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<td>DRA</td>
<td>design reference architecture</td>
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<td>ECLSS</td>
<td>environmental control and life support system</td>
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<td>EDL</td>
<td>entry, descent, and landing</td>
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<td>EML</td>
<td>Earth-Moon Lagrange point</td>
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<td>GC</td>
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<td>GEO</td>
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<td>geostationary transfer orbit</td>
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<td>IMLEO</td>
<td>initial mass in low Earth orbit</td>
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<td>ISRU</td>
<td>in-situ resource utilization</td>
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<td>ISS</td>
<td>International Space Station</td>
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<td>LEO</td>
<td>low Earth orbit</td>
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<td>LLO</td>
<td>low lunar orbit</td>
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<td>LMO</td>
<td>low Mars orbit</td>
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<td>LSP</td>
<td>lunar south pole</td>
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<td>MOI</td>
<td>Mars orbit insertion</td>
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<td>NEO</td>
<td>near-Earth object</td>
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<td>PAC</td>
<td>Pacific Ocean splashdown</td>
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<td>PCO</td>
<td>Phobos capture orbit</td>
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<td>PHOB</td>
<td>Phobos</td>
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<tr>
<td>TEI</td>
<td>trans-Earth injection</td>
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<td>TLMLEO</td>
<td>total launch mass to low Earth orbit</td>
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<td>TMI</td>
<td>trans-Mars injection</td>
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<td>TOF</td>
<td>time of flight</td>
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I. Introduction

FUTURE human space exploration will need to be as self-sustainable as possible as we seek to explore beyond low Earth orbit (LEO). The final report of the Augustine Committee in 2009 [1] and the subsequent space policy speech by President Obama in 2010 [2] reaffirmed that Mars is the ultimate goal of human spaceflight and presented a plan for NASA that follows the "Flexible Path to Mars" option, which takes an evolutionary approach to a variety of destinations such as lunar orbit, near-Earth objects (NEOs), Lagrange points, and the moons of Mars, followed by human landings on the lunar surface and/or Martian surface. In the decades to come, space exploration is expected to transition from a set of isolated missions to an intricately-linked campaign, which will then require mission architectures to be tightly integrated, involving multiple destinations with diverse objectives and spanning many years. As budgets are constrained and destinations are far away from home, a well-planned logistics strategy becomes imperative. A logistics infrastructure network in space with appropriate supply chain management is a potential enabler of sustainable space exploration, just as it has served our terrestrial life well for centuries.

As shown in Fig. 1, past human space exploration programs fall into two different types of logistics paradigms. The Apollo program sent six missions to the lunar surface between 1969 and 1972, each of which was independent and self-contained. Those missions were based on a "carry-along" approach where all vehicles and resources traveled with the crew at all times. On the other hand, the International Space Station (ISS) logistics strategy has been based on regular resupply flights by various vehicles such as the American Space Shuttle, the Russian Progress and Soyuz, the European ATV, the Japanese HTV, and the commercial Dragon and Cygnus. This type of strategy is suitable for long-term missions conducted relatively close to a resupply source such as Earth (cf. people replenishing their pantries regularly from the nearby grocery store).

With the advent of a new era of human space exploration, the research presented in this paper originates from the question of what the next-generation space logistics paradigm should look like. The problem is complex, especially if in-situ resources on the Moon and Mars can be utilized for propellant and life support. In-situ resource utilization (ISRU) implies that the key to success comes from a "travel light and live off the land" strategy [3]. While the answer is expected to lie in some
complex combination of the "carry-along" and "resupply" strategies, a quantitative framework is necessary to provide a scientific underpinning for the hypothesis that not all resources have to come from Earth.

The purpose of this study is to develop a comprehensive graph-theoretic modeling framework to quantitatively evaluate and optimize space exploration logistics with optional ISRU from a network perspective. Network flow is a branch of graph theory that can be applied to space logistics strategy selection. This paper presents a network flow model that provides a mathematical representation of crew and cargo in-space transportation as well as local resource production (ISRU), followed by a case study of human exploration of Mars. Results are benchmarked against NASA’s current Mars Design Reference Architecture 5.0.

II. Background

A. Mars Design Reference Architecture 5.0

NASA has developed a set of design reference architectures (DRA) for Moon, Mars, and Asteroid missions. DRAs are used to help define the current "best" strategy for human exploration missions and architectures and are constantly updated as technology and knowledge improve. They also serve as a benchmark against which alternative architectures can be measured. As of mid-2014, the most recent publication for Mars missions is Mars DRA 5.0 and the associated addenda [4, 5]. This design reference architecture describes the spacecraft and missions which could be used for the first three excursions to the surface of Mars. The Mars exploration architecture is heavily based on lunar concepts from the Constellation program, including the Ares V heavy lift launch vehicle, but also
includes advanced technology concepts such as nuclear thermal rockets (NTRs) for interplanetary propulsion, zero-loss cryogenic coolers for propellant storage, aerocapture as the Mars arrival capture method, ISRU for Mars ascent propellant production, and nuclear fission reactors for surface power on Mars.

B. Cislunar Propellant and Logistics Infrastructure

More recently, concepts for a cislunar propellant and logistics infrastructure and transportation architectures have been proposed [6]. "Cislunar space" is a term that is used to describe the space between the Earth and the Moon with the potential to exploit lunar resources to refuel spacecraft in cislunar space. Cislunar space is taken to include LEO because LEO is closer to the lunar surface than to the Earth’s surface in terms of propulsive energy required. A potential cislunar infrastructure includes a propellant depot, a reusable lunar lander (RLL), a propellant tanker, and an orbital transfer vehicle (OTV) with aerobraking capability. Such an infrastructure would be "game-changing" in that it would fundamentally affect the architecture of future space exploration campaigns, providing greater and potentially cheaper access to space beyond LEO. If operational costs and risks can be managed, this concept could be a significant improvement over the current strategy for Mars exploration described in Mars DRA 5.0.

Various lunar ISRU systems have been proposed such as hydrogen reduction, methane carbothermal reduction, molten electrolysis (electrowinning), volatile extraction, and polar water ice extraction [7-12]. ISRU options on Mars include the Sabatier reaction, reverse water gas shift reaction, and atmosphere electrolysis. The two moons of Mars, Phobos and Deimos, the dwarf planet, Ceres, and near-Earth asteroids could also be sources of raw materials for ISRU [13, 14]. As such, ISRU, or the ability to produce water, gases, and propellants on other planetary bodies, along with an in-space logistics infrastructure to transport and store those resources, is one of the most interesting key concepts for future human space exploration.

C. Literature Review

Although a substantial body of research exists on terrestrial transportation networks and supply chain logistics in business and military applications, space logistics is an emerging topic in recent
years. While the general literature in the field of space logistics is described in detail elsewhere [15], this section briefly reviews the current modeling frameworks for space logistics architectures.

The direct predecessor of this paper is a mathematical model for interplanetary logistics developed by Taylor et al. [16–18]. This prior work solved a vehicle design and routing concurrent optimization problem using a combinatorial integer programming model. One of the limitations that the authors identify is the computational complexity due to the integer programming formulation. Due to the nature of integer programming, it was only able to solve a problem of limited size: a two-week lunar sortie in a time-expanded Earth-Moon network with 6 static nodes. It is impractical to apply this technique to a much larger, more complex problem such as Mars exploration with more nodes and much longer mission duration at this time, because of the size of the resulting time-expanded network. The other limitation is that there are only two types of commodities that travel through the network: cargo and vehicles.

Simultaneously with Taylor’s work, efforts have been made to develop a discrete event simulation software called SpaceNet [19–21]. SpaceNet is a software tool that models space exploration from a supply chain and logistics perspective within a discrete event simulation environment to support campaign analyses and trade studies [22]. In the current version, however, one needs to pre-define a transportation network and a mission sequence as inputs, and SpaceNet can only determine the propulsive and logistical feasibility and the optimal manifesting strategy for a given exploration campaign.

Other past studies put more emphasis on an exploration system’s architecture rather than the underlying logistics network. Bounova et al. presented a method to generate a large number of space transportation architectures using Object Process Networks (OPN) and how those OPN-generated architectures could be evaluated and down-selected using an integration tool [23]. OPN is a graphical meta-language that represents a system architecture in terms of a network of objects and processes. While this work took a graph-theoretic approach, the network graph represents information flows and not physical locations and transportation between them. In the OPN framework, physical locations and paths are fixed for each transportation architecture, and there is no flexibility in network selection.
Komar et al. presented a centralized, integrated framework of parametric models for performing architecture definition and assessment, known as the EXploration Architecture Model for IN-space and Earth-to-orbit (EXAMINE) [24]. This method claims to provide a flexible, modular capability to execute the architecture definition and assessment process faster and more consistently than the distributed team approach that has been prevalent at NASA and other organizations. However, the user needs to define the concept of operations and waypoints at physical locations arbitrarily at the beginning of the analysis. Selection of waypoints cannot be optimized within the EXAMINE framework.

