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ORIGINAL PAPER

A game-theoretic approach to optimizing the scale of incorporating renewable sources of energy and electricity storing systems in a regional electrical grid

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Abstract The problem of developing a decision support system for estimating a) the scale of incorporating available renewable sources of energy (such as solar and wind energy) in a part of a country's electrical grid (called a regional electrical grid further in this paper), and b) the scale of storing electricity in this (regional) electrical grid to make these renewable sources of electric power competitive with traditional power generators (such as fossil-fuel and nuclear ones) and to reduce the cost of acquiring electricity from all the electric power generating facilities in the grid is considered. In the framework of this system, renewable sources of energy are viewed as electricity generating facilities under both existing and expected electricity prices, and the uncertainty of energy supply from them and the uncertainty of the grid customer demand for electricity during every 24 h are taken into account. A mathematical model underlying the system allows one to study the interaction of all the grid elements as a game with a finite (more than three) number of players on a polyhedron of connected player strategies (i.e., strategies that cannot be chosen by the players independently of each other) in a finite-dimensional space. It is shown that solving both parts of the problem under consideration is reducible to finding Nash equilibrium points in this game.

Keywords Electrical grid · Equilibrium points · n-Person game · Polyhedron · Renewable sources of energy · Storing electricity

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1 Introduction

Problems associated with incorporating renewable sources of energy and systems for effectively storing electricity in electrical grids can be divided into two groups. The first group is formed by problems of developing

a) effective physical and chemical methods for transforming wind, solar, and other types of energy received from renewable sources of energy into electricity and those for storing electricity in large, medium, and small volumes, and
b) particular systems for transforming energy from renewable sources of energy into electricity and systems and devices for storing electricity that function based on these methods.

The second group of these problems implies effectively incorporating in the grid both systems for transforming energy from renewable sources of energy into electricity and electricity storage systems in electrical grids, functioning, in particular, in every developed country. This means that both types of the systems should become an effective tool for smoothing fluctuations of the customer demand for electricity (which substantially effect production schedules of base load power plants) and making renewable sources of energy competitive with fossil-fuel and nuclear base load power stations.

The present paper concerns a systems approach to studying the second group of the problems by means of a) mathematical models designed to describe the functioning of renewable sources of energy and electricity storing systems as part of a country's electrical grid, and b) mathematical methods for both analyzing the expediency and determining optimal scales of incorporating renewable sources of energy and the storing systems in particular electrical grids.

A description of the functioning of an electrical grid of a country is a complicated task, and studying this system presents considerable difficulties. Unstable world prices on fossil fuels [1,2], which are used for generating electric energy at many power stations, an uncertain customer demand for electricity during different day, evening, and night hours [3], a rapidly growing deployment of renewable sources of energy, especially those of wind and solar energy, for obtaining electricity [4–6], and the uncertainty caused by the liberalization of laws governing the operation of the electrical grid [7,8] represent only a few major factors that should be taken into consideration in choosing directions of keeping the grid responsive to public needs, as well as to those of both private and public sectors of the country's economy in general.

The physics of electricity production by base load power plants (including fossil-fuel and nuclear ones) imposes certain constraints on the production schedule for every generating facility, and the problem of finding an optimal supply from the base load power plants, taking into account the above uncertainties [9,10], is a challenge.

Currently, there are two viewpoints on how to operate base load power plants under deregulation measures that have been in force in many countries. The first one implies a) signing both long-term and interruptible agreements with large customers, such as industries, businesses, and utility companies for supplying them on a permanent basis with a particular volume of electric energy for each such customer at a fixed price while offering the "overproduced" volume of electricity for sale via auctions on electricity spot markets [11–13], and b) deploying peaking power plants to meet the demand for electric energy that exceeds the expected level of its consumption. The second one consists of selling most of the electricity produced via wholesale and retail markets in which retailers compete for buying electricity for their customers. Approaches to operating base load power plants reflecting both viewpoints have merits and deficiencies [14,15], and a lot depends on what amounts of money the electricity producers and the consumers would agree to receive and to pay, respectively, for the electricity to be supplied and to be bought.

Under either viewpoint, it is clear that effectively storing electricity can substantially change the way base load power plants operate, since both electricity producers and the grid customers can store electricity and benefit from doing this on account of smoothing fluctuations of the grid customer demand. It seems to be the most important with respect to renewable sources of energy, first of all, to those transforming the wind and solar power into electric power, which can be deployed by both the producers and the grid customers. Moreover, the ability to store electricity is the key to making the renewable sources of energy competitive with such traditional facilities producing electricity as fossil-fuel and nuclear base load power plants.

The aim of this paper is to propose mathematical tools underlying decision support systems for analyzing a part of a country's electrical grid, which is formed by a set of generating facilities (both traditional and those transforming solar and wind power into electric power), a set of electricity storing facilities, a set of large customers (businesses, industries, electric energy retailers, etc.), and a transmission company. (For the sake of simplifying the terminology, a part of a country's electrical grid under consideration in this paper is further called a regional electrical grid though it is clear that such a grid can provide services for several geographic regions.) Particularly, such tools should allow one a) to find an optimal constant hourly production volume for every base load power plant deployed in the grid under consideration, along with optimal volumes of electricity to be bought by the grid customers every hour during every 24 h, and b) to evaluate an optimal scale of using both the available facilities transforming wind and solar energy into electricity and the available electricity storage facilities for each type of the customers in the grid under consideration under both the existing and any expected prices for electricity from all the available sources of electric power in the grid.

The interaction of all the electricity producers serving the regional grid customers, the transmission company, and the (large) electricity consumers (who can receive electricity from wind and solar sources of energy and who have access to electricity storing facilities) is the subject of the mathematical modeling of the grid functioning undertaken in this paper. It is shown that under some natural, verifiable assumptions,

a) this interaction can be described as a game with a finite (more than three) number of players on polyhedra of player strategies being vectors in a finite-dimensional space, where strategies of some of the players are connected, i.e., cannot be chosen by the players independently of each other,
b) the above optimal values of the (constant) electricity production volumes either are components of equilibrium points in this game or are determined by these components, and

c) the optimal volumes of electricity to be stored by all the interested grid customers under both the existing and expected electricity prices from all the sources available in the grid (both traditional and renewable ones) are also components of the equilibrium points of the above game.

Mathematical models for optimizing energy systems in general and those for finding optimal solutions to a number of problems arising in studying electrical grids in particular, as part of energy systems engineering, including models for studying the functioning of these systems under uncertainty, have intensively been developed [16–39]. There are numerous publications, including surveys, relating to various aspects of the functioning of both the grid as a whole and its particular parts.

Though the present paper concerns a game-theoretic approach to studying particular issues associated with electrical grids, the author believes that providing a brief review of publications relating to the use of renewable sources of energy and storing systems in electrical grids in this paper makes more sense than providing any detailed review of applications of only game-theoretic approaches to studying electrical grids. The reason for this choice of the review structure is dictated by the fact that game-theoretic works with respect to electrical grids focus mostly on studying electricity spot markets (where electricity is sold and bought) and on studying issues associated with investing in the development of new power plants and systems for storing electricity [16–20].

For instance, a theoretical framework is offered in [16] for explaining two phenomena in electricity markets: a) the excess of marginal costs by spot prices, and b) the excess of spot prices by forward prices. The authors attempt to explain the coexistence of price markups and forward premiums in the market by the presence of frequent start-ups and shutdowns of the generators and ramping constraints (making the supply curve dynamic), along with the dependence of regulating the power system on the future production of the designated generators. A survey of approaches to designing economic mechanisms of the electricity market functioning for markets with uniform price auctions, pay-as-bid auctions, and the Vickrey auctions is presented in [17]. The survey suggests that a two-to three stage market in which both the day-ahead market and the forward sales are run as either the Vickrey auctions or the uniform price actions is preferable for all the market participants. The problem of reconciling the major “ideological” difference existing between two approaches to modeling multi-bid electricity markets—a) the theoretical one that assumes that the companies generating electricity provide piece-wise and differentiable supply functions, with a specific amount that they would like to supply at each particular price, and b) the practical one in which the supply functions have the form of a ladder of steps—is studied in [18]. Under the practical viewpoint, the market should be modeled as a discrete unit auction, which causes the existence of only mixed-strategy equilibria in the bidding game, whereas under the theoretical approach, an equilibrium in pure strategies can be achieved. The authors propose sufficient conditions for the convergence of a pure-strategy equilibrium in the game with particular stepped offer functions to the equilibrium in the game with continuous ones.

A dynamic game is proposed to use in [19] for describing the competition of several electricity producers each of which makes a decision on investing in new capacities (or in new facilities) in a certain period of time in an attempt to maximize the expected utility under uncertainty. One of the uncertainties is associated with an unknown demand for electricity (in a particular region), and another one deals with the fact that producers with smaller volumes of production make their investment decisions only after such decisions have been made by larger producers. In [20], the competition of several producers is modeled by a non-cooperative static game in which the uncertainty is modeled by a tree of possible random events, taking into account the aging of the existing generating facilities and the existence of two types of demand (a base demand and a pick demand).

