Streamwise Vorticity Enhanced Compressible Mixing Downstream of Lobed Mixers

by

David Early Tew

B.S. Aerospace Engineering, University of Michigan (1990)

S.M. Aeronautics and Astronautics, Massachusetts Institute of Technology (1992)

Submitted to the Department of Aeronautics and Astronautics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 1997

© Massachusetts Institute of Technology 1997. All rights reserved.

Author
Department of Aeronautics and Astronautics
October 30, 1996

Certified by
Professor Ian A. Waitz
Assistant Professor of Aeronautics and Astronautics
Thesis Supervisor

Certified by
Professor Edward M. Greitzer
H. N. Slater Professor of Aeronautics and Astronautics

Certified by
Dr. Choon S. Tan
Principal Research Engineer

Accepted by
Professor Jaime Peraire
Chairman, Department Graduate Committee

MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

FEB 10 1997

LIBRARIES

LIBRARIES

Streamwise Vorticity Enhanced Compressible Mixing Downstream of Lobed Mixers

by

David Early Tew

Submitted to the Department of Aeronautics and Astronautics
on October 30, 1996, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Abstract

An efficient and flexible mixer-ejector performance model was developed from a physical understanding of the streamwise vorticity enhanced compressible mixing process obtained in a series of experiments.

In these experiments, forced mixer performance was shown to be sensitive to the trailing edge static pressure ratio. If the static pressures were unmatched at the trailing edge, trough separations sometimes occurred; the separation both reduced the streamwise circulation shed by the mixer and reduced the effective interface area between the two streams. Streamwise vorticity was also shown to be effective in augmenting the mixing rate downstream of lobed mixers in compressible regimes. Compressibility had three effects on the streamwise vorticity enhanced mixing process. First, the reduced shear layer growth rates associated with high convective Mach number regimes give the streamwise vorticity more time to stretch the interface between the two streams, and hence, the vorticity was more effective in augmenting the mixing rate in compressible regimes than in incompressible ones. Second, unmatched static pressures at the trailing edge resulted in a displacement of the vortex sheets downstream of the trailing edge; this displacement has a subsequent impact on the development of the cross-flow interface. Third, vorticity was generated downstream of the mixer trailing edge by the interaction of trailing edge shocks and density gradients across the mixing layer. Depending on the density ratio between the streams, this vorticity generation may either enhance or reduce the mixing rate.

From the physical understanding of the mixing process obtained in these experiments, the a mixer-ejector model was developed to predict pumping and thrust performance given mixer and duct geometry, the secondary and primary nozzle pressure ratios, and the total temperature ratio between the streams. The model is efficient and flexible; it is capable of computing the flow field 100 times faster than three-dimensional Navier-Stokes codes. This speed and flexibility enabled design trade and optimization studies to be conducted.

Using this model with the experimental duct and mixer geometries, quantitative ejector pumping and thrust performance predictions were made. For square lobe mixers with a height to wavelength ratio of 1.25, a 10% pumping augmentation and a 1% thrust augmentation was associated with the streamwise circulation shed by the 15° forced mixer at flow conditions similar to those anticipated for application in noise suppressor nozzles for supersonic transport aircraft. However, for square lobe mixers with a height to wavelength ratio of 2.75, no pumping or thrust augmentation was associated with streamwise circulation.

Thesis Supervisor: Professor Ian A. Waitz

Title: Assistant Professor of Aeronautics and Astronautics

Acknowledgments

The completion of this thesis would not have been possible without the guidance and support of a number of people. Foremost, I would like to thank Prof. **Ian Waitz** for his advice and friendship. I would also like to thank Prof. **Edward Greitzer** for keeping me focused on the bottom-line, and Dr. **Choon Tan** for his reminders to keep the big picture in mind. Additionally, I would like to thank Prof. **Frank Marble** for his advice and encouragement.

I would also like to thank Dr. **James Hermanson**, Mr. **Frank Zumpano**, Dr. **Roy Guile**, and Mr. **James MacLeod** at United Technologies Research Center for their efforts. Without their assistance, this project would not have been possible.

Additionally, I would also like to thank my colleagues in mixer-ejector research for their help and friendship—**Brian Teeple**, **Greg Stanislaw**, and **John Kerwin**.

I would also like to thank my officemates—**Luc Fréchette**, **Yang-Sheng Tzeng**, **Fouzi Al-Essa**, **Greg Stanislaw**, and **John Kerwin**—for letting me take over no fewer than two desks and the office terminal for months on end as my thesis was nearing completion and for putting up with the coffee grinder. (A special thanks to **Yang** for spending my penny collection.)

I would also like to thank my many friends and intramural teammates—too numerous to mention—in the Gas Turbine Lab and Aero-Department for their friendship.

Last but not least, I would like to thank **Patia** and my family for their encouragement and support. This thesis is dedicated to them.

This research was supported by NASA Grant NAG-1-1511 with Dr. **Jack M. Seiner** as the technical monitor and by Pratt and Whitney with Dr. **Edwin J. Kawecki** as the technical monitor. This support is gratefully acknowledged.

Contents

Nomenclature	8
1 Introduction	11
1.1 Motivation	11
1.2 Background	12
1.2.1 Lobed mixers	13
1.2.2 Shear layers	15
1.2.3 HSR mixer-ejectors	16
1.3 Objective	17
1.4 Approach	17
1.5 Contributions	18
1.6 Overview of thesis	18
2 Description of Experiments	20
2.1 Overall Approach	20
2.2 Facility	21
2.2.1 Nozzle designs	24
2.2.2 Mixer designs	24
2.2.3 Mixing duct	28
2.3 Diagnostic techniques	30
2.3.1 Mie scattering	30
2.3.2 Pitot-static pressure surveys	31
2.3.3 Wall static pressure measurements	33
2.3.4 Schlieren photographs	34
2.3.5 Fluorescent dye surface flow visualization	34
2.4 Performance Metrics	36
2.4.1 Mixing Performance Metrics	36
2.4.2 Ejector Performance Metrics	37

2.5	Chapter Summary	37
3	Lobed Mixer Performance in Compressible Mixing Layers	39
3.1	Qualitative Description of Compressible Mixer Flows	39
3.2	Convoluted plate performance in compressible flow	44
3.3	Forced mixer performance in compressible flow	47
3.4	Implications of mixer separation	52
3.5	Chapter Summary	56
4	Impact of Streamwise Vorticity in Compressible Mixing Layers	70
4.1	Planar Shear Layers	70
4.2	Convoluted Shear Layers	74
4.3	Streamwise vorticity enhanced mixing layers	81
4.3.1	Mixing augmentation associated with streamwise vorticity	81
4.3.2	Identification of the mixing mechanism associated with streamwise vorticity	83
4.3.3	Vortex mixing model	89
4.3.4	Impact of compressibility on mixing augmentation associated with Γ^*	98
4.4	Chapter Summary	110
5	Development of a Streamwise Vorticity Enhanced Mixer-Ejector Model	111
5.1	Summary	111
5.2	Lobed mixer model	112
5.3	Static pressure matching control volume	115
5.4	Vortex interface model	118
5.5	Differential control volume model	119
5.6	Integrated model/data comparisons	122
5.6.1	Pitot mixedness comparisons	123
5.6.2	Wall static pressure comparisons	123
5.7	Chapter Summary	132
6	Quantitative Assessment of the Impact of Streamwise Vorticity on Supersonic/Subsonic Mixer-Ejector Performance	135
6.1	Impact of streamwise vorticity on performance	135
6.2	Impact of losses on augmented ejector performance	136
6.2.1	Impact of mixer drag	136
6.2.2	Impact of duct skin friction	138
6.3	Impact of duct length on augmented ejector performance	139
6.4	Impact of primary Mach number	142

6.5	Optimal operating point for maximum thrust, maximum pumping	142
6.6	Chapter summary	144
7	Optimal Design of Lobed Mixers for Maximum Ejector Pumping and Thrust	146
7.1	Impact of lobe angle and lobe height on mixing rate	146
7.2	Estimation of lobed mixer drag	147
7.3	Optimal lobe design for maximum ejector pumping and thrust	151
7.4	Chapter summary	157
8	Summary, Conclusions, and Recommendations	158
8.1	Summary	158
8.2	Conclusions	159
8.3	Recommendations for Future Work	160
A	Experimental Test Matrix	162
A.1	Measurement locations	162
A.1.1	Static pressure data	162
A.1.2	Pitot pressure data	162
A.2	Experimental Test Conditions	163
A.2.1	Subsonic-subsonic data	165
A.2.2	$M_p = 1.3$ nozzle data	168
A.2.3	$M_p = 1.6$ nozzle data	176
A.2.4	$M_p = 2.4$ nozzle data	185
B	Mie diagnostic system	191
B.1	Description of technique	191
B.2	Description of image correction procedures	192
B.2.1	Background and Response Transformations	192
B.2.2	Perspective Transformatation	192
B.2.3	Scaling transformation	195
C	Pressure measurement: diagnostic system and data analysis	197
C.1	Instrumentation	197
C.2	Wall static pressure measurements	197
C.2.1	Calculated quantities	198
C.2.2	Error Analysis	198
C.3	Pitot/static pressure surveys	199
C.3.1	Calculated quantities	202

C.3.2	Error analysis	203
D	Derivation of Vortex Model	204
D.1	Description of model	204
D.2	Inputs/Outputs	205
D.3	Formulation of vortex model	205
D.4	Derivation of model	209
D.5	Modelling of lobed mixer flows	214
D.5.1	Specification of initial vorticity distribution	214
D.5.2	Boundary conditions	215
D.5.3	Compressibility effects	216
D.6	Calculation of effective interface length	218
D.7	Sensitivity studies	223
D.7.1	Sensitivity of M_{sc} and L_{eff} to Γ^*	223
D.7.2	Sensitivity of M_{sc} and L_{eff} to δ'_{vel}	223
D.7.3	Sensitivity of M_{sc} and L_{eff} to ϵ^*	225
E	Derivation of static pressure matching control volume	229
E.1	Description of Model	229
E.1.1	Input/Output	229
E.1.2	Assumptions	230
E.2	Derivation of model	230
E.3	Sensitivity Studies	231
F	Derivation of baroclinic torque circulation model	237
G	Derivation of differential control volume	239
G.1	Description of model	239
G.2	Inputs/Outputs	239
G.3	Derivation of model	240
G.4	Sensitivity studies	241
G.4.1	Sensitivity of the mixing rate to L_{eff}	241
G.4.2	Sensitivity of the axial distributions of the mixing measures to Pr	241
G.4.3	Sensitivity of the fully-mixed conditions and axial distributions of the mixing measures to M_p and M_s	243
H	Model/data static pressure comparisons	250
H.1	Flat plate	250

H.2	Convoluted plate	250
H.3	15° forced mixer	250
H.4	25° forced mixer	255

List of Figures

1-1	Conceptual HSCT mixer-ejector noise suppression system [28].	11
1-2	Typical lobed mixer[33].	12
1-3	Water tunnel measurements of molecular mixedness downstream of a flat plate, convoluted plate, and forced mixer for a velocity ratio of 0.67[19].	14
1-4	Incompressible/compressible interface length comparison[11, 33].	15
1-5	Compressible($M_c > 0$) planar shear layer growth rate normalized by the equivalent incompressible growth rate($M_c = 0$) versus M_c [22, 10].	17
2-1	Planar shear layer operating map. M_c , r , and s contours are drawn as functions of the two inflow Mach numbers for equal total temperature matched static pressure streams.	22
2-2	Contours of $\log_{10}(\frac{\frac{\partial \delta'}{\partial x}}{\frac{\partial \delta'}{\partial M_c}})$	22
2-3	Supersonic Shear Layer Facility in the Gas Dynamics Lab at the United Technologies Research Center.	23
2-4	Schematic of the Supersonic Shear Layer Facility with the $M_p = 2.4$ nozzle and the 15° forced mixer as it was configured for the supersonic-subsonic experiments.	25
2-5	Schematic of the lobed mixers' trailing edge in the UTRC Compressible Mixing Facility. All three lobed mixers have the same trailing edge shape. Specifications: $\lambda = 2.0$ cm, $\frac{h}{\lambda} = 1.25$, corner radius/ $\lambda = 0.125$	26
2-6	Convoluted plate.	26
2-7	15° forced mixer.	27
2-8	25° forced mixer.	27
2-9	Axial forced mixer and convoluted plate circulation distribution[21].	29
2-10	Instantaneous image, mean image, and passive scalar field 2λ downstream of the 15° forced mixer with $M_1 = 2.4$, $M_2 = 1.5$, $M_c = 0.30$, and $r = 0.80$	31
2-11	Measure Normalized Pitot pressure distributions 3, 6, 10, and 21 wavelengths downstream of the 25° forced mixer with $M_p = 1.65$, $M_s = 0.14$, and $\frac{P_t}{P_p} = 0.85$	32
2-12	Schlieren photograph of flow over the 15° forced mixer with $M_p = 3.2$. Flow is from right to left.	35

2-13	Surface streamline patterns on the 15° forced mixer at $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$. Note that the flow on the vertical mixer walls leaves the mixer at the lobe angle.	35
2-14	Separated wall streamlines downstream of the 15° forced mixer at $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$	36
2-15	Four mixing measures vs. normalized distance from the trailing edge of the flat plate for $M_p = 1.6$, $M_s = 0.3$, $\frac{p_s}{p_p} = 1$, $Sc = 1$, and $Pr = 1$	38
2-16	Normalized mixing measure versus pumping for $NPR = 4.0$, $SNPR = 1.0$, $\frac{Tt_s}{Tt_p} = 1.0$, $M_p = 1.6$ and, $\frac{A_s}{A_p} = 1.0$	38
3-1	Schlieren photographs of mixer flows over each of the four tested mixers. $M_p = 1.6$ for the flat plate, convoluted plate and 25° forced mixer. $M_p = 3.2$ for the 15° forced mixer. The flow direction is from right to left.	41
3-2	Schematics highlighting the key features of supersonic lobed mixer flows	42
3-3	Normalized Pitot pressure contours 1λ downstream of the 15° forced mixer. The supersonic stream is on top, and the subsonic is on the bottom.	43
3-4	Surface streamline patterns on the 15° and 25° forced mixers at $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_1} = 1.0$ Note the separation in the supersonic trough on the 25° mixer.	45
3-5	Incompressible convoluted plate trough boundary layer and freestream streamline	46
3-6	Convoluted plate surface streamlines. $M_p = 1.6$ and $M_s = 0.3$. The supersonic side of the mixer is on top, and the subsonic is on the bottom.	46
3-7	Supersonic convoluted plate trough boundary layer and trough flow streamlines	46
3-8	Regions of attached and separated converging-diverging nozzle flow per Zukoski's nozzle separation criteria[15].	48
3-9	15° forced mixer performance map. Cases with separated trough flow are marked with x 's, and cases with attached flow are marked with o 's.	50
3-10	25° forced mixer performance map. Cases with separated trough flow are marked with x 's, and cases with attached flow are marked with o 's.	51
3-11	Normalized Pitot pressure fields at the first survey location downstream of each of the lobed mixers with $M_p = 1.6$ and $M_s \approx 0.2$. (Survey locations: $x_{CP}^* = 1.2$, $x_{FM1}^* = 1.7$, $x_{FM2}^* = 2.9$.)	53
3-12	Interface length at first survey plane versus the inflow secondary/primary static pressure ratio— $M_t = 1.6$, $M_s \approx 0.2$	54
3-13	Momentum mixing parameter versus $\frac{p_s}{p_p}$ for $M_p = 1.6$ and $M_s \approx 0.2$	55
4-1	Composite Schlieren photograph of a planar shear layer with $M_p = 1.6$ and $M_s = 0.7$, $\frac{p_s}{p_p} = 1$. The flow is from right to left.	72

4-2	Instantaneous Mie images of a planar shear layer at two different times with $M_p = 1.6$ and $M_s = 0.3$	73
4-3	Mean Mie images of a planar shear layer with $M_p = 1.6$ and $M_s = 0.3$	74
4-4	Methanol droplet concentration profiles in a planar shear layer with $M_p = 1.6$ and $M_s = 0.3$	75
4-5	Compressible($M_c > 0$) planar shear layer growth rates normalized by their equivalent incompressible growth rate($M_c = 0$) versus M_c as measured in the current experiments and by Papamoschou and Roshko[22]. Dimotakis'[10] fit to Papamoschou's data is shown as well as a best line fit to the current data.	76
4-6	Instantaneous Mie images of the mixing layer downstream of the convoluted plate mixer with $M_p = 1.6$ and $M_s = 0.3$	78
4-7	Instantaneous cross-flow Mie image 2λ from the trailing edge of the convoluted plate with $M_p = 1.6$ and $M_s = 0.3$	79
4-8	Mean Mie images 1, 2, and 3 wavelengths downstream of the convoluted plate with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$	79
4-9	Normalized Pitot pressure fields at two axial locations downstream of the flat and convoluted plates with $M_p = 1.6$, $M_s = 0.2$, and $\frac{p_s}{p_p} = 0.9$	80
4-10	Mean interface length versus x^* for each of the mixers with $M_p = 1.6$, $M_s = 0.2$, $\frac{p_s}{p_p} = 0.9$	82
4-11	Pitot mixedness versus x^* for each of the mixers with $M_p = 1.6$, $M_s = 0.2$, $\frac{p_s}{p_p} = 0.9$	82
4-12	15° forced mixer/convoluted plate Pitot mixedness comparison at 20λ from the trailing edges.	83
4-13	25° forced mixer/15° forced mixer Pitot mixedness comparison at 20λ wavelengths from the trailing edges.	84
4-14	Instantaneous spanwise Mie images downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.3$	85
4-15	Instantaneous Mie images 2 wavelengths downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$	86
4-16	Mean Mie images 1, 2, and 3 wavelengths downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$	87
4-17	Mean Mie images 2 wavelengths downstream of the three lobed mixers with $M_p = 2.4$, $M_s = 1.6$, and $\frac{p_s}{p_p} = 1.0$	87
4-18	Normalized Pitot pressure fields at two axial locations downstream of the 15° and 25° forced mixers with $M_p = 1.6$, $M_s = 0.2$, and $\frac{p_s}{p_p} = 0.9$	88
4-19	Vortex paths from 0 to 2λ and scalar field at 2λ for $\Gamma^* = 0.5$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$	91

4-20 Modelled axial scalar mixedness distribution downstream of all 4 mixers with $\delta'_{ve} = 0.1$ and $\epsilon^* = 0.25$.	92
4-21 Vortex model and Mie data cross-flow interface structure comparison. Cross-flow planes are 1λ , 2λ , and 3λ downstream of the 15° mixer with $M_1 = 2.4$ and $M_2 = 1.6$. $M_c = 0.22$, $r = 0.80$, $s = 0.70$. Model parameters: $\Gamma^* = 0.5$, $\delta'_{vel} = 0.02$, and $\epsilon^* = 0.05$.	93
4-22 Vortex model and Mie data cross-flow interface structure comparison. Cross-flow planes are 1λ , 2λ , and 3λ downstream of the 15° mixer with $M_1 = 1.6$ and $M_2 = 0.3$. $M_c = 0.56$, $r = 0.21$, $s = 0.67$. Model parameters: $\Gamma^* = 0.5$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.	94
4-23 Convoluted plate model/Mie mean interface length comparison for $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 0.9$.	96
4-24 15° forced mixer model/Mie mean interface length comparison for $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 0.9$.	96
4-25 25° forced mixer model/Mie mean interface length comparison for $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 0.9$.	97
4-26 Normalized mass-averaged total pressures 20λ downstream of the trailing edge of the flat plate, convoluted plate and 15° forced mixer at $M_c = 0.58$, $r = 0.20$ and $M_c = 0.43$, $r = 0.25$.	100
4-27 Scaling mixing augmentation associated with the streamwise circulation shed by the 15° forced mixer versus the convective Mach number and distance from the trailing edge for a velocity ratio of 0.2	101
4-28 Interface deflection due to unmatched primary/secondary static pressures at the trailing edge of the mixer	101
4-29 Matched pressure trough area ratio versus trailing edge static pressure ratio for $M_p = 1.6$ and $M_s = 0.3$.	102
4-30 Fully mixed scalar fields 2λ from the trailing edge with $\Gamma^* = 0.5$; $\delta'_{vel} = 0.10$; $\epsilon = 0.25$; and secondary/primary trough width ratios of 0.80, 1.00, 1.25.	103
4-31 Scalar mixedness at 20λ versus $\frac{w_s}{w_p}$. $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.	104
4-32 Schlieren photograph of 15° forced mixer with $M_p = 1.6$ and $M_s = 0.7$ with an oblique shock at the trailing edge of the primary trough.	105
4-33 Cartoon illustrating generation of streamwise vorticity by the baroclinic torque associated with the interaction of the trough shock and a density gradient across the mixing layer. Note that in this schematic the secondary/primary density ratio is less than 1.	105
4-34 Schematic illustrating vorticity generation associated with a shock interacting with a density gradient	105

4-35 Shock/mixer circulation ratio vs. secondary/primary density ratio for 15° forced mixer with $M_p = 1.6$.	107
4-36 Fully mixed scalar fields 1, 2, and 3λ downstream of the 15° forced mixer with $\Gamma^* = 0.5$, $\delta'_{vel} = 0.1$, and $\epsilon^* = 0.25$.	108
4-37 Modelled M_{scalar} distributions downstream of the 15° forced mixer ($\Gamma^* = 0.5$) with $M_p = 1.6$; $M_s = 0.3$, and $s = 0.7, 1.0, 2.0$. The modelled scalar mixedness is also plotted for the baseline convoluted plate ($\Gamma^* = 0.01$). For all cases, $\delta'_{vel} = 0.1$, and $\epsilon^* = 0.25$.	109
5-1 Schematic of a mixer-ejector noise suppressor model with locations of sub-models indicated.	113
5-2 Schematic of mixer control volume models	114
5-3 Cartoon of expansion process which matches the stream static pressures when $\frac{p_a}{p_p} < 1$.	116
5-4 Cartoon of compression process which matches the stream static pressures when $\frac{p_a}{p_p} > 1$.	116
5-5 Comparison between modelled matched point Cp_{super} and measured Cp_{super} at the first pressure tap downstream of the trailing edge of each of the mixers. $M_p = 1.3$.	117
5-6 Comparison between modelled matched point Cp_{super} and measured Cp_{super} at the first pressure tap downstream of the trailing edge of each of the mixers. $M_p = 1.6$.	117
5-7 Comparison between modelled matched point Cp_{super} and measured Cp_{super} at the first pressure tap downstream of the trailing edge of each of the mixers. $M_p = 2, 4$.	118
5-8 Modelled effective interface length distribution downstream of all 4 mixers with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_a}{p_p} = 1.0$.	120
5-9 15° forced mixer effective interface length distribution as a function of δ'_{vel} and x^* .	120
5-10 Modelled effective interface length distribution downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_a}{p_p} = 1.0$.	121
5-11 Stream Mach number distributions downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_a}{p_p} = 1.0$.	121
5-12 Modelled and measured Pitot mixedness versus distance from the trailing edge of all four mixers for $M_p = 1.6$, $M_s = 0.3$ and $\frac{p_a}{p_p} = 0.9$.	124
5-13 Flat plate duct exit Cp_{super} model/data comparisons. The error bars on the experimental measurements are smaller than the symbols.	126
5-14 Flat plate subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$.	126
5-15 Convoluted plate duct exit Cp_{super} model/data comparisons.	128
5-16 Convoluted plate subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$.	128
5-17 Separated side and lower wall streamlines downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.2$.	130

5-18	Attached side and lower wall streamlines downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.4$	131
5-19	15° forced mixer duct exit Cp_{super} model/data comparisons.	133
5-20	15° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$	133
5-21	Cp_{super} versus secondary/primary mass flow ratio for all mixers at $M_p = 1.6$	134
6-1	Secondary/primary mass flow ratio and gross thrust coefficient versus NPR. $SNPR = 1.0$, $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, and $\frac{T_{t_p}}{T_{t_s}} = 3.0$	137
6-2	Mixing rate/mixer drag trade study. Mass flow ratio versus thrust coefficient. $NPR = 3.0$, $SNPR = 1.0$, $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, $\frac{T_{t_p}}{T_{t_s}} = 3.0$	139
6-3	Impact of duct skin friction. Mass flow ratio versus thrust coefficient. $NPR = 3.0$, $SNPR = 1.0$, $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, $\frac{T_{t_p}}{T_{t_s}} = 3.0$	140
6-4	Secondary/primary mass flow ratio and gross thrust coefficient versus mixing duct length. $NPR = 3.0$, $SNPR = 1.0$, $\frac{T_{t_p}}{T_{t_s}} = 3.0$, and $\frac{A_s}{A_p} = 1.0$	141
6-5	Gross thrust coefficient and pumping for the FP , CP , and 15° FM versus the primary Mach number.	143
6-6	Ejector performance parameters versus the matched point compound flow indicator. $NPR = 3.0$, $SNPR = 1.0$, $\frac{T_{t_s}}{T_{t_p}} = 1.0$, $\frac{A_s}{A_p} = 1.0$	143
7-1	Scalar fields 2λ downstream of square lobe forced mixers with $h^* = 0.5$, $\epsilon^* = 0.25$, and $\delta'_{vel} = 0.1$	148
7-2	Passive scalar fields 2λ downstream of square lobe forced mixers with $h^* = 1.25$, $\epsilon^* = 0.25$, and $\delta'_{vel} = 0.1$	148
7-3	Passive scalar fields 2λ downstream of square lobe forced mixers with $h^* = 2.00$, $\epsilon^* = 0.25$, and $\delta'_{vel} = 0.1$	149
7-4	Passive scalar fields 2λ downstream of square lobe forced mixers with $h^* = 2.75$, $\epsilon^* = 0.25$, and $\delta' = 0.1$	149
7-5	Scalar mixedness at 20λ versus Γ^* and h^* with $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$	150
7-6	Scalar mixing augmentation at 20λ versus Γ^* and h^* with $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$	150
7-7	Forced mixer wetted area in square wavelengths as a function of lobe height to wavelength ratio and lobe angle.	152
7-8	Convolute plate wetted area in square wavelengths as a function of lobe height to wavelength ratio and the length of the straight section in wavelengths. ($\alpha_{CP} = 16.5^\circ$).	152
7-9	Mass flow ratio/thrust operating map for optimization study	154
7-10	Mixer drag coefficients for the six mixer geometries evaluated in the optimization study ($C_f = 0.001$).	155
7-11	Pumping/thrust operating map for the six mixers with $C_f = 0.001$	156

7-12 Pumping/thrust operating map for the six mixers with $C_f = 0.005$	156
B-1 Background, response, uncorrected flow, and corrected flow images 1λ downstream of the 15° forced mixer with $M_1 = 2.4$ and $M_2 = 1.6$	193
B-2 Untransformed and transformed grids 1λ downstream of the 15° forced mixer. The 2 is on the tunnel centerline.	194
B-3 Object-lens-image system.	194
B-4 Illustration of image transformation process.	195
B-5 Corrected mean mie and passive scalar fields 1λ down stream of the 15° forced mixer with $M_1 = 2.4$ and $M_2 = 1.6$	196
C-1 Inflow quantities with typical error bars for supersonic-subsonic inflow conditions. . .	199
C-2 Sample axial Cp_{super} distributions with typical error bars for supersonic-subsonic inflow conditions.	200
C-3 Sample axial Φ distributions with typical error bars for supersonic-subsonic inflow conditions.	200
C-4 Near field(high density) and far field(low density) Pitot survey grids.	201
C-5 Normalized Pitot pressure distribution 4 duct heights downstream of the flat plate with $M_p = 1.6$ and $M_s = 0.3$	203
D-1 Cartoon illustrating relationship between distance and time in the slender body approximation for lobed mixer flows[33].	206
D-2 Incompressible/compressible comparison[11, 33]	207
D-3 Discrete vortex/spectral code streamwise circulation comparison for $\Gamma^* = 0.5$, $\delta'_{vel} = 0.04$, $\epsilon^* = 0.25$, and $h^* = 1.0$	208
D-4 Discrete vortex/spectral code scalar mixedness comparison for $\Gamma^* = 0.5$, $\delta'_{vel} = 0.1$, $\epsilon^* = 0.25$, and $h^* = 1.0$	209
D-5 Cartoon illustrating the specification of the initial thickness of the boundary layer .	211
D-6 Normalized Pitot and velocity similarity profiles for $M_p = 1.6$, and $M_s = 0.3$	213
D-7 Contours of the ratio of the velocity to Pitot thickness of a planar shear layer as functions of the stream Mach numbers for equal total temperature streams.	214
D-8 Single square lobe vorticity distribution with evenly spaced vortices.	215
D-9 Normalized circulations along the vertical wall of a square lobe mixer with $h^* = 1.25$ and $\epsilon^* = 0.25$	216
D-10 Satisfaction of symmetry/sidewall boundary conditions via the method of images. . .	217
D-11 Scalar distribution(η) about a single vortex.	219
D-12 Dual vortex scalar field.	220

D-13 Fully-mixed scalar fields 1, 2, 3, 10, and 20λ downstream of a square lobe forced mixer with $h^* = 1.25$, $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.	221
D-14 Scalar mixedness distribution downstream of a square lobe forced mixer with $h^* = 1.25$, $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.	222
D-15 Mean interface length distribution downstream of a square lobe forced mixer with $h^* = 1.25$, $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.	222
D-16 Scalar fields 2λ downstream of a square lobe forced mixer with $h^* = 1.25$; $\epsilon^* = 0.25$; $\delta'_{vel} = 0.10$; and $\Gamma^* = 0.01, 0.25, 0.50, 0.75, 1.0$	223
D-17 Scalar mixedness at 20λ versus Γ^* for $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.	224
D-18 Mean effective interface length versus Γ^* for $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.	224
D-19 Scalar fields 2λ downstream of a square lobe forced mixer with $h^* = 1.25$; $\Gamma^* = 0.5$; $\epsilon^* = 0.25$; and $\delta'_{vel} = 0.02, 0.04, 0.10, 0.14, 0.20$.	225
D-20 Scalar mixedness at 20λ versus δ' for $\Gamma^* = 0.01, 0.50, 0.90$.	226
D-21 Mean effective interface length versus δ'_{vel} for $\Gamma^* = 0.01, 0.50, 0.90$.	226
D-22 Scalar fields 2λ downstream of a square lobe forced mixer with $h^* = 1.25$; $\Gamma^* = 0.5$; $\delta'_{vel} = 0.10$; and $\epsilon^* = 0.10, 0.20, 0.30, 0.40$.	227
D-23 Scalar mixedness at 20λ versus ϵ^* for $\Gamma^* = 0.50$, $\delta'_{vel} = 0.1$, and $h^* = 1.25$.	228
D-24 Mean effective interface length versus ϵ^* for $\Gamma^* = 0.50$, $\delta'_{vel} = 0.1$, and $h^* = 1.25$.	228
E-1 Matched/inflow area averaged static pressure and matched secondary/primary area ratio vs. M_s for $\frac{p_s}{p_p} = 0.8, 0.9, 1.0, 1.1$. ($M_p = 1.6$ and $\frac{A_s}{A_p} _{te} = 1.0$.)	232
E-2 Matched Mach numbers vs. M_s for $\frac{p_s}{p_p} = 0.8, 0.9, 1.0, 1.1$. ($M_p = 1.6$ and $\frac{A_s}{A_p} _{te} = 1.0$.)	233
E-3 Matched/inflow area averaged static pressure and matched secondary/primary area ratio vs. M_s for $M_p = 1.3, 1.6, 2.4$ and $\frac{A_s}{A_p} _{te} = 1.0$.	235
E-4 Matched Mach numbers vs. M_s for $M_p = 1.3, 1.6, 2.4$ and $\frac{A_s}{A_p} _{te} = 1.0$.	236
F-1 Angle definitions and region in which circulation is generated.	237
G-1 Normalized axial pressure distribution for $L^*_{eff} = 1.0, 2.0, 4.0, 8.0$ with $M_p = 1.6$, $M_s = 0.3$, $\frac{A_s}{A_p} = 1.0$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$	242
G-2 Entropy/pressure paths for $M_p = 1.6$, $M_s = 0.3$, $\frac{p_s}{p_p} = 1.0$, $\frac{T_{t_s}}{T_{t_p}} = 1.0$, $\frac{A_s}{A_p} = 1.0$ and $0.5 < Pr < 2.0$.	242
H-1 Flat plate subsonic wall Cp_{super} model/data comparisons for $M_p = 1.6$.	251
H-2 Flat plate subsonic wall Cp_{super} model/data comparisons for $M_p = 2.4$.	252
H-3 Convoluted plate subsonic wall Cp_{super} model/data comparisons for $M_p = 1.6$.	252
H-4 Convoluted plate subsonic wall Cp_{super} model/data comparisons for $M_p = 1.6$.	253
H-5 Convoluted plate subsonic wall Cp_{super} model/data comparisons for $M_p = 2.4$.	253

