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Uncertainty, Flexibility, Valuation & Design: How 21st Century Information & Knowledge Can Improve 21st Century Urban Development

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Abstract:

The 21st century presents humankind with perhaps its greatest challenge since our species almost went extinct some 70,000 years ago in Africa. A big part of meeting that challenge lies in how the urbanization of three billion additional people (equal to the entire world population in 1960) will be accomplished between now and mid-century, on top of necessary renewal and renovation of the earth's existing cities. China alone will urbanize 300 million more people between now and 2030. (That is equal to the entire population of the U.S., the world's third most populous country, and just 20 years!) This is development on a scale and pace that is an order of magnitude greater than the past century, in a world resource and climate environment that is near the breaking point, in a context of greater technological, financial, and economic uncertainty than ever before.

To meet this challenge will require that we use the best tools in our kit, including ones that have become available to us only in this new knowledge and information-based century. Technology got us here, and technology will be key to getting us through. In this paper we will review and synthesize two important methodological developments in our profession that can help infrastructure and real estate physical development (i.e., urban development) to be accomplished more effectively and efficiently in a world of uncertainty. The first methodological development is the honing of real options theory and methodology for practical application to identify and evaluate sources of flexibility in the design and operation of capital projects. The second development is the marriage of digital data compilation of property transactions records with the honing of econometric analysis methodology to allow the practical quantification of real estate and infrastructure asset price dynamics. We argue that this latter development provides the key input to the former development, enabling a much more complete and rigorous treatment of design and evaluation problems for urban development. We also argue that an engineering systems approach to option modeling is likely to find better traction in actual professional practice than the economic theoretical models that have dominated the academic literature. We provide a concrete example by applying the suggested approach to the Songdo New City development in Korea.

The result can be better informed design and valuation, more efficient urban development laced with greater flexibility to avoid the worst down-side outcomes and to take advantage of the best up-side opportunities, saving vital resources of capital, land, raw materials, and energy. Finally, we argue that a global, thought-leadership institution such as the RICS can and should play a leadership role in supporting and promulgating the new information bases and interdisciplinary educational formations (property, land, construction) that must underpin the successful dissemination of such 21st century tools of analysis.

**Uncertainty, Flexibility, Valuation & Design:
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*“Airplanes are interesting toys, but of no military value.”
–Maréchal Ferdinand Foch, Professor of Strategy, École Supérieure de Guerre.*

*“Stocks have reached what looks like a permanently high plateau.”
–Irving Fisher, Professor of Economics, Yale University, 1929.*

“It’s really hard to design products by focus groups. A lot of times, people don’t know what they want until you show it to them.” –Steve Jobs.

“It is not the strongest species that survive, nor the most intelligent, but the most responsive to change.” –Charles Darwin.

The 21st century presents humankind with perhaps its greatest challenge since our species may have approached extinction in Africa some 70,000 years ago.¹ Population will likely peak by the middle of this century at around nine billion, almost a quarter more than today and three times what it was when the co-authors of this paper first entered MIT as undergraduates half a century ago. For the first time in history, half the human race now lives in cities, and while the world rural population is already essentially at its peak, urban population will double, adding another three billion people to cities by the middle of the century. In China alone, in just the next 20 years, 300 million people will become urbanized, equal to the entire (not just urban) population of the United States, the world’s third most populous country. Over just the next 10 years The Economist Intelligence Unit predicts China will invest over USD 11 trillion on urban housing alone (almost the magnitude of the entire current U.S. annual GDP). This is development on a scale and at a pace that is an order of magnitude greater than in the past century, which was itself already earth-shaking in many ways. Development in the first half of the 21st century will require massive investment in infrastructure, housing, and commercial real

¹ Genetic evidence suggests that our ancestors may have been reduced at that time to perhaps barely over 1,000 breeding pairs, a level perilously close to the point at which extinction would almost surely have resulted. It is not known exactly why or how this “bottleneck” in human evolution occurred, but a result is that at the genetic level our species exhibits much less diversity than most species.

estate. And this development must occur in a world where resources are constrained as never before. Furthermore, it is now clear that climate change will be an increasing factor throughout this century. Development must occur in the context of a need to transform production and consumption patterns and technology to prevent further damage to the environment not least being a possibly catastrophic escalation of global warming. This will require a revolution in energy sources and usage. Against these challenges, humanity is now armed with an incredible level and growth in technology and information, and in global productivity and wealth, as well as supra-national institutions, far beyond what previous centuries have had at their disposal. The challenge is unprecedented, but so is our means to address it. They say that “change is the only constant”, but what we are facing in the coming decades is not just revolutionary change, but a degree of uncertainty that is perhaps unprecedented in human history. If you think the first decade of the 21st century was volatile, our guess is, “*you ain’t seen nothin’ yet!*”

What is the meaning of this challenge for we who are professionals with important responsibility in the planning and design, construction, and financing of the infrastructure and real estate that will define the built environment of the 21st century? This question may have many answers, but among them are likely to be words such as “urgency” and “humility”. Urgency because we are literally in a race between the unleashed explosive forces and effects of rampant development on the one hand and the advance of rational controlling and guiding powers that will be necessary to shape these forces for the good on the other hand. Humility because we cannot possibly know what the future will bring, and we have made major mistakes in the past. Infrastructure and real estate are huge fixed investments. (For our purposes in this paper we will define “infrastructure”, like real estate, to refer to long-lived, capital-intensive, spatially-fixed real assets, whether in the private or public sector.) The quantity, quality, type, and location of such real asset placements in the 20th century have not always proved to have been optimal or wise, in retrospect, even when they seemed so logical at the time when

they were made.² The as yet unknowns that await us in the 21st century loom even larger than those that humbled the planners and builders of the last century.³

With this in mind, it is the thesis of this paper that the design and development of the built environment in this century must be done with a greater understanding and appreciation of the value of, and need for, *flexibility*, than has heretofore been the case. Flexibility, and particularly as implemented by the explicit inclusion of options, or “optionality”, within major development projects, can be a major rational response to the great uncertainty and change that this century will certainly witness. Flexibility in design and decision-making can enable more efficient and effective use of scarce resources. It can enable developers and users of the built environment to take advantage of unexpected upside opportunities while also facilitating the avoidance of untoward downside events.

More specifically, we write this paper with the intent of weaving together three strands or themes that have arisen in the past couple decades in both the academic literature and professional practice of the engineering, design, and financial communities that focus on real estate and infrastructure development. We believe that, applied synergistically in an integrated fashion, these three strands can raise the professional level of practice in real estate and infrastructure design and investment so as to take advantage of opportunities for flexibility. The three strands we are referring to are:

- 1. Real Options Models:** The development in the financial and real estate academic literature of sophisticated, rigorous economic models of the valuation and optimal

² Many of the massive “slum clearance” so-called “urban renewal” projects undertaken in the United States in the 1950s and ‘60s are an obvious example that comes to mind. To go back even earlier, it would seem that the U.S. built about twice as much railroad mileage as it soon actually needed, as over 100,000 miles of track have been abandoned beginning in the early to mid-20th century, more abandoned than the entire existing railroad mileage in all but one other country (Russia). (Of course this does not *necessarily* imply that all of that abandoned track should never have been developed in the first place.)

³ See Macomber (2011) for an important perspective on the big picture of how and where capital can interact with design to intervene most effectively in the great global urbanization process that is challenging humanity.

exercise of “options”, that is, the right without obligation to undertake (or delay or abandon) a physical capital investment, such as buildings or highways, in an environment of uncertainty.

2. **Monte Carlo Simulation:** The development and honing of practical engineering analysis tools to model and value decision flexibility in the design and operation of physical assets in a context of uncertainty, in a manner that can be effectively communicated to and used by professional decision makers.
3. **Real Asset Pricing Data & Indexing:** The development of vast electronic databases of housing and commercial property asset transaction prices and appraised valuations which, combined with advances in econometric methodology and computation (and supplemented by some evidence regarding privatized infrastructure investment performance), allowing much more complete and rigorous quantification of the aforementioned economic and engineering models applied to real estate and infrastructure development projects.

We will present a brief review of each of these three developments, and then we aim to show how the three are mutually reinforcing and synergistic, with the economic real options model providing important theoretical underpinning to a type of Monte Carlo simulation model that can be more useful than the economic model in design practice, and with the asset pricing data and indexing advances resolving a key and fundamental “GIGO” (*garbage in, garbage out*) input problem for both types of models by enabling better quantification of the relevant volatility and uncertainty. We will apply these tools to a stylized but realistic example of a major development project that may be prototypical of much of the new urbanization of the 21st century in the emerging market countries, that of the Songdo International Business District development in the Republic of Korea. Finally, we will argue that there may be an important role for high quality global professional standards institutions

such as the RICS to help promulgate and guide the further development and use of these types of information sources and analytical tools.

The paper is organized into five sections. We begin with sections reviewing each of the three aforementioned methodological developments as they may be applied to real estate and infrastructure development. A fourth section then synthesizes the three and applies them to the Songdo example. A fifth section provides our concluding remarks.

1. Economic Real Options Models

The concept of “options” is most traditional in the field of finance, where “call options” and “put options” on stocks have been traded for decades. Fundamentally, an option is a “*right without obligation*”, for example, a call option gives its owner the right to buy a specified stock at a specified price, but the option owner does not have to exercise that right. The object to which the right applies is called the underlying asset of the option. A put option gives the right to sell its underlying asset at a specified price.

A real option is an option in which the underlying asset is a physical asset or collection of physical assets, such as mines or drilling platforms, factories, commercial buildings, or infrastructure facilities such as highways or ports. In the sorts of real options that are the focus of the current paper, the exercise of the call option on the asset is associated with the construction of the asset. The exercise price (or “strike price”) of the option is the construction cost of the project (exclusive of land cost). If the ownership of an undeveloped parcel of land conveys the real option rights to develop the land, then the value of that option is the value of the land parcel (and vice versa, in essence, the value of the land

is the value of the real option). The value of the vacant land is completely based on the nature and timing of the development decision that can (but need not, if it is an “option”) occur for the land.

The economics-based real option model of land value and optimal development arose from two strands of the academic literature. In economics departments and finance departments in business schools, mathematical models of financial options were developed in the late 1960s and 1970s, most famously with the 1972 Black-Scholes Model of so-called European calls and puts (options that can only be exercised on their expiration dates), for which the Nobel Prize in economics was awarded to Myron Scholes and Robert Merton in 1997. In a famous 1977 article MIT professor Stewart Myers coined the term “real options” when he extended the financial call option model to the value of “growth opportunities” a corporation holds in the investment projects they could implement (at their discretion). By the 1980s the real options concept was being applied to capital budgeting and project development investment decisions by authors such as McDonald and Siegel, with an influential book published by Dixit and Pindyck in 1994. A key focus was on the “value of waiting” to invest, and the need for the project developer to see more than just a zero net present value (NPV) comparing the construction cost of the project to the anticipated value of its benefit. In essence, options have value because of their non-obligatory nature. They enable the option holder to take advantage of upside possibilities that arise in the future, while not forcing the option holder to enter into an unfavorable outcome when downside eventualities occur.⁴ This makes options a bit of an oddity in the financial world: assets whose values are *increased* with greater volatility (i.e., with greater risk). Greater volatility means a greater downside possibility, but also a greater upside. An option allows one to take

⁴ An analogy has been made using a surfing metaphor. The surfer paddles out beyond the breakers and waits. He doesn't take the first swell that comes along (usually), because there may be a better wave beyond that one, or the next. This is analogous to not building a development just as soon as its value exceeds only its construction cost. But the surfer doesn't wait forever either. A good enough swell will make it worthwhile to give up the opportunity to keep waiting for an even better wave. The ability to wait and select the wave of his choice is a big part of what makes the surfing fun, what gives the surfing its “value”. This is analogous to the option value.

advantage of the upside without exposure to the downside. Hence, volatility is a “friend” to an option holder.

The second strand of the real options literature came out of urban economics and real estate, with the publication in 1984 by Sheridan Titman of “Urban Land Prices Under Uncertainty”, the first explicit presentation of the call option model of land value as described above, showing that uncertainty in the real estate market increased land value but delayed land development. Dennis Capozza and Robert Helsley extended the classical “monocentric city model” to include the consideration of uncertainty, a fusion of spatial and financial economics that showed that optimal development under uncertainty would produce a denser and higher-rent city. Joe Williams and others extended the model to consider optimal development density as well as timing, and there has been a thriving real options literature in the real estate and urban economics fields, as well as in capital budgeting and corporate finance, ever since the 1980s and continuing into the 2000s. More recently there has been a focus on empirical verification of the theoretical real option model, and several careful studies have generally confirmed the empirical validity of the real option approach as a model of land value and development.⁵

The key insight from the economic real option model for purposes of the present paper is that *flexibility contains value* in the context of physical capital investments, and this value is *greater* the *more uncertainty* or volatility exists in the value of the underlying assets being developed. In essence, an option (or more generally, “optionality” in development projects) provides flexibility. Options allow

⁵ See for example recent articles by Cunningham (2006, 2007), Schwartz & Torous (2007), Clapp & Lindenthal (2009), and Bulan *et al* (2009), which built on much earlier work by Quigg (1993). Also worthy of note was a series of articles by Steve Grenadier applying the real option model to optimal leasing and to help explain the over-building phenomenon or tendency toward “development cascades” in real estate. Indeed, there are by now literally hundreds of academic articles applying real option theory to real estate and capital budgeting. And more recently a few articles have applied real options theory specifically to infrastructure projects, such as Chiara *et al* (2007), Rose (1998), and Smit & Trigeorgis (2009). All the references cited or mentioned in this paper are contained in the bibliography at the back, but we are not claiming this does more than highlight a few of the more famous articles, and we are leaving out many fine contributions.

the developer to build later (or sooner), to build more (or less) on a given site, or to build different types of structures. From a systems perspective, it is sometimes useful to view these types of flexibility as providing options *on* the system. (This is in distinction from options *in* the system, which have to do with a more micro-level of design and operational decision-making, where the economic model may be less useful.) The real options literature proposes a number of ways in which major capital projects often contain flexibility that can be evaluated as options. For example, Trigeorgis (1996) enumerates the following taxonomy:

- Option to Defer
- Time-to-build Option (Staged Investment)
- Option to Alter Operating Scale (Expansion, Contraction)
- Option to Abandon
- Option to Switch (Outputs or Inputs)
- Growth Options
- Multiple Interacting Options

The importance of the economic model's insight about the value of flexibility cannot be overstated. But the practical importance of the economic real option model is not just in this insight, but also in its ability to *quantify* the value of flexibility in development projects, at least at an approximate and broad-brush or large-scale level, in a very *rigorous* manner, based on economic science. To see this, consider the theoretical basis of the economic options model.

It is often believed that the theoretical underpinnings of the economic option model make it only valid or relevant in circumstances where the underlying assets are traded in "perfect markets", that is, highly competitive and frictionless markets where assets are homogeneous, can trade in any fractional shares long or short, all prices are immediately publicly quoted, and transactions are free and

immediate. But in fact the theoretical underpinnings of the economic option model are much more broadly relevant than in such a rarified circumstance.

It is true that the original derivations of the option valuation model in the financial economic literature employed arbitrage analysis, in which the mathematical formulas were derived by imposing no-arbitrage conditions on the price the option could command. As an “arbitrage” involves the making of certain money risklessly, it is presumed that any such opportunities will be quickly seized and, in a competitive market, prices would adjust until no arbitrage is possible. The no-arbitrage condition is thus a powerful descriptor of equilibrium prices in the markets relevant for trading the option. But equilibrium pricing is a deeper and broader concept than the no-arbitrage condition. In that broader context one may view the no-arbitrage condition as merely a technical device for arriving at an equilibrium pricing model. If the relevant markets are so complete and frictionless as to enable true arbitrage trading to occur, then indeed such trading will “enforce” in reality the equilibrium price derived by the model. (In the real world no markets are quite as “perfect” as the mathematical models assume, but some come close enough to make arbitrage trading a major reality, such as some markets for stocks, bonds, commodities, foreign exchange, and various derivatives based on these underlying assets.)

