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Analysis of Reversible Ejectors and Definition of an Ejector Efficiency

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Abstract

Second Law analyses of ejector performance have rarely been conducted in literature. Many measures of ejector efficiency have not always been clearly defined and the rationale underlying and justifying current performance metrics is often unclear. One common means of assessing performance is to define a thermodynamically reversible reference process against which real processes may be benchmarked. These reversible processes represent the thermodynamic limit of real ejector performance. In this paper parameters from real and reversible processes are compared and performance metrics are defined. In particular, the entrainment ratio of real devices is compared to the reversible entrainment ratio and denoted the reversible entrainment ratio efficiency. An efficiency comparing the ejector performance to that of a turbine-compressor system is also investigated, as is an exergetic efficiency. A rigorous analysis of performance metrics reported in the literature is undertaken. Graphical illustrations are provided to support intuitive understanding of these metrics. Analytical equations are also formulated for ideal-gas models. The performance metrics are then applied to existing experimental data to illustrate the difference in their numerical values. The reversible entrainment ratio efficiency η_{RER} is shown to always be lower than the turbine-compressor efficiency η_{TER} . For general air-air and steam-steam ejectors, the exergetic efficiency η_X is very close in numerical value to the reversible entrainment ratio efficiency, η_{RER} .

Keywords: Ejector, Thermocompressor, Pressure exchange, Second Law of Thermodynamics, Isentropic efficiency

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Nomenclature

Symbols

c_p	specific heat capacity at constant pressure [kJ/kg·K]
ER	entrainment ratio [-]
h	specific enthalpy [kJ/kg]
$\tilde{h}$	specific molar enthalpy [kJ/kmol]
$h_{E,D}$	specific enthalpy of the entrained fluid at the discharge [kJ/kg]
$h_{M,D}$	specific enthalpy of the motive fluid at the discharge [kJ/kg]
j	dissipation [J]
$\dot{m}$	mass flow rate [kg/s]
M	molar mass [kg/kmol]
$\dot{N}$	molar flow rate [kmol/s]
p	partial pressure [bar or kPa]
P	absolute pressure [bar or kPa]
$\dot{Q}$	rate of heat transfer [W]
R	gas constant [kJ/kg·K]
RER	Reversible Entrainment Ratio [-]
s	specific entropy [kJ/kg·K]
$\tilde{s}$	specific molar entropy [kJ/kmol·K]
$\dot{S}_{gen}$	entropy generation rate [kW/K]
$\bar{s}_{gen}$	dimensionless entropy generation [-]
T	temperature [°C or K]
v	specific volume [m ³ /kg]
x	dryness fraction of steam [-] or specific exergy [J/kg]
y	useful work done [J]

Greek

γ	ratio of specific heats
Δ	change
η	efficiency [-]
Π_c	compression ratio (P_D/P_E) [-]
ϕ	relative humidity [-]
ω	absolute humidity [kg H ₂ O/kg carrier gas]
ξ	ratio of moles of vapor to moles carrier gas [-]

Subscripts

c	compression
cg	carrier gas
comp	compressor
D	discharge fluid
E	entrained fluid
f	saturated liquid
g	saturated vapour
irrev	associated with irreversible dissipation

M	motive fluid
NE	nozzle exit
q	associated with heat transfer
RDP	reversible discharge pressure
RER	reversible entrainment ratio
RHE	reversible heat engine
s	isentropic
TER	turbine-compressor entrainment ratio
turb	turbine
vap	vapor
X	exergetic
0	ambient

Superscripts

rev	reversible
sat	saturated

1. Introduction

Steady flow ejectors are devices without moving parts that typically combine two fluids, one of high total pressure and one of low total pressure, to obtain a fluid at an intermediate total pressure at the discharge. The fluid with high total pressure, known as the motive fluid, is expanded through a nozzle to high velocity and low pressure (point NE in Fig. 1). The low pressure at the nozzle exit provides the driving force for suction of the low pressure fluid, known as the entrained fluid, at pressure P_E , such that $P_M > P_E > P_{NE}$. The motive and entrained streams undergo mixing, brought about by shear forces between the fluids, before the resulting mixture is diffused to attain the discharge pressure, P_D , Fig. 1.

[Figure 1 about here.]

The most intuitive although approximative manner to comprehend ejector operation is to analyse performance using simple 1-D analyses. Examples of excellent work in this regard have been done by Elrod [1], Keenan *et al.* [2], Chou *et al.* [3] and Arbel *et al.* [4]. Ejectors are relatively simple in design and can provide reliable operation at low capital cost. These advantages have led to ejectors being used in processes requiring the drawing of a vacuum (*e.g.* extraction of non-condensable gases from condensers) or the compression of a gas (*e.g.* within refrigeration cycles). In thermal desalination, ejectors are commonly employed to compress and entrain water vapor. Such devices are known as thermal vapor compressors. The increased use of thermal vapor compressors for energy intensive processes of thermal desalination, in particular multi-effect distillation [5–8], has brought about renewed interest in the efficiency of ejectors. Despite the interest in energy efficiency, scant attention has been paid to the definition and justification of the utility of ejector performance metrics employed to date. Bulusu *et al.* [9] presented common measures of ejector performance, whilst Arbel *et al.* [4] performed an analysis of entropy generation within ejectors. Nonetheless, there remains no universally accepted efficiency to characterise and compare the performance of such devices.

In this paper, ejector performance is evaluated by benchmarking real ejector processes against carefully defined thermodynamically reversible processes. In Section 2, a systematic methodology is presented to define such processes. One important parameter arising from this analysis is the reversible entrainment ratio, *i.e.* the maximum theoretical entrainment ratio achievable by a thermodynamically reversible ejector. The variation in the reversible entrainment ratio with an ejector’s operating conditions is investigated in Section 3. Then, in Section 4 ejector performance metrics are explained and justified by means of analytical equations or graphical methods. Finally, in Section 5, the numerical values of these metrics are computed for experimental data available in open literature. The exergetic efficiency η_X of an ideal gas ejector with inlet fluids at the same temperature is shown to be identical to the reversible entrainment ratio efficiency η_{RER} .

2. Description of Reversible Ejector Processes

This section establishes a basis for benchmarking real device performance against the performance of ideal, thermodynamically reversible processes. In order to effectuate this comparison, the following steps are necessary:

1. Identify the input quantities, output quantities and equations describing a thermodynamically reversible process.
2. Define a thermodynamically reversible reference process, against which real processes may be compared.
3. Develop a performance metric based on a comparison of parameters between the real and reversible processes. (4)

A control volume model of an ejector process is presented in Fig. 2. The inlet stream of higher pressure is known as the motive flow, the low pressure inlet stream is termed the entrained flow and the outlet stream is termed the discharge flow. The control volume in both real and reversible cases is considered externally adiabatic ($\dot{Q}_0 = 0$). Due to the high flow speeds within ejectors, the rate of heat loss from the device to ambient is taken to be small relative to changes of enthalpy of the streams within. The nomenclature of Fig. 2 shall henceforth be used in referring to the mass flow rates and state properties of inlet and outlet streams. All analyses shall consider only equilibrium states of inlet and outlet fluids where the fluids are at conditions of stagnation (zero kinetic energy). Thus, all temperatures and pressures are total rather than static temperature and pressures.

[Figure 2 about here.]

