This practice Second Midterm is made up of some questions from the final Spring 2004 final exam plus some other problems. This year the flavor of the exam will be a little different, more like the first exam. Note both Max Flow and Gomery Cuts are NOT on the set of Practice Problems but are covered on the exam.

**Problem 1**
Find the shortest path from node 1 to node 4 using Dijkstra’s algorithm in the graph below. Show your work. Be sure to clearly state the order in which nodes are selected and the distance labels of nodes at each iteration.

![Figure 1. A shortest path Network](image)

**Problem 2 (Game Theory).**
The following matrix defines the row player payoffs for a 2-person zero-sum game.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

a. Formulate a linear program that will find an optimal mixed strategy for the column player.

b. Use the graphical method to determine the optimal randomized strategy and optimal payoff for the column player. If the column player plays this strategy, what is the expected payoff to the column player? What is the expected payoff to the row player?
Problem 3
The graph below describes a minimum cost flow problem. The number associated with each arc is the cost of the arc. In addition, there is an upper bound of 4 on each arc flow. The supplies are as follows: \( b(1) = 6; \ b(2) = b(3) = b(4) = 0; \ b(5) = -6. \) Formulate this minimum cost flow problem as an LP.

![Network for a minimum cost flow problem](image.png)

Figure 3. The network for a minimum cost flow problem.

Problem 4
Consider the following knapsack, capital budgeting problem.

Maximize: \[ 10 x_1 + 9 x_2 + 8 x_3 + 7 x_4 + 6 x_5 \]
Subject to: \[ 15x_1 + 13 x_2 + 11x_3 + 10x_4 + 9x_5 \leq 30. \]
\[ x_j \in \{0, 1\} \text{ for } j = 1 \text{ to } 5. \]

List 5 possible cover constraints for this problem.

Problem 5
While running a branch and bound algorithm, eventually one gets to the node 27

Corresponds to the problem:

Minimize: \[ c_1 x_6 + c_2 x_7 + 15 \]
Subject to: \[ a_1 x_6 + a_2 x_7 \geq 15 \]
\[ x_6 \in \{0, 1\}, x_7 \in \{0, 1\} \]

The current incumbent has an objective value of 34. Note that the objective is to minimize.

In the following, give example values for the parameters \( c_1, c_2, a_1, \) and \( a_2 \) that satisfy the conditions given (assume integer values for all parameters).

a. Give conditions on the parameters so that node 27 of the tree is fathomed because of infeasibility.
b. Give conditions on the parameters so that node 27 of the tree is fathomed because of integrality.

c. Give conditions on the parameters so that node 27 of the tree is fathomed because of the bound.

d. Give conditions of the parameters so that node 27 of the tree is not fathomed.

Problem 6:
Houseco Developers is considering erecting three office buildings. The time required to complete each and the number of workers required to be on the job at all times are shown in Table 1. Once a building is completed, it brings in the following amount of rent per year: building 1, $50,000; building 2, $30,000; building 3: $40,000. Houseco faces the following constraints:

   a. During each year, 60 workers are available.
   b. At most, one building can be started during any year.
   c. Building 2 must be completed by the end of year 4.

Formulate an IP that will maximize the total rent earned by Houseco through the end of the fourth year.

Table 1

<table>
<thead>
<tr>
<th>Building</th>
<th>Duration of Project (Years)</th>
<th>Number of Workers Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>20</td>
</tr>
</tbody>
</table>

Problem 7:
A certain scooter requires 3 parts: the base, the stem, and a set of wheels. The production work required to produce these parts can be done on 3 different machines: a thingamajig, a blackbox, or a whatchamacallit. The parts can be processed on any of the machines (i.e., the machines can be run in parallel). The basic data are as follows:

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>Number of Machines</th>
<th>Bases per machine per hour*</th>
<th>Stems per machine per hour*</th>
<th>Sets of wheels per machine per hour*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thingamajig</td>
<td>4</td>
<td>10</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Blackbox</td>
<td>5</td>
<td>20</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Whatchamacallit</td>
<td>2</td>
<td>30</td>
<td>80</td>
<td>50</td>
</tr>
</tbody>
</table>

*This rate is the number of parts the machine can make if devoted exclusively to that particular part.

a. Let $B_i$ = number of bases per hour made using a machine of type $i$, let $S_i$ = number of stems per hour made using a machine of type $i$, and let $H_i$ = number of sets of wheels per hour made using a machine of type $i$, where $i = \{T$ (thingamajig), $X$ (blackbox), $W$ (whatchamacallit) $\}$. Formulate an LP that allocates work to the machines in order to obtain the maximum number of completed scooters per hour. Let $C$ be the number of
completed scooters per hour. Thus the number of bases, stems, and sets of wheels completed per hour is also C. (You may assume that it is exactly C.)

The objective function is: Max C. What are the constraints? The only decision variables should be BT, BX, BW, ST, SX, SW, HT, HX, HW and C. You may assume that all variables are permitted to be fractional.

b. Suppose that you wanted to add the constraint that a blackbox could make bases, or it could make stems, or it could make wheels, but it would not be permitted to make two of these item types. Define new variables yB, yS and yH, where yB = 1 if all five blackboxes makes bases, yS = 1 if all five blackboxes makes stems, and yH = 1 if all five blackboxes makes wheels. The variables are 0 otherwise. Modify your formulation from part a to incorporate this new constraint using only variables defined in part a and the variables yB, yS and yH.

NOTE: The models for problems 1 and 4 should not involve any non-linear constraints. In particular, no constraint should involve a term that includes two decision variables being multiplied together. No credit will be given for non-linear constraints.

Problem 8: Game Theory (From Last Years Midterm)

After publishing his latest book, Bill earned $500,000, which he wants to invest in stocks. Bill believes the investment will be affected by whoever wins the Presidential election in November. He has narrowed down his investment to three stocks, labeled C, D and E. In this election, the candidates are denoted as B, K, and N.

The performance of each stock under each candidate is show in the table below:

<table>
<thead>
<tr>
<th>Candidates</th>
<th>B</th>
<th>K</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.5</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>D</td>
<td>1.1</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>E</td>
<td>0.9</td>
<td>1.2</td>
<td>1.4</td>
</tr>
</tbody>
</table>

For example, the table says that if Bill invests $1,000 in stock D, and candidate K is elected, then Bill’s position in stock D will be worth $1,300 after one year.

Bill is by nature a pessimist, and assumes that no matter how he invests his money, the candidate least favorable to his investments will win. So, he wants to invest conservatively, so as to maximize his payment over the next year in the worst possible case. We will call his objective the maximin objective, since his goal is to maximize the payment in the worst case.

a. If Bill were required to invest all his money in one stock, which stock would he invest in?
b. Let \( p_C, p_D, \) and \( p_E \) denote the proportion of the money invested in stocks \( C, D, \) and \( E. \) Formulate an LP to compute the optimal maximin strategy for Bill. It must be in the form of a linear program.

c. Suppose that the pessimistic Bill decides that candidate \( N \) really has no chance whatsoever, and so eliminates that column from consideration. Under this scenario Stock \( E \) is dominated by Stock \( D. \) Determine what proportion of his $500,000 Bill should allocate to Stocks \( C \) and \( D \) for his optimal maximin portfolio. Please show your work.

You may find it helpful to show your work using 2-dimensional graphical analysis.