But even in the absence of perfect conditions and arbitrage trading, markets still exist and often function well, and find (or tend toward) equilibrium between demand and supply. Thus, an equilibrium price model will be a “good” model even in less than perfect markets where literal arbitrage is not possible, such as real estate markets. The option pricing models derived using the no-arbitrage condition describe equilibrium pricing of the options, prices that balance supply and demand, whether or not such equilibrium is “enforced” by arbitrage trading. Indeed, such models in the finance literature have often also been derived using explicit equilibrium models such as the Capital Asset Pricing

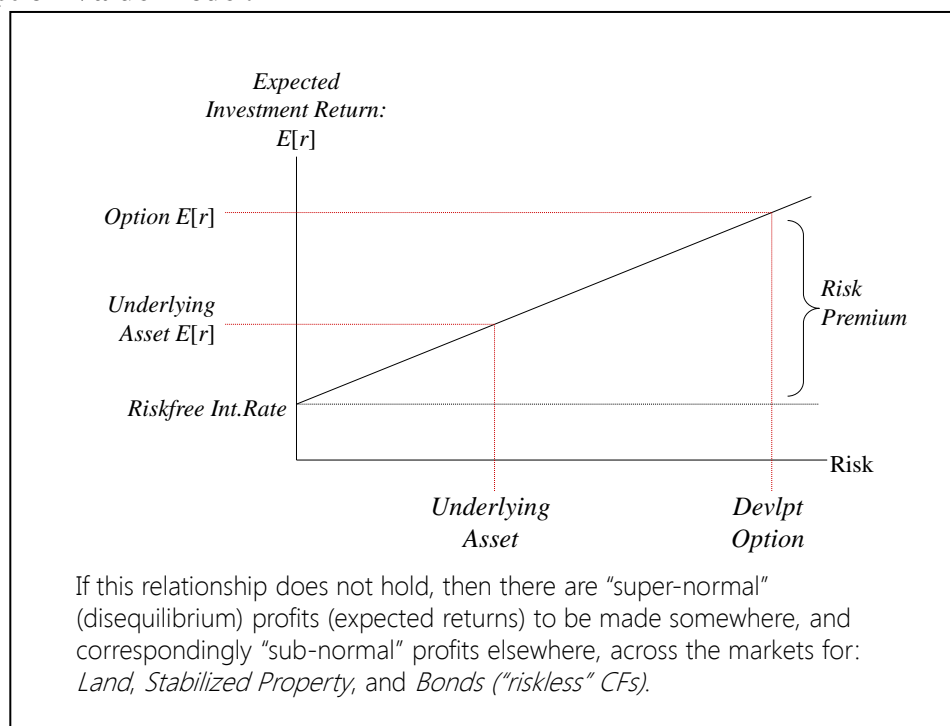
Model (CAPM). The way to look at the economic option price models in a context of well-functioning but imperfect markets such as real estate is that the models provide a good guide for what the option would likely sell for. While any given deal may deviate from the model's prediction, the model will tend to predict the correct price on average.

In fact, we can go even farther. The economic option value models have an important type of validity in a context where the relevant market is not very good at all, or even non-existent (such as, perhaps, some very unique underlying assets or some types of infrastructure assets that are not traded in the private sector at all). In this context, there is no market value or exchange value for the option, but there may still be a type of "opportunity cost". Resources will be expended to acquire and exercise the option, and it will produce an asset that has value. One may define the option value in such circumstances based on *normative* considerations, and view the valuation model from such a normative rather than *positive* (or empirical predictive) perspective. From this perspective the economic option value model may be viewed simply as implementing the "Law of One Price", requiring that the option and its underlying asset and riskless bonds all provide expected returns on the same "Security Market Line." That is, each investment must provide the *same* going-in expected investment return risk premium (above the risk-free interest rate) *per unit of risk*. This would seem to be quite a reasonable basis on which to define a normative definition of value for the option, in relation to its underlying asset and construction cost.

This "Law of One Price" essence of the theoretical underpinnings of the economic option value model is depicted in Exhibit 1. The horizontal axis measures risk in whatever way the capital market cares about risk and thereby reflects risk in asset prices. The vertical axis measures expected investment returns. The straight line, a security market line (SML), indicates the expected returns that will provide all assets, including both underlying assets and derivatives such as options, with the same

expected return risk premium per unit of risk. The economic option value model simply indicates the option price that will cause the option to provide such an investment return expectation. Any other price would cause the option to either provide more, or less, return premium per unit of risk than is “normal” (as indicated by equilibrium or “average” prices of assets in the capital market). If nothing else, such a “miss-pricing” would seem to be unfair in some sense (or to some party).

**Exhibit 1:
The “Law of One Price” and the Security Market Line: The Theoretical Underpinning of the Economic Option Value Model.**

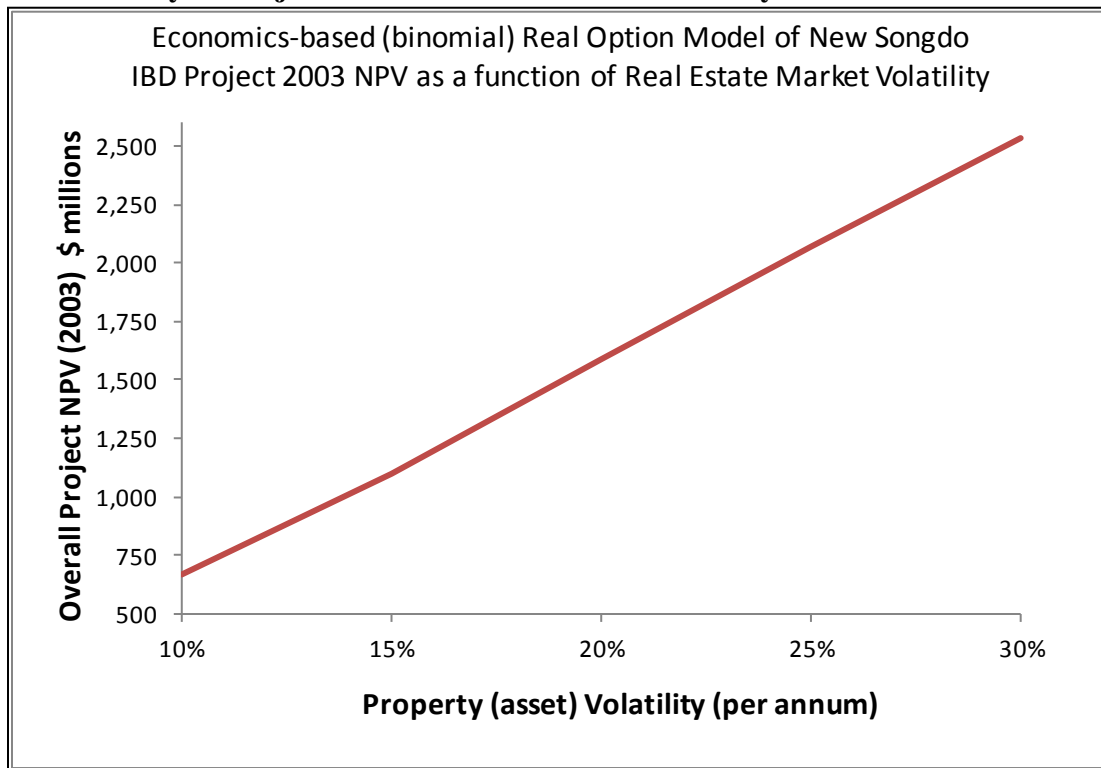


The real option valuation model is unquestionably one of the major discoveries in the economics of finance and real estate in the past generation. And yet, interestingly, it is very little used in the actual real world of professional practice, either in valuation or in project investment analysis and decision-making. This is in spite of the fact that it would seem, in principle, to be of considerable practical import in both of those fields. And it is also in spite of the fact that financial option models have achieved widespread practical use in the investment world. For example, stock options and derivatives are regularly valued and traded with the aid of option valuation models, and whole new

indices and major investment products have been based on such models (for example, the “VIX” index tracking the volatility in the stock market). In our view, there appear to be two major reasons for the lack of practical usage of the real option model, at least regarding application to real estate and infrastructure development projects: difficulty quantifying the model’s required inputs, and the complexity or opacity of the models regarding practical decision making. We will attempt to make a contribution to both of these issues in the present paper.

The problem with the quantification of the inputs to the real option model is summarized by the classic expression: “*Beware of GIGO!*” (*Garbage in; Garbage out*). The economic real option model requires several quantitative inputs, but perhaps the most problematic for application to real estate and infrastructure projects has been the metric that the model uses to measure the relevant uncertainty regarding the underlying asset’s value. In the most basic and potentially widespread applications, this input represents the volatility in the underlying asset’s value, that is, the longitudinal standard deviation in the returns to an investment in the underlying asset. Option value can be extremely sensitive to this input. For example, drawing on our Songdo project example application that we will elaborate on in Section 4, Exhibit 2 shows how development project values can vary several-fold within a reasonable range of input values for underlying asset volatility. For financial option model applications in the securities investment world of publicly-traded homogeneous assets, such as shares of stocks or commodities or foreign exchange, a wealth of data has long been available to quantify actual historical volatility and asset price dispersion, giving investors and traders confidence to use the option models. But until recently there has been no such similar empirical data on asset prices in the world of private markets trading unique whole assets, such as real estate. We will discuss the recent developments in this field in Section 3 below.

Exhibit 2: Sensitivity of Project Value as a Function of Volatility of Assets to be Built.



The second problem underlying the lack of widespread usage of the real option model in professional practice relates to the nature of the model itself. The derivation of the model as an artifact of the discipline of economics is both a strength and a weakness regarding practical adoption. The strength is the aforementioned rigor of the model, the clarity and elegance of its theoretical underpinnings in the concept of the Law of One Price. But the model contains two weaknesses that undercut its widespread use. One is that its mathematics tends to appear complex and opaque to the uninitiated and non-specialized, i.e., to most people who aren't card-carrying economists. The mechanics of the model, in terms of formulae or algorithms, typically lack much intuitive appeal for many potential users and decision makers. As a result, they have difficulty trusting or understanding the meaning of its prescriptions, and they tend to shy away from its use. Another problem is that while the mechanics of the model may appear complex, the essential nature and assumptions of the model actually often make it overly simplistic or inexact in terms of representing specific real world design

and investment decision choices and behaviors. The model tends to apply “*at 30,000 feet*”. This relates to our previous point about modeling “*options on*” as distinct from “*options in*” the building and infrastructure systems that compose the actual “bricks and mortar” of the development projects. The economic real option model may be good for evaluating the option to build a particular building, but it is less well suited to modeling the design question of whether to build in the structural capability to expand the building vertically from its initial 30 stories to a possible subsequent 50 stories without disrupting the occupants of the initial 30 stories, for example. It may be good for elucidating in principle how a developer should decide optimally to pull the trigger on a development (in terms of a critical or “hurdle” value of the asset to be built compared to its construction cost, for example), but it may not be good at modeling how the decision makers actually make such a decision, including consideration of available debt and joint-venture partner financing terms, likelihood of initial anchor tenant leasing, observable vacancy levels or lease-up rates in the local market, knowledge about specific competitive projects, and so forth.

This second problem in the use of the real options model derives from its very nature as an artifact of the economics discipline, and as a result, we suggest that to address this issue one must move beyond economics, to a fundamentally different type of model. We call this the “engineering model” of flexibility in development projects, and its major tool (in our current context) is the use of simple Monte Carlo simulation modeling in Excel®. This approach is admittedly less rigorous (for example, it cannot root its prescriptions in equilibrium theory or the Law of One Price), but it can better address the problems of opacity and over-simplicity for modeling specific decisions relating to flexibility not just “on” but also “in” development projects. It has obtained more widespread usage in actual practice, so far not much in real estate and infrastructure projects, but in manufacturing and natural resource extraction. Furthermore, we view the economics model and the engineering model as complements to each other. The economics model can be used to help calibrate and confirm the

engineering model by applying them side-by-side to simple problems to which they can both be applied (such as the valuation of a basic “option on” a particular building project). It is to the engineering model that we turn in the next section.

2. Engineering Models & Monte Carlo Simulation

The engineering models as we understand them are true complements to the economic models for the analysis of real options. That is, they provide solutions to valuation issues that the economic analyses as we know them cannot handle. On the other hand, they lack the strong theoretical underpinnings of the economic analysis. Engineering models provide insights and information that the economic models cannot, and vice versa. And the engineering models are in some circumstances able to communicate more effectively to decision makers. Consequently, we argue that a full analysis of an important valuation can often profitably use both what we label as the engineering and the economic approaches – as we do in our example application in section 4 to the valuation for the New Songdo City project.

Engineering models represent a conceptually different approach to the valuation of projects than the financial models. They address different issues from different perspectives. There are many aspects to the contrast in approaches, as we discuss further on. For the moment, an immediate distinction is that whereas the economic model tends to apply “*at 30,000 feet,*” as we said previously, engineering models can look at much more detail, consider the realities on the ground much more closely – perhaps they fly at 3,000 feet (but let’s not push the analogy too far).

This difference in perspective has important consequences for the analysis. Because engineering models are sensitive to important details, they cannot rely on the crucial simplifying

assumptions that form the basis for mathematical solutions used in the economic real option models. To make this point explicitly, consider two features of engineering models. They generally consider that:

- Uncertain processes generally change over time and commonly feature jumps. For example, the analysis might reflect the common assumption that demand for a service or product initially grows rapidly and then later more slowly as demand saturates. It might also factor in the likelihood of one or more sudden jumps as associated with changes in legislation (to impose environmental standards or carbon taxes, say, that would increase the value of an investment in “green” infrastructure) or in the structure of the marketplace (as by the creation of a free trade area or the arrival of a competitor).
- Projects frequently involve a variety of options that might be exercised in any order. Significant infrastructure projects involve options to expand locally (adding capacity to what exists), geographically (extending the network), and technologically (adopting new technologies). Just like moves on a chessboard, these options can be exercised in many ways, although some evidently must precede others.⁶

By contrast, economic models typically presume that the uncertainties stem from a stationary stochastic process and focus on implications of a single option. Such assumptions constitute the basis

⁶ For example, the New Songdo City project includes options to expand horizontally onto as yet undeveloped plots; options to expand locally, as for the convention center designed for possible modular expansion on its site; and options to change technologically, for example by shifting the water supply from reservoirs to desalination. Or consider investments in urban water supply, such as those of Hyflux, based in Singapore. The company delivers Build-Own-Operate plants worldwide. For any particular city it must consider that demand over the next generation will expand in some fashion (and thus could benefit from pre-building certain components of the system in advance of need, to enable easy future expansion), and it must also recognize that the technology and the energy cost of water purification may change substantially (which means that it should be open to new forms and location for future plants, as a switch from reservoirs to desalination implies a shift from facilities at higher altitudes to ones seaside).

of the widely used lattice method for analyzing options pioneered by Cox and Ross (1979).⁷ As the engineering models go beyond simplified situations that enable direct analytical solutions, they thus rely on “brute force” approaches – they develop solutions by looking at all the possibilities they can. This is why what we call the “engineering” approaches to valuation generally rely on Monte Carlo simulation.

While Monte Carlo simulation is a necessary feature of the engineering model of valuation, it is not sufficient. An approach to valuation can use simulation (see Hoesli *et al* 2006) without therefore being what we would call an engineering valuation. Monte Carlo simulation is just a tool that can be used in many contexts. It is simply a process that considers the set of possible outcomes for the range of possible scenarios, and then derives appropriate measures of value from these results. In other words, the engineering approach to valuation is not just a mathematical method. The crux of the engineering model lies in how it frames the problem of valuing projects.