2.1. Equations Describing a Thermodynamically Reversible Process

Taking a control volume approach, the First and Second Laws of Thermodynamics may be written for a real ejector process involving pure and identical fluids at the ejector inlet: First Law of Thermodynamics for an adiabatic process:

$$\dot{m}_M h_M + \dot{m}_E h_E = \dot{m}_D h_D \quad (1)$$

$$h_M + \text{ER} \cdot h_E = (1 + \text{ER})h_D \quad (2)$$

The entrainment ratio, ER is defined as the ratio of entrained mass flow to motive mass flow,

$$\text{ER} = \frac{\dot{m}_E}{\dot{m}_M} \quad (3)$$

Second Law of Thermodynamics for an adiabatic process:

$$\dot{m}_M s_M + \dot{m}_E s_E + \dot{S}_{\text{gen}} = \dot{m}_D s_D \quad (4)$$

$$s_M + \text{ER} \cdot s_E + \frac{\dot{S}_{\text{gen}}}{\dot{m}_M} = (1 + \text{ER})s_D \quad (5)$$

Noting that no entropy is generated during a reversible process, the equations for an adiabatic and reversible process may be formulated as follows:

$$h_M + \text{RER} \cdot h_E = (1 + \text{RER})h_D^{\text{rev}} \quad (6)$$

$$s_M + \text{RER} \cdot s_E = (1 + \text{RER})s_D^{\text{rev}} \quad (7)$$

Reversible entrainment ratio RER is defined as the ratio of entrained mass flow to motive mass flow in the reversible process:

$$\text{RER} = \left(\frac{\dot{m}_E}{\dot{m}_M} \right)^{\text{rev}} \quad (8)$$

For a pure fluid, the thermodynamic state is fully specified at the control volume inlets and outlet once two state properties (*i.e.* h and s) are known. An equation of state may be written relating the temperature T or the pressure P to the specific enthalpy and entropy. The specific enthalpy and entropy of the discharge stream is simply the mass weighted average of the inlet streams, see Fig. 3. Of course, according to Eq. (6) and (7), the entrainment ratio, ER, is the mass ratio relevant to this weighting. Hence, on a diagram of specific enthalpy versus specific entropy, the discharge fluid state must lie on the line joining the states of the inlet fluids, as shown in Fig. 3. The next step is to define the different possible reversible reference processes.

[Figure 3 about here.]

2.2. Definition of Thermodynamically Reversible Reference Processes

Six state variables and one mass flow rate ratio describe the process of Fig. 2. For a real process there are two equations, Eq. (2) and Eq. (5), including one further unknown, $\frac{\dot{S}_{\text{gen}}}{\dot{m}_M}$. This value of this entropy generation term is dictated by the quality of design of the device within the control volume. For a reversible process, there are two associated equations, Eq. (6) and Eq. (7). Consequently, given 7 variables and 2 equations, there are five degrees of freedom that must be specified in order to define a reversible reference process. There are therefore 21 (7-choose-5) different ways in which a reference process could be defined. A select number of combinations are discussed in the following subsections.

2.2.1. Reversible Discharge Pressure Reference Process

In a paper elucidating the sources of irreversibility within steady flow ejectors, Arbel *et al.* [4] investigated ejector performance in great detail by comparing real processes to a reversible reference process, described in Table 3:

[Table 1 about here.]

In essence, Arbel *et al.*'s reference processes asks: for fixed inlet fluid states and a fixed entrainment ratio, what is the maximum discharge pressure achievable if the process is reversible? On an $h - s$ diagram, Arbel's process may be represented as:

[Figure 4 about here.]

Discharge enthalpy in both the real and reference case is a weighted average of the inlet fluid enthalpies. Since by definition of this reference process, the real and reversible entrainment ratios are equal, the discharge specific enthalpy is also equal in the real and reversible

cases, Eq. (2). In the real process, entropy is generated and thus the discharge specific entropy is greater than the weighted mean of the entropy of the inlet streams.

For applications where the inlet states are fixed, as is the relative mass flow of motive and entrained fluid available, this reference process may be employed to determine how close the actual discharge pressure achieved is to the reversible limit.

2.2.2. Reversible Entrainment Ratio Reference Process

In many situations, the inlet states to the ejector are known, as is a desired discharge pressure¹. Such is the case when one has steam available at a fixed state and wishes to compress vapor, also at a known state, to a desired pressure. The efficiency of the ejector will determine the mass flow of vapor one can entrain per unit mass flow of steam (*i.e.* the entrainment ratio). In such a scenario, the reference process described in Table 4 is of relevance:

[Table 2 about here.]

On an $h - s$ diagram, the reversible entrainment ratio reference process may be represented as:

[Figure 5 about here.]

The real discharge pressure in Fig. 5 is the same as that in Fig. 4. However, the discharge state is different. In the reversible entrainment ratio reference process, the reversible and real discharge pressure are set as equal. Employing the reversible entrainment ratio reference process, one may compare the real entrainment ratio to the entrainment ratio achieved in a reversible process.

2.2.3. Summary and Further Reference Processes

The above two reference processes are only two of the 21 possible reference processes for a dual inlet, pure and identical fluid ejector. In particular, knowledge of the entrainment ratio is interesting from a cost perspective as it quantifies the mass of fluid entrained per unit mass of motive fluid supplied. The reversible discharge pressure reference process is also considered due to the notable work presented on this process by Arbel *et al.* [4]. One could also consider a reference process whereby the motive and discharge state are fixed along with the entrainment ratio, and the real entrained pressure and temperature are compared to the entrained pressure and temperature obtained in a reversible process. Having established a basis for the definition of such processes, further permutations are left to the reader bearing in mind that the choice of reference process must be guided by the intended application of the ejector.

¹Where one is dealing with an ejector involving fluids of identical chemical composition, it is equivalent to define either the discharge pressure or the saturation temperature at that pressure, as is done by Al Khalidy and Zayonia [10] and by Elrod [1].

2.3. Interpretation of a Thermodynamically Reversible Ejector Process

Here a thermodynamically reversible ejector process is further elucidated. Importantly, this illustration does not attempt to explain how a reversible process could be achieved in a real steady-flow ejector. Rather, it points out how, theoretically, a reversible process could take place within the control volume of an ejector. For simplicity, let us consider an ejector process with pure fluids at the inlet, both of identical chemical composition. The a reversible ejector process can now be interpreted as the sum of two processes. The first process establishes mechanical equilibrium and the second establishes thermal equilibrium. The process that establishes mechanical equilibrium at the discharge pressure, P_D is in fact a turbine-compressor process. The motive and entrained fluids are respectively expanded and compressed through an adiabatic and isentropic turbine and compressor, Fig. 6.

[Figure 6 about here.]

The thermal equilibrating process may be represented by a reversible heat engine that transfers heat (and entropy) from the the hotter of the fluids M' or E' to the colder fluid, allowing them to reach an equilibrium temperature T_D . Of course, work is produced by the heat engine to supply the compression process. It is therefore clear, that in a reversible ejector process, the work available for compression is greater than that available in a turbine-compressor process. Consequently, the reversible entrainment ratio will always be greater than or equal to the turbine-compressor entrainment ratio. In the case where the fluid exiting the turbine and the compressor in Fig. 6 are at the same temperature, no work will be done by the reversible heat engine. Consequently, in this special case, the reversible entrainment ratio and the turbine-compressor entrainment ratio will be identical. The equilibrium process paths of Fig. 6 are represented in Fig. 7.

[Figure 7 about here.]

Until this stage, processes involving only pure and identical inlet fluids have been considered. However, the same analysis can be generalised to cover processes such as the compression of a moist air stream using high pressure steam. In such cases, additional variables, such as humidity ratio, are required to fully define the equilibrium state at any point.

