Suggested Reading

Scheck Chapter 7, Goldstein Sections 12-1 to 12-3.

Problems

Problem 9.1 (25 pts)

A book can be found in one of the orientations as shown in Figures 1a-d.

a. Write down the triplets of Euler angles that correspond to the rotations that start in position 1a and finish in positions 1b, 1c and 1d respectively.

b. Find triplets of Euler’s angles that correspond to rotations 1b to 1c and 1c to 1d.

Problem 9.2 (25 pts)

Consider a uniform rotationally symmetric three-dimensional ellipsoid with half-axes of \((a, a, b)\) and mass \(M\). A small mass \(m << M\) collides with the ellipsoid completely inelastically and attaches
itself to the ellipsoid. Before the collision the mass \( m \) travelled with speed \( v \) in the direction parallel to the \( y \) axis and with impact parameter \( \rho_1 \) along \( x \)-axis and \( \rho_2 \) along \( z \)-axis, see Figure 2.

Describe the motion of the ellipsoid after the collision. Assume that the mass \( m \) is so small that the rotational inertia of the ellipsoid remains unchanged.

\[
\begin{align*}
\text{Problem 9.3 (25 pts)}
\end{align*}
\]

Mass points of mass \( m \) are mounted on an elastic massless string of tension \( T \). The masses have equal distance to each other \( a \). There are \( N \) masses. The masses can move in the direction perpendicular to the rest position of the string but not along the length of the string, see Figure 3.

\[
\begin{align*}
\text{a. Calculate the increase of the length of the string between subsequent mass points } i \text{ and } i + 1 \\
\text{as a function of transverse displacement } q_i \text{ and } q_{i+1} \text{ for small displacements.}
\end{align*}
\]

\[
\begin{align*}
\text{b. Write kinetic and potential energy for the system of } N \text{ points. Write Lagrangian for the system.}
\end{align*}
\]

\[
\begin{align*}
\text{c. Convert the discrete Lagrangian to the Lagrangian density of a similar continuous system by evaluating a limit where the number of points } N \text{ is increased while the total length of the string is being kept constant.}
\end{align*}
\]
d. Write the equations of motion for the scalar field describing the transverse displacements $q(x,t)$.

**Problem 9.4 (25 pts)**

In class we showed that the wave equation in solid is:

$$\frac{\partial^2 \eta_i}{\partial t^2} - \frac{Y}{\lambda} \nabla^2 \eta_i = 0$$

where $\eta_i(x,t)$ is the displacement of the solid molecules at $(x,t)$. Show the wave equation has solution

$$\vec{\eta} = \vec{A} \sin(\vec{k} \cdot \vec{x} + \omega t + \phi)$$

and find a relation between $\omega$ and $\vec{k}$. 