In response to the limitations of the previous models, Arney et al. developed an improved architecture modeling framework using graph theory [25]. This framework uses a network graph to express physical locations as nodes and different means of transport between nodes as arcs. It also allows the user to specify nodes that are reused over time so that assets can be prepositioned for subsequent flights or reused over multiple flights. This capability is equivalent to the time-expanded network in Taylor’s work [16–18] but is more flexible and computationally efficient, while still requiring a manual specification by the user. The benefit of this method is that it can explore a broader architecture-level design space and rapidly compare different system architecture options. However, the architecture focus of this framework introduces limitations that limit the trade space of logistics strategies from the beginning. Certain logistics strategies would be precluded by the user-specified scenarios, because defining an architecture is coupled with defining a transportation strategy or paths on which vehicle, crew, and cargo travel.

Although these frameworks are useful in their respective contexts, they require the user to somewhat arbitrarily predetermine a logistics network as inputs to the models. This is where optimization comes into play. A well-informed network selection should be made upstream in concept development. Taylor’s framework is capable of automated optimization of a logistics network, but due to the integer programming formulation, which is conventional for a class of vehicle routing problems (VRPs), it is not applicable to a complex supply chain network for long-term exploration campaigns. Particularly in the case where optional ISRU and in-space infrastructures are used, additional complexity arises. Therefore, a new approach is needed that is capable of solving a larger
optimization problem for logistics network selection within a reasonable time. Once this method has been established, it can serve as a front end to the aforementioned frameworks thus effectively providing a network auto-generation capability.

D. Modeling Space Logistics as an Interdependent Network Flow

The graph-theoretic approach essentially models the movement of cargo or commodities in a flow network. Figure 2 shows an example of the Earth-Moon-Mars logistics network, including representative nodes in the cislunar and Martian systems. Arcs connecting nodes represent possible movements or transports between two locations. While a space mission objective can generally be stated in any language depending on the point of view (e.g., science, engineering, etc.), it can be translated, from a logistics perspective, into a set of demands at certain nodes in a network. For example, the mission objective of Mars DRA 5.0 from a logistics perspective is simply to send a crew of six to the Martian surface and to bring them back to Earth after 540 days of surface stay on Mars.

In conventional network flow modeling, the propellant required for a mission is likely to be modeled as arc costs. In spaceflight, however, the fraction of propellant mass is significantly greater than that of terrestrial transports, and the propellant for all subsequent stages is regarded as payload on the current arc. Furthermore, ISRU allows resources to be generated at other locations than the Earth’s surface. For these reasons, resources required should be treated as commodities included in the flow variables rather than as costs resulting from the flow of primary commodities. The network flow model must be able to deal separately with materials that are only available on Earth (e.g., science payload) and resources that are also available at other locations. Therefore, the problem is formulated as a multi-commodity network flow.

Given a mission objective in the form of a set of demands at destination nodes, the network flow model is tasked to find the best route(s) in the network that satisfies those demands while meeting certain constraints. In other words, the optimization result will figure out which nodes and arcs to use. This result can be interpreted as "where to deploy what", providing insights on the best transportation architecture and infrastructure concept. To explore a broader solution space, it is
important for the network graph to include as many nodes and arcs as possible (see Fig. 2).

III. Network Flow Modeling Framework

A. Network Flow Overview

Network flow problems provide versatile applications not only in industrial logistics. While an intuitive application is the distribution of a single homogeneous product from plants (origins) to consumer markets (destinations), many other problems that are initially not cast as networks can be transformed into a network format. The minimum cost flow problem, which is the most fundamental of all network flow problems, determines a least cost shipment of a single commodity through a network in order to satisfy demands at certain nodes from available suppliers at other nodes. The fundamentals of network flows can be found in Ahuja et al. [26]. The basic notations are as follows.

Let $G = (\mathcal{N}, \mathcal{A})$ be a directed network defined by a set $\mathcal{N}$ of nodes and a set $\mathcal{A}$ of directed arcs. Each node $i \in \mathcal{N}$ is associated with a number $b_i$ representing its supply/demand. If $b_i > 0$, node $i$ is a supply node; if $b_i < 0$, node $i$ is a demand node with a demand $-b_i$; and if $b_i = 0$, node $i$ is
a potential transshipment node. Each arc \((i, j) \in \mathcal{A}\) is associated with a cost \(c_{ij}\) that denotes the cost per unit flow on that arc. The flow cost is assumed to vary linearly with the amount of flow. The decision variables are the arc flows represented by \(x_{ij}\). The minimum cost flow problem is an optimization model formulated as follows:

Minimize

\[
J = \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}
\]  

subject to

\[
\sum_{j:(i,j) \in \mathcal{A}} x_{ij} - \sum_{j:(j,i) \in \mathcal{A}} x_{ji} \leq b_i \quad \forall \ i \in \mathcal{N}
\]

\[
l_{ij} \leq x_{ij} \leq u_{ij} \quad \forall \ (i,j) \in \mathcal{A}
\]

where \(l_{ij}\) and \(u_{ij}\), respectively, denote a lower bound and a maximum capacity of arc \((i,j)\). The constraints in Eqs. (2a) and (2b) are referred to as mass balance constraints and flow bound constraints, respectively. Two relevant generalizations of the minimum cost flow problem are also described below.

1. Generalized Flow Problems

   In the minimum cost flow problem described above, one very fundamental, yet almost invisible, assumption is that flow on every arc is conserved, that is, the amount of flow on any arc that leaves its head node equals the amount of flow that arrives at its tail node. This assumption is very reasonable in many application settings. Other practical contexts, however, violate this conservation assumption. In generalized flow problems, arcs might "consume" or "generate" flow. To address these situations, a positive multiplier \(\mu_{ij}\) is associated with every arc \((i,j)\) of the network, assuming that if \(x_{ij}\) units are sent from node \(i\) along arc \((i,j)\), then \(\mu_{ij} x_{ij}\) units arrive at node \(j\).

2. Multi-Commodity Flow Problems

   In many application contexts, more than one commodity, each governed by its own network flow constraints, share the same network. If the commodities do not interact in any way, then this problem can be solved as a set of independent single-commodity problems. In some situations,
however, the individual commodities share the common arc capacities, that is, each arc has a capacity that restricts the total flow of all commodities on that arc (*bundle constraints*). To collectively accommodate multiple commodities, the above equations can be rewritten by using the vector notation (bold letters) instead of scalar expressions. If $k$ is the number of different commodities to be considered, each vector is a $k$-dimensional column vector.

**B. GMCNF: Generalized Multi-Commodity Network Flow Formulation**

This section describes the GMCNF methodology for generalized multi-commodity network flows. In this paper, multiple commodities interact with each other in several ways. This network can be referred to as the generalized multi-commodity network flow \([15, 27]\). The individual commodities can mutually affect each other in a way that is more than just sharing the common arc capacities. Interdependent network flows consider not only a gain/loss of each individual commodity but also a gain/loss of a commodity caused by the existence of another commodity and even a transformation between different commodities. For example, the amount of propellant consumed is driven by the total mass (not only the propellant itself), and food is consumed and turned into waste by the crew. This can all be mathematically implemented by multiplying a flow variable vector by a square matrix (equivalent to a scalar multiplier $\mu_{ij}$ in generalized flows). The interactions between different commodities appear in the off-diagonal entries of this matrix.

