Education and the Family: Some Notes

The goal is here is to understand the relationship between educational investment, family decision-making and credit constraints.

1 The Basic Model

Production Technology: Consider a world where there is only one final good but two types of human inputs—skill and unskilled labor. The final good is produced by combining the two types of inputs using a technology:

\[ y = f(H, L, \alpha) \]

where \( H \) and \( L \) are, respectively, the amount of skill and the amount of unskilled labor used in production, and \( \alpha \) is a parameter describing the nature of the technology. We assume that the production function exhibits constant returns with respect to the two inputs together but diminishing returns with respect to each individual input. This formulation also imposes the restriction that all levels of human capital are perfect substitutes—one of the insights of the paper by Mookherjee and Ray, cited above, is that relaxing this assumption introduces an additional source of persistent inequality.

Labor Supply: Each person in the economy is assumed to own one unit of unskilled labor. In addition they own a certain number of units of skills, which correspond to the amounts they invested in human capital.

The Life Cycle: Agents in this economy live for three periods. In the first period of their lives there is no consumption or work: all they do is acquire skills. The second period is when people work and earn an income. We assume that labor has no disutility associated with it. This income can be spent on current consumption or saved and invested for consumption in the next period, when they no longer have wage income. In the second period of their lives they also give birth to exactly one child. The population therefore stays unchanged over time and each cohort is assumed to be size one.

Human Capital: Human capital is produced using a combination of human capital and unskilled labor. The cost of acquiring \( h \) units of human capital is \( \gamma s(h, h^-) \) units of human capital and \( (1 - \gamma) s(h, h^-) \) units of unskilled labor, where \( 0 \leq \gamma \leq 1 \) and \( h^- \) is the level of human capital of the parent of the person who is acquiring the human capital. We assume that \( \frac{\partial s}{\partial h} > 0, \frac{\partial^2 s}{\partial h^2} > 0, \frac{\partial s}{\partial h^-} < 0, \frac{\partial^2 s}{\partial h^-^2} > 0, \) and \( \frac{\partial S(h)}{\partial h} > 0, \) where \( S(h) = s(h, h) \). The second and third assumptions tell us that the cost function is increasing and convex, therefore ruling out non-convexities. The next two assumptions capture the idea that the family atmosphere matters—children of skilled parents find it easier to acquire skills—though at a diminishing rate. The last assumption imposes the condition that the increased cost from increasing \( h \) is not outweighed by the reduction in...
cost coming from the fact that the next generation will now have more education and therefore find it easier to educate their children.

**Markets:** Throughout this paper we will make the assumption of perfectly competitive markets for labor and skills, with the price of labor at time $t$ being $w^L_t$ and that of skill $w^H_t$. Except in the next section we will make the extreme assumption that there are no capital markets and no assets other than human capital.

**Policy:** Policy instruments, we assume that there is an educational subsidy of $c_0 + c_1 E$, where $E = (\gamma w^H_t + (1 - \gamma) w^L_t) s(h_{t+1}, h_t)$ is the amount the family spends on education. This is financed partly or entirely by a tax on the earning members of society that is partly lump-sum and partly a function of the taxpayer’s human capital: $T = \tau_0 + \tau_1 h_t w^H_t$: Lump-sum taxes are of course standard while the tax on human capital earnings will turn out to be a simple way to introduce redistributive taxation. In order to get sharper results and limit the number of cases, we will, for the most part, focus on the case where we start from a situation where the family was already spending some amount on education and then look at the effect of very small changes in the taxes and subsidies. This allows us to avoid the issue of corner solutions.  