3. Reversible Entrainment Ratio Calculations for Different Fluids

At present, the performance of ejectors is typically predicted using graphical methods or by semi-empirical correlations [11, 12]. These methods allow the calculation of the entrainment ratio as a function of inlet and outlet fluids' thermodynamic states. Whilst the reversible entrainment ratio does not provide much information about the entrainment ratio of real devices, it does provide an upper bound upon the entrainment ratio achievable. In this section, trends in the reversible entrainment ratio with inlet fluid conditions and discharge pressure are illustrated for ideal gases (air-air), steam-steam and steam-moist-carrier-gases ejectors.

3.1. Air-air

3.1.1. Derivation of the Reversible Entrainment Ratio for Ideal Gas Ejectors

For two reasons, the ideal gas ejector model is central to the understanding of ejector operation and performance characterization. First, the ideal gas assumptions allow analytical expressions to be derived for ejector performance. Secondly, for performance characterization of ejectors, the ideal gas model is appropriate for many ejector processes, as will be seen in Section 3.2.

Here an analytical expression is derived for the reversible entrainment ratio of an ideal gas process. The specific heat capacities of the gases at constant pressure are assumed to be equal and constant. The equation of state for an ideal gas is:

$$Pv = RT \quad (9)$$

where P is the total pressure, v the specific volume, R the ideal gas constant on a unit mass basis and T the total temperature. Equation (2) for ideal gases takes the following form for a reversible ejector process:

$$c_p T_M + \text{ER} \cdot c_p T_E = (1 + \text{ER})c_p T_D \quad (10)$$

Rearranging this equation, the discharge temperature may be written in terms of the inlet conditions and the reversible entrainment ratio:

$$T_D = \frac{\text{ER} \cdot T_E + T_M}{1 + \text{ER}} \quad (11)$$

Rearranging Eq. (11) the following dimensionless equations are obtained:

$$\frac{T_M}{T_D} = \frac{1 + \text{ER}}{1 + \text{ER} \cdot \frac{T_E}{T_M}} \quad (12)$$

$$\frac{T_D}{T_E} = \frac{\frac{T_M}{T_E} + \text{ER}}{1 + \text{ER}} \quad (13)$$

Next, rearranging Eq. (7), the Second Law may be written as follows:

$$\text{ER} = \frac{s_M - s_D^{\text{rev}} + \frac{\dot{S}_{\text{gen}}}{\dot{m}_M}}{(s_D^{\text{rev}} - s_E)} \quad (14)$$

Here we seek an expression linking the change in entropy to the state properties T and P . The following expression describes a differential change in entropy of an ideal gas:

$$ds = c_p \frac{dT}{T} - \frac{v}{T} dP \quad (15)$$

$$ds = c_p \frac{dT}{T} - R \frac{dP}{P} \quad (16)$$

Now the entrainment ratio may be written as follows:

$$\text{ER} = \frac{\frac{\dot{S}_{\text{gen}}}{\dot{m}_{\text{M}}} + \int_{\text{D}}^{\text{M}} c_p \frac{dT}{T} - \int_{\text{D}}^{\text{M}} R \frac{dP}{P}}{\int_{\text{E}}^{\text{D}} c_p \frac{dT}{T} - \int_{\text{E}}^{\text{D}} R \frac{dP}{P}} \quad (17)$$

For constant specific heats Eq. (17) results in the following relation describing the reversible entrainment ratio:

$$\text{ER} = \frac{\frac{\dot{S}_{\text{gen}}}{\dot{m}_{\text{M}}} + c_p \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - R \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)}{c_p \ln\left(\frac{T_{\text{D}}}{T_{\text{E}}}\right) - R \ln\left(\frac{P_{\text{D}}}{P_{\text{E}}}\right)} \quad (18)$$

Dividing the right hand side of Eq. (18) by the ideal gas constant, R , renders all terms dimensionless. Note also the definition, purely for convenience, of the dimensionless entropy generation rate.

$$\text{ER} = \frac{\bar{s}_{\text{gen}} + \frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)}{\frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{D}}}{T_{\text{E}}}\right) - \ln\left(\frac{P_{\text{D}}}{P_{\text{E}}}\right)} \quad (19)$$

$$\bar{s}_{\text{gen}} = \frac{\dot{S}_{\text{gen}}}{\dot{m}_{\text{M}} R} \quad (20)$$

Eq. (12), (13) and Eq. (19) allow the entrainment ratio to be expressed as a function of the following dimensionless parameters:

$$\text{ER} = \text{fn}(T_{\text{E}}/T_{\text{M}}, P_{\text{E}}/P_{\text{M}}, P_{\text{D}}/P_{\text{M}}, \gamma, \bar{s}_{\text{gen}}) \quad (21)$$

The reversible entrainment ratio, RER , can now be defined in terms of the inlet fluid states and the discharge pressure using Eq. (19).

$$\text{RER} = \frac{\frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)}{\frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{D}}}{T_{\text{E}}}\right) - \ln\left(\frac{P_{\text{D}}}{P_{\text{E}}}\right)} \quad (22)$$

3.2. Steam-steam Reversible Entrainment Ratios

In this section, the reversible entrainment ratio is calculated for steam-steam ejectors. As a first step, let us ask how well steam-steam ejectors can be modelled using ideal gas approximations. We note that steam-steam ejectors are typically supplied with supersaturated motive steam. This is done to avoid condensation within the ejector and a subsequent loss of mass via a condensate stream.

Steam behaves like an ideal gas at conditions of high temperature or low pressure. Supersaturated steam is very well approximated by ideal gas assumptions at pressures below approximately 10 bar. Above this pressure, ideal gas assumptions are only valid at higher levels of superheat. In Fig. 8, the saturation curve of pure water is plotted on axes of specific enthalpy versus specific entropy. The ratio of specific heats, γ , is calculated at a variety of points within the superheated region and plotted to demonstrate that γ is approximately constant in the low pressure superheated zone.

[Figure 8 about here.]

The entrained pressure and discharge pressure for steam ejectors are typically close to or below atmospheric pressure. Therefore, if the motive steam pressure is below about 10 bar, Eq. (12), (13) and (21) are approximately applicable in describing the inlet and outlet states of typical steam-steam ejectors.

The exact calculation of RER for steam is relatively straightforward. In each case the equations solved are Eq. (6) & (7). (If the state lies under the saturation curve, T and P would be insufficient to determine h and s . Instead, T and x , the dryness fraction, or P and x would be required.) To graphically explain trends in the reversible entrainment ratio of a steam-steam ejector, Fig. 9 is provided. Figure 9 shows the states of the motive, discharge and entrained fluids on a specific enthalpy versus specific entropy diagram. The state of the discharge fluid is given by the intersection of the isobar at the discharge pressure and a line joining the motive and entrained states (again see Sect. 2 for derivation). The entrainment ratio is represented by the length of the line from the motive to the discharge state, $|MD|$, divided by the length of the line from the discharge to the entrained inlet state, $|DE|$, Eq. (23).

$$\text{RER} = \frac{|MD|}{|DE|} \quad (23)$$

[Figure 9 about here.]

For a fixed state of the entrained fluid, it may be shown that a motive saturated steam pressure exists that maximizes RER. In Fig. 10, RER is plotted as a function of the motive steam pressure, choosing inlet fluid dryness fractions of 100% for illustrative purposes. RER increases logarithmically until a motive steam pressure of approximately 15 bar is reached. Beyond a motive saturated steam pressure of 15 bar, the rate of increase in RER reduces until an optimal motive steam pressure is reached. This optimum pressure corresponds to the point at which the tangent to the saturation curve is parallel to the slope of the isobar at the discharge state (see Fig. 9). This conclusion is reached by considering the point at which the distance $|MD|$ divided by $|DE|$ is maximised subject to point D lying at the intersection of the isobar at the discharge pressure and a line joining states M and E. The optimal pressure of the saturated motive fluid therefore depends upon the discharge pressure. As the discharge pressure changes, the slope of the isobar changes and thus the point at which the tangent to the saturation curve is tangential to the isobar at the discharge pressure also changes.