As shown in Fig. 3, the flow on each arc is split into two parts: $x_{ij}^+$ represents the outflow from node $i$ and $x_{ij}^-$ represents the inflow into node $j$. The unit cost associated with the flow is also split and denoted by $c_{ij}^+$ and $c_{ij}^-$. Using this notation, the GMCNF model is formulated as follows:

Minimize

$$J = \sum_{(i,j) \in A} \left( c_{ij}^+ x_{ij}^+ + c_{ij}^- x_{ij}^- \right)$$

(3)
subject to

\[ \sum_{j: (i,j) \in A} A^+_{ij} x^+_{ij} - \sum_{j: (j,i) \in A} A^-_{ij} x^-_{ij} \leq b_i \quad \forall i \in \mathcal{N} \]  

(4a)

\[ x^-_{ij} = B_{ij} x^+_{ij} \quad \forall (i, j) \in \mathcal{A} \]  

(4b)

\[ C^+_{ij} x^+_{ij} \leq d^+_{ij} \quad \text{and} \quad C^-_{ij} x^-_{ij} \leq d^-_{ij} \quad \forall (i, j) \in \mathcal{A} \]  

(4c)

\[ l^+_{ij} \leq x^+_{ij} \leq u^+_{ij} \quad \text{and} \quad l^-_{ij} \leq x^-_{ij} \leq u^-_{ij} \quad \forall (i, j) \in \mathcal{A} \]  

(4d)

The GMCNF model introduces three types of matrix multiplications: a \textit{flow equilibrium matrix} \( A^\pm_{ij} \), a \textit{flow transformation matrix} \( B_{ij} \), and a \textit{flow concurrency matrix} \( C^\pm_{ij} \). Each of the constraints in Eqs. (4a)-(4c) involves their respective matrix multiplication and Eq. (4d) represents the flow bound constraints for both outflow and inflow.

As can be seen from Eqs. (4a) and (4b), \( A^+_{ij} x^+_{ij} \) is consumed or generated at node \( i \) to send out \( x^+_{ij} \) into arc \((i, j)\), \( x^+_{ij} \) is transformed into \( x^-_{ij} = B_{ij} x^+_{ij} \) on the arc itself, and \( A^-_{ij} x^-_{ij} \) is received at node \( j \). Also in the flow concurrency constraints in Eq. (4c), the relationship between commodities traveling together on the arc is self-constrained such that the dot product with \( C^\pm_{ij} \) is less than or equal to \( d^\pm_{ij} \). If \( k \) different commodities are considered, that is, the flow vector has \( k \) components, then the \( A^\pm_{ij} \) and \( B_{ij} \) matrices must be \( k \)-by-\( k \) square matrices while the \( C^\pm_{ij} \) matrix is a \( n_{ij} \)-by-\( k \) matrix, where \( n_{ij} \) is the number of flow concurrency constraints on arc \((i, j)\). The off-diagonal entries of \( A^\pm_{ij} \) and \( B_{ij} \) and the non-zero entries of \( C^\pm_{ij} \) indicate that there are interactions between commodities.

With this modification, the GMCNF model can handle the flow gain/loss due to the interaction between commodities, transformation between commodities, mass balance at nodes, and flow concurrency on arcs. This model can be applied to any problem that is translatable into a network flow problem in which multiple commodities interact with each other in various ways [28]. Note that because of its linear structure, this model can be formulated as a linear programming (LP) problem. One advantage of the LP formulation is that it can be quickly solved and that if solved, the solution...
is guaranteed to be optimal. This advantage of LP allows a large number of runs without much computational effort, which is more difficult in Taylor’s integer programming (IP) formulation [18].

1. **Flow Equilibrium Matrix A**

The $A$ matrix in Eq. (4a) can be used to introduce the commodities that are consumed or generated by sending out or receiving the flow at a specific node but do not travel on the arc themselves. Examples of this include the electricity consumed for pumping groundwater to the surface, the workforce required for loading/unloading a freighter at a seaport, or the fees paid for using a bank ATM. As discussed in the next section, resource processing is modeled using a "graph loop" in this study and the $A$ matrix plays a key role in graph loops required to enable an ISRU plant.

2. **Flow Transformation Matrix B**

The $B$ matrix in Eq. (4b) can describe the flow gain/loss and transformation between commodities on arcs. Unless the $B$ matrices are all identity matrices, strict flow conservation no longer exists. Examples of this include propulsive burns, resource boil-off, crew consumables consumption and waste generation, ISRU resource production, and so on. When there are multiple flow transformation events on a single arc $(i, j)$, $B_{ij}$ can be the product of multiple matrices with left-multiplication:

$$B_{ij} = B_{ij}^{(n)} \cdots B_{ij}^{(2)} B_{ij}^{(1)}$$

where $B_{ij}^{(n)}$ is a flow transformation matrix for the $n$th flow transformation event on arc $(i, j)$. In general, a non-diagonal matrix is not commutative. Therefore, if there are interactions between different commodities, that is, $B$ matrices have off-diagonal entries, then the order of matrix multiplications must be exactly the sequence of transformation events.

3. **Flow Concurrency Matrix C**

The $C$ matrix in Eq. (4c) can be used to enforce the flow concurrency on a single arc. In other words, it can handle the situation that a commodity traveling on an arc needs a certain amount of
another commodity (or commodities) to travel along with it. In Eq. (4c), $d_{ij}^+$ on the right-hand side is a set of constants. The bundle constraints in the classical multi-commodity flows can also be enforced by setting $C_{ij}$ to a vector of ones and $d_{ij}$ to a bundle capacity. The $C$ matrix is, for example, used to enforce the relationships between crew size and transfer vehicle mass, propellant mass and structure mass, and total spacecraft mass and aeroshell mass.

C. Other Concepts in Graph Theory

In addition to the three matrix multiplications, the GMCNF model also introduces two other concepts in graph theory that help in formulating the network flow problem addressed in this paper.

1. Graph Loop

In graph theory, a graph loop is an arc that connects a node to itself (Fig. 4(a)). It is also called a self-loop or a "buckle". This can be used for modeling a generic plant as a resource processing facility. A resource processing facility is likely to be modeled as a node, as opposed to a typical arc modeling transportation. In the GMCNF model, however, a resource processing facility is modeled as a graph loop. The $ABC$ matrices discussed above are also applicable to graph loops and resource processing such as ISRU can be represented by a flow equilibrium matrix $A_{ii}$ and a flow transformation matrix $B_{ii}$ (details will follow later).

2. Multigraph

In graph theory, an undirected graph that has no loops and no more than one arc between any two different nodes is called a simple graph. As opposed to a simple graph, a multigraph refers to a graph in which multiple arcs (also called "parallel arcs") are permitted between the same end nodes. Thus, as shown in Fig. 4(b), two nodes may be connected by more than one arc. For example, there are multiple options for in-space transportation between the same nodes such as chemical and nuclear thermal rockets, each of which has a different $I_{sp}$. Aerocapture adds an option at Earth/Mars arrival. Moreover, the trade-offs between $\Delta V$ and TOF (time of flight) must be explored. All these parameters are implemented through different $ABC$ matrices on each arc. Therefore such multiple choices can be embedded by allowing parallel arcs between the same end
nodes. By implementing parallel arcs that represent these trades, the optimization of network flow will automatically yield a solution to the trade-off problem. This concept is also used in Arney’s framework [25].

IV. Case Study: Human Exploration in the Earth-Moon-Mars System

To demonstrate the GMCNF methodology, a sample human exploration of Mars based on NASA’s Mars DRA 5.0 [4] is presented. One of the goals of this case study is to discuss the potential benefits of utilizing lunar resources in the cislunar network for Mars exploration. To this end, this section explores logistics strategies with ISRU options for human exploration of Mars with the same objective as Mars DRA 5.0 and compares the best GMCNF results with the DRA 5.0 scenario. A sensitivity analysis to find the key drivers and thresholds follows.

A. Case Study Summary and Problem Definition

Figure 5 shows a notional network graph that shows only the relationship between nodes and arcs from the logistics network in Fig. 2. This network graph is composed of 16 nodes and 598 arcs allowing both self-loops (teardrop-shaped arcs) and parallel arcs between the same end nodes.

First, the commodities considered in the flow variable vector and the objective function need to be defined. In the GMCNF model, everything that travels on the network (even the crew) is regarded as an individual commodity. Table 1 lists the 20 different commodities considered in this analysis. The flow and demand of these commodities are all measured in mass (kilograms). The commodity "crewRe" represents crew returning to Earth, distinguished from "crew" traveling outbound. A self-loop representing Mars surface stay transforms "crew" into "crewRe" using the
The "Resources" category includes major propellants and fuels as well as crew provisions and waste. The "Infrastructure" category includes habitation facilities as well as ISRU plants and spares. The "Transportation" category includes vehicles, propulsive elements, and non-propulsive elements. This case study allows three types of propulsion systems: LOX/LH$_2$, LOX/LCH$_4$, and NTR. Inert mass (engines) and tanks are separately defined for each propulsion system and propellant combination.

While solar electric propulsion is another attractive option especially for cargo missions, it is beyond the scope of the present study because low-thrust trajectory requires a quite different way of defining arc parameters.

