Suggested Reading
Scheck Chapter 1

Problems

Problem 1.1 (20 pts)
A block of mass $m$ slides down a frictionless track with a vertical loop as shown in Figure 1. The block is released from rest at a height $H$ above the bottom of the loop. Gravitational acceleration is $g$ and the radius of the loop is $R$.

a. What is the magnitude and direction of the velocity and the force on the mass at the bottom of the loop (point $A$).

b. Assume that the mass $m$ moves sufficiently fast such that it remains on the track all the way to the topmost position (point $B$). What is the magnitude and direction of the velocity and the force on the mass at that point?

c. What is the minimum height $H_{\text{min}}$ such that the mass still remains on the track at point $B$?

d. Assume that the mass was released at some height $H_C < H_{\text{min}}$ and that it lost contact with the track exactly at point $C$ located at an angle $\theta = 45^\circ$ from the horizontal. Find $H_C$.

e. What is the trajectory followed by the mass immediately after it lost contact with the loop? Find $x(t)$ and $y(t)$ assuming that the mass lost contact at $t = 0$. 
Problem 1.2 (20 pts)
Consider the motion of a material point where

\[ x(t) = h_1 \sin(\omega t + \gamma_1) \]
\[ y(t) = h_2 \sin(\omega t + \gamma_2) \]

where \( h_1, h_2, \gamma_1, \gamma_2 \) are constant.

a. Show that the trajectory is always an ellipse with a center at \((x, y) = (0, 0)\).
b. Show that the hodograph of the trajectory is also an ellipse. The hodograph is a plot of the velocity of particle as a function of time. The velocity is represented by the vector from the origin to the point on the hodograph.

Problem 1.3 (20 pts)
Find the trajectory \( y(x) \) of a dog \( D \) following a hare \( H \). The hare is running along a straight line with constant velocity \( \vec{c} \). The dog is following the hare with constant speed with direction of the velocity vector always pointing towards the hare. At \( t = 0 \) the dog was at a distance \( a \) from the hare. Use the coordinate system shown in Figure 2.

\begin{center}
\includegraphics[width=0.5\textwidth]{figure2}
\end{center}

Problem 1.4 (20 pts)
Consider tidal forces in the Earth-Moon system. Write an expression for the equilibrium form of the surface of an ocean covering the whole Earth applying conditions required for static equilibrium at a given instance. What is the maximum and minimum deviation of this equilibrium surface from an ideal sphere and where do they occur?
Problem 1.5 (20 pts)

Find the solutions to the equation of motion of mass $m$ under the influence of force

$$\ddot{\mathbf{r}}(\mathbf{r}) = k^2 \mathbf{r}$$

where $k^2$ is a constant and $k^2 > 0$