H-6	15° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p \approx 0.6$.	254
H-7	15° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 1.6$.	254
H-8	15° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 2.4$.	255
H-9	25° forced mixer duct exit Cp_{super} model/data comparisons.	256
H-10	25° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$.	256
H-11	25° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p \approx 0.6$.	257
H-12	25° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 1.6$.	257
H-13	25° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 2.4$.	258

List of Tables

2.1	Lobed mixer dimensions.	28
2.2	Estimated mixer streamwise circulations(Γ^*) and trailing edge lengths($L_{TE}^* = \frac{L_{TE}}{\lambda}$).	28
4.1	Mixer outflow/inflow total pressure ratio per the oblique shock loss model for three of the mixers at $M_p = 1.3$ and $M_p = 1.6$	99
5.1	Duct/mixer parameters for experimental comparisons	123
6.1	Flow and duct geometry parameters for mixer drag coefficient and duct wall skin friction coefficient impact studies.	137
6.2	Flow and duct geometry parameters for duct length impact study.	140
6.3	Comparison between the pumping and thrust performance of the FP , CP , and 15° at their optimal lengths.	141
6.4	Flow and duct geometry parameters for M_p impact study.	142
7.1	Flow and duct geometry parameters for mixer optimization study.	153
A.1	Subsonic wall static tap locations.	163
A.2	Pitot survey locations.	163
A.3	Nozzle number key	164
A.4	Mixer number key	164
A.5	Subsonic Pitot survey case number designations.	165
A.6	Subsonic static pressure case number designations.	165
A.7	Subsonic-subsonic Pitot survey data	166
A.8	Subsonic-subsonic wall static pressure data	167
A.9	$M_p = 1.3$ Pitot survey case number designations.	168
A.10	$M_p = 1.3$ static pressure case number designations.	168
A.11	$M_p = 1.3$ Pitot survey data	169
A.12	$M_p = 1.3$ wall static pressure data	170
A.13	$M_p = 1.6$ Pitot survey case number designations.	176

A.14 $M_p = 1.6$ static pressure case number designations.	176
A.15 $M_p = 1.6$ Pitot survey data	177
A.16 $M_p = 1.6$ wall static pressure data	179
A.17 $M_p = 2.4$ Pitot survey case number designations.	185
A.18 $M_p = 2.4$ static pressure case number designations.	185
A.19 $M_p = 2.4$ Pitot survey data	186
A.20 $M_p = 2.4$ wall static pressure data	187

Nomenclature

Roman

<i>A</i>	Cross-sectional area
<i>C</i>	Coefficient, constant
<i>CP</i>	Convoluted plate mixer
<i>EPNLdB</i>	Estimated Perceived Noise Level in decibels
<i>FM1</i>	15° forced mixer
<i>FM2</i>	25° forced mixer
<i>FP</i>	Flat plate mixer
<i>HSCT</i>	High Speed Civil Transport
<i>h</i>	Lobe peak to peak height
<i>L</i>	Cross-flow interface length
<i>M</i>	Mach number
<i>M</i>	Mixedness
<i>p</i>	Static pressure
<i>Pit</i>	Pitot pressure
<i>Pt</i>	Total pressure
<i>NPR</i>	Primary stream mixing duct nozzle pressure ratio
<i>r</i>	Secondary to primary velocity ratio
<i>Re</i>	Reynold's number
<i>s</i>	Secondary to primary density ratio
<i>Sc</i>	Schmidt number
<i>SNPR</i>	Secondary stream mixing duct nozzle pressure ratio
<i>t</i>	Time
<i>Tt</i>	Total temperature
<i>U</i>	Axial velocity
$\bar{U}$	Mean axial velocity
<i>u</i>	Axial velocity

v	Complex cross-flow velocity
x	Distance from mixer trailing edge

Greek

α	Lobe half angle
β	Matched to inflow primary area ratio
δ	Shear layer thickness
ϵ	Boundary layer thickness
γ	Ratio of specific heats
Γ	Streamwise-circulation shed from half-wavelength span
λ	Lobe wavelength
ν	Viscosity
Φ	Momentum mixing parameter
Ψ	Normalized mass-averaged total pressure
ξ	Similarity variable
ζ	Complex cross-flow position of vortex in cross-flow plane

Subscripts

c	Convective
D	Drag
eff	Effective interface length
f	skin friction
j	Vortex number
k	Vortex number
p	Primary stream quantity
pit	Pitot
s	Secondary stream quantity
sc	scalar
t	Turbulent
Γ	Circulation

Superscripts

$*$	Non-dimensional quantity
-----	--------------------------

† **Scalar field similarity variable**

Chapter 1

Introduction

1.1 Motivation

Recently efforts have been underway to develop a supersonic civilian transport aircraft for trans-Pacific operation that will be both economically competitive with contemporary subsonic aircraft and environmentally acceptable [1, 9, 25]. In order for this aircraft to be deemed environmentally acceptable, it must abide by the take-off noise levels mandated by the Federal Aviation Administration's FAR36 Stage 3 community noise regulations[1, 9].

With the current engines, it is anticipated that in order to satisfy the noise regulations, over 20 EPNdB of engine jet noise must be suppressed. Furthermore, this suppression must be done in a manner which minimizes engine thrust loss. Specifically, a goal of 4 EPNdB per percent thrust loss has been established[25]. This goal cannot be met with current technology[9, 25]. Hence, NASA's High Speed Research Program has launched an effort to develop a jet noise suppresser for use on the proposed supersonic civilian transport aircraft[12, 17, 32, 31, 33].

One promising noise suppression concept is an acoustically treated ejector[1, 9, 25]. This ejector

Figure 1-1: Conceptual HSCT mixer-ejector noise suppression system [28].

Figure 1-2: Typical lobed mixer[33].

would be located aft of the engine and would entrain atmospheric air into an acoustically treated duct to mix with the engine exhaust air to reduce the velocity of the jet leaving the engine nacelle, as illustrated in Figure 1-1. This reduction in velocity would decrease the intensity of the turbulent mixing of the jet with ambient air and thereby reduce the radiated noise[31]. In an effort to augment the mixing rate of the entrained(secondary) and engine(primary) streams and hence minimize the required length of the ejector duct, a lobed mixer would be employed to enhance the mixing rate between the two streams. Enhanced mixing rates are crucial to the performance of the aircraft since 1 pound of ejector weight can add more than 10 pounds to the gross take-off weight of the four engine aircraft.

1.2 Background

The concept of a lobed mixer-ejector-noise-suppressor was developed in the early 1960's when the devices were considered for use in the suppression of the jet noise of low-bypass ratio turbofan

engines for subsonic aircraft. However, due to the development of high-bypass ratio turbofans, the technology was never widely utilized[27].

More recently, lobed mixers have been investigated for use in the rapid mixing of the hot engine core stream with cool ambient air in order to reduce the infrared signature of engines[23]. Additionally, lobed mixers are again being utilized in mixer-ejector noise suppressors for subsonic aircraft to allow older turbofans meet increasingly stringent noise regulations.

In an effort to develop a methodology for optimizing the design of lobed mixers, an ongoing research program at the MIT Gas Turbine and Aero-Environmental Research Laboratories has been directed at developing an understanding of the fluid dynamic mechanisms through which these devices enhance mixing.

1.2.1 Lobed mixers

Lobed mixers, such as that shown in Figure 1-2, augment the mixing rate between two co-flowing streams via two mechanisms: the increased initial interface length associated with their convoluted trailing edge, and the stretching of the mean cross-flow interface by streamwise vorticity with distance from the trailing edge by streamwise vorticity[19]. In Figure 1-3, a mixing augmentation associated with the increased initial interface length and an additional augmentation associated with the presence of streamwise vorticity are demonstrated. In this figure, water tunnel measurements of the scalar mixedness downstream of a flat plate, forced mixer, and a convoluted plate mixer (with the same trailing edge shape as the forced mixer but shedding little streamwise vorticity) at a low speed stream to high speed stream velocity ratio of 0.67 are plotted versus distance from the trailing edge. A comparison between the mixedness curves downstream of the convoluted and flat plates illustrates that a mixing augmentation is associated with the additional interface length provided by the convoluted trailing edge. Additionally, a comparison between the mixedness curves downstream of the convoluted plate and forced mixer indicates that a mixing augmentation is associated with the streamwise vorticity shed by the forced mixer. (Both mixers in the comparison had the same trailing edge shape; they differed only the magnitudes of the streamwise circulations they shed.)

The mixing augmentation associated with the presence of streamwise vorticity has been shown to be due to the stretching of the mean cross-flow interface with distance from the trailing edge[23, 11, 30]. In Euler computations, Elliott[11] and Tew[30] have demonstrated that for the mixer geometries tested, the initial rate of stretching of the mean cross-flow interface is proportional to the streamwise circulation shed from a half wavelength span of the mixer. Furthermore, computations by Elliott suggest that the inflow Mach number has little effect on the evolution of the cross-flow interface downstream of a lobed mixer with equal velocity inflow streams[11]. This point is illustrated in Figure 1-4, where the interface length is plotted versus distance from the trailing edge for two cases with equal Mach number inflow streams—0.5 and 2.0.

Figure 1-3: Water tunnel measurements of molecular mixedness downstream of a flat plate, convoluted plate, and forced mixer for a velocity ratio of 0.67[19].

Figure 1-4: Incompressible/compressible interface length comparison[11, 33].

For a particular trailing edge geometry, the shed streamwise circulation may be estimated by a simplified scaling law developed by Barber *et al.* [2].

$$\Gamma = \frac{\Gamma}{U h} \approx C \tan \alpha, \quad (1.1)$$

where C is a function of the lobe trailing edge geometry. ($C = 2.0$ for square lobe geometries.) In Euler computations Elliott demonstrated that over the range of Mach numbers he examined ($M < 2.0$), compressibility had only a marginal effect on the generation of streamwise circulation by the mixers, and hence he showed that the above scaling law can be used to estimate the shed streamwise circulation in compressible flow regimes[11].

If a velocity difference exists between the two streams flowing over the mixers, an additional spanwise component of the vorticity is shed. The presence of the spanwise vorticity leads to the development of a turbulent shear layer between the two streams.

1.2.2 Shear layers

The growth rate of shear layers as a function of the inflow conditions has been the focus of a number of previous investigations [22, 5, 10]. This previous research has demonstrated that the growth rate of the Pitot thickness of shear layers between two co-flowing streams of fluid with matched inflow static pressures in a constant pressure mixing duct and without embedded streamwise vorticity is a

function of three non-dimensional parameters:

1. the velocity ratio(r),
2. the density ratio(s), and
3. the convective Mach number(M_c).

The density and velocity ratios are the ratio of the respective properties of the inflow streams. The convective Mach number is the parameter introduced by Papamoschou and Roshko[22] to correlate the impact of compressibility on the growth rate of the shear layers.

In supersonic jets, the compressibility of the shear layer is measured by the jet Mach number. In these flows, increasing compressibility, or increasing jet Mach number, results in a decrease in the growth rate of the mixing layer at the boundary of the jet. In two-dimensional, constant pressure, confined mixing layers, Papamoschou and Roshko demonstrated that the parameter which correlates the impact of compressibility is the Mach number in a reference frame convecting with structures in the shear layer—the convective Mach number(M_c).

$$M_c = \frac{U_p - U_s}{\bar{a}_p + \bar{a}_s} \quad (1.2)$$

The influence of M_c on the growth rate of planar shear layers is illustrated in Figure 1-5. In this figure(adapted from [22]), the planar shear layer compressible growth rate normalized by the growth rate of an incompressible shear layer($M_c = 0$) with the same velocity and density ratios is plotted versus the convective Mach number. An 80% reduction in the normalized growth rate is observed as the convective Mach number increases from 0.2 to 1.0.

This 80% decrease in the mixing rate of two-coflowing streams with increasing compressibility is of concern in the HSR mixer-ejector development program as the convective Mach number in the ejector is expected to be between 0.5 and 1.0.

1.2.3 HSR mixer-ejectors

In the late 1980's, the renewed interest in developing a supersonic civilian transport aircraft spurred the current interest in the development of a mixer-ejector-noise-suppressor which operates in compressible(high M_c) mixing regimes.

Several proof-of-concept studies were conducted to assess whether a mixer-ejector could indeed meet the thrust and noise goals established by the HSR program. In one of these recent studies, Tillman *et al.* observed a 75% increase in secondary stream pumping relative to an ejector with a conventional planar mixer[32]. Furthermore, ejector shroud exit total temperature and pressure surveys suggested that more mixing had occurred downstream of the lobed mixer. In another set of supersonic ejector experiments, Lord *et al.* demonstrated that the estimated pumping levels

Figure 1-5: Compressible($M_c > 0$) planar shear layer growth rate normalized by the equivalent incompressible growth rate($M_c = 0$) versus M_c [22, 10].

required for a 20 EPNLdB reduction in jet noise could be achieved with a lobed mixer[17]. However, now in the mid 1990's, the noise and thrust goals established by the HSR program have yet to be simultaneously obtained.

To date, the ejector development effort has been largely based on empirical techniques. A number of large parametric tests and three-dimensional Navier-Stokes computations have been conducted in which a number of mixer and ejector design parameters have been varied. The sole focus of these experiments has been on the thrust and noise performance of the devices, and little effort has been expended on understanding the fluid physics of the compressible mixing process.

1.3 Objective

In light of the need for a rapid and physically-based ejector performance model, the objective of this investigation was to develop a streamwise vorticity enhanced compressible mixer ejector model from a physical understanding of the fluid mechanics of ejector flows.

1.4 Approach

In order to achieve this objective,

1. An understanding of streamwise vorticity enhanced, compressible ejector flows was obtained through a series of experiments in which both the level of streamwise circulation and the compressibility of the flow were independently controlled. The focus of this work was on the impact of compressibility on the mixing augmentation associated with streamwise vorticity, since previous research has provided an understanding of the incompressible streamwise vorticity enhanced mixing process.
2. A computationally efficient (relative to three-dimensional Navier-Stokes codes) mixer-ejector performance model was developed and compared to experimental data.
3. Several trade studies were conducted with the model to determine the sensitivity of the ejector performance to key design parameters.

1.5 Contributions

The main contributions of this thesis are

1. The key lobed mixer performance drivers in supersonic-subsonic flows were identified. Lobed mixer performance was sensitive to the trailing edge static pressure ratio; this sensitivity increased with increasing lobe angle.
2. The effectiveness of the streamwise vorticity in enhancing mixing was shown to decrease with increasing lobe height to wavelength ratio, and with increasing shear layer growth rate.
3. Compressibility was shown to have a three-fold impact on the mixing augmentation associated with streamwise vorticity–
 - (a) Slower shear layer growth rates lead to an increased effectiveness of the vorticity.
 - (b) Streamwise vorticity may be generated downstream of the mixer by the interaction of trailing edge shocks with a density gradient across the mixing layer.
 - (c) The vortex sheets shed from the vertical side walls of the mixer may be displaced by a mismatch in static pressures at the trailing edge of the mixer.
4. A computationally efficient streamwise-vorticity-enhanced compressible mixer-ejector model was developed which has enabled design trade and optimization studies to be conducted.

1.6 Overview of thesis

Chapter 2 contains a description of the experiments. The controlled parameters are defined; the facility and diagnostic tools are described. Chapter 3 contains an assessment of forced mixer performance in compressible mixing layers. In Chapter 4, a quantitative assessment of the impact

of streamwise vorticity on compressible mixing layers is presented; the vortex interface model is introduced and comparisons are made to experimental Mie scattering data. Chapter 5 contains a description of the streamwise vorticity enhanced mixer-ejector model; comparisons are made to experimental static pressure data. In Chapter 6, the impact of streamwise vorticity on supersonic-subsonic ejector performance is assessed. Chapter 7 contains a sample lobed mixer optimization study. The final conclusions of this work are presented in Chapter 8.

Chapter 2

Description of Experiments

In this chapter, the procedure through which the impact of compressibility on the vorticity enhanced mixing process was assessed is described. The experimental tools–facility, mixers, and diagnostic techniques–with which the experimental assessment was made are then introduced. The mixer and ejector performance information extracted from each of the techniques is presented. Lastly, the four normalized mixing measures utilized to assess the impact of streamwise vorticity on the mixing rate are defined as well as the two ejector performance parameters of interest to the noise suppressor development program.

2.1 Overall Approach

A series of experiments were conducted with lobed mixers to assess the impact of streamwise vorticity in compressible ($M_c > 0.3$) mixing environments. In order to make this assessment, both the amount of streamwise vorticity shed into the flow and the compressibility of the flow were controlled–

- The level of streamwise vorticity shed into the mixing layers was controlled via the mixer geometry. Three lobed mixers with the same trailing edge geometry were designed and fabricated to shed three different levels of streamwise circulation. A comparison between the mixing rates downstream of the three mixers was used to quantify the mixing augmentation associated with streamwise vorticity.
- The compressibility of the flow was measured by the inflow convective Mach number, which was controlled by varying the inflow Mach numbers of the co-flowing streams. A comparison between the mixing augmentations associated with streamwise vorticity in compressible and incompressible mixing environments was used to assess the impact of compressibility.

In order to isolate the impact of the convective Mach number, the two stream Mach numbers were varied in such a way that the velocity ratio remained constant but the convective Mach number

changed. This is illustrated in Figure 2-1. On this map contours of the planar mixing layer parameters M_c , r , and s are plotted as functions of the inflow Mach numbers for equal total temperature, matched static pressure streams.

Note that since it was not possible to vary the total temperature ratio between the two streams in these experiments, it was not possible to independently control all three planar mixing layer parameters. Hence, as the convective Mach number was varied while holding the velocity ratio constant, the density ratio also changed. However, using correlations of compressible shear layer growth rates[10], it can be shown that in flow regimes of interest to this investigation, the influence of the density ratio on the growth rate of planar shear layers is much less than that of convective Mach number. This point is illustrated in Figure 2-2 where contours of the ratio of the derivative of the planar shear layer growth rate with respect to the density ratio to the derivative of the planar shear layer growth rate with respect to the convective Mach number are drawn. The sensitivity of the planar shear layer growth rate to s is one to two orders of magnitude less than its sensitivity to M_c in the Mach number regime of interest. Hence, to assess the impact of M_c , only the velocity ratio needed to be controlled.

An additional effort was made to assess the impact of the secondary to primary static pressure ratio at the trailing edge of the mixers on the performance of lobed mixers. In practical supersonic-subsonic mixer ejector flows, the static pressures of the two streams at the mixer trailing edge need not be matched, and as will be seen in Chapter 3, forced mixer performance is sensitive to the trailing edge pressure matching conditions.

2.2 Facility

The experiments were conducted in the Supersonic Shear Flow Facility in the Gas Dynamics Lab at the United Technologies Research Center. This facility was designed for continuous two-stream operation with unheated air at Mach numbers up to 3.5 and at stagnation pressures up to 8 atm.

The tunnel was driven by a series of compressors which supplied 27 atm air at up to 9 kg/sec. The 27 atm air was throttled through two regulators to each of the two inflow streams. Total pressures in the plenum chambers ranged between 1 and 5 atm in the high speed stream and 0.1 to 1 atm in the low speed stream. Additionally, a vacuum source was applied downstream of the test section during the experiments to provide back pressures as low as 0.25 atm.

A photograph of the facility is shown in Figure 2-3. The flow passes from the plenum chambers on the left of the window through the mixing duct to the diffuser on the right. Optical access was provided through the two visible side windows and additional windows on the upper walls of the mixing duct. Pressure taps were located in the plenum chambers and on the upper and lower mixing duct walls visible through the window.

Figure 2-1: Planar shear layer operating map. M_c , r , and s contours are drawn as functions of the two inflow Mach numbers for equal total temperature matched static pressure streams.

Figure 2-2: Contours of $\log_{10}\left(\frac{\frac{\partial \rho'}{\partial n}}{\frac{\partial M_c}{\partial n}}\right)$

Figure 2-3: Supersonic Shear Layer Facility in the Gas Dynamics Lab at the United Technologies Research Center

The relative locations and dimensions of the major facility components are indicated in the schematic shown in Figure 2-4. In the remainder of this section, the designs of the individual facility components (nozzles, mixers, and duct) are reviewed.

2.2.1 Nozzle designs

In order to control the inflow Mach numbers of the two streams, six nozzles were designed and constructed for these experiments: five converging-diverging supersonic nozzles and one converging subsonic nozzle.

The supersonic converging-diverging nozzles were designed for inflow Mach numbers of 1.2, 1.5, 1.6, 2.4, and 3.2 using the method of characteristics[26] with a correction to the nozzle contour to account for the growth of the boundary layers through the nozzle[14]. The $M_p = 2.4$ nozzle is depicted in the facility schematic shown in Figure 2-4.

The subsonic converging nozzle was designed for use in the lower stream. In order to alleviate, mixing duct separation problems, it was designed such that the subsonic stream area was continually decreasing upstream of the trailing edges of the mixers, as shown in Figure 2-4.

The six nozzles were used to configure the tunnel for either supersonic-supersonic or supersonic-subsonic experiments. The tunnel was also run with two subsonic streams by keeping the total pressure in the supersonic plenum low enough to prevent the nozzle from starting. The tunnel was only run subsonically with the lowest Mach number supersonic nozzle.

2.2.2 Mixer designs

In order to control the shed streamwise circulation introduced into the mixing layers, four mixers were designed: three lobed and one planar. Two of the lobed mixers were forced mixers, designed to shed streamwise circulation. The third lobed mixer had the same trailing edge shape as the two forced mixers, but was designed to shed little circulation.

A schematic of the trailing edge profiles of the lobed mixers is shown in Figure 2-5. All three mixers have the same trailing edge shape. The height to wavelength($\frac{h}{\lambda}$) ratio was 1.25. The corners of the mixer were rounded to prevent flow separation. The trailing edge thickness was 0.08 cm. The lobe wavelength was 2.0 cm. Five lobes spanned the 10 cm test section as shown in Figure 2-5. The penetration($\frac{h}{h_{duct}}$) of the lobed mixers was 0.25.

The mixers were design to sustain a load of 8 atm. They were 8.4 mm thick at their leading edges and tapered to a trailing edge thickness of 0.76 mm, as illustrated in the profiles depicted in Figures 2-6-2-8.

The two forced mixers shown in Figures 2-7 and 2-8 had lobe angles of 15° and 25° respectively. Γ^* for these geometries was estimated using the scaling law for square lobe mixers developed by

Figure 2-4: Schematic of the Supersonic Shear Layer Facility with the $M_p = 2.4$ nozzle and the 15° forced mixer as it was configured for the supersonic subsonic experiments.

Figure 2-5: Schematic of the lobed mixers' trailing edge in the UTRC Compressible Mixing Facility. All three lobed mixers have the same trailing edge shape. Specifications: $\lambda = 2.0 \text{ cm}$, $\frac{h}{\lambda} = 1.25$, corner radius/ $\lambda = 0.125$.

Figure 2-6: Convoluted plate.

Figure 2-7: 15° forced mixer.

Figure 2-8: 25° forced mixer.

Table 2.1: Lobed mixer dimensions.

Dimension	cm	λ
Wavelength(λ)	2.03	1
Height(h)	2.54	1.25
Corner radius(r_c)	0.25	0.12
Edge thickness(t_{edge})	0.08	0.04

Table 2.2: Estimated mixer streamwise circulations(Γ^*) and trailing edge lengths($L_{TE}^* = \frac{L_{TE}}{\lambda}$).

Mixer	α_{TE}	Γ^*	L_{TE}^*
Flat plate(FP)	0	0	1
Convolute plate(CP)	0	0.03	3.3
15° forced mixer(FM1)	15	0.5	3.3
25° forced mixer(FM2)	25	0.9	3.3

Barber *et al.*[2] given in Eqn. 2.1.

$$\Gamma^* = 2 \tan \alpha \quad (2.1)$$

The non-dimensional streamwise circulations are 0.5 and 0.9 for the 15° and 25° degree lobes respectively. The third lobed mixer, the convoluted plate, has the same trailing edge shape, but since the flow is turned back into the axial direction before leaving the trailing edge of the mixer, it sheds relatively little streamwise circulation. In subsonic computations by O’Sullivan[21] with a similar convoluted plate geometry, the non-dimensional streamwise circulation generated by the mixer was 0.03—due to incomplete turning of the flow into the axial direction before the trailing edge. In Figure 2-9, the axial distributions of the streamwise circulation obtained by O’Sullivan are plotted for a forced mixer and a convoluted plate geometry. The estimated streamwise circulations and mixer trailing edge lengths are listed in Table 2.2. A comparison between the mixing performance of the convoluted plate and that of the two forced mixers was used to delineate the impact of streamwise vorticity on the mixing process.

2.2.3 Mixing duct

The mixing duct for the supersonic-subsonic tests was square in cross-section. It spanned 10.4 cm by 10.4 cm. The mixer ramp leading edge low speed to high speed stream area ratios was 1.23. (The mixer trailing edge low speed to high speed stream area ratio was 1.0.) The mixing duct for the supersonic-supersonic tests spanned 10.4 cm by 8.9 cm, and the low speed to high speed stream leading edge area ratio for these tests was 0.91. The mixing duct extended 64 cm (6 duct heights or 31λ) in the axial direction. The cross-flow area of the mixing duct was constant for both

Figure 2-9: Axial forced mixer and convoluted plate circulation distribution[21].

configurations.

2.3 Diagnostic techniques

In order to measure the mixing rate, assess mixing performance, and ascertain the structure of the mixing layer, a number of diagnostic techniques were employed in this investigation. These techniques included–

- Mie scattering imaging from methanol droplets,
- Pitot/static pressure surveys,
- Wall static pressure measurements,
- Schlieren photographs, and
- Fluorescent dye surface flow visualization.

Each of these techniques is briefly described in this section, as well as the quantitative or qualitative mixer and/or mixing performance measures derived from them.

2.3.1 Mie scattering

Planar Mie scattering imaging from methanol condensate was used to assess the instantaneous and time-mean structure of the mixing layers.

A passive scalar seeding technique was employed [7, 8]. The supersonic stream was seeded with methanol liquid upstream of the plenum chamber. The liquid evaporated and then recondensed as the flow expanded through the nozzle. The liquid methanol droplets in the high speed stream served as scattering sites for incident laser light. The seeded stream appeared as a uniform fog. The methanol to seeded stream mass flow ratio was 0.005.

To obtain information about the instantaneous structure of the mixing layer, and for guidance in interpreting the time-mean images, single pulsed images were obtained. A single pulsed image is shown 2λ downstream of the 15° forced mixer in Figure 2-10a. In the image, the high speed stream is light gray, and the low speed stream is black. The instantaneous interface between the streams is apparent. Note the presence of small scale(relative to λ) structures in the interface.

In order to assess the mean structure of the mixing layer and to determine the passive scalar field, multiple pulsed cross-flow images were obtained. A multiple pulsed image is shown in Figure 2-10b. In this image, the high speed stream is light gray; the low speed stream is black; and the shear layer between the two streams is dark gray. (Note that the thickness of the shear layer is of the order of the scale of the small scale structures in the instantaneous image.) This time-mean image was obtained by averaging over 1800 laser pulses and has been corrected for background scattered

Figure 2-10: Instantaneous image, mean image, and passive scalar field 2λ downstream of the 15° forced mixer with $M_1 = 2.4$, $M_2 = 1.5$, $M_c = 0.30$, and $r = 0.80$.

light and laser sheet intensity variations using the procedure outlined in Appendix B. However, variations in the scattered intensities of the two streams are still visible—due to imperfect correction and density variations in the seeded stream.

In order to further correct for these variations and thus calculate the passive scalar field, the mean Mie images were scaled between their maximum unmixed unseeded stream intensities and minimum unmixed seeded stream intensities using the procedure discussed in Appendix B. The passive scalar(η) varies between 0 in the unmixed low speed stream and 1 in the unmixed high speed stream. A passive scalar field is also shown in Figure 2-10c. In this field, the high speed stream is white; the low speed stream is black; and the shear layer is gray.

2.3.2 Pitot-static pressure surveys

Pitot and static pressure surveys were used to calculate the Pitot mixedness and the mass averaged total pressure in axial planes downstream of the mixers. A brief summary of the measurement technique and data analysis procedures are contained in this section; details of the analysis procedures are presented with a comprehensive error analysis in Appendix C.

The surveys were conducted in cross-flow planes downstream of each of the mixers with a rake of 14 Pitot probes and one static probe. The Pitot probes were spaced 7 mm apart and spanned from within 6 mm of the subsonic wall to within 3 mm of the supersonic wall. Seventy percent of the duct cross-section was covered in each survey. Within 10λ of the mixer trailing edges, measurements were taken at forty stations, yielding a survey density of 11 points per wavelength. At distances greater

Figure 2-11: Measure Normalized Pitot pressure distributions 3, 6, 10, and 21 wavelengths downstream of the 25° forced mixer with $M_p = 1.65$, $M_s = 0.14$, and $\frac{P_s}{P_p} = 0.85$.

than 10λ , 20 points were taken per survey, yielding a survey density of 5 points per wavelength. Wall static pressure measurements were also taken in this plane on three of the four test section walls.

In Figure 2-11 Pitot pressure distributions are shown at four axial locations downstream of the 25° forced mixer. Additionally, a cross-flow static pressure distribution was calculated from the wall and rake static pressure measurements using the procedure described in Appendix C. From the Pitot and static pressure distributions, Mach number and total pressure distributions were calculated.

Two mixing measures were calculated from the Pitot/static pressure surveys—a normalized Pitot mixedness and a normalized mass average total pressure. The Pitot mixedness is the area averaged Pitot mixedness and is analogous to the Pitot thickness utilized in many planar shear layer studies[22,

5]. For the matched total temperature streams of interest in these experiments, the normalized mass averaged total pressure is a direct measure of the entropy rise associated with the mixing process.

The Pitot mixedness was calculated by area averaging the Pitot pressures measured across the plane. In order to isolate the change in Pitot pressure associated with the mixing of the two streams from that associated with skin friction on the walls of the test section, the integration was performed only over the center wavelength span and was conducted outside the subsonic and supersonic wall boundary layers.

In order to compare the mixing performance of a given mixer at different inflow conditions, the area averaged Pitot pressures were normalized using the mixer trailing edge area average Pitot pressure and the fully mixed Pitot pressure determined by a mixing out the two uniform inflow streams in a constant area control volume analysis. This normalized mixing parameter is defined by

$$\mathcal{M}_{pitot}(x) = \frac{\bar{P}_{pit}^A|_{TE} - \bar{P}_{pit}^A(x)}{\bar{P}_{pit}^A|_{TE} - \bar{P}_{pit}^A|_{mixed}}. \quad (2.2)$$

$\mathcal{M}_{pitot}$ varies between 0 at the trailing edge(TE) and 1 at the fully mixed state.