**Preferences:** We allow people to get utility both from private and collective (family-level) outcomes. Private outcomes includes both material consumption and symbolic consumption. Material consumption is the consumption of the one final good when one is middle-aged and when one is old: $U^M_t$, the utility from material consumption accruing to a person of generation $t$, is $U^M_t(c_t, p_{t+1})$, where $c_t$ is his consumption in middle-age and $p_{t+1}$ his consumption in the last period of his life. Symbolic consumption, covers things like the “warm-glow” of giving to one’s children (Andreoni, 1989), pride in having children who are well-educated or rich and the pleasure of being able to say that one’s children go to an expensive school. In other words, the utility from private outcomes may take the form:

$$U^S_t = U^S_t(h_t, h_{t+1}, c_{t+1}, (\gamma w^H + (1 - \gamma) w^L)S(h_{t+1})),\$$

where $h_t$ is the human capital level of current generation, $h_{t+1}$ is that of the next (“my son has a Ph.D”), $c_{t+1}$ is the consumption level of the next generation (“my daughter drives a Porsche” and $(\gamma w^H + (1 - \gamma) w^L)S(h_{t+1})$ is the amount spent on the education (“my son goes to Exeter”). Total private utility is therefore $U^P_t = mU^M_t + sU^S_t$.

The utility from collective outcomes, is given by

$$U^C_t = \sum_{s=1}^{\delta_s} U^P_{t+s} + \sum_{s=1}^{\delta_s} U^C_{t+s}.\$$

Finally let $U$ represent an individual’s total utility, i.e. $U_t = U^P_t + cU^C_t$.

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1This formulation does however come with the cost that there is no way to distinguish between educational subsidies and general income subsidies since even if the subsidy were earmarked for education, the family could always cut back on what it was already spending on education.
While this formulation is relatively general, it still imposes a number of stringent restrictions. In particular we have assumed additive separability at a number of different levels and we have ruled out children caring about the welfare parents.

A special case of this is the standard Barro-Becker formulation of altruism: This is the case where \( c = 1, \delta_1 = \delta_1 = \delta \) and \( \delta_s = \delta_s = 0 \) for \( s > 1 \). We will call this case perfect altruism to distinguish it from the case where either \( \delta_s \neq \delta_s \), or at least one of \( \delta_s \) and \( \delta_s \neq 0 \), for \( s > 1 \), which we call imperfect altruism. And even more special case is one where symbolic consumption is ruled out i.e. \( s = 0 \) and \( m = 1 \). It is easily checked that this yields the standard Barro-Becker preferences, which is a utility function of the form

\[
U_t = \sum_{s=0}^{\delta} \delta^s U^M(c_{t+s}, p_{t+s+1}). \tag{1}
\]

**Intergenerational Contracting:** Since each generation may have to rely on educational investment decisions taken by previous generations, intergenerational contracts can potentially play an important role in this analysis. Suppose, for example, generation \( t \) has to rely on generation \( t - 1 \) for its human capital investment. But if generation \( t - 1 \) does not care about the well-being of the next generation, why would it invest? One possible solution is that generation \( t \) contracts with the previous generation to provide it with old-age care \( (p_t) \) in return for the human capital investment \( (h_t) \). This would clearly provide some incentives to invest. However, the amount of old-age care generation \( t \) is happy to provide will depend on how much he gets to consume, \( c_t \), which, given that he has a budget constraint, clearly depends on how much he will have to invest in the human capital of the next generation \( (h_{t+1}) \). In other words, it will depend on the contract between generation \( t \) and generation \( t + 1 \), which, in turn, will depend on the contract between generations \( t + 1 \) and \( t + 2 \), etc.

The same issues arise even when there is some altruism. The basic point is that the preferences of the current generation over future outcomes can be very different from that of the generation who has control over that decision. Of course, there could be contracts (implicit or otherwise) between generations which would minimize these issues but there are obvious difficulties with contracts that span many generations.

The fact that there are externalities across contracts, suggests that the ideal contract may be one that covers all the generations. Under such a contract, \( \{p_t, c_t, h_{t+1}\}_{t=0}^{\infty} \) will be simultaneously chosen to maximize \( \Sigma_{t=0}^{\infty} \lambda^t U_t \). Obviously, one should not think of this as a real contract: It is perhaps best thought of as a norm that binds all generations of particular dynasty. This is what we will call a **complete contract**.

