Households in developing countries have incomes that are variable and risky. How do they cope with such risk?
Ways to cope:
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- 
- 
We will start by seeing how much households can achieve by saving. Saving is a way for an individual to transfer resources into the future.

1 Savings: A simple model with certainty
Imagine you can live for 2 periods. In the first period you earn $y_1$, in the second period your earn $y_2$. You can save or borrow in period 1.
Maximization problem:
\[ \text{Max} u(c_1) + \beta u(c_2) \]
such that:
\[ c_1 = y_1 - S \]
\[ c_2 = y_2 + RS \]
where $R$ is the growth interest rate.
What is the solution of this problem?

If $\beta R = 1$, what does this imply? $\beta$ is the value of consumption tomorrow, relative to today. Economists often use a related concept, the discount rate, defined by:

$$\beta = \frac{1}{1 + \delta}$$

If the $\delta = r$, $\beta R = 1$, therefore $c_1 = c_2$.

This is the permanent income hypothesis: if the discount rate is equal to the interest rate and the income stream is certain, the consumption should be equal over the lifecycle.

We can now use the budget constraint to recover $S$, and $c_1 = c_2 = c$.

2 Savings of a rainy (or dry...) day: Introducing uncertainty

Let’s use the same model, but think of it as describing a shorter horizon (i.e., one year). We now introduce uncertainty: $y_1$ is known but $y_2$ is uncertain. We will assume it can be high ($y_H$) with probability $p$ and low ($y_L$) with probability $1 - p$.

Maximization problem:

$$\text{Max} u(c_1) + \beta E[u(c_2)]$$

such that:

$$c_1 = y_1 - S$$
$$c_2 = y_2 + RS$$

Note that we now have the expectation of future consumption in the maximization problem. I do not know how much consumption I will be able to afford. On the other hand, we know that the budget constraint will be satisfied with certainty.

Specifically,

- with probability $p$, $c_2$ will be:
- with probability $p$, $c_2$ will be:

Now replace $c_1$ and $c_2$ with their values from the budget constraints in the maximization problem.
Max\(u(c_1) + \beta[pu(y_H + RS) + (1 - p)u(y_L + RS)]\)

FOC:
\[
\beta R = \frac{u'(c_1)}{pu'(y_H + RS) + (1 - p)u'(y_L + RS)}
\]

which can be rewritten:
\[
\beta R = \frac{u'(c_1)}{E[u'(c_2)]}
\]

The first order condition resembles the one in section 1, except that we now have an expectation. Note that in general it does not imply that \(c_1 = E(c_2)\) even if \(\beta R = 1\).

However, consider the special case of a quadratic utility function:
\[
u(c) = ac - 0.5bc^2
\]

The FOC becomes:
\[
\beta R = \frac{a - bc_1}{E[a - bc_2]}
\]

if \(\beta R = 1\) we get
\[
c_1 = E(c_2)
\]

Consumption is a marginal (a celebrated result due to Hall).

We can now determine the level of \(c_1\).

First combine the two budget constraints. We obtain:
\[
\begin{align*}
c_2 + Rc_1 &= y_2 + Ry_1 \\
\frac{c_1 + c_2}{1 + r} &= y_1 + \frac{y_2}{1 + r}
\end{align*}
\]

which we can rewrite:
\[
\begin{align*}
c_1 + \frac{c_2}{1 + r} &= y_1 + \frac{y_2}{1 + r}
\end{align*}
\]

Take expectation at time 1:
\[
\begin{align*}
c_1 + \frac{E[c_2]}{1 + r} &= y_1 + \frac{E[y_2]}{1 + r}
\end{align*}
\]
\[ c_1 + \frac{c_1}{1+r} = y_1 + \frac{E[y_2]}{1+r} \]

\[ c_1(\frac{2+r}{1+r}) = y_1 + \frac{E[y_2]}{1+r} \]

We are now in a position to consider how a household will react to an increase in income depending on its source.

1. Compare two households who face the same income process. Household 1 received the high value in period 1, household 2 received the low value in period 1. To simplify assume that \( y_H = y_L + 1 \).

\[ c_1^1 - c_2^1 = \]

2. Now compare two households who face a different income process. For household 1, \( y_H \) and \( y_L \) are always one unit higher than for household 2

\[ c_1^1 - c_1^2 = \]

This is the second important result: The propensity to consume out of permanent income change should be higher than the propensity to consume out of a temporary change in income. The propensity to consume out of a permanent change in income should be 1. If the horizon is infinite, the propensity to consume out of a transitory change in income should be 0. It follows immediately that: the propensity to save out of permanent income should be close to 0, and the propensity to save out of transitory income should be close to 1 (with a long horizon).

3 Testing this Model: Savings and Rainfall in Thailand

The paper by Chris Paxson in the reading packet tests this proposition, using data from rice farmers in Thailand. She seeks to run the regression

\[ S_{irt} = \alpha_0 + \alpha_1 Y_{irt}^P + \alpha_2 Y_{irt}^T + Controls + \epsilon_{sirt} \]

Where \( i \) is the individual, \( r \) is the region, \( t \) is the time period, \( S_{irt} \) is the savings rate, \( Y_{irt}^P \) is the permanent income, and \( Y_{irt}^T \) is the transitory income.
What does she expect to find?

What is the main problem she faces in implementing this equation?

How can she construct measures of $Y_{irt}^P$ and $Y_{irt}^T$?

Idea: the income of a rice farmer is essentially determined by the amount of rainfall (more rainfall is better). But the exact amount of rainfall in a given season is unpredictable, and in particular is not serially correlated: a good rainfall this season does not predict how much rainfall you will get next season, once you control for the region’s average rainfall.

Therefore, deviation from the norm should be a good predictor of:

So she can run a regression of income on rainfall ($X_{irt}^T$) and characteristics that will help predict the permanent income ($X_{irt}^T$).

$$Y_{irt} = \beta_t + \beta_0 r + X_{irt}^P \beta_1 + X_{irt}^T \beta_2 + \epsilon_{irt}$$

She then uses the fact that:

- rainfall predicts only the transitory portion of the income
- the other variables predict permanent portion of the income

to construct:

$$\hat{Y}_{irt}^P =$$

$$\hat{Y}_{irt}^T =$$

$$\hat{\epsilon}_{irt} =$$

She then runs the regression:

$$S_{irt} = \alpha_0 + \alpha_1 \hat{Y}_{irt}^P + \alpha_2 \hat{Y}_{irt}^T + Controls + \epsilon_{irt}$$

See the handout: what are the results?

4 Introducing borrowing constraints

You will see very soon that households may not be able to borrow. How much can they smooth income?
They can accumulate assets in good time (through savings), and run them down in bad times. For example, if you call $x_t$ the “cash on hand” available to a household at date $t$ (the sum of accumulated asset + current income), it can be shown that a simple rule of thumb is very close to the best a household can do: Consume everything if cash on hand is below some threshold, otherwise save a fraction of what’s above the surplus.

For example, for a i.i.d. income of mean 100:

$$c_t = x_t \text{ if } x_t < 100$$

$$c_t = x_t - (x_t - 100) * 0.7 \text{ if } x_t \geq 100$$

How much smoothing can they achieve in this way? Look at figures 6.8 and 6.9 in handout (simulations by Deaton). What are the main remarks?

- There are times when assets run out and consumption can drop dramatically. Can households do better, and achieve consumption smoothing through mutual insurance?

5 Thinking about savings constraints

Households do not always have great savings instruments (they are not served by banks, banks do not want small accounts). The returns to their savings may be low (lower than the discount rate). They may save in inefficient instruments. For example, in bullocks!