It is important note that the normalized Pitot mixedness is not a monotonically increasing function of distance from the trailing edge for all possible combinations of inflow Mach numbers and static pressure ratios. However, for all inflow conditions of interest to this investigation, $\mathcal{M}_{pit}$ monotonically increases from 0 at the unmixed trailing edge to 1 at the fully mixed conditions.

The mass averaged total pressure was also calculated by integrating the Mach number and total pressure distributions across the plane. This integration was also performed only over the center wavelength span and outside the wall boundary layers.

In order to compare the mixing performance of a given mixer at different inflow conditions, the mass averaged total pressures were normalized using the mixer trailing edge mass average total pressure and the fully mixed mass averaged total pressure determined by a mixing out the two uniform inflow streams in a constant area control volume analysis. This normalized mixing parameter is defined by

$$\Psi(x) = \frac{\bar{P}_t^m|_{TE} - \bar{P}_t^m(x)}{\bar{P}_t^m|_{TE} - \bar{P}_t^m|_{mixed}}. \quad (2.3)$$

Ψ varied between 0 at the trailing edge(TE) and 1 at the fully mixed state.

2.3.3 Wall static pressure measurements

The wall static pressure measurments were used to estimate the cross-flow area averaged static pressure. Static pressure taps were located on the supersonic and subsonic walls. Additionally, during some experiments, pressure taps were located on the side walls. The locations of these taps

are tabulated in Appendix A.

The cross-flow area averaged static pressure was used to calculate a normalized momentum mixing parameter and the supersonic pressure coefficient. The momentum mixing parameter is defined by

$$\Phi(x^*) = \frac{p(x^*) - \bar{p}_{in}}{p_{mix} - \bar{p}_{in}} \quad (2.4)$$

The fully mixed static pressure, $\bar{p}_{mix}$, was determined in a control volume analysis. Φ also varies between 0 at the inflow and 1 at the fully mixed conditions. The supersonic pressure coefficient is defined by

$$Cp_{super}(x^*) = \frac{p(x^*) - p_p}{P_{t_p} - p_p} \quad (2.5)$$

It is 0 at the primary stream leading edge.

2.3.4 Schlieren photographs

Schlieren photographs were used to assess mixer performance and estimate the vertical extent of the mixing layers. A Schlieren photograph of the 15° forced mixer with attached trough flow is shown in Figure 2-12. Note the thinning of the inflow boundary layer on the lobe crest, and the presence of an oblique shock at the trailing edge of the supersonic mixer troughs. (Lobed mixer performance in compressible mixing layers is discussed in greater detail in Chapter 3.)

2.3.5 Fluorescent dye surface flow visualization

A fluorescent dye surface flow visualization technique was employed to estimate the streamwise circulation shed by the lobed mixers, and to determine whether the mixing duct flow had separated.

To estimate the shed circulation, the surface of the mixers was painted with a mixture of oil, kerosene, and fluorescent dye; the tunnel was then started and run until the mixture had dried. A typical unseparated mixer flow pattern is shown in Figure 2-13. Surface streamlines on the vertical walls of the mixers were used to estimate the flow angle and thus the shed streamwise circulation using the Barber *et al.*[2] scaling law given in Eqn. 2.1.

To determine if the mixing duct flow was separated the mixing duct walls were painted with the dye mixture; the tunnel was started and run until the mixture had dried. A typical separated wall streamline pattern is shown in Figure 2-14.

Figure 2-12: Schlieren photograph of flow over the 15° forced mixer with $M_p = 3.2$. Flow is from right to left.

Figure 2-13: Surface streamline patterns on the 15° forced mixer at $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$. Note that the flow on the vertical mixer walls leaves the mixer at the lobe angle.

Figure 2-14: Separated wall streamlines downstream of the 15° forced mixer at $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$.

2.4 Performance Metrics

In this section the ejector and mixing performance metrics utilized to quantitatively assess the impact of streamwise vorticity are summarized.

2.4.1 Mixing Performance Metrics

Four mixing performance metrics were utilized in this study. One metric was calculated from the vortex code introduced in Chapter 4, two from the Pitot/static surveys and one from the wall static pressure measurements. The four mixing performance parameters are

- Scalar mixedness($\mathcal{M}_{sc}$)
- Pitot mixedness($\mathcal{M}_{pitot}$)
- Normalized mass-averaged total pressure(Ψ)
- Momentum mixing parameter(Φ)

All four of these parameters vary between 0 at the unmixed trailing edge and 1 at the fully mixed conditions.

In Figure 2-16, the four normalized mixing measures are plotted versus the normalized distance from the trailing edge of the flat plate mixer. All four parameters vary between 0 and 1. At fixed inflow conditions and fixed Schmidt and Prandtl numbers, three of the normalized mixing measures may be written as a function of the fourth. Hence, if any one of the measures is known at some point in the mixing duct, the other three measures are known as well.

2.4.2 Ejector Performance Metrics

Two ejector performance metrics were utilized to assess the impact of streamwise vorticity on ejector performance—

1. Pumping($\frac{\dot{m}_s}{\dot{m}_p}$), and
2. the gross thrust coefficient(C_{TG}).

For an ejector of fixed secondary to primary area ratio operating at fixed pressure and total temperature ratios, the ejector pumping is a measure of the mixedness of the secondary and primary streams at the exit of the ejector. An increase in pumping is associated with an increase in mixedness at the duct exit. This point is illustrated in Figure 2-16 where the secondary to primary mass flow ratio is plotted versus the normalized mass average total pressure at the duct exit for fixed nozzle pressure and total temperature ratios and a fixed secondary to primary area ratio. (In order to generate this figure, the ejector duct length was increased to increase the mixedness at the duct exit.)

In an actual mixer-ejector, the ultimate performance parameter of interest is its noise suppression relative to the unsuppressed primary in $\Delta EPNLdB$. The ejector pumping, or exit mixedness, is a useful indicator of this suppression. As the pumping and exit mixedness increase, the mean ejector exit velocity decreases; an increasing noise suppression is associated with the decreasing exit velocity.

The gross thrust coefficient is a measure of the thrust augmentation or thrust penalty associated with the presence of the mixer-ejector. The gross thrust coefficient is defined by

$$C_{TG} = \frac{\int U d\dot{m} - \int U_s d\dot{m}_s}{\dot{m}_p U_{ip}} \quad (2.6)$$

The duct exit velocity is given by U ; U_s is the forward flight velocity. The ideal primary exit velocity that would be obtained if the primary stream were expanded to the ambient static pressure is given by U_{ip} . If $C_{TG} < 1$, a thrust penalty is associated with the presence of the ejector, and if $C_{TG} > 1$, a thrust augmentation is associated with the presence of the ejector.

2.5 Chapter Summary

In this chapter, the procedure utilized for the assessment of the impact of compressibility on the mixing augmentation associated with streamwise vorticity was described. The tools with which the experimental assessment was made were presented. Lastly, the four normalized mixing measures and two ejector performance measures used to measure the impact of streamwise vorticity on the mixing rate and ejector performance were introduced.

Figure 2-15: Four mixing measures vs. normalized distance from the trailing edge of the flat plate for $M_p = 1.6$, $M_s = 0.3$, $\frac{p_s}{p_p} = 1$, $Sc = 1$, and $Pr = 1$.

Figure 2-16: Normalized mixing measure versus pumping for $NPR = 4.0$, $SNPR = 1.0$, $\frac{T_{t_s}}{T_{t_p}} = 1.0$, $M_p = 1.6$ and $\frac{A_s}{A_p} = 1.0$.

Chapter 3

Lobed Mixer Performance in Compressible Mixing Layers

This chapter contains an assessment of lobed mixer performance in supersonic flows. The criteria utilized to assess performance are the effective length of the interface at the trailing edge of the mixers and the shed streamwise circulation. (Recall that the two mixing mechanisms associated with streamwise vorticity are the increased initial interface length due to their convoluted trailing edge and the interface stretching associated with the streamwise vorticity.)

In the discussion, supersonic lobed mixer flows are first qualitatively described. Second, a more quantitative assessment of the ability of convoluted plate mixers to generate increased initial interface length without shedding streamwise vorticity is presented. Third, the ability of forced mixers to generate streamwise vorticity in supersonic mixing layers is assessed; an attempt is made to develop a quantitative forced mixer separation criteria. Last, an assessment is made of the implications of the reduced circulation and initial interface lengths associated with the trough separations on the mixing rate downstream of lobed mixers.

3.1 Qualitative Description of Compressible Mixer Flows

This section contains a description of the mixer flow field with particular emphasis on situations where at least of the inflow streams is supersonic, since much previous research has been conducted on subsonic mixer flows. This description is provided to familiarize the reader with the key supersonic mixer flow phenomena which impact performance. The performance implications of these phenomena are addressed in Sections 3.2, 3.3, and 3.4.

The current experiments were configured such that the mixers were located downstream of the throat of the primary nozzle. In this configuration, the primary flow into the mixers was supersonic,

as is evident in the Schlieren photographs of flow over each of the mixers shown in Figure 3-1. In these photographs, the flow direction is from right to left. Shock waves and Mach waves are visible in the supersonic(upper) stream. The key flow features to be described are the mixer boundary layers, leading edge shock, and trough or trailing edge shock. The locations of each of the features are indicated in the schematics in Figure 3-2. Each of these features are described in more detail and their performance implications identified in the remainder of this section.

Lobe boundary layers The inflow primary stream boundary layer is evident in the Schlieren photographs shown in Figure 3-1. This boundary layer thinned to less than 10% of its initial thickness on the supersonic lobe peaks. (Compare the visible thickness of the inflow boundary layers to those on the supersonic peaks.) The thinning of the peak boundary layers is associated with acceleration of the inflow boundary layer fluid into the lobe troughs. O'Sullivan has shown that the presence of the boundary layer fluid in the forced mixer troughs is detrimental to the mixer performance since it fills the mixer trough and reduces the effective angle of the lobe, thereby reducing the shed circulation[21]. The presence of a thick(relative to the vertical sidewalls) boundary layer in the supersonic trough of the 15° forced mixer is evident in the normalized Pitot pressure field 1λ downstream of the mixer shown in Figure 3-3.

Leading edge shock One flow feature evident in all the lobed mixer Schlieren photographs at the leading edges of the lobed mixers is an oblique shock. This shock is a three-dimensional bow shock. The bow shock turns the supersonic inflow around the lobe peak and into the lobe troughs. This shock is both a loss generating mechanism and an implicit mixing mechanism, as the flow must be turned to generate the streamwise vorticity and increase the trailing edge interface length.

Trough shock In the supersonic/subsonic mixing layers of interest in the current study, the pressures of the two inflow streams need not be matched, since an oblique shock or expansion fan may be located at the trailing edge. A trailing edge expansion fan can be observed in the Schlieren photograph of the flat plate in Figure 3-1.

On the other hand an oblique shock is apparent at the trailing edge of the 15° forced mixer. This shock turns the over-expanded trough flow toward the axial direction and increases the static pressure of the flow leaving the primary trough. When the shock is located at the trailing edge of the mixer, the flow inside the trough remains attached and the mixer is effective in generating streamwise vorticity and increasing the interface length between the streams. In Figure 3-4a, surface flow patterns on the 15° forced mixer are shown. The streaklines on the vertical walls of the 15° forced mixer left the mixer at approximately the metal angle. Hence, at these flow conditions, the trough flow remained attached, and the mixer was effective in generating streamwise vorticity.

If the flow in the primary troughs is sufficiently over-expanded relative to the subsonic stream,

(a) Flat Plate

(b) Convoluted Plate

(c) 15° forced mixer

(d) 25° forced mixer

Figure 3-1: Schlieren photographs of mixer flows over each of the four tested mixers. $M_p = 1.6$ for the flat plate, convoluted plate and 25° forced mixer. $M_p = 3.2$ for the 15° forced mixer. The flow direction is from right to left.

Figure 3-2: Schematics highlighting the key features of supersonic lobed mixer flows

Figure 3-3: Normalized Pitot pressure contours 1λ downstream of the 15° forced mixer. The supersonic stream is on top, and the subsonic is on the bottom.

this trailing edge shock will move into the trough and separate the flow. The shock may be observed in the primary trough in the Schlieren photograph of the 25° forced mixer. The presence of the shock inside the supersonic trough separated the trough flow as may be seen in the surface flow pattern on the 25° forced mixer shown in Figure 3-4b. Decreased mixing rates were associated with the presence of this primary trough separation, as will be shown in Section 3.4.

3.2 Convoluted plate performance in compressible flow

The convoluted plate mixer was designed to increase the initial interface length between the streams but to shed little streamwise vorticity. A schematic of the convoluted plate mixer utilized in these experiments is shown in Figure 2-6.

In incompressible convoluted plate flows, the trough flow is initially turned off axis over the ramp portion of the trough. The flow is then gradually turned back into the axial direction through the straight section of the mixer. Incompressible convoluted plate trough flow streamlines are sketched in the cartoon shown in Figure 3-5. As indicated in the cartoon, and in Navier-Stokes computations by O'Sullivan[21], for the convoluted plate geometry used in these experiments, the trough flow will not turn completely back into the axial direction through the straight section and some streamwise circulation will be shed. For the experimental geometries, the level of the streamwise circulation shed is estimated to be $\Gamma^* \approx 0.03$.

In the supersonic convoluted plate flows, however, the freestream trough flow turns through an expansion fan followed by a turn back into the axial direction through an oblique shock located at the onset of the straight section. (The shock in the convoluted plate trough may be seen in the Schlieren photograph in Figure 3-1.) However, these supersonic trough flows have subsonic, higher temperature, and lower density boundary layers which see nearly the same pressure gradient as the supersonic free stream. Hence, through the oblique shock which turns the freestream fluid into the axial direction, the lower density boundary layers turn beyond the axial direction. (The component of the pressure gradient across the oblique shock perpendicular to the density gradient through the boundary layer produces a baroclinic torque which will generate streamwise vorticity within the trough boundary layer.) This baroclinic torque generates a flow of boundary layer fluid up the vertical wall of the mixer. This flow of the boundary layer fluid up the wall may be seen in the dye photograph of the surface streamlines on the convoluted plate shown in Figure 3-6. The overturning of the boundary layer fluid through the oblique shock is also illustrated in the cartoon in Figure 3-7.

As shown by the surface streamlines in Figure 3-6, downstream of the shock, some turning of the boundary layer fluid back into the axial direction did occur. However, in the vicinity of the supersonic lobe peak, the oblique trough shock passed near the trailing edge of the mixer, and in

(a) 15° forced mixer

(b) 25° forced mixer

Figure 3-4: Surface streamline patterns on the 15° and 25° forced mixers at $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_2}{p_1} = 1.0$. Note the separation in the supersonic trough on the 25° mixer.

Figure 3-5: Incompressible convoluted plate trough boundary layer and freestream streamline

Figure 3-6: Convoluted plate surface streamlines. $M_p = 1.6$ and $M_s = 0.3$. The supersonic side of the mixer is on top, and the subsonic is on the bottom.

Figure 3-7: Supersonic convoluted plate trough boundary layer and trough flow streamlines

this region the magnitude of the vorticity shed was comparable to that shed by the forced mixers in the study. However, the sense of the vorticity was opposite that shed by the forced mixers since the sidewall boundary layer fluid was exiting in the upward direction.

As the inflow supersonic Mach number was decreased, the trough shock became more vertical and moved away from the trailing edge decreasing the circulation shed by the convoluted plate. However, if the supersonic Mach number was increased, the trough shock angle decreased and the intersection of the trough shock and vertical side wall moved closer to the bottom of the trough. This movement of the shock shifted downward the region on the vertical side wall in which significant levels of streamwise vorticity were shed.

The streamwise vorticity that was shed locally impacted the mixing rate. (The impact of this convoluted plate vorticity will be discussed in Chapter 4.) However, despite the shedding of significant levels of vorticity near the intersection of the shock with the trailing edge; the convoluted plate is estimated to have shed $\approx \frac{1}{5}$ of the circulation that was shed by the 15° forced mixer.

3.3 Forced mixer performance in compressible flow

In this section, a quantitative assessment of forced mixer performance in compressible mixing layers is made using Pitot pressure data. The two performance measures of interest are the streamwise circulation(Γ^*) and the effective trailing edge interface length.

The mixer flow feature which most significantly influenced the performance of the forced mixers was the trough shock. For a fixed lobe geometry and primary inflow Mach number, the location of the shock was a function of the secondary/primary trailing edge static pressure ratio. For extremely low static pressure ratios, no trailing edge shock was present and no turning of the supersonic trough flow occurred at the trailing edge. As the secondary/primary static pressure ratio is increased a shock of increasing strength formed at the supersonic trough exit. This shock turned the supersonic trough flow back toward the axial direction. If the static pressure ratio was increased further, this shock moved back into the lobe trough causing the flow to separate.

The trough flow phenomena described here is analogous to an over-expanded converging-diverging nozzle. If the nozzle exit static pressure is matched to ambient, no turning of the nozzle flow occurs at the nozzle exit. As the ambient static pressure is increased relative to the nozzle exit pressure, a shock forms at the trailing edge which turns the nozzle flow inward and increases the exit stream static pressure. As the ambient static pressure increases, this shock becomes stronger and steeper until it eventually moves back into the nozzle causing the boundary layer to separate.

Zukoski proposed an empirical method to predict the location of the shock and associated boundary layer separation in the nozzle[15]. He suggests that the ratio of the static pressure at the

Figure 3-8: Regions of attached and separated converging-diverging nozzle flow per Zukoski's nozzle separation criteria[15].

separation point to the ambient static pressure is given by

$$\frac{p_{sep}}{p_{ambient}} = \frac{1}{1 + \frac{M_{sep}^2}{2}}, \quad (3.1)$$

where M_{sep} is the Mach number just upstream of the separation. Furthermore, the static pressure at the separation is related to the total pressure by

$$\frac{Pt}{p_{sep}} = \frac{Pt}{p_{ambient}} \frac{p_o}{p_{sep}} = \left(1 + \frac{\gamma - 1}{2} M_{sep}^2\right)^{\frac{\gamma}{\gamma - 1}}. \quad (3.2)$$

Eqns. 3.1 and 3.2 may be solved for M_{sep} and $\frac{p_{sep}}{p_o}$ for a given $\frac{Pt}{p_o}$. In Figure 3-8, the separation Mach number is plotted versus $\frac{Pt}{p_{ambient}}$. If the design(ideal) nozzle Mach number is less than M_{sep} , the nozzle flow will be attached. If the design nozzle Mach number is greater than M_{sep} the nozzle flow is predicted to separate. The separated region is marked in gray, and the attached region in white in Figure 3-8.

Zukoski's nozzle separation criteria may be combined with a mixer trough flow model to derive a forced mixer separation criteria. A simplified model of the trough flow was developed to estimate the primary trough Mach number(M_{ptr}). If the modelled trough Mach number is greater than the separation Mach number predicted by Zukoski's criteria, the trough flow is predicted to separate.

The mixer primary trough flow model assumes–

1. The trough flow first passes through a leading edge shock at one-half the ramp metal angle.
2. The trough flow then undergoes a Prandtl-Meyer expansion at the trough metal angle.

The first assumption represents a simplified model of the three-dimensional bow shock at the leading edge of the ramp. The trough flow passes through a portion of this shock, and hence it is not isentropic. On the other hand this shock in front of the trough will not be as strong as a two-dimensional oblique shock at the ramp angle. The actual flow lies somewhere in between the no-shock and two-dimensional shock extremes. In the simplified trough model, the trough flow is assumed to pass through an oblique shock at one-half the mixer ramp angle; this model approximates the average of the two extreme cases. After passing through the leading edge oblique shock, the trough flow is assumed to undergo a Prandtl-Meyer expansion at the trough metal angle. The reduction in the effective lobe angle associated with the presence of boundary layer fluid in the troughs is not modelled. Hence, the trough expansion model will over-predict the trough Mach number.

In Figure 3-9a, the boundary between the separated and attached trough flow regions as predicted by Zukoski's criteria and the mixer trough flow model described above is marked with the solid line. The predicted separated supersonic trough region is the gray region just below the solid line.

The solid line represents the lowest $\frac{Pt_p}{p_s}$ at which the supersonic trough flow will be attached for a given inflow primary Mach number. As this ratio is increased further, the supersonic trough flow will continue to become more robust. The shock at the trough trailing edge will diminish in strength and the trough boundary layers will become thinner. However, as this ratio is increased, at a fixed secondary Mach number, the static pressure in the primary trough($p_{p_{tr}}$) increases relative to the secondary stream total pressure(Pt_s). Along the dashed line in Figure 3-9a, the primary trough static pressure is equal to the secondary total pressure for a secondary Mach number of 0.3; the secondary trough flow will be separated. The region of subsonic trough separation is the gray region above the dashed line. If the secondary Mach number is increased, the $\frac{Pt_p}{p_s}$ at which $\frac{p_{p_{tr}}}{Pt_s} = 1$ increases, and the area of the region in which both troughs are predicted to be attached increases.

In Figure 3-10a-b, the predicted primary and secondary trough separation regions are drawn for the 25° forced mixer for primary Mach numbers between 1.25 and 2.5 at secondary Mach numbers of 0.2 and 0.7. The steeper trough angle of this mixer relative to the 15° mixer results in an increased trough Mach number and lower trough static pressure. Hence, the supersonic trough is predicted to separate at a higher $\frac{Pt_p}{p_s}$. On the other hand, the lower static pressure in the supersonic trough for the 25° mixer relative to the 15° mixer for a fixed Pt_p also increases the $\frac{Pt_p}{p_s}$ at which the secondary trough is predicted to separate for a given secondary Mach number. In effect, the predicted region of attached trough flow shifts up the $\frac{Pt_p}{p_s}$ axis with increasing trough angle.

Several comparisons between the predicted mixer performance maps and experimental data were

(a) $M_s = 0.2$

(b) $M_s = 0.7$

Figure 3-9: 15° forced mixer performance map. Cases with separated trough flow are marked with x's, and cases with attached flow are marked with o's.

(a) $M_s = 0.2$

(b) $M_s = 0.7$

Figure 3-10: 25° forced mixer performance map. Cases with separated trough flow are marked with x's, and cases with attached flow are marked with o's.

made using the Pitot pressure surveys taken at $\approx 1\lambda$ from the mixer trailing edges. In Figure 3-11, The measured normalized Pitot pressure contours are shown just downstream of the trailing edge of the three forced mixers with $M_p = 1.6$ at three different $\frac{P_{t_p}}{p_s}$, or $\frac{p_s}{p_p}$. (As $\frac{P_{t_p}}{p_s}$ increases, $\frac{p_s}{p_p}$ decreases.) The color scale varies between white for the unmixed supersonic stream and black for the unmixed subsonic stream.

From these near-mixer surveys, the supersonic trough performance of the mixers may be ascertained. The approximate trailing edge location is marked in the normalized Pitot pressure fields. For $\frac{p_s}{p_p} = 0.90$, the flow was attached in the supersonic troughs of all the mixers. As the static pressure ratio was increased, the trailing edge shock moved into the troughs of the two forced mixers and began to separate the flow in the trough boundary layer.

In Figure 3-9, for each of the tested $\frac{P_{t_p}}{p_s}$ and $M_{primary}$, points with separated trough flow (as ascertained from the Pitot surveys 1λ from the mixer) are marked with $\circ$'s and points with separated trough flow are marked with $\times$'s. As may be seen in the Figure, the trough separation criteria works well for the 15° mixer.

In Figure 3-10 the separated and attached 25° forced mixer cases are marked with $\times$'s and $\circ$'s respectively. On the other hand, the separation criteria does not work as well for the 25° mixer, since the supersonic trough separation line is lower than predicted.

Potential reasons for the discrepancy include

- An over prediction of the leading edge shock strength.
- An over estimation of the effective trough flow angle. (The metal angle is currently used.)

3.4 Implications of mixer separation

The separation of the primary stream over the mixer due to the over-expansion of the trough flow had two detrimental effects on mixer performance:

1. Decreased the effective interface length between the streams at the trailing edge of the mixers; and
2. Decreased streamwise circulation introduced into the flow by the mixer due to the turning the flow back toward the axial direction through the oblique shock in the trough.

The impact of the trough separations on the interface length was measured from the surveys in Figure 3-11. In Figure 3-12, the mean interface length in the survey plane is plotted versus $\frac{p_s}{p_p}$. As $\frac{p_s}{p_p}$ increased, the trough shock moved farther into the lobed mixer troughs reducing the effective trailing edge length of the mixer. The presence of a trough separation reduced the circulation shed by the mixer in Figure 3-4b because the flow turned back into the axial direction through the shock.

Figure 3-11: Normalized Pitot pressure fields at the first survey location downstream of each of the lobed mixers with $M_p = 1.6$ and $M_s \approx 0.2$. (Survey locations: $x_{CP}^* = 1.2$, $x_{FM1}^* = 1.7$, $x_{FM2}^* = 2.9$.)

Figure 3-12: Interface length at first survey plane versus the inflow secondary/primary static pressure ratio— $M_p = 1.6$, $M_s \approx 0.2$.

The impact of this decreased effective interface length and decreased circulation is illustrated in Figure 3-13. In this Figure, the momentum mixing parameter at 30λ is plotted versus the trailing edge secondary to primary static pressure ratio for each of the mixers at $M_p = 1.6$ and $M_s \approx 0.2$. (The same inflow Mach numbers at which the Pitot surveys shown in Figure 3-11 were taken.) The momentum mixedness downstream of the convoluted plate and forced mixer were actually seen to increase slightly with increasing $\frac{p_s}{p_p}$. However, Φ downstream of the two forced mixers decreased with increasing static pressure ratio and decreased downstream of the 25° mixer.

At low static pressure ratio, the 25° mixer supersonic flow was attached and the mixer was effective in generating streamwise vorticity and maintaining the increased interface length as may be seen in the Pitot surveys in Figure 3-11. However, as the static pressure ratio increased, the trailing edge shock moved into the trough and separated the flow decreasing the shed circulation and initial interface length as shown in Figures 3-11 and 3-12. As $\frac{p_s}{p_p}$ was increased, the shock moved farther into the trough, increasing the severity of the separation and the resulting mixing performance penalty.

Figure 3-13: Momentum mixing parameter versus $\frac{p_s}{p_p}$ for $M_p = 1.6$ and $M_s \approx 0.2$.

3.5 Chapter Summary

In this chapter, an assessment was made of the ability of lobed mixers to generate streamwise vorticity in compressible (supersonic-subsonic) mixing layers. Mixer performance was shown to be sensitive to the trailing edge static pressure ratio and this sensitivity was seen to increase with increasing lobe angle. An empirical mixer separation criteria was developed to predict the ranges of static pressure ratios and lobe angles at which the primary and secondary trough flows would be attached. Last, a decreased mixing rate was associated with the presence of a primary trough separation on the 25° forced mixer.

Chapter 4

Impact of Streamwise Vorticity in Compressible Mixing Layers

This chapter contains a discussion of the impact of streamwise vorticity in compressible mixing layers. The discussion begins with a presentation of Mie, Pitot, and wall static pressure data—first for the flat and convoluted plates and then for the forced mixers. The mixing mechanism associated with streamwise vorticity in compressible mixing layers is shown to be the same as it is in incompressible layers—the stretching of the cross-flow interface.

From the physical understanding developed in the experiments and a number of previous investigations[23, 11, 21, 30], a model of the vorticity enhanced mixing process is developed. Relative to three-dimensional Navier-Stokes, the model is computationally efficient and flexible and has enabled a more detail assessment of the impact of compressibility on the vorticity enhanced mixing process. Using this model in conjunction with experimental data, the impact of compressibility on the streamwise vorticity enhanced mixing process is further elucidated.

4.1 Planar Shear Layers

In this section using Mie, Schlieren and Pitot data, the mixing associated with a mean flow velocity difference or spanwise vorticity is quantified. Comparisons are drawn between the current data and that published by Papamoschou and Roshko[22].

The interface between the two co-flowing streams of fluid with a velocity difference between them is inviscidly unstable. Small perturbations in the interface grow with distance from the trailing edge. This instability(the Kelvin-Helmholtz instability) leads to the growth of waves in the interface and to the subsequent development of a shear layer.

If the Reynold's number based on ΔU and the shear layer layer thickness(δ) is greater than

10^4 , the shear layer will be turbulent[10]. All mixing layers investigated in the current study, augmented and unaugmented, were turbulent. Turbulent structures in the both the mixing and inflow boundary layers are visible in the typical composite Schlieren photograph of a planar shear layer shown in Figure 4-1. The flow direction is from right to left. The boundary layer is visible on the supersonic(primary) side of the mixer, and from this boundary layer, a shear layer grows with distance from the trailing edge of the mixer. Furthermore, the shear layer grows at nearly a constant rate with distance from the trailing edge even in a constant-area, and hence, non-constant-pressure mixing duct.

In the Schlieren photograph, no large-scale(on the order of the shear layer thickness) two-dimensional structures are apparent. However, in Figure 4-2, instantaneous Mie images are shown downstream of the flat plate mixer. In these images, interface structures of the order of the shear layer thickness are visible. The existence of these large-scale structures is consistent with Prandtl's mixing length hypothesis which is used in the development of the interface vortex and differential control volume models. The scale of these structures increases with distance from the trailing edge as does the shear layer in the Schlieren photographs.

In Figure 4-3, these instantaneous images of the large-scale structures are averaged. When the time-mean concentration of methanol droplets is plotted across the shear layer, as shown in Figure 4-4, a classical error function concentration profile results as would be predicted by Goertler's model of incompressible mixing layers with constant turbulent Schmidt number. Goertler's model will be used to estimate the turbulent viscosity in the interface vortex and differential control volume models for both compressible and incompressible mixing layers.

Much previous research has been directed at quantifying the growth rate of shear layers in constant-pressure mixing layers as functions of the inflow Mach numbers, total temperatures, molecular weights, and ratios of the specific heats[5, 10, 22]. In sum, the growth rate of planar shear layers has been found to be a function of three parameters: the velocity ratio(r), the convective Mach number(M_c), and the density ratio(s):

$$\delta'_{pit} = f(M_c) C_{pit} \frac{(1-r)(1+\sqrt{s})}{(1+r\sqrt{s})} [22], \quad (4.1)$$

where

$$f(M_c) = 0.8 \exp -3M_c^2 + 0.2 [10]. \quad (4.2)$$

In Figure 4-5, the measured growth rate of the Pitot thickness of a compressible($M_c > 0.3$) shear layer normalized by the correlated incompressible growth rate($M_c = 0$), is plotted versus M_c along with the Dimotakis[10] correlation. Indeed the correlation captures the trend indicated by the data. However, of interest to this investigation is the accuracy with which the growth rate correlation can

Figure 4-1: Composite Schlieren photograph of a planar shear layer with $M_p = 1.6$ and $M_s = 0.7$, $\frac{p_s}{p_p} = 1$. The flow is from right to left.

(a) Time 1

(b) Time 2

Figure 4-2: Instantaneous Mie images of a planar shear layer at two different times with $M_p = 1.6$ and $M_s = 0.3$.

Figure 4-3: Mean Mie images of a planar shear layer with $M_p = 1.6$ and $M_s = 0.3$.

predict the actual growth rate of the shear layers; the maximum relative error in the Dimotakis correlation is 141%.

In the current study, in order to assess the applicability of Dimotakis' correlation, Pitot surveys were conducted downstream of flat plate mixers at a range of convective Mach numbers to measure the growth rate of the Pitot thickness of shear layers in the non-constant pressure mixing ducts of these experiments. In Figure 4-5, measured compressible Pitot thickness growth rates normalized by their respective incompressible growth rates are plotted versus the convective Mach number. As observed by Papamoschou [22], a decrease in the normalized growth rate with increasing convective Mach number ($0.1 < M_c < 0.9$) is apparent. However, the rate of decrease of the normalized growth rate is roughly half that observed by Papamoschou. Furthermore, the normalized growth rate varies approximately linearly with M_c over the measured range of convective Mach numbers.