This is obviously an extreme assumption. One alternative would be to go to other extreme—simply assume that intergenerational contracts are impossible, i.e. **no contracting**. Then each generation would have to be assigned control rights over a set of decisions. We impose the sensible restriction that this set should not include decisions about actions taken when they are not living. We define the **potential control set** of generation \( t \) \( D_t = \{p_{t-1}, p_t, p_{t+1}, c_{t-1}, c_t, c_{t-1}, h_t, h_{t+1}, h_{t+2}\} \).
The actual control rights would be an allocation of a subset of $D_t$ to generation $t$ in such a way that each generation gets to take the exact corresponding set of decisions.\footnote{In other words if generation $t$ gets to decide on a vector $X_t = \{p_t, c_t, h_t\}$ (say) then generation $t + s$ will get to decide on a vector $X_{t+s}$.}

A third alternative, which allows some scope for contracting, is to assume bi-generational contracting—we assume that generation $t$ and $t + 1$ are jointly allocated control over a subset of $D_t \cup D_{t+1}$ in such a way that each pair of generation gets to take the exact corresponding set of decisions. They then choose the contract to maximize a weighted average of their utilities, $U_t + \lambda U_{t+1}$, taking as given the contract between generations $t$ and $t-1$. Note that this makes no contracting a special case of bi-generational contracting where $\lambda = 0$.

### 2 The Benchmark: The Beckerian Model

This is the case of our model defined by standard preferences, i.e., no symbolic consumption and perfect altruism combined with perfect credit markets and no contracting. The first two assumptions imply that the preferences of each generation is given by 1. Perfect credit markets amount to assuming that anyone can borrow and lend as much as they want at the market interest rate $r$. Under this assumption, each dynasty faces the intertemporal budget constraint:

$$
\omega_{t+1} = r_t(\omega_t - c_t - p_t + w_t^H) + h_t(1-\tau)w_t^H - (\gamma w_t^H + (1-\gamma)w_t^L) s(h_{t+1}, h_t)(1-e_1) + e_0 - \tau_0), \forall t.
$$

where $\omega_t$ is the starting wealth of the $t^{th}$ generation. The term $(\gamma w_t^H + (1-\gamma)w_t^L) s(h_{t+1}, h_t)(1-e_1)$ represents the investment in the next generation’s human capital measured in units of the good.

Since there is no contracting, each generation is assigned control rights over its own consumption when middle-aged ($c_t$) and old ($p_{t+1}$), and the education of its child ($h_{t+1}$). Maximizing the utility function given in 1 under 2 gives us:

$$
\partial U_m(c_t, p_{t+1})/\partial c_t = \delta r_t \partial U_m(c_{t+1}, p_{t+2})/\partial c_{t+1}
$$

$$
\partial U_m(c_t, p_{t+1})/\partial p_{t+1} = \delta r_t \partial U_m(c_{t+1}, p_{t+2})/\partial p_{t+2}
$$

$$
(\gamma w_t^H + (1-\gamma)w_t^L) s_1(h_{t+1}, h_t)(1-e_1)
$$

$$
= \frac{1}{r_t^2} [w_{t+1}^H (1-\tau_1) - (\gamma w_{t+1}^H + (1-\gamma)w_{t+1}^L)s_2(h_{t+2}, h_{t+1})(1-e_1)].
$$

Note that $h_{t+1}$ is completely determined by the last equation, taking as given the current endowment $h_t$ and its future expected level, $h_{t+2}$. Remarkably no utility term enters this equation. This has two implications: First, there are no income effects, since the choice of $h_{t+1}$ is unaffected by whether the marginal utility of consumption is high or low. Second, parental preferences do not affect
the decision to invest in human capital. Both these results follow from the fact that under perfect capital markets, the decision to invest in human capital can be analyzed entirely in terms of its net present value of income—the person who invests in his son’s education today can immediately borrow back the implied increase in his son’s future income and consume it.\footnote{However, while there are no income effects in this economy, there can be a parental human capital effect which in the data may look like a wealth effect: for example, if \( s_{12} < 0 \), an increase in \( h_t \), keeping \( h_{t+2} \) fixed, lowers the marginal cost of investing in education (without affecting the benefits) and therefore raises \( h_{t+1} \). In fact, the amount of human capital in all generations will go up. The increase in \( h_{t+1} \) causes \( h_{t+2} \) to go up, which causes \( s_2(h_{t+2}, h_{t+1}) \) to go down, which encourages further increases in \( h_{t+1} \), etc.}

**Observation 1:** In a model with no symbolic consumption, perfect altruism, no contracting and perfect credit markets, investment in human capital neither depends on how much money the parents have nor on their preferences.