The engineering model of value has three particular features that make it useful and interesting:

1. It builds upon the perspective of the investor/developer team that is designing and implementing a project. It does this by building on the spreadsheet models of cash flows that the development team will be using in any case. This provides both an easy link between the DCF analyses and option valuation as well as great transparency for the analysis, and thus easier acceptance by the users.
2. The engineering model pays particular attention to the specific idiosyncratic risks associated with specific features of any projects – issues that would be of less interest or

⁷ The efficiency of the lattice model depends on the possibility that alternative paths of development recombine so that the number of states to be examined expands linearly with the number of periods instead of exponentially, that is (1, 2, 3, 4, 5...) instead of (1, 2, 4, 8, 16...). This is only possible if we posit path independence in terms of the outcomes associated with each path, and this implies that we can effectively only value a single option. Luenberger (1998) provides a good exposition of this approach.

even beyond the ken of investors in a portfolio of projects such as REITs. Notably, the engineering model considers the value of important design details associated with a project, its “*options in*” the design, as defined by Wang and de Neufville (2006). These are the various forms of flexibility built into a project – such as the extra strength built into a bridge so that it can be eventually be double-decked should that seem desirable.⁸

3. The engineering model focuses on providing information on the distribution of the potential outcomes associated with optionality in a project. In addition to presenting a single value or price of an option, the model readily provides information on such features such as the value-at-risk, the upside potential or value-at-gain, maximum and minimum outcome values, and so on.

We motivate and describe these perspectives in detail next, and then illustrate their application through a simple example project.

It is important to reiterate in this context that economic real options analysis does not have much traction for many developers of infrastructure. Many have simply not heard of the approach. And of those that have been exposed to the concept, many find the concepts and mathematics lacking in transparency – and thus have little confidence in the approach. If we wish to address these practitioners, to have access to this market, then we need to have some tools that address the valuation of options in a way that they can understand.

As noted, engineering models of valuation explicitly attempt to frame the issues from the perspective of the developers and similar decision-makers. For starters, they build on the financial

⁸ The use of flexibility in designing projects is becoming increasingly common, as detailed in de Neufville and Scholtes (2011). The George Washington Bridge in New York and the Ponte do 25 Abril in Lisbon were both originally built with the strength to carry a second deck, and both got theirs almost a generation later. Buildings are often built with the extra strength to add extra stories (Guma *et al*, 2009), highways laid out with extra width to enable expansion or the installation of metro lines (as done for the access corridor to the Washington/Dulles airport), and in Japan JR East built some of its Shinkansen bullet train viaducts with the strength to receive not just one but two extra levels as necessary.

spreadsheets since these appear to constitute the *lingua franca* for ordinary financial analysis. We see how this approach has the significant advantages of establishing credibility and of providing transparency. Decision-makers can examine the assumptions in a form to which they are accustomed. They can include as much detail as they want. They can easily see the effect of different assumptions by changing the size or timing of cash flows, “pressing the button,” to effect a recalculation of the spreadsheet, so that different results appear almost immediately.⁹

From the perspective of the analysts, the use of spreadsheets enables them to model a project in a useful level of detail, with all the requisite subtleties. They can project demand according to any trend they wish, and can easily incorporate any kind of jumps or other step changes, thus reflecting the possible opening of complementary projects, the enactment of new regulations, step changes in the fee structures and so on. The engineering approach can thus model projects as designers and decision makers in the real world actually perceive and deal with them.

The focus on the realities of particular projects further distinguishes the engineering models from the economic models of valuation. We can view this as an extension of the interest in communicating with the investor/developer teams organizing and deciding on projects. These project developers may have limited ability to diversify over a portfolio of projects, and may thus need to pay considerable attention to idiosyncratic risks.¹⁰ And the realism is bolstered by focusing the analysis on a valuation framework that highlights a distribution of *ex post* outcome results, showing the decision maker not just the expected or most likely outcome but the nature and extent of the “tails” (upside &

⁹ Of course, spreadsheets also have their limitations. In some circumstances the particular nature or complexity of the design and operational decisions that need to be examined may require more specialized software tools. But many of these tools integrate with common “*lingua franca*” packages such as Excel®, and we find it most useful to attempt as much as possible to keep high level analysis and decision-oriented valuations at a level of simplicity and detail that can be modeled directly in Excel or similar spreadsheet products.

¹⁰ Consider the case of the GMR group, based in India. In recent years it has been a leading developer of airports (greenfield new airport for Hyderabad, total reconstruction of New Delhi airport, active participation in the development of Istanbul/Sabiha Gokcen second airport). This portfolio of projects, large compared to competitors, is too small or targeted to represent much by way of diversification that would eliminate idiosyncratic risks in this business.

downside). Economics options models can also indicate ex post outcome distributions, but with less flexibility and customizability to the specific design and decision parameters of a given project, and we find that the simulation approach often renders results distributions that seem more credible or meaningful to decision makers. Engineering models, inherently based on simulation analysis readily fulfill this need as they routinely supply information on the distribution of the possible values associated with the optionality features of a project.

To illustrate the nature and use of engineering models of evaluation, we apply the approach here to a conceptually simple case study that illustrates its range and power. The case concerns the development of a multi-story parking garage. It was inspired by an actual situation, the development of a similar facility for the Bluewater Shopping Centre in Kent, England, which is one of the largest in the UK and in Europe. The interesting feature of this parking facility is that it was built to enable future vertical expansion. The developers/designers sized and built its foundations and columns strong enough to add on several extra floors. In our terminology, this parking facility included an “*option in*,” an option that only existed because the engineers had built this flexibility into the project. The option value of the parking garage of course depends in the first instance on the success of the entire Bluewater project: good growth makes it attractive for the owner of the project to add on extra floors.

The exposition of this case highlights the features of the engineering model of the valuation of flexibility in design. Full analytical details are available in de Neufville *et al* 2006. The textbook by de Neufville and Scholtes (2011) provides extensive discussion of the subtleties of the use of the engineering model and its application to the case study of the parking garage.

The engineering model of valuation starts with a spreadsheet of the cash flows for the project. It does not require any special format. It builds on whatever arrangement that the project development team may be using. The common essential feature is that the spreadsheet provides some sort of

projection of demand for and revenues from the service as it evolves over time, together with a projection of investment and operating costs over the same period. For simplicity, our example supposes a 20-year leasehold and considers annual revenues and expenses, but in principle the engineering model considers as much detail as desired.

By building on the spreadsheets used by the investor/developer team, the analysis immediately builds rapport with that group. There is no need to adopt new assumptions, to challenge the vision and perceptions of the developers. This approach furthermore builds in transparency in the process of valuing the optionality of the project. The project team can easily explore the implications of any of the special features of the project that they consider to be important, and that they have therefore already embedded into the spreadsheet model.

We now need to consider carefully the three aspects that give rise to the value of the flexibility or optionality of a project. First there is the uncertainty itself: the risks and opportunities that may occur determine whether an option is exercised and thereby generates extra value for the project. Second, there is the range of decision choices or options available to the developers or operators of a system. The engineering model offers the analyst a great capacity to customize the decision choices, for example in the case of the parking garage we could specify that the owners could add either a single level at a time, or might be restricted to adding two at once. Third, there is the timing of exercising the option: it makes a difference when an option might be exercised, and we must address this issue carefully and realistically. The engineering model enables the analyst to represent both these features generally in greater detail than in an economic option value model. It therefore enables a much more realistic and accurate assessment of the opportunities.

When we represent the evolution of uncertainty in the spreadsheet, we have basically no restrictions on the trend of this evolution or what we enter in any period for the demand, revenues,

costs or whatever element of uncertainty we wish to model. For example, in the case of the parking garage we posited that the growth in demand over the first half of the project would increase exponentially as the Shopping Centre developed, but would then level off significantly as saturation set in. This is a standard kind of assumption for growth in demand but is of course only one possibility; other trend lines can be examined equally easily. It is also easy to impose jumps in the trends of any parameter, either explicitly at a specific time (as when it is known that a new lease agreement will take effect) or at some random time determined by the simulation process. In short, the spreadsheet format permits us to analyze a wide range of forms for the distribution of uncertainty.

The greater facility of the engineering model to deal with realistic trends and disruptions, or any sort of custom-tailored random evolution in the relevant parameters, is particularly valuable at the more micro-level of the design and operation of physical components within an engineering system or a real estate or infrastructure asset. This is one reason why the approach is more appropriate for addressing “options *in*” such systems or assets, as distinct from options *on* them. In general a well-crafted engineering model pays little or no extra cost for dealing with whatever the analyst wishes to include. It deals with numbers in the cells equally, no matter how they got there.

The engineering model constructs a scenario over the life of the project year by year. Starting from the beginning, it defines the value of any of the parameters it is considering by drawing from the particular distribution assigned by the analyst to that year. This distribution may be a simple trend with a standard deviation; it may be a combination of a number of factors, such as the probability of population growth and the probability of an economic recession; or some other factor. In this way it can develop literally thousands of scenarios very quickly. In our case, the analysis of the performance of the parking garage over 20 periods for 10,000 scenarios takes just a few seconds on a laptop. In the engineering model, the process of exploring the effects of uncertainty is fast and reliable.

Turning now to the matter of exercising options embedded in the model, the engineering model mimics the anticipated decision processes of the project managers over the lifetime of the project. That is, based on what the analysts and the investor/developer team decide would or should be the behavior of managers over time, the engineering model embeds “decision rules” in the spreadsheet. For instance, a rule we adopted for the parking facility was “add an extra floor to the garage if demand exceeded capacity over the preceding two years” – the idea being that managers would not react too quickly to spikes in demand, but might delay their response until the traffic growth appeared to be established. Note that this approach is essentially descriptive – it does not pretend to be optimal. It should be developed by consultation with the actual decision makers in the real world. To implement the decision rules that trigger the exercise of options, the engineering model embeds them as “if” statements in appropriate cells of the spreadsheet. Thus as the simulation unfolds a scenario period by period, in each period an “if” statement checks to see if the evolution of the parameter (the demand for example) has satisfied the condition requiring the exercise of the option. If the condition is met, the “if” statement triggers the action, making the associated changes (such as increasing the capacity of the garage, and incurring the cost of the investment associated with the expansion). Note that in this arrangement there is no requirement that the decision rule be constant over the entire life of the project. Indeed, our analysis of the garage case embed different “if” statements for different periods. For example, one version required greater growth to trigger expansion in the last years of the project, another even closed off the option (it might not be sensible to expand the facility in the last years of the lease-hold). In short, here again the engineering model offers great flexibility in describing the potential evolution of the project and assists in thinking about management policies.

The engineering model adopts this descriptive approach to specifying the exercise of options out of necessity. The possible complexity of a realistic design and decision situation with the evolution of the relevant parameters and factors precludes optimization in a formal or rigorous sense, but is also

not limited by the simplifying assumptions necessary to derive such formal optimality. Instead, the engineering model enables the analyst to mimic actual real world behavior closely. Further, as simulation models do not have to presume path independence for outcomes, the engineering model of valuation can consider multiple options exercised over the period of analysis. The result is a reasonable approach that can deal with multiple options under all kinds of circumstances.

The results coming from the engineering model of value consist of a distribution of possible “future values” of a project with optionality. Each scenario for the combination of uncertainties leads to a set of possible exercises of the options and thus a value for the project derived in the usual way from the associated cash flow. The resulting values are effectively *ex post* values, the outcomes of particular “future histories” generated in the simulation. However, a useful metric for quantifying these *ex post* results is often a “net present value” (NPV) computation, that is, a discounting of the *ex post* results back to a valuation as of the time of inception of the project.¹¹

Combining these individual results, for however many thousand simulations the analysis involves, gives the distribution of project results. These can be summarized in terms of their central tendency by the mean of that *ex post* outcome distribution, a sort of expected value to represent a “value with options” for the project. However, in our experience, the distribution of the outcome values is often more interesting than its average or central tendency. Indeed, the expected value of the project may easily not be usefully representative of its economic results. In general, the distribution may be skewed with fat tails on the downside or upside. In either case, the average value may be unrepresentative of the median or modal possible outcomes. For example, investments in start-up IT companies have led to very impressive returns (Microsoft, Google, Facebook and so on) but the vast majority of them are failures in which the investors “lose their shirts”. Insofar as investor/developer

¹¹ One could as easily employ a “net terminal value” metric, but NPV is the more common and widely accepted perspective.

teams care about the distribution of possible returns (either because they are risk averse or because they are on the contrary willing to bet on possible extraordinary gains) then the engineering model of valuation has the merit of focusing conveniently on the distribution of possible values of a project.

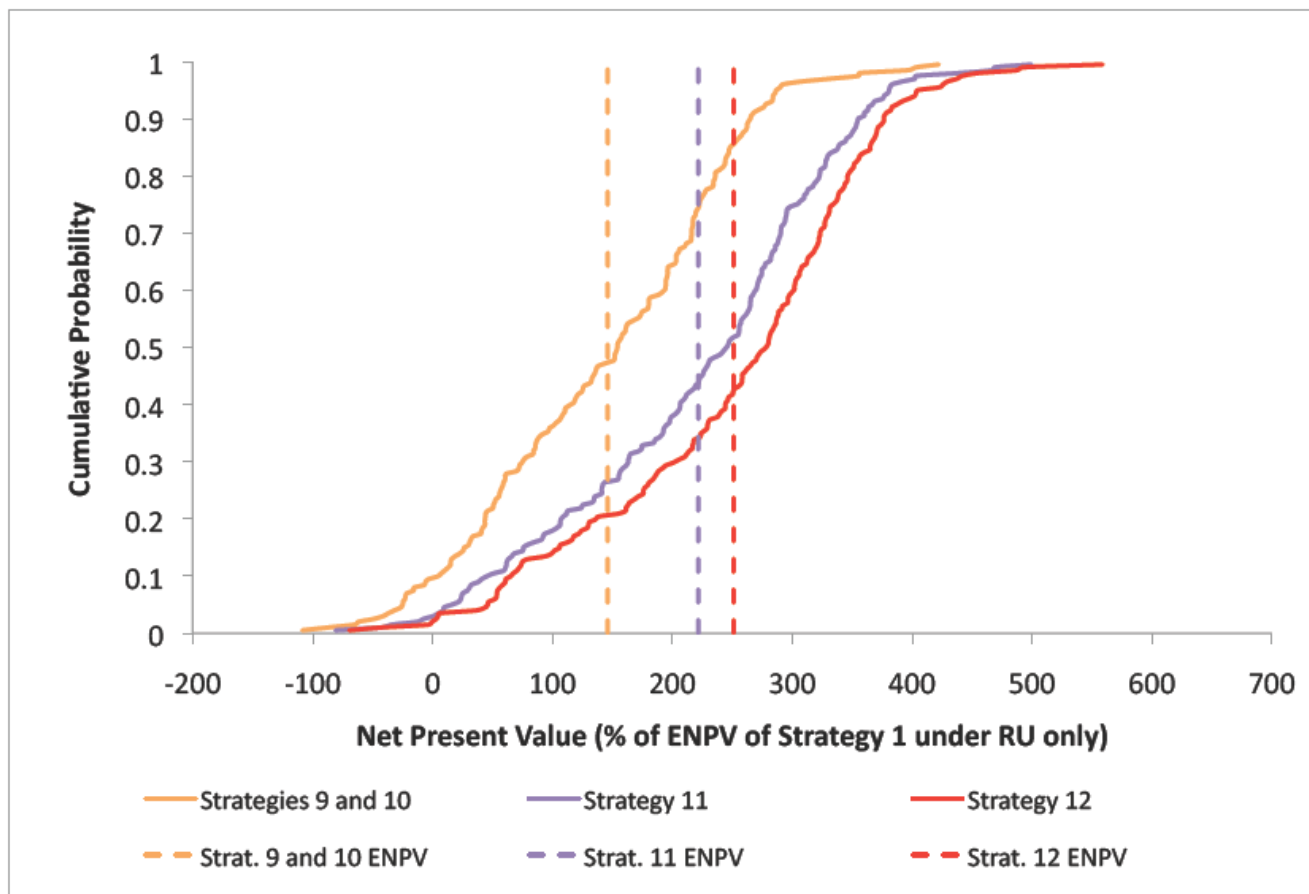
At this point it is important to stress that engineering models can have an important role in uncovering significant sources of value – and thus of greatly improving design. Here's how it works: the exploration of the value of various possible flexibilities in the design – that is, of options – leads to the identification of sources of value that the designers did not originally recognize. Indeed, designers often, perhaps habitually, focus on creating the details of a project that clients have specified to them. Designers do not usually challenge the specifications, and therefore may not go out of their way to explore alternative specifications for a project. When we use engineering models to value flexible designs that could fulfill alternative specifications, we frequently uncover flexible design alternatives that could provide much greater value, and therefore lead the design team to significantly more valuable configurations of a project. In this regard, there is a rich and varied experience with engineering models of value for projects with optionality in all kinds of investments in infrastructure, in the natural resource extraction industry (oil, gas, minerals), in major industries (automobile manufacture, satellite development), and in product development. We have shared in many such cases and can report that the use of the engineering model often uncovers significant sources of value.