In the last several years, papers on using game-theoretic approaches to studying various aspects of "smart" grid such as, for instance, energy consumption schedules [21,22], demand-side management problems [23,24], including those in which the presence of systems for storing electricity is taken into consideration [25], were published. However, none of all the above publications deals with problems under consideration in the present paper. It is for this reason, the author decided to provide a brief review of a set of examples from three groups of typical publications on issues more closely related to the subject of the present paper than those in the publications [21–25] to let the reader form the impression on what concerns researchers in the field the most.

The first group of the publications includes those dealing with configuring electrical grids, including micro grids, as an electrical network, with estimating the perspectives for the use of traditional and renewable sources of energy in the grid, and with planning the work of the grids. An approach to configuring large-scale electric power networks such as a national electrical grid by simulating their functioning with the Plexos tool (a tool for simulating financial markets) combined with the multi-objective optimization evolutionary algorithm MOOEA (a heuristic stochastic software developed by the authors) is proposed in [26]. Optimal management strategies for a Micro Grid—which includes several sources of power and storage devices, and interacts with a grid under such uncertainties as the load forecast, grid bid changes, and outputs of solar and wind sources—are found in [27] by solving a nonlinear problem of minimizing the total operational cost of the Micro Grid. An optimal proportion between the rate of fossil-fuel generated energy and that generated by renewable sources of energy is proposed to find by solving an optimal control problem in which a) the above rates are decision variables whose dynamics is described by two nonlinear, first-order differential equations, and b) the functional reflects the difference between the actual power generation and the demand for this power for the planning period, the cost of the control, and the gap between the targeted and the actual generation level of energy [28]. A stochastic dynamic programming model in combination with Monte Carlo simulations is used in [29] for planning the work of a generation company possessing the Willingston Reservoir, an electricity storage with the capacity of 40,000 million m^3, for the five-to-six year time planning horizon with a discrete monthly time steps. Linear programming is proposed for optimizing the integration of two electrical grids into one, depending on the transmission capacity between them [30].

The second group of publications includes those related to storing electricity. The state of the art of technologies for storing electricity obtained from the wind power is reviewed in [31]. The review is done particularly, from the viewpoint of choosing the sizes and the placement of storing systems in integrating them into utility grids to balance the benefits and the cost. Several mathematical models, including those for optimizing the sizes of the storage systems that take into consideration the accuracy of the wind power forecast, along with charging-discharging rules, and consider the power deviation from the next hourly average as the cost function to be minimized, are mentioned in that review. A review of the available electricity storage systems and the problem of optimally sizing an energy storage system taking into account the reliability requirements for isolated grids with wind power being the only source of energy are considered in [32]. This problem is formulated as the one of minimizing the annual capital and maintenance cost of the storage system under constraints of the balance kind, and it is solved by means of sequential simulations with the use of a generic algorithm. A review of recent advances and contemporary trends of storing electricity technologies in Micro Grids (as electric power systems that may unite both distributed energy resources, customers, and electricity storage systems capable of functioning either separately from utility grids or as part of them) is presented in [33]. Both commercial and non-commercial models for calculating optimal sizes of the electricity storages, along with corresponding software for simulation modeling, are mentioned, and the authors' viewpoint of their applicability to optimizing the size of a Micro Grid is provided.

The third group of publications includes those considering particular parts of the grid, including renewable sources of energy incorporated there. The functioning of a power generating system within a grid with an energy storage system supplied by wind power as a part of its back-up system in modeled in [34]. A two-stage stochastic optimization problem in which the uncertainty of wind power (simulated by means of the interval simulation technique to describe the wind power characteristics) is formulated to optimize the total electricity demand in an electrical grid. A survey of the use of mathematical modeling in the optimization of the power generation and supply is presented in [35]. Among the models sited in the survey, there are stochastic programming, robust stochastic programming, multi-objective optimization ones, and equilibrium models that have become deployed in energy planning and managing electricity supply networks. As shown in [36], in the framework of a power supply and demand management in an islanded Micro Grid under uncertainties associated with the power generation by renewable sources of energy, the Micro Grid central controller can solve a problem with a convex goal function and a nonlinear system of constraints to minimize the energy cost in determining an optimal consumption and generation of power. Results of a simulation modeling of two scenarios of the forecasted development of the electricity market in India for 2017 under the expected growth of using the renewable sources of energy (called High and Low Renewable scenarios) are presented in [37]. These results are used by the authors to offer their viewpoint on several issues, including the impact of the operating costs for renewable sources of energy on the total cost of the energy generation, on the transmission system, etc. A brief review of numerous simple mathematical models describing physical and chemical principles of functioning for various renewable sources of energy such as wind energy systems, Micro Hydro Power systems, solar photovoltaic systems, biogas, and biomass gasifier systems are presented in [38]. Besides these models a) mathematical descriptions of various economic criteria in optimization problems related to unit sizing of the integrated renewable energy systems and cost optimization in these systems, b) citations to scientific publications in which these optimization problems are considered, and c) brief, one-sentence descriptions of the essence of these publications (as the authors see it) are provided in [38]. The sited publications study, in particular, artificial intelligence approaches, multi-objective approaches, developing iterative algorithms, simulation, and probabilistic approaches, as well as available software utilizing these approaches.

Since every electricity storage system can formally be viewed as an inventory system, one may question whether the well-developed theory of inventory systems—especially of those with the so-called power demand [39], which may allow one to determine the size of the storing system—should or can be used in modeling the functioning of a regional electrical grid. However, one should bear in mind that the replenishment patterns in electricity storages are difficult to describe other than those randomly formed, so as an inventory system, an electricity storage system has both a random demand and a random replenishment. In the present paper, an attempt to determine the demand patterns and the replenishment strategy is undertaken (with respect to electricity storage systems) in the framework of finding an optimal configuration of a regional electrical grid, and these patterns can further be used for developing any detailed models for electricity storage systems as such, including those considering these systems as inventory ones.

All the models cited in the above and problems formulated on their basis focus mostly on the optimal use of particular technologies, facilities, and devices being part of the grid, including those for storing electricity. As far as the author is aware, a game-theoretic approach has never been used for analyzing the potential of the current suppliers for electrical grids with particular configurations to compete with other suppliers of electricity (from renewable sources of energy). Nor has it been used for finding directions and the scale of possible reconfigurations of the grids, especially on account of incorporating renewable sources of energy and storing systems in them. To a certain extent, this state of affairs may reflect two phenomena:

a) the current dominance of one of the above two viewpoints on how to operate base load power plants, that is, to "tune them on the wholesale and retail markets" in which electricity is sold and bought, rather than to "match" them with prospective customers under long-term contractual agreements, and
b) considerable difficulties of modeling the grids as large-scale systems in forms properly reflecting basic technological and possible business relations among all its elements while allowing one to solve optimization problems formulated with the use of these models by available techniques and software with a needed accuracy and in an acceptable time.

The author believes that the approach to studying electrical grids in the framework of particular classes of games with many (more than three) players, proposed in this paper, is novel and has several helpful features. First, it allows all interested parties—large grid customers, generating facilities, and the transmission companies—to choose their business strategies while taking into account rapid developments of renewable sources of energy and electricity storing systems and their growing accessibility for both households and neighborhoods. Second, it allows one to have a different look at the electricity markets and better evaluate their structure and dynamics. Third, games on polyhedral sets of player strategies, underlying the proposed approach, allow both the grid customers and electricity producers effectively calculate their Nash equilibrium strategies to estimate long-term policies to pursue in determining possible configurations and vital parameters of the grids. Fourth, this approach demonstrates the power of linear programming techniques—possessing enormous computational potential—in solving n-person games on polyhedral sets of player strategies in general and in analyzing large-scale economic problems in the energy sector associated with producing and supplying electricity in particular.

Though the presented paper concerns a practical problem, it is purely theoretical, since its goal is to demonstrate the power of optimization techniques in developing tools for analyzing practical large-scale economic problems, particularly those associated with producing and supplying electricity in a region. Any model examples of using these tools would demonstrate no more than how a particular piece of software implementing the tools really functions, which may present interest mostly from the viewpoint of transforming the tools into a research or a commercial instrument for studying electrical grids in principle and for estimating the quality of their implementation in particular. Also, one should bear in mind that collecting the real data for the presented game model is not an easy task, especially taking into account that most of the data has commercial value and may not be available other than for particular businesses.

2 The problem statement and basic assumptions

Let us consider a regional electrical grid which includes [15]

a) a company (or a group of companies acting as one legal entity) which is (is considered as) a base load power plant, producing and selling electricity to the customers of the grid (the generator further in this paper),
b) a company (or a group of companies acting as one legal entity) offering services associated with transmitting electric energy via (high voltage) transmission lines (the transmission company),
c) companies providing electricity for individual end users in industrial and residential areas via (low voltage) distribution lines (the utility companies),
d) industrial enterprizes and large businesses that receive electricity from the generator under direct supply agreements (the industrial customers),
e) groups of end users that are licensed to operate the (low voltage) distribution lines via which electricity is delivered to them directly, rather than via utility companies (the groups of advanced customers, in particular, micro grids),
f) individual end users that can receive electricity via the above utility companies only (the households), and
g) a company (or a group of companies acting as one legal entity) which is (is considered as) a peaking power plant producing and selling electricity to all the above customers when the actual electricity demand of the customers in the grid (to be covered by the other sources of electricity in the grid) at a particular moment exceeds the expected one (the peaking power facility (or plant) further in this paper).