A linear correlation for the ratio of the compressible to incompressible growth rate was derived from the current data and is plotted in Figure 4-5. The maximum relative error between this correlation and current experimental data is 16%. This correlation served as an empirical basis for the turbulent viscosity models in the interface vortex and differential control volume models presented in Sections 4.3.3 and 5.5.

4.2 Convoluted Shear Layers

The convoluted plate mixer tested in the current study increased the initial interface length of the two co-flowing streams by a factor of 3.3 but introduced little streamwise vorticity, since the majority of the flow left the trailing edge of the mixer in approximately the axial direction. In the presence of a velocity difference between the streams, spanwise vorticity was shed, and a shear layer grew on

(a) $x^* = 0.4$

(b) $x^* = 2.5$

Figure 4-4: Methanol droplet concentration profiles in a planar shear layer with $M_p = 1.6$ and $M_s = 0.3$.

Figure 4-5: Compressible ($M_c > 0$) planar shear layer growth rates normalized by their equivalent incompressible growth rate ($M_c = 0$) versus M_c as measured in the current experiments and by Papamoschou and Roshko[22]. Dimotakis'[10] fit to Papamoschou's data is shown as well as a best line fit to the current data.

the convoluted interface between the two streams.

In Figure 4-6, three instantaneous spanwise Mie images are shown. The three images are aligned with the center of a supersonic peak, the center of a supersonic trough and the vertical side wall of the convoluted plate mixer. The trailing edge of the mixer is located at $x^* = 0$; the flow direction is from left to right. The vertical locations of the peak and trough of the mixer are indicated with dotted lines.

The growth of the structures in the shear layer with distance from the trailing edge is evident in both the peak and trough views. In a comparison of these images, a large difference between the initial size of the shear layer structures is apparent. The scale of the structures at the supersonic peak is $\approx \frac{1}{20}\lambda$, while the boundary layer structures in the trough are $\approx \frac{1}{4}\lambda$. (Recall the thinning of the peak boundary layer evident in the Schlieren photograph in Figure 3-1 associated with the acceleration of the inflow boundary layer fluid into the troughs.) McCormick also observed a similar disparity between the size of the peak and trough boundary layers in subsonic lobed mixer flows[20].

In the supersonic trough image, the convection of the low speed fluid into the image plane near the peak of the mixers is seen. This low speed fluid appears as the black patch growing with x^* just below the dotted line indicating the peak of the mixer. The convection of this low speed fluid into the center of the supersonic trough was due to the streamwise vorticity shed in the vicinity of the intersection of the trough shock with the mixer trailing edge. (Recall the discussion of convoluted plate performance in Section 3.2.)

In Figure 4-7, an instantaneous axial Mie image is shown 2λ from the trailing edge of the convoluted plate. In this image, structures of the order of the shear layer thickness are distributed along the entire interface.

In Figure 4-8, axial time-mean Mie images are shown 1, 2, and 3 wavelengths downstream of the convoluted plate mixer. These time-mean images have been averaged over 1800 laser pulses. The white supersonic stream is on top and the black subsonic stream is on the bottom. The gray shear layer is located between the two streams. Comparing these images, the growth of the shear layer with increasing x^* is evident, as is some local interface stretching in the vicinity of the supersonic peak. This local interface stretching is associated with the streamwise vorticity shed in the vicinity of the intersection of the trough shock and trailing edge.

In Figure 4-9, normalized pitot pressure fields are plotted at approximately 1 and 20 lobed mixer wavelengths downstream of the flat and convoluted plate mixers. The normalized Pitot pressure varies between 0 at the inflow Pitot pressure of the subsonic stream and 1 at the inflow Pitot pressure of the supersonic stream. The color scale varies between white for a normalized Pitot pressure of 1 and black for a normalized Pitot pressure of 0.

From these normalized Pitot pressure fields (and two additional ones at approximately 5 and 10λ), the length of the mean interface (defined as the 0.5 contour) was calculated and is plotted

(a) Supersonic peak

(b) Supersonic trough

(c) Vertical Wall

Figure 4-6: Instantaneous Mie images of the mixing layer downstream of the convoluted plate mixer with $M_p = 1.6$ and $M_s = 0.3$.

Figure 4-7: Instantaneous cross-flow Mie image 2λ from the trailing edge of the convoluted plate with $M_p = 1.6$ and $M_s = 0.3$.

Figure 4-8: Mean Mie images 1, 2, and 3 wavelengths downstream of the convoluted plate with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_\lambda}{p_p} = 1.0$

(a) FP: $x^* = 2$

(b) CP: $x^* = 1$

(c) FP: $x^* = 20$

(d) CP: $x^* = 19$

Figure 4-9: Normalized Pitot pressure fields at two axial locations downstream of the flat and convoluted plates with $M_p = 1.6$, $M_s = 0.2$, and $\frac{P_a}{P_p} = 0.9$.

versus distance from the trailing edge in Figure 4-10. The length of the interface downstream of the flat plate was roughly a constant 1λ . The interface length downstream of the convoluted plate varied from approximately the length of the trailing edge to 1λ . This decrease in interface length occurred as the thickness of the shear layer on the mean interface approached $\frac{1}{2}\lambda$.

Additionally, from these normalized Pitot pressure fields, a normalized mixing measure is calculated—the Pitot mixedness.

$$\mathcal{M}_{pitot}(x) = \frac{\bar{P}_{pit}^A|_{TE} - \bar{P}_{pit}^A(x)}{\bar{P}_{pit}^A|_{TE} - \bar{P}_{pit}^A|_{mixed}}. \quad (4.3)$$

This measure varies between 0 at the mixer trailing edge and 1 at the fully mixed conditions. The Pitot mixedness is plotted versus distance from the trailing edge in Figure 4-11. The rate of increase of $\mathcal{M}_{pitot}$ downstream of the *CP* was initially seen to be ≈ 3 times that downstream of the *FP*; the ratio of the trailing edge lengths of the two mixers was 3.3. The rate of increase of $\mathcal{M}_{pitot}$ remained constant downstream of the flat plate as the time-mean interface length is constant. However, downstream of the convoluted plate, since the length of the time-mean interface decreased, the rate of increase of $\mathcal{M}_{pitot}$ dropped as well.

4.3 Streamwise vorticity enhanced mixing layers

This section contains a discussion of the impact of streamwise vorticity in compressible mixing layers. First, the mixing augmentation associated with streamwise vorticity in compressible mixing layers is discussed. Next, a discrete vortex model of the streamwise vorticity enhanced mixing process is introduced. Using this model and experimental data, the impact of compressibility on the mixing augmentation associated with streamwise vorticity is further described.

4.3.1 Mixing augmentation associated with streamwise vorticity

In this section, in order to assess the magnitude of the mixing augmentation associated with streamwise vorticity in compressible mixing layers, comparisons between the measured Pitot mixednesses 20λ downstream of the three lobed mixers are made.

In Figure 4-12a, the Pitot mixedness 20λ downstream of the 15° forced mixer is plotted versus the Pitot mixedness the same distance downstream of the convoluted plate at the same inflow Mach numbers and the same inflow secondary to primary static pressure ratio. If a mixing augmentation is associated with the presence of the 15° forced mixer relative to the convoluted plate at a particular set of inflow conditions, the point will fall in the white region.

As may be seen in Figure 4-12a, a mixing augmentation was associated with the presence of the 15° mixer in most cases; however, at some inflow conditions the 15° mixer did not perform as well as

Figure 4-10: Mean interface length versus x^* for each of the mixers with $M_p = 1.6$, $M_s = 0.2$, $\frac{P_s}{P_p} = 0.9$.

Figure 4-11: Pitot mixedness versus x^* for each of the mixers with $M_p = 1.6$, $M_s = 0.2$, $\frac{P_s}{P_p} = 0.9$.

Figure 4-12: 15° forced mixer/convoluted plate Pitot mixedness comparison at 20λ from the trailing edges.

the convoluted plate. In Figure 4-12b, the mixing comparison is made only for the inflow conditions at which the primary stream was determined to be attached to the mixer in a Pitot survey 1λ from the mixer trailing edge. As may be seen in 4-12b, a mixing augmentation was associated with the presence of the 15° forced mixer, and with the streamwise circulation it shed, for all the cases in which the primary flow was known to be attached to the mixer.

In Figure 4-13a, the Pitot mixedness 20λ downstream of the 25° forced mixer is plotted versus the Pitot mixedness 20λ downstream of the 15° forced mixer. A mixing augmentation was associated with the 25° mixer relative to the 15° mixer in roughly half of the test cases. In Figure 4-13b, the Pitot mixedness comparisons are made only for those cases which were determined to be attached in a Pitot survey 2λ from the mixer trailing edge. As may be seen in the Figure, a mixing augmentation is associated with the presence of the additional circulation shed by the 25° mixer for all the cases in which the primary flow was known to be attached to the mixer.

4.3.2 Identification of the mixing mechanism associated with streamwise vorticity

In the previous section, the mixing augmentation associated with streamwise vorticity in compressible mixing layers was illustrated. In this section, the fluid dynamic mixing mechanism associated with the vorticity is described.

In Figure 4-14, two spanwise instantaneous Mie images centered on a supersonic peak are shown

Figure 4-13: 25° forced mixer/15° forced mixer Pitot mixedness comparison at 20λ wavelengths from the trailing edges.

downstream of the 25° forced mixer. As was observed downstream of the flat and convoluted plates, structures in the interface grow with distance from the trailing edge of the mixer. In the near-mixer image, the convection of high speed fluid into the center of the subsonic trough by the streamwise vorticity may be observed. (Note the white region near the bottom dotted line growing in height with distance from the trailing edge.)

In Figure 4-15, three instantaneous cross-flow Mie images are shown 2λ from the trailing edge of the 15° forced mixer with $M_p = 1.6$ and $M_s = 0.3$. The unsteady movement of the interface is apparent upon comparing these images; the scale of the movement is of the order of the scale of structures seen in the interface.

In order to determine the full extent of this unsteady movement, these instantaneous images were averaged over 1800 laser pulses. Three mean images 1, 2, and 3λ downstream of the 15° forced mixer produced by this averaging process are shown in Figure 4-16. In these images, the gray shear layer is located between the white supersonic stream and black subsonic stream. The thickness of this shear layer increased with distance from the trailing edge as was observed downstream of the convoluted plate. Additionally, upon comparison of the three images, a cross-stream convection (swirling) of the two streams is observed. This wavelength scale cross-stream movement of the two streams was not observed downstream of the convoluted plate and is the fluid dynamic mechanism through which streamwise vorticity augments the mixing rate between the two streams; this cross-stream swirling distorts and stretches the mean interface between the streams and increases the area across which

Figure 4-14: Instantaneous spanwise Mie images downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.3$.

Figure 4-15: Instantaneous Mie images 2 wavelengths downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$

species, momentum and energy exchange occurs.

In Figure 4-17, mean Mie images are shown 2λ downstream of the three lobed mixers with $M_p = 2.4$ and $M_s = 1.6$. Comparing these images, the level of cross-stream convection and the rate of interface stretching are seen to increase with the shed streamwise circulation (Γ^*).

In Figure 4-18, normalized Pitot pressure fields are shown at two locations downstream of the 15° and 25° forced mixers. The inflow conditions at which these surveys were taken are the same as those at which the surveys downstream of the flat and convoluted plates presented in Figure 4-9 were conducted. The primary Mach number was 1.6; the secondary Mach number was 0.3, and the secondary to primary inflow static pressure ratio was 0.9. Again, in these normalized Pitot fields an increased rate of interface stretching is associated with increasing Γ^* .

Figure 4-16: Mean Mie images 1, 2, and 3 wavelengths downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$

Figure 4-17: Mean Mie images 2 wavelengths downstream of the three lobed mixers with $M_p = 2.4$, $M_s = 1.6$, and $\frac{p_s}{p_p} = 1.0$

(a) 15° FM: $x^* = 2$

(b) 25° FM: $x^* = 3$

(c) 15° FM: $x^* = 20$

(d) 25° FM: $x^* = 21$

Figure 4-18: Normalized Pitot pressure fields at two axial locations downstream of the 15° and 25° forced mixers with $M_p = 1.6$, $M_s = 0.2$, and $\frac{p_1}{p_p} = 0.9$.

From these normalized Pitot pressure fields and surveys at two additional locations, the mean interface length between the two streams was calculated; it is plotted versus distance from the trailing edge in Figure 4-10. In this Figure, the interface lengths at the survey planes closest to the two forced mixers are greater than the trailing edge length of the mixers. The interface growth between the trailing edge and the first survey plane was due to the stretching of the interface by streamwise vorticity. As the thickness of the shear layer on the interface approached $\frac{1}{2}\lambda$, the mean interface length dropped to the planar shear layer interface length; a similar decrease in interface length was observed downstream of the convoluted plate.

In Figure 4-11, the normalized Pitot mixedness distributions downstream of each of the mixers with $M_p = 1.6$ and $M_s = 0.3$ are plotted. The distributions downstream of the two forced mixers are similar to that downstream of the convoluted plate; all three are characterized by an initial region of rapid mixing followed by a transition to a region of slower mixing as the thickness of the shear layers approaches $\frac{1}{2}\lambda$. Comparing the mixedness profiles downstream of each of the mixers, it is evident that a mixing augmentation is associated with the increased interface length of the lobed mixers, and an additional augmentation is associated with the presence of streamwise circulation. For the mixer geometries and flow conditions at which these measurements were taken, at 20λ from the mixer trailing edge, a 31% increase in the Pitot mixedness was associated with the increased initial interface length of the convoluted plate. An additional 8% augmentation was associated with the streamwise circulation generated by the 15° mixer, and an additional 5% increase in Pitot mixedness was associated with the extra circulation generated by the 25° forced mixer.

However, it will be demonstrated in the remainder of this thesis that the mixing augmentation associated with streamwise circulation is a function of the mixer height to wavelength ratio, the shear layer growth rate, and the trailing edge boundary layer thickness. The impacts of these three parameters on the augmentation associated with Γ^* will be illustrated with the aid of a discrete vortex model developed from the physical understanding of the mixing process gained from these experiments and from the research of a number of previous investigators[11, 19, 21, 23, 33, 20, 30].

4.3.3 Vortex mixing model

This section contains a description of a discrete vortex mixing model developed to help quantify the mixing augmentation associated with streamwise vorticity and to quantify the impact of compressibility on that augmentation. A full derivation and assessment of the sensitivity of the model predictions to the input parameters may be found in Appendix D. Following the description, comparisons are made between the models predictions and experimental Mie and Pitot pressure data.

A series of point vortices are used to model the cross-stream velocity field downstream of the mixers. The approach taken is similar to that developed by Qiu [23]. A slender body approximation is made in which a steady three-dimensional flow field is modelled by an unsteady two-dimensional

one. In this approximation, time in the unsteady two-dimensional field corresponds to the distance from the trailing edge divided by the mean axial velocity($\frac{x}{U}$).

A series of vortices are initially positioned to model the streamwise vorticity distribution shed by the lobed mixer. The interface between the streams is the curve connecting these vortices. This interface evolves with distance from the trailing edge as the vortices convect in the induced velocity field as calculated from

$$v_k^*(\zeta_k^*, t^*) = \left(\frac{i}{2\pi} \right) \sum_{j=1}^N \Gamma_j^{**} \frac{\zeta_k^* - \zeta_j^*}{|\zeta_k^* - \zeta_j^*|^2} \left(1 - e^{\frac{-Re_\Gamma}{2i} (\zeta_k^* - \zeta_j^*)^2} \right). \quad (4.4)$$

The complex positions of the vortices are given by ζ_j ; the normalized circulations are given by Γ_j^{**} . Re_Γ is the circulation Reynold's number; it is the total streamwise circulation normalized by the turbulent viscosity. A passive scalar is also tracked in order to calculate the scalar mixedness and the effective interface length for the calculation of the rate of transport of mass, momentum, or energy in the differential control volume analysis discussed in Chapter 5.

In Figure 4-19a, the paths in a cross-flow plane that these vortices take over the first two wavelengths downstream of the trailing edge of the 15° forced mixer with a shear layer growth rate(δ'_{vel}) of 0.10 and an initial boundary layer thickness(ϵ^*) of 0.25 are drawn; the curve connecting each of the vortices at 2λ is marked in bold. In order to determine L_{eff} , a passive scalar is tracked. In Figure 4-19b, the fully mixed scalar field is shown downstream of the same mixer and at the same flow conditions for which the paths in Figure 4-19a are drawn. From these scalar fields, the scalar mixedness(M_{sc}) is calculated as a function of distance from the trailing edge. M_{sc} varies between an unmixed value of 0 and a fully mixed value of 1.

In Figure 4-20, the modelled scalar mixedness is plotted versus distance from the trailing edge for all four mixers investigated experimentally with $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$. The modelled trends are similar to those previously observed downstream of subsonic mixers [11, 19, 23, 30]. An increasing initial mixing rate is associated with a increasing shed streamwise circulation, and a decrease in the mixing rate occurs as the thickness of the shear layer on the mean interface approaches a half-wavelength. The scalar mixedness is non-zero at the trailing edge due to the finite initial thickness of the mixing layer.

In Figure 4-21, a comparison between the modelled fully mixed scalar fields and the measured passive scalar fields are shown 1, 2, and 3 wavelengths from the trailing edge of the 15° forced mixer with $M_p = 2.4$ and $M_s = 1.6$. In the model, the shed circulation($\Gamma^* = 0.5$) was estimated using the Barber *et al.* scaling law[2]; the shear layer growth rate($\delta'_{vel} = 0.02$) was estimated using the correlation introduced in Section 4.1; and the thickness of the boundary layer($\epsilon^* = 0.05$) at the trailing edge was estimated from the Mie image at 1λ . Furthermore, in the model, the cross-flow was assumed to be incompressible and neither of the two compressibility effect models to be discussed

Figure 4-19: Vortex paths from 0 to 2λ and scalar field at 2λ for $\Gamma^* = 0.5$, $\delta'_{vet} = 0.10$, and $\epsilon^* = 0.25$.

Figure 4-20: Modelled axial scalar mixedness distribution downstream of all 4 mixers with $\delta'_{ve} = 0.1$ and $\epsilon^* = 0.25$.

in Section 4.3.4 were utilized.

In Figure 4-22, another comparison is made between the modelled fully mixed scalar fields and the measured scalar fields downstream of the 15° forced mixer. In this comparison, the inflow Mach numbers are 1.6 and 0.3. The shear layer growth rate as predicted by the scaling law in Section 4.1 and trailing edge boundary layer thickness as estimated from the first Mie image are five times that of the test case presented in Figure 4-21. Once again, the cross-flow is assumed incompressible.

(a) Mie: $x^* = 1$

(b) Mie: $x^* = 2$

(c) Mie: $x^* = 3$

(d) Model: $x^* = 1$

(e) Model: $x^* = 2$

(f) Model: $x^* = 3$

Figure 4-21: Vortex model and Mie data cross-flow interface structure comparison. Cross-flow planes are 1λ , 2λ , and 3λ downstream of the 15° mixer with $M_1 = 2.4$ and $M_2 = 1.6$. $M_c = 0.22$, $r = 0.80$, $s = 0.70$. Model parameters: $\Gamma^* = 0.5$, $\delta'_{vel} = 0.02$, and $\epsilon^* = 0.05$.

(a) Mie: $x^* = 1$

(b) Mie: $x^* = 2$

(c) Mie: $x^* = 3$

(d) Model: $x^* = 1$

(e) Model: $x^* = 2$

(f) Model: $x^* = 3$

0

0.5

1

Figure 4-22: Vortex model and Mie data cross-flow interface structure comparison. Cross-flow planes are 1λ , 2λ , and 3λ downstream of the 15° mixer with $M_1 = 1.6$ and $M_2 = 0.3$. $M_c = 0.56$, $r = 0.21$, $s = 0.67$. Model parameters: $\Gamma^* = 0.5$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.

As may be noted in comparison between the scalar fields in Figures 4-21 and 4-22, there is a significant qualitative difference in their structure. The shear layer between the mixed streams is much thicker and grows faster in Figure 4-22 than in Figure 4-21. In Figure 4-21, the streamwise vorticity is more effective in stretching the cross-flow interface between the two streams. The thicker shear layer and reduced interface stretching in Figure 4-22 are associated with the increased rate of diffusion of the streamwise vorticity due to the more rapid growth of the shear layer. Hence, in the high diffusion case, the streamwise vorticity is less effective in augmenting the mixing rate. However, it is important to note that the overall mixing rate is higher due to the more rapid growth of the shear layer. This point is illustrated in one of the vortex model sensitivity studies presented in Section D.7.2.

In Figures 4-23 to 4-25, comparisons are made between the measured and modelled mean interfaces. The experimental mean interface is defined as the length of the 0.5 contour in the mean the normalized Pitot pressure fields. The modelled mean interface is the length of the contour between the primary stream and mixed region in the scalar field. The model captures the decrease of mean interface length as the shear layer thickness approaches $\frac{1}{2}\lambda$ as well as the interface stretching associated with streamwise vorticity.

Figure 4-23: Convoluted plate model/Mie mean interface length comparison for $M_p = 1.6$, $M_s = 0.3$, and $\frac{\rho_s}{\rho_p} = 0.9$.

Figure 4-24: 15° forced mixer model/Mie mean interface length comparison for $M_p = 1.6$, $M_s = 0.3$, and $\frac{\rho_s}{\rho_p} = 0.9$.

Figure 4-25: 25° forced mixer model/Mie mean interface length comparison for $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 0.9$.

4.3.4 Impact of compressibility on mixing augmentation associated with Γ^*

This section contains a discussion of three compressibility effects in planar shear layers. First, a decreasing mixing rate is associated with an increasing convective Mach number. Second, the movement of the interface, and the streamwise vorticity on that interface, due to a mismatch of the stream static pressures is addressed. Third, a model for the vorticity generation downstream of the mixer associated with shock-density gradient interactions is presented.

Decreasing shear layer growth rate In section 4.1, a decreasing planar shear layer growth rate was associated with increasing compressibility as measured by the convective Mach number. For lobed mixer flows, this decreasing mixing rate with increasing M_c has been indirectly observed in Pitot/static pressure survey data.

In Figure 4-26, the normalized mass average total pressure (Ψ) is plotted 20λ downstream of the *FP*, *CP*, and 15° *FM* at two different convective Mach numbers at approximately the same velocity ratio. The mass averaged total pressure was calculated by integrating the Mach number and total pressure distributions across the plane. To isolate the calculation of Ψ from losses associated with skin friction on the walls of the test section, the integration was performed only over the center wavelength span and outside the subsonic and supersonic wall boundary layers. Furthermore, to isolate the mass-averaged *Pt* loss associated with the mixing process from that associated with the flow over the mixers themselves, an oblique shock loss model was used to estimate the reduction in mass-averaged total pressure over the mixers. In this model, half the uniform supersonic inflow was assumed to pass through an oblique shock at the leading edge of the forced mixer and convoluted plate, while the other half was assumed to expand isentropically into the lobe troughs. Additionally, for the convoluted plate, the expanded trough flow was assumed to pass through an oblique shock at the onset of the straight section as the flow turned back into the axial direction.

The mixer outflow to inflow mass-averaged total pressure ratios estimated with this model are listed in Table 4.1. The flat plate mixer operates shock free; hence, no reduction in total pressure is predicted. Also, no reduction in total pressure is predicted over the 15° forced mixer; for the Mach numbers investigated, the total pressure loss over an oblique shock at the 8° exterior wedge angle is insignificant. For the convoluted plate with a Mach 1.3 inflow, the losses through both sets of shocks were negligible; however, for a Mach 1.6 inflow, a 4 percent drop in total pressure was estimated to occur through the oblique shocks at the onset of the straight section of the mixer. This loss in total pressure over the mixer represented 25% of the mixing loss that occurred over the first 20λ of the mixing duct.

The data presented in Figure 4-26 indicate that in mixing layers downstream of lobed mixers a mixing penalty is indeed associated with increased convective Mach number at constant velocity

Table 4.1: Mixer outflow/inflow total pressure ratio per the oblique shock loss model for three of the mixers at $M_p = 1.3$ and $M_p = 1.6$.

Mixer	$M_p = 1.3$	$M_p = 1.6$
Flat Plate	1.0	1.0
Convoluted Plate	0.96	1.0
15° Forced Mixer	1.0	1.0

ratio.

In Section 4.3.3, an increasing importance of streamwise vorticity has been associated with a decreasing shear layer growth rate, and in Section 4.1 a decreasing shear layer growth rate has been related to an increasing convective Mach number at a constant velocity ratio. Hence, one would expect an increasing importance of streamwise vorticity with increasing M_c .

In Figure 4-27, the modelled scalar mixing augmentation associated with the streamwise circulation shed by the 15° forced mixer is plotted versus M_c and x at a fixed velocity ratio. The mixing augmentation at a particular axial location and convective Mach number is defined by

$$\Delta M_{sc}|_{\Gamma^*=0.5} = \frac{M_{sc}|_{\Gamma^*=0.5} - M_{sc}|_{\Gamma^*=0.01}}{M_{sc}|_{\Gamma^*=0.01}}. \quad (4.5)$$

In Figure 4-27, the mixing augmentation associated with streamwise vorticity at any location downstream of the mixer increases with increasing convective Mach number.

Movement of vorticity due to mismatched stream static pressures In supersonic-supersonic and supersonic-subsonic mixing layers, the static pressures of the two inflow streams need not be matched at the trailing edge of the mixers. If the pressures are unmatched, some expansion or compression of the two streams occurs in order to match the stream static pressures. This pressure matching process results in area changes in both streams and in displacement of the interface and the streamwise vorticity on that interface as shown in Figure 4-28. In this figure, the trailing edge static pressure in the gray(subsonic) stream is higher than that in the white(supersonic) stream. Downstream of the trailing edge, a mean upward displacement of the interface will occur as well as a decrease in the primary to secondary trough stream width ratio.

A quasi-one dimensional control volume model of this process was developed; a full derivation of the model appears in Appendix E. In this model, the inflow conditions of both streams are specified. The conservation of mass, momentum, and energy are solved with the constraints that the secondary stream is isentropic and that the stream static pressures are matched at the exit of the control volume.

In order to assess the impact of the displacement of the vortex sheets on the mixing rate, several test cases with the pressure matching and interface vortex models were conducted. In Figure 4-29,

Figure 4-26: Normalized mass-averaged total pressures 20λ downstream of the trailing edge of the flat plate, convoluted plate and 15° forced mixer at $M_c = 0.58$, $r = 0.20$ and $M_c = 0.43$, $r = 0.25$.

Figure 4-27: Scaling mixing augmentation associated with the streamwise circulation shed by the 15° forced mixer versus the convective Mach number and distance from the trailing edge for a velocity ratio of 0.2

Figure 4-28: Interface deflection due to unmatched primary/secondary static pressures at the trailing edge of the mixer

Figure 4-29: Matched pressure trough area ratio versus trailing edge static pressure ratio for $M_p = 1.6$ and $M_s = 0.3$.

for an initially symmetric lobed mixer, the resulting matched pressure, secondary/primary trough width ratio is plotted versus the secondary/primary static pressure ratio for $M_p = 1.6$ and $M_s = 0.3$. The secondary/primary trough width ratio plotted in Figure 4-29 is calculated assuming that the vortex sheets shed from the vertical walls of the mixer remain vertical during the pressure matching process.

In Figure 4-30, the passive scalar fields are drawn 2λ downstream of a forced mixer with $\Gamma^* = 0.5$ and secondary to primary trough width ratios of 0.8, 1.0, and 1.25. At inflow Mach numbers of 1.6 and 0.3, these trough width ratios represent the pressure matched plane width ratios downstream of a symmetric lobed mixer with trailing edge secondary to primary static pressure ratios of 0.8, 1.0 and 1.3. In Figure 4-30, several qualitative differences between the scalar fields are evident. The displacement of the section of the mixing layer shed from the vertical side walls of the mixer is

Figure 4-30: Fully mixed scalar fields 2λ from the trailing edge with $\Gamma^* = 0.5$; $\delta'_{v\epsilon,l} = 0.10$; $\epsilon = 0.25$; and secondary/primary trough width ratios of 0.80, 1.00, 1.25.

apparent. The resulting asymmetry in the vorticity distribution causes a bulk downward convection of the vorticity when $\frac{w_s}{w_p} < 1$ and a bulk upward convection of the vorticity when $\frac{w_s}{w_p} > 1$.

However, despite the qualitative differences between the scalar fields shown in Figure 4-30, the trough width ratio has no significant impact on the predicted scalar mixedness distributions downstream of the mixers. In Figure 4-31, the scalar mixedness 20λ from the mixer trailing edge is plotted versus the secondary to primary trough width ratio: no significant variation in the mixedness with $\frac{w_s}{w_p}$ is apparent.

Generation of vorticity downstream of mixer In the supersonic-subsonic mixer flows investigated in these experiments, shocks located in the vicinity of the mixer trailing edge interacting with the density gradient across the mixing layer can generate levels of streamwise circulation comparable to that shed by the mixer. In this section, an analytical model for the vorticity generation is derived and implemented in the discrete vortex model. The predicted scalar fields are then compared with experimental Mie data.

In Figure 4-34, a Schlieren photograph of flow over the 15° forced mixer with $M_p = 1.6$, $M_s = 0.7$, and $\frac{p_s}{p_p} = 1.0$ is shown. In the photograph, an oblique shock is located at the trailing edge of the supersonic trough. This shock turns the primary trough flow back towards the axial direction.

Both the angle of this shock and the direction of the primary flow downstream of it are functions of the secondary/primary static pressure ratio and the primary Mach number. Over the range of primary Mach numbers investigated, when the trailing edge static pressures were matched, the primary flow turned back into approximately the axial direction. (The direction of the primary

Figure 4-31: Scalar mixedness at 20λ versus $\frac{w_s}{w_p}$. $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.

flow was estimated from the angle of the bottom of the mixing layer in the Schlieren photographs.) As the secondary/primary static pressure ratio was decreased, the turning of the primary trough flow became less abrupt. On the other hand, as the static pressure ratio was increased, the shock strengthened and eventually moved back up into the trough causing the flow to separate.

If a density gradient exists across the mixing layer, the interaction of the shock and density gradient will result in the generation of streamwise circulation. For the supersonic-subsonic, equal total temperature streams of these experiments, the secondary/primary density and velocity ratios (s and r) were less than one. In mixing layers where $s < 1$, the mechanism for the generation of the circulation is illustrated in Figure 4-34. In this Figure, an oblique shock located at the trailing edge of the primary trough turns the primary flow back into the axial direction and increases the static pressure of the stream. The secondary stream sees approximately the same pressure gradient as the primary, and this pressure gradient turns the secondary flow farther off axis increasing the streamwise circulation.

An analytical model of this shock/mixing layer interaction was derived in order to estimate the streamwise circulation generated downstream of the trailing edge of the mixers. (The full derivation of this model is contained in Appendix F; a summary is contained in the next few paragraphs.)

The rate of change of the streamwise circulation with distance from the trailing edge of the mixer

Figure 4-32: Schlieren photograph of 15° forced mixer with $M_p = 1.6$ and $M_s = 0.7$ with an oblique shock at the trailing edge of the primary trough.

Figure 4-33: Cartoon illustrating generation of streamwise vorticity by the baroclinic torque associated with the interaction of the trough shock and a density gradient across the mixing layer. Note that in this schematic the secondary/primary density ratio is less than 1.