A salient example of our experience with engineering models is the work we did recently with BP. This relates to the design of their offshore development of infrastructure for one of their oil fields, as reported by Lin *et al* 2011. The case involved the choice of the design and the location of a series of oil platforms and the associated sub-sea connections between wellheads. The total cost of this infrastructure would easily exceed \$5 billion, depending on what BP eventually decides to build, on what options they choose to exercise. The major uncertainties around the project concern the price of

oil; the quality of the oil, specifically as regards its viscosity, which determines its flow; and ultimately on the amount of oil that is economic to recover, which depends on the price of oil, its quality, and also on the geology of the field. The analysis considered both “*options on*” and “*options in*” the project. The former involved decisions about when to exercise the option to install additional platforms, the latter the ability to connect the platforms with different parts of the oil field, as determined by the design of the sub-sea connections. The analysis used a spreadsheet backed by extensive models of the performance of the system for recovering oil. That is, we obtained the costs and revenues in each cell for each period by calling upon a reasonably detailed petroleum engineering model of the complex system associated with the recovery of oil and gas.

By using the engineering model for the valuation of options, we were able to uncover several designs for the system that had great value and which had not been previously considered. Exhibit 3 illustrates the results. Each curve represents the target curve, or cumulative distribution of the possible values of different design strategies. These curves came directly from the application of the engineering option model to a particular design. The simulation analysis for each design indicated the range and distribution of possible outcome values, and permitted us to calculate an expected value. The “option value” associated with any design is then the difference between its expected value and that of a design without options. Given that each design involves many different options, and that managers would exercise different ones depending on the scenario, the “option value” for any design cannot be associated with any single option, it is a value ascribed to a set of options. The curves in Exhibit 3 indicate the performance of the three best alternative designs that our team uncovered. The curve furthest to the right offers the best set of outcomes and should be preferred. While the overall level of benefits is confidential, Exhibit 3 indicates about 150% increase in value on a multi-billion dollar project. This is tremendous.

Exhibit 3: Results from Application of Engineering Model to Development of Infrastructure for an Oil Field



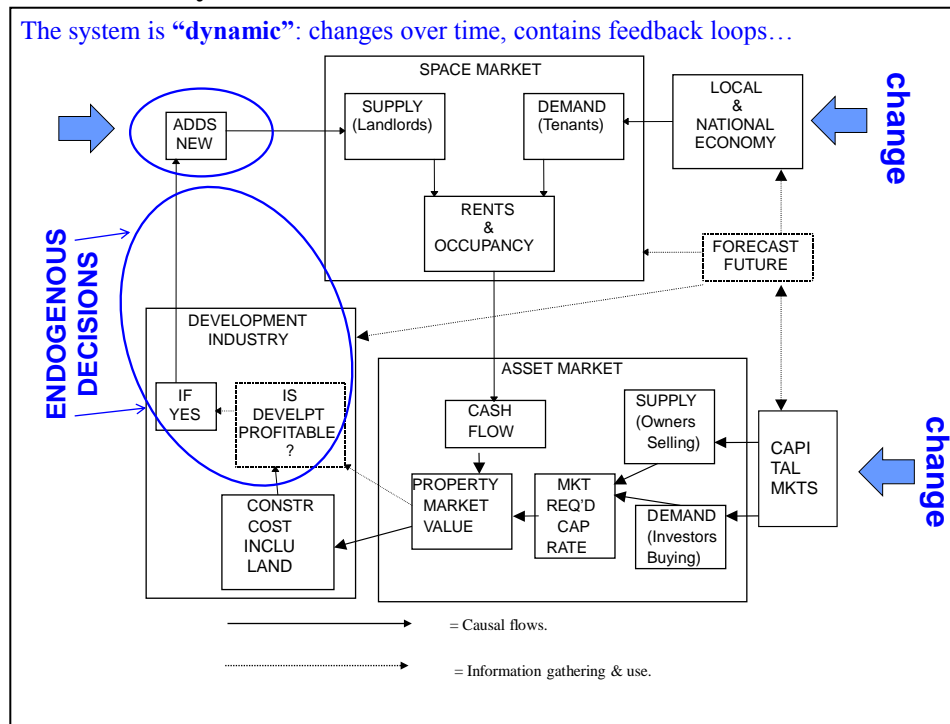
3. Quantifying Real Asset Uncertainty: Private Market Transaction Price Data & Indices

It is apparent that both the economics-based real option model and engineering-based simulation models are very sensitive to the quantitative assumptions about the magnitude of uncertainty as represented by the volatility in the underlying asset value. Yet until recently there has been a paucity of data to help quantify this vital input. As we have noted, a wealth of data on the historical returns to stocks, bonds, commodities, and foreign exchange has facilitated the practical application of option models in the finance industry. But most real estate assets trade, at the underlying level, in private search markets for whole (unique) assets rather than public auctions markets for

homogeneous shares or units. There has always been skepticism that one can simply assume that real estate assets have the same type and magnitude of volatility as stocks. And similar concerns have also applied regarding infrastructure assets, most of which have at least until recently not been privately owned or traded at all. But recent decades have seen tremendous development in our ability to quantify the underlying volatility necessary for modeling the value of flexibility in real estate and infrastructure projects. Perhaps most fundamental has been the advent of rich datasets of property returns and price dynamics, at first based on appraisals and more recently based directly on transaction prices in the private property market. Also important, however, has been the blossoming and maturation of real estate and infrastructure sectors within the stock market, including most prominently the so-called “REITs” (Real Estate Investment Trusts), essentially “pure play” stocks, publicly traded but holding nothing but real estate investment assets. The infrastructure privatization movement that largely began in the 1980s has also enriched the record of traded infrastructure assets. In this section we will briefly review these developments as they bear upon the flexibility modeling discussed previously. We will sketch the major sources and types of uncertainty and volatility in real asset values over time.

Let us begin by stepping back and considering the fundamental sources of the uncertainty that underlies the volatility in asset values that we seek to quantify. A good way to do this is to envision the entire “system” within which real estate or infrastructure physical assets are embedded. Exhibit 4 presents a picture of the essentials of this “real asset system”. There are three major parts to the system: two different types of markets, and a productive industry that links them.

Exhibit 4: The Real Asset System



The large encompassing box at the top of the exhibit is the “space market”. This is the most fundamental part of the system, its *raison d’être*. This is the market for the use of the built space or facility. On the demand side of this market are potential tenants or other types of users of the buildings and infrastructure assets (travelers using bridges, shippers using ports, shoppers using retail stores, dwellers using apartment flats, and so forth). It is fundamentally by serving such demand, and only by that service, that real assets add value and make a positive contribution to the economy and society in which they are located, and this is true whether the assets are owned in the private or public sector. Underlying the demand side of the space market is the local and national (or even global) economy, which determines the amount, type, and location of spatially-fixed physical assets that will be useful and of value. On the supply side of the space market are landlords or other types of owners (public or private) controlling and managing the operation and usage of the built assets. The equilibrium between supply and demand in the space market determines the price of usage and the physical amount or rate of usage of the real assets (such as occupancy rate in buildings, or flow-through rate in transport

infrastructure). The combination of this price (such as rent or tolls) and physical usage rate determines the magnitude of the benefit flows attributable to the real assets in the space market.¹² This annual benefit flow (a net income stream in the case of priced assets) is the output from the space market to the asset market.

The asset market is where the ownership of real assets is traded. For our purposes it may also be thought of as where such assets are evaluated even if they are publicly owned and not traded. Most fundamentally, the asset market is where future streams of cash flows (or, more fundamentally, user benefit flows) are evaluated and (often) traded. Mechanically, the asset market converts such future streams into a present value. It does this by forecasting what the future benefit flow streams are expected to be, making a judgment about their risk, and discounting to a present certain value to reflect time and risk. The asset market is a branch of the capital markets, where money and long-lived assets are traded. The capital market determines the opportunity cost of capital (OCC), including the price of time and risk. Even for public sector investments in assets that will not be traded, the capital for the investments comes ultimately from the capital markets, displaces private investment on the margin, and therefore faces the same OCC from a fundamental economic or social perspective.¹³ The OCC relevant to each project serves as the discount rate for converting forecasted future cash flows into present value. As a shortcut pricing convention in the asset market, one often thinks of a simple “direct capitalization rate” (or “cap rate” in American real estate parlance), which is an initial annual net income yield rate, that one can use to quickly value assets by simply dividing their current annual net income by their cap rate. While the cap rate is just a stylized short cut, it can be thought of as reflecting

¹² In the case of asset usage that is not priced, such as roadways that are not tolled, one must quantify a social “shadow price” reflecting the marginal social benefit of the asset usage, in order to quantify the benefit flow from a public or social perspective.

¹³ Keep in mind that the OCC reflects the risk and cash (or benefit) flow timing for each specific project. For public entities the social or economic OCC may differ from the accounting cost of capital that may be what matters from a budgetary perspective.

the pricing in the asset market, reflecting the equilibrium between supply and demand of and for investment in capital assets of the type being traded in the asset market in question. As in the case of the space market, the underlying source of this supply and demand is at least partly exogenous to the asset market itself, reflecting the broader global capital and money markets, which in turn are products and parts of the world economy and information system.

In the real asset system depicted in Exhibit 4 we view the output of the asset market as being the prices (values) of existing (or potential proposed) real assets, including also the price of land sites necessary for the construction of spatially fixed real assets. These asset prices are a key input into the third major element in the overall system, which is the physical development industry, which governs the construction of new real assets. The development industry takes financial capital (money) from the capital markets and converts it into physical capital (spatially-fixed real assets) that add to the stock of supply in the space market, thereby completing the central loop in the overall system. This central loop is characterized by a negative feedback mechanism that acts to help keep the entire system in balance, a bit like a thermostat in a heating/cooling system. The negative feedback occurs as the development industry makes a crucial comparison: between on the one hand the expected value of the benefits of the development represented by the price the proposed new real assets could fetch in the asset market, and on the other hand the costs of the development (including the opportunity cost of land and the necessary profit expectation for the developers and their financial backers). Only if and when the benefit exceeds the cost (i.e., there appears to be sufficient profit) will development be undertaken. Thus, if supply overshoots demand in the space market, this will tend to depress prices (rents) and/or average usage (occupancy), which will reduce the benefit (income) flows achievable by the real assets, which will in turn reduce their valuations (prices) in the asset market (holding cap rates constant), thereby tending to make the benefit/cost comparison in the development industry come up less favorable, resulting in a cutoff or cutback in the supply of new real asset stock into the space market.

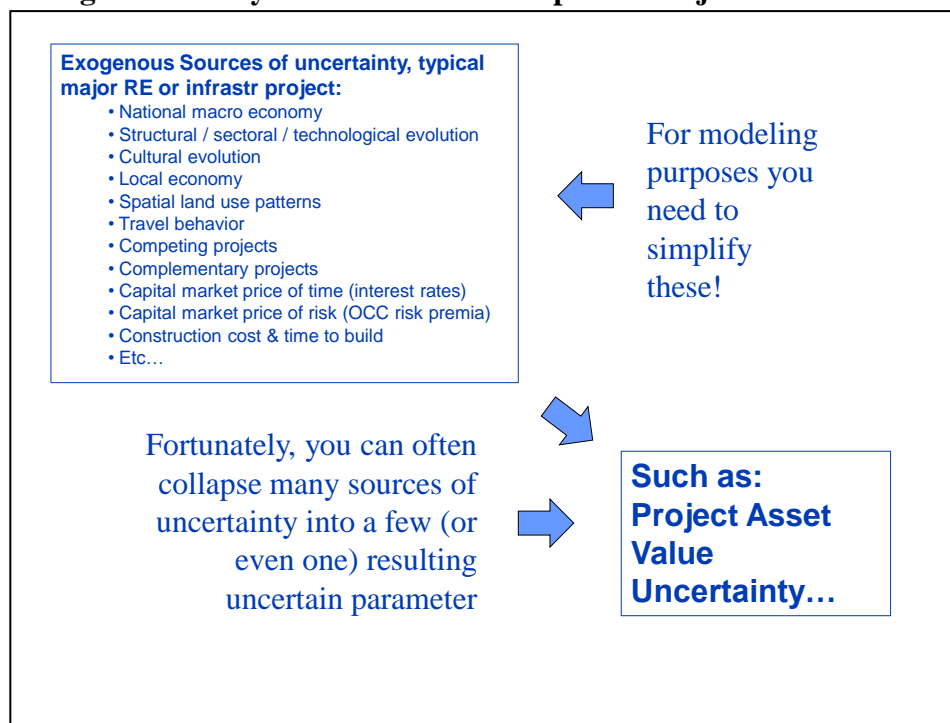
Note however, that this negative feedback loop is not perfect. It is a bit of a “ratchet”, in that real assets tend to be very long-lived. Once built, they are “out there” for a long time, even if demand for their usage declines. With a well functioning capital market and development industry the physical supply of real assets increases much more easily or quickly than it decreases.

Exhibit 4 highlights the sources of uncertainty in the real asset system we have just described. In the first instance, uncertainty exists in the two markets. What will be the prices and usage levels and therefore the magnitude of income or benefit flows coming out of the real assets in the space market? What will be the future expectations and the relevant interest rates and OCC and cap rates applied to the current and expected future benefit flows in the valuation of the real assets in the asset market? In both these cases, a fundamental source of the uncertainty is exogenous to the real asset system itself. As indicated in the exhibit, it is *change* in the (largely external) sources of usage demand and of capital supply and demand (including the forecasting of relevant future scenarios by players in those external systems) that governs the demand in the space market and the pricing and capital flow in the asset market. But there is also an internal source of uncertainty within the real asset system itself, because the system is dynamic and contains the development industry that produces the physical assets that compose the system. We have noted the negative feedback loop that tends to keep the system in balance, but we also noted that this loop is not perfect. It is not perfect in part because no one can have a “crystal ball”, and we have noted the external sources of change acting on the other parts of the system, change that cannot be perfectly foreseen. It is also not perfect because of the aforementioned “ratchet” or asymmetry between additions to, versus subtractions from, the stock of real assets (long-lived assets once built do not go away).

But there is another source of imperfection in the system’s negative feedback loop, and this is a source that has only received serious attention among economists in recent decades. Here we are

speaking of human behavioral characteristics that may cause a tendency toward certain types of systematic or persistent errors in economic and business decision-making. For example, people may tend to be overly optimistic; they may tend to feel more confident than they actually should be about their knowledge and their decisions. They may over-rate small probabilities and under-rate larger probabilities of negative events. They may also exhibit “herding” or “contagion” type behaviors, and be overly conservative in some ways, irrationally shying away from or delaying admission of losses. Furthermore, these behavioral phenomena are quite widespread. It would appear that in the real asset system they operate not only in the development industry but also in both of the other two main parts of the system, the space market and asset market. One result can be positive feedback loops that can occur in either the space market or the asset market. An upsurge in prices (caused perhaps by an exogenous shock) can cause a herd to push up prices further, potentially causing the price rise itself to fuel a further increase. Clearly bubbles can form and burst in asset markets, and also in some types of space markets.