Assumption 1 Under the regulation and deregulation measures that are in force in the country, each industrial customer, each utility company, and each group of advanced customers have access to a fleet of equipment to store electricity up to a needed volume. Also, each of these customers of the grid has access to a fleet of equipment for transforming solar and wind energy into electric energy, which is a property of some legal entities (the suppliers further in this paper) and can provide the needed volume of electricity to the customers. No grid customer may resell (either via a wholesale electricity market or via a retial electricity market) excessive volumes of electricity that may eventually be in its possession (beyond volumes that the customer decides to store) though each customer can return electricity received from the suppliers to these suppliers at no cost to the suppliers.

Assumption 2 In the framework of the grid, no base load power plants other than those being part of the generator, no peaking power plants other than those being part of the peaking power facility, and no other transmission enterprizes other than the transmission company compete for the customers of the regional grid. Any demand for electricity that exceeds the baseload production of the generator that cannot be covered by any available sources of electricity other than by peaking power plants is to be covered by the peaking power facility.

It is important to emphasize the difference between two logically possible approaches to understanding the interaction of all electricity consumers and the generator in a regional electrical grid. In the framework of the first approach, all the consumers attempt to meet their electricity demands from the base load power plants that comprise the generator and from renewable sources of energy (directly or with the deployment of electricity storage facilities available to them in both cases). They use electricity produced by the peaking power facility only when the actual electricity demand exceeds the expected one and cannot be covered by any other manner. In contrast, under the second approach, the peaking power facility is considered a part of the generator. The major reason not to consider peaking power plants as base load power plants is associated with much higher prices per unit volume of electricity produced by peaking power plants, compared to those for electricity produced by base load power plants (most of which are either nuclear or coal-operated facilities).

Assumption 3 The following information is known:

a) a minimum and a maximum average hourly electricity demands for each industrial customer for every twenty-four-hour period of time,
b) a minimum and a maximum aggregate hourly electricity demands for each utility company and for each group of advanced customers (which are based on such estimates for individual end users (the households) in both cases) for every 24 h,
c) a maximum capacity of the generator for producing electricity, along with the volume of losses in transmitting a unit volume of electricity via transmission lines to each category of customers within the grid,
d) the costs of (the expenses associated with) operating the equipment for transforming solar and wind energy into a unit volume of electric energy, and
e) the costs of (the expenses associated with) operating the equipment for storing electricity per unit volume.

One needs to determine

- an optimal (constant) hourly baseload production volume for each base load power plant being part of the generator,
- optimal prices for a unit volume of the electricity produced by the generator for each group of customers within the grid on an hourly basis,
- optimal prices for transmitting a unit volume of electricity from the generator to each customer that receives electricity from the generator directly, and
- the scale of incorporating in the grid facilities for storing electricity for each category of customers within the grid, along with that of incorporating facilities for transforming solar and wind energy into electricity, under both the existing and any expected prices for electricity that can be supplied by these facilities.

Let [40]

m be the number of industrial customers within the grid,

n be the number of utility companies that have access to the (low voltage) distribution lines via which individual end users of the grid receive electricity,

r be the number of groups of advanced customers that have licences to operate the existing (low voltage) distribution lines directly, rather than via utility companies,

$Y^g(l)$ be the volume of electric energy produced by the generator in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^g(l)$ be the volume of electric energy produced by the generator that is bought by industrial customer i, $i \in \overline{1,m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$z_j^g(l)$ be the volume of electric energy produced by the generator that is bought by utility company j, $j \in \overline{1,n}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$u_k^g(l)$ be the volume of electric energy produced by the generator that is bought by group of advanced customers k, $k \in \overline{1,r}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$.

Assumption 4 [15] The loss of power in transmitting electricity to all the above large customers for the period of time from hour $l-1$ to hour l within a certain "working segment" of power produced by the generator can be described by a piece-wise linear function of the volume of electricity produced by the generator

$$\max_{\lambda_l \in \overline{1,\Lambda_l}} \left(a_{\lambda_l} + b_{\lambda_l} Y^g(l)\right), \quad l \in \overline{1,24},$$

where $a_{\lambda_l} + b_{\lambda_l} Y^g(l)$ are linear functions of the variables $Y^g(l)$, $\lambda_l \in \overline{1,\Lambda_l}$, $a_{\lambda_l}, b_{\lambda_l} \in R^1$, $l \in \overline{1,24}$.

This assumption allows one to describe the volume of the electric energy that is to be produced by the generator during the period of time from hour $l-1$ to hour l (to provide the customers with the above volumes of electricity $y_i^g(l)$, $i \in \overline{1,m}$, $z_j^g(l)$, $j \in \overline{1,n}$, and $u_k^g(l)$, $k \in \overline{1,r}$, $l \in \overline{1,24}$) as follows:

$$Y^g(l) = \left(\sum_{i=1}^{m} y_i^g(l) + \sum_{j=1}^{n} z_j^g(l) + \sum_{k=1}^{r} u_k^g(l) \right) + \max_{\lambda_l \in \overline{1, \Lambda_l}} \left(a_{\lambda_l} + b_{\lambda_l} Y^g(l) \right),$$
$$l \in \overline{1, 24}, \quad (1)$$

Remark 1 As is well known, the power loss in transmitting power in an electrical grid from a generating facility to a customer is formed by several ingredients, and the loss in high voltage transmission lines is the major one. According to Ohm's law, the power loss in a particular transmission line is proportional to the square of the power transmitted (provided the transmission voltage is kept constant), and the proportionality coefficient depends on the length of the transmission line (along which the power travels), the diameter of the conductor, the properties of the conductor material (resistance) that is used in the transmission line, and the value of the above high transmission voltage. Since the production of each base load power plant is to be set at a particular volume (and, as mentioned earlier, finding its optimal production is one of the aims of developing the mathematical model under consideration in the present paper), one can consider a) a certain segment of production volumes containing the above particular production volume, say, $[c_{min},\ c_{max}]$ and b) the functions $\max_{\lambda_l \in \overline{1, \Lambda_l}} \left(a_{\lambda_l} + b_{\lambda_l} Y^g(l) \right)$ as piece-wise approximations of the convex functions $f_l : R^1 \to R^1,\ f_l(Y^g(l)) = (Y^g(l))^2/k,\ l \in \overline{1, 24}$ on the segment $[c_{min},\ c_{max}]$, where $k >> 1$, and $1/k$ equals the loss of power in transmitting a unit of electric power volume via the transmission line of the transmission company. The functions $f_l,\ l \in \overline{1, 24}$ describe the loss of power in the transmission line associated with transmitting electric energy from the generator to the grid customers (under the above assumption on the transmission voltage in the high voltage transmission lines) in the period of time from hour $l-1$ to hour $l, l \in \overline{1, 24}$. It is clear that such an approximation can be done with any accuracy.

Let

$p_i^y(l)$ be the price at which a unit volume of electric energy is sold by the generator to industrial customer $i,\ i \in \overline{1, m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$p_j^z(l)$ be the price at which a unit volume of electric energy is sold by the generator to utility company $j,\ j \in \overline{1, n}$, in the period of time from hour $l-1$ to hour $l, l \in \overline{1, 24}$,

$p_k^u(l)$ be the price at which a unit volume of electric energy is sold by the generator to group of advanced customers $k,\ k \in \overline{1, r}$ in the period of time from hour $l-1$ to hour $l, l \in \overline{1, 24}$,

$y^g = \left(y_1^g(1), \dots, y_1^g(24); y_2^g(1), \dots, y_2^g(24); \dots; y_m^g(1), \dots, y_m^g(24) \right)$ be the vector whose component $y_i^g(l)$ is the volume of electric energy that is bought by industrial customer i from the generator in the period of time from hour $l-1$ to hour $l, i \in \overline{1, m}, l \in \overline{1, 24}$,

$p^y = \left(p_1^y(1), \dots, p_1^y(24); p_2^y(1), \dots, p_2^y(24); \dots; p_m^y(1), \dots, p_m^y(24) \right)$ be the vector whose component $p_i^y(l)$ is the price at which a unit volume of electric energy is sold to industrial customer i by the generator in the period of time from hour $l-1$ to hour $l, i \in \overline{1, m}, l \in \overline{1, 24}$,

$z^g = \left(z_1^g(1), \ldots, z_1^g(24); z_2^g(1), \ldots, z_2^g(24); \ldots; z_n^g(1), \ldots, z_n^g(24)\right)$ be the vector whose component $z_j^g(l)$ is the volume of electric energy that is bought by utility company j from the generator in the period of time from hour $l-1$ to hour l, $j \in \overline{1,n}$, $l \in \overline{1,24}$,

$p^z = \left(p_1^z(1), \ldots, p_1^z(24); p_2^z(1), \ldots, p_2^z(24); \ldots; p_n^z(1), \ldots, p_n^z(24)\right)$ be the vector whose component $p_j^z(l)$ is the price at which a unit volume of electric energy is sold to utility company j by the generator in the period of time from hour $l-1$ to hour l, $j \in \overline{1,n}$, $l \in \overline{1,24}$,

$u^g = \left(u_1^g(1), \ldots, u_1^g(24); u_2^g(1), \ldots, u_2^g(24); \ldots; u_r^g(1), \ldots, u_r^g(24)\right)$ be the vector whose component $u_k^g(l)$ is the volume of electric energy that is bought by group of advanced customers k from the generator in the period of time from hour $l-1$ to hour l, $k \in \overline{1,r}$, $l \in \overline{1,24}$,

$p^u = \left(p_1^u(1), \ldots, p_1^u(24); p_2^u(1), \ldots, p_2^u(24); \ldots; p_r^u(1), \ldots, p_r^u(24)\right)$ be the vector whose component $p_k^u(l)$ is the price at which a unit volume of electric energy is sold to group of advanced customers k by the generator in the period of time from hour $l-1$ to hour l, $k \in \overline{1,r}$, $l \in \overline{1,24}$.