Figure 4-34: Schematic illustrating vorticity generation associated with a shock interacting with a density gradient

is found from Kelvin's theorem:

$$\frac{D\Gamma}{D\left(\frac{t}{U}\right)} = - \int \int \left(\frac{\nabla \rho \times \nabla p}{\rho^2} \right) \cdot \hat{n} dA. \quad (4.6)$$

If the trough flow is assumed to leave the mixer at the metal angle(α) and turn back into the axial direction through an oblique shock of angle θ with respect to the axial direction and if the static pressure and density are assumed to vary linearly across the shock and mixing layer respectively, Eqn. 4.6 may be integrated from the trailing edge of the mixer to the downstream end of the shock to calculate the total circulation generated by the shock. The ratio of this shock to mixer circulation is given by Eqn. 4.7.

$$\frac{\Gamma_{shock}}{\Gamma_{mixer}} = \frac{-8}{\gamma M_p^2} \frac{s-1}{(s+1)^2(r+1)^2} \left(\frac{p_2}{p_1} - 1 \right) \frac{1}{\tan \alpha (\tan \alpha + \tan \theta)}, \quad (4.7)$$

where $\frac{p_2}{p_1}$ is the static pressure ratio across an oblique shock with an inflow Mach number of M_p and a flow turning angle of θ .

In Figure 4-35, the ratio of the total shock to mixer circulation is plotted versus the density ratio for a lobe angle of 15° ; a primary Mach number of 1.6; and velocity ratios of 0.2, 0.4, 0.6, 0.8, 1.0. From Figure 4-35 and Eqn. 4.7, it may be seen that if $s < 1$, circulation is generated in the same sense as that generated by the mixer. On the other hand, if $s > 1$, circulation is generated in the opposite sense as that generated by the mixer.

In order to assess the impact of this circulation generation on the mixing rate downstream of these mixers, the baroclinic torque model was implemented in the discrete vortex model. The location of the shock was determined from a simplified model of the trough flow. Given the two stream Mach numbers and total temperature ratio, the circulations of the individual vortices were adjusted by Eqn. 4.7 as they passed through the shock.

Three test cases were conducted with the model with a primary Mach number of $M_p = 1.6$; a secondary Mach number of 0.3, and primary to secondary total temperature ratios(Θ) of 1.0, 1.5, 3.0. The low total temperature ratio case has a density ratio of 0.7 and is representative of the conditions at which experimental Mie data was taken; circulation will be generated in the sense of that shed by the mixer. The middle total temperature ratio case has a density ratio of 1.0, and no circulation is generated. The secondary to primary density ratio is 2.0, and circulation will be generated in the opposite sense of that generated by the mixer.

In Figure 4-36, the fully-mixed scalar fields are shown at 1, 2, and 3λ from the trailing edge at density ratios of 0.7 and 2.0. (The fully mixed scalar fields are shown at a density ratio of 1.0 in Figure 4-22.) In comparison between the scalar fields, a decreasing primary to secondary total temperature ratio and the corresponding decrease in the secondary to primary density ratio is seen to result in an increasing effectiveness of the mixer as additional circulation is generated downstream of

Figure 4-35: Shock/mixer circulation ratio vs. secondary/primary density ratio for 15° forced mixer with $M_p = 1.6$.

the trailing edge. This additional circulation increases the rate of stretching of the interface. On the other hand, an increase in Θ and a corresponding increase in s results in the generation of circulation in the sense opposite of that shed by the mixer. This shock-generated circulation counteracts that generated by the mixer; a decrease in the rate at which the interface is stretched results.

In Figure 4-37, the axial scalar mixedness distributions are plotted for the three test cases and a convoluted plate ($\Gamma^* = 0$) baseline. In this figure, the trend of a increasing mixing rate with decreasing density ratio suggested by the scalar fields in Figure 4-36 is shown; the scalar mixedness at 20λ decreases by $\approx 15\%$ as the density ratio increases from 0.7 to 2.0.

Also in Figure 4-37, the mixing rate downstream of the convoluted plate mixer is still less than that downstream of the forced mixer with the highest density ratio. The model predicts that the forced mixer mixing rate will always be equal to (if all the vorticity is cancelled) or higher than that downstream of the convoluted plate rate.

In the experiments, all measurements were taken at one total temperature ratio; hence, no direct comparison of the modelled trends in Figures 4-36 and 4-37 and experimental data may be performed. However, qualitative comparisons may be drawn between the modelled scalar fields at density ratios of 0.7 and 1.0 and the Mie data obtained at $s = 0.7$ shown in Figure 4-22.

The $s = 1.0$ model case represents an incompressible model of the mixing process; circulation is generated by the mixer only. As seen in Figure 4-22, this incompressible model of the interface

Figure 4-36: Fully mixed scalar fields 1, 2, and 3λ downstream of the 15° forced mixer with $\Gamma^* = 0.5$, $\delta'_{vel} = 0.1$, and $\epsilon^* = 0.25$.

Figure 4-37: Modelled M_{scalar} distributions downstream of the 15° forced mixer($\Gamma^* = 0.5$) with $M_p = 1.6$; $M_s = 0.3$, and $s = 0.7, 1.0, 2.0$. The modelled scalar mixedness is also plotted for the baseline convoluted plate($\Gamma^* = 0.01$). For all cases, $\delta'_{vel} = 0.1$, and $\epsilon^* = 0.25$.

qualitatively captures the evolution of the interface. However, the scalar fields in Figure 4-36 for $s = 0.7$ bear closer resemblance to the Mie images in Figure 4-22.

4.4 Chapter Summary

In this chapter, an assessment was made of the impact of compressibility on the vorticity enhanced mixing process. The mixing mechanism associated with streamwise vorticity in compressible flows was shown to be the same as it is in incompressible flows—the stretching of the cross-flow interface with distance from the trailing edge.

From the physical understanding gained from the experiments and previous incompressible lobed mixer research[23, 11, 21, 30], a discrete vortex model of the evolution of the interface was developed; this model runs 100 times faster than three-dimensional Navier-Stokes. The speed and flexibility of this model has enabled a more detailed investigation of the impact of compressibility on the vorticity enhanced mixing process than was possible experimentally.

Compressibility has been shown to have a three-fold impact on the mixing augmentation associated with streamwise vorticity. First, the decreased shear layer growth rates associated with increasing convective Mach number leads to an increased effectiveness of streamwise vorticity. Second, streamwise vorticity may be generated downstream of the mixing by the interaction of the trailing edge shock with a density gradient across the mixing layer. If the subsonic to supersonic density ratio is less than one, vorticity is generated in the sense of that shed by the mixer—a mixing augmentation results. If the density ratio is greater than one, vorticity is generated in the opposite sense of that shed by the mixer—a mixing penalty results. Third, the vortex sheets may be displaced by an area adjustment of the two streams associated with a mismatch of the stream static pressures. This displacement has little impact on the mixing augmentation associated with streamwise vorticity.

Chapter 5

Development of a Streamwise Vorticity Enhanced Mixer-Ejector Model

In order to make ejector pumping and thrust predictions, the vortex model was linked to a series of control volume models. This chapter contains a description of this integrated model. Comparisons are made to experimental Pitot and wall static pressure data. The model to be described runs 100 times faster than three-dimensional Navier-Stokes computations. Its speed and flexibility have enabled design trade and optimization studies to be conducted. Several example trade studies are presented in Chapter 6; a sample mixer optimization study is then presented in Chapter 7.

5.1 Summary

The objectives of developing the streamwise vorticity enhanced mixer-ejector model were to devise a fast(relative to 3D N-S) method to predict the pumping augmentation associated with streamwise vorticity and to develop a quantitative model to assess sensitivity of compressible ejector pumping performance to key design parameters. The approach taken was to model only the physical processes essential to making useful performance predictions.

The mixer-ejector model is composed of four sub-models:

- A mixer/inlet model, to determine the streamwise circulation and to estimate the trailing edge conditions given the mixer geometry, leading edge flow conditions, and mixer drag coefficient;
- A stream static pressure matching control volume, to match the stream static pressures downstream of the mixer trailing edge by adjusting their areas and Mach numbers given the trailing

edge area ratio and the trailing edge flow conditions;

- A vortex model, to calculate the effective interface length as a function of distance from the trailing edge given pressure matched flow conditions;
- A differential control volume model, to determine the static and total pressure distribution through the duct given the pressure matched flow conditions, duct geometry, and effective interface length distribution.

A schematic of a mixer-ejector is drawn in Figure 5-1 with the locations of the different sub-models indicated.

The inputs required by the complete model are the mixer and duct geometries and the inflow conditions. Given the inflow conditions, the duct static and total pressure distributions are solved in a time marching fashion. To calculate the pumping for a given geometry and nozzle pressure ratios, the secondary mass flow must be iterated upon.

The key assumptions and limitations of each of the four sub-models are addressed in the detailed derivations of the models in the Appendices. The remainder of this chapter contains descriptions of each of the sub-models and comparisons between their predictions and experimental data.

5.2 Lobed mixer model

The lobed mixer sub-model is composed of two components. Barber *et al.*'s[2] streamwise circulation model is used to calculate the shed circulation given the inner lobe angle and trailing edge shape. Two one-dimensional control volumes (one for each stream) are used to calculate the trailing edge conditions given the leading edge conditions, the trailing edge to leading and area ratios, and the mixer drag coefficient.

The inputs required to the mixer sub-model are

- the mixer trailing edge shape,
- the mixer inner angles,
- the mixer drag coefficient,
- the trailing edge to leading edge area ratios, and
- the leading edge flow conditions,

The outputs of the mixer sub-model are

- the shed streamwise circulation(Γ^*), and
- the one-dimensional trailing edge flow conditions.

Figure 5-1: Schematic of a mixer-ejector noise suppressor model with locations of sub-models indicated.

Figure 5-2: Schematic of mixer control volume models

The normalized streamwise vorticity distribution must be ascertained either experimentally or with Euler or Navier-Stokes computations over the lobes. However, the distribution for an attached mixer is not expected to significantly change with changing inflow conditions.

Streamwise Circulation The non-dimensional circulation shed from a one-half wavelength span of a mixer is estimated using the Barber *et al.*[2] model as defined in Eqn. 2.1.

For the approximately square lobe mixers utilized in the current study, all the trough flow is assumed to leave at the mixer inner angle. This assumption is supported by the fluorescent dye mixer flow visualization pictures shown in Figure 3-4 with attached trough flow. Hence, the streamwise vorticity shed is constant along the vertical side walls. For more complicated mixer geometries, Euler or Navier-Stokes computations over the mixers may be required to estimate the trailing edge streamwise vorticity distribution.

Trailing edge flow conditions One-dimensional trailing edge flow conditions for each stream are calculated from the leading edge conditions given the trailing edge to leading edge area ratios and the mixer drag coefficients for each stream. The models are sketched in the schematic shown in Figure 5-2.

For all of the model/data comparisons presented in this thesis, the mixer drag coefficients were assumed to be 0. For the flat plate mixer, the secondary trailing edge to leading edge area ratio ($\frac{A_{s2}}{A_{s1}}$) is 0.83, and the primary trailing edge to leading edge area ratio ($\frac{A_{p2}}{A_{p1}}$) is 1. For the lobed mixers, the secondary area ratio is 0.73, and the primary area ratio is 1.07.

No direct assessment of the lobed mixer control volume models was performed with the experimental data obtained. However, a comparison of the combined mixer and pressure matching models

with experimental static pressure data is contained in Section 5.3.

5.3 Static pressure matching control volume

In the supersonic-subsonic mixing layers investigated in these experiments, the individual stream static pressures need not be matched at the trailing edge of the mixers. If the pressures are unmatched, some expansion or compression of the two streams will occur downstream of the trailing edge in order to match the two stream static pressures.

If the subsonic to supersonic stream static pressure ratio is less than 1, the supersonic stream expands into the subsonic stream, and both streams are accelerated as the static pressures are matched, as illustrated in the cartoon in Figure 5-3. If the subsonic-supersonic static pressure ratio is greater than 1, the subsonic stream expands into the supersonic and both streams are decelerated as the stream static pressures match, as sketched in Figure 5-4

A two-stream, quasi-one-dimensional control volume model of the pressure matching process was developed. In the model, the inflow conditions are specified, the subsonic stream is assumed to be isentropic, and the static pressures of the two streams are matched at the exit. Furthermore, the pressure matching is assumed to occur in a constant area duct, and in the fully integrated model, the pressure matching region is assumed to be infinitesimally thin. In Appendix E, the model is derived, and the sensitivity of the matched point conditions to the trailing edge static pressure ratio and the two trailing edge Mach numbers is assessed.

In Figures 5-5, 5-6, and 5-7, comparisons between the predicted matched supersonic pressure coefficient and a subsonic wall measurement are presented. The supersonic pressure coefficient at a particular axial location in the duct is defined by Eqn. 5.1.

$$Cp_{super}(x^*) = \frac{p(x^*) - p_p|_{LE}}{Pt_p - p_p|_{LE}}. \quad (5.1)$$

The comparisons are made for each mixer at primary Mach numbers of 1.3, 1.6, and 2.4. For leading edge secondary/primary static pressure ratios less than 1.2, the model works well for all mixers at all primary Mach numbers. For static pressure ratios greater than 1.2, the model over predicts the static pressure rise. This over prediction may be the result of separations associated with shocks moving into the mixers and eventually the nozzle. (Recall the mixer performance discussion in Chapter 3.) The implications of these errors in Cp_{super} predictions on the ability of the model to make ejector performance predictions are discussed in Section 5.6.2.

Figure 5-3: Cartoon of expansion process which matches the stream static pressures when $\frac{p_t}{p_p} < 1$.

Figure 5-4: Cartoon of compression process which matches the stream static pressures when $\frac{p_t}{p_p} > 1$.

Figure 5-5: Comparison between modelled matched point Cp_{super} and measured Cp_{super} at the first pressure tap downstream of the trailing edge of each of the mixers. $M_p = 1.3$.

Figure 5-6: Comparison between modelled matched point Cp_{super} and measured Cp_{super} at the first pressure tap downstream of the trailing edge of each of the mixers. $M_p = 1.6$.

Figure 5-7: Comparison between modelled matched point Cp_{super} and measured Cp_{super} at the first pressure tap downstream of the trailing edge of each of the mixers. $M_p = 2, 4$.

5.4 Vortex interface model

Given the matched point flow conditions from the differential control volume and the shed circulation and vorticity distribution, the vortex interface model introduced in Chapter 4 and derived in Appendix D is used in the integrated ejector model to calculate the effective interface length (L_{eff}) between the two streams. This effective interface length is then used in the differential control volume, where the local rate of transport of momentum, energy, and the passive scalar is proportional to L_{eff} .

The effective interface length as a function of distance from the trailing edge is calculated from the M_{sc} distribution by

$$L_{eff}^* = \frac{\frac{\partial M_{sc}}{\partial x^*}}{\frac{\partial M_{sc}}{\partial x^*}|_{FP}}. \quad (5.2)$$

As may be seen in Eqn. 5.2, L_{eff} at a particular axial location downstream of a forced mixer is the ratio of its scalar mixing rate to the scalar mixing rate downstream of a flat plate mixer at the same inflow conditions. The L_{eff} distributions downstream of each of the 4 mixers calculated from the M_{sc} distributions in Figure 4-20 are plotted in Figure 5-8.

In order for the vortex code be useful as a design/optimization tool, a specific effort was under-

taken to make the code as computationally efficient as possible. To this end, only a half-wavelength span of vortices were tracked as the rest of the mixing layer may be constructed from symmetry. Boundary condition sensitivity studies were conducted to determine the minimum number of image lobes required to adequately satisfy the wall/symmetry boundary conditions. Additionally, a procedure was derived[13] to automatically calculate the positions of the image vortices so that it was only necessary to track the positions of the half-wavelength span of vortices.

In the simulations performed with the experimental mixer geometries(h^*), run times on an IBM RS6000 varied between 3 minutes for the convoluted plate and 300 minutes for the 25° with a shear layer growth rate of $\delta'_{vel} = 0.02$. Over the range of growth rates of main interest($\delta'_{vel} \approx 0.1$) in the experiments, run times were of the order of 10 minutes.

Despite the effort to make the vortex code run as fast as possible, it remains the slowest component of the ejector model by at least a factor of 10. Hence, in the interest of computational efficiency, when a large number of runs were to be performed with the ejector model for a given mixer, a look-up table for L_{eff} was constructed. If the cross-flow is assumed incompressible, the effective interface length downstream of a given mixer is a function of only δ'_{vel} and ϵ^* . If the trailing edge boundary layer thickness is assumed constant, a look-up table for the effective interface length as a function of δ'_{vel} and x^* may be constructed. In Figure 5-9, the look-up table constructed for the 15° forced mixer is plotted.

5.5 Differential control volume model

Given the matched point flow conditions from the static pressure matching control volume and the effective interface length distribution from the interface vortex model, the differential control volume calculates the axial total and static pressure distributions.

In the differential control volume, the two co-flowing streams interact via the exchange of momentum, thermal energy, and a passive scalar. The mass, momentum, and energy conservation equations are integrated from the unmixed state at the exit of the pressure matching control volume to the duct exit. This control volume is described in more detail and the sensitivity of its predictions to key parameters are assessed in Appendix G.

In Figure 5-11, as an example of the output of the differential control volume, the four normalized mixing measures are plotted versus distance from the trailing edge of the 15° forced mixer with inflow Mach numbers of 1.6 and 0.3 and an inflow secondary to primary static pressure ratio of 0.9. In Figure 5-11, the primary and secondary stream Mach number distributions are plotted for the same inflow conditions.

Figure 5-8: Modelled effective interface length distribution downstream of all 4 mixers with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$.

Figure 5-9: 15° forced mixer effective interface length distribution as a function of δ'_{vel} and x^* .

Figure 5-10: Modelled effective interface length distribution downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$.

Figure 5-11: Stream Mach number distributions downstream of the 15° forced mixer with $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 1.0$.

5.6 Integrated model/data comparisons

In this section, for the purpose of validating the ejector model, comparisons are made between the model predictions and experimental Pitot and wall static pressure data. Comparisons are first made to the normalized Pitot mixedness at four axial planes downstream of each of the mixers with inflow Mach numbers of 1.6 and 0.3 and a leading edge static pressure ratio of 0.9. Second, comparisons are made between subsonic wall static pressure measurements and the predicted static pressure distributions at a primary Mach number of 1.3 and over a range of secondary Mach numbers and inflow static pressure ratios. Additional comparisons are made to subsonic wall static pressure measurements at primary Mach numbers of ≈ 0.6 , 1.6 and 2.4 in Appendix H.

In all the comparisons that are presented in this section and in Appendix H,

1. The modelled lobed mixers are square lobe mixers without rounded corners ($h^* = 1.25$).
2. The boundary layer thickness at the trailing edge of the mixers is 0.25 wavelengths.
3. The shear layer growth rate calculated from the modified Papamoschou and Roshko scaling law at the pressure matched point is used to calculate Re_T .
4. Re_T is constant through the duct.
5. The cross-flow is assumed to be incompressible.
6. The inflow conditions are completely specified by the experimental measurements and the outflow/mixing duct conditions are predicted.
7. The duct geometric parameters, mixer drag coefficients, and mixing duct wall skin friction utilized in the comparisons are tabulated in Table 5.1.

The procedure in which the model/data comparisons are made is as follows—

1. Given the leading edge static pressures and plenum total pressures from the experimental data, the leading edge Mach numbers and static pressure ratio are calculated.
2. Two one-dimensional control volumes are then utilized to calculate to trailing edge Mach numbers and static pressure ratio given the trailing edge to leading edge area ratios in each stream and the mixer drag coefficient.
3. The trailing edge conditions and area ratio are then input into the pressure matching control volume, which calculates the pressure matched Mach numbers and stream area ratio. This pressure matching region is assumed to be infinitesimally thin.
4. Given the pressure matched conditions, the growth rate of the velocity thickness of the shear layer is calculated. If a look-up table has been generated for a particular mixer geometry, the

Table 5.1: Duct/mixer parameters for experimental comparisons

Parameter	FP	CP	15° FM	25° FM
$\frac{A_s}{A_p} _{LE}$	1.46	1.46	1.46	1.46
$\frac{A_{TE}}{A_{LEp}}$	1.00	1.07	1.07	1.07
$\frac{A_{TE}}{A_{LEs}}$	0.83	0.73	0.73	0.73
$\frac{A_s}{A_p} _{TE}$	1.07	1.00	1.00	1.00
$\frac{h_{duct}}{\lambda}$	5	5	5	5
$\frac{w_{duct}}{\lambda}$	5	5	5	5
$\frac{L_{duct}}{\lambda}$	30	30	30	30
$C_{D mixer}$	0	0	0	0
$C_{F ductwall}$	0.001	0.001	0.001	0.001
Γ^*	0.00	0.01	0.50	0.90

effective interface length distribution is then interpolated from this table. If a table has not been generated, the vortex code is run.

5. Given the pressure matched conditions and the effective interface length distribution from the look-up table or vortex code, the differential control volume is run to determine the local mixing duct and duct exit conditions.

5.6.1 Pitot mixedness comparisons

In Figure 5-12, the modelled and measured normalized Pitot mixednesses are plotted versus distance from the trailing edge for $M_p = 1.6$, $M_s = 0.3$, and $\frac{p_s}{p_p} = 0.9$. The uncertainty bars on the measurements are slightly smaller than the plotting symbols. The measured and modelled Pitot mixing rates downstream of the flat plate are comparable; although, the model overpredicts the mixedness at the exit of the pressure matching control volume. Downstream of the lobed mixers, the model captures the trend of a rapid initial mixing rate followed by a region of slower mixing after the shear layers from adjacent lobes have merged. The model has also captured the trend of an increasing mean mixing rate with increasing shed streamwise circulation. However, at these inflow conditions the model overpredicts the mean mixing rate downstream of all the mixers. As will be demonstrated in the next section, comparisons between the model and static pressure data are better than the Pitot mixedness comparison in Figure 5-12. Probe blockage effects are suspected to be the source of the discrepancy.

5.6.2 Wall static pressure comparisons

In this section, comparisons are drawn between the modelled and measured static pressure distributions. The experimental measurements used in the comparisons were taken on the centerline of the

Figure 5-12: Modelled and measured Pitot mixedness versus distance from the trailing edge of all four mixers for $M_p = 1.6$, $M_s = 0.3$ and $\frac{P_s}{P_p} = 0.9$.

subsonic(secondary) wall. Comparisons are made using the supersonic pressure coefficient defined in Eqn. 5.1. In this section, for three of the mixers, the modelled and measured pressure coefficients at the duct exit are compared for four different primary Mach numbers(except for the flat plate) at a range of inflow secondary Mach numbers and static pressure ratios. Additionally, the axial static pressure distributions through the mixing duct are compared for $M_p = 1.3$. (In Appendix H, the modelled and measured duct static pressured distributions are compared for primary Mach numbers of ≈ 0.6 , 1.6, and 2.4. Additionally, comparisons are made at all Mach numbers for the separation-plagued 25° forced mixer.)

Flat plate In Figure 5-13, the modelled and measured duct exit supersonic pressure coefficients(Cp_{super}) are compared downstream of the flat plate mixer at primary Mach numbers of 1.3, 1.6, and 2.4. For a fixed primary Mach number, the model is seen to capture the trend of increasing Cp_{super} with increasing secondary to primary mass flow ratio. However, for $M_p = 1.6$, the model underpredicts the pressure rise through the duct. If the growth rate constant in the shear layer model is increased from 0.14 to 0.20, the predictions may be brought into agreement with the data, which indicates that the source of the discrepancy may be the growth rate model.

The pressure coefficient plotted in Figure 5-13 and defined in Eqn. 5.1 may also be written as

$$Cp_{super} = \frac{\frac{1}{NPR}(1 + \frac{\gamma-1}{2}M_p^2)^{\frac{\gamma}{\gamma-1}} - 1}{(1 + \frac{\gamma-1}{2}M_p^2)^{\frac{\gamma}{\gamma-1}} - 1}. \quad (5.3)$$

Hence, at a fixed primary Mach number Cp_{super} is only a function of the primary nozzle pressure ratio(NPR). When the model is used in an iterative fashion to make ejector performance predictions, the nozzle pressure ratios are given and the secondary to primary mass flow ratio(pumping) is the performance parameter of interest. In Figure 5-13, if Cp_{super} is taken to be the independent variable, the consistent underprediction of by Cp_{super} by 0.02 at $M_p = 1.6$ may be interpreted as an overprediction of the secondary to primary mass flow ratio(coincidentally) by 0.02, or roughly 5%. At the other two Mach numbers the slopes of the predicted Cp_{super} curves are roughly the same as the $M_p = 1.6$ curve, and hence, at these Mach numbers, the relationship between errors in Cp_{super} and errors in pumping will very close to the one derived above.

In Figure 5-14, modelled and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the flat plate mixer at a range of secondary Mach numbers and secondary to primary static pressure ratios($0.2 < M_s < 0.4$ and $0.7 < \frac{p_s}{p_p} < 1.3$). The experimental measurements are marked with x's, and the measurements at one set of inflow conditions are connected by the dotted line. The predicted Cp_{super} distributions are plotted with the solid line.

The lowest measured Cp_{super} distribution in Figure 5-14 was taken at the lowest inflow secondary Mach number and lowest secondary/primary static pressure ratio. Moving up the figure from measured distribution to measured distribution, both the secondary Mach number and static pressure ratio are increasing. All the data in Figure 5-14 were taken at a fixed primary mass flow; hence, the secondary to primary mass flow ratio increases with increasing secondary Mach number and static pressure ratio.

At low secondary to primary mass flow ratios(and low secondary to primary static pressure ratios), both streams are accelerated into the mixing duct and a rapid drop in Cp_{super} occurs. After this rapid drop in static pressure associated with the initial area adjustment of the two streams, the mixing process ensues. A rise in Cp_{super} through the duct is associated with this mixing process. As the mass flow ratio(and $\frac{p_s}{p_p}$) was increased, the magnitude of the initial drop in Cp_{super} decreased as well as the rate of rise of Cp_{super} associated with the mixing process. At the highest secondary to primary mass flow ratios, the inflow secondary to primary static pressure ratio was greater than 1 and both streams were decelerated into the duct; an initial rapid rise in Cp_{super} was observed. At these high static pressure ratio cases, the subsequent rate of static pressure rise associated with the mixing was low enough that the pressure drop associated with the duct wall skin friction dominated.

Figure 5-13: Flat plate duct exit Cp_{super} model/data comparisons. The error bars on the experimental measurements are smaller than the symbols.

Figure 5-14: Flat plate subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$.

Convolutd plate In Figure 5-15, the modelled and measured duct exit supersonic pressure coefficients(Cp_{super}) are compared downstream of the convoluted plate mixer at primary Mach numbers of $\approx 0.6, 1.3, 1.6$, and 2.4 . In these comparisons, the interface vortex model has been used to calculate the axial L_{eff} distribution. The local rate of transport of momentum and energy in the differential control volume is proportional to the local effective interface length. For a fixed primary Mach number, the model is seen to capture the trend of increasing Cp_{super} with increasing secondary to primary mass flow ratio.

In Figure 5-16, modelled and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the convoluted plate mixer at a range of secondary Mach numbers and secondary to primary static pressure ratios($0.2 < M_s < 0.4$ and $0.7 < \frac{p_s}{p_p} < 1.3$). The experimental measurements are marked with x's, and the measurements at one set of inflow conditions are connected by the dotted line. The predicted Cp_{super} distributions are plotted with solid lines.

The static pressure measurements were taken using the same procedure with which the flat plate measurements were taken, and the overall trends described above in the flat plate discussion hold for the convoluted plate as well. However, there are two major differences between the convoluted plate mixing layers and those downstream of the flat plate–non-uniform(slowing) mixing rates associated the variation of the effective interface length with distance from the trailing edge and the presence of duct secondary stream separations at the low secondary to primary mass flow ratio cases.

At the low mass flow ratio cases, the ejector model predicts an initially more rapid rise in Cp_{super} in comparison to the flat plate. This initially rapid rise slows as the effective interface length decreases with distance from the trailing edge. However, at the lowest mass flow ratio the data predicts on opposite trend: an initially slower rise in Cp_{super} followed by a region in which Cp_{super} increases more rapidly. The source of the discrepancy between the model and data is an unmodelled secondary stream separation.

At the lowest mass flow ratios, the static pressure rise through the duct was several times the dynamic pressure of the inflow secondary stream. At these conditions, the secondary stream separates from the lower duct wall. The presence of this separation may be ascertained from fluorescent dye flow visualization on the mixing duct lower and side walls. In Figure 5-17, the side and lower duct walls are shown downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.2$. Two separated regions in the corners formed by the lower duct wall and two side walls are visible. For comparison, attached duct flow streamlines are shown also downstream of the 25° mixer in Figure 5-18. (The images shown are downstream of the 25° ; however, similar separated patterns have been observed downstream of the convoluted plate. Furthermore, the measured static pressure distribution downstream of the 25° mixer at the flow conditions at which these dye photos were taken displays the same trends as the low mass flow ratio distributions downstream of the convoluted plate–slow initial rise in Cp_{super} followed by a more rapid rise in Cp_{super} .)

Figure 5-15: Convolved plate duct exit $C_{p_{super}}$ model/data comparisons.

Figure 5-16: Convolved plate subsonic wall $C_{p_{super}}$ model/data comparisons for $M_p = 1.3$.

The separated region in Figure 5-17 acted as a blockage and reduced the effective cross-flow area of the duct. The resulting partially blocked compound subsonic flow had a lower local static pressure than would an unblocked(attached) flow with an equivalent level of mixedness.

Throughout the mixing duct, the higher total pressure mixing layer continually grows in cross-flow extent and eventually impinges on the separated region. The impingement of this high total pressure fluid on the separated regions effectively reattaches the flow. This reattachment was observed as a rapid rise in static pressure since in the reattachment region the mixing process increased the local static pressure of the duct via two mechanisms: by decelerating the supersonic stream and by increasing the effective cross-flow area of the duct. This reattachment process was also observed via duct flow visualization. This reattachment typically occurred within the first 20λ downstream of the mixers, since no regions of separated flow were detected in Pitot surveys at 20λ from the trailing edge of the mixers in any of surveys conducted.

Nevertheless, despite the presence of these duct separations in the low mass flow ratio cases, the model appears to capture the overall pressure rise through the duct. As the mass flow ratio is increased, the severity of these separations decreases and the model more accurately captures the change in Cp_{super} through the duct.

15° forced mixer In Figure 5-19, the modelled and measured duct exit supersonic pressure coefficients(Cp_{super}) are compared downstream of the 15° mixer at primary Mach numbers of ≈ 0.6 , 1.3, 1.6, and 2.4. In these comparisons, the interface vortex model has been used to calculate the axial L_{eff} distribution. The local rate of transport of momentum and energy in the differential control volume is proportional to the local effective interface length. For a fixed primary Mach number, the model is seen to capture the trend of increasing Cp_{super} with increasing secondary to primary mass flow ratio.

In Figure 5-20, modelled and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the 15° forced mixer at a range of secondary Mach numbers and secondary to primary static pressure ratios($0.2 < M_s < 0.4$ and $0.7 < \frac{P_s}{P_p} < 1.3$). The experimental measurements are marked with x's, and the measurements at one set of inflow conditions are connected by the dotted line. The predicted Cp_{super} distributions are plotted with the solid line.

The static pressure measurements were taken using the same procedure with which the flat and convoluted plate measurements were taken, and the overall trends described above in the flat plate discussion also hold for this mixer. Additionally, the separated regions observed downstream of the convoluted plate mixer at low secondary to primary mass flow ratios are also observed downstream of this mixer at similar inflow conditions.

The major difference between this mixer and the convoluted plate is the more rapid initial mixing rate downstream of this mixer associated with the stretching of the cross-flow interface by

(a) Side wall

(b) Bottom wall

Figure 5-17: Separated side and lower wall streamlines downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.2$.

(a) Side wall

(b) Bottom wall

Figure 5-18: Attached side and lower wall streamlines downstream of the 25° forced mixer with $M_p = 1.6$ and $M_s = 0.4$.

the streamwise vorticity.

