1 Balance sheets and asset prices

- Kiyotaki and Moore (1997)

- Mechanism: balance sheet effects + forward looking prices $\Rightarrow$ amplification

- Risk neutral consumers and entrepreneurs with preferences

  \[ \sum \beta^t c_t \]

- Two goods: consumption good, capital in fixed supply $\bar{k}$, never depreciates

- Relative price of the capital good $q_t$
• Entrepreneurs ("farmers") flow of funds

\[ c_t^E + q_t k_{t+1} \leq n_t + \beta b_{t+1} \]

• Net-worth dynamics

\[ n_t = (a + q_t) k_t - b_t \]

• Collateral constraint

\[ b_{t+1} \leq q_{t+1} k_{t+1} \]

• Inalienable human capital of entrepreneurs necessary to produce \( a \) (a form of limited enforcement)
• Alternative use for capital: concave production function controlled by the consumers ("gatherers")

\[ \tilde{y}_t = G(\tilde{k}_t) \]

• Market clearing

\[ k_t + \tilde{k}_t = \bar{k} \]

• Optimality condition for the use of capital in the G sector (unconstrained)

\[ q_t = \beta \left[ q_{t+1} + G'(\tilde{k}_{t+1}) \right] \]

• Initial conditions: \( k_0 \) and \( b_0 \)
Suppose initial conditions such that entrepreneurs repay, i.e. \( \exists \) equilibrium with

\[
q_0 k_0 \geq b_0
\]

Some results:

- the entrepreneurs are constrained and consume \( c_t^E = 0 \) for the first \( T \) periods (\( T \) could be zero)

- After \( T \) they are unconstrained and the price is equal to

\[
q_t = q^* = \frac{\beta}{1 - \beta} a
\]
and capital stock invested in entrepreneurial firms is $k_{t+1} = k^*$, such that

$$a = G' \left( \bar{k} - k^* \right)$$
• In all previous periods $k_{t+1} < k^*$ and $q_t < q^*$

• Find sequence that satisfies

$$q_t = \beta \left[ q_{t+1} + G' \left( \bar{k} - k_{t+1} \right) \right]$$

and

$$q_t k_{t+1} = (a + q_t) k_t - b_t + \beta q_{t+1} k_{t+1}$$

up to period $T - 1$, and the second as $\geq$ from $T$ onwards
Check optimality

\[ V_t(n_t) = \max_{c_t^E, k_{t+1}, b_{t+1}} c_t^E + \beta V_{t+1}((a + q_{t+1}) k_{t+1} - b_{t+1}) \]

\[ c_t^E + q_t k_{t+1} \leq n_t + \beta b_{t+1} \]
\[ b_{t+1} \leq q_{t+1} k_{t+1} \]

- FOC

\[ 1 \leq \lambda_t \]
\[ \lambda_t q_t = \beta (a + q_{t+1}) V'_{t+1} + \mu_t q_{t+1} \]
\[ \lambda_t \beta = \beta V'_{t+1} + \mu_t \]

- Envelope

\[ V'_{t+1} = \lambda_t \]
• Decreasing sequence of $\lambda_t$ that converges to $\lambda_t = 1$ in finite time

$$q_t \lambda_t = \beta \left( a + q_{t+1} \right) \lambda_{t+1} + \mu_t q_{t+1}$$

$$\mu_t = \beta \lambda_t - \beta \lambda_{t+1}$$

$$q_t = \beta \left( a \frac{\lambda_{t+1}}{\lambda_t} + q_{t+1} \right) < \beta \left( a + q_{t+1} \right)$$

fine as long as

$$\beta \frac{a + q_{t+1}}{q_t} > 1$$

and delivers

$$\lambda_t = \frac{\beta a}{q_t - \beta q_{t+1}} \lambda_{t+1} = \frac{\beta a}{q_t - \beta q_{t+1}} \frac{\beta a}{q_{t+1} - \beta q_{t+2}} \cdots \frac{\beta a}{q^* - \beta q_{T-1}}$$
Finding an equilibrium

- Balance sheet relation:

\[ k_1 = \frac{(a + q_0) k_0 - b_0}{q_0 - \beta q_1} = \frac{(a + q_0) k_0 - b_0}{\beta G' \left( \bar{k} - k_1 \right)} \]

increasing relation between asset price \( q_0 \) and investment in entrepreneurial sector

- Asset pricing relation: given \( k_1 \) find sequence \( \{k_t\}_{t=2}^\infty \) that satisfies

\[ k_{t+1} = \frac{(a + q_t) k_t - b_t}{q_t - \beta q_{t+1}} = \min \left\{ \frac{a k_t}{\beta G' \left( \bar{k} - k_{t+1} \right)}, k^* \right\} \]
and find

$$q_0 = \sum_{t=0}^{\infty} \beta^{t+1} G' \left( \bar{k} - k_{t+1} \right)$$

increasing relation between investment in entrepreneurial sector and asset price $q_0$
• Introduce a temporary shock to productivity