While one can define any objective function under which the network flow is optimized, this case study minimizes the total launch mass to LEO (TLMLEO). Initial mass in LEO (IMLEO) is often used as a measure of the mission cost as a widely accepted surrogate for estimating it. However, IMLEO is typically assessed on a mission-by-mission basis. Therefore, to avoid confusion, this paper uses TLMLEO, which is taken to include all of the masses that have to be launched from the Earth’s surface to enable the resulting architecture.

It should be noted that TLMLEO is not the only figure of merit upon which to optimize the logistics strategy. While it is useful for estimating the launch cost, it does not represent the design,
development, test, and evaluation (DDT&E) costs of the various components such as ISRU systems and in-space depots. Furthermore, the increased number of rendezvous and refueling events is likely to contribute adversely to safety and reliability. The added complexity and risk of the logistics network should also be accounted for. Given these considerations, the resulting strategy must be evaluated from multiple perspectives or the problem must be formulated as a multi-objective optimization. For the purpose of demonstrating GMCNF, however, this paper focuses on TLMLEO as a single-objective.

Using the notation of the GMCNF model in Eq. (3), TLMLEO can be represented by setting $c_{ij}^\pm = 0$ for all arcs $(i, j)$ except for $c_{ij}^+ = 1$ for an arc from KSC to LEO. Note that if ISRU is used, TLMLEO in this analysis also includes the ISRU systems and associated spares that are pre-deployed so that a fair comparison can be made.

Second, the supply/demand at each node $b_i$ needs to be set. In the network in Fig. 5, the Kennedy Space Center (KSC) can provide all the commodities in Table 1 with the exception of "crewRe" (which can only come from Mars surface). The potential ISRU nodes include the lunar south pole (LSP), Phobos (PHOB), Deimos (DEIM), and Gale Crater (GC) on Mars. These nodes can provide resources only if an ISRU system is deployed there. Surface manufacturing of spares and infrastructure has also been proposed [7] but this case study only considers resource production. Raw materials are assumed to be unlimited at these nodes but the amount of resources actually

<table>
<thead>
<tr>
<th>Crew</th>
<th>Resources</th>
<th>Infrastructure</th>
<th>Transportation</th>
</tr>
</thead>
<tbody>
<tr>
<td>crew</td>
<td>hydrogen</td>
<td>habitat</td>
<td>vehicle</td>
</tr>
<tr>
<td>crewRe</td>
<td>oxygen</td>
<td>plantISRU</td>
<td>inertLOXLH2</td>
</tr>
<tr>
<td></td>
<td>water</td>
<td>sparesISRU</td>
<td>inertLOXLCH4</td>
</tr>
<tr>
<td></td>
<td>methane</td>
<td></td>
<td>inertNTR</td>
</tr>
<tr>
<td></td>
<td>carbonDioxide</td>
<td></td>
<td>tankLOX</td>
</tr>
<tr>
<td></td>
<td>food</td>
<td></td>
<td>tankLH2</td>
</tr>
<tr>
<td></td>
<td>waste</td>
<td></td>
<td>tankLCH4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>aeroshell</td>
</tr>
</tbody>
</table>
produced is limited up to the capacity of the ISRU system.

Setting the demand in a network is essentially identical to defining the mission from a logistics perspective. For example, a stipulated demand for "plantISRU" at LSP corresponds to a lunar mission to send an ISRU plant to the lunar south pole. In this study, a human exploration mission to Mars based on Mars DRA 5.0 is used as a case study and so the demand is determined by reference to Mars DRA 5.0 [4]. Logistically speaking, the mission objective of Mars DRA 5.0 is to send a crew of six with a surface habitat (SHAB) to the Martian surface and to bring the crew back to Earth after 540 days of stay on Mars. This can be translated into a network flow problem by setting the demand for "habitat" at GC and the demand for "crewRe" at the Pacific Ocean splashdown (PAC) point, instead of specifying the demand for "crew" at GC. Setting the demand for "crew" at GC only considers the outbound portion and would be interpreted as a one-way mission [29]. Unlike the crew, the SHAB is not brought back to Earth and could be useful for subsequent human presence on Mars even though such subsequent use is outside the scope of this case study. The total mass of the surface systems is assumed to be 51,700 kg (extracted from [4]), and this value is used in a lump as a demand for "habitat" at GC. There is no need to explicitly set other demands such as crew provisions or ISRU spares because these commodities are implicitly demanded through the three matrix multiplications. Orbital and Lagrange nodes are all potential transshipment nodes and therefore \( b_i = 0 \) for those nodes.

B. Examples of Use of ABC Matrices

This section presents the selected examples of use of the flow equilibrium matrix \( A \), the flow transformation matrix \( B \), and the flow concurrency matrix \( C \). Other examples are provided in [15, 27]. The key parameters necessary in determining these matrices are given in Table 2 and Fig. 6. Table 2 summarizes the parameters and assumptions that are used in this analysis [4, 30]. Figures 6(a) and 6(b) provide the \( \Delta V \) values in km/s units and the times of flight (TOFs) in Earth days, respectively, based on [6, 27, 31]. Note that these \( \Delta V \) values assume high thrust propulsion, the Oberth maneuver at Earth flyby, and launch during the appropriate launch windows. Shaded cells represent aerobraking options, in which \( \Delta V \) for arrival capture can be saved but an aeroshell is
needed, which adds to vehicle mass. Also, while the $\Delta V$ and TOF for Mars transfer are time-variant, this study assumes a "fast" trajectory of 180 days as with Mars DRA 5.0.

1. **ISRU Resource Production**

ISRU resource production is modeled as a self-loop. An ISRU system generally includes a power system, a tanker, and a terrain management vehicle as well as a plant. Once an ISRU system is deployed, operational maintenance tasks arise. In this study it is assumed that the ISRU system is automated or teleoperated with robots and that both the maintenance requirement and the resource productivity are linearly scalable with respect to the size of the system. While a previous study showed that ISRU plants actually follow economies of scale [8, 10], this study takes advantage of LP formulation by assuming linear scalability. Let $\alpha$ and $\beta$ be the proportional constants for maintenance requirement (ISRU spares) and resource productivity, respectively. If an ISRU plant with a mass of $m_{plant}$ is used for a duration of $\Delta t_{ii}$, it requires $\alpha \Delta t_{ii} m_{plant}$ of spares and produces $\beta \Delta t_{ii} m_{plant}$ of resources. This can be modeled using the $A$ and $B$ matrices as follows:

$$
\begin{pmatrix}
\text{plant} \\
\text{spares} \\
\text{resources}
\end{pmatrix}_{ii}^{\pm} = \begin{pmatrix}
1 & 0 & 0 \\
\alpha \Delta t_{ii} & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}_{ii}^{\pm} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
\beta \Delta t_{ii} & 0 & 1
\end{pmatrix}_{ii}
$$

(6)

Note that only relevant variables are shown and that for the other variables, the $A$ and $B$ matrices are identity matrices.

2. **Propulsive Burn**

The *propellant mass fraction* is the ratio between mass of the propellant used and the initial mass of the vehicle. From the rocket equation, the propellant mass fraction $\phi$ on arc $(i, j)$ is derived as:

$$
\phi_{ij} = 1 - \exp \left( - \frac{\Delta V_{ij}}{I_{sp} g_0} \right)
$$

(7)

where $\Delta V_{ij}$ is the change in the vehicle's velocity on arc $(i, j)$, $I_{sp}$ is the specific impulse dependent on propulsion technology and fuel choice, and $g_0$ is the standard gravity constant. Using the propellant
mass fraction $\phi$, the propellant consumption to traverse an arc from $i$ to $j$ can be represented as:

$$
\mathbf{x}_{ij}^\pm = 
\begin{bmatrix}
\text{propellant} \\
\text{dry mass}
\end{bmatrix}_{ij}^\pm
$$

$$
\mathbf{B}_{ij} = 
\begin{bmatrix}
-\phi & 1 - \phi \\
1 & 0
\end{bmatrix}_{ij}
$$

Since the three types of propulsion systems (LOX/LH$_2$, LOX/LCH$_4$, and NTR) are considered in this analysis, "propellant" in the above equation includes hydrogen, oxygen, and methane while "dry mass" includes all the other commodities, including the vehicle dry mass. Note that each propellant is consumed fractionally according to its proper stoichiometric mixture ratio.