[Figure 10 about here.]

[Figure 11 about here.]

In Fig. 11, the optimal motive steam pressure is plotted for a fixed entrained fluid state and discharge pressure. The results are primarily of academic interest given that the optimal motive steam pressures are well beyond the typical pressures used in practical applications.

In Fig. 12 RER is plotted as a function of the discharge steam pressure. It is seen that RER drops drastically with an increasing compression ratio. Often, the motive steam may be superheated and therefore it is interesting to consider the effect of the degree of superheating on RER. Figure 13 highlights the improvement in RER due to superheating at constant pressure of the motive steam.

[Figure 12 about here.]

[Figure 13 about here.]

In contrast to Fig. 10 and Fig. 12, the rate of change in RER is small with changes in the inlet motive steam temperature. Again, this trend may be visualised making use of Fig. 9. As the motive steam is superheated following an isobaric process, both its specific enthalpy and its specific entropy increase, although its specific enthalpy increases at a greater rate. This results in an increasing reversible entrainment ratio. The apparent linearity of Fig. 13 may be explained by the fact that the isobars in the superheated region of Fig. 9 are each approximately linear. Thus, the increase in reversible entrainment ratio, according to Eq. (23), will be linear.

Finally, we consider how RER can be maximized when there is an upper limit on the motive steam temperature. To do this we may plot RER at constant temperature against motive steam specific entropy, as is done in Fig. 14.

[Figure 14 about here.]

The line plotted in Fig. 14 is a line of constant temperature. The reversible entrainment ratio reaches a maximum when $x=1$, *i.e.* when the steam is in a saturated condition. Consider the entrainment of saturated steam at state E by saturated motive steam at state M (see Fig. 15). If the condition of the motive steam is brought below the saturation curve, the ratio of the line |MD| to |DE| decreases. Thus, because the isobars on a $h-s$ diagram of steam converge as the state moves from that of saturated steam to that of a wet mixture, RER using saturated motive steam will always be superior to that using a wet mixture at the same temperature. In the superheated region of the $h-s$ diagram, isobars and isotherms do not coincide. If state M moves along an isotherm into the superheated region, RER is severely compromised, as the isotherm corresponding to state M converges rapidly with and meets the isobar corresponding to the discharge state D. In real steam-steam ejectors, slightly superheated motive steam may preferentially be used to saturated steam in order to discourage the formation of droplets in the discharge, which may need to be removed using a mist eliminator.

[Figure 15 about here.]

As a final caveat, maximizing RER does not in any way maximize the exergetic efficiency of a real device. The RER represents a perfect and reversible process that is 100% efficient from a Second Law perspective. Rather, the purpose of this section was to comprehend the upper thermodynamic limit on the entrainment ratio.

3.3. Steam-Moist-Gas Reversible Entrainment Ratios

Multipressure humidification-dehumidification (HDH) desalination cycles, which previously have been proposed by the current authors, make use of ejectors to pressurize a moist carrier gas [13–15]. It is therefore of interest to consider the effect upon the reversible entrainment ratio, RER, of using carrier gases of different types. In the case of a steam-moist-carrier-gas ejector, the entrainment ratio is defined as follows:

$$\text{ER} = \frac{(1 + \omega_E)\dot{m}_{\text{cg}}}{\dot{m}_M} \quad (24)$$

where ω_E is the humidity ratio of the entrained fluid. In the case of humidification-dehumidification desalination, the quantity of water vapor entrained is of importance, rather than the total amount of fluid entrained. For this reason, it is useful to define the vapor entrainment ratio. Of course, the vapor entrainment ratio is equivalent on a mass and molar basis as long as the motive fluid is steam.

$$\text{ER}_{\text{vap}} = \frac{\omega_E \dot{m}_{\text{cg}}}{\dot{m}_M} = \text{ER} \frac{\omega_E}{1 + \omega_E} \quad (25)$$

In this section the entrained and discharge fluids are treated as ideal mixtures of ideal gases, within which the enthalpy and entropy of the carrier gas and steam are calculated at their partial pressures. The ratio of the number of moles of carrier gas and water vapor present in a moist carrier gas mixture along with the relative humidity are given as follows:

$$\xi = \frac{p_{\text{vap}}}{P - p_{\text{vap}}} = \frac{\dot{N}_{\text{st}}}{\dot{N}_{\text{cg}}} = \frac{\phi p_{\text{sat}}}{P - \phi p_{\text{sat}}} \quad (26)$$

When comparing the reversible entrainment ratio of different moist carrier gases, we do so at equal entrained fluid temperature, pressure and relative humidity. Therefore, it is sensible to consider the governing equations on a molar basis, since ξ is constant for all carrier gases under these conditions.

First Law of Thermodynamics (reversible process):

$$\dot{N}_{\text{st},M} \tilde{h}_{\text{st},M} + \dot{N}_{\text{cg}} \tilde{h}_{\text{cg},E} + \dot{N}_{\text{st},E} \tilde{h}_{\text{st},E} = \dot{N}_{\text{st},D} \tilde{h}_{\text{st},D}^{\text{rev}} + \dot{N}_{\text{cg}} \tilde{h}_{\text{cg},D}^{\text{rev}} \quad (27)$$

where $\dot{N}$ is a molar flow rate and $\tilde{h}$ is a molar enthalpy. According to a mole balance on steam:

$$\dot{N}_{\text{st},D} = \dot{N}_{\text{st},M} + \dot{N}_{\text{st},E} \quad (28)$$

$$\tilde{h}_{\text{st},M} + \frac{\text{RER}_{\text{vap}}}{\xi_E} \tilde{h}_{\text{cg},E} + \text{RER}_{\text{vap}} \tilde{h}_{\text{st},E} = (1 + \text{RER}_{\text{vap}}) \tilde{h}_{\text{st},D}^{\text{rev}} + \frac{\text{ER}_{\text{vap}}^{\text{rev}}}{\xi_E} \tilde{h}_{\text{cg},D}^{\text{rev}} \quad (29)$$

Second law of Thermodynamics (reversible process):

$$\dot{N}_{\text{st},M} \tilde{s}_{\text{st},M} + \dot{N}_{\text{cg}} \tilde{s}_{\text{cg},E} + \dot{N}_{\text{st},E} \tilde{s}_{\text{st},E} = \dot{N}_{\text{st},D} \tilde{s}_{\text{st},D}^{\text{rev}} + \dot{N}_{\text{cg}} \tilde{s}_{\text{cg},D}^{\text{rev}} \quad (30)$$

$$\tilde{s}_{\text{st},M} + \frac{\text{RER}_{\text{vap}}}{\xi_E} \tilde{s}_{\text{cg},E} + \text{RER}_{\text{vap}} \tilde{s}_{\text{st},E} = \quad (31)$$

$$(1 + \text{RER}_{\text{vap}}) \tilde{s}_{\text{st},D}^{\text{rev}} + \frac{\text{RER}_{\text{vap}}}{\xi_E} \tilde{s}_{\text{cg},D}^{\text{rev}} \quad (32)$$

Equations of State (reversible process):

$$\tilde{h}_{\text{st,D}}^{\text{rev}} = \text{fn}(p_{\text{st,D}}^{\text{rev}}, \tilde{s}_{\text{st,D}}^{\text{rev}}) \quad (33)$$

$$\tilde{h}_{\text{cg,D}}^{\text{rev}} = \text{fn}(p_{\text{cg,D}}^{\text{rev}}, \tilde{s}_{\text{cg,D}}^{\text{rev}}) \quad (34)$$

$\tilde{s}$ is the molar enthalpy. The partial pressures of the steam, $p_{\text{st,D}}$, and carrier gas, $p_{\text{cg,D}}$, both sum to the total discharge pressure, P_{D} . RER and RER_{vap} are now plotted in Fig. 16 and Fig. 17 as a function of motive steam pressure, setting x_{M} and ϕ_{E} to unity for illustrative purposes.²

[Figure 16 about here.]

