**Reaction-Diffusion equation**

\[
\frac{dC_i}{dt} = \Phi_i([C_j] \text{ reactions involving } i) + D_i \nabla^2 C_i
\]

- **Is it possible to have a stable fixed point** \( C_i^* \), **where the concentrations are uniform in space**, **which becomes unstable at finite wave-lengths?**

- **Linearize around the fixed point** \( C_i(\vec{r},t) = C_i^* + \varepsilon_i(\vec{r},t) \)

\[
\frac{d\varepsilon_i}{dt} = \sum_j \sum_k M_{ij} \varepsilon_j + D_i \nabla^2 \varepsilon_i
\]

- **Stability of the uniform state implies that all eigen-values of** \( M \) **are negative.**

- **Fourier transform** \( C_i(\vec{r},t) = \int d\vec{k} \ e^{i\vec{k} \cdot \vec{r}} \hat{C}_i(\vec{k},t) \)

\[
\frac{d\hat{C}_i}{dt} = \sum_j (\hat{M}_{ij} - \delta_{ij} D_i k^2) \hat{C}_i
\]

- **Is it possible for the above matrix to have a positive eigen-value at finite \( k \)?**

- **For 1 species, this is not possible because** \( \lambda(k) = \lambda(0) - D k^2 \) **is even more negative.**

- **It is possible for 2 and more species.**