3. Aeroshell

For Mars arrival or Earth arrival, aerobraking, which takes advantage of atmospheric drag, is available with an aeroshell. Since aerobraking is an option, this is modeled as a parallel arc separately from an arc with propulsive capture. Let $\theta$ denote the aeroshell mass fraction, meaning that when a spacecraft with a mass of $m_{sc}$ performs aerobraking, it must have an aeroshell with a mass of $\theta m_{sc}$ as a concurrent flow requirement. Then

$$
\mathbf{x}_{ij}^- = 
\begin{bmatrix}
\text{spacecraft} \\
\text{aeroshell}
\end{bmatrix}_{ij}^-
$$

$$
\mathbf{C}_{ij}^- = 
\begin{bmatrix}
\theta & -1
\end{bmatrix}_{ij}^-
$$

$$
d_{ij}^- = 0
$$

which, in light of Eq. (4c), is equivalent to

$$
[\theta m_{sc}]_{ij}^- \leq [m_{aeroshell}]_{ij}^-
$$

Since $m_{sc}$ is the total mass being decelerated, "spacecraft" in Eq. (9) refers to all the commodities except for the "aeroshell" itself. Note that aerobraking is performed at arrival, so that this is an "inflow" concurrency constraint with a superscript of $-$ (see Fig. 3). The relationship between propellant and structural mass can also be constrained in the same manner.
Table 2 Summary of parameters and assumptions used in the analysis [4, 30]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assumed value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mission data</strong></td>
<td></td>
</tr>
<tr>
<td>Number (quantity)</td>
<td>Crew</td>
</tr>
<tr>
<td>Mass (dry), kg</td>
<td>100</td>
</tr>
<tr>
<td><strong>Propulsion system</strong></td>
<td>LOX/LH$_2$</td>
</tr>
<tr>
<td>Specific impulse, s</td>
<td>450</td>
</tr>
<tr>
<td>Mixture ratio</td>
<td>5.88</td>
</tr>
<tr>
<td><strong>Inert mass fraction</strong></td>
<td>LOX/LH$_2$</td>
</tr>
<tr>
<td>For in-space</td>
<td>0.10</td>
</tr>
<tr>
<td>For descent</td>
<td>0.30</td>
</tr>
<tr>
<td>For ascent</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>Propellant and fuel</strong></td>
<td>LH$_2$</td>
</tr>
<tr>
<td>Boil-off rate, %/day</td>
<td>0.127</td>
</tr>
<tr>
<td>Tank mass fraction</td>
<td>0.18</td>
</tr>
<tr>
<td><strong>Crew life support</strong></td>
<td></td>
</tr>
<tr>
<td>Daily usage, kg/day</td>
<td>Oxygen</td>
</tr>
<tr>
<td>Daily output, kg/day</td>
<td>–</td>
</tr>
<tr>
<td><strong>Other assumptions</strong></td>
<td></td>
</tr>
<tr>
<td>Oxygen leak rate from vehicle/habitat</td>
<td>0.05% of pressurized volume per day</td>
</tr>
<tr>
<td>Aeroshell mass fraction</td>
<td>37% of total mass being braked</td>
</tr>
<tr>
<td>ISRU resource production rate</td>
<td>10 kg per year plant mass</td>
</tr>
<tr>
<td>ISRU spares mass required</td>
<td>10% of plant mass per year</td>
</tr>
<tr>
<td>From</td>
<td>TOF [days]</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
</tr>
<tr>
<td>KSC</td>
<td></td>
</tr>
<tr>
<td>PAC</td>
<td></td>
</tr>
<tr>
<td>LEO</td>
<td>1</td>
</tr>
<tr>
<td>GTO</td>
<td>1</td>
</tr>
<tr>
<td>GEO</td>
<td>1</td>
</tr>
<tr>
<td>EML1</td>
<td>3</td>
</tr>
<tr>
<td>EML2</td>
<td>4</td>
</tr>
<tr>
<td>EML4/5</td>
<td>4</td>
</tr>
<tr>
<td>LLO</td>
<td>4</td>
</tr>
<tr>
<td>LSP</td>
<td></td>
</tr>
<tr>
<td>DCO</td>
<td>180</td>
</tr>
<tr>
<td>DEIM</td>
<td></td>
</tr>
<tr>
<td>PCO</td>
<td>180</td>
</tr>
<tr>
<td>PHOB</td>
<td></td>
</tr>
<tr>
<td>LMO</td>
<td>180</td>
</tr>
<tr>
<td>GC</td>
<td></td>
</tr>
</tbody>
</table>

(a) Arc ΔV cost, km/s

(b) Arc time of flight (TOF), Earth days

Fig. 6 ΔV values and times of flight used in the analysis [4, 6, 27, 31].
C. Model Validation

Before network flow optimization, a model validation through comparison of TLMLEO with Mars DRA 5.0 is presented in this section. Note that TLMLEO corresponds to the total IMLEO in Mars DRA 5.0. If the model returns a similar TLMLEO under the same logistical conditions as assumed in DRA 5.0, it is likely that the model is an accurate representation of logistics in space or at least that the model gives a fair comparison with DRA 5.0.

For the in-space transportation system for crew and cargo, Mars DRA 5.0 conducted top-level performance assessments of both NTR and advanced chemical propulsion in terms of total IMLEO and the total number of Ares V launches, and concluded that NTR was the preferred transportation technology for both the crew and the cargo vehicles, while retaining chemical/aerocapture as a backup option [4]. Both propulsive and aerocapture orbit insertions are considered for the cargo missions while only propulsive orbit insertions are allowed for the crewed vehicles. A LOX/LCH\textsubscript{4} rocket is used for the Mars ascent/descent vehicle. DRA 5.0 also considers the use of atmospheric acquisition ISRU on Mars for ascent oxidizer production and life support. The ISRU plant is made up of solid oxide CO\textsubscript{2} electrolyzers (SOCEs) that convert CO\textsubscript{2} into O\textsubscript{2}. The LCH\textsubscript{4} fuel that is required for ascent and the hydrogen that is reacted with Mars-produced O\textsubscript{2} to make up H\textsubscript{2}O are both brought from Earth.

The GMCNF model is validated under these logistical strategies as well as other parameters and assumptions that are given in Table 2 and Fig. 6. Table 3 provides a comparison of TLMLEO and its breakdown by category (see Table 1) between Mars DRA 5.0 and the GMCNF model. For both in-space propulsion options, the GMCNF model shows close agreement with Mars DRA 5.0 (only +0.92\% for NTR and −0.25\% for chemical/aerocapture) and therefore the model is expected to give reasonably reliable results under various scenarios described in the following sections. For later reference, it should be noted that the NTR case saves approximately 400 metric tons (32\%) in TLMLEO compared to the chemical/aerocapture case.
Table 3 Comparison of TLMLEO and its breakdown in metric tons [4]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>TLMLEO</th>
<th>Crew</th>
<th>Resources</th>
<th>Infrastructure</th>
<th>Transportation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRA 5.0 – NTR</td>
<td>848.7</td>
<td>0.6</td>
<td>436.6</td>
<td>52.8</td>
<td>358.8</td>
</tr>
<tr>
<td>GMCNF – NTR</td>
<td>856.6</td>
<td>0.6</td>
<td>444.5</td>
<td>53.2</td>
<td>358.4</td>
</tr>
<tr>
<td>DRA 5.0 – chemical/aero</td>
<td>1251.8</td>
<td>0.6</td>
<td>901.1</td>
<td>52.8</td>
<td>297.3</td>
</tr>
<tr>
<td>GMCNF – chemical/aero</td>
<td>1248.7</td>
<td>0.6</td>
<td>909.0</td>
<td>53.2</td>
<td>285.9</td>
</tr>
</tbody>
</table>

D. Baseline Problem

First, the baseline problem is defined and solved. In the baseline problem, the logistical demand remains the same but the constraints on propulsion systems are relaxed such that (1) the combined use of chemical and nuclear thermal rockets is allowed (NTR is never allowed for descent/ascent), and (2) aerocapture can be used along with NTR for cargo flights while crewed vehicles must perform propulsive orbit insertions at Mars arrival. Also, the following assumptions on ISRU availability/technology are introduced: (3) lunar ISRU can produce O$_2$ from regolith or H$_2$O from water ice at a rate of 10 kilograms per year per unit plant mass while requiring spares of 10% of plant mass per year, and (4) Mars ISRU can acquire CO$_2$ from the atmosphere or H$_2$O from water ice with the same production rate and spares requirement as those for lunar ISRU. Mars CO$_2$ can be converted into CH$_4$ and H$_2$O via the Sabatier reaction or can be converted into O$_2$ via solid oxide electrolysis. Additionally, electrolysis of H$_2$O and pyrolysis of CH$_4$ are assumed to be available along with lunar/Mars ISRU. All these chemical reactions are modeled as an optional self-loop.