Assumption 5 The generator can sell electric energy at any (competitive) prices as long as these prices do not violate the consumer rights of the grid customers. Moreover, these prices can be different for different categories of the customers at any particular period of time, and the prices for each customer from the above three groups of customers within the grid can be different in different periods of time within any 24 h. All the energy sold by the generator as a whole is that consumed (bought) by the customers within the grid.

Remark 2 If the generator is comprised of several generating facilities, one can easily determine a baseload supply of electricity for each of these facilities (that secure the optimal supply of electricity by the generator as a whole) by introducing corresponding relations of the balance type into the mathematical description of the set of the generator strategies (these relations are presented in Concluding Remark 1).

Assumption 6 The baseload supply of the generator is sought within a certain "working segment" of the electricity production volumes, and within the segment, expenses of the generator associated with producing electric energy can be described by a piecewise linear function.

Remark 3 Certainly, the description of the generator expenses associated with producing electric energy as functions of the production volumes in the form of a piecewise linear function is a simplification of the grid regularities, since, generally, these expenses are suggested to be described by non-decreasing convex functions for each base load power plant [41].

However, the reasoning presented in Remark 1 regarding a piece-wise approximation of the functions f_l (at least within the above "working segment" of the electricity production volumes for the generator, mentioned in Assumption 6) is applicable in choosing a particular form of a formalized description of the generator expenses associated with the electricity production. So in the reasoning to follow, the generator expenses associated with producing electricity are considered to be described by piece-wise linear functions in the form

$$\max_{\mu_l \in \overline{1, \Gamma_l}} \left(c_{\mu_l} + b_{\mu_l} Y^g(l) \right), \quad l \in \overline{1, 24},$$

where $c_{\mu_l} + d_{\mu_l} Y^g(l)$ are linear functions of the variables $Y^g(l),\ c_{\mu_l},\ b_{\mu_l} \in R^1,\ l \in \overline{1, 24}$ (within the above segment of the production volume $[c_{min},\ c_{max}]$).

Assumption 7 The transmission company charges both the generator and each of the (three) groups of large grid customers for transmitting a unit volume of electricity. The transmission prices are constant for both the generator and for the customers during the 24 h period of time though they can be different for different customers. The capacity of the transmission lines is considered to be sufficient to meet the estimated maximum demands of all the grid customers that use these lines for receiving electricity bought from the generator.

3 The mathematical formulation of the problem

Let us first consider mathematical models describing the functioning of the grid elements.

Industrial customers

Let

$y_i^{dem}(l)$ be the electricity demand of industrial customer $i,\ i \in \overline{1, m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$\overline{y}_i^{dem}(l)$ be the estimate of the maximal electricity demand of industrial customer $i,\ i \in \overline{1, m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$\underline{y}_i^{dem}(l)$ be the estimate of the minimal electricity demand of industrial customer $i,\ i \in \overline{1, m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$y_i^w(l)$ be the volume of electric energy that is received by industrial customer $i,\ i \in \overline{1, m}$ from wind energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$y_i^{w-st}(l)$ be the part of the volume of electric energy that is received by industrial customer $i,\ i \in \overline{1, m}$ from wind energy and that goes to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$y_i^{w-dir}(l)$ be the part of the volume of electric energy that is received by industrial customer $i,\ i \in \overline{1, m}$ from wind energy and that is used by this customer directly, beginning from the moment of receiving this energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$y_i^s(l)$ be the volume of electric energy that is received by industrial customer $i,\ i \in \overline{1, m}$ from solar energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$y_i^{s-st}(l)$ be the part of the volume of electric energy that is received by industrial customer $i,\ i \in \overline{1, m}$ from solar energy and that goes to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

$y_i^{s-dir}(l)$ be the part of the volume of electric energy that is received by industrial customer $i,\ i \in \overline{1, m}$ from solar energy and that is used by this customer directly, beginning from the moment of receiving this energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^g(l)$ be the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from the generator, in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{g-st}(l)$ be the part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from the generator and that goes to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{g-dir}(l)$ be the part of the volume of electric energy which is received by industrial customer i, $i \in \overline{1,m}$ from the generator and that is used by this customer directly, beginning from the moment of receiving this energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{st}(l)$ be the volume of electric energy that is available to industrial customer i, $i \in \overline{1,m}$ from its storage system in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{dem-st}(l)$ be the volume of electric energy that is consumed by industrial customer i, $i \in \overline{1,m}$ from its storage system in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{st}(0)$ be the volume of electric energy that is present in the storage system of industrial customer i, $i \in \overline{1,m}$ at the beginning of the 24 h period of time,

$y_i^{st^+}$ be the maximum volume of electric energy that can be stored in the storage system of industrial customer i, $i \in \overline{1,m}$,

$\underline{y}_i^{st}$ be the minimal hourly volume of electric energy that is to be present in the storage system of industrial customer i, $i \in \overline{1,m}$,

$\triangle_i^y$ be the volume of (average hourly) energy loss associated with storing electricity in the storage system of industrial customer i, $0 < \triangle_i^y < 1$, $i \in \overline{1,m}$,

λ_i^{yw} be the (average hourly) expenses of industrial customer i, $i \in \overline{1,m}$ that are associated with receiving a unit volume of electric energy from wind energy,

λ_i^{ys} be the (average hourly) expenses of industrial customer i, $i \in \overline{1,m}$ that are associated with receiving a unit volume of electric energy from solar energy, and

π_i^y be the (average hourly) expenses of industrial customer i, $i \in \overline{1,m}$ that are associated with operating its storage system per unit volume of electricity available to this customer.

The functioning of the storage facility of industrial customer i, $i \in \overline{1,m}$ can be described by the following model of the balance kind:

$$y_i^{st}(l) = y_i^{st}(0) + \sum_{x=1}^{l} \left(y_i^{g-st}(x) + y_i^{s-st}(x) + y_i^{w-st}(x)\right) - \sum_{x=1}^{l} (y_i^{dem-st}(x) + \triangle_i^y x),$$

$$\underline{y_i^{st}} \le y_i^{st}(l) \le y_i^{st^+}, \quad l \in \overline{1,24}.$$

Further, let

$\underline{y}_i^w(l)$ be the estimate of the minimal volume of electricity produced from wind energy that industrial customer i, $i \in \overline{1,m}$ agrees to receive from the supplier, $l \in \overline{1,24}$, and

$\underline{y}_i^s(l)$ be the estimate of the minimal volume of electricity produced from solar energy that industrial customer i, $i \in \overline{1,m}$ agrees to receive from the supplier, $l \in \overline{1,24}$.

Then the functioning of industrial customer i, $i \in \overline{1,n}$ (from the viewpoint of receiving and consuming electric energy) can be described by the following mathematical model:

$$\begin{aligned}
&\underline{y}_i^w(l) \le y_i^w(l),\\
&\underline{y}_i^s(l) \le y_i^s(l),\\
&\underline{y}_i^{dem}(l) \le y_i^{dem}(l) \le \overline{y}_i^{dem}(l),\\
y_i^{st}(l) &= y_i^{st}(0) + \sum_{x=1}^{l}\left(y_i^{g-st}(x) + y_i^{s-st}(x) + y_i^{w-st}(x)\right)\\
&\quad -\sum_{x=1}^{l}(y_i^{dem-st}(x) + \triangle_i^y x),\\
\underline{y}_i^{st} &\le y_i^{st}(l) \le y_i^{st^+},\\
y_i^{dem}(l) &= y_i^{w-dir}(l) + y_i^{s-dir}(l) + y_i^{g-dir}(l) + y_i^{dem-st}(l),\\
y_i^w(l) &= y_i^{w-dir}(l) + y_i^{w-st}(l),\\
y_i^s(l) &= y_i^{s-dir}(l) + y_i^{s-st}(l),\\
y_i^g(l) &= y_i^{g-dir}(l) + y_i^{g-st}(l),\\
i &\in \overline{1,m},\ l \in \overline{1,24}.
\end{aligned} \tag{2}$$

Here, it is assumed that system (2) is compatible, which can be verified by a simple technique, proposed in [42].

