Properties of the Laplace transform:

- **Definition:** \( f(t) \sim F(s) = \int_0^\infty f(t)e^{-st}dt \) for \( \text{Re } s > > 0 \).
- **Linearity:** \( af(t) + bg(t) \sim aF(s) + bG(s) \).
- **\( \mathcal{L}^{-1} \):** \( F(s) \) essentially determines \( f(t) \).
- **s-shift rule:** \( e^{at}f(t) \sim F(s - a) \).
- **s-derivative rule:** \( tf(t) \sim -F'(s) \).
- **t-derivative rule:** \( f'(t) \sim sF(s) - f(0+) \).

Formulas for the Laplace transform:

\[
\begin{align*}
1 & \sim \frac{1}{s} \\
\cos(\omega t) & \sim \frac{s}{s^2 + \omega^2} \\
\sin(\omega t) & \sim \frac{\omega}{s^2 + \omega^2} \\
t^n & \sim \frac{n!}{s^{n+1}}
\end{align*}
\]

1. Compute the Laplace transform of \( f(t) = 3te^{-2t} + t^2\sin(2t) \)

2. Compute the inverse Laplace transform of \( \frac{s + 1}{s^2 + 4} \) and of \( \frac{s}{s^2 + 3s + 2} \).

Describe some other functions (not continuous) with the same Laplace transforms as these.

3. Solve \( \dot{x} + 2x = e^{-t} \), \( x(0) = 1 \) using the Laplace transform. Then identify in your solution the transient and the particular solution \( x_p \) given by the Exponential Response Formula \( x_p = e^{rt}/p(r) \) for a solution to \( p(D)x = e^{rt} \).

4. Start with the formula for the Laplace transform of \( \cos(\omega t) \) and verify the formula for the Laplace transform of \( \sin(\omega t) \) using the \( t \)-derivative rule.