Using MATLAB 8.3 (R2014a) with CPLEX 12.6 on an Intel® Core™ i7-2640M CPU at 2.80 GHz, one run of the optimization model takes approximately 12 seconds for preprocessing and 1.2 seconds for optimization (TLMLEO minimization). Figure 7 shows the flow of crew and cargo in the baseline solution, where arrows represent the direction of the flow. Bold self-loops indicate that ISRU and other chemical reactions are performed. The resulting TLMLEO turns out to be 271.8 metric tons. Compared to Mars DRA 5.0, the baseline solution saves 68.0% from the reference NTR scenario and 78.3% from the reference chemical/aerocapture scenario. For each of the arcs used in the baseline solution, Fig. 8 indicates the propulsion system and propellant origin as well as lists
the amount of each commodity transported.

![Commodity flow and arcs in use](image1.png)

**Fig. 7 Baseline solution: commodity flow and arcs in use.**

![Commodity flow breakdown in metric tons](image2.png)

**Fig. 8 Baseline solution: commodity flow breakdown in metric tons.**

First, the most significant change from Mars DRA 5.0 is that the baseline solution makes extensive use of lunar ISRU. Table 4 provides details on the use of ISRU in the baseline solution.
For both lunar and Mars ISRU, H\textsubscript{2}O is produced from water ice, which is partly used for propellant by way of electrolysis/liquefaction and partly for crew life support. Note that lunar regolith and the Mars atmosphere are not utilized. The route that the crew takes to get to Mars can be identified by looking at the flow path of "crew" in isolation. Likewise, the return route back to Earth is represented by the flow path of "crew\text{Re}". Figures 7 and 8 show that for the outbound leg, the crew stops over at geostationary transfer orbit (GTO) and Earth-Moon Lagrange point 2 (EML2) to refuel the vehicle with lunar oxygen and hydrogen instead of a direct flight between LEO and low Mars orbit (LMO) as proposed in DRA 5.0. On the return, however, the crew transfers directly from LMO to GTO for aerocapture and subsequent splashdown at PAC.

Second, it is found that all transfers are performed using LOX/LH\textsubscript{2} and that NTR is not used. Despite the superiority of NTR over chemical/aerocapture in DRA 5.0, it turns out that chemical propulsion using LOX/LH\textsubscript{2} is totally dominant in the baseline solution. This is because lunar ISRU produces a massive amount of oxygen and hydrogen, which more than makes up for the relatively low specific impulse of the chemical rocket. Among all arcs in the network, launch from KSC to LEO is by far the most energy-costly arc (\(\Delta V = 9.5\) km/s). This cannot be avoided. Due to the high \(\Delta V\) requirement to get out of the Earth’s deep gravity well and dense atmosphere, LEO is often referred to as being "halfway to anywhere in the solar system." Therefore, one wants to reach low Earth orbit as lightly as possible, and it is not difficult to imagine that in-situ resources available on orbit from elsewhere could help dramatically reduce the launch payload from the ground. There are two more points to note about propulsion system selection: (1) while the crew vehicle is not allowed to perform aerobraking at Mars arrival, everything else is transported to LMO using aerocapture, including the surface habitat, ISRU plant and spares, and the Earth return stage with propellant and

<table>
<thead>
<tr>
<th></th>
<th>LSP</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISRU plant deployed, kg</td>
<td>60,415</td>
<td>2,360</td>
</tr>
<tr>
<td>H\textsubscript{2}O produced (780 days), kg</td>
<td>1,291,056</td>
<td>50,428</td>
</tr>
<tr>
<td>Spares mass required (780 days), kg</td>
<td>12,911</td>
<td>504</td>
</tr>
</tbody>
</table>
crew provisions for the return leg; (2) while DRA 5.0 uses oxygen produced on Mars and methane brought from Earth for Mars ascent, the optimized solution relies completely on Mars-produced oxygen and hydrogen.

Lastly, the logistics paradigm is discussed. One thing to note is that the baseline solution only provides a flow network where the flow converges and diverges at each node, and it does not limit itself to any concrete transportation architecture or operation that implements these flows. Depending on how to interpret and translate the optimized flows, there might be different architectures and strategies to achieve the same flow. Flow convergence/divergence at a node could imply that some logistics infrastructure should be established at the node (a "service station" style). Another interpretation could be that two vehicles rendezvous, share the flight, and separate at some point (a "pickup bus" style). With that in mind, the resulting network flow must subsequently be translated into a potential mission architecture.

### Table 5 Baseline solution: total traffic at each node (outflow and inflow) in metric tons

<table>
<thead>
<tr>
<th>Node</th>
<th>LEO</th>
<th>GTO</th>
<th>EML2</th>
<th>LLO</th>
<th>LSP</th>
<th>LMO</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total outflow</td>
<td>590.4</td>
<td>453.6</td>
<td>878.4</td>
<td>998.1</td>
<td>1376.3</td>
<td>244.3</td>
<td>52.2</td>
</tr>
<tr>
<td>Total inflow</td>
<td>595.1</td>
<td>487.9</td>
<td>878.5</td>
<td>998.1</td>
<td>246.6</td>
<td>253.0</td>
<td>113.9</td>
</tr>
</tbody>
</table>

The resource distribution can be seen from Fig. 8. Earth hydrogen is launched to LEO and used for a transfer from LEO to GTO. Mars oxygen and hydrogen are used for Mars ascent. Lunar resources are distributed via low lunar orbit (LLO) and EML2 for wide use, including transfers in cislunar space, trans-Mars injection (TMI), Mars descent, and even trans-Earth injection (TEI) at Mars. Furthermore, the traffic (i.e., outflow and inflow) at each node in the baseline solution is listed in Table 5. Note that the inflow into LEO (595.1 metric tons) includes a TLMLEO of 271.8 metric tons in it. Considering that EML2 has high traffic and is the last stop before TMI, it seems intuitively reasonable that a propellant depot is placed at EML2. One possible transportation scenario is illustrated in Fig. 9. Between GTO and EML2, both outbound and return transfers are performed using lunar propellants. Therefore, one possibility is that an orbital transfer vehicle (OTV) fueled with lunar propellants serves as a shuttle running back and forth on this arc. Also,
LMO is where lunar oxygen and hydrogen wait for the crew vehicle’s arrival, which might encourage the deployment of another depot at LMO. Note that this is just one example among others that interprets the network flow of the baseline solution.

![Diagram of baseline solution]  
*Fig. 9 Baseline solution: one possible transportation scenario.*

To summarize, it is found that compared to Mars DRA 5.0, the baseline solution improves TLMLEO by a factor of more than 2, primarily by taking advantage of lunar ISRU. Following NASA’s Flexible Path idea, EML1/2 has received more attention in recent years as a potential location for an exploration gateway platform [32, 33]. It is interesting that the result obtained here merely by optimizing the logistics network in terms of TLMLEO is consistent with the gateway concept.

By the nature of this static flow analysis, however, the present GMCNF model does not consider the logical order of events; all the flows occur simultaneously. The propellant used to deliver the ISRU plant to a destination surface comes from the ISRU plant itself, which can only come later after the plant is actually deployed at the destination and operates for a while. For this reason, it is fair to interpret that TLMLEO used in this analysis represents the total launch mass for not the first mission but the nth mission in a campaign of recurring human missions to Mars. Note that this comparison is still valid because TLMLEO is not different between the first mission and the
Table 6 Baseline scenario: mission sequence

<table>
<thead>
<tr>
<th>Mission/event description</th>
<th>Arcs used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. [Uncrewed] Deliver ISRU plants to LSP/GC (one launch window before crew)</td>
<td>KSC-LEO-GTO-EML2</td>
</tr>
<tr>
<td>2. [Uncrewed] Conduct ISRU water production at LSP/GC (780 days)</td>
<td>LSP loop/GC loop</td>
</tr>
<tr>
<td>3. [Uncrewed] Deliver lunar propellant to EML2 and LEO</td>
<td>LSP-LLO-EML2-LEO</td>
</tr>
<tr>
<td>4. [Uncrewed] Send OTV with lunar propellant to GTO</td>
<td>EML2-GTO</td>
</tr>
<tr>
<td>5. [Crewed] Launch to LEO; refuel with lunar propellant</td>
<td>KSC-LEO</td>
</tr>
<tr>
<td>6. [Crewed] Transfer to GTO; rendezvous with OTV</td>
<td>LEO-GTO</td>
</tr>
<tr>
<td>7. [Crewed] Transfer to EML2; refuel with lunar propellant</td>
<td>GTO-EM2L</td>
</tr>
<tr>
<td>8. [Crewed] Perform TMI; transfer to LMO (180 days); perform MOI</td>
<td>EML2-LMO</td>
</tr>
<tr>
<td>9. [Crewed] Perform EDL; stay on Mars (540 days)</td>
<td>LMO-GC; GC loop</td>
</tr>
<tr>
<td>10. [Crewed] Ascend to LMO with Mars propellant; refuel with lunar propellant</td>
<td>GC-LMO</td>
</tr>
<tr>
<td>11. [Crewed] Perform TEI; transfer to GTO (180 days); perform aerocapture</td>
<td>LMO-GTO</td>
</tr>
<tr>
<td>12. [Crewed] Perform reentry; splashdown to PAC</td>
<td>GTO-PAC</td>
</tr>
</tbody>
</table>

Furthermore, this result must add a caveat that it can change greatly depending on the input parameters and assumptions as well as the objective functions. For this reason, the following sections attempt several different settings, based on the baseline problem presented in this section. Variant problems are solved that have different conditions and parameters in: (1) propulsion system, (2) ISRU availability, and (3) ISRU productivity, focusing on how TLMLEO and the network topology vary with these factors. While not discussed in this paper, some other system parameters (e.g., inert mass fraction, tank mass fraction, ECLSS closure level, etc.) can also have a significant impact on the results, even to the level that the resulting strategy changes qualitatively.