Let

$$\begin{aligned}
y_i^g &= \left(y_i^g(1),\ldots,y_i^g(24)\right);\ y_i^w = \left(y_i^w(1),\ldots,y_i^w(24)\right);\ y_i^s = \left(y_i^s(1),\ldots,y_i^s(24)\right);\\
p_i^y &= \left(p_i^y(1),\ldots,p_i^y(24)\right);\ \lambda_i^{yw}(av) = (\lambda_i^{yw},\ldots,\lambda_i^{yw}),\ \lambda_i^{ys}(av) = (\lambda_i^{ys},\ldots,\lambda_i^{ys});\\
y_i^{st} &= (y_i^{st}(1),\ldots,y_i^{st}(24)),\ \lambda_i^{yw}(av),\lambda_i^{ys}(av) \in R_+^{24},\ i \in \overline{1,m},
\end{aligned}$$

so that $y^g = (y_1^g, y_2^g, \ldots, y_m^g)$, $p^y = (p_1^y, p_2^y, \ldots, p_m^y)$, and let

$$\begin{aligned}
\tilde{y} &= \left(y^g; y_1^w, y_2^w, \ldots, y_m^w; y_1^s, y_2^s, \ldots, y_m^s; y_1^{st}, y_2^{st}, \ldots, y_m^{st}\right),\\
\tilde{p}^y &= (p^y; 0, 0\ldots, 0; 0, 0\ldots, 0; 0, 0\ldots, 0),\\
\tilde{q}^y &= \left(0; \lambda_1^{yw}(av), \ldots, \lambda_m^{yw}(av); \lambda_1^{ys}(av), \ldots, \lambda_m^{ys}(av); \pi_1^y, \ldots, \pi_m^y\right),
\end{aligned}$$

so that $\langle y^g, p^y\rangle = \langle \tilde{y}, \tilde{p}^y\rangle$. Finally, let s_i^y be the hourly price that industrial customer i pays the transmission company for a unit volume of electric energy transmitted to this customer during the 24 h period of time, $i \in \overline{1,m}$, and let $\tilde{\epsilon}^y = (\epsilon^y, \ldots, \epsilon^y; 0 \ldots, 0)$, where $\tilde{\epsilon}^y$ is the vector of the same dimension as $\tilde{y}$ with all non-zero components equalling 1, where $\epsilon^y \in R^{24}_+$ is the vector with all the components equalling 1.

Then the goal function of industrial customer i, $i \in \overline{1,m}$ can be written as

$$\sum_{l=1}^{24} \left(y_i^g(l) p_i^y(l) + y_i^w(l) \lambda_i^{yw}(av) + y_i^s(l) \lambda_i^{ys}(av) + y_i^g(l) s_i^y + y_i^{st}(l) \pi_i^y \right)$$
$$= \left\langle \tilde{p}^y, \tilde{y} \right\rangle_i + \left\langle \tilde{\epsilon}^y, \tilde{y} \right\rangle_i s_i^y + \left\langle \tilde{q}^y, \tilde{y} \right\rangle_i \to \min_{(\tilde{y})_i}, \quad i \in \overline{1,m}, \tag{3}$$

where $\langle \tilde{y}, \tilde{p}^y \rangle_i$ means a part of the scalar product of the vectors $\tilde{y}$, and $\tilde{p}^y$ relating to industrial customer i, $i \in \overline{1,m}$, and $\langle \tilde{\epsilon}^y, \tilde{y} \rangle_i$, $\langle \tilde{q}^y, \tilde{y} \rangle_i$ have the similar meaning with respect to transmitting electricity to industrial customer i and to the electric energy received by industrial customer i from the wind, solar, and storage systems available to this customer.

Utility companies

The functioning of utility company j, $j \in \overline{1,n}$ can be described by a similar system of constraints binding the variables and parameters

$$\begin{aligned}
&z_j^w(l),\ z_j^{w-st}(l),\ z_j^{w-dir}(l),\ z_j^s(l),\ z_j^{s-st}(l),\ z_j^{s-dir}(l),\\
&z_j^g(l),\ z_j^{g-dir}(l),\ z_j^{g-st}(l),\ z_j^{st}(l),\\
&z_j^{dem-st}(l),\ z_j^{st}(0),\ \underline{z}_j^{st},\ z_j^{st^+},\ \triangle_j^z,\ \lambda_j^{zw},\ \lambda_j^{zs},\\
&\pi_j^z,\ z_j^{dem}(l),\ \overline{z}_j^{dem}(l),\ \underline{z}_j^{dem}(l),\ \underline{z}_j^w(l),\ \underline{z}_j^s(l),\ j \in \overline{1,n},\ l \in \overline{1,24},
\end{aligned}$$

that have the same meaning that do the (corresponding to them) variables and parameters that are used for describing the functioning of industrial customer i, $i \in \overline{1,m}$. A detailed description of all these variables, along with that of the functioning of the storage facilities that utility company j uses, is presented in [15].

Let

$$\begin{aligned}
&z_j^g = \left(z_j^g(1), \ldots, z_j^g(24)\right);\ z_j^w = \left(z_j^w(1), \ldots, z_j^w(24)\right);\ z_j^s = \left(z_j^s(1), \ldots, z_j^s(24)\right);\\
&p_j^z = \left(p_j^z(1), \ldots, p_j^z(24)\right);\ \lambda_j^{zw}(av) = (\lambda_j^{zw}, \ldots, \lambda_j^{zw}),\ \lambda_j^{zs}(av) = (\lambda_j^{zs}, \ldots, \lambda_j^{zs});\\
&z_j^{st} = (z_j^{st}(1), \ldots, z_j^{st}(24)),\ \lambda_j^{zw}(av), \lambda_j^{zs}(av) \in R^{24}_+, \quad j \in \overline{1,n},
\end{aligned} \tag{4}$$

so that $z^g = (z_1^g, z_2^g, \ldots, z_n^g)$, $p^z = (p_1^z, p_2^z, \ldots, p_n^z)$, and let

$$\begin{aligned}
\tilde{z} &= \left(z^g; z_1^w, z_2^w, \ldots, z_n^w; z_1^s, z_2^s, \ldots, z_n^s; z_1^{st}, z_2^{st}, \ldots, z_n^{st}\right),\\
\tilde{p}^z &= (p^z; 0, 0 \ldots, 0; 0, 0 \ldots, 0; 0, 0 \ldots, 0),\\
\tilde{q}^z &= \left(0; \lambda_1^{zw}(av), \ldots, \lambda_n^{zw}(av); \lambda_1^{zs}(av), \ldots, \lambda_n^{zs}(av); \pi_1^z, \ldots, \pi_n^z\right),
\end{aligned}$$

so that $\langle z^g, p^z\rangle = \langle \tilde{z}, \tilde{p}^z\rangle$, where the sense of all the components of the vectors $\tilde{z},\ \tilde{p}^z,\ z^g,\ p^z$, is completely identical to that of the corresponding components of the vectors $\tilde{y},\ \tilde{p}^y,\ y^g,\ p^y$, respectively, and the corresponding components of the vectors from (4) satisfy the system of constraints that is completely identical to system (2) and is assumed to be compatible. Here, $\pi_j^z,\ j \in \overline{1,n}$ have the meaning similar to that of $\pi_i^y,\ i \in \overline{1,m}$.

Finally, let s_j^z be the hourly price that utility company j pays the transmission company for a unit volume of electric energy transmitted to this untility company j during the 24 h period of time, and let $\tilde{\epsilon}^z = (\epsilon^z, \dots, \epsilon^z; 0 \dots, 0)$, where $\tilde{\epsilon}^z$ is the vector of the same dimension as $\tilde{z}$ with all non-zero components equalling 1, where $\epsilon^z \in R_+^{24}$ is the vector with all the components equalling 1.

With the use of this notation, the goal function of utility company $j,\ j \in \overline{1,n}$ can be written as

$$\sum_{l=1}^{24}\left(z_j^g(l)p_j^z(l) + z_j^w(l)\lambda_j^{zw}(av) + z_j^s(l)\lambda_j^{zs}(av) + z_j^g(l)s_j^z + z_j^{st}(l)\pi_j^z\right)$$
$$= \left\langle \tilde{p}^z, \tilde{z}\right\rangle_j + \left\langle \tilde{\epsilon}^z, \tilde{z}\right\rangle_j s_j^z + \left\langle \tilde{q}^z, \tilde{z}\right\rangle_j \to \min_{(\tilde{z})_j}, \quad j \in \overline{1,n}. \tag{5}$$

Groups of advanced customers

The functioning of group of advanced customers $k,\ k \in \overline{1,r}$ can be described by a system of constraints binding the variables and parameters

$$\begin{aligned}
&u_k^w(l),\ u_k^{w-st}(l),\ u_k^{w-dir}(l),\ u_k^s(l),\ u_k^{s-st}(l),\ u_k^{s-dir}(l),\\
&u_k^g(l),\ u_k^{g-dir}(l),\ u_k^{g-st}(l),\ u_k^{st}(l)\\
&u_k^{dem-st}(l),\ u_k^{st}(0),\ \underline{u}_k^{st},\ u_k^{st^+},\ \triangle_k^u,\ \lambda_k^{uw},\ \lambda_k^{us},\\
&\pi_k^u,\ u_k^{dem}(l),\ \overline{u}_k^{dem}(l),\ \underline{u}_k^{dem}(l),\ \underline{u}_k^w(l),\ \underline{u}_k^s(l),\ k \in \overline{1,r}, \quad l \in \overline{1,24},
\end{aligned}$$

that have the same meaning that do the (corresponding to them) variables and parameters that are used for describing the functioning of industrial customer $i,\ i \in \overline{1,m}$. A detailed description of all these variables, along with that of the functioning of the storage facilities that group of advanced customers k uses, is presented in [15].