• Productivity is

\[ a + \Delta a \]

for first period only

• This would have no effect in frictionless benchmark (purely forward looking)

• Here it shifts the BS relation to the right

\[ k_1 = \frac{(a + \Delta a + q_0) k_0 - b_0}{q_0 - \beta q_1} \]
• Backward looking effect of net worth on investment

• ...+amplification due to forward looking element

• Questions:
  
  – here shock is completely unexpected
  
  – what happens if state contingency allowed?
  
  – do entrepreneurs want to insure (hedge)?
  
  – if yes, why they do not do it?
Suppose state contingent contracts allowed at date \(-1\)

State \(s\) realized at date \(0\):

- \(s_g\) : productivity\(= a\)
- \(s_b\) : productivity\(= a + \Delta a \ (\Delta a < 0)\)

State contingent enforcement constraint

\[
b_0(s) \leq q_0(s) k_0
\]
• Question 1: what happens to effect of shocks if firms decide to choose max borrowing in all $s$?

• Question 2: will firms even choose max borrowing?
Question 1: Now no feedback effect (vertical BS curve)

$$\beta G' (\bar{k} - k_1) k_1$$

so total effect is

$$\frac{\Delta k_1}{k_0} = \frac{1}{\beta G'' - G''' k_1} \Delta a$$

instead of

$$\frac{\Delta k_1}{k_0} = \frac{1}{\beta G'' - G''' k_1} (\Delta a + \Delta q_0)$$
Question 2:

Check optimality

Entrepreneurs problem at $t = -1$

$$\max_{k_0,b_0(s)} \sum_s \pi(s) \beta V_0 \left( (a(s) + q_0(s)) k_0 - b_0(s) \right)$$

$$q_{-1} k_0 \leq n_{-1} + \beta \sum_s \pi(s) b_0(s)$$

$$b_0(s) \leq q_0(s) k_0$$

- FOC

$$\lambda_{-1} q_{-1} = \beta \sum \pi(s) (a(s) + q_0(s)) V_0'(s) + \sum \pi(s) \mu(s) q_0(s)$$

$$\lambda_{-1} \pi(s) \beta = \beta \pi(s) V_0'(s) + \pi(s) \mu(s)$$
• Can we have $\mu(s) = 0$?

• Answer: yes if

$$V'_0(s) = \lambda_{-1}$$

• Substituting

$$\lambda_{-1} q_{-1} = \beta \sum \pi(s)(a(s) + q_0(s)) \lambda_0(s) + \beta \sum \pi(s)(\lambda_{-1} - \lambda_0(s)) q_0(s)$$

yields

$$\lambda_{-1} = \frac{\beta \sum \pi(s)a(s) \lambda_0(s)}{q_{-1} - \beta \sum \pi(s) q_0(s)}$$
• Recall that
\[ V'_0(s) = \lambda_0(s) = \frac{\beta a}{q_0(s) - \beta q_1(s)} \frac{\beta a}{q_1(s) - \beta q_2(s)} \cdots \frac{\beta a}{q^* - \beta q_{T(s)-1}(s)} \]

• Equilibrium construction: try
\[ b_0(s) = q_0(s) k_0 \text{ for all } s \]

• If shocks $\Delta a < 0$ realized then $q_0(s)$ lower

• Check if
\[ \lambda_0(s_1) < \frac{\beta \sum \pi(s) a(s) \lambda_0(s)}{q^{-1} - \beta \sum \pi(s) q_0(s)} \]
• If not then look for equilibrium where

\[ b_0(s_g) = q_0(s_g)k_0 \]
\[ b_0(s_b) \leq q_0(s_b)k_0 \]

Equilibrium with spare debt capacity

• Crucial point: Entrepreneurs risk neutral \( \neq \) no hedging demand

• Value function \((V'_0(s))\) depends on asset prices!

• Even if \( \pi(s_b) \) is very small we have a lower bound on how big can be the capital destruction in a crisis
• Broader question

• Why “entrepreneurs” (i.e. potentially financially constrained agents) do not insure?

• Classic example of lack of state contingency (actually of “wrong” state contingency): dollarization of liabilities in emerging economies
\[ \frac{q_0 k_1}{n_0} = \frac{q_0}{q_0 - \beta q_1} = \frac{\sum_{t=0}^{\infty} \beta^{t+1} G'(\bar{k} - k_{t+1})}{\beta G'(\bar{k} - k_1)} \]