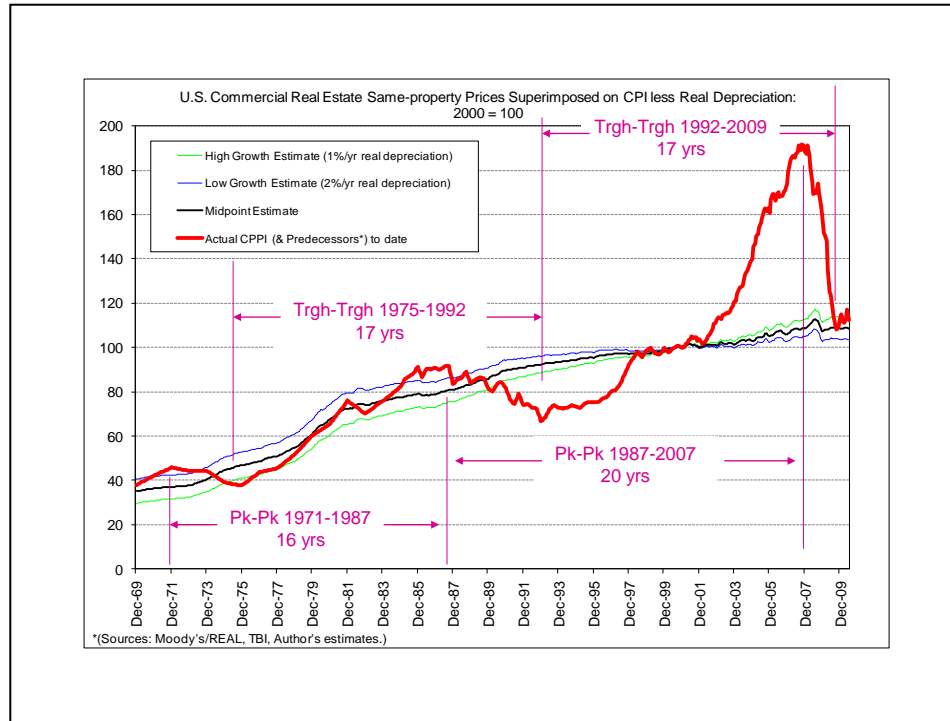
Exhibit 5: Modeling Uncertainty in Real Asset Development Projects



Thus, there are both exogenous and endogenous sources of uncertainty that cause volatility in real asset prices and valuations. And these sources are many and difficult to predict. Exhibit 5 presents a “laundry list” of typical sources of uncertainty (and some of these should be viewed as being possibly either exogenous or endogenous, with the list being only representative not exhaustive). But the point of Exhibit 5 is that, while one should certainly consider specific and underlying sources of likely future change, it is often possible for practical modeling purposes to collapse all such fundamental sources into one or a few key resulting variables. In particular, the most all-encompassing and often most useful such variable is the volatility in the real asset value, that is, the longitudinal variation in the price change (or value change) of the asset being developed. This volatility will typically reflect and be caused by all or most of the underlying specific sources of uncertainty. The lesson in the laundry list in the upper-left of Exhibit 5 (and in our prior discussion in this section) is to be humble, and not be surprised if the real asset volatility is substantially higher than you might have initially supposed!

With the above as background, let us consider what the recent databases and econometric discoveries can tell us about the nature and magnitude of uncertainty and volatility in real asset values relevant for quantifying models of the value of flexibility in development projects. As noted, a central challenge has been to put together a good empirical history of asset prices in the private property market. Thus, a good place to begin is with the big picture of the history of U.S. commercial real estate prices for “major assets” (aka “institutional property”, generally large assets held by professional investment institutions such as pension funds). A picture of this history spanning four decades is shown in Exhibit 6, highlighting the long-term secular trend in (same-property) prices, as well as the three major asset-pricing cycles that have occurred during that history.

Exhibit 6: The secular trend and long cycle in U.S. commercial property major assets



The first serious attempt to track investment property prices was the development of the NCREIF Property Index (NPI) in 1982. This index is based on appraised values rather than on transactions prices, which raises some problems, but it was a path-breaking and important starting point, and researchers have developed ways to “unsmooth” or “de-lag” such appraisal-based indices to improve their accuracy (Geltner *et al*, 2003). More recently, the first regularly published transactions based index of commercial property in the U.S. was developed at the MIT Center for Real Estate, based on the NCREIF properties that sell out of the index. This so-called “TBI” (for Transactions Based Index) has been published since 2006 but its history goes back to 1984 at the quarterly frequency (see Fisher *et al*, 2007). Another index developed in 2006 is the Moody’s/REAL Commercial Property Price Index (CPPI), which has been a monthly-frequency index beginning in 2001 reflecting a much broader and larger dataset of property transactions based on the Real Capital Analytics Inc (RCA) transactions database. (This database is still limited to the “institutional” or “major asset” category of properties, that is, not reflecting small “mom-&-pop” properties that number

in the millions in the U.S. but amount to only a fraction of the dollar volume of sales or value of real estate assets). The Moody's index is a *repeat-sales* index, a methodology popularized by Case and Shiller (1987) based on the percentage change in the prices of the same properties sold more than once at different points in time. This type of index is very analogous to a stock market price index, in that it reflects the actual round-trip investment experiences of investors buying and selling properties in the asset market. Exhibit 6 reflects a splicing together of these three indices (and some predecessor funds), to provide a reasonably fine-grained and consistent history since 1969.¹⁴ While 40 years is less than we would like, and less than what is available for stocks and bonds, it is longer than most investment and construction project planning horizons and long enough to give a meaningful picture of the overall typical price dynamics of commercial real estate assets in the private property market.¹⁵ Moreover, the quantity and quality of the transactions price data has greatly improved in recent years, along with the capability of econometric methods for digesting such data and producing accurate price indices. This is what has permitted the development of the TBI and Moody's indices.

There are two striking features of the price history in Exhibit 6. First is the long-run secular trend. This appears to be slightly less than the rate of inflation. The black line in Exhibit 6 traces a path that is 1.5 percent per year *less* than the U.S. Consumer Price Index (CPI). This line appears to represent a long-run sustainable price trajectory for same-property (aging) assets. Of course, this reflects the average context for major commercial property assets in the United States, a land-rich country where the historical period in question witnessed major improvements in transportation and

¹⁴ The index in Exhibit 6 uses the "best" available index over each span of time, starting (in reverse order) with the monthly Moody's index since 2000, continuing back with the quarterly TBI through 1984, and then an annual-frequency version of unsmoothed/de-lagged NPI back to that index's inception in 1978, with similar predecessor series based on early institutional comingled funds prior to that. Throughout, the series is constructed to reflect "same property" price movements, that is, to include the effects of the aging and depreciation of existing structures, excluding major renovation/rehabilitation projects which require new capital infusion and therefore would obfuscate returns on pre-existing investments.

¹⁵ There are a few *ad hoc* historical studies that provide much longer series, including a more-than-three-century history of annual price changes on the Herengracht canal in Amsterdam (see Eichholtz & Geltner, 2002), and in general these longer histories seem broadly consistent with the type of dynamics displayed in Exhibit 6.

telecommunications technology and infrastructure, resulting in a flattening of rent gradients and relative decline in location rent premiums in central places. The needs and preferences of real estate space users also evolved substantially over the historical period in question, as for example “Class A” office buildings that could get by with copper wiring and a neat lobby in the 1970s by the 2000s needed to include fiber-optic cable, atriums, physical fitness centers, natural lighting, and perhaps a heliport on the roof! The result was (as is typical) substantial depreciation in the value of the built structure due to functional as well as physical and economic obsolescence. Thus, it is not surprising that the long-run secular trend in asset values reflects “real depreciation”, price growth slightly less than inflation.

The second striking feature in Exhibit 6’s pricing history is its strongly cyclical nature. The 40-year history traces three strong and clear asset pricing cycles, each one of notably similar length, with a period between 16 and 20 years (peaks in 1971, 1987, and 2007; and troughs in 1975, 1992, and 2009). Each of these cycles was quite dramatic at the time when it turned down, reflecting peak-to-trough declines in same-property real (inflation adjusted) average pricing on the order of 40 percent (as a fraction of the peak value). The persistence of this cyclicity is notable, particularly considering that the period of the cycle is shorter than the average real estate professional’s personal career span and hence the upswings that lead to the downturns must occur in the presence of considerable institutional memory of the last crash. The cycles seem to be associated with increasing use of debt and a relaxation of lending standards. While each cycle is unique in terms of its specific historical causes and characteristics, there does seem to be a strong “rhyme and meter” in the commercial property market. Conflated with the cyclicity, there is an average annual volatility in the Exhibit 6 index on the order of 10 percent, somewhat less than the U.S. stock market’s 15-20 percent long-run average annual volatility range (depending on what stocks one includes and what span of history one includes), and

the property index displays considerable serial correlation or inertia (much more than stock market indices).¹⁶

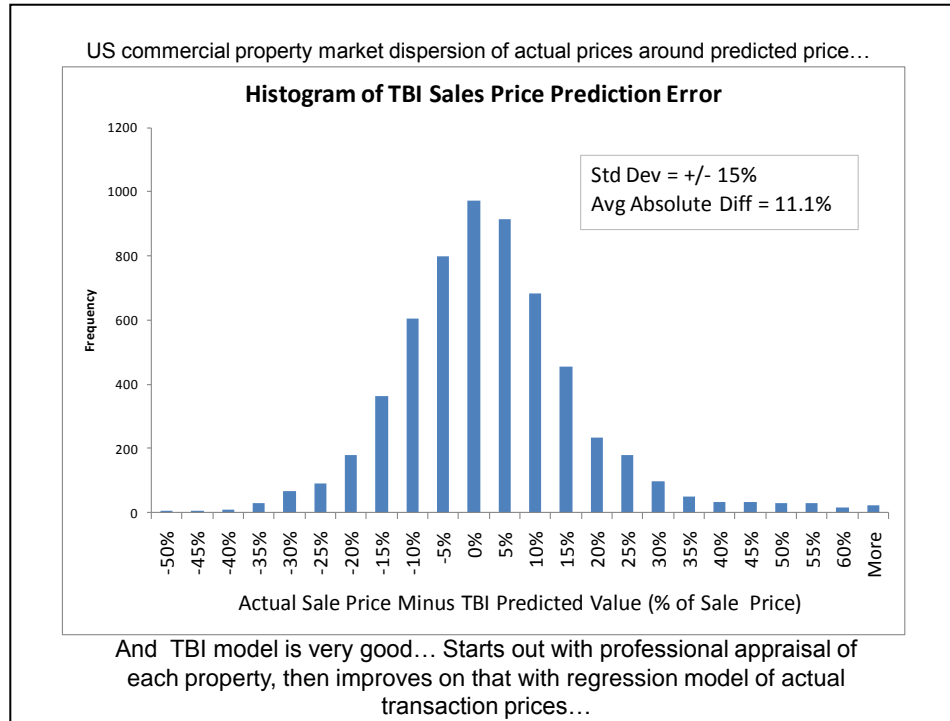
While the type of aggregate or overall price dynamics portrayed in Exhibit 6 is very important for understanding the nature and magnitude of volatility in real assets, this level of analysis is not the only one that is important for modeling uncertainty in the valuation of individual assets or projects. Exhibit 7 depicts the dispersion of individual asset prices around the prices predicted by the regression model that underlies the transactions based index (TBI) noted earlier. In other words, the dispersion shown in the exhibit is *in addition* to the aggregate volatility represented by the index. It represents a type of transaction price “noise”, reflecting the fact that it is difficult to precisely know the value of any individual asset at any given point in time, due to the uniqueness of each asset and the thinness of trading in the real property asset market. As a result, in any transaction price (or any valuation estimate) of an individual asset at a given point in time there will be random dispersion around some unobservable “true value” (or *ex ante* price expectation). This is a source of uncertainty that is akin to volatility, but that is non-temporal, that is, not substantially a function of the passage of time.¹⁷ As shown in Exhibit 7, the indication from the TBI data is that this type of random dispersion may typically be on the order of +/-15% (standard deviation) around the average or expected price.¹⁸

¹⁶ Both the volatility and the autocorrelation were considerably greater during the most recent decade, 2000-2010. During that decade the annual calendar year volatility in the Moody’s index was 15%. This of course reflects the great financial crisis of 2008-09.

¹⁷ A slight exception to this point is the fact that a prior transaction price does in principle provide some information about the value of the asset in question, and thus, the longer it has been since the asset previously transacted, the greater may be the price noise component depicted in Exhibit 7. This type of “heteroskedasticity” is explicitly accounted for in the Case-Shiller type of repeat-sales indices such as the Moody’s index noted earlier.

¹⁸ As we will note shortly, the 15% standard error figure is also supported by analysis of the larger RCA dataset underlying the Moody’s index. However, there is reason to believe large scale “mass appraisal” type models such as those used to construct indices like the TBI and CPPI would not be as accurate as more specifically targeted valuation estimates for individual subject properties. Evidence from other studies suggests that valuation noise may be closer to a 10% standard error in more targeted analyses. (See for example Crosby *et al* 1998, and Diaz and Wolverton 1998.)

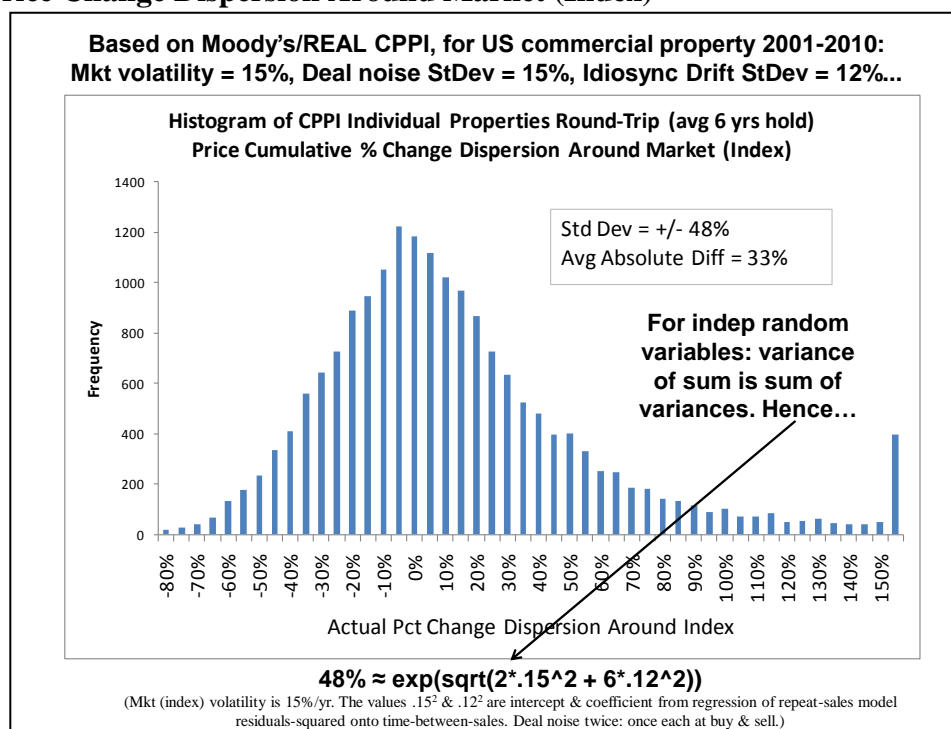
Exhibit 7: US commercial property market dispersion of actual prices around predicted price



In addition to the aggregate temporal volatility portrayed by indices, and the non-temporal valuation noise that occurs when (but only when) an asset is transacted or valued, another component of asset value volatility that is important in flexibility valuation is the temporal idiosyncratic “drift” (or “idiosyncratic risk” or “specific risk”) that causes individual asset values to gradually diverge from aggregate indices or market average valuations. This reflects the fact that individual buildings or assets experience value evolution that is specific to themselves that is unrelated to the market as a whole, or is cancelled out in the aggregate average price evolution represented by the index. This source of volatility is captured in the residuals from the repeat-sales regressions that underlie indices such as the Moody’s, and is depicted in Exhibit 8. As noted in the exhibit, the dispersion in the residuals of a repeat-sales index reflects the combination of the non-temporal deal noise discussed previously as well as the temporally-accumulating idiosyncratic drift. By regressing the squared residuals onto a constant and the time between the two sales within each of the repeat-sale pairs (that make up the observations on which the repeat-sales regression is estimated), we can identify how much of the residual dispersion

is due to each of these components. It turns out that the estimate of the magnitude of the non-temporal deal noise in the RCA database is very similar to that in the TBI residuals noted previously, approximately a 15 percent standard error (this despite the fact that the regression models and index methodologies are rather different, and the estimation database is different). The estimate of the rate of idiosyncratic risk or drift accumulation is approximately 12 percent per year (or more precisely, the variance accumulates at the rate of 12 percent-squared per year¹⁹).