Let

$$\begin{aligned}
&u_k^g = \left(u_k^g(1), \dots, u_k^g(24)\right);\ u_k^w = \left(u_k^w(1), \dots, u_k^w(24)\right);\ u_k^s = \left(u_k^s(1), \dots, u_k^s(24)\right);\\
&p_k^u = \left(p_k^u(1), \dots, p_k^u(24)\right);\ \lambda_k^{uw}(av) = (\lambda_k^{uw}, \dots, \lambda_k^{uw}),\ \lambda_k^{us}(av) = (\lambda_k^{us}, \dots, \lambda_k^{us});\\
&u_k^{st} = (u_k^{st}(1), \dots, u_k^{st}(24)),\ \lambda_k^{uw}(av), \lambda_k^{us}(av) \in R_+^{24}, \quad k \in \overline{1,r},
\end{aligned} \tag{6}$$

so that $u^g = (u_1^g, u_2^g, \dots, u_r^g),\ p^u = (p_1^u, p_2^u, \dots, p_r^u)$, and let

$$\tilde{u} = \left(u^g; u_1^w, u_2^w, \ldots, u_r^w; u_1^s, u_2^s, \ldots, u_r^s; u_1^{st}, u_2^{st}, \ldots, u_r^{st}\right),$$
$$\tilde{p}^u = (p^u; 0, 0 \ldots, 0; 0, 0 \ldots, 0; 0, 0 \ldots, 0),$$
$$\tilde{q}^u = \left(0; \lambda_1^{uw}(av), \ldots, \lambda_r^{uw}(av); \lambda_1^{us}(av), \ldots, \lambda_r^{us}(av); \pi_1^u, \ldots, \pi_r^u\right),$$

so that $\langle u^g, p^u \rangle = \langle \tilde{u}, \tilde{p}^u \rangle$, where the sense of the components of the vectors $\tilde{u}, \tilde{p}^u, u^g, p^u$ is completely identical to that of the corresponding components of the vectors $\tilde{y}, \tilde{p}^y, y^g, p^y$, respectively, and the corresponding components of the vectors from (6) satisfy the system of constraints that is completely identical to system (2) and is assumed to be compatible. Here, $\pi_k^u, k \in \overline{1, r}$ have the same meaning that do $\pi_i^y, i \in \overline{1, m}$ and $\pi_j^z, j \in \overline{1, n}$.

Finally, let s_k^u be the hourly price that group of advanced customers k pays the transmission company for a unit volume of electric energy transmitted to this group of advanced customers during the 24 h period of time, and let $\tilde{\epsilon}^u = (\epsilon^u, \ldots, \epsilon^u; 0 \ldots, 0)$, where $\tilde{\epsilon}^u$ is the vector of the same dimension as $\tilde{u}$ with all non-zero components equalling 1, where $\epsilon^u \in R_+^{24}$ is the vector with all the components equalling 1.

With the use of this notation, the goal function of group of advanced customers $k, k \in \overline{1, r}$ can be written as follows:

$$\sum_{l=1}^{24} \left(u_k^g(l) p_k^u(l) + u_k^w(l)\lambda_k^{uw}(av) + u_k^s(l)\lambda_k^{us}(av) + u_k^g(l)s_k^u + u_k^{st}(l)\pi_k^u\right)$$
$$= \left\langle \tilde{p}^u, \tilde{u} \right\rangle_k + \left\langle \tilde{\epsilon}^u, \tilde{u} \right\rangle_k s_k^u + \left\langle \tilde{q}^u, \tilde{u} \right\rangle_k \to \min_{(\tilde{u})_k}, \quad k \in \overline{1, r}. \tag{7}$$

The transmission company

Let

θ^y be the vector whose component θ_i^y is the hourly price for a unit volume of electric energy that the transmission company charges the generator for the use of the transmission lines to transmit electricity to industrial customer $i, i \in \overline{1, m}$, and it is assumed that this price remains the same for the generator during 24 h,

θ^z be the vector whose component θ_j^z is the hourly price for a unit volume of electric energy that the transmission company charges the generator for the use of the transmission lines to transmit electricity to utility company $j, j \in \overline{1, n}$, and it is assumed that this price remains the same for this utility company during 24 h,

θ^u be the vector whose component θ_k^u is the hourly price for a unit volume of electric energy that the transmission company charges the generator for the use of the transmission lines to transmit electricity to group of advanced customers $k, k \in \overline{1, r}$, and it is assumed that this price remains the same for this group of advanced customers during 24 h,

θ be the price for a unit volume of the electric energy lost in transmitting electricity to the grid customers via the (high voltage) transmission line that the transmission company charges the generator, and it is assumed that this price is known and remains the same during 24 h,

$\tilde{s}^y$ be the vector whose component s_i^y is the hourly price which industrial customer i as a customer of the grid pays the transmission company for a unit volume of electric energy transmitted to this customer, $i \in \overline{1,m}$, and it is assumed that this price remains the same for this industrial customer during 24 h,

$\tilde{s}^z$ be the vector whose component s_j^z is the hourly price which utility company j as a customer of the grid pays the transmission company for a unit volume of electric energy transmitted to this utility company, $j \in \overline{1,n}$, and it is assumed that this price remains the same for this utility company during 24 h,

$\tilde{s}^u$ be the vector whose component s_k^u is the hourly price which group of advanced customers k as a customer of the grid pays the transmission company for a unit volume of electric energy transmitted to this group of advanced customers, $k \in \overline{1,r}$, and it is assumed that this price remains the same for group of advanced customers k during 24 h.

Taking into account *Assumption 4*, the goal function of the transmission company can be described as a bilinear function of the vector variables $\tilde{y}$, $\tilde{z}$, $\tilde{u}$, and components of the vector variables θ^y, θ^z, θ^u, $\tilde{s}^y$, $\tilde{s}^z$, $\tilde{s}^u$

$$\left(\sum_{i=1}^{m}\langle\tilde{\epsilon}^y,\tilde{y}\rangle_i\theta_i^y+\sum_{j=1}^{n}\langle\tilde{\epsilon}^z,\tilde{z}\rangle_j\theta_j^z+\sum_{k=1}^{r}\langle\tilde{\epsilon}^u,\tilde{u}\rangle_k\theta_k^u\right)$$
$$+\theta\sum_{l=1}^{24}\max_{\lambda_l\in\overline{1,\Lambda_l}}\left(a_{\lambda_l}+b_{\lambda_l}Y^g(l)\right)$$
$$+\left(\sum_{i=1}^{m}\langle\tilde{\epsilon}^y,\tilde{y}\rangle_i s_i^y+\sum_{j=1}^{n}\langle\tilde{\epsilon}^z,\tilde{z}\rangle_j s_j^z+\sum_{k=1}^{r}\langle\tilde{\epsilon}^u,\tilde{u}\rangle_k s_k^u\right)\rightarrow\max_{(\theta^y,\ \theta^z,\ \theta^u,\ \tilde{s}^y,\tilde{s}^z,\tilde{s}^u)}. \tag{8}$$

The generator

Using the notation from the previous section, one can easily conclude that the total revenue of the generator can be described by the bilinear function $\langle p^y, y^g\rangle + \langle p^z, z^g\rangle + \langle p^u, u^g\rangle$ of the vector variables y^g, z^g, u^g and p^y, p^z, p^u. Then the functioning of the generator within 24 h (from the viewpoint of producing and selling electric energy to the grid customers) can be described by the following mathematical model of the balance kind:

$$\langle\epsilon,Y^g\rangle-\left(\langle\epsilon^y,y^g\rangle+\langle\epsilon^z,z^g\rangle+\langle\epsilon^u,u^g\rangle\right)-\langle\epsilon,MAX_{loss}(Y^g)\rangle=0,$$
$$H_{min}\le\langle\epsilon,Y^g\rangle\le H_{max},$$
$$\zeta(y^g,z^g,u^g,p^y,p^z,p^u)=\langle y^g,p^y\rangle+\langle z^g,p^z\rangle+\langle u^g,p^u\rangle-\langle\epsilon,MAX_{expen}(Y^g)\rangle$$
$$-\Psi(Y^g,y^g,z^g,u^g)), \tag{9}$$

where $Y^g = (Y^g(1), \ldots, Y^g(24))$, ϵ, ϵ^y, ϵ^z, ϵ^u are vectors of corresponding dimensions whose all components equal 1, H_{min} and H_{max} are the minimal and the maximal technologically possible production capacities of the generator within 24 h, respectively, and the function in the last equation describes the generator's profit within 24 h.

$$MAX_{loss}(Y^g) = \left(\max_{\lambda_1 \in \overline{1,\Lambda_1}} (a_{\lambda_1} + b_{\lambda_1} Y^g(1)), \ldots, \max_{\lambda_{24} \in \overline{1,\Lambda_{24}}} (a_{\lambda_{24}} + b_{\lambda_{24}} Y^g(24)) \right),$$

$$MAX_{expen}(Y^g) = \left(\max_{\mu_1 \in \overline{1,\Gamma_1}} (c_{\mu_1} + d_{\mu_1} Y^g(1)), \ldots, \max_{\mu_{24} \in \overline{1,\Gamma_{24}}} (c_{\mu_{24}} + d_{\mu_{24}} Y^g(24)) \right),$$

the function $\Psi(Y^g, y^g, z^g, u^g)$ is the function describing the generator expenses associated with transmitting electric energy to the grid customers, which can be written in the form

$$\Psi(Y^g, y^g, z^g, u^g) = \Psi(Y^g, \tilde{y}, \tilde{z}, \tilde{u})) = \theta \sum_{l=1}^{24} \max_{\lambda_l \in \overline{1,\Lambda_l}} \left(a^l_{\lambda_l} + b^l_{\lambda_l} Y^g(l) \right)$$
$$+ \left(\sum_{i=1}^{m} \langle \tilde{\epsilon}^y, \tilde{y} \rangle_i \theta^y_i + \sum_{j=1}^{n} \langle \tilde{\epsilon}^z, \tilde{z} \rangle_j \theta^z_j + \sum_{k=1}^{r} \langle \tilde{\epsilon}^u, \tilde{u} \rangle_k \theta^u_k \right). \quad (10)$$