All mixers In this section in order to assess the ability of the model to predict the difference in mixing rates between the three mixers, the modelled and predicted Cp_{super} distributions are compared downstream of the flat plate, convoluted plate, and 15° forced mixers in Figure 5-21. The model is seen to capture the trend of increasing Cp_{super} with increasing mass flow ratio for all three mixers as well as capturing the differences between the mixing rates. At high mass flow ratios (and high $\frac{\rho_e}{\rho_p}$), the model overpredicts the Cp_{super} downstream of the 15° forced mixer. In these flow regimes, lobe trough flow separation, as predicted by the criteria in Chapter 3, is likely. This unmodelled separation reduces the shed streamwise circulation and trailing edge interface length and thereby reduces the mixing rate downstream of the mixer.

5.7 Chapter Summary

In this chapter, an integrated mixer-ejector performance model was developed. Comparisons to experimental Pitot and wall static pressure data were made. This model is computationally efficient and flexible—running 100 times faster than three-dimensional Navier-Stokes codes. Its speed and flexibility have enabled design trade and optimization studies to be conducted. In Chapter 6, several trade studies are presented to illustrate the utility of the model, and Chapter 7 contains a sample mixer optimization study.

Figure 5-19: 15° forced mixer duct exit Cp_{super} model/data comparisons.

Figure 5-20: 15° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$.

Figure 5-21: C_{p_super} versus secondary/primary mass flow ratio for all mixers at $M_p = 1.6$.

Chapter 6

Quantitative Assessment of the Impact of Streamwise Vorticity on Supersonic/Subsonic Mixer-Ejector Performance

This chapter contains several trade studies that were conducted with the integrated ejector performance model. A quantitative assessment is made of the impact of streamwise vorticity on the pumping and thrust of supersonic-subsonic ejectors. The impact of streamwise vorticity on the performance of ideal supersonic-subsonic ejectors is first assessed. These ideal ejectors have frictionless walls and drag-free mixers. The impact of both mixer drag and duct wall skin friction on the performance of augmented ejectors is then evaluated. At a constant skin friction coefficient, an assessment is then made of the impact of mixing duct length. Additionally, an evaluation of the impact of the primary Mach number is presented. Lastly, an optimal operating point for maximum pumping and thrust is defined.

6.1 Impact of streamwise vorticity on performance

In this section, using the mixer-ejector model, an assessment of the impact of streamwise vorticity on ejector performance is presented. The mixer geometries utilized are the flat plate, convoluted plate, and 15° forced mixer evaluated in the experiments. In the comparisons made in this section, the mixer drag coefficient(C_D) and mixing duct wall skin friction coefficient(C_f) are assumed to be zero.

In Figure 6-1, the gross thrust coefficient(C_{TG}), secondary to primary mass flow ratio($\frac{\dot{m}_s}{\dot{m}_p}$), and the momentum mixing parameter(Φ) are plotted versus the primary nozzle pressure ratio(NPR). The secondary nozzle pressure ratio($SNPR$) is held constant at 1; the primary to secondary total temperature ratio($\frac{T_{t_p}}{T_{t_s}}$) is 3. The secondary to primary area ratio($\frac{A_s}{A_p}$) is 1; the duct height(h_{duct}) is 5 wavelengths; and the duct length (L_{duct}) is 30 wavelengths.

Each of the three mixers were run with primary nozzle pressure ratios ranging from 2.5 to the NPR at which the ejector compound chokes at the exit of the pressure matching region. At NPR 's above this choking point, both compound supersonic and compound subsonic exit solutions are physically possible and additional modelling needs to be employed to determine which solution occurs and it may be necessary to fit a shock in the ejector mixing duct[29]. As may be noted in Figure 6-1, the forced mixer chokes at a lower NPR than the convoluted plate which in turn chokes at a lower NPR than the flat plate. This difference in choking NPR is due to the increased mixing rates downstream of the forced mixer and convoluted plate. These increased mixing rates result in increased pumping at a fixed NPR, and this increased pumping results in a greater total mass flow which causes the convoluted plate and forced mixer to choke at lower NPR's. The increased pumping associated with the CP and 15° FM may be seen in Figure 6-1.

Additionally, in Figure 6-1 a thrust augmentation relative to the FP is seen for the CP ; an additional augmentation relative to the CP is associated with the 15° FM. In both cases, the thrust augmentation is due to the increased leading edge suction associated with the increased pumping of these two mixers.

Also plotted in Figure 6-1 is the momentum mixing parameter(Φ) which varies between 0 at the mixer trailing edge and 1 at the fully-mixed conditions. For the duct lengths investigated ($\frac{L_{duct}}{h_{duct}} = 6.0$), the two streams were not completely mixed at the exit of the duct at any of the investigated flow conditions. Hence, additional pumping and thrust could be obtained by increasing the duct length.

6.2 Impact of losses on augmented ejector performance

In this section, the impact of losses associated with mixer drag and mixing duct skin friction on ejector performance are evaluated. The flow conditions and duct geometry parameters utilized for these two loss investigations are tabulated in Table 6.1.

6.2.1 Impact of mixer drag

In order to assess the trade between the increased mixing rates associated with streamwise circulation and mixer drag, a cost/benefit study was performed with the model. A series of simulations were conducted with three of the experimental mixer geometries in which the mixer drag coefficient(C_D)

Figure 6-1: Secondary/primary mass flow ratio and gross thrust coefficient versus NPR. $SNPR = 1.0$, $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, and $\frac{Tt_p}{Tt_s} = 3.0$.

Table 6.1: Flow and duct geometry parameters for mixer drag coefficient and duct wall skin friction coefficient impact studies.

Parameter	Value
NPR	3.0
$SNPR$	1.0
$\frac{Tt_p}{Tt_s}$	3.0
$M_{primary}$	1.6
$\frac{A_s}{A_p}$	1.0
L_{duct}^*	30
h_{duct}^*	5
C_D	variable
C_f	variable

was varied between 0.00 and 0.05.

In Figure 6-2, the predicted ejector mass flow ratio and gross thrust coefficient are plotted for each simulation performed. Lines of constant mixer geometry and lines of constant C_D are drawn between each of the operating points. Along a line of constant mixer geometry, decreasing pumping and thrust performance are associated with an increasing C_D . Along a line of constant drag coefficient, both thrust and pumping augmentations are associated with the increased trailing edge length of the CP , and additional thrust and pumping augmentations are associated with the streamwise vorticity generated by the $15^\circ FM$.

From Figure 6-2, the trade between the generation of circulation and any additional mixer drag associated with its generation may be assessed. For instance, if a convoluted plate(circulation-free) mixer could be designed with a C_D of 0.02, in order not to see a thrust penalty associated with the generation of streamwise circulation, the forced mixer must have a C_D of less than 0.025. (If the drag coefficient of the forced mixer were 0.025, a 7% pumping augmentation would be achieved with no change in the ejector thrust.) Furthermore, if the drag coefficient of the forced mixer were greater than 0.045, both a thrust and pumping penalty relative to the convoluted plate mixer with $C_D = 0.02$ would be associated with the generation of circulation. It remains then to estimate the drag coefficient for convoluted plate and forced mixer geometries. (A preliminary attempt to do so is made in Chapter 7.)

6.2.2 Impact of duct skin friction

In order to assess the impact of mixing duct wall skin friction on the performance of augmented mixer-ejectors, a series of simulations were conducted for the flat plate, convoluted plate and 15° forced mixer. In these simulations the duct wall skin friction coefficient(C_f) was varied between 0.00 and 0.01; the mixer drag coefficients were zero.

In Figure 6-3, the predicted secondary to primary mass flow ratio and gross thrust coefficient are plotted for each of the simulations conducted. Lines of constant mixer geometry and lines of constant skin friction coefficient are drawn between each of the points. As may be seen in Figure 6-3, both decreased pumping and decreased gross thrust coefficient are associated with increased C_f . At low skin friction coefficients, a pumping and thrust augmentation are associated with the increased interface length of the CP and additionally with the streamwise vorticity shed by the 15° forced mixer. However, as the skin friction coefficient increases, the pressure drop associated with the wall friction grows relative to the pressure rise associated with the mixing process, and both the pumping and thrust augmentations associated with the CP and $15^\circ FM$ diminish.

Figure 6-2: Mixing rate/mixer drag trade study. Mass flow ratio versus thrust coefficient. $NPR = 3.0$, $SNPR = 1.0$, $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, $\frac{T_{t_p}}{T_{t_s}} = 3.0$.

6.3 Impact of duct length on augmented ejector performance

In order to assess the impact of duct length on augmented ejector performance, a number of simulations were performed with the flat plate, convoluted plate, and 15° forced mixer with duct lengths ranging between 1 and 40 duct heights. The flow conditions and duct geometry parameters at which the simulations were conducted are tabulated in Table 6.2.

In Figure 6-4, the pumping and gross thrust coefficient are plotted for each of the mixers versus $\frac{L_{duct}}{h_{duct}}$. Increasing the duct length has two counterbalancing effects on the pumping and thrust performance of ejectors. On one hand, an increasing duct length gives the two streams more time to mix. This increased mixedness at the exit of the duct results in greater pumping and thrust (due to the increased leading edge suction). On the other hand, increasing the duct length increases the total skin friction force on the mixing streams. This skin friction force, acting opposite to the flow direction, acts to reduce the pumping and thrust of the ejector.

Due to these two counterbalancing effects, maxima in the thrust and pumping performance distributions of each of the mixers plotted in Figure 6-4 are observed. The increased mixing rates associated with the CP and 15° forced mixer decrease the optimal duct length for maximum pumping. In Table 6.3, the performance of the three mixers at their respective optimal pumping lengths

Figure 6-3: Impact of duct skin friction. Mass flow ratio versus thrust coefficient. $NPR = 3.0$, $SNPR = 1.0$, $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, $\frac{T_{t_p}}{T_{t_s}} = 3.0$.

Table 6.2: Flow and duct geometry parameters for duct length impact study.

Parameter	Value
NPR	3.0
$SNPR$	1.0
$\frac{T_{t_p}}{T_{t_s}}$	3.0
$M_{primary}$	1.6
$\frac{A_s}{A_p}$	1.0
h_{duct}^*	5
L_{duct}^*	variable
C_D	0.00
C_f	0.00275

Figure 6-4: Secondary/primary mass flow ratio and gross thrust coefficient versus mixing duct length. $NPR = 3.0$, $SNPR = 1.0$, $\frac{T_{t_p}}{T_{t_s}} = 3.0$, and $\frac{A_s}{A_p} = 1.0$.

Table 6.3: Comparison between the pumping and thrust performance of the *FP*, *CP*, and 15° at their optimal lengths.

Mixer	L_{duct}^*	$\frac{\dot{m}_s}{\dot{m}_p}$	C_{TG}	$L_{duct}^*/L_{duct}^* _{CP}$	$\frac{\dot{m}_s}{\dot{m}_p}/\frac{\dot{m}_s}{\dot{m}_p} _{CP}$	$C_{TG}/C_{TG} _{CP}$
FP	9	0.42	0.94	1.8	0.88	0.98
CP	5	0.48	0.96	1	1	1
15° FM	3	0.52	0.98	0.6	1.08	1.02

are compared. For each of the mixers, the optimal duct length for maximum pumping is tabulated as well as the pumping and gross thrust coefficient at the optimal duct length. Additionally, the optimal lengths, mass flow ratios, and thrust coefficients of the flat plate and forced mixer are normalized by the performance parameters of the convoluted plate.

At the inflow conditions listed in Table 6.2, for an ejector that is 55% of the length of the optimal flat plate ejector length, the *CP* ejector pumps 12% more secondary air and generates 2% more gross thrust. Additionally, for an ejector that is 52% of the length of the optimal *CP* ejector, the *FM* ejector pumps 8% more secondary fluid and generates 2% more thrust than the *CP* ejector.

Table 6.4: Flow and duct geometry parameters for M_p impact study.

Parameter	Value
NPR	3.0
$SNPR$	1.0
$\frac{T_{t_p}}{T_{t_s}}$	3.0
$M_{primary}$	variable
$\frac{A_s}{A_p}$	1.0
h_{duct}^*	5
L_{duct}^*	30
C_D	0.00
C_f	0.00275

6.4 Impact of primary Mach number

In order to assess the impact of primary Mach number on the pumping and thrust performance of augmented ejectors, a series of simulations were performed in which M_p was varied. The ejector configuration and flow parameters at which these simulations were performed are tabulated in Table 6.4. For the three mixer geometries evaluated (FP , CP , and $15^\circ FM$), the secondary to primary mass flow ratio increases with M_p . However, a maximum in the gross thrust coefficient is observed for all three mixers. The maximum is the result of a balance between two competing effects associated with increasing M_p : increasing leading edge suction associated with the increased entrainment, and increasing skin friction.

6.5 Optimal operating point for maximum thrust, maximum pumping

In this section, an ideal operating point is defined for maximum pumping and thrust for an ejector of fixed pressure, temperature, and area ratios. If the duct length to height ratio of a frictionless constant area ejector operating at a fixed NPR , $SNPR$, M_p , $\frac{T_{t_p}}{T_{t_s}}$, and $\frac{A_s}{A_p}$ is increased, the pumping and thrust of the device continually increase until the duct compound chokes in the pressure matching region. The increase in pumping and thrust associated with increasing duct length is due to the increased mixedness of the two streams at the duct exit. The pumping and gross thrust coefficient are plotted versus the compound flow indicator at the end of the pressure matching region in Figure 6-6. In this sensitivity study, the duct is the shortest at the highest compound flow indicator and longest where the indicator is 0. An ideal ejector operating in the subsonic mode at the compound choked point is entraining the maximum secondary mass flow and generating the maximum leading edge suction for the given flow conditions and secondary to primary area ratio.

Increased NPR , $SNPR$, or mixedness at the duct exit would not result in increased pumping

Figure 6-5: Gross thrust coefficient and pumping for the *FP*, *CP*, and 15° *FM* versus the primary Mach number.

Figure 6-6: Ejector performance parameters versus the matched point compound flow indicator. $NPR = 3.0$, $SNPR = 1.0$, $\frac{T_{t_2}}{T_{t_p}} = 1.0$, $\frac{A_2}{A_p} = 1.0$

or thrust but would result in the ejector shifting to the supersonic mode of operation. This shift to the supersonic mode is not desired for noise and operability reasons. If the ejector is operating in the supersonic mode, either the exit will be supersonic or a compound shock will be located in the duct. The supersonic exit would be excessively noisy and the shock would tend to move around in the duct as NPR , $SNPR$, and $\frac{A_*}{A_p}$ change through the flight envelope. Hence, for some NPR , $SNPR$, and $\frac{A_*}{A_p}$ it may be possible to mix too well.

This analysis suggests that the best place to operate the ejector is close to the stability boundary between the subsonic and supersonic modes of operation. The pumping and thrust increase rapidly as this boundary is approached from the subsonic side. However, the price paid for this extra pumping and thrust is an increased likelihood of a shift to the supersonic mode. (As the compound choked point is approached, the ejector becomes increasingly sensitive to changes in NPR and $SNPR$.)

6.6 Chapter summary

In this Chapter, at $NPR = 3.0$, $SNPR = 1.0$, and $\frac{T_{t_p}}{T_{t_e}} = 3.0$ an ejector pumping and thrust augmentation was associated with the streamwise circulation shed by a drag-free 15° forced mixer with $h^* = 1.25$ relative to a drag-free convoluted plate mixer with the same trailing edge shape. A sensitivity analysis of the impact of mixer drag was then conducted to demonstrate that if this forced mixer could be designed so that $C_{D|FM} = C_{D|CP} + 0.005$, a 7% pumping augmentation would be obtained with no loss in thrust. The next step to be taken in this analysis is to estimate the mixer drag coefficients.

A sensitivity study of the impact of duct skin friction on pumping and thrust performance was then performed. In this analysis, three mixers were evaluated in an ejector of constant geometry in which the duct wall skin friction coefficient was varied between 0.00 and 0.01. As the skin friction coefficient was increased, a decrease in the pumping and thrust augmentations associated with streamwise vorticity was observed. With $C_f = 0.00$, a 9% pumping augmentation was associated with the vorticity, and with $C_f = 0.01$, a 6% pumping augmentation was associated with the vorticity.

An additional study was conducted to assess the impact of duct length on ejector performance. Due to the counter-balancing effects of increased mixedness and increased skin friction associated with increasing duct length, an optimal ejector length for maximum pumping was observed for each of the mixers. When the optimal convoluted plate and forced mixer ejectors are compared, the forced mixer pumps 8% more secondary fluid and generates 2% more thrust than the convoluted plate ejector in an ejector that is 52% of the length of the CP ejector.

An optimal operating point for maximum thrust and pumping has been identified—the compound choked point. The pumping and thrust increase rapidly as this point is approached from the subsonic

side. However, the price paid for this extra pumping and thrust is an increased likelihood of a shift to the supersonic mode.

Chapter 7

Optimal Design of Lobed Mixers for Maximum Ejector Pumping and Thrust

In this chapter, a lobed mixer design is optimized for a compressible mixer-ejector. The optimal lobed mixer design is defined as the one which provides the maximum pumping and maximum thrust. Two lobed mixer geometric design parameters were varied in the optimization study—the lobe height and the lobe angle. For simplicity, the mixers were symmetric (equal width secondary and primary trough widths) and of a square lobe design.

In the study, the impact of lobe angle(streamwise circulation) and lobe height on the mixing rate were evaluated using the discrete vortex model for mixers with height to wavelength ratios between 0.5 and 2.75 and lobe angles between 0 and 27 degrees. For these geometries, the mixer drag coefficients were estimated assuming that skin friction on the mixer was the only loss source. Using the ejector model, a sensitivity study was performed in which the mixer geometry and mixer drag coefficient were varied. Lastly, using the mixer skin friction loss estimates in the ejector model, an optimal mixer design was selected from an array of six mixer geometries.

7.1 Impact of lobe angle and lobe height on mixing rate

The two lobe design parameters varied in this optimization study are the lobe angle and the lobe height to wavelength ratio. Increasing the lobe angle increases the amount of streamwise circulation generated by the mixer. For the square lobe mixers of interest here, Barber *et al.*[2] demonstrated that the streamwise circulation non-dimensionalized by the lobe height and mean velocity can be

approximated by

$$\Gamma^* = 2 \tan \alpha. \quad (7.1)$$

Increasing the lobe height to wavelength ratio increases the initial interface length. For square lobe mixers, the trailing edge length non-dimensionalized by the lobe wavelength is given by

$$L_{TE}^* = 2h^* + 1. \quad (7.2)$$

The increased initial trailing edge length and the further increase in interface length due to the streamwise vorticity are the two principal mixing mechanisms associated with lobed mixers.

In order to assess the impact of Γ^* and h^* on the mixing rate downstream of lobed mixers, a series of runs were performed with the vortex model in which both the lobe height and streamwise circulation (lobe angle) were varied. The height to wavelength ratio was varied between 0.5 and 2.75, and Γ^* was varied between 0.01 and 1.0. In all the simulations, the boundary layer thickness was held constant at 0.25 wavelengths, and the shear layer growth rate was a constant 0.1. Both of these values were typical of the what was observed in the experiments discussed in Chapters 2-5.

In Figures 7-1 to 7-4, the fully mixed scalar fields are drawn 2λ downstream of the trailing edges of the forced mixers. The lines connecting the vortices which compose the interface are also drawn. At constant lobe height, the rate of interface stretching increases with increasing Γ^* . In addition, at constant streamwise circulation, the interface length at 2λ normalized by the initial length of the trailing edge decreases with increasing h^* .

In Figure 7-5, the scalar mixedness 20λ from the mixer trailing edges is plotted as a function of the lobe height and streamwise circulation. The scalar mixedness at 20λ increases with both h^* and Γ^* ; the maximum mixedness occurs at the maximum investigated h^* and Γ^* . In Figure 7-6 the scalar mixing augmentation associated with Γ^* as defined in Eqn. 4.5 is plotted versus Γ^* and h^* ; the mixing augmentation associated with streamwise vorticity decreases with increasing h^* .

7.2 Estimation of lobed mixer drag

In the optimization study presented in Section 7.3, the only loss source associated with the mixer geometries is skin friction drag. In this section, a procedure is developed for calculating the mixer skin friction drag coefficients for forced mixer and convoluted plate geometries from their wetted areas and skin friction coefficients.

Figure 7-1: Scalar fields 2λ downstream of square lobe forced mixers with $h^* = 0.5$, $\epsilon^* = 0.25$, and $\delta'_{vel} = 0.1$.

Figure 7-2: Passive scalar fields 2λ downstream of square lobe forced mixers with $h^* = 1.25$, $\epsilon^* = 0.25$, and $\delta'_{vel} = 0.1$.

Figure 7-3: Passive scalar fields 2λ downstream of square lobe forced mixers with $h^* = 2.00$, $\epsilon^* = 0.25$, and $\delta'_{vel} = 0.1$.

Figure 7-4: Passive scalar fields 2λ downstream of square lobe forced mixers with $h^* = 2.75$, $\epsilon^* = 0.25$, and $\delta' = 0.1$.

Figure 7-5: Scalar mixedness at 20λ versus Γ^* and h^* with $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.

Figure 7-6: Scalar mixing augmentation at 20λ versus Γ^* and h^* with $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.

The mixer drag coefficients for each stream are calculated from

$$C_{D|primary} = C_f \frac{A_{wet}^*}{A_{duct}^*} \frac{1}{1 + \frac{A_s}{A_p}}, \quad (7.3)$$

$$C_{D|secondary} = C_{D|primary} \frac{A_s}{A_p} \quad (7.4)$$

$$(7.5)$$

The skin friction coefficient is given by C_f . A_{wet}^* and A_{duct}^* are the wetted area of the mixer and the inflow cross-flow areas of the duct in units of λ^2 where λ is the lobe wavelength. $\frac{A_s}{A_p}$ is the ratio of the secondary to primary inflow duct areas. In this estimate of the drag coefficient, mixer form drag has been neglected.

The wetted area of one side of a square lobe mixer with a straight extension is given by

$$A_{wet}^* = \frac{(h^*)^2}{2 \tan \alpha} + \frac{h^*}{2 \sin \alpha} + 2l_s^* h^*. \quad (7.6)$$

h^* is the height to wavelength ratio; α is the lobe half angle; and l_s^* is the length of the straight extension section of the mixer in wavelengths. For forced mixers, $l_s = 0$.

In Figure 7-7, the wetted area of a single side of a square lobe forced mixer as calculated from Eqn. 7.6 is plotted as a function of h^* and α . The wetted area increases with increasing lobe height and increases with decreasing lobe angle. For a fixed h^* , the wetted area increases with decreasing lobe angle since a longer ramp is required to obtain the desired height. The wetted area also increases with h^* at a fixed lobe angle.

In Figure 7-8, the wetted area of a single side of a square lobe convoluted plate mixer as calculated from Eqn. 7.6 is plotted as a function of h^* and l_s . For all the convoluted plates in Figure 7-8, the interior angle of the ramp is the same as it was in the experimental geometry ($\alpha = 16.5^\circ$). The wetted area increases with both h^* and l_s .

7.3 Optimal lobe design for maximum ejector pumping and thrust

Six different lobed mixer geometries were evaluated. These six different mixers were of three different lobe heights: 0.5, 1.25, and 2.75. For each lobe height, two mixers were evaluated—a convoluted plate and a 15° forced mixer shedding $\Gamma^* = 0.5$.

The optimization was conducted at only one set of flow conditions and for one particular duct geometry. The geometric and flow parameters are tabulated in Table 7.1. Additionally, in order to minimize the complexity of the optimization, the duct walls were frictionless.

In order to assess the sensitivity of the mixing rate to the mixer geometries and mixer drag,

Figure 7-7: Forced mixer wetted area in square wavelengths as a function of lobe height to wavelength ratio and lobe angle.

Figure 7-8: Convoluted plate wetted area in square wavelengths as a function of lobe height to wavelength ratio and the length of the straight section in wavelengths. ($\alpha_{CP} = 16.5^\circ$).

Table 7.1: Flow and duct geometry parameters for mixer optimization study.

Parameter	Value
NPR	3.0
$SNPR$	1.0
$\frac{T_{t_p}}{T_{t_s}}$	3.0
$M_{primary}$	1.8
$\frac{h_s}{h_p}$	1.0
h_{duct}^*	5
L_{duct}^*	30
C_D	variable
C_f	0

a series of simulations were performed using the ejector model with all six of the mixers in which the mixer drag coefficients were varied between 0.0 and 0.05. In Figure 7-9, the predicted ejector pumping and gross thrust coefficient are plotted for each of the simulations performed. Lines of constant mixer geometry and constant drag coefficient are drawn between the points. For each of the geometries, an increase of 0.01 in the drag coefficient resulted in a decrease of 0.02 in both pumping and gross thrust coefficient. Moving along a line of constant drag coefficient, with $h^* = 0.5$ and $h^* = 1.25$, a pumping increase of 0.05 and a thrust coefficient increase of 0.01 are associated with the addition of streamwise circulation ($\Gamma^* = 0.5$) to the mixing layer. However, when h^* is increased to 2.75, no pumping or thrust augmentation is associated with the addition of streamwise vorticity.

Estimates of the mixer drag coefficients were then made using the procedure outlined in Section 7.2; the calculated drag coefficients for the six mixers are plotted in Figure 7-10 for a mixer skin friction coefficient of 0.001. These drag coefficients were used in the ejector model with the flow and duct parameters tabulated in Figure 7.1 to predict the pumping and thrust coefficients for the six mixer geometries. An additional set of simulations was performed for the same six mixer geometries with $C_f = 0.005$.

In Figure 7-11, the operating points in a pumping/thrust plane are plotted for each of the six mixers with $C_f = 0.001$. Comparing the performance of the six mixers, the two $h^* = 2.75$ mixers have the best pumping and thrust performance.

In Figure 7-12, the operating points in a pumping/thrust plane are plotted for each of the six mixers with $C_f = 0.005$. At this higher skin friction coefficient, the 15° forced mixer with $h^* = 1.25$, and therefore with less wetted area than the $h^* = 2.75$ mixers and with a small enough h^* so that the streamwise vorticity is effective, has the best pumping and thrust performance.

Figure 7-9: Mass flow ratio/thrust operating map for optimization study

Figure 7-10: Mixer drag coefficients for the six mixer geometries evaluated in the optimization study($C_f = 0.001$).

Figure 7-11: Pumping/thrust operating map for the six mixers with $C_f = 0.001$.

Figure 7-12: Pumping/thrust operating map for the six mixers with $C_f = 0.005$.

7.4 Chapter summary

In this chapter, a design trade has been associated with increasing lobe h^* . Increasing h^* increases the initial trailing edge length which in turn increases the mixing rate downstream of the mixers. This increased mixing rate results in an increased pumping and thrust. However, as h^* is increased the wetted area of the mixers increases as well; this increase in wetted area leads to an increase in mixer drag. The increased drag reduces the pumping and thrust of the ejector.

It has been demonstrated that one method of increasing the mixing rate without increasing mixer skin friction drag is via the introduction of streamwise vorticity. The streamwise vorticity then stretches the interface downstream of the trailing edge of the mixer. In this optimization study, for $C_f = 0.005$, the forced mixer with $h^* = 1.25$, pumps 3% more fluid and generates 1% more thrust than the convoluted plate mixer with $h^* = 2.75$. However, additional losses are associated with the turning, or mixing out, of the cross-flow momentum associated with the streamwise vorticity. No model for these non-axial flow losses has been utilized in this optimization study.

Chapter 8

Summary, Conclusions, and Recommendations

This chapter contains a summary of the experiments, modelling efforts and trade studies which comprise this work. Conclusions are drawn and recommendations for future research are made.

8.1 Summary

A series of experiments with lobed mixers in compressible mixing layers has been conducted to assess the ability of forced mixers to generate streamwise vorticity and generate increased initial interface length in compressible mixing layers, to quantify the mixing augmentation associated with streamwise vorticity in compressible mixing layers, and to understand the underlying physics of the vorticity generation and its role in the mixing process.

Based on the physical understanding gained from these experiments a streamwise vorticity enhanced ejector performance model was developed. This model can be used to predict ejector performance (pumping and gross thrust) given the nozzle pressure ratios, the secondary to primary temperature ratios and the mixer and duct geometries.

The ejector model was used to perform several design trade studies: an assessment of the impact of streamwise vorticity on ejector performance, an assessment of the impact of mixer and duct losses, and an assessment of the mixing rate/mixer loss trade between mixer height and streamwise circulation. Additionally, for a fixed NPR, SNPR, and $\frac{T_{t_2}}{T_{t_1}}$, the model was used to determine an optimal operating point for maximum thrust and pumping.

8.2 Conclusions

The major conclusions of this study are–

1. Forced mixer performance is sensitive to the trailing edge static pressure ratio in supersonic/subsonic applications. A mismatched static pressure at the trailing edge of the mixers can separate the trough flow. These trough separations both reduce the streamwise circulation shed by the mixers and reduce the initial interface length between the two streams, and hence, a mixing performance penalty is associated with the presence of these separations.
2. An empirical forced mixer separation criteria was developed to predict the range of trailing edge static pressure ratios for which the trough flow will be attached. The range of static pressure ratios at which the primary trough is predicted to be attached diminishes with increasing lobe angle.
3. Streamwise vorticity is effective in augmenting the mixing rate in compressible mixing layers; the mechanism for the augmentation is the same as it is in incompressible flows; convection associated with the streamwise vorticity stretches the cross-flow interface with distance from the trailing edge.
4. The effectiveness of streamwise vorticity decreases with increasing shear layer growth rate. Due to the higher diffusion rate, the vorticity diffuses more rapidly.
5. Compressible streamwise vorticity enhanced mixing layers differ from incompressible mixing layers in three ways:
 - Slower shear layer growth rates in compressible layers lead to a relative increase in the effectiveness of streamwise vorticity.
 - Streamwise vorticity may be generated downstream of the mixer trailing edge by baroclinic torque associated with the interaction of shocks with density gradients. The shock generated vorticity may either augment or slow the mixing rate. If the secondary to primary density ratio is less than 1, this downstream vorticity augments the mixing rate. If the density ratio is greater than 1, this vorticity will slow the mixing rate.
 - Vortex sheets may be displaced by a mismatch in stream static pressures at the trailing edge. Displacement of the vortex sheets associated with a mismatch in the trailing edge static pressures does not significantly impact the mixing augmentation associated with streamwise vorticity.
6. A computationally efficient and flexible vorticity-enhanced mixer-ejector model has been developed which has enabled optimization studies to be conducted. Several design trade and optimization studies were conducted to illustrate the utility of the model.

- (a) The vorticity enhanced ejector model predicts that for the experimental mixer and duct geometries with no duct wall skin friction a 10% percent increase in pumping and a 1% increase in gross thrust is associated with the streamwise circulation shed by the 15° forced mixer at $NPR = 3.0$, $SNPR = 1.0$, and $\frac{T_{t_p}}{T_{t_e}} = 3.0$.
- (b) Due to the increased entrainment associated with the higher mixing rate, the ejector with the 15° forced mixer compound chokes at a lower NPR than the ejector with the convoluted plate.
- (c) For the experimental mixer and duct geometries and with a skin friction coefficient of 0.001, the model predicts that the 15° forced mixer pumps 8% more secondary fluid and generates 2% more thrust than the CP ejector, and this greater performance is realized in a duct that is 50% shorter.
- (d) The optimal operating point for a constant area ejector operating at a fixed NPR , $SNPR$, and $\frac{T_{t_p}}{T_{t_e}}$ is the compound choked point. At this point, the ejector is entraining the maximum secondary mass flow and generating the maximum leading edge suction.
- (e) A procedure for optimizing the design of lobed mixers for supersonic-subsonic ejectors has been developed. Two sample optimizations were performed using six mixers, with skin friction as the only mixer loss source; the optimizations were performed with mixer skin friction coefficients of 0.001 and 0.005. At the lower friction coefficient, the two tested mixers with the highest h^* pumped the most secondary fluid and generated the most thrust. (At the highest tested h^* , streamwise vorticity was not effective in augmenting ejector performance.) At the higher skin friction coefficient ($C_f = 0.005$), the ejector with the 15° forced mixer with the mid-range h^* performed the best.

