C. I. Lewis's S1 and S2

Lewis formulated five systems of modal logic of increasing strength: S1 to S5. Only the last two are “normal” systems, in a sense to be defined, but the weaker systems are of some interest. His idea was not to build modal logic on top of the standard extensional propositional logic, but to give axiom system from the ground up. He thought that Russell’s analysis of implication as material implication was unsatisfactory, and should be replaced by a different account. The standard account of negation, conjunction and disjunction were okay, in his view, and of course the material conditional is definable in terms of these other truth functional connectives, but Lewis thought of his strict implication as an alternative to the material conditional.

The Lewis systems were first formulated in the early years of modern symbolic logic, not long after the publication of Russell and Whitehead’s *Principia Mathematica*. The method was purely syntactic. Systems of axioms and rules were rigorously formulated, but the semantic interpretation was entirely informal.

I will give Lewis’s original axiomatization for S1, then state one additional axiom that, when added to S1 yields S2. Then I will give a more modern formulation for S2 that is equivalent to Lewis’s system.

The primitive symbols are propositional variables ‘p’, ‘q’, ‘r’, etc., parentheses, and three connectives: ‘~’ for negation, ‘∧’ for conjunction (Lewis used a dot), and ‘◊’ for possibility. The axioms use a defined connective, ‘⇒’ for strict implication, which has the following definition: \((\phi \Rightarrow \psi) =_{df} \sim\diamond (\phi \land \sim \psi)\). (Lewis used a hook symbol)

**Axioms**

1. \((p \land q) \Rightarrow (q \land p)\)
2. \((p \land q) \Rightarrow p\)
3. \(p \Rightarrow (p \land p)\)
4. \(((p \land q) \land r) \Rightarrow (p \land (q \land r))\)
5. \(p \Rightarrow \sim \sim p\)
6. \(((p \Rightarrow q) \land (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)\)
7. \((p \land (p \Rightarrow q)) \Rightarrow q\)

**Rules**

1. Substitution of provably necessary equivalents
2. Uniform substitution of sentences for sentence letters
3. Adjunction (If \(|- \phi \text{ and } |- \psi\) , then \(|- (\phi \land \psi)\) )
4. Strict detachment (If \(|- \phi \text{ and } |- (\phi \Rightarrow \psi)\) , then \(|- \psi\) )
S2 has all of the axioms and rules of S1, plus one additional axiom:

8. \((p \Rightarrow \neg p) \Rightarrow (p \Rightarrow q)\)

An Alternative Formulation of S2

Axiom schemata

A1. All tautologous sentences
A2. \(\Box \phi \supset \phi\)
A3. \(\Box(\phi \supset \psi) \supset (\Box \phi \supset \Box \psi)\)

Rules
R1. If \(|- \phi \supset \psi\), and \(|- \phi\), then \(|- \psi\)
R2. If \(\phi\) is an axiom, then \(|- \Box \phi\)
R3. If \(|- \Box(\phi \supset \psi)\), then \(|- (\Box \phi \supset \Box \psi)\)
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