### 3 Beyond the Beckerian Model

#### 3.1 The role of credit constraints

Let us now consider a world where there are no credit markets, or at least the existing credits markets are too inefficient to be of relevance to most people. At this point we impose no restrictions on the class of preferences beyond what has already been imposed in section 2. In this world, people invest in their children’s schooling using their family resources. Moreover, investing in education is the only investment opportunity available to tfamily. There are, however, also taxes and subsidies, just as in the previous section. Therefore, its budget constraint in any period \( t \) can be written as:

\[
 w_t^L + h_t w_t^H (1 - \tau_1) - (\gamma w_t^H + (1 - \gamma) w_t^L) s(h_{t+1}, h_t)(1 - c_1) - p_t + c_0 - \tau_0 = c_t. \tag{6}
\]

This constraint holds for every \( t \). Moreover, consumption is assumed to be always non-negative in order to make the credit constraint meaningful.

This assumption rules out the possibility that teh decision to invest in human capital could be evaluted in terms of its effect on the present value of income—changing the level of investment must necessarily alter the distribution of consumption over time and/or across people. Marginal utilities of consumption must therefore enter the calculation that determines investment in education, and since changes in income have an effect on the marginal utility of consumption, they will affect investment in human capital. More specifically, from 6 it is clear that any increase in current income (say \( c_0 \) or \( w_t^L \) has gone up) must either go into a higher \( h_{t+1} \) or a higher \( c_t \) or \( p_t \). Suppose \( h_{t+1} \) did not go up. Then at least one of \( c_t \) and \( p_t \) would have to go up and the corresponding marginal utilities would have to go down. Yet, because of our separability assumptions, the marginal gain from additional investment in human capital is unchanged and therefore the new outcome with \( h_{t+1} \) unchanged and \( c_t \) or \( p_t \) having gone up, cannot be an optimum. Therefore \( h_{t+1} \) must go up. An increase in income
must be associated with an increase in investment in human capital. Moreover the extent to which it would go up will clearly depend on the relative weight given to these various outcomes in the collective preferences and therefore will typically depend on parental preferences.

This logic is quite easy to formalize, but the wide range of preferences permitted here makes it notationally cumbersome. We therefore limit ourselves to stating the implied result informally:

**Observation 2:** As long as credit markets are imperfect, two dynasties with different income levels and/or different preferences will typically invest different amounts in the human capital. This remains true even if there is perfect altruism and no symbolic consumption.

### 3.2 The role of non-standard preferences

In this section we go back to the assumption of perfect credit markets but now expand the set of preferences to allow for symbolic consumption and imperfect altruism. This is not entirely straightforward, in particular because even small departures from the Beckerian formulation can lead to unexpected complexities.

To see this combine the Barro-Becker assumption about preferences (i.e. the utility function of generation $t$ is $\sum_{s=0}^{\infty} \delta^s U^M(c_{t+s}, p_{t+s+1})$) with the assumption of bi-generational contracting (instead of no contracting) Let generations $t$ and $t+1$ have control over $c_t, p_{t+1}$ and $h_{t+1}$. Then generations $t$ and $t+1$ will want to maximize $\sum_{s=0}^{\infty} \delta^s U^M(c_{t+s}, p_{t+s+1}). + \lambda \sum_{s=1}^{\infty} \delta^{s-1} U^M(c_{t+s}, p_{t+s+1}),$ which gives them an effective utility function

$$U^M(c_t, p_{t+1}) + (\lambda + \delta) \sum_{s=0}^{\infty} \delta^s U^M(c_{t+s}, p_{t+s+2}).$$

This utility function builds in a form of hyperbolic discounting. As is well-known from the literature (Harris-Laibson (2000)) this generates decision rules that are not time-consistent and therefore the current generations in taking their decisions have to take account of the reaction function of the next set of decision-makers. The Beckerian model avoids this issue by making the very specific assumption that $\lambda = 0$.