The goal function of the generator can be written as

$$\zeta(y^g, z^g, u^g, p^y, p^z, p^u) = \sum_{i=1}^{m} \langle \tilde{y}, \tilde{p}^y \rangle_i + \sum_{j=1}^{n} \langle \tilde{z}, \tilde{p}^z \rangle_j + \sum_{k=1}^{r} \langle \tilde{u}, \tilde{p}^u \rangle_k$$
$$- \sum_{l=1}^{24} \max_{\mu_l \in \overline{1,\Gamma_l}} \left(c_{\mu_l} + d_{\mu_l} Y^g(l) \right) - \theta \sum_{l=1}^{24} \max_{\lambda_l \in \overline{1,\Lambda_l}} \left(a_{\lambda_l} + b_{\lambda_l} Y^g(l) \right)$$
$$- \left(\sum_{i=1}^{m} \langle \tilde{\epsilon}^y, \tilde{y} \rangle_i \theta^y_i + \sum_{j=1}^{n} \langle \tilde{\epsilon}^z, \tilde{z} \rangle_j \theta^z_j + \sum_{k=1}^{r} \langle \tilde{\epsilon}^u, \tilde{u} \rangle_k \theta^u_k \right) \to \max_{(\tilde{p}^y, \tilde{p}^z, \tilde{p}^u)}, \quad (11)$$

where the inequalities for the prices $p^y \leq \hat{p}^y,\ p^z \leq \hat{p}^z,\ p^u \leq \hat{p}^u$, in which the vectors $\hat{p}^y, \hat{p}^z, \hat{p}^u$ reflect the current status of the electricity market, hold.

The game model of the interaction of the grid elements

Let us now consider the interaction of the generator with all the above large grid customers that receive electric energy from the generator, as well as from the available renewable sources of energy (directly and via electricity storing facilities), and with the transmission company. This interaction can be viewed as a $(m+n+r+2)$-person game with the payoff functions described by (3), (5), (7), (8), and (11) [15]

$$\sum_{i=1}^{m} \langle \tilde{p}^y, \tilde{y} \rangle_i + \sum_{j=1}^{n} \langle \tilde{p}^z, \tilde{z} \rangle_j + \sum_{k=1}^{r} \langle \tilde{p}^u, \tilde{u} \rangle_k - \theta \sum_{l=1}^{24} \max_{\lambda_l \in \overline{1,\Lambda_l}} \left(a_{\lambda_l} + b_{\lambda_l} Y^g(l) \right)$$
$$- \sum_{l=1}^{24} \max_{\mu_l \in \overline{1,\Gamma_l}} \left(c_{\mu_l} + d_{\mu_l} Y^g(l) \right)$$
$$- \left(\sum_{i=1}^{m} \langle \tilde{\epsilon}^y, \tilde{y} \rangle_i \theta^y_i + \sum_{j=1}^{n} \langle \tilde{\epsilon}^z, \tilde{z} \rangle_j \theta^z_j + \sum_{k=1}^{r} \langle \tilde{\epsilon}^u, \tilde{u} \rangle_k \theta^u_k \right) \to \max_{(\tilde{p}^y, \tilde{p}^z, \tilde{p}^u)},$$

$$\left(\sum_{i=1}^{m}\langle\tilde{\epsilon}^{y},\tilde{y}\rangle_{i}\theta_{i}^{y}+\sum_{j=1}^{n}\langle\tilde{\epsilon}^{z},\tilde{z}\rangle_{j}\theta_{j}^{z}+\sum_{k=1}^{r}\langle\tilde{\epsilon}^{u},\tilde{u}\rangle_{k}\theta_{k}^{u}\right)+\theta\sum_{l=1}^{24}\max_{\lambda_l\in\overline{1,\Lambda_l}}\left(a_{\lambda_l}+b_{\lambda_l}Y^{g}(l)\right)$$
$$+\left(\sum_{i=1}^{m}\langle\tilde{\epsilon}^{y},\tilde{y}\rangle_{i}s_{i}^{y}+\sum_{j=1}^{n}\langle\tilde{\epsilon}^{z},\tilde{z}\rangle_{j}s_{j}^{z}+\sum_{k=1}^{r}\langle\tilde{\epsilon}^{u},\tilde{u}\rangle_{k}s_{k}^{u}\right)\to\max_{(\theta^{y},\,\theta^{z},\,\theta^{u},\,\tilde{s}^{y},\tilde{s}^{z},\tilde{s}^{u})},$$
$$\langle\tilde{p}^{y},\tilde{y}\rangle_{i}+\langle\tilde{\epsilon}^{y},\tilde{y}\rangle_{i}\,\tilde{s}_{i}^{y}+\langle\tilde{q}^{y},\tilde{y}\rangle_{i}\to\min_{(\tilde{y})_i},\ i\in\overline{1,m},$$
$$\langle\tilde{p}^{z},\tilde{z}\rangle_{j}+\langle\tilde{\epsilon}^{z},\tilde{z}\rangle_{j}\,\tilde{s}_{j}^{z}+\langle\tilde{q}^{z},\tilde{z}\rangle_{j}\to\min_{(\tilde{z})_j},\ j\in\overline{1,n},$$
$$\langle\tilde{p}^{u},\tilde{u}\rangle_{k}+\langle\tilde{\epsilon}^{u},\tilde{u}\rangle_{k}\,\tilde{s}_{k}^{u}+\langle\tilde{q}^{u},\tilde{u}\rangle_{k}\to\min_{(\tilde{u})_k},\ k\in\overline{1,r},$$
$$(\tilde{y},\ \tilde{z},\ \tilde{u},\ Y^{g})\in\Omega,\ (\tilde{p}^{y},\ \tilde{p}^{z},\ \tilde{p}^{u})\in M,\ (\theta_1^{y},\ldots,\theta_m^{y},\ \theta_1^{z},\ldots,\theta_n^{z},\ \theta_1^{u},\ldots,\theta_r^{u})\in T,$$
$$(s_1^{y},\ldots,s_m^{y},\ s_1^{z},\ldots,s_n^{z},\ s_1^{u},\ldots,s_r^{u})\in S, \tag{12}$$

where $\Omega,\ M,\ T,\ S$ are polyhedra formed by systems of linear constraints, including (1), (2), as well as by systems analogous to (2) for the variables $\tilde{z}$ and $\tilde{u}$ (which are presented in [15]). As before, here $\tilde{\epsilon}^{y}=\left(\epsilon^{y},\ldots,\epsilon^{y};0,\ldots,0\right)$, $\tilde{\epsilon}^{z}=\left(\epsilon^{z},\ldots,\epsilon^{z};0,\ldots,0\right)$, $\tilde{\epsilon}^{u}=\left(\epsilon^{u},\ldots,\epsilon^{u};0,\ldots,0\right)$ are vectors whose all non-zero components equal 1, and $\langle\tilde{\epsilon}^{y},\tilde{y}\rangle_{i}$, $\langle\tilde{\epsilon}^{z},\tilde{z}\rangle_{j}$, $\langle\tilde{\epsilon}^{u},\tilde{u}\rangle_{k}$ are parts of the scalar products $\langle\tilde{\epsilon}^{y},\tilde{y}\rangle$, $\langle\tilde{\epsilon}^{z},\tilde{z}\rangle$, $\langle\tilde{\epsilon}^{u},\tilde{u}\rangle$, respectively, relating to industrial customer i, utility company j, and group of advanced customers k, $i\in\overline{1,m},\ j\in\overline{1,n},\ k\in\overline{1,r}$, respectively, whereas, as before, ϵ^{y}, ϵ^{z}, and ϵ^{u} are vectors of corresponding dimensions with all the components equal 1, whereas $\tilde{\epsilon}^{y}$, $\tilde{\epsilon}^{z}$, $\tilde{\epsilon}^{u}$ have the same dimensions as do the vectors $\tilde{y},\ \tilde{z},\ \tilde{u}$, respectively.

Though finding equilibria in this game by known game theory methods presents considerable difficulties, it turns out that this game possesses certain features allowing one to develop effective methods for finding these equilibria (in solvable games) [15].

Components of an equilibrium point of (solvable) game (12) determine a) (corresponding to this equilibrium point) the optimal hourly volumes of electricity to be bought by each of the large customers of the grid (i.e., by industrial customers, utility companies, and groups of advanced customers) and the optimal hourly production volume of electricity for the generator, b) the optimal hourly volumes of electricity to be received by each of the large customers of the grid from the suppliers (i.e., from the electricity producers who transform wind and solar energy into electric energy), c) the optimal hourly volumes of electricity to be stored by each large customer of the grid, d) the optimal hourly prices for electricity to be paid by the large customers to the generator, as well as those to be paid to the suppliers, within 24 h, and e) the optimal hourly prices to be paid to the transmission company both by the generator and by each large customer. Since the (solvable) game may have more than one equilibrium point, one should bear in mind that, generally, there could be more than one optimal set of the above volumes and prices to choose from (each set corresponding to each equilibrium point).