[Figure 17 about here.]

The results of Fig. 16 collapse onto a single curve in Fig. 17. Surprisingly, RER_{vap} appears to be almost independent of carrier gas. Finally, in Fig. 18 the reversible vapor entrainment ratio is plotted versus the entrained fluid inlet temperature, maintaining a relative humidity of unity.

[Figure 18 about here.]

The increase in RER_{vap} with temperature of the entrained moist carrier gas is explained by the decrease in value of the specific heat of the entrained fluid per mole of vapor.

4. Development of Ejector Performance Metrics

When one seeks to evaluate the thermodynamic ideality of a process, performance metrics are often chosen that compare useful work done in a real process to that in a defined reference process. However, the identification of work done within an ejector is not a trivial matter. In this section, these dilemmas are discussed and suggestions are made as to how performance metrics can be defined.

The crux of difficulties in evaluating work done within an ejector is that one cannot easily distinguish between useful flow work, heat transfer and dissipation. A similar difficulty is very clearly explained by Casey and Fesich [16] in relation to diabatic turbochargers, where heat transfer between the compressor and the turbine is significant. Maintaining the nomenclature of Casey and Fesich, the Gibbs equation may be integrated along the process path of a single fluid stream:

$$h_2 - h_1 = \int_1^2 v dp + \int_1^2 (T ds)_q + \int_1^2 (T ds)_{\text{irrev}} = y_{12} + q_{12} + j_{12} \quad (35)$$

where y_{12} is the useful flow work done in the process, q_{12} is the reversible heat transfer and j_{12} is the dissipation, or in other words, the lost work. Whilst the change in enthalpy of the

²Note: Although not graphed here, the variation of reversible entrainment ratio with compression ratio for helium, air and carbon dioxide carrier gases is qualitatively similar to that obtained for steam-steam ejectors.

fluid is fixed by the end states, the useful flow work, the heat transferred and the dissipation are all path dependent. Only in the special case of an adiabatic and reversible process can the useful flow work be equated with the change in enthalpy of the fluid:

$$h_2 - h_1 = \int_1^2 v dp \quad (36)$$

In reality, the motive and entrained streams in an ejector remain neither adiabatic nor distinct throughout the process. In real ejector processes, both heat transfer and dissipation are experienced by the motive and entrained streams during mixing. Consequently, the useful flow work can only be evaluated if the process path is known. Unfortunately, ejector mixing processes are characterized by a high degree of macroscopic disorder, involving heat transfer with finite temperature differences, forces that are not fully restrained and irreversible mixing. Intermediate states cannot accurately be represented on a process path of one intensive quantity against another. Consequently, there is no process path or equation in the form of Eq. (35) that can be applied to actual ejector processes.

Despite the difficulty of identifying useful work done in an ejector, there remain two broad approaches to characterising performance:

1. Comparisons may be made between terms representing useful work done in real and reference processes, provided that a clear rationale is given as to why such a metric compares different systems at different operating points. (Some performance metrics may only evaluate the performance of a part of the entire ejector process, as shall be seen with the turbine-compressor or compression efficiency.)
2. An exergy based approach may be taken. A performance metric may be defined once the useful exergetic output, the exergetic input, and a reference state for the process have been decided upon.

Next, four efficiencies based on the comparison of real and reversible processes are presented, followed by an exergetic efficiency.

4.1. Reversible Entrainment Ratio Efficiency

The definition of reversible entrainment ratio efficiency was provided in work by Elrod [1]. Al-Khalidy *et al.* [10] referenced the work of Elrod and also employed this efficiency. The reversible entrainment ratio efficiency η_{RER} quantifies how close the actual entrainment ratio of an ejector is to the theoretical maximum, for fixed conditions of operation. Its value is therefore always between zero and unity. RER is calculated based upon the reversible process described in Section 2.2.2.

$$\eta_{\text{RER}} = \frac{\text{ER}}{\text{RER}} \quad (37)$$

For a process involving fluids with identical ratios of specific heats and behaving as ideal gases, the RER may be derived based upon the ideal gas analysis conducted in Section 3.1.1.

The ratio of the real to reversible entrainment ratio may be written as follows:

$$\frac{\text{ER}}{\text{RER}} = \left(\frac{\bar{s}_{\text{gen}} + \frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)}{\frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)} \right) \left(\frac{\frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)}{\frac{\gamma}{\gamma-1} \ln\left(\frac{T_{\text{M}}}{T_{\text{D}}}\right) - \ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)} \right) \quad (38)$$

One interesting case of ejector operation to consider is where the inlet temperatures of both ideal gases are equal. According to the First Law, the discharge temperature must equal the inlet temperatures in both real and reversible cases. Equation (22) then simplifies to a ratio of the natural logarithm of the expansion ratio $P_{\text{M}}/P_{\text{D}}$ to the ratio of the natural logarithm of the compression ratio $P_{\text{D}}/P_{\text{E}}$.

$$\text{RER} = \frac{\ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)}{\ln\left(\frac{P_{\text{D}}}{P_{\text{E}}}\right)} \quad (39)$$

and the ratio of ER to RER simplifies to:

$$\frac{\text{ER}}{\text{RER}} = \frac{\ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right) - \bar{s}_{\text{gen}}}{\ln\left(\frac{P_{\text{M}}}{P_{\text{D}}}\right)} \quad (40)$$

Finally, as a general comment, the principles of operation of steady flow ejectors impose further limits upon ejector performance that are not captured by a control volume analysis whereby the control volume surrounds the ejector. These internal limits depend both upon ejector geometry and operational conditions. Some of the most enlightening work in this regard has been conducted by Chou *et al.* [3] and Arbel *et al.* [4]. The same is true for the reversible discharge pressure efficiency, to be presented in the following section.

4.2. Reversible Discharge Pressure Efficiency

The reversible discharge pressure efficiency quantifies how close the actual rise in pressure of the entrained fluid is to the theoretical maximum pressure rise. As with the reversible entrainment ratio efficiency, it is bounded between zero and unity. The theoretical rise in pressure is calculated based upon Section 2.2.1. This efficiency is introduced and discussed in detail by Arbel *et al.* [4].

$$\eta = \frac{P_{\text{D}} - P_{\text{E}}}{P_{\text{D}}^{\text{rev}} - P_{\text{E}}} \quad (41)$$

4.3. Turbine-Compressor Efficiency

The turbine-compressor efficiency [9, 17–19] (also known as the turbomachinery analogue efficiency), allows the comparison of an ejector with a mechanically coupled turbine and compressor. The efficiency is defined as the ratio of the entrainment ratio in a real ejector to the entrainment ratio achieved in a turbo-charger exhausting to the same discharge pressure,

P_D , as the ejector, and within which the turbine and compressor are adiabatic and isentropic (Fig. 19).

$$\eta_{\text{TER}} = \frac{\text{ER}}{\text{TER}} \quad (42)$$

[Figure 19 about here.]