We now consider a model where there is symbolic consumption of educational spending and potentially imperfect altruism.

$$U^P_t = U^M(c_t, p_{t+1}) + U^S((\gamma w^H_t + (1 - \gamma) w^E_t)s(h_{t+1}, h_t)).$$

$$U^C_t = \sum_{s=1}^{\infty} \delta_s U^P_{t+s} + \sum_{s=1}^{\infty} \delta_s U^C_{t+s}.$$  

$$U_t = U^P_t + cU^C_t$$

It is implicitly assumed here that there are no taxes and subsidies. To complete the description of the model, assume bi-generational contracting with generation $t$ and $t+1$ having control over $c_t, p_{t+1}$ and $h_{t+1}$. As noted above, this subsumes
the case of no contracting. Maximizing $U_t + \lambda U_{t+1}$ subject to the budget constraint (Equation 6), tells us that at an interior maximum it must be the case that:

$$\frac{\partial U^M(c_t, p_{t+1})}{\partial c_t} \times$$

$$[w_{t+1}^H(1 - \tau_1) - (\gamma w_{t+1}^H + (1 - \gamma)w_{t+1}^L)s_2(h_{t+2}, h_{t+1})(1 - e_1) -$$

$$r_1(\gamma w_t^H + (1 - \gamma)w_t^L)s_1(h_{t+1}, h_t)(1 - e_1)]$$

$$= -U^S((\gamma w_t^H + (1 - \gamma)w_t^L)s(h_{t+1}, h_t))(\gamma w_t^H + (1 - \gamma)w_t^L)s_1(h_{t+1}, h_t)(1 - e_1)$$

Note that in writing down this condition we have not had to deal with the time consistency issues. This because we assume both perfect credit markets and $s_{t2} = 0$. If $s_{t2}$ were not equal to zero, the marginal cost of educational investment in the future would depend on how much the current generation invested and therefore the current generation’s investment decision would have to take account of the responsiveness of future investment to current investment.

It follows from equation 7 that as long as $U^M$ and $U^S$ are subject to diminishing marginal utility, any increase in family income (say $h_t$ goes up) necessarily must increase $h_{t+1}$, Moreover parental preferences, given by the nature of the $U^M$ and $U^S$ functions, clearly plays a role. Moreover it is evident that the result here is uninfluenced by the nature and form of altruism that we have assumed, since these terms do not enter the above condition. The key condition is that $U^P_t \neq 0$: When $U^P_t = 0$, the above condition reduces to equation 4, i.e. we collapse back to the benchmark case. Once again, this remains true whether we have perfect or imperfect altruism and whether we have no contracting ($\lambda = 0$) or bi-generational contracting. Moreover since symbolic consumption of other people’s material consumption is effectively just a form of altruism, adding this type of symbolic consumption does not alter the result. Finally, while not shown here, it is easy to show that the same result (that we get same results as the benchmark case unless there is symbolic consumption of investment in human capital) holds in the case of multigenerational contracting including the case of complete contracts.

This leads us to:

**Observation 3:** Given perfect credit markets, there can be income effects and parental preference effects on investment in human capital if and only if there is symbolic consumption of investment in human capital.

4 What have we learnt

It follows from the three observations above that the evidence of income effects on educational investment (see Jacoby, 1994; Glewwe and Jacoby, 2000; and Carvalho, 2000) can be interpreted as evidence for credit constraints if and only if we are prepared to assume that there is no symbolic consumption of educational investment in itself. They also tell us that there is parallel result for parental preferences and if we had good measures of shocks to parental
preferences, we could use them to study credit constraints in exactly the same way as we use income effects.