8.251 Homework 2

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**Problem 1.** (15 points) More on Lorentz transformations.
Show that the set of four objects $\frac{\partial}{\partial \xi^\mu}$ transform under Lorentz transformations in the same way as any four objects $p_\mu$ defining a four-vector with lower indices do (it suffices to verify this for boosts). Thus, partial derivatives with respect to conventional upper-index coordinates $x^\mu$ behave as a four-vector with lower indices – as reflected by writing it as $\partial_\mu$.

**Problem 2.** (20 points) Maxwell equations in four dimensions.
(a) We said in class that the vanishing of the object $T_{\mu\lambda\nu}$

$$T_{\mu\lambda\nu} \equiv \partial_\mu F_{\lambda\nu} + \partial_\lambda F_{\nu\mu} + \partial_\nu F_{\mu\lambda} = 0,$$

encodes two of the Maxwell equations. Show that $T_{\mu\lambda\nu}$ is totally antisymmetric. How many independent equations do we get by setting this object to zero? Show explicitly that the two source-free Maxwell equations emerge precisely.

(b) We said in class that

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \frac{4\pi}{c} j^\mu,$$

encodes the other Maxwell equations. Show this explicitly.

**Problem 3.** (20 points) E&M in three dimensions.
(a) Consider both the standard Maxwell equations and the force law in four dimensions and find the reduced equations in three dimensions $(t, x, y)$ by assuming that there cannot be forces in the $z$ direction, and that no field can depend on the $z$-direction.

(b) Repeat the analysis of three-dimensional E&M by starting with the covariant equations beginning with $A^\mu = (\Phi, A^1, A^2)$ and examining $F_{\mu\nu}$, the Maxwell equations (shown in the problem above) and the equation of motion, as discussed in the previous homework, problem 4.

**Problem 4.** (30 points) Gravitational field of a point mass in compactified five dimensional world.
Consider five dimensional space-time with space coordinates $(x, y, z, w)$ not yet compactified and consider a point mass of mass $M$ located at the origin $(x, y, z, w) = (0, 0, 0, 0)$.

(a) Find the gravitational potential $V(r)$ due to this point mass. Here $r = (x^2 + y^2 + z^2 + w^2)^{1/2}$, and your answer should be in terms of $G^{(5)}$. You may use the equation $\nabla^2 V = 4\pi G^{(5)} \rho_M$, and the divergence theorem. Get your constants right – for this you will need to find out the volume of the three sphere.

Now let $w$ become a circle with radius $a$ keeping the same mass at the same point.

(b) Write an exact expression for the gravitational potential $V(x, y, z, 0)$. This potential is in fact a function of $R \equiv (x^2 + y^2 + z^2)^{1/2}$, and can be written as an infinite sum.

(c) Show that for $R \gg a$ the above gravitational potential takes the form of a four-dimensional gravitational potential, with Newton’s constant $G^{(4)}$ given in terms of $G^{(5)}$ as calculated in class. [Hint: turn the infinite sum into an integral].

These results confirm both the relation between the four and five dimensional Newton constants for a compactification, and the emergence of a four-dimensional potential at distances large compared to the compactification size.