Here is a summary of some terminology concerning frames, models and systems

A system is characterized by a class of models iff the system is sound and complete relative to that class of models.

That is, system S is characterized by class of models M if and only if

(1) every S-consistent set of sentences is simultaneously satisfied in some world of some model in M.

and

(2) every set of sentences that is satisfied in some world of some model in M is S-consistent.

A system S is characterized by a class of frames iff it is characterized by the class of all models that are models on one of the frames in the class.

A model is a model for a system S iff only S-consistent sets are satisfied in any world in the model. A frame is a frame for a system iff every model on that frame is a model for S.

The canonical model for a system S is the model that is defined as in the completeness proof that we discussed. That is, it is a model \( \langle W, R, v \rangle \) in which

(1) \( W \) is the set of all S-maximal-consistent sets of sentences

(2) \( R \) is defined as follows: \( xRy \) iff \( \{ \phi : \Box \phi \in x \} \subseteq y \)

(3) For all sentence letters \( \phi \), \( v_x(\phi) = 1 \) iff \( \phi \in x \),

The canonical model for any system S has these two distinctive features:

(1) the unit class of the canonical model for S characterizes S, since every S-consistent set of sentences is satisfied in some world of the canonical model.

(2) the canonical model is a distinguished model, which means that for any two distinct worlds in the model, there is a sentence that is true in one, and false in the other.

A system S is called a canonical system iff the frame for the canonical model for S is a frame for S. (For any system S, every set of sentences that is satisfied in a world in the canonical model for S will be S-consistent, but in a noncanonical system, we can hold the frame of the canonical
model fixed, and vary the interpretation function so as to make some S-inconsistent sentences true in some world of the frame.) Most systems we consider a canonical, but a few are not.

Soundness and completeness are defined relative to a class of models, or a class of frames, but we can also define notions of absolute completeness and incompleteness. Every normal modal system is characterized by some classes of models, since trivially, S will be sound and complete relative to the class of models that are models for S (as well as by the unit class of its canonical model). But not every system will be characterized by some class of frames. A system is absolutely incomplete if there is no class of frames that characterizes it.

For any system S, there is a class of frames that are the frames for S, but there may be sentences that are valid, relative to the class of all models defined on the frames for S that are not theorems of S. Or to put it in terms of satisfiability and consistency, there may be sentences that are S-consistent, but that cannot be satisfied in any model on a frame for S. How can this happen? Since a frame can be a frame for S only if every model on that frame is a model for S, a frame has to meet a more stringent condition to be a frame for S than it does to be a frame of some model for S. The additional restriction may yield a class of frames that validates some additional sentences beyond those validated by the class of all models for S.

An example of a noncanonical system is KW, defined as follows: K + (☐(☐ϕ ⊃ ϕ) ⊃ ☐ϕ)

An example of an incomplete system is KH, a system that is slightly weaker than KW, defined as follows: K + (☐(☐ϕ ≡ ϕ) ⊃ ☐ϕ)

The class of frames for KW is the same as the class of frames for the KH, but KH is strictly weaker. So there are sentences (1) that are not theorems of KH, but (2) that are valid in all frames that are frames for KH. So KH is absolutely incomplete because there is no class of frames that is characterized by KH.