8.3 Recommendations for Future Work

Suggestions for improvement and extension of the current work include—

1. A more thorough assessment of the losses associated with the generation of streamwise vorticity in compressible mixing layers needs to be made in order to fully determine the performance trade between the enhanced mixing rates associated with streamwise vorticity and the losses associated with its generation.
2. With these more accurate loss models an evaluation of the trade between mixer height and streamwise circulation can be performed.
3. A method of determining the streamwise vorticity distribution of non-square mixers should be developed.

4. An more thorough experimental assessment of the impact of the baroclinic torque generated vorticity on the mixing process need be conducted to quantify the mixing augmentation or mixing penalty associated with its generation.
5. A more direct link between the vortex model and differential control volume should be developed so that the local shear layer growth rates may be used to estimate the local diffusion rates.

Appendix A

Experimental Test Matrix

This Appendix contains the experimental test conditions and measurement locations. The measurement locations for the wall static pressure measurements and Pitot surveys are tabulated in Section A.1. The experimental test conditions are tabulated in Section A.2.

A.1 Measurement locations

This section contains the measurement locations for the wall static pressure measurements and Pitot surveys.

A.1.1 Static pressure data

A series of 9 static pressure taps were spaced axially along the center of subsonic wall from 2 to 30λ . Since the mixers were of various lengths, the distance from the wall taps to the mixer trailing edge varied from mixer to mixer. The distances from each of the wall pressure taps to the trailing edges are tabulated in Table A.1.

Additional mixing duct static taps were spaced across the duct in line with centerline static tap numbers 1, 3, 4, and 7. Static pressure taps were also located at the leading edges of the mixers. During some tests, additional static measurements were taken on the mixing duct supersonic and side walls. The flow conditions at which the static pressure measurements were taken are contained in Sections A.2.1-A.2.4.

A.1.2 Pitot pressure data

Pitot and static pressure surveys were taken with a rake of 14 Pitot probes and one static probe. Surveys were conducted in as many as 4 planes downstream of the mixers. These survey locations

Table A.1: Subsonic wall static tap locations.

Tap Number	$x^*(FP)$	$x^*(CP)$	$x^*(15^\circ FM)$	$x^*(25^\circ FM)$
1	1.8	1.6	2.0	3.2
2	3.2	2.9	3.4	4.5
3	4.6	4.4	4.8	6.0
4	8.4	8.1	8.6	9.8
5	12.2	11.9	12.4	13.5
6	15.9	15.6	16.1	17.3
7	19.6	19.4	19.8	21.0
8	23.4	23.1	23.6	24.8
9	27.2	26.9	27.4	28.5

Table A.2: Pitot survey locations.

Survey Number	$x^*(FP)$	$x^*(CP)$	$x^*(15^\circ FM)$	$x^*(25^\circ FM)$
1	1.8	1.6	2.0	3.2
2	4.6	4.4	4.8	6.0
3	8.4	8.1	8.6	9.8
4	19.6	19.4	19.8	21.0

are tabulated for each of the mixers in Table A.2. The flow conditions at which these surveys were conducted are listed in Sections A.2.1-A.2.4.

A.2 Experimental Test Conditions

This section contains a list of the experimental test conditions at which data was obtained. The test conditions are first divided by primary Mach number: subsonic, 1.3, 1.6, and 2.4. The conditions are then subdivided by the diagnostic technique utilized: Pitot surveys, or wall static pressure measurements.

A series of 4 numbers are used to designate a specific Pitot pressure data set—the nozzle number, the mixer number, the rake position(survey) number, and the case number. The nozzle numbers for each of the respective nozzles are tabulated in Table A.3. The mixer numbers are listed for each mixer in Table A.4. The axial distances from the mixers' trailing edges corresponding to the rake position(survey) number are listed in Table A.2. The case numbers corresponding to the different series of tests with each nozzle are explained in Sections A.2.1, A.2.2, A.2.3, and A.2.4.

A series of 3 numbers are used to designate a particular static pressure data set—the nozzle number, the mixer number, and the case number. The nozzle and mixer numbers utilized are the same as those used to designate a Pitot data set(See Tables A.3 and A.4). The case numbers for each nozzle are explained in Sections A.2.1, A.2.2, A.2.3, and A.2.4.

Also tabulated for each Pitot survey plane or static pressure point are

Table A.3: Nozzle number key

M_p	Nozzle #
1.3	1
1.6	2
2.4	3

Table A.4: Mixer number key

Mixer	Mixer #
FP	0
CP	1
15° FM	3
25° FM	2

- M_p , the leading edge primary Mach number;
- M_s , the leading edge secondary Mach number;
- $\frac{p_s}{p_p}$, the leading edge secondary to primary static pressure ratio;
- NPR , the ratio of the primary stream total pressure to the static pressure at the end of the mixing duct;
- $SNPR$, the ratio of the secondary stream total pressure to the static pressure and the end of the mixing duct;
- NPR^* , the ratio of the primary stream total pressure to the ambient static pressure at the exit of the vacuum pump downstream of the mixing duct;
- $SNPR^*$, the ratio of the primary stream total pressure to the ambient static pressure at the exit of the vacuum pump downstream of the mixing duct.

All tests were conducted with equal total temperature air-air streams.

A.2.1 Subsonic-subsonic data

This section contains a list of all the subsonic-subsonic test conditions at which static and Pitot pressure data was obtained. Brief summaries of the tests corresponding to the different Pitot and static cases is contained in Tables A.5 and A.6. The Pitot survey conditions are tabulated in Table A.7, and the static pressure conditions are listed in Table A.8.

Table A.5: Subsonic Pitot survey case number designations.

Subsonic Pitot case #	Summary
5	$NPR^* = 0.40$, $SNPR^* = 0.33$
6	$NPR^* = 0.39$, $SNPR^* = 0.31$
7	$NPR^* = 0.38$, $SNPR^* = 0.28$

Table A.6: Subsonic static pressure case number designations.

Subsonic static case #	Summary
4	Range of NPR^* , $SNPR^*$

Table A.7: Subsonic-subsonic Pitot survey data

Subsonic-subsonic Pitot survey data										
Nozzle	Mixer	Rake	Case	M_p	M_s	$\frac{P_t}{P_o}$	NPR	SNPR	NPR*	SNPR*
1	0	4	5	0.62	0.20	1.03	1.34	1.09	0.40	0.33
1	0	4	6	0.65	0.17	1.03	1.34	1.06	0.39	0.31
1	0	4	7	0.71	0.10	1.03	1.35	0.99	0.38	0.28
1	1	1	5	0.69	0.19	1.09	1.34	1.09	0.41	0.33
1	1	4	5	0.68	0.19	1.09	1.34	1.10	0.40	0.33
1	1	4	6	0.71	0.14	1.08	1.34	1.04	0.39	0.31
1	1	4	7	0.83	0.06	1.10	1.36	0.96	0.39	0.27
1	2	1	5	0.66	0.19	1.07	1.32	1.08	0.40	0.33
1	2	4	5	0.66	0.19	1.07	1.34	1.09	0.40	0.33
1	2	1	6	0.70	0.14	1.07	1.33	1.04	0.39	0.31
1	2	4	6	0.69	0.15	1.06	1.33	1.05	0.39	0.31
1	2	1	7	0.76	0.08	1.07	1.34	0.99	0.38	0.28
1	2	4	7	0.75	0.10	1.06	1.34	0.99	0.38	0.28
1	3	1	5	0.65	0.19	1.09	1.30	1.09	0.39	0.33
1	3	4	5	0.68	0.20	1.09	1.34	1.10	0.41	0.34
1	3	4	6	0.73	0.16	1.09	1.35	1.05	0.40	0.31
1	3	1	7	0.71	0.09	1.08	1.30	1.00	0.37	0.29
1	3	4	7	0.84	0.09	1.11	1.39	0.98	0.39	0.28

Table A.8: Subsonic-subsonic wall static pressure data

Subsonic-subsonic static pressure data										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
1	1	4	1	0.33	0.08	1.01	1.06	0.99	0.35	0.33
1	1	4	2	0.40	0.08	1.02	1.08	0.99	0.30	0.27
1	1	4	3	0.65	0.17	1.07	1.25	1.03	0.35	0.29
1	1	4	4	0.63	0.18	1.07	1.24	1.04	0.35	0.30
1	1	4	5	0.62	0.22	1.09	1.24	1.07	0.36	0.31
1	1	4	6	0.61	0.27	1.11	1.24	1.13	0.36	0.33
1	1	4	7	0.60	0.31	1.14	1.24	1.18	0.36	0.35
1	1	4	8	0.60	0.35	1.18	1.24	1.24	0.37	0.37
1	1	4	9	0.58	0.38	1.22	1.22	1.32	0.37	0.40
1	2	4	1	0.31	0.07	1.01	1.05	0.99	0.36	0.34
1	2	4	2	0.37	0.08	1.01	1.07	0.99	0.31	0.28
1	2	4	3	0.63	0.16	1.06	1.24	1.02	0.35	0.29
1	2	4	4	0.62	0.19	1.06	1.24	1.04	0.35	0.30
1	2	4	5	0.59	0.25	1.08	1.24	1.10	0.36	0.32
1	2	4	6	0.58	0.31	1.11	1.24	1.17	0.37	0.35
1	2	4	7	0.57	0.36	1.14	1.24	1.25	0.37	0.37
1	2	4	8	0.58	0.40	1.20	1.25	1.34	0.38	0.41
1	3	4	1	0.39	0.08	1.02	1.07	0.99	0.30	0.28
1	3	4	2	0.67	0.14	1.07	1.24	1.00	0.33	0.27
1	3	4	3	0.65	0.15	1.07	1.24	1.01	0.34	0.27
1	3	4	4	0.59	0.16	1.06	1.21	1.03	0.33	0.28
1	3	4	5	0.56	0.23	1.08	1.21	1.09	0.34	0.31
1	3	4	6	0.55	0.29	1.10	1.21	1.14	0.34	0.33
1	3	4	7	0.55	0.33	1.13	1.20	1.20	0.35	0.35
1	3	4	8	0.54	0.35	1.16	1.20	1.24	0.35	0.37
1	3	4	9	0.52	0.39	1.20	1.19	1.31	0.36	0.40

A.2.2 $M_p = 1.3$ nozzle data

This section contains a list of all the $M_p = 1.3$ test conditions at which static pressure and Pitot pressure data was obtained. Brief summaries of the tests corresponding to the different Pitot and static cases are contained in Tables A.9 and A.10. The Pitot survey conditions are tabulated in Table A.11, and the static pressure conditions are listed in Table A.12.

Table A.9: $M_p = 1.3$ Pitot survey case number designations.

$M_p = 1.3$ Pitot case #	Summary
1	$NPR^* = 0.78, SNPR^* = 0.25$
2	$NPR^* = 0.78, SNPR^* = 0.29$
3	$NPR^* = 0.78, SNPR^* = 0.47$
4	$NPR^* = 1.16, SNPR^* = 0.43$

Table A.10: $M_p = 1.3$ static pressure case number designations.

$M_p = 1.3$ static case #	Summary
1	$\frac{P_s}{P_p} = 1.0$ (Range of $NPR^*, SNPR^*$)
2	$NPR^* = 0.77, 0.19 < SNPR^* < 0.52$
3	$NPR^* = 1.19, 0.18 < SNPR^* < 0.67$

Table A.11: $M_p = 1.3$ Pitot survey data

$M_p = 1.3$ Pitot survey data										
Nozzle	Mixer	Rake	Case	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
1	0	4	1	1.30	0.04	0.88	2.50	0.79	0.78	0.25
1	0	4	2	1.30	0.14	1.01	2.45	0.90	0.78	0.29
1	0	4	3	1.30	0.28	1.60	2.12	1.29	0.78	0.47
1	0	4	4	1.31	0.21	1.02	3.22	1.20	1.16	0.43
1	1	1	1	1.30	0.09	1.05	2.45	0.93	0.77	0.29
1	1	4	1	1.30	0.05	0.91	2.38	0.78	0.77	0.25
1	1	4	2	1.30	0.11	1.02	2.39	0.88	0.78	0.29
1	1	4	3	1.31	0.28	1.59	2.23	1.34	0.78	0.47
1	1	4	4	1.31	0.21	1.02	3.14	1.18	1.16	0.43
1	2	1	1	1.30	0.03	0.84	2.47	0.75	0.77	0.23
1	2	4	1	1.30	0.08	0.85	2.44	0.75	0.77	0.24
1	2	1	2	1.30	0.10	1.03	2.36	0.89	0.77	0.29
1	2	4	2	1.30	0.14	1.04	2.34	0.89	0.78	0.30
1	2	1	3	1.30	0.28	1.58	2.04	1.23	0.77	0.47
1	2	4	3	1.30	0.27	1.57	2.13	1.27	0.78	0.46
1	2	4	4	1.30	0.22	1.01	3.33	1.24	1.15	0.43
1	3	1	1	1.30	0.13	1.05	2.35	0.90	0.77	0.30
1	3	4	1	1.30	0.07	0.82	2.45	0.73	0.77	0.23
1	3	4	2	1.30	0.14	1.00	2.36	0.87	0.78	0.29
1	3	1	3	1.31	0.29	1.60	2.04	1.23	0.77	0.47
1	3	4	3	1.30	0.29	1.59	2.23	1.36	0.78	0.47
1	3	4	4	1.30	0.22	1.01	3.38	1.27	1.16	0.43

Table A.12: $M_p = 1.3$ wall static pressure data

$M_p = 1.3$ static pressure data										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_e}$	NPR	SNPR	NPR*	SNPR*
1	0	1	1	1.25	0.10	1.03	2.11	0.84	0.53	0.21
1	0	1	2	1.25	0.16	0.97	2.39	0.91	0.59	0.23
1	0	1	3	1.25	0.20	1.02	2.44	0.99	0.65	0.26
1	0	1	4	1.25	0.24	1.01	2.52	1.02	0.73	0.30
1	0	1	5	1.25	0.26	1.00	2.55	1.04	0.81	0.33
1	0	1	6	1.25	0.31	1.01	2.62	1.09	0.95	0.40
1	0	1	7	1.26	0.34	1.01	2.70	1.13	1.08	0.45
1	0	1	8	1.26	0.35	1.01	2.74	1.15	1.18	0.50
1	0	1	9	1.26	0.37	1.00	2.82	1.19	1.31	0.55
1	0	1	10	1.26	0.40	1.00	2.92	1.25	1.47	0.63
1	0	1	11	1.26	0.41	1.00	3.00	1.29	1.58	0.68
1	0	1	12	1.30	0.40	1.01	3.23	1.32	1.74	0.71
1	0	2	1	1.26	0.10	0.63	3.12	0.76	0.77	0.19
1	0	2	2	1.26	0.14	0.71	3.05	0.84	0.77	0.21
1	0	2	3	1.26	0.17	0.81	2.90	0.92	0.77	0.24
1	0	2	4	1.26	0.20	0.87	2.80	0.96	0.77	0.26
1	0	2	5	1.26	0.21	0.92	2.71	0.99	0.77	0.28
1	0	2	6	1.26	0.23	0.96	2.64	1.01	0.77	0.29
1	0	2	7	1.26	0.23	0.98	2.60	1.01	0.77	0.30
1	0	2	8	1.26	0.24	1.00	2.56	1.02	0.77	0.31
1	0	2	9	1.26	0.25	1.02	2.51	1.03	0.77	0.31
1	0	2	10	1.26	0.26	1.05	2.46	1.03	0.77	0.32
1	0	2	11	1.26	0.26	1.07	2.42	1.04	0.77	0.33
1	0	2	12	1.26	0.27	1.09	2.38	1.05	0.77	0.34
1	0	2	13	1.26	0.29	1.14	2.31	1.06	0.77	0.36
1	0	2	14	1.26	0.30	1.19	2.24	1.08	0.77	0.37
1	0	2	15	1.26	0.31	1.27	2.11	1.09	0.77	0.40
1	0	2	16	1.26	0.32	1.34	2.03	1.11	0.77	0.42
1	0	2	17	1.26	0.34	1.41	1.97	1.14	0.77	0.45
1	0	2	18	1.26	0.36	1.39	2.02	1.18	0.77	0.45
1	0	2	19	1.26	0.39	1.45	2.00	1.22	0.77	0.47

$M_p = 1.3$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
1	0	2	20	1.26	0.41	1.50	1.97	1.26	0.77	0.49
1	0	2	21	1.26	0.44	1.57	1.97	1.34	0.77	0.52
1	0	3	1	1.25	0.09	0.39	6.70	1.00	1.19	0.18
1	0	3	2	1.25	0.14	0.44	4.87	0.83	1.19	0.20
1	0	3	3	1.25	0.18	0.50	4.13	0.81	1.19	0.23
1	0	3	4	1.25	0.20	0.61	3.75	0.90	1.19	0.29
1	0	3	5	1.26	0.26	0.73	3.48	1.02	1.19	0.35
1	0	3	6	1.26	0.30	0.82	3.19	1.07	1.19	0.40
1	0	3	7	1.26	0.31	0.87	3.06	1.09	1.19	0.42
1	0	3	8	1.26	0.33	0.92	2.93	1.11	1.19	0.45
1	0	3	9	1.26	0.34	0.96	2.84	1.13	1.19	0.47
1	0	3	10	1.26	0.34	0.97	2.81	1.13	1.19	0.48
1	0	3	11	1.26	0.34	0.99	2.77	1.13	1.19	0.49
1	0	3	12	1.26	0.35	1.00	2.75	1.14	1.19	0.49
1	0	3	13	1.26	0.35	1.01	2.72	1.15	1.19	0.50
1	0	3	14	1.26	0.36	1.04	2.68	1.16	1.19	0.52
1	0	3	15	1.26	0.37	1.08	2.63	1.19	1.19	0.54
1	0	3	16	1.26	0.39	1.11	2.59	1.21	1.19	0.55
1	0	3	17	1.26	0.39	1.13	2.57	1.22	1.19	0.57
1	0	3	18	1.26	0.40	1.15	2.55	1.24	1.19	0.58
1	0	3	19	1.26	0.41	1.17	2.54	1.27	1.19	0.59
1	0	3	20	1.26	0.43	1.20	2.53	1.31	1.19	0.61
1	0	3	21	1.26	0.46	1.28	2.52	1.42	1.19	0.67
1	1	1	1	1.26	0.11	1.02	2.07	0.81	0.52	0.20
1	1	1	2	1.26	0.16	1.02	2.26	0.90	0.60	0.24
1	1	1	3	1.26	0.18	1.01	2.33	0.91	0.66	0.26
1	1	1	4	1.26	0.21	1.01	2.39	0.95	0.73	0.29
1	1	1	5	1.26	0.22	1.03	2.37	0.96	0.80	0.32
1	1	1	6	1.26	0.23	1.01	2.43	0.97	0.86	0.34
1	1	1	7	1.26	0.25	0.98	2.52	0.99	0.92	0.36
1	1	1	8	1.26	0.28	1.00	2.56	1.04	0.99	0.40
1	1	1	9	1.26	0.30	1.01	2.60	1.06	1.05	0.43
1	1	1	10	1.26	0.32	1.00	2.65	1.08	1.14	0.47

$M_p = 1.3$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
1	1	1	11	1.26	0.34	1.01	2.71	1.12	1.22	0.51
1	1	1	12	1.26	0.36	1.01	2.80	1.18	1.38	0.58
1	1	2	1	1.26	0.11	0.58	3.17	0.70	0.77	0.17
1	1	2	2	1.26	0.14	0.71	2.87	0.79	0.77	0.21
1	1	2	3	1.26	0.17	0.79	2.72	0.83	0.77	0.24
1	1	2	4	1.26	0.18	0.87	2.61	0.88	0.77	0.26
1	1	2	5	1.26	0.19	0.92	2.53	0.91	0.77	0.28
1	1	2	6	1.26	0.20	0.95	2.50	0.93	0.77	0.29
1	1	2	7	1.26	0.21	0.96	2.47	0.93	0.77	0.29
1	1	2	8	1.26	0.21	0.98	2.45	0.95	0.77	0.30
1	1	2	9	1.26	0.22	1.00	2.43	0.95	0.77	0.30
1	1	2	10	1.26	0.22	1.01	2.42	0.96	0.77	0.31
1	1	2	11	1.26	0.23	1.03	2.40	0.97	0.77	0.31
1	1	2	12	1.26	0.23	1.04	2.38	0.98	0.77	0.32
1	1	2	13	1.26	0.24	1.07	2.36	1.00	0.77	0.33
1	1	2	14	1.26	0.26	1.11	2.31	1.02	0.77	0.34
1	1	2	15	1.27	0.29	1.18	2.25	1.06	0.77	0.36
1	1	2	16	1.27	0.32	1.22	2.23	1.10	0.77	0.38
1	1	2	17	1.27	0.34	1.27	2.19	1.14	0.77	0.40
1	1	2	18	1.27	0.37	1.32	2.15	1.17	0.77	0.42
1	1	2	19	1.27	0.39	1.41	2.08	1.23	0.77	0.46
1	1	2	20	1.27	0.41	1.49	2.07	1.30	0.78	0.49
1	1	2	21	1.27	0.44	1.60	2.08	1.43	0.78	0.54
1	1	3	1	1.26	0.10	0.36	5.84	0.80	1.19	0.16
1	1	3	2	1.26	0.15	0.47	4.11	0.75	1.19	0.22
1	1	3	3	1.26	0.17	0.56	3.74	0.81	1.19	0.26
1	1	3	4	1.26	0.21	0.64	3.44	0.86	1.19	0.30
1	1	3	5	1.26	0.25	0.74	3.16	0.93	1.19	0.35
1	1	3	6	1.26	0.28	0.83	2.98	0.99	1.19	0.40
1	1	3	7	1.26	0.30	0.91	2.83	1.04	1.19	0.44
1	1	3	8	1.26	0.32	0.97	2.74	1.08	1.19	0.47
1	1	3	9	1.27	0.33	1.01	2.69	1.11	1.19	0.49
1	1	3	10	1.27	0.34	1.03	2.66	1.12	1.19	0.50

$M_p = 1.3$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_o}$	NPR	SNPR	NPR*	SNPR*
1	1	3	11	1.27	0.35	1.06	2.63	1.14	1.19	0.52
1	1	3	12	1.27	0.36	1.09	2.60	1.17	1.19	0.54
1	1	3	13	1.27	0.37	1.14	2.53	1.20	1.19	0.57
1	1	3	14	1.27	0.38	1.19	2.46	1.22	1.19	0.59
1	1	3	15	1.27	0.41	1.28	2.43	1.32	1.20	0.65
1	2	1	1	1.26	0.09	1.05	1.87	0.75	0.49	0.20
1	2	1	2	1.26	0.15	1.02	2.15	0.84	0.57	0.22
1	2	1	3	1.26	0.18	1.01	2.24	0.88	0.63	0.25
1	2	1	4	1.26	0.22	1.03	2.28	0.92	0.71	0.29
1	2	1	5	1.26	0.22	1.04	2.27	0.93	0.77	0.31
1	2	1	6	1.26	0.29	0.94	2.54	0.96	0.90	0.34
1	2	1	7	1.26	0.34	1.00	2.59	1.07	1.00	0.41
1	2	1	8	1.26	0.35	1.00	2.66	1.10	1.08	0.45
1	2	1	9	1.26	0.38	1.00	2.82	1.18	1.23	0.52
1	2	1	10	1.26	0.38	1.00	2.89	1.21	1.33	0.56
1	2	2	1	1.26	0.17	0.72	2.62	0.73	0.77	0.22
1	2	2	2	1.26	0.19	0.78	2.62	0.79	0.78	0.24
1	2	2	3	1.26	0.23	0.86	2.56	0.87	0.77	0.26
1	2	2	4	1.26	0.25	0.90	2.52	0.90	0.77	0.28
1	2	2	5	1.26	0.26	0.94	2.46	0.92	0.77	0.29
1	2	2	6	1.26	0.26	1.00	2.38	0.94	0.78	0.31
1	2	2	7	1.26	0.24	1.05	2.30	0.95	0.77	0.32
1	2	2	8	1.26	0.25	1.07	2.27	0.97	0.77	0.33
1	2	2	9	1.26	0.25	1.11	2.24	0.98	0.77	0.34
1	2	2	10	1.27	0.26	1.16	2.18	1.00	0.78	0.36
1	2	2	11	1.27	0.28	1.22	2.12	1.02	0.78	0.37
1	2	2	12	1.27	0.30	1.28	2.08	1.06	0.78	0.40
1	2	2	13	1.27	0.32	1.39	1.98	1.11	0.78	0.43
1	2	2	14	1.27	0.34	1.46	1.93	1.14	0.78	0.46
1	2	2	15	1.27	0.37	1.55	1.90	1.22	0.78	0.50
1	2	2	16	1.27	0.39	1.60	1.90	1.26	0.78	0.52
1	2	2	17	1.26	0.09	0.33	5.30	0.67	1.20	0.15
1	2	3	1	1.26	0.16	0.43	3.74	0.63	1.20	0.20

$M_p = 1.3$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_e}$	NPR	SNPR	NPR*	SNPR*
1	2	3	2	1.26	0.20	0.49	3.58	0.69	1.19	0.23
1	2	3	3	1.26	0.25	0.57	3.38	0.77	1.20	0.27
1	2	3	4	1.26	0.29	0.72	3.14	0.91	1.20	0.35
1	2	3	5	1.26	0.30	0.80	3.00	0.97	1.20	0.39
1	2	3	6	1.26	0.32	0.86	2.91	1.02	1.20	0.42
1	2	3	7	1.26	0.34	0.92	2.84	1.07	1.19	0.45
1	2	3	8	1.26	0.35	0.95	2.79	1.10	1.19	0.47
1	2	3	9	1.26	0.36	0.98	2.78	1.13	1.19	0.48
1	2	3	10	1.26	0.36	0.99	2.77	1.14	1.19	0.49
1	2	3	11	1.26	0.37	0.99	2.77	1.15	1.19	0.50
1	2	3	12	1.26	0.38	1.02	2.76	1.17	1.19	0.51
1	2	3	13	1.26	0.39	1.05	2.75	1.22	1.19	0.53
1	2	3	14	1.27	0.40	1.09	2.73	1.25	1.19	0.55
1	2	3	15	1.27	0.41	1.14	2.71	1.31	1.19	0.58
1	2	3	16	1.27	0.41	1.20	2.57	1.30	1.19	0.61
1	2	3	17	1.27	0.42	1.27	2.49	1.34	1.20	0.64
1	3	1	1	1.26	0.10	1.03	1.86	0.73	0.49	0.19
1	3	1	2	1.26	0.19	1.01	2.17	0.86	0.60	0.24
1	3	1	3	1.26	0.22	1.01	2.25	0.89	0.65	0.26
1	3	1	4	1.26	0.25	1.01	2.32	0.93	0.72	0.29
1	3	1	5	1.26	0.27	1.00	2.38	0.95	0.79	0.32
1	3	1	6	1.26	0.28	1.00	2.43	0.98	0.85	0.34
1	3	1	7	1.26	0.31	1.01	2.50	1.02	0.92	0.38
1	3	1	8	1.26	0.34	1.01	2.58	1.08	1.00	0.42
1	3	1	9	1.26	0.35	1.00	2.64	1.10	1.06	0.44
1	3	1	10	1.26	0.36	1.01	2.69	1.13	1.11	0.47
1	3	1	11	1.26	0.37	1.00	2.74	1.16	1.18	0.50
1	3	1	12	1.26	0.37	1.01	2.73	1.15	1.25	0.53
1	3	1	13	1.26	0.38	1.00	2.81	1.18	1.33	0.56
1	3	1	14	1.26	0.39	1.00	2.89	1.22	1.42	0.60
1	3	2	1	1.26	0.17	0.62	2.83	0.69	0.77	0.19
1	3	2	2	1.26	0.18	0.71	2.74	0.75	0.77	0.21
1	3	2	3	1.26	0.20	0.77	2.65	0.80	0.77	0.23

M_p = 1.3 static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
1	3	2	4	1.26	0.23	0.86	2.52	0.85	0.77	0.26
1	3	2	5	1.26	0.23	0.95	2.36	0.88	0.77	0.29
1	3	2	6	1.26	0.23	0.98	2.32	0.90	0.77	0.30
1	3	2	7	1.26	0.23	0.99	2.31	0.91	0.77	0.30
1	3	2	8	1.26	0.24	1.00	2.30	0.91	0.77	0.31
1	3	2	9	1.26	0.24	1.02	2.28	0.92	0.77	0.31
1	3	2	10	1.26	0.25	1.05	2.26	0.94	0.77	0.32
1	3	2	11	1.26	0.26	1.10	2.22	0.98	0.77	0.34
1	3	2	12	1.26	0.28	1.14	2.20	1.00	0.77	0.35
1	3	2	13	1.26	0.32	1.21	2.18	1.07	0.77	0.38
1	3	2	14	1.27	0.36	1.30	2.15	1.16	0.77	0.42
1	3	2	15	1.27	0.37	1.40	2.06	1.19	0.77	0.45
1	3	2	16	1.27	0.38	1.47	2.00	1.23	0.77	0.47
1	3	2	17	1.27	0.41	1.54	2.02	1.32	0.77	0.51
1	3	2	18	1.27	0.43	1.61	2.03	1.40	0.77	0.53
1	3	3	1	1.26	0.09	0.36	8.28	1.14	1.19	0.16
1	3	3	2	1.26	0.17	0.43	6.29	1.05	1.19	0.20
1	3	3	3	1.26	0.24	0.52	3.66	0.76	1.19	0.25
1	3	3	4	1.26	0.27	0.65	3.21	0.84	1.19	0.31
1	3	3	5	1.26	0.30	0.76	3.03	0.94	1.19	0.37
1	3	3	6	1.26	0.34	0.89	2.88	1.05	1.19	0.44
1	3	3	7	1.26	0.36	0.94	2.82	1.10	1.19	0.47
1	3	3	8	1.26	0.36	0.97	2.79	1.13	1.19	0.48
1	3	3	9	1.26	0.37	0.99	2.77	1.14	1.19	0.49
1	3	3	10	1.26	0.37	1.00	2.75	1.15	1.19	0.50
1	3	3	11	1.26	0.37	1.02	2.74	1.17	1.19	0.51
1	3	3	12	1.26	0.39	1.06	2.71	1.20	1.19	0.53
1	3	3	13	1.26	0.39	1.09	2.69	1.24	1.19	0.55
1	3	3	14	1.26	0.40	1.12	2.67	1.27	1.19	0.57
1	3	3	15	1.27	0.41	1.17	2.64	1.31	1.19	0.59
1	3	3	16	1.27	0.42	1.24	2.60	1.38	1.19	0.63
1	3	3	17	1.26	0.43	1.29	2.57	1.42	1.19	0.66

A.2.3 $M_p = 1.6$ nozzle data

This section contains a list of all the $M_p = 1.6$ test conditions at which static pressure, Pitot pressure, and Mie data was obtained. Brief summaries of the tests corresponding to the different Pitot, and static cases are contained in Tables A.13, and A.14. The Pitot survey conditions are tabulated in Table A.15, and the static pressure conditions are listed in Table A.16.

Table A.13: $M_p = 1.6$ Pitot survey case number designations.

$M_p = 1.6$ Pitot case #	Summary
1	$NPR^* = 1.31, SNPR^* = 0.25$
2	$NPR^* = 1.31, SNPR^* = 0.31$
3	$NPR^* = 1.31, SNPR^* = 0.38$
4	$NPR^* = 1.70, SNPR^* = 0.43$

Table A.14: $M_p = 1.6$ static pressure case number designations.