E. Propulsion System

This section solves the problems with some limitations or changes in the propulsion system as follows: (a) LOX/LH$_2$ not allowed, (b) aerocapture not allowed for orbital transfer, (c) lightweight aeroshell (aeroshell mass fraction of 15% as opposed to 37%), and (d) reusable TMI/TEI stage (no need to jettison). Note that other parameters and assumptions remain the same.
Table 7 compares the results with the baseline solution in terms of TLMLEO and the size of ISRU plant deployed at LSP and GC. Also, Figs. 10(a)-10(d) show the resulting network graphs.

As discussed earlier, Mars DRA 5.0 selected NTR as a leading propulsion system option because of its high specific impulse capability, which is twice that of a LOX/LH$_2$ chemical rocket. However, as opposed to DRA 5.0, LOX/LH$_2$ turns out to be dominant in the baseline solution. This is primarily because ISRU provides an abundance of oxygen, which greatly reduces the propellant that must be brought from Earth. To investigate the ISRU compatibility with NTR, case (a) prohibits use of LOX/LH$_2$. It is found that this case still uses lunar ISRU but it is greatly scaled down, and TLMLEO turns out to increase by 56.5% from the baseline. While this case still looks much improved from the DRA 5.0 NTR scenario by making more use of ISRU, it falls far short of the baseline scenario for the following reason. Although ISRU can produce hydrogen from water ice, it simultaneously produces much more oxygen (8 times more than hydrogen by mass). If NTR is used, only hydrogen is consumed, and a large amount of oxygen is left unused. On the other hand, since a LOX/LH$_2$ propulsion system burns hydrogen and oxygen at a mixture ratio of 5.88, it can consume hydrogen and oxygen in a well-balanced manner that is synchronized with ISRU water production.

Case (b) prohibits use of aerocapture for orbital transfer. This turns out to encourage the scale-up of Mars ISRU by a factor of more than 5. However, TLMLEO increases by 24%. On the other hand, case (c) assumes a lightweight aeroshell with a mass fraction of 15% of the vehicle mass, as assumed in [6, 30]. It is interesting that this change also encourages the scale-up of Mars ISRU because the lightweight aeroshell enables more ISRU plant to be delivered to Mars. TLMLEO decreases by 23.7% relative to the baseline.

Up to this point, it is assumed that each TMI/TEI module is jettisoned after it performs its burn. Case (d) allows for optional reuse of these modules. Within the scope of this comparison, this case only improves TLMLEO by 5.2%. However, this study considers only a single round-trip mission to Mars as a case study. If the subsequent missions could reuse the TMI/TEI modules just by refueling them in space thus avoiding to launch another set of vehicle stages, it would help reduce the launch mass from the ground in subsequent missions while it requires repositioning of those modules to the appropriate positions for next use.
Table 7 Summary of the solutions with various settings on propulsion system

<table>
<thead>
<tr>
<th>Scenario</th>
<th>TLMLEO, mt</th>
<th>ISRU plant, kg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LSP</td>
</tr>
<tr>
<td>DRA 5.0 – NTR</td>
<td>848.7</td>
<td>–</td>
</tr>
<tr>
<td>GMCNF Baseline</td>
<td>271.8 (± 0.0%)</td>
<td>60,415</td>
</tr>
</tbody>
</table>

**Propulsion system**

(a) No LOX/LH$_2$  
(b) No aerocapture  
(c) Lightweight aeroshell  
(d) Reusable TMI/TEI stage

<table>
<thead>
<tr>
<th>Scenario</th>
<th>TLMLEO, mt</th>
<th>ISRU plant, kg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LSP</td>
</tr>
<tr>
<td>(a) No LOX/LH$_2$</td>
<td>425.5 (+56.5%)</td>
<td>4,458</td>
</tr>
<tr>
<td>(b) No aerocapture</td>
<td>337.0 (+24.0%)</td>
<td>65,390</td>
</tr>
<tr>
<td>(c) Lightweight aeroshell</td>
<td>207.5 (−23.7%)</td>
<td>61,813</td>
</tr>
<tr>
<td>(d) Reusable TMI/TEI stage</td>
<td>257.7 (−5.2%)</td>
<td>75,401</td>
</tr>
</tbody>
</table>

**F. ISRU Availability**

This section adds some variations to ISRU availability. The baseline problem assumes that several ISRU options are available on the Moon (LSP) and Mars (GC) such as surface regolith, water ice, and atmosphere. In the baseline problem, ISRU on Phobos/Deimos is not considered because it is controversial, even though there is evidence that water ice may exist in the interior of Phobos/Deimos and the possibility of ISRU is discussed in [13, 14]. ISRU in a microgravity environment might be a challenge; however, it is worthwhile evaluating the potential utility of Phobos/Deimos by assuming the resource availability and ISRU feasibility. Therefore, this section discusses four variant problems as follows. Case (e) assumes that ISRU is available at all four locations: the Moon (LSP), Mars (GC), Phobos (PHOB), and Deimos (DEIM). Case (f) assumes ISRU on Mars (GC) only. Furthermore, the baseline scenario and other cases considered up to this point mainly utilize water ice because it provides both oxygen and hydrogen. Therefore, cases (g) and (h) assume the unavailability of water ice (H$_2$O) and only allow for oxygen production from surface regolith and/or Mars atmosphere. In case (g), ISRU (O$_2$) is available on the Moon (LSP) and Mars (GC), and in case (h), ISRU (O$_2$) is available at all four locations.

Table 8 compares the results with the baseline solution in terms of TLMLEO and the size of ISRU plants deployed at each ISRU node. Also, Figs. 10(e)-10(h) show the resulting network graphs. In case (e), all the four ISRU nodes are used. The baseline scenario relies heavily on lunar
ISRU, whereas this case relies on a more complex network with a relatively even distribution of ISRU systems between the four locations. TLMLEO improves by 18.5% with the introduction of Phobos/Deimos ISRU while risk should be a major issue in such a complex logistics network. The network flow shows that Mars ISRU resources are used for life support during the human stay on Mars and as propellant for Mars ascent, while the lunar/Phobos/Deimos resources are used for in-space transportation. Putting aside the actual feasibility/reasonability from other perspectives, it is interesting that part of the resources produced on Deimos are delivered back to GTO and LEO, and wait to be used for the crew and cargo outbound trip. Though it seems strange and non-intuitive at first, this is true at least computationally because in terms of $\Delta V$, LEO is closer to Deimos than to the lunar surface and even Earth’s surface.

Case (f) assumes that ISRU is only available on Mars. TLMLEO increases by as much as 49% from the baseline just by losing lunar ISRU. In other words, this result shows that the contribution of lunar ISRU in the baseline solution is significant. Compared to the DRA 5.0 NTR scenario, this case allows for extensive use of ISRU including water ice as well as use of LOX/LH2 for Mars descent/ascent, and as a result, TLMLEO reduces by more than half.