Exhibit 8: Histogram of CPPI Individual Properties Round-Trip (avg 6 yrs hold) Cumulative Percentage Price Change Dispersion Around Market (Index)



We have now discussed and in some measure quantified five different types or sources of uncertainty or volatility in real asset values over time: the long-term secular trend rate in asset price growth, the long-run cyclicity in the asset market, the aggregate market-wide annual volatility exhibited in the market price index, the non-temporal price or valuation noise component, and the

¹⁹ In other words, as noted in the Exhibit, the 48% standard deviation in the repeat-sales residuals from the regression model can be decomposed as the square root of the sum of twice the non-temporal dispersion whose variance is 15 percent-squared (once for each transaction in the transaction pair) plus six times the annual idiosyncratic drift variance of 12 percent-squared per year (reflecting the average holding period between the repeat-sales of approximately six years).

temporally accumulating idiosyncratic drift of individual property prices around the aggregate or market price evolution. But there is at least one additional source or type of uncertainty, and this is perhaps the most difficult to observe and measure empirically. Asset valuations over time occasionally reflect relatively rare and difficult to predict major market-wide historical “jumps” or “black swan” type events, reflecting the “fat tails” of a non-Gaussian stochastic process. There is some reason to believe such “events” tend to occur more on the down side, value drops, rather than on the up side. Such an event is probably well represented in the U.S. commercial property asset market by the great financial crisis of 2008-09. The effect of such a “fat tail” event is graphically illustrated in Exhibit 9, which also serves to portray another important, and relatively recently matured source of data, individual REIT share price evolution histories from the stock market.²⁰

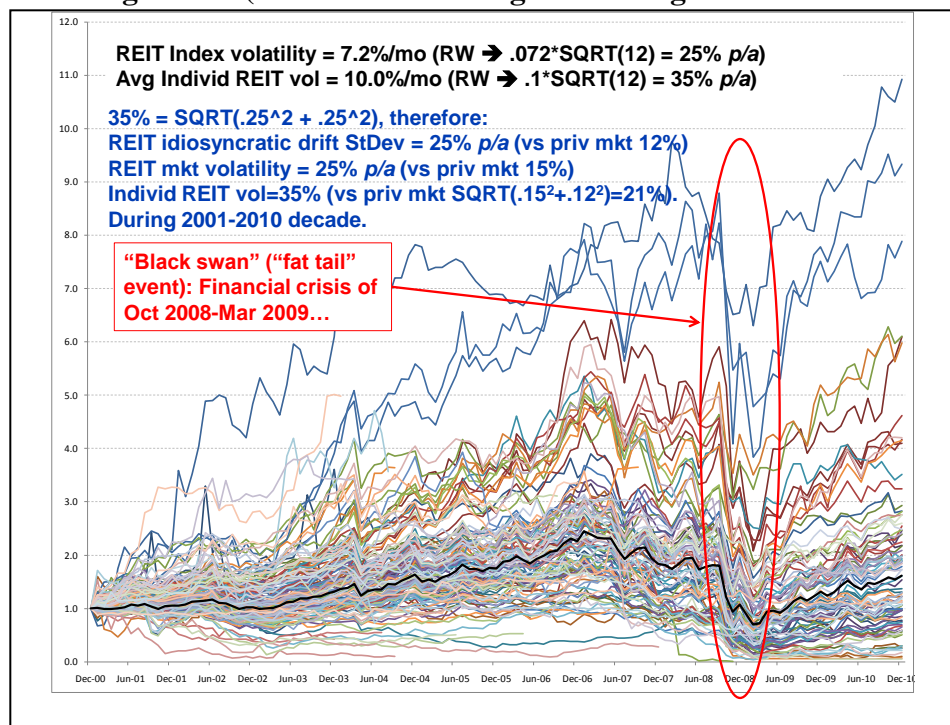
Exhibit 9 depicts the price evolution of all 111 U.S. REITs that were publicly traded and tracked by the CompuStat database as of December 2000. The exhibit shows the evolution of each of these individual REIT’s common stock share prices relative to what they started out at in December 2000, through December 2010. During that period, 47 REITs disappeared or dropped out of the database, so that only 63 REITs finished the decade. (The disappearances were due to de-listings and buy-outs, as well as to mergers and acquisitions. Very few if any REITs were delisted as a result of bankruptcy.)

As noted previously, REITs in the U.S. are relatively “pure plays” that are limited to investing in real estate assets. (They are not “merchant builders” who simply reflect the construction industry.) As such, REITs represent a way to understand the price dynamics and evolution of real estate asset values over time. While each individual REIT typically holds numerous individual properties, and this

²⁰ The U.S. REIT industry greatly expanded and matured during the 1990s, making it for the first time a robust and well followed sector of the stock market, as well as an important source of capital and active player in the private property market (as REITs buy, sell, and develop major properties in that market).

should reduce REIT volatility due to the diversification effect in the property portfolios, in fact REITs tend to specialize, and they also reflect management and “entity level” risk. The result is that the stock market tends to price individual REITs as single entities, as if each REIT is an individual asset in itself, albeit a somewhat different type of asset than an individual property. Stock market valuation is a different type of process than private market asset valuation. Furthermore, REITs are typically levered, so their stock price movements may be magnified by the effect of their capital structure. For all these reasons, REIT share price dynamics represent a somewhat “different animal” compared to the private asset market valuations we have been analyzing previously in this section and which are more directly relevant to the modeling of flexibility in development projects. Nevertheless, the stock market is a highly efficient and liquid asset trading arena characterized by very fast price discovery. And as REITs are essentially and fundamentally “real estate assets”, it is relevant and enlightening to see what REIT valuation dynamics look like over time.

Exhibit 9: Individual Price Histories of 111 REIT stocks (relative to value as of December 2000 = 1.00) from 2001 through 2010. (Black line is unweighted average across all the REITs.)



With this in mind, we note that the (unweighted) average annualized volatility across all the REITs in Exhibit 9 was 35 percent. This reflects the combination of aggregate REIT “market” risk plus individual REIT specific risk (or idiosyncratic volatility). Taking the average REIT return represented by the black line in the exhibit as representing the aggregate market (that is, the REIT market or REIT “sector” within the stock market), we note that this had an annualized volatility of 25 percent. This suggests that individual REIT idiosyncratic volatility also averaged about 25 percent (as $\text{SQRT}(0.25^2 + 0.25^2) = 0.35$). Thus, both in the aggregate, and regarding individual asset idiosyncratic returns, REITs appear to be more volatile than private real assets as traded directly in the private market. In part this may reflect the leverage in the REITs. In part it may reflect (and help to support) the different functioning of a public auction share market as compared to a private search whole-asset market, including the type of liquidity that public exchanges are designed to provide.²¹

Perhaps of more interest in our current context is the fact that the history of REIT share price movements during the 2000s decade depicted in Exhibit 9 presents a dramatic and graphic picture of the major “black swan” event of the 2008-09 financial crisis that we noted earlier. This is seen in the gigantic “downdraft” that pulled virtually all REITs down together and drastically during the October 2008-March 2009 period. During that period, REITs prices plunged farther than they ever had in the history of the industry, and they did so all together almost in unison, in effect, idiosyncratic drift was temporarily cast aside as all assets moved as one. There then occurred a quick and strong recovery, as the stock market apparently perceived that as regards most REITs, Mark Twain’s famous remark was applicable: “Reports of my death are greatly exaggerated!”

²¹ Comparisons between public and private markets are fraught with “apples-vs-oranges” issues. One such is in comparing “liquidity”. This term tends to mean different things between the two market structures, and investors and traders tend to care about “liquidity” in different ways and for different reasons. For example, small REITs may suffer an illiquidity discount in their equity share values, even though they are by some definitions more liquid than large privately held properties that reap a premium in their valuations because they are viewed as being “more liquid” in the institutional asset market. We should also note, again, that the 2000s were a relatively volatile decade by historical standards. (Although, as we suggested at the outset of this paper, greater volatility may be characteristic of this century.)

The stock market also can provide quantitative empirical evidence about the magnitude of asset value volatility among infrastructure assets. Up to now our discussion about quantifying the key asset value uncertainty relevant for major urban development projects has focused entirely on real estate. But the magnitude and often pioneering nature of such projects means that they almost always involve substantial infrastructure components, possibly including transport, water, power, and telecommunications. During most of the 20th century most infrastructure assets were publicly owned and not traded at all, making it difficult to estimate the magnitude of valuation risk in a manner comparable to that for real estate assets. However, since the latter decades of the 20th century there has developed a growing movement to privatize major infrastructure assets. In a recent study, Rothballer and Kaserer (2011) have performed a comprehensive analysis of the volatility of an exhaustive universe of over 1400 publicly listed infrastructure firms worldwide. In general, they find that such firms display levels of individual stock risk (that is, total risk including systematic plus idiosyncratic volatility) similar to the average in the stock market. Interestingly, they find that infrastructure stocks tend to display greater idiosyncratic risk than the typical stock, with less systematic or market risk (resulting in total risk that is about average). They find average individual infrastructure firm annual volatility on the order of 41% (measured in local currency). Among the infrastructure sectors examined, transport firms (airports, ports, highways, railroads, pipelines) showed 40% volatility, telecommunications firms 51% volatility, and utilities (electricity, water, gas) 33% volatility (including only 30% for water companies).

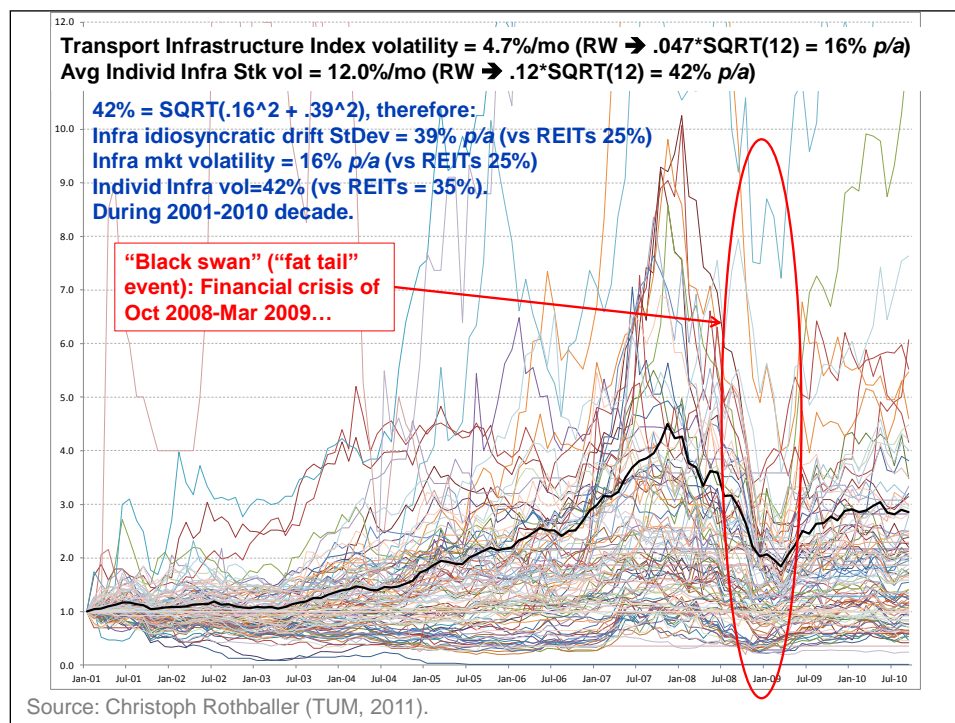
For applicability to specific project analyses the Rothballer-Kaserer figures bear the same sorts of caveats as our previous discussion of REIT volatility for real estate projects. Stock market based volatilities reflect the levered equity of firms that may hold multiple individual assets (including green-field development projects as well as stabilized income-producing assets), that employ considerable debt in the firm's capital structure, and which are viewed by stock market investors as individual

entities with possibly strong active management level risk components. While we don't have private infrastructure valuation series to confirm, we would speculate based on the real estate analogy that the resulting stock market based infrastructure volatility is probably greater than that of passive individual infrastructure assets valued privately or in a direct private context (as distinct from a highly liquid public stock exchange). Nevertheless, the comparison between infrastructure stocks and REITs is at least strongly suggestive of the relative nature of infrastructure and real estate risk and uncertainty. Exhibit 10 is constructed for transport infrastructure firms from the Rothballer-Kaserer data in a manner very analogous to Exhibit 9 previously described for REITs, and presented on the same scale.²² As in the REIT Exhibit 9, the heavy black line in the infrastructure chart in Exhibit 10 represents the systematic or aggregate "sector risk" component. This is simply an index of the equal-weighted average across all of the transport firms in the chart. This transport infrastructure index volatility is lower than the corresponding U.S. REIT sector risk (16% versus 25% annualized volatility during the 2001-10 period covered). But in contrast, the individual firm volatility is larger for the transport firms than for the REITs (42% versus 35%).²³ At a minimum, the suggestion is that infrastructure components of major urban development projects probably are subject to at least approximately as much uncertainty and value volatility as are the corresponding real estate assets of such projects. Thus our previous discussion of real estate uncertainty quantification may be generally applicable to infrastructure as well at least at a broad-brush level.

²² The authors thank Christoph Rothballer for producing the chart in Exhibit 10.

²³ It should be kept in mind that the infrastructure stock sample examined by Rothballer and Kasserer is international, including many stocks in smaller stock markets and emerging economies that tend to be more volatile than the U.S. stock market in which all of the REITs included in Exhibit 9 are traded.

Exhibit 10: Individual Price Histories of 143 Transport Infrastructure Stocks (relative to value as of December 2000 = 1.00) from 2001 through September 2010. (Black line is unweighted average across all the stocks.)



This completes our “tour” of the nature and sources of real asset uncertainty and volatility, and the recently available or recently matured data sources that give us a better foundation than ever before for quantifying the uncertainty input that is vital in the flexibility valuation models discussed in Sections 1 and 2 of this paper. While it is important to understand and consider all of these types and sources of real asset volatility, it is equally important to note that in actual modeling practice it will not always be necessary or desirable to separately and explicitly model each of the six sources of volatility described in this section. We often gain little additional insight by separately modeling all of these different stochastic components. Quantitative analysis of development projects is always a mixture of “art” and “science”, and both of these perspectives favor parsimony and simplicity. But we believe that the depth and sophistication of empirical knowledge about real asset price dynamics that is now possible (and getting better all the time) can substantially improve the efficacy and accuracy of the

modeling and analysis of the value of flexibility in development projects. To make the points we have presented in these first three sections of this paper more concrete, and to show an illustrative application of how they may be woven together and applied in an actual real world based example, we present a specific case in the next section.

4. Putting It All Together: An Example Based on the Songdo IBD Development Project

“I don’t know what proportion is art, and what proportion is science, but it seems to me it’s all engineering.”

–Professor Richard de Neufville (responding to a question in class).

Perhaps the best way to clarify and make more concrete the main points in the preceding sections, and to show how they may come together and reinforce each other, is to show how they may be applied to a specific project and example design question in the real world. The example we discuss in this section is one we have worked with in our classes at MIT. We like it in part because it is iconic of the type of large-scale Asian urbanization that we noted at the outset of this paper will be so dominant in the development of the world’s built environment in the first half of this century. Our example project is the New Songdo City (NSC) in Korea, and in particular its International Business District (IBD) component which is one of the most ambitious and sophisticated real estate (and infrastructure) development projects ever undertaken anywhere. But while this example is at a very mega scale, please keep in mind that the general principles and analytical approach we are suggesting in this paper can be applied at various scales. To relate this example to our discussion of simulation modeling in Section 2, what we are illustrating here is a high level, relatively simple representation of the project that can be communicated to top decision makers or financial backers. More detailed engineering models would then be applied to more specific design and operational aspects. The project

design element that we will focus on in this example is the staging or phasing of the overall project development program, a crucial element in the implementation of a project of such scale and complexity as the Songdo IBD.