Here, TER stands for Turbo Entrainment Ratio or Turbine-Compressor Entrainment Ratio. There is an important difference between the reversible entrainment ratio RER and the turbo entrainment ratio TER. The RER is calculated based on a process whereby the discharge is a single stream homogeneous in total temperature, total pressure and chemical composition. The TER is based upon a process whereby the inlet streams remain distinct at the discharge, *i.e.* the streams are not required to be in thermal or chemical equilibrium. Consequently, the value of TER will always be less than or equal to RER. A corollary of this is that η_{TER} is not bounded by an upper limit of 100%, as a real ejector could be imagined that achieves an entrainment ratio greater than that of a turbo, through the exploitation of the thermal or chemical disequilibrium of the fluids. Of course, a real steady-flow ejector could not in practice achieve a turbine-compressor entrainment efficiency beyond 100%. There is no obvious means by which a disequilibrium in chemical potential between the streams may be exploited. The question as to how thermal disequilibria may be exploited is more complex. As shall be seen in Sect. 5, for ejectors designed to date, the difference in the value of η_{TER} and η_{RER} is small. Finally, η_{TER} may also be computed for a system comprising an independent turbine and compressor with the role of expanding one fluid and compressing a second, if the isentropic (also known as adiabatic) efficiencies of the turbine and compressor are known. Bulusu *et al.* [9] demonstrates the following equality:

$$\eta_{\text{TER}} = \frac{\text{ER}}{\text{TER}} = \eta_{\text{turb},s} \eta_{\text{comp},s} \quad (43)$$

where $\eta_{\text{turb},s}$ and $\eta_{\text{comp},s}$ are the isentropic efficiencies of the turbine and compressor. Consequently, it may be said that an ejector with a turbine-compressor efficiency of 35% will provide the same performance as a turbine and compressor with isentropic efficiencies of 50% and 70% respectively, for example.

4.4. Compression Efficiency

Al-Najem *et al.* [5] presented the following measure of ejector performance, which here is denoted the compression efficiency:

$$\eta_c = \frac{(\dot{m}_M + \dot{m}_E)(h_D - h_E)}{\dot{m}_M(h_M - h_{M,NE})} \quad (44)$$

where $h_{M,NE}$ represents the static enthalpy at the exit of the motive fluid nozzle. Al-Najem *et al.* mention that this efficiency compares the compression work done in the ejector to the kinetic energy available from the motive stream. In a steady flow ejector, the nozzle through which the motive fluid is accelerated is adiabatic. The quantity $h_M - h_{M,NE}$ represents the kinetic energy gained by the motive flow from the nozzle inlet to the exit. In an adiabatic mechanical compressor, the compression work done is given by $\dot{m}_E(h_D - h_E)$. Based upon this

assertion, it seems likely that the quantity $h_D - h_E$ was selected to represent the compression work done in an ejector. However, in an ejector, heat transfer occurs between the motive and entrained streams during mixing. Therefore, the change in enthalpy $h_D - h_E$ is due to a combination of heating/cooling and work. Indeed, the useful compression work done in an ejector requires the thermodynamic states describing the process path of the entrained fluid to be known. In conclusion, based upon the review of literature conducted, the rationale behind and the utility of the compression efficiency is unclear.

4.5. Exergetic Efficiency

Exergetic analysis is a means of evaluating ejector performance from a Second Law point of view. The premise of exergetic efficiency is to compare the useful exergetic output of a system or component to the exergetic input. First, for an exergetic efficiency to be meaningful, an environmental dead state must be defined. Let us consider an ejector operating within an environment at T_0 and P_0 , as in Fig. 2. $\dot{Q}_0$ represents the rate of heat input from the surroundings. For a steady-state process, the First and Second Laws take the following form:

$$(h_M - h_{D,M}) + \dot{Q}_0 = \text{ER} (h_{D,E} - h_E) \quad (45)$$

$$(s_M - s_{D,M}) + \frac{\dot{Q}_0}{T_0} + \dot{S}_{\text{gen}} = \text{ER} (s_{D,E} - s_E) \quad (46)$$

where states D,M and D,E represent the state of the motive and entrained fluids in the discharge stream, calculated at their respective partial pressures. If the motive and entrained fluids do not differ in chemical composition, these states will be identical. Multiplying the Second Law by T_0 and subtracting the resulting equation from the First Law, the ejector process may be described in terms of flow exergy, x , a state variable.

$$(h_M - h_D) - T_0 (s_M - s_D) = (h_D - h_E) - T_0 (s_D - s_E) + T_0 \dot{S}_{\text{gen}} \quad (47)$$

$$-\Delta \dot{x}_M = \text{ER} \Delta \dot{x}_E + T_0 \dot{S}_{\text{gen}} \quad (48)$$

$$\Delta \dot{x}_E = (h_D - h_E) - T_0 (s_D - s_E) \quad (49)$$

$$\Delta \dot{x}_M = (h_D - h_M) - T_0 (s_D - s_M) \quad (50)$$

The next step is to identify the exergetic output from and exergetic input to the system. This choice is somewhat subjective and depends on how the purpose of the ejector is defined. It may be argued that the role of an ejector is to transfer exergy from the motive fluid to the entrained fluid. This is somewhat analogous to a compressor or an extraction fan, where the device's role is to transfer exergy to a fluid. The value (or output) of the ejector process, in exergetic terms, may be measured by the change in exergy of the entrained fluid. The price paid to implement the process, again in exergetic terms, is the change in exergy of the motive fluid. Hence, the following formulation for exergetic efficiency, identical to that employed by Al-Najem *et al.* [5], is obtained:

$$\eta_X = \frac{\text{ER} \Delta \dot{x}_E}{-\Delta \dot{x}_M} \quad (51)$$

Considering Eq. (47) it is clear that η_X also quantifies the exergetic losses per unit of exergetic input to the ejector process, Eq. (52).

$$1 - \eta_X = \frac{T_0 \dot{S}_{\text{gen}}}{\dot{m}_M [(h_D - h_M) - T_0 (s_D - s_M)]} \quad (52)$$

4.5.1. Comparison of the Exergetic and Reversible Entrainment Ejector Efficiencies

An analytical expression for the exergetic efficiency may be obtained when the inlet fluids are ideal gases of identical and constant specific heats. If the further restriction of having inlet fluids at the same temperature is imposed, the discharge enthalpy becomes independent of the entrainment ratio, Eq. (11). The discharge temperature equals the inlet temperature, and thus outlet and inlet specific enthalpies are also equal, Eq. (10). Consequently, the exergetic ejector efficiency takes the following form.

$$\eta_X = \text{ER} \frac{(h_D - T_0 s_D) - (h_E - T_0 s_E)}{(h_M - T_0 s_M) - (h_D - T_0 s_D)} \quad (53)$$

$$= \text{ER} \frac{-T_0 s_D + T_0 s_E}{-T_0 s_M + T s_D} \quad (54)$$

$$= \text{ER} \frac{s_D - s_E}{s_M - s_D} \quad (55)$$

$$= \text{ER} \frac{\ln \frac{P_D}{P_E}}{\ln \frac{P_M}{P_D}} \quad (56)$$

From Eq. (18), we can obtain the following expression for ER:

$$\text{ER} = \frac{\ln(\frac{P_M}{P_D}) - \frac{\dot{S}_{\text{gen}}}{R\dot{m}_M}}{\ln(\frac{P_D}{P_E})} \quad (57)$$

Upon substituting Eq. (57) into Eq. (56), the exergetic ejector efficiency is found to be equal to the reversible entrainment ratio for an ideal gas ejector with identical specific heats and identical inlet temperatures:

$$\eta_X = \frac{\ln(\frac{P_M}{P_D}) - \frac{\dot{S}_{\text{gen}}}{R\dot{m}_M}}{\ln(\frac{P_M}{P_D})} = \eta_{\text{RER}} \quad (58)$$

5. Comparison of Ejector Performance Metrics using Experimental Data

Now, having rigourously defined measures of ejector performance we may put these measures of efficiency to use in order to answer the following questions:

1. How do the numerical values of these efficiencies compare to one another?
2. What are the values of efficiency achieved by devices documented in literature?