Remark 4 Game (12), describing the interaction of all the grid customers, the generator, and the transmission company, does not, generally, reflect the fact that the prices at which the generator charges the customers for the electricity supplied to them depend on the prices at which the transmission company charges the generator for transmitting electricity to the grid customers. Indeed, the model does not take into consideration the fact that if the above transmission prices for the generator are high, the generator may need to increase the electricity prices for the customers. This increase, in turn, may affect the volumes of electricity that the customers may decide to purchase from the generator, as well as both the revenue of the transmission company (if the customers decide the decrease the volumes of electricity purchased from the generator) and its prices for transmitting electricity to the customers. However, the model to a certain extent acceptably covers the following two important cases of the above interaction: a) The case in which the transmission company charges only the customers for the electricity transmitted (to them), which means that the equalities $\theta^y = 0, \theta^z = 0, \theta^u = 0$ hold, and b) the case in which the polyhedra S and T are described by the same system of linear inequalities. In the latter case, one should bear in mind that by solving Game 12, the generator tries to estimate parameters of its potential contractual agreements with the grid customers. By doing so the generator may assume that the customers would agree that the prices for transmitting electricity to them from the generator, which the generator pays to the transmission company, and the ones at which the customers are charged by the transmission company for the electricity received may differ as long as they are chosen from the same set of these prices (being the polyhedron $S = T$) [40].

To reflect the above dependence between the prices described by the vectors $\tilde{p}^y, \tilde{p}^z, \tilde{p}^u$ and $\theta^y, \theta^z, \theta^u$, one should study games with a more complicated structure of the payoff function of the generator than that in Game 12. More complicated games should also be studied if one takes into consideration that expenses of the transmission company for transmitting electricity from the generator to the grid customers may also depend on the volume of the electricity transmitted, as well as on its services rendered to both each particular large grid customer and customers other than large grid customers. However, studying such games requires an appropriate justification (besides simple curiosity), and, in any case, the consideration of these games lies beyond the scope of the present paper.

4 Concluding Remarks

1. The proposed mathematical model of interacting the generator with the transmission company, and with the three groups of (large) grid customers allows one to describe a hypothetical case of supply-demand relations between all the interacting participants of the grid that assumes that both the supply and the demand can change every hour. However, the proposed model allows one to calculate an optimal schedule of the electricity supply in the framework of which a) the generator produces the same amount of electricity hourly (though not necessarily the same amount for different base load power plants comprising the generator if it consists of more than one base load power plant), i.e., when all the base load power plants that comprise the generator produce the same amount of electricity hourly combined, and b) all the electricity sold by the generator is consumed by the customers (directly or via their electricity storages). To this end, one should include the equalities

$$Y^g(l) = Y^g(l+1), \quad l \in \overline{1, 23} \tag{13}$$

in the description of the set Ω (which is assumed to be compatible in solving game (12), including the case in which the system of constraints (13) is included in the description of the set Ω).

Here, all the base load power plants comprising the generator produce the same (though, generally, different for different base load plants) volume of electricity combined hourly, whereas the peaking power plants comprising the peaking power facility take care of the excess of the hourly customer demand over the hourly supply that is provided by the base load power plants. Also, one can see that this (equilibrium) hourly supply from the base load power plants (calculated as a result of solving game (12)) is balanced by the storage facilities available to the customers so that if this supply exceeds any current customer demand, electricity produced by these plants may go to these storage facilities.

As mentioned in Remark 2, for each generating facility ξ, $\xi \in \overline{1, \Psi}$ that is part of the generator, a production schedule corresponding to an optimal supply of electricity by the generator as a whole can be found by modifying the system of constraints describing the set Ω. This is possible since a part of coordinates of every vector from Ω forms a vector whose coordinates reflect the production strategies of the generator.

Let

$Y^{g\xi}(l)$ be the volume of electricity produced by facility ξ (which is a base load power plant) as part of the generator, $\xi \in \overline{1, \Psi}$, in the period of time from hour $l-1$ to hour l, $l \in \overline{1, 24}$,

H^{ξ}_{min} be the technologically possible minimal volume of electricity production by base load power plant ξ, $\xi \in \overline{1, \Psi}$,

H^{ξ}_{max} be the technologically possible maximal volume of electricity production by base load power plant ξ, $\xi \in \overline{1, \Psi}$.

Then the following constraints should be added to system (9):

$$\begin{aligned}
&Y^{g\xi}(l-1) = Y^{g\xi}(l),\ \xi \in \overline{1, \Psi}, \quad l \in \overline{1, 24},\\
&H^{\xi}_{min} \le Y^{g\xi}(l) \le H^{\xi}_{max},\ \xi \in \overline{1, \Psi}, \quad l \in \overline{1, 24}\\
&\sum_{\xi=1}^{\Psi} Y^{g\xi}(l) = Y^g(l), \quad l \in \overline{1, 24},\\
&\sum_{\xi=1}^{\Psi} H^{\xi}_{min} = H_{min}, \sum_{\xi=1}^{\Psi} H^{\xi}_{max} = H_{max},
\end{aligned}$$

2. Based upon the optimal volumes and prices for electricity for each utility company from all the sources of electricity available in the grid, each utility company can then easily determine the prices for the households that this company supplies with electricity. One should, however, bear in mind that each household and each group of households may use individual devices (systems) to transform solar and wind energy into electricity, as well as individual devices (systems) for storing electricity. This may

substantially affect the volumes of electricity that the corresponding utility company should buy from both the generator and the suppliers [15].

3. The structure of the goal function of the transmission company depends on its contractual agreements with the generator, industrial customers, utility companies, and groups of advanced customers. Generally, the transmission company can charge both the generator and each large customer of the grid under consideration that uses high voltage lines to receive electricity from the generator. In this paper, the goal function of the transmission company reflects this particular case, i.e., the transmission company charges both the generator and all the customers of the grid that use the transmission lines though at different rates, which may depend on the transmission distance.

4. One should bear in mind that in Game (12), the use of renewable sources of energy and storage systems is considered a possible strategy of each (large) customer of the grid. However, proceeding from its expenses associated with producing electricity (described earlier in this paper), the generator may develop (or purchase) its own electricity storage facility (or even both an electricity storage facility and systems for transforming wind and solar energy into electricity) or at least may explore the expediency of having them in use. Both options may be explored in the framework of the proposed model, describing the interaction of all the grid elements, as long as the generator properly forms the system of constraints describing the vectors $\tilde{p}^y$, $\tilde{p}^z$, and $\tilde{p}^u$. (It is easy to modify game (12) to take into account the possibility of incorporating in the model, for instance, a description of the functioning of a hydro-electricity storage facility that the generator may be interested in using.)

The conventional wisdom suggests that, currently, storing electricity in large volumes by the grid customers is not economical, and this viewpoint substantially affects the way the electrical grid operates [43–45]. However, it is unclear why electricity should necessarily be stored in large volumes in particular storages while the same volumes of electric energy could be stored in a set of distributed, relatively small storages like accumulator batteries. These batteries can collectively be used by a certain number of neighboring households, which may turn out to be economical even if the use of the same small storages by individual households under the current prices of the available storage devices is not. So quantitatively evaluating how much of electric energy (if any) would be reasonable to store by the households (when the electricity offered by the generator is relatively cheap) and then to release (during the hours when it is expensive) makes sense under various possible schemes of using relatively small electricity storage facilities [46] and under both the existing and any expected prices for the electricity storage systems and devices. Certainly, the effectiveness of each type of electricity storing systems much depends on the volume of electricity that is lost there as a result of storing electricity, i.e., on the coefficient of the storing system performance, which should be analyzed for each type of electricity storages in adopting decisions on their deployment.

Finally, in this paper, the functioning of the storage facilities that the (large) grid customers can use was described by a simplified mathematical model, corresponding mostly to large storage facilities, such as, for instance, water reservoirs (in which the potential energy of the water pumped into the reservoir can be transformed into electric energy). However, the physics of the functioning of, say, chemical batteries suggests that this model can be viewed as an acceptable first approximation of the description of the functioning for other storage systems. The reasons for choosing this simplified model were a) to bring into step the mathematical description of the storage facilities and the relations of the balance kind that are used in the other parts of the model, and b) to be able to mathematically formulate the problem of finding optimal (equilibrium) strategies of both the producers of electricity and the (large) grid customers that interact within the grid as a problem from a class of large-scale ones that can effectively be solved. As mentioned earlier and shown in [47], with the use of this model, (solvable) games describing the above interaction can be solved on the basis of linear programming techniques, which have high computational potential.

5. One can easily be certain that by introducing additional variables and constraints in the proposed game model, one can extend this model to cover the option of buying electricity by the large grid customers in electricity markets and delivering it to the customers either via the same transmission company or via a different transmission company. However, other game-theoretic problems solvable by different optimizations techniques may be formulated based on other models describing the interaction of the large grid customers with both the electricity producers and the electricity market [23,48,49].

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