The Songdo IBD is the central core of the New Songdo City development, which is being built on reclaimed land south of Incheon, the major port just west of Seoul. NSC is linked to the new Incheon International Airport, which is only 15 minutes away by super-highway over a new bridge. The development is a key part of Korea's national strategy to build a post-industrial economy based on headquarters and information functions. The IBD is targeted to become a major business hub of northeast Asia, and the development is designed to appeal to an international population. (NSC has been designated a "Free Economic Zone" by the Korean Government, to avoid many restrictions that apply elsewhere in the country.) The IBD master plan, designed by renowned architectural firm Kohn, Pedersen & Fox (KPF), covers 1500 acres and envisions 100 million square feet of built space including 40 MSF office, 35 MSF residential, 10 MSF retail, 5 MSF hotel, and 10 MSF of civic uses including world-class schools and hospitals. When completed the project is expected to house 65,000 residents and be the site of 300,000 jobs. (See Exhibit 11.) The Songdo IBD features world-class international architecture and urban design, and aims to be one of the "greenest" cities in the world, targeting 80 MSF of LEED certified projects, and including 600 acres of open space (including a 100-acre central park), public transit, bikeways, and greywater systems. Originally projected to have a total development cost of USD 20 billion, the project is now estimated to cost over \$35 billion. The iconic 1000-foot KPF/Heerim designed North East Asia Trade Tower (NEATT) mixed-use skyscraper at the center of the CBD was completed in 2011. By 2010 over USD 3 billion funding had been secured and invested, and the success of the initial developments was providing additional cash flow to finance further development. A world-class convention center and over 4 MSF of residential and retail

development had been completed and successfully sold and several major office projects were nearing completion. The project could be substantially complete by as early as 2016.

Exhibit 11: Artists impression of Songdo International Business District at completion



For our example application of flexibility modeling, we will step back in time, to late 2002, when the project was still in its inception. At that point, the physical plan and components of the project had been pretty well established, and a key issue was the programming of the development. How should the project be staged? Should it proceed all at once as fast as possible? Or should it be broken into phases, and if so, how many phases? By breaking a large development program into explicit phases, additional flexibility is obtained. There are more opportunities to delay or abandon or simply to fine-tune exactly when and how the project proceeds. How valuable is such flexibility? How sensitive is this value to the degree of phasing (the number of phases)? These are the program design questions that our illustrative example analysis will focus on.

The first step is to build a model of the project that reflects the real option that actually exists in the ability to delay or even abandon portions of the project. In fact, only such a model can provide a rigorous and realistic evaluation of the project. Traditional DCF modeling, based on a single projected

(most likely?) cash flow stream, deterministically (but artificially) fixed in time, cannot possibly realistically model the net present value (NPV) and meaningful rate of return for such a large and complex project as the Songdo IBD.

Exhibit 12: Songdo IBD Project Initial Cash Flow Projection as of 2002 (stylized)

| Songdo IBD Project Initial Cash Flow Projection as of 2002 (stylized): | | | |
|---|--------------------------|----------------|----------------|
| Year: | Costs (USD billions): | Income & Sales | Net Cash Flow: |
| 2003 | \$257 | \$0 | -\$257 |
| 2004 | \$354 | \$406 | \$52 |
| 2005 | \$1,618 | \$841 | -\$777 |
| 2006 | \$2,637 | \$1,788 | -\$849 |
| 2007 | \$3,121 | \$2,109 | -\$1,012 |
| 2008 | \$4,153 | \$3,852 | -\$301 |
| 2009 | \$4,163 | \$3,881 | -\$282 |
| 2010 | \$2,447 | \$2,373 | -\$74 |
| 2011 | \$831 | \$1,920 | \$1,089 |
| 2012 | \$0 | \$7,316 | \$7,316 |
| 2013 | \$0 | \$6,090 | \$6,090 |
| 2014 | \$0 | \$4,488 | \$4,488 |
| Undiscounted Sum | \$19,579 | \$35,063 | \$15,484 |
| NPV @ 20% OCC: | | | \$1,291 |
| IRR: | | | 29.35% |

The table in Exhibit 12 portrays an approximate and stylized representation of the overall project cash flows as of our subject point in time.²⁴ This is the type of projection on which a traditional DCF investment analysis would be performed.²⁵ How might such a traditional analysis have looked at that time? The Songdo IBD project was pioneering in several ways. Not only was it of an unprecedented scale, being built on a new site and untested location, but it was to be led by an international developer. In fact, for the first time in Korean history, a foreign developer would be allowed to purchase and own the land beneath the project. To incentivize such pioneering foreign participation and investment and jump-start the strategically important project, the Korean Government

²⁴ Source: Kang (2004). DISCLAIMER: Please note that while the numbers in this exhibit, and in the entire illustrative analysis to follow, are believed to be broadly similar to the actual projections used by the principals at the time, these are hypothetical numbers provided for illustrative purposes only, to demonstrate the methodological points of the current paper. The analysis herein should not be taken as an exact historical account of the Songdo IBD project in the real world, nor as an implication of either good or bad decision making on the part of any of the principals.

²⁵ The costs in Exhibit 12 include land costs at the generally pre-fixed prices being offered by the government to the developer. Thus, the NPV of the net cash flow stream in Exhibit 12 is net of land cost.

was in effect offering the developer an option to purchase land at pre-fixed prices that were designed to make the project economically appealing. In the early 2000s it had not been easy to find an international developer willing to take on the project. Considering the above, it seems likely that the project would have presented the developer with what would pencil out at a positive NPV. Indeed, we see that the projected net cash flows in Exhibit 12 provide a positive NPV of some \$1.3 billion using a 20% discount rate. Such a return was in fact a typical expected return for a speculative development project in the early 2000s, in the U.S. However, for such a pioneering and overseas investment (from the perspective of an American developer), in a country that was at that time still perceived as an “emerging market” and not that far removed from a major financial and economic crisis (which had occurred in 1998), it seems plausible that the opportunity cost of capital (OCC) could have been realistically regarded by the developer as substantially greater than 20 percent. Indeed, the projected cash flows in Exhibit 12 present a going-IRR of over 29 percent. Thus, at a 30% OCC the project does not provide a positive NPV. Assuming a hurdle rate somewhere between 20% and 30% the project probably pencils out acceptably, and this would be a typical type of economic reasoning behind the decision to undertake such a project. But such an approach is admittedly simplistic and incomplete, and it does not allow for a rigorous exploration of differing design options, including the value of flexibility in the project, such as a consideration of the staging of the project

To do a more complete and realistic analysis and evaluation of the investment presented to the developer by the Songdo project we need to explicitly recognize the optionality that is in the project. The cash flow stream shown in Exhibit 12, while it may be a good representation of the *expected* cash flows (the mean of the cash flows’ *ex ante* probability distribution) as of the initial decision time (end of 2002 or beginning of 2003), does not suggest the optionality that actually exists. Once the decision trigger is pulled and the project is begun, the development project is still not irreversibly committed to incur the entire cash outflows in the cost column of Exhibit 12 at the times indicated there. Suppose it

were. Then we would discount those costs to present value (as of end of 2002) at a low OCC reflecting little financial risk in that cash flow stream.²⁶ In 2002 this would have meant a discount rate around 5.5%. This would give the cost stream a present value of \$15.2 billion. But under such an irreversibility assumption, in order to be consistent, we should discount the benefit stream of expected cash inflows at an OCC reflecting the risk in the real estate market, the volatility in the rents and values of the assets to be built. In Korea at that time typical expected returns on real estate investments (at the property level, that is, unlevered) were on the order of 14%. Discounting the inflows at 14% produces a present value of only \$13.8 billion (even though the undiscounted sum is over \$35 billion). Thus, to be consistent with economic theory, assuming an irreversible commitment to the project as projected in Exhibit 12, the NPV facing the developer would actually have been negative \$1.4 billion!

To construct a more complete and realistic model of the investment value of the Songdo project as of the beginning of 2003, including consideration of the flexibility value to delay or conceivably abandon later phases of the project, we first built an economic real option model of the project, of the general type described in Section 1 of this paper. In this case, we employed the binomial lattice type model that is common and widely taught in academia.²⁷

²⁶ Even if the cost projection is not fixed by contract, it typically would vary *ex post* primarily as a result of engineering and construction cost factors that are generally not highly correlated with financial markets, hence giving such a cash flow stream little systematic risk of the type that is priced in the capital markets. (For example, using a CAPM based model of the relevant OCC, the “beta” of the construction cost cash flow stream would be very low, calling for very little risk premium in the applicable discount rate.)

²⁷ This type of option valuation model is relatively intuitive and can be easily programmed in Excel®. The procedure requires specifying the volatility and the net cash yield or payout ratio of the underlying real estate assets once they would be built and operating. While the model is often specified using a mathematical device known as “risk neutral dynamics”, we employed a more intuitive (mathematically equivalent) form of the model that employs certainty-equivalence discounting. In this form the dynamics of asset valuation (the underlying real estate asset future return probability distribution) is expressed in “real” terms, reflecting actual expected price trends and probabilities for the assets to be built, enabling us to more directly use the empirical knowledge about real estate markets described in Section 3, and enabling the model to show the actual expected returns for the option investments and the actual probabilities of exercise and of ex post value distributions.

As noted in previous sections, a crucial input to the modeling of the value of flexibility is the quantitative estimate of the magnitude of uncertainty. In the case of the economic real option model, this boils down essentially to the assumed volatility of the real assets of the type that will be built by the development project. Here is where we can use the relatively recent data and discoveries about real asset volatility discussed in Section 3. First, recall that the long-run history of major assets in the private market represented back in Exhibit 6 indicated an annual volatility in the aggregate in the private market in the U.S. of around 10%. However, we noted that during the most recent decade (which is when the most data and best quality data is available) the annual market volatility was 15%, and it may be that this decade will be more indicative of the magnitude of volatility typical in the 21st century. Furthermore, due to the substantial positive serial correlation (inertia) in the market index, and its strong cyclical tendency over the long run, the effective volatility relevant for medium to long-run investments is probably greater than the simple annual volatility. A final consideration is that, at least as of the early 2000s, volatility in Korean asset markets probably tended to be greater than in the U.S. (at any rate, greater than the nationwide aggregate U.S. market, due to greater size and diversity within the U.S. national market²⁸). For all of these reasons it would seem reasonable to take the 15% number rather than the 10% number as our indication of relevant asset market volatility for our Songdo modeling purposes. In addition, we noted in our discussion of Exhibit 8 that typical idiosyncratic volatility has been found to be 12% per year in the U.S. data. While ideally we would like a comparable Korean data source, for present purposes we may content ourselves with the assumption that there is no obvious reason why this parameter would be different between Korea and the U.S.²⁹ Putting the market and idiosyncratic volatility together, we arrive at a real asset volatility estimate of

²⁸ For example, comparisons between U.S. and British national commercial property indices reveal greater volatility in the U.K., probably at least partly reflecting the fact that the U.K. is a smaller less diverse market.

²⁹ This assumption is also supported by the fact that the idiosyncratic volatility estimate is rather robust in the U.S., tending to be nearly the same in a number of different markets and sub-indices, when estimated in the same manner based on residuals from repeat-sales indices.

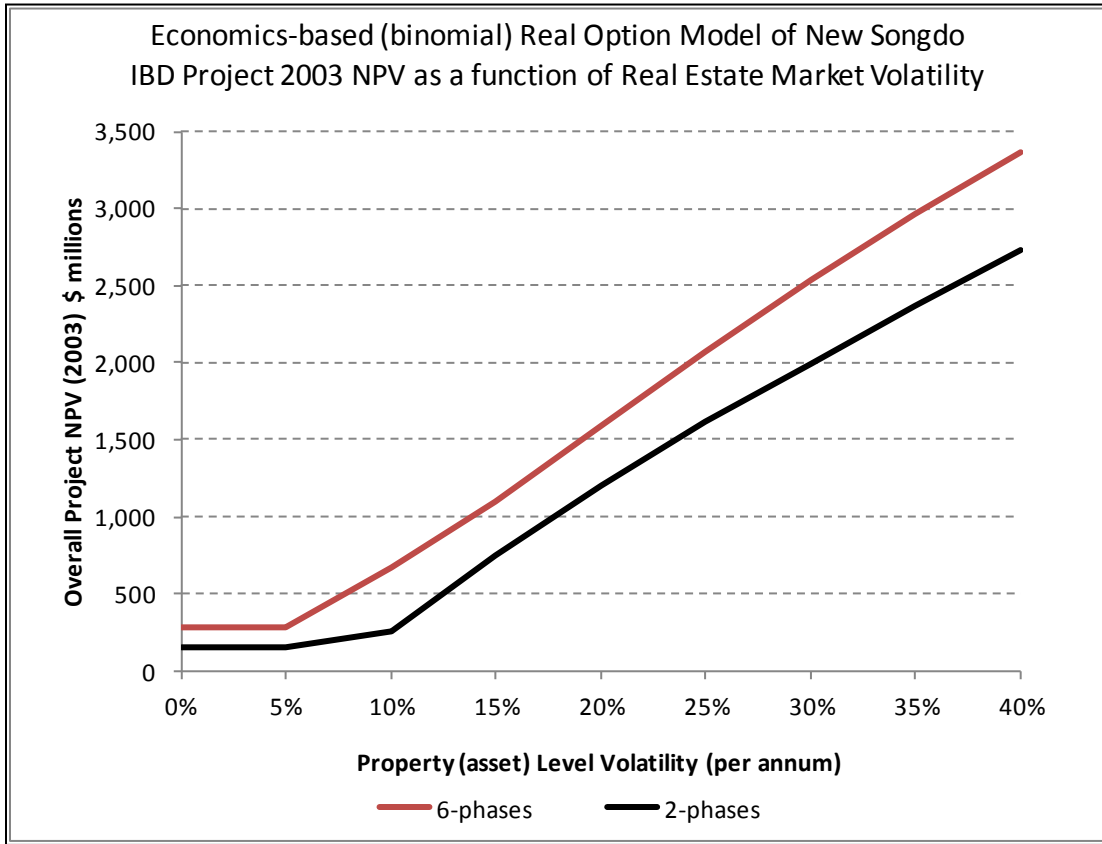
approximately 20% per annum: $\text{SQRT}(0.15^2 + 0.12^2) = 0.192$. While 20% is a good point estimate, in implementing the model we also conduct sensitivity analysis by examining alternative volatility input values.

In applying the real option model to the Songdo project we considered two design scenarios: a two-phase program and a six-phase program, to explore the nature and value of staging flexibility for the project. The result is shown in Exhibit 13 as a function of the assumed volatility of the underlying real estate assets that would be built in the project. Assuming a real asset volatility of 20% the value of the option (i.e., the NPV of the development project net of land costs) is approximately \$1.2 billion with the minimal staging flexibility suggested by having only two phases, but \$1.6 billion with the more substantial flexibility by dividing the project into six phases, a difference worth some \$400 million or about a quarter of the more flexible project's NPV.³⁰ The positive NPV in 2003 was therefore roughly 6%-8% of the projected undiscounted total development costs (including the pre-specified land cost), a substantial incentive for the pioneering developer, but not outrageous by any means. The binomial model also reveals that the actual going-in expected return for the development project investment was on the order of 50% per annum for the first phase, considering the optionality (45% in the less flexible case).³¹ And the model reveals that with the six-phase staging it is optimal to begin development right away in 2003 (as was in fact done), whereas with only two phases (which includes a much larger first phase) it would not be optimal to commence development right away but rather to wait initially before committing to such a large initial single phase.

³⁰ The actual project was in fact divided into six phases. To model the 2-phase alternative, we started from the actual six phases and collapsed the first three into a larger first phase and the latter three into a large second (final) phase.