Cases (g) and (h) assume that water ice is not available. This could be the case if water ice rich landing coordinates are excluded on the Moon and on Mars. In both cases, TLMLEO increases significantly from the baseline scenario. One interesting thing is that as shown in Figs. 10(g) and 10(h), NTR comes into use for part of in-space transportation. At first glance, it seems paradoxical that the hydrogen-fueled NTR is used despite the fact that ISRU only provides oxygen in these cases while NTR is not used in the baseline scenario and other cases where ISRU provides hydrogen through water. However, the overall scale-down of ISRU implies that ISRU oxygen is of low value relative to ISRU water, and that NTR comes to play an important role by taking advantage of its high $I_{sp}$. Another thing to note is that lunar ISRU is not used in case (h), which indicates that lunar oxygen is dominated by Phobos/Deimos oxygen for Mars missions if assuming an equal level of productivity despite differences in the local environment.
Table 8 Summary of the solutions with various settings on ISRU availability

<table>
<thead>
<tr>
<th>Scenario</th>
<th>TLMLEO, mt</th>
<th>ISRU plant, kg</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LSP</td>
<td>GC</td>
<td>PHOB</td>
<td>DEIM</td>
</tr>
<tr>
<td>DRA 5.0 – NTR</td>
<td>848.7</td>
<td>–</td>
<td>1,131</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>GMCNF Baseline (LSP/GC)</td>
<td>271.8 (± 0.0%)</td>
<td>60,415</td>
<td>2,360</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td><strong>ISRU availability options</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e) LSP/GC/PHOB/DEIM</td>
<td>221.6 (−18.5%)</td>
<td>25,676</td>
<td>7,047</td>
<td>8,994</td>
<td>9,363</td>
</tr>
<tr>
<td>(f) GC only</td>
<td>404.9 (+49.0%)</td>
<td>–</td>
<td>14,012</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(g) LSP/GC (no H₂O)</td>
<td>416.3 (+53.2%)</td>
<td>7,874</td>
<td>1,674</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(h) LSP/GC/PHOB/DEIM (no H₂O)</td>
<td>356.7 (+31.2%)</td>
<td>–</td>
<td>1,674</td>
<td>786</td>
<td>3,747</td>
</tr>
</tbody>
</table>
Fig. 10 Cases (a)-(h): resulting network graphs.
Lastly, this section performs a sensitivity analysis to identify the solution dependence on ISRU productivity. The baseline problem assumes a linear ISRU resource production rate of 10 kg per year per unit plant mass. A previous study showed that ISRU plants follow economies of scale, meaning that larger plants would be less costly to achieve the same total production rate [8, 10]. With that, a production rate of 10 kg/year/kg might be too optimistic especially for smaller plants, even with technological advancement in the future. Therefore, this section investigates how the resulting strategy varies with the production rate, particularly in the case of a lower ISRU productivity. It is easy to imagine that ISRU is no longer beneficial when the production rate falls below a certain threshold.

Figure 11 plots the optimized TLMLEO (on the left axis) and ISRU system mass at LSP and GC (on the right axis) with respect to the ISRU resource production rate, which varies from 0 to 12 kg/year/kg. The point at which the ISRU system mass drops to zero is the threshold that determines whether ISRU is a worthwhile investment or not. For the lunar ISRU, the threshold is found at 1.9; for Mars ISRU, the threshold is found at 0.6. Therefore, for the production rate between 0 and 0.5, both lunar and Mars ISRU have no benefit in this context, and TLMLEO remains unchanged. Since this study assumes a production period of 780 days (about 2 years), an ISRU plant with a production rate of 0.5 or below cannot even produce its own mass in resources. Once ISRU comes into use at a production rate of 0.6, TLMLEO simply decreases monotonically with increasing productivity, which is intuitive. Between 0.6 and 1.8, only Mars ISRU is used. Lunar ISRU becomes beneficial above 1.9. Above 1.9, as the production rate increases, the lunar ISRU system mass goes up and down in a zigzag manner while the overall trend is up. This zigzag pattern is caused by the "network topology". Before and after each discontinuous point, major and minor changes occur in the transportation strategy, including route selection, propulsion system selection with optional aerobraking, propellant origin selection, and repositioning and reuse, at the level of individual commodities. For example, at an ISRU production rate of 3.5, the crew vehicle performs the TMI burn at GTO using NTR, while at a production rate of 3.6 or above, the crew vehicle performs TMI at EML2 using LOX/LH\textsubscript{2} with lunar resources. Above 3.6, lunar ISRU is strongly
favored over Mars ISRU. On the other hand, in each continuous segment (e.g., from 8.6 to 10.6), the ISRU system can be downsized with improved productivity while keeping the same network topology (transportation strategy).

![Graph showing TLMLEO and ISRU system mass with respect to ISRU productivity.](image)

**Fig. 11** Optimized TLMLEO and ISRU system mass with respect to ISRU productivity (ISRU only at LSP and GC).

To summarize, as the ISRU productivity improves, TLMLEO monotonically decreases while the optimized ISRU system mass exhibits a complex pattern along with the change in the network topology. The dominant propulsion system shifts from high $I_{sp}$ NTR towards ISRU-friendly LOX/LH$_2$. Lunar ISRU comes into use at a production rate of 1.9, and Mars ISRU at 0.6. Below these thresholds, the cost exceeds the benefit. Note that this result is found in the context of sending a single manned mission to Mars.

V. Conclusion

This paper develops an interdependent network flow modeling method used to determine optimal logistics network and transportation strategy for space exploration with optional in-situ resource utilization (ISRU). A complex interplanetary supply chain network for long-term exploration goes beyond previous work, particularly in the case where optional ISRU brings additional complexity.
to the network selection problem. Extending the classical network flow theory, this study developed a novel network flow model (the GMCNF method) by introducing three types of matrix multiplications (flow equilibrium, flow transformation, and flow concurrency) as well as allowing self-loops associated with nodes (graph loops) and parallel arcs between the same end nodes (multigraph). With this modification, the model can handle multiple commodities that interact with each other in various ways. A linear programming (LP) formulation allows a large number of runs without much computational effort, which is helpful for system-level trade studies during the mission concept development phase.

As a case study to demonstrate the GMCNF model, this paper applied it to human exploration of Mars. The baseline problem was solved first and the result was compared with NASA’s Mars Design Reference Architecture (DRA) 5.0. It was found that the baseline solution improves total launch mass to LEO (TLMLEO) by 68% from DRA 5.0, once the transportation and ISRU infrastructures are deployed and operational in the lunar vicinity and on the lunar surface. As opposed to DRA 5.0, chemical propulsion using LOX/LH$_2$ turned out to be the preferred transportation technology, synchronized with lunar ISRU water production as well as a cislunar logistics network. Further analysis on the propulsion systems and ISRU availability yielded the following key findings: (1) LOX/LH$_2$ is much more compatible with ISRU water production than NTR; (2) the use of aerocapture makes a significant contribution to reducing TLMLEO, encouraging the development of lightweight aeroshell/thermal protection system; (3) among several ISRU options, lunar water ice is the most valuable resource for Mars missions. ISRU productivity analysis revealed threshold values of the production rate (1.9 for lunar ISRU and 0.6 for Mars ISRU) where the lunar/Mars ISRU benefit exceeds the cost in terms of overall launch mass. That being said, a caveat must be added that these results can change greatly depending on parameters and assumptions used in the model as well as figures of merit upon which to optimize the logistics network and that reliance on TLMLEO as an exclusive figure of merit may yield misleading conclusions. The impact of the sensitivity of these results to the parameters and assumptions may outweigh the impact of introducing ISRU. Meanwhile, the emphasis of this paper is on demonstrating that the GMCNF method is able to provide an initial guide for logistics architecture including ISRU infrastructures in the early
phase of mission concept design.

The current GMCNF model will be able to serve as a front-end tool to the existing frameworks (e.g., SpaceNet), providing a network auto-generation capability. From a methodology perspective, future work will address the current limitations of the GMCNF model that primarily arise in three areas: risk analysis, model linearity, and time evolution of network topology. The current model assumes that all transports occur with certainty and that all demands are purely deterministic. Network robustness should be addressed so that the risks of node/arc failures could be considered in the optimization. One possible way of doing this is to introduce stochastic uncertainty on arcs and loops. The second limitation of the current model is model linearity. In reality, some commodities come in discrete sizes but in this study, they are all linearly scalable. Extending the model to the nonlinear regime would allow for a high fidelity model that might be required for detailed design and planning. The third limitation is that the current model simulates a static network flow for a given snapshot of supply/demand. The capability to optimize the time evolution of the network would enable optimization of the investment sequence and timing (e.g., staged deployment of infrastructure). This would then also enable a preprocessor for SpaceNet in a clear determination of a sequence of discrete flights with cargo manifest. From an application perspective, future work is targeted at analyzing a larger-scale space exploration campaign involving multiple destinations and spanning many years in consideration of additional options (e.g., solar electric propulsion, distant retrograde orbit) and other constraints (e.g., volume and power). Also, more detailed ISRU modeling and target setting for technology development can be another future work.

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