To answer the first question, we can simply compare a sample of fictitious ejector performance data. For example, we can compare the disparity in efficiency definitions over a range of compression ratios, P_D/P_E . Let us consider a single case involving the compression of saturated water vapor using high pressure saturated steam. Let us choose $P_M=10$ bar and $P_E=50$ kPa. Let us fix the performance of the real ejector by fixing ER (Note: we could also theoretically fix the real outlet state or the dimensionless entropy generation $\bar{s}_{\text{gen}}$). Now, in Fig. 20 the range of efficiencies are plotted versus the compression ratio, P_D/P_E . The compression efficiency is omitted due to its lack of rigour and its requirement for knowledge of a thermodynamic state internal to the ejector control volume.

[Figure 20 about here.]

To answer our second question, we must extract data from literature. One drawback of using such data is that the performance of ejectors analysed in literature is not necessarily representative of the state of the art ejectors employed in industry. Furthermore laboratory tests may not have been performed at the ejectors' design points of operation. With this caveat in mind, the efficiencies of an air-air and a steam-steam ejector are analysed, acknowledging that the conclusions drawn may not be generalised to cover all ejectors. Amenable data for air-moist-carrier-gas ejectors was not available. Data for the air-air ejector was extracted from work by Bulusu *et al.* [9], Fig. 21 and Table 1 in Appendix 1. Data for the steam-steam ejector is from work by Eames *et al.* [20], Fig. 22 and Table 2 in Appendix 2. One should be aware that the degree of superheat of the motive steam is not obvious from the reported work and motive steam has been assumed to be saturated at the entrance to the ejector.

[Figure 21 about here.]

Note how, according to Eq. (58), the exergetic and reversible entrainment ratios are identical in value. The discrepancy in the turbine-compressor efficiency compared to the former two definitions is due to the fact that entropy generated in reaching thermal equilibrium of the fluids is unaccounted for.

[Figure 22 about here.]

Note that critical conditions of operation typically refer to the mode of ejector operation where the discharge pressure is such that choking of the entrained fluid occurs within the ejector. This choking is due to the formation of a hypothetical throat for the entrained fluid by the motive fluid expanding from the nozzle. For given inlet fluid conditions, the critical point of operation corresponds to the point where the entrained mass flow rate is

maximum. The occurrence of choking within steady flow ejectors is dealt with in great detail by Chou *et al.* [3]. In Fig. 22 as in Fig. 21, the exergetic and reversible entrainment ratio efficiencies are similar in value although both demonstrate an increasing trend. The turbine-compressor efficiency is also close, and as expected, is greatest in value. Meanwhile the reversible discharge pressure efficiency differs significantly in value. This may be explained in two ways. Firstly, the reversible processes employed for the reversible discharge pressure and the reversible entrainment ratio are different. Secondly, the ratio of mass flows, employed in the reversible entrainment ratio, and the ratio of pressure rises of the entrained fluid, employed in the reversible discharge pressure clearly do not scale in the same manner.

6. Conclusions

In this paper, various definitions of ejector efficiency have been evaluated. When defining or employing an ejector efficiency, it is crucial to describe the physical significance of the particular efficiency along with its utility. The following conclusions have been reached:

1. A reversible entrainment ratio may be calculated for any steady-flow ejector process, by employing the the First and Second Laws of thermodynamics, provided the thermodynamic states of the inlet fluids and the total pressure of the discharge fluid, all at equilibrium conditions, are known for a real ejector process.
2. The reversible entrainment ratio of an ideal gas ejector may be formulated analytically as a function of the inlet (motive-to-entrained) temperature and pressure ratios, the compression (discharge-to-entrained) pressure ratio and the ratio of specific heats. Calculation of the real entrainment ratio also requires knowledge of a further state variable describing the discharge (or alternatively the dimensionless entropy generation).
3. For steam-steam ejector applications, the inlet and outlet ejector states are shown to be accurately represented by ideal gas equations for pressures below 10 bar. Higher pressure motive steam may also be modelled as an ideal gas, provided the level of superheat is sufficient.
4. Where the motive fluid employed in a ejector is saturated steam, there exists an optimal pressure that maximizes the reversible entrainment ratio.
5. Where the temperature of motive steam is limited, the reversible entrainment ratio is maximized by employing saturated motive steam at a pressure equal to the saturation pressure at the maximum allowable temperature.
6. The reversible vapor entrainment ratio (ratio of vapor entrained to motive vapor) for steam-moist-carrier gas ejectors is insensitive to the choice of carrier gas, if the inlet relative humidity is maintained constant.
7. The turbine-compressor entrainment ratio differs from the reversible entrainment ratio only in that the discharge fluids are unmixed and not brought into thermal and chemical equilibrium.
8. The reversible entrainment ratio RER is always greater than or equal to the turbine-compressor entrainment ratio TER .
9. The reversible entrainment ratio efficiency η_{RER} is always lower than the turbine-compressor efficiency η_{TER} .

10. For general air-air and steam-steam ejectors, the exergetic efficiency η_X is very close in numerical value to the reversible entrainment ratio efficiency η_{RER} .
11. The exergetic efficiency η_X for an ideal gas ejector with inlet fluids at the same temperature is identical to the reversible entrainment ratio efficiency η_{RER} .
12. The reversible entrainment ratio efficiency η_{RER} is advantageous compared to the exergetic efficiency η_X as its value does not depend upon the choice of a reference temperature.

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Appendix - Experimental Data Tables

Table 1: Experimental air-air ejector data extracted from work by Bulusu *et al.*, [9]