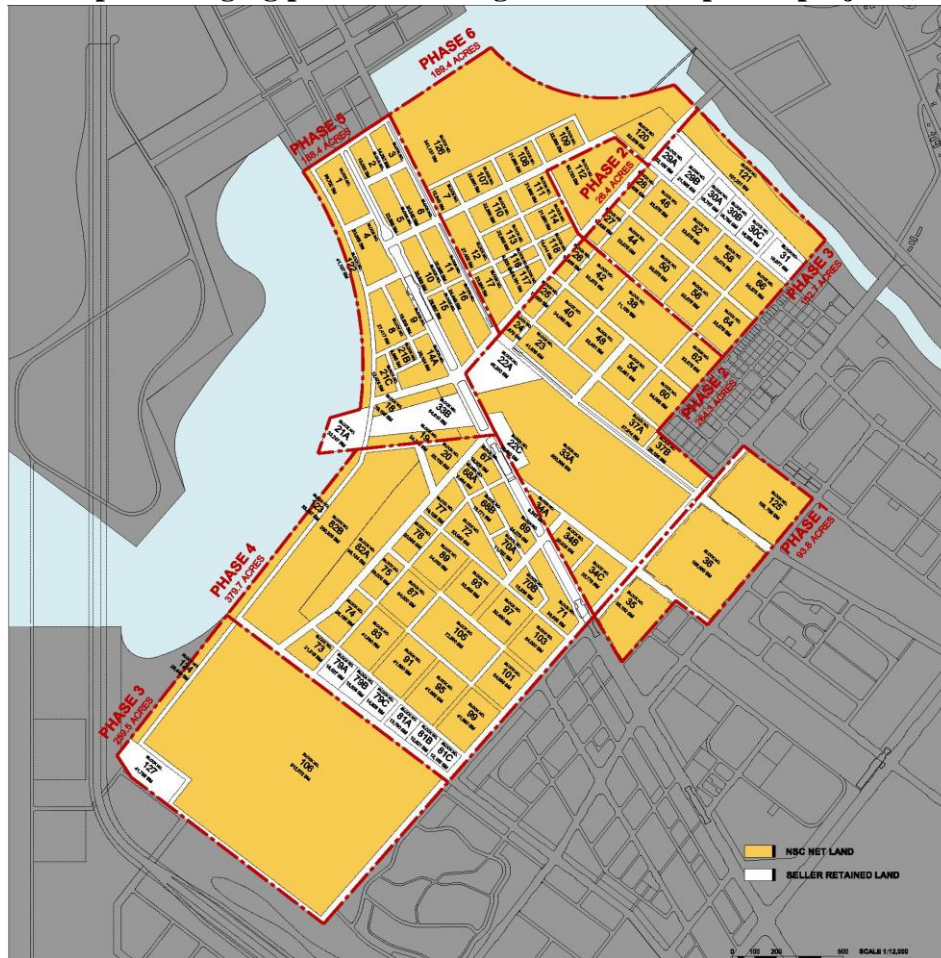
³¹ The option model quantifies a different OCC for each phase as a function of the current value of the underlying assets and the remaining time left in the option, reflecting different amount of risk in each such "state of the world". In other words, there is no single correct OCC or discount rate for a project like Songdo, as the risk effectively changes with time and the situation in the underlying real asset market. What the economic model is telling us is that, at the very beginning, the risk was very high, but so was the expected return.

Exhibit 13: Effect of Real Asset Volatility and Project Staging on NPV Based on Economic Real Option Model for Songdo International Business District Project as of End of 2002.



While the economic real option model is certainly of interest in its own right, as noted in Section 1 it is difficult for decision makers to relate to this type of model, and it has limited flexibility to realistically reflect specific aspects of the project and ways that decisions will likely realistically be made regarding the execution of the various phases of the project. Both of these shortcomings can be addressed by use of a Monte Carlo simulation model more in the classical tradition of decision analysis, the “engineering model” approach described in Section 2 of this paper. The economic model can be used to help calibrate and confirm the economic validity of the engineering model, even as the latter model opens up new intuitive perspectives on the investment decision and the program design. High level screening models of the type developed in this case can be implemented in basic Excel®, using the Data Table (What-if Analysis) utility to implement the random number generation based Monte Carlo simulation.

Exhibit 14: Initial 6-phase staging plan of the Songdo IBD development project (2002)



In the present example we constructed two different versions of the project program, both based on and consistent with the original projected cash flows shown previously in Exhibit 12. In one version we grouped the cash flows into only two phases. The other version reflected the six phases that the actual real world project eventually adopted, as indicated in Exhibit 14. Each of those six phases involved the complete development of a specified non-overlapping set of land parcels (at a pre-specified price from the Korean Government), involving a mixed-use component of the overall project master plan. To construct the two-phase version, we combined the first three, and the last three, of those six phases into two much larger phases (the same as in the previously-described binomial model). In all cases the phases add up to the same projected cash flows and time line consistent with the original base case “deterministic” (most likely) cash flow projection of Exhibit 12.

For simplicity we keep the base case cost projections, but we model the uncertainty and volatility in the real asset market by allowing the outcome real estate sales and income values to vary randomly, a different random *ex post* history generated in each of 2000 runs of the Monte Carlo simulation.³² As a benchmark, we model an “inflexible” (and therefore unrealistic) program by forcing the original base-case development timing no matter what random future history occurs. To model the effect of flexibility we build optionality and decision rules into the Excel workbook, applicable in either staging program (2-phase or 6-phase). The major decision rule extrapolates the current randomly generated real estate market outcome forward as of each potential phase start date and on that basis computes a projected IRR for that phase. If the projected IRR is above an input 30% hurdle, then the phase is triggered. (The level of the hurdle is an input parameter that can be changed.) Otherwise the phase is delayed, and the projection and hurdle criterion is checked again the next year (after a new random real estate market result is generated). If the hurdle is not exceeded after three years (four in the case of the 2-phase program), then that next phase and all subsequent phases are abandoned. Although simplified, this structure is a reasonable model of how development project decision-making actually occurs in the real world, and reflects the essence of the process for the purposes of the screening model. An advantage of the simulation approach over the economic model is that one can model uncertainty in more nuanced and multi-faceted ways, and we can model the staging decision making process more realistically. In the present example we contented ourselves with including only the two major forms of real estate market uncertainty: asset value volatility and pricing noise. As suggested in Section 3, these two sources of volatility are often sufficient to capture most of the

³² We experimented with a more complex model that allowed for randomness in development costs, but the results were essentially the same as reported here.

essential effects of uncertainty.³³ Based on the findings and analysis reported in Section 3 and discussed earlier, we set these parameters at 20% for the asset volatility and 10% for the noise.³⁴

In addition, unlike the economic option model, the engineering model of project NPV requires that we input a discount rate, presumably an opportunity cost of capital (OCC) relevant for the development project. From a rigorous economic perspective this is a simplification, as discussed above, as there is not a single OCC for a complex project with optionality. But it well reflects the way investors think and make decisions in the real world. In the context of the simulation model, this discount rate is a device to compute what is effectively an *ex post* NPV, that is, what the NPV (as of project inception in 2003) would be given a particular randomly generated future history. The discount rate so employed, along with other inputs in the model (such as the hurdle criterion for phase commencement), can be calibrated so that the simulation model results are broadly consistent with the more rigorous but more abstract economic real option model result noted in Exhibit 13. Based on such calibration and our discussion at the outset of this section regarding the political and economic context of the Songdo project in 2003, we set the discount rate at 25%.

Let us now summarize the screening model simulation. Two thousand random “future histories” (unfolding commencing in 2003) of the underlying real asset market are generated based on the 20% volatility and 10% noise uncertainty parameters. In each of these “histories” an *ex post* NPV for the project is computed using the 25% discount rate. This NPV is computed in each history for each of three different development program flexibility structures: (1) No flexibility (the project is built out according to the Exhibit 12 base case projection no matter how the history unfolds); (2)

³³ The model we used is also designed to easily represent one-time “black swan” type events, though we have not explored this issue in the present research.

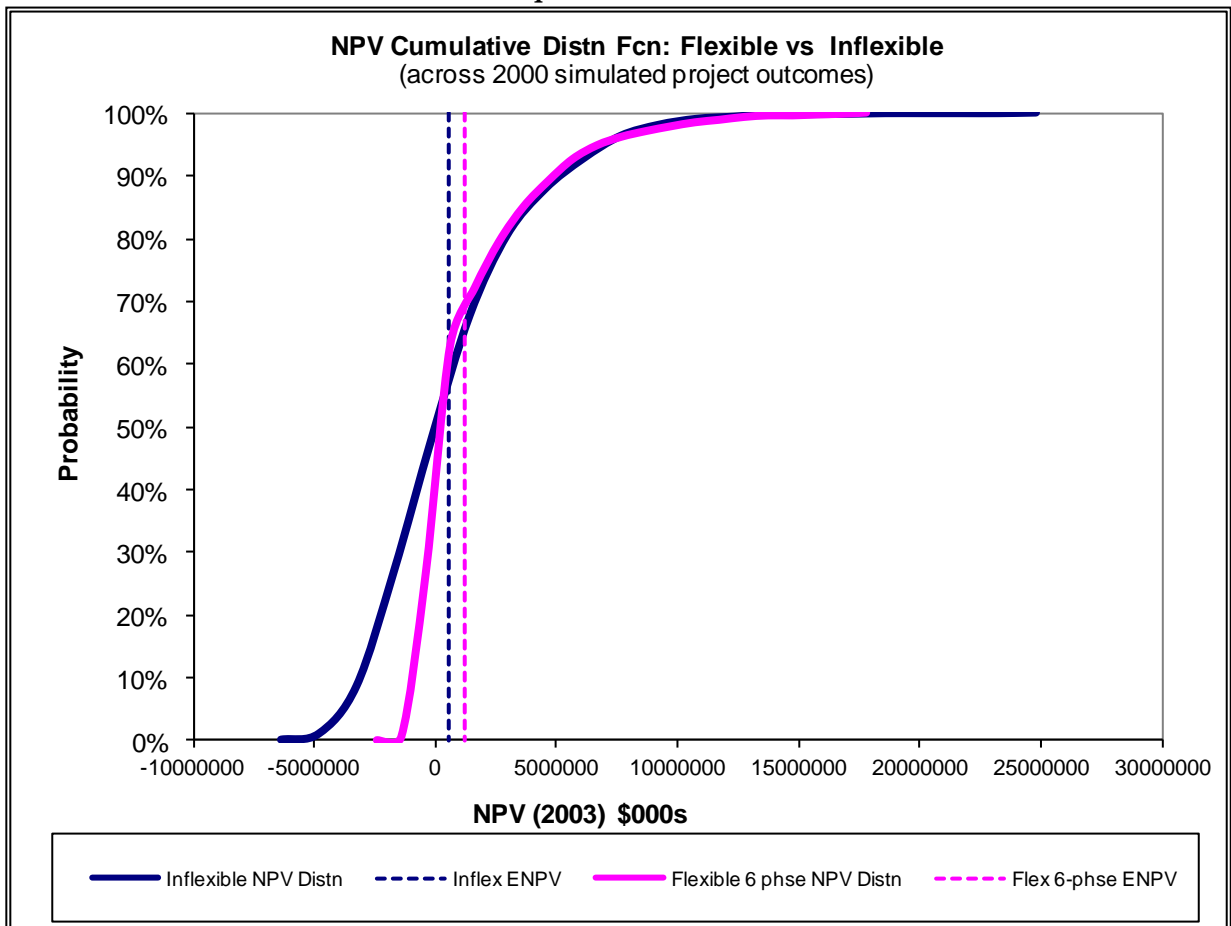
³⁴ In Section 3 we reported evidence of somewhat greater noise, around 15% standard deviation. However, this reflects residuals from a “mass appraisal” type model (or repeat-sales index), and we noted that it is likely that valuation estimation noise can be reduced somewhat below that when applied to specific assets, and we noted that there is some evidence for the 10% figure.

Flexibility with possibility for delay or abandonment at the beginning of each of two phases (based on the 30% expected return hurdle); (3) Flexibility as above only with the six phases that the actual real world project was broken into.

Exhibit 15: Engineering Model Results Summary for NSC

| Simulation Results NPV Statistics (USD millions): | | | |
|---|------------|-------------------|-------------------|
| Across 2000 simulated histories | Inflexible | Flexible 2 phases | Flexible 6 phases |
| Mean (ENPV) | \$554 | \$1,040 | \$1,224 |
| Maximum | \$24,759 | \$24,759 | \$17,816 |
| Minimum | -\$6,372 | -\$3,058 | -\$2,400 |
| Std.Dev | \$3,408 | \$2,739 | \$2,743 |
| Pct abandoned | NA | 9% | 0% |

Exhibit 16: Engineering Model Results: Expectation & Cumulative Probability of *ex post* Songdo IBD Net Present Value discounted at 25% per annum



The results of the simulation analysis are indicated in Exhibits 15 and 16. Across the 2000 histories, the (unweighted) average *ex post* NPV (@ the 25% discount rate) is over \$1.2 billion for the (actual real world) six-phase flexible case, almost \$200 million less at slightly over \$1.0 billion for the 2-phase flexible case, and less than \$600 million for the inflexible case. Using these average *ex post* NPV results, the engineering model provides valuations as of the end of 2002 that are broadly similar to those of the economic real option model (whose valuations are *ex ante*), but the engineering model's average NPVs are a bit lower than the economic model's. To us, this makes sense (and indeed this is part of the calibration of the engineering model³⁵), as the economic model presumes *optimal* behavior in terms of development timing and/or abandonment. It is more likely that the real world is not quite so optimal, and is better reflected by the decision process represented in the simulation model (to wit: commitment to each phase based on a projected IRR exceeding a typical hurdle rate, based on the then-prevailing real estate market).

As noted in Section 2, an important focus of the engineering model is that it presents not just a single *ex ante* valuation result, but an entire distribution of *ex post* results across the 2000 random histories. Thus, we can compare design alternatives not just on the basis of their average or statistical expectation of their NPVs, but also by the other characteristics of the Monte Carlo generated probability distribution of the outcomes, such as the range or standard deviation, as well as the extremes or tails of the distribution including possible skewness. This is shown graphically in Exhibit 16, which reveals how the flexibility to delay or abandon phases of the project cuts off the left-hand tail of the possible outcomes, leaving the project outcome distribution with more of a positive skew as upside outcomes can be taken advantage of while downside outcomes are avoided or mitigated through the technique of delaying or abandoning development phases. In our experience, decision makers and

³⁵ As noted, the economics-based real option model is used to confirm and calibrate the engineering-oriented simulation model. (See also Masunaga, 2007.)

investment backers of development projects can relate to this kind of model better than they can to the more abstract economic real option models, though as suggested, it is wise for analysts to consider both and to use insights from the economic model to help hone the simulation model.

5. Concluding Thoughts: A Role for the RICS?

The preceding sections have taken you on quite a tour. We have reviewed and attempted to synthesize three major strands of academic literature and professional practice related to the design and evaluation of investment and development in real assets, relevant for both real estate and infrastructure projects. We have presented a concrete example of how this knowledge and toolkit can be applied to analyze a major urban development project, the famous Songdo IBD project in Korea. But now we should emphasize that the intellectual enterprise that is reported on in this paper is still very much a work in process. Indeed, we feel that the profession is only at the outset of the development of the necessary data, knowledge, and tools to really improve the design and implementation of the great urban development that is ongoing apace (without waiting for us, that's for certain!), in Asia, the Middle East, and elsewhere. There is certainly a need for more practice and honing of the art of applying the new data and tools described herein to real world projects, such as our Songdo example, but extending much further and wider into different types and scales of design questions and development problems. As described in Section 2, there is a rich and varied experience with simulation modeling of flexibility in industrial and natural resource extraction industry examples. This suggests scope for extending and furthering this “art” in the real estate and infrastructure development field.

We would also like to humbly propose that there may be an important role for international professional organizations such as the RICS in the advance of this process. The proper and productive use of tools and techniques such as described in this paper requires a relatively high level of education.

It requires that subjects such as option theory and simulation and the econometrics of asset pricing analysis be effectively taught to the professionals in the relevant fields of real estate, architecture, engineering, and construction. Furthermore, the type of massive electronic data collection, compilation, and analysis that we described in Section 3 relevant to the quantification of uncertainty has to date been advanced largely only in the U.S. and a few other countries (such as most notably, although to varying degrees and in some different ways, Australia, Netherlands, Hong Kong, Singapore, and the U.K.). There is a great need for this type of sophisticated and high quality empirical information infrastructure in the major emerging economies, including China, India, and many others. We would challenge the RICS to consider how it might play a useful role in this endeavor, perhaps by promulgating or advocating for desirable data collection and compilation systems, as well as by fostering educational programs.

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