P_M [kPa]	P_E [kPa]	P_D [kPa]	P_D^{rev} [kPa]	T_M [K]	T_E [K]	T_D [K]	T_D^{rev} [K]	ER [-]	RER [-]	PI_c [-]	$\bar{s}$ [-]	η_{RDP} [%]	η_{RER} [%]	η_{TME} [%]	η_X [%]
769	27	105	620	293.7	293.7	292.3	293.2	0.068	1.46	3.92	1.90	13.18	4.69	7.51	4.69
677	30	104	538	293.7	293.7	292.5	293.3	0.079	1.50	3.48	1.77	14.64	5.28	8.18	5.28
601	36	104	492	293.7	293.1	292.6	293	0.076	1.65	2.90	1.67	14.92	4.62	6.85	4.62
538	42	103	453	293.7	293.1	292.7	293.1	0.072	1.84	2.45	1.58	14.87	3.93	5.60	3.93
485	48	103	417	293.7	293.1	292.8	293.1	0.071	2.03	2.14	1.49	14.94	3.47	4.78	3.47
437	63	103	387	294.3	292.6	293.4	292.9	0.067	2.94	1.63	1.41	12.34	2.27	2.96	2.27
394	68	103	354	294.3	292.6	293.5	292.9	0.066	3.28	1.51	1.32	12.09	2.00	2.54	2.00
356	72	103	320	294.3	292.6	293.6	292.9	0.072	3.48	1.43	1.22	12.43	2.08	2.58	2.08
322	75	102	294	294.3	292.6	293.7	292.9	0.068	3.75	1.36	1.12	12.36	1.82	2.22	1.82
292	78	102	268	294.3	292.6	293.7	292.9	0.069	3.94	1.31	1.03	12.58	1.74	2.08	1.74
264	79	102	237	294.3	292.0	293.7	292.5	0.098	3.76	1.29	0.92	14.44	2.60	3.05	2.59
238	83	102	218	294.3	292.0	293.8	292.4	0.093	4.14	1.23	0.83	14.05	2.26	2.60	2.25
215	87	102	201	294.3	292.0	293.8	292.4	0.085	4.66	1.17	0.73	13.29	1.83	2.06	1.82
194	90	102	184	294.3	292.0	293.9	292.4	0.076	5.20	1.13	0.63	12.66	1.46	1.61	1.45
174	93	102	167	294.3	292.0	294.0	292.3	0.072	5.96	1.09	0.53	11.86	1.21	1.32	1.21
155	95	102	150	294.3	292.0	294.0	292.3	0.065	6.45	1.07	0.42	11.67	1.00	1.06	1.00
129	97	102	127	294.3	291.5	294.0	291.9	0.060	5.72	1.04	0.24	14.05	1.04	1.08	1.03

Table 2: Experimental steam-steam ejector data extracted from work by Eames *et al.*, [20]

P_M [kPa]	P_E [kPa]	P_D [kPa]	P_D^{rev} [kPa]	T_D [K]	T_D^{rev} [K]	ER [-]	RER [-]	$\bar{s}$ [-]	η_{RDP} [%]	η_{RER} [%]	η_{TME} [%]	η_X [%]
199	0.87	3.40	27.5	344	304	0.59	3.1	0.59	9.5	19.1	21.3	19.9
232	0.87	3.70	34.0	349	306	0.54	3.0	0.54	8.5	18.2	20.4	19.1
270	0.87	4.40	44.7	355	310	0.47	2.6	0.47	8.1	17.9	20.3	18.8
313	0.87	5.10	62.1	362	313	0.39	2.4	0.39	6.9	16.1	18.6	17.2
362	0.87	5.40	89.3	370	314	0.31	2.4	0.31	5.1	13.0	15.0	14.0
199	1.04	3.60	35.6	349	305	0.50	3.3	0.50	7.4	15.0	16.5	15.8
232	1.04	4.10	48.4	356	308	0.42	3.0	0.42	6.5	13.8	15.3	14.7
270	1.04	4.60	63.8	362	310	0.36	2.8	0.36	5.7	12.7	14.2	13.6
313	1.04	5.10	89.0	369	312	0.29	2.7	0.29	4.6	10.8	12.2	11.7
362	1.04	5.70	123.6	376	315	0.23	2.5	0.23	3.8	9.1	10.4	10.0
199	1.23	3.80	47.7	355	305	0.40	3.6	0.40	5.5	11.0	12.0	11.9
232	1.23	4.19	63.0	361	307	0.34	3.4	0.34	4.8	10.0	11.0	10.9
270	1.23	4.70	85.0	367	310	0.28	3.1	0.28	4.1	8.9	9.9	9.8
313	1.23	5.30	105.6	372	312	0.25	2.9	0.25	3.9	8.6	9.7	9.5
362	1.23	6.00	154.3	380	315	0.18	2.7	0.18	3.1	6.7	7.6	7.5

Figure 1: Schematic diagram of a steady flow ejector

Figure 2: Control volume for an ejector process

Figure 3: Graphical representation of the reversible entrainment ratio

Figure 4: Graphical representation of the reversible discharge pressure reference process

Figure 5: Graphical representation of the reversible entrainment ratio reference process

Figure 6: Schematic diagram of a thermodynamically reversible ejector for fluids of identical chemical composition

Figure 7: Process paths for a thermodynamically reversible ejector

Figure 8: Lines of constant ratio of specific heats, γ , for steam

Figure 9: Specific enthalpy-entropy diagram for pure water

Figure 10: Reversible entrainment ratio versus motive steam pressure for a steam-steam ejector. ($P_D=15$ kPa, $P_E=10$ kPa, $x_M=1$, $x_E=1$)

Figure 11: Optimal motive steam pressure of a steam-steam ejector

Figure 12: Reversible entrainment ratio versus discharge pressure for a steam-steam ejector. ($P_M=10$ bar, $P_E=10$ kPa, $x_M=1$, $x_E=1$)

Figure 13: Reversible entrainment ratio versus motive steam superheat for a steam-steam ejector. ($P_D=15$ kPa, $P_M=10$ bar, $P_E=10$ kPa, $x_E=1$, $T_{M,sat} = 180^\circ\text{C}$)

Figure 14: Reversible entrainment ratio versus motive steam specific entropy for a steam-steam ejector, with motive steam at constant temperature. ($T_M=179.9^\circ\text{C}$, $P_{\text{sat},M}=10$ bar, $P_D=15$ kPa, $P_E=10$ kPa, $x_E=1$)

Figure 15: Isotherms on a specific enthalpy-entropy diagram for pure water

Figure 16: Reversible entrainment ratio versus motive steam pressure for steam-moist-carrier-gas ejectors. ($P_D=75$ kPa, $P_E=50$ kPa, $T_E=50^\circ\text{C}$, $x_M=1$, $\phi_E=1$)

Figure 17: Reversible vapor entrainment ratio versus motive steam pressure for steam-moist-carrier-gas ejectors. ($\Pi_c=1.5$, $P_E=50$ kPa, $T_E=50$ °C, $x_M=1$, $\phi_E=1$)

Figure 18: Reversible vapor entrainment ratio versus entrained saturated carrier gas temperature for a steam-moist-carrier-gas ejector. ($P_D=75$ kPa, $P_E=50$ kPa, $\Pi_i=20$, $x_M=1$, $\phi_E=1$)

Figure 19: Isentropic expansion and compression processes

Figure 20: Comparison of efficiency definitions for a steam-steam ejector over a range of compression ratios

Figure 21: Comparison of efficiency definitions using data from an air-air ejector, [9, 21]. Chamber throat diameter, 0.88 inches; Diffuser exit diameter, 3.02 inches; Nozzle throat diameter, 0.5625 inches; Nozzle exit area 0.735 inches; Plenum diameter for secondary flow, 3.25 inches.

Figure 22: Comparison of efficiency definitions using data from a steam-steam ejector operating at conditions of critical back pressure [20]. Chamber throat diameter, 18 mm; Diffuser exit diameter, 40 mm; Nozzle throat diameter, 2 mm; Nozzle exit area 8 mm; Plenum diameter for secondary flow, 24 mm.

Table 3: Reversible discharge pressure process

Fixed	Output
$P_E = P_E^{\text{rev}}$	P_D^{rev}
$T_E = T_E^{\text{rev}}$	T_D^{rev}
$P_M = P_M^{\text{rev}}$	
$T_M = T_M^{\text{rev}}$	
ER = RER	

Table 4: Reversible entrainment ratio process

Fixed	Output
$P_E = P_E^{\text{rev}}$	RER
$T_E = T_E^{\text{rev}}$	T_D^{rev}
$P_M = P_M^{\text{rev}}$	
$T_M = T_M^{\text{rev}}$	
$P_D = P_D^{\text{rev}}$	