$M_p = 1.6$ static case #	Summary
1	$NPR^* = 0.65, 0.26 < SNPR^* < 0.21$
2	$NPR^* = 0.97, 0.21 < SNPR^* < 0.33$
3	$NPR^* = 1.31, 0.20 < SNPR^* < 0.42$
4	$NPR^* = 1.69, 0.39 < SNPR^* < 0.51$
5	$\frac{P_s}{P_p} = 1.0$ (Range of $NPR^*, SNPR^*$)

Table A.15: $M_p = 1.6$ Pitot survey data

$M_p = 1.6$ Pitot survey data										
Nozzle	Mixer	Rake	Case	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	0	1	1	1.64	0.19	0.86	4.41	0.86	1.30	0.25
2	0	3	1	1.66	0.16	0.90	4.62	0.92	1.32	0.26
2	0	4	1	1.65	0.13	0.89	4.42	0.87	1.32	0.26
2	0	1	2	1.64	0.26	1.03	4.30	1.02	1.30	0.31
2	0	3	2	1.66	0.20	1.05	4.28	1.00	1.32	0.31
2	0	4	2	1.64	0.17	1.02	4.32	0.99	1.33	0.30
2	0	1	3	1.64	0.28	1.22	3.77	1.08	1.30	0.37
2	0	3	3	1.65	0.26	1.28	3.70	1.09	1.32	0.39
2	0	4	3	1.64	0.23	1.27	3.93	1.14	1.33	0.39
2	0	1	4	1.64	0.31	1.07	4.57	1.15	1.69	0.43
2	0	3	4	1.65	0.27	1.09	4.44	1.11	1.72	0.43
2	0	4	4	1.65	0.23	1.12	4.96	1.27	1.73	0.44
2	1	1	1	1.65	0.14	0.93	4.05	0.83	1.31	0.27
2	1	2	1	1.65	0.16	0.95	3.81	0.81	1.31	0.28
2	1	3	1	1.65	0.17	0.95	3.90	0.83	1.31	0.28
2	1	4	1	1.65	0.13	0.89	3.90	0.77	1.33	0.26
2	1	1	2	1.65	0.19	1.09	3.85	0.94	1.32	0.32
2	1	2	2	1.64	0.19	1.11	3.61	0.91	1.31	0.33
2	1	3	2	1.65	0.19	1.04	3.72	0.87	1.32	0.31
2	1	4	2	1.64	0.16	1.02	3.76	0.86	1.34	0.31
2	1	1	3	1.64	0.25	1.25	3.79	1.10	1.32	0.38
2	1	3	3	1.64	0.25	1.25	3.51	1.01	1.32	0.38
2	1	4	3	1.64	0.23	1.27	4.18	1.22	1.33	0.39
2	1	1	4	1.64	0.23	1.11	4.22	1.07	1.68	0.43
2	1	2	4	1.64	0.25	1.12	3.97	1.02	1.71	0.44
2	1	3	4	1.65	0.25	1.10	4.09	1.03	1.72	0.43
2	1	4	4	1.64	0.23	1.11	4.86	1.24	1.74	0.44
2	2	1	1	1.65	0.14	0.85	3.97	0.75	1.31	0.25
2	2	2	1	1.64	0.19	0.89	3.98	0.81	1.32	0.27
2	2	3	1	1.64	0.10	0.87	4.00	0.77	1.31	0.25
2	2	4	1	1.64	0.13	0.90	3.76	0.75	1.33	0.27

M_p = 1.6 Pitot survey data (Continued)										
Nozzle	Mixer	Rake	Case	M_p	M_s	$\frac{P_a}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	2	1	2	1.64	0.20	1.10	3.76	0.94	1.31	0.33
2	2	2	2	1.64	0.21	1.05	3.72	0.89	1.32	0.32
2	2	3	2	1.64	0.18	1.01	3.78	0.86	1.33	0.30
2	2	4	2	1.64	0.18	1.05	3.71	0.88	1.34	0.32
2	2	1	3	1.64	0.24	1.26	3.59	1.04	1.31	0.38
2	2	3	3	1.64	0.23	1.26	3.57	1.03	1.33	0.38
2	2	4	3	1.64	0.23	1.25	3.72	1.06	1.34	0.38
2	2	3	4	1.64	0.25	1.07	4.07	1.01	1.72	0.43
2	2	4	4	1.64	0.24	1.12	4.68	1.20	1.74	0.44
2	3	1	1	1.64	0.17	0.94	3.85	0.82	1.30	0.28
2	3	2	1	1.65	0.19	0.97	3.81	0.84	1.32	0.29
2	3	3	1	1.65	0.15	0.89	3.83	0.76	1.32	0.26
2	3	4	1	1.65	0.16	0.91	3.85	0.78	1.34	0.27
2	3	1	2	1.64	0.20	1.10	3.70	0.92	1.30	0.32
2	3	2	2	1.64	0.20	1.08	3.62	0.88	1.30	0.32
2	3	3	2	1.65	0.18	1.00	3.70	0.83	1.32	0.30
2	3	4	2	1.65	0.18	1.02	3.77	0.86	1.34	0.31
2	3	1	3	1.64	0.22	1.26	3.55	1.02	1.30	0.38
2	3	2	3	1.64	0.24	1.26	3.43	0.99	1.30	0.38
2	3	3	3	1.64	0.24	1.26	3.41	0.98	1.32	0.38
2	3	4	3	1.64	0.24	1.29	3.55	1.05	1.34	0.39
2	3	1	4	1.64	0.25	1.09	4.05	1.02	1.69	0.43
2	3	2	4	1.64	0.26	1.11	3.93	1.01	1.71	0.44
2	3	3	4	1.65	0.26	1.10	4.01	1.01	1.72	0.44
2	3	4	4	1.65	0.24	1.11	4.74	1.21	1.75	0.44

Table A.16: $M_p = 1.6$ wall static pressure data

$M_p = 1.6$ static pressure data										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	0	1	1	1.64	0.17	1.78	2.41	0.98	0.65	0.26
2	0	1	2	1.32	0.30	1.41	2.04	1.07	0.65	0.34
2	0	2	1	1.64	0.18	0.94	4.23	0.90	0.97	0.21
2	0	2	2	1.64	0.20	1.00	4.00	0.91	0.97	0.22
2	0	2	3	1.64	0.22	1.07	3.77	0.92	0.97	0.24
2	0	2	4	1.64	0.23	1.16	3.56	0.94	0.97	0.26
2	0	2	5	1.64	0.25	1.23	3.41	0.96	0.97	0.27
2	0	2	6	1.64	0.28	1.28	3.32	0.99	0.97	0.29
2	0	2	7	1.64	0.30	1.36	3.17	1.02	0.97	0.31
2	0	2	8	1.64	0.31	1.45	3.02	1.03	0.97	0.33
2	0	3	1	1.65	0.21	0.69	5.32	0.84	1.31	0.20
2	0	3	2	1.65	0.24	0.79	4.89	0.88	1.31	0.24
2	0	3	3	1.65	0.28	0.89	4.56	0.94	1.31	0.27
2	0	3	4	1.65	0.29	0.93	4.42	0.96	1.31	0.28
2	0	3	5	1.65	0.31	0.98	4.27	0.98	1.31	0.30
2	0	3	6	1.65	0.32	1.00	4.22	0.99	1.31	0.31
2	0	3	7	1.65	0.31	1.04	4.01	0.98	1.31	0.32
2	0	3	8	1.65	0.31	1.06	3.94	0.98	1.31	0.33
2	0	3	9	1.65	0.31	1.08	3.89	0.99	1.31	0.33
2	0	3	10	1.65	0.32	1.13	3.77	1.01	1.31	0.35
2	0	3	11	1.65	0.34	1.16	3.73	1.04	1.31	0.36
2	0	3	12	1.65	0.37	1.22	3.62	1.07	1.31	0.39
2	0	3	13	1.64	0.40	1.31	3.46	1.11	1.31	0.42
2	0	4	1	1.63	0.37	0.93	4.72	1.08	1.69	0.39
2	0	4	2	1.63	0.39	0.97	4.60	1.10	1.69	0.40
2	0	4	3	1.63	0.40	1.00	4.48	1.12	1.69	0.42
2	0	4	4	1.63	0.40	1.02	4.42	1.13	1.69	0.43
2	0	4	5	1.63	0.41	1.04	4.34	1.14	1.69	0.44
2	0	4	6	1.63	0.42	1.09	4.21	1.16	1.69	0.46
2	0	4	7	1.63	0.43	1.13	4.11	1.18	1.69	0.48
2	0	4	8	1.63	0.44	1.17	4.02	1.21	1.69	0.51

$M_p = 1.6$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_r}$	NPR	SNPR	NPR*	SNPR*
2	0	5	1	1.63	0.39	0.99	4.48	1.11	1.69	0.42
2	0	5	2	1.63	0.35	0.98	4.33	1.05	1.48	0.36
2	0	5	3	1.63	0.34	0.98	4.24	1.02	1.38	0.33
2	0	5	4	1.64	0.26	0.96	4.10	0.92	1.18	0.27
2	0	5	5	1.64	0.21	0.98	4.04	0.91	1.04	0.24
2	0	5	6	1.63	0.16	0.99	3.98	0.90	0.91	0.21
2	0	5	7	1.63	0.00	0.95	3.63	0.78	0.78	0.17
2	1	1	1	1.64	0.16	1.40	2.44	0.77	0.65	0.21
2	1	1	2	1.63	0.20	1.64	2.48	0.94	0.65	0.25
2	1	2	1	1.65	0.12	0.88	3.72	0.72	0.97	0.19
2	1	2	2	1.65	0.13	0.92	3.65	0.74	0.97	0.20
2	1	2	3	1.65	0.14	0.95	3.63	0.77	0.97	0.21
2	1	2	4	1.65	0.17	0.97	3.70	0.81	0.97	0.21
2	1	2	5	1.65	0.18	1.00	3.65	0.82	0.98	0.22
2	1	2	6	1.64	0.20	1.04	3.58	0.84	0.98	0.23
2	1	2	7	1.64	0.21	1.07	3.52	0.85	0.98	0.24
2	1	2	8	1.64	0.22	1.10	3.48	0.87	0.98	0.24
2	1	2	9	1.64	0.25	1.15	3.40	0.90	0.98	0.26
2	1	2	10	1.64	0.33	1.29	3.28	1.01	0.98	0.30
2	1	3	1	1.65	0.16	0.73	4.72	0.77	1.31	0.21
2	1	3	2	1.65	0.21	0.86	4.31	0.84	1.31	0.25
2	1	3	3	1.65	0.23	0.90	4.19	0.86	1.31	0.27
2	1	3	4	1.65	0.24	0.93	4.14	0.88	1.31	0.28
2	1	3	5	1.65	0.25	0.96	4.06	0.89	1.31	0.29
2	1	3	6	1.65	0.26	0.98	4.01	0.90	1.31	0.30
2	1	3	7	1.65	0.26	1.00	3.96	0.91	1.31	0.30
2	1	3	8	1.65	0.26	1.02	3.91	0.92	1.31	0.31
2	1	3	9	1.65	0.27	1.05	3.85	0.93	1.31	0.32
2	1	3	10	1.65	0.28	1.08	3.77	0.95	1.31	0.33
2	1	3	11	1.65	0.31	1.15	3.64	0.98	1.31	0.35
2	1	3	12	1.64	0.33	1.23	3.52	1.03	1.31	0.38
2	1	4	1	1.65	0.18	0.62	5.51	0.77	1.69	0.24
2	1	4	2	1.65	0.20	0.70	5.12	0.81	1.69	0.27

$M_p = 1.6$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	1	4	3	1.65	0.24	0.81	4.73	0.87	1.69	0.31
2	1	4	4	1.65	0.29	0.92	4.43	0.95	1.69	0.36
2	1	4	5	1.65	0.30	0.94	4.36	0.96	1.69	0.37
2	1	4	6	1.65	0.31	0.97	4.30	0.97	1.70	0.38
2	1	4	7	1.65	0.31	0.98	4.26	0.98	1.70	0.39
2	1	4	8	1.65	0.31	1.00	4.23	0.99	1.70	0.40
2	1	4	9	1.65	0.32	1.01	4.19	1.00	1.70	0.40
2	1	4	10	1.65	0.32	1.04	4.13	1.01	1.70	0.41
2	1	4	11	1.65	0.34	1.10	3.98	1.04	1.70	0.44
2	1	4	12	1.65	0.36	1.15	3.87	1.07	1.69	0.47
2	1	4	13	1.65	0.37	1.20	3.76	1.09	1.69	0.49
2	1	5	1	1.65	0.31	1.00	4.20	0.98	1.69	0.40
2	1	5	2	1.65	0.29	0.99	4.14	0.95	1.57	0.36
2	1	5	3	1.65	0.27	0.99	4.05	0.93	1.44	0.33
2	1	5	4	1.64	0.24	0.98	3.93	0.88	1.25	0.28
2	1	5	5	1.64	0.19	0.97	3.74	0.82	1.04	0.23
2	1	5	6	1.64	0.10	1.00	3.20	0.71	0.79	0.18
2	2	1	1	1.64	0.00	1.13	2.87	0.72	0.65	0.16
2	2	1	2	1.64	0.09	1.36	2.82	0.85	0.65	0.20
2	2	1	3	1.63	0.16	1.54	2.57	0.90	0.65	0.23
2	2	1	4	1.62	0.17	1.58	2.47	0.91	0.65	0.24
2	2	2	1	1.64	0.15	0.83	3.65	0.68	0.98	0.18
2	2	2	2	1.64	0.16	0.88	3.57	0.71	0.98	0.19
2	2	2	3	1.64	0.17	0.93	3.53	0.73	0.98	0.20
2	2	2	4	1.64	0.19	0.96	3.53	0.77	0.97	0.21
2	2	2	5	1.64	0.20	0.97	3.54	0.78	0.97	0.22
2	2	2	6	1.64	0.21	0.98	3.54	0.79	0.98	0.22
2	2	2	7	1.64	0.21	1.00	3.54	0.80	0.98	0.22
2	2	2	8	1.64	0.23	1.02	3.54	0.82	0.98	0.23
2	2	2	9	1.64	0.24	1.06	3.53	0.86	0.98	0.24
2	2	2	10	1.64	0.25	1.07	3.53	0.87	0.98	0.24
2	2	2	11	1.64	0.26	1.10	3.53	0.90	0.98	0.25
2	2	3	1	1.65	0.20	0.63	4.32	0.62	1.30	0.19

$M_p = 1.6$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	2	3	2	1.65	0.20	0.79	4.02	0.71	1.31	0.23
2	2	3	3	1.65	0.21	0.85	3.92	0.76	1.31	0.25
2	2	3	4	1.65	0.23	0.90	3.85	0.79	1.31	0.27
2	2	3	5	1.65	0.24	0.93	3.84	0.82	1.31	0.28
2	2	3	6	1.65	0.25	0.96	3.83	0.84	1.31	0.29
2	2	3	7	1.65	0.26	0.97	3.82	0.86	1.31	0.29
2	2	3	8	1.65	0.27	0.99	3.80	0.87	1.31	0.30
2	2	3	9	1.65	0.27	1.00	3.79	0.88	1.30	0.30
2	2	3	10	1.65	0.28	1.02	3.77	0.89	1.30	0.31
2	2	3	11	1.65	0.28	1.04	3.74	0.90	1.30	0.31
2	2	3	12	1.65	0.29	1.06	3.71	0.91	1.31	0.32
2	2	3	13	1.64	0.31	1.13	3.63	0.97	1.31	0.35
2	2	3	14	1.64	0.33	1.18	3.59	1.01	1.31	0.37
2	2	3	15	1.64	0.34	1.23	3.51	1.03	1.31	0.38
2	2	4	1	1.65	0.24	0.65	4.56	0.68	1.70	0.25
2	2	4	2	1.65	0.24	0.73	4.40	0.73	1.70	0.28
2	2	4	3	1.65	0.27	0.81	4.26	0.80	1.70	0.32
2	2	4	4	1.65	0.28	0.92	4.04	0.86	1.70	0.36
2	2	4	5	1.65	0.29	0.95	3.99	0.88	1.70	0.37
2	2	4	6	1.65	0.29	0.97	3.97	0.89	1.70	0.38
2	2	4	7	1.65	0.31	0.99	3.99	0.92	1.70	0.39
2	2	4	8	1.65	0.31	1.00	3.98	0.93	1.70	0.40
2	2	4	9	1.65	0.32	1.02	3.94	0.95	1.70	0.41
2	2	4	10	1.65	0.34	1.10	3.81	1.00	1.70	0.45
2	2	4	11	1.64	0.37	1.19	3.68	1.06	1.70	0.49
2	2	5	1	1.65	0.31	1.00	3.97	0.93	1.70	0.40
2	2	5	2	1.65	0.29	1.00	3.87	0.90	1.48	0.34
2	2	5	3	1.64	0.26	0.99	3.74	0.86	1.25	0.29
2	2	5	4	1.64	0.23	0.99	3.59	0.82	1.05	0.24
2	2	5	5	1.64	0.12	0.97	3.12	0.68	0.79	0.17
2	3	1	1	1.64	0.00	1.13	2.60	0.64	0.65	0.16
2	3	1	2	1.64	0.10	1.51	2.48	0.83	0.65	0.22
2	3	1	3	1.64	0.17	1.62	2.47	0.91	0.65	0.24

$M_p = 1.6$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	3	2	1	1.65	0.18	0.94	3.71	0.78	0.97	0.21
2	3	2	2	1.65	0.19	0.96	3.67	0.79	0.97	0.21
2	3	2	3	1.65	0.19	1.00	3.61	0.81	0.97	0.22
2	3	2	4	1.65	0.20	1.02	3.57	0.82	0.97	0.22
2	3	2	5	1.65	0.21	1.05	3.51	0.84	0.97	0.23
2	3	2	6	1.65	0.22	1.10	3.44	0.86	0.97	0.24
2	3	2	7	1.65	0.24	1.18	3.32	0.90	0.97	0.26
2	3	2	8	1.64	0.28	1.28	3.20	0.95	0.97	0.29
2	3	3	1	1.65	0.18	0.70	4.44	0.69	1.31	0.20
2	3	3	2	1.65	0.23	0.79	4.20	0.75	1.31	0.23
2	3	3	3	1.65	0.25	0.87	4.06	0.81	1.31	0.26
2	3	3	4	1.65	0.26	0.91	4.00	0.83	1.31	0.27
2	3	3	5	1.65	0.27	0.94	3.95	0.85	1.31	0.28
2	3	3	6	1.65	0.27	0.97	3.90	0.87	1.30	0.29
2	3	3	7	1.65	0.28	1.00	3.85	0.89	1.31	0.30
2	3	3	8	1.65	0.28	1.03	3.79	0.90	1.31	0.31
2	3	3	9	1.65	0.28	1.06	3.72	0.91	1.31	0.32
2	3	3	10	1.65	0.29	1.12	3.62	0.94	1.31	0.34
2	3	3	11	1.65	0.31	1.19	3.51	0.98	1.31	0.36
2	3	3	12	1.65	0.34	1.33	3.26	1.04	1.30	0.41
2	3	4	1	1.65	0.31	0.88	4.28	0.88	1.69	0.35
2	3	4	2	1.65	0.32	0.95	4.16	0.93	1.69	0.38
2	3	4	3	1.65	0.33	0.96	4.14	0.94	1.69	0.38
2	3	4	4	1.65	0.33	0.98	4.10	0.95	1.69	0.39
2	3	4	5	1.65	0.33	0.99	4.08	0.96	1.69	0.40
2	3	4	6	1.65	0.33	1.00	4.05	0.96	1.69	0.40
2	3	4	7	1.65	0.34	1.03	4.01	0.97	1.69	0.41
2	3	4	8	1.65	0.34	1.05	3.96	0.98	1.69	0.42
2	3	4	9	1.65	0.35	1.11	3.85	1.02	1.69	0.45
2	3	4	10	1.65	0.36	1.17	3.73	1.05	1.69	0.48
2	3	4	11	1.65	0.37	1.20	3.67	1.06	1.69	0.49
2	3	5	1	1.65	0.33	1.00	4.06	0.96	1.69	0.40
2	3	5	2	1.65	0.31	0.99	3.98	0.93	1.55	0.36

$M_p = 1.6$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
2	3	5	3	1.65	0.27	0.96	3.91	0.86	1.34	0.29
2	3	5	4	1.65	0.25	1.00	3.74	0.86	1.18	0.27
2	3	5	5	1.65	0.22	0.98	3.67	0.82	1.05	0.23
2	3	5	6	1.65	0.17	0.96	3.51	0.76	0.91	0.20
2	3	5	7	1.64	0.09	0.97	3.10	0.67	0.78	0.17

A.2.4 $M_p = 2.4$ nozzle data

This section contains a list of all the $M_p = 2.4$ test conditions at which static and Pitot pressure data was obtained. Brief summaries of the tests corresponding to the different Pitot and static are contained in Tables A.17 and A.18. The Pitot survey conditions are tabulated in Table A.19, and the static pressure conditions are listed in Table A.20.

Table A.17: $M_p = 2.4$ Pitot survey case number designations.

$M_p = 2.4$ Pitot case #	Summary
1	$NPR^* = 2.91$, $SNPR^* = 0.20$
2	$NPR^* = 2.91$, $SNPR^* = 0.22$
3	$NPR^* = 2.90$, $SNPR^* = 0.36$

Table A.18: $M_p = 2.4$ static pressure case number designations.

$M_p = 2.4$ static case #	Summary
1	$\frac{p_s}{p_p} = 1.0$ (Range of NPR^* , $SNPR^*$)
2	$NPR^* = 2.92$, $0.16 < SNPR^* < 0.45$

Table A.19: $M_p = 2.4$ Pitot survey data

$M_p = 2.4$ Pitot survey data										
Nozzle	Mixer	Rake	Case	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
3	0	4	1	2.42	0.07	1.06	11.62	0.81	2.91	0.20
3	0	4	2	2.42	0.12	1.13	11.39	0.86	2.91	0.22
3	0	4	3	2.41	0.29	1.74	8.57	1.07	2.90	0.36
3	1	4	1	2.44	0.13	0.96	9.65	0.61	2.91	0.18
3	1	4	2	2.43	0.16	1.06	9.56	0.67	2.92	0.20
3	1	4	3	2.41	0.31	1.75	9.28	1.17	2.92	0.37
3	2	4	1	2.43	0.17	0.99	9.37	0.62	2.90	0.19
3	2	4	2	2.42	0.18	1.11	9.51	0.71	2.90	0.22
3	2	4	3	2.41	0.28	1.80	8.22	1.06	2.90	0.37
3	3	4	1	2.42	0.14	0.95	9.14	0.58	2.91	0.19
3	3	4	2	2.42	0.16	1.05	9.01	0.64	2.91	0.21
3	3	4	3	2.41	0.31	1.75	7.85	0.99	2.91	0.37

Table A.20: $M_p = 2.4$ wall static pressure data

$M_p = 2.4$ static pressure data										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
3	0	1	1	2.43	0.13	1.07	11.82	0.84	2.10	0.15
3	0	1	2	2.40	0.16	1.06	12.23	0.90	2.57	0.19
3	0	1	3	2.41	0.16	1.04	12.57	0.89	2.65	0.19
3	0	1	4	2.41	0.17	1.02	12.89	0.90	2.84	0.20
3	0	1	5	2.41	0.18	1.02	12.98	0.91	2.99	0.21
3	0	1	6	2.41	0.18	1.02	13.16	0.92	3.12	0.22
3	0	1	7	2.41	0.19	1.02	13.19	0.93	3.27	0.23
3	0	1	8	2.41	0.20	1.01	13.32	0.94	3.45	0.24
3	0	1	9	2.41	0.20	1.01	13.47	0.94	3.58	0.25
3	0	1	10	2.41	0.21	1.01	13.49	0.95	3.71	0.26
3	0	1	11	2.40	0.24	1.02	13.58	0.98	4.13	0.30
3	0	1	12	2.40	0.25	1.00	13.78	0.98	4.35	0.31
3	0	1	13	2.40	0.26	1.02	13.71	1.00	4.60	0.34
3	0	1	14	2.40	0.26	1.00	13.94	1.00	4.73	0.34
3	0	2	1	2.40	0.16	0.80	14.21	0.78	2.92	0.16
3	0	2	2	2.40	0.17	0.90	13.64	0.86	2.92	0.18
3	0	2	3	2.40	0.17	0.94	13.34	0.87	2.92	0.19
3	0	2	4	2.40	0.17	1.00	12.95	0.90	2.92	0.20
3	0	2	5	2.40	0.18	1.06	12.37	0.92	2.92	0.22
3	0	2	6	2.40	0.18	1.12	11.77	0.93	2.92	0.23
3	0	2	7	2.40	0.22	1.24	10.85	0.95	2.92	0.25
3	0	2	8	2.40	0.25	1.36	9.89	0.96	2.92	0.28
3	0	2	9	2.39	0.28	1.45	9.37	0.99	2.92	0.31
3	0	2	10	2.39	0.30	1.53	8.88	1.01	2.92	0.33
3	0	2	11	2.39	0.34	1.59	8.58	1.03	2.92	0.35
3	0	2	12	2.39	0.36	1.63	8.32	1.04	2.91	0.36
3	0	2	13	2.38	0.40	1.69	8.03	1.06	2.91	0.39
3	0	2	14	2.38	0.43	1.74	7.80	1.09	2.91	0.41
3	0	2	15	2.37	0.48	1.85	7.42	1.15	2.91	0.45
3	1	1	1	2.41	0.21	1.00	9.50	0.66	1.90	0.13
3	1	1	2	2.41	0.23	0.99	10.11	0.71	2.09	0.15

$M_p = 2.4$ static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
3	1	1	3	2.40	0.23	1.00	10.38	0.73	2.26	0.16
3	1	1	4	2.40	0.24	1.00	10.56	0.75	2.45	0.17
3	1	1	5	2.40	0.25	1.00	10.75	0.76	2.64	0.19
3	1	1	6	2.40	0.26	1.00	10.83	0.78	2.87	0.21
3	1	1	7	2.40	0.27	1.00	10.98	0.79	3.07	0.22
3	1	1	8	2.40	0.26	1.01	10.81	0.78	3.23	0.23
3	1	1	9	2.40	0.27	0.99	11.00	0.78	3.41	0.24
3	1	1	10	2.40	0.28	0.98	11.34	0.80	3.63	0.26
3	1	1	11	2.40	0.29	0.99	11.37	0.82	3.96	0.29
3	1	1	12	2.40	0.30	1.00	11.43	0.83	4.48	0.33
3	1	1	13	2.40	0.31	0.91	12.68	0.85	4.62	0.31
3	1	1	14	2.40	0.30	0.99	11.59	0.84	4.62	0.33
3	1	2	1	2.40	0.24	0.75	12.10	0.64	2.96	0.16
3	1	2	2	2.40	0.24	0.81	12.12	0.70	2.96	0.17
3	1	2	3	2.40	0.25	0.85	11.93	0.73	2.96	0.18
3	1	2	4	2.40	0.26	0.91	11.54	0.75	2.96	0.19
3	1	2	5	2.40	0.26	0.94	11.26	0.76	2.96	0.20
3	1	2	6	2.40	0.27	0.99	10.87	0.78	2.96	0.21
3	1	2	7	2.40	0.27	1.05	10.47	0.80	2.96	0.22
3	1	2	8	2.40	0.28	1.10	10.27	0.82	2.96	0.24
3	1	2	9	2.40	0.31	1.18	9.84	0.85	2.96	0.26
3	1	2	10	2.40	0.33	1.24	9.56	0.88	2.96	0.27
3	1	2	11	2.40	0.36	1.32	9.31	0.92	2.96	0.29
3	1	2	12	2.39	0.39	1.42	8.82	0.96	2.96	0.32
3	1	2	13	2.39	0.40	1.52	8.45	1.00	2.96	0.35
3	1	2	14	2.38	0.42	1.60	8.11	1.02	2.96	0.37
3	1	2	15	2.38	0.43	1.69	7.80	1.06	2.96	0.40
3	1	2	16	2.37	0.44	1.77	7.59	1.09	2.96	0.43
3	1	2	17	2.37	0.45	1.80	7.51	1.11	2.96	0.44
3	2	1	1	2.40	0.22	1.01	8.17	0.58	1.73	0.12
3	2	1	2	2.40	0.24	1.00	8.52	0.61	1.86	0.13
3	2	1	3	2.40	0.26	0.98	8.89	0.62	2.04	0.14
3	2	1	4	2.40	0.29	0.99	9.36	0.67	2.22	0.16

M_p = 2.4 static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
3	2	1	5	2.40	0.31	0.99	9.67	0.70	2.42	0.17
3	2	1	6	2.40	0.32	0.99	9.87	0.72	2.58	0.19
3	2	1	7	2.40	0.33	0.99	10.02	0.74	2.74	0.20
3	2	1	8	2.40	0.34	0.99	10.16	0.75	2.86	0.21
3	2	1	9	2.40	0.35	1.00	10.24	0.76	2.99	0.22
3	2	1	10	2.40	0.35	1.00	10.24	0.76	3.14	0.23
3	2	1	11	2.40	0.36	0.99	10.51	0.78	3.35	0.25
3	2	1	12	2.40	0.37	1.00	10.64	0.80	3.56	0.27
3	2	1	13	2.40	0.38	1.00	10.77	0.81	3.77	0.28
3	2	1	14	2.40	0.39	1.00	10.89	0.83	4.02	0.31
3	2	1	15	2.40	0.40	1.00	11.07	0.84	4.39	0.33
3	2	1	16	2.40	0.40	1.00	11.18	0.85	4.64	0.35
3	2	2	1	2.40	0.29	0.56	13.53	0.55	2.95	0.12
3	2	2	2	2.40	0.29	0.66	13.75	0.66	2.95	0.14
3	2	2	3	2.40	0.31	0.74	10.88	0.59	2.96	0.16
3	2	2	4	2.40	0.31	0.80	10.29	0.60	2.95	0.17
3	2	2	5	2.40	0.31	0.87	10.10	0.64	2.95	0.19
3	2	2	6	2.40	0.33	0.93	10.24	0.70	2.95	0.20
3	2	2	7	2.40	0.34	0.99	10.24	0.75	2.95	0.22
3	2	2	8	2.40	0.36	1.04	10.11	0.79	2.95	0.23
3	2	2	9	2.40	0.38	1.13	9.90	0.85	2.95	0.25
3	2	2	10	2.39	0.30	1.51	8.55	0.96	2.95	0.33
3	2	2	11	2.39	0.31	1.54	8.44	0.97	2.95	0.34
3	2	2	12	2.39	0.32	1.60	8.27	1.00	2.95	0.36
3	2	2	13	2.38	0.33	1.68	7.95	1.02	2.95	0.38
3	2	2	14	2.38	0.34	1.73	7.79	1.03	2.95	0.39
3	2	2	15	2.38	0.37	1.75	7.71	1.04	2.95	0.40
3	2	2	16	2.38	0.38	1.79	7.55	1.06	2.95	0.41
3	2	2	17	2.38	0.39	1.83	7.41	1.07	2.95	0.43
3	2	2	18	2.38	0.40	1.86	7.30	1.08	2.95	0.43
3	3	1	1	2.41	0.19	1.02	8.25	0.59	1.80	0.13
3	3	1	2	2.41	0.22	1.01	8.73	0.62	1.96	0.14
3	3	1	3	2.40	0.25	1.00	9.12	0.65	2.09	0.15

M_p = 2.4 static pressure data (Continued)										
Nozzle	Mixer	Case	Point	M_p	M_s	$\frac{P_s}{P_p}$	NPR	SNPR	NPR*	SNPR*
3	3	1	4	2.40	0.24	1.03	8.99	0.66	2.23	0.16
3	3	1	5	2.40	0.28	1.02	9.51	0.70	2.47	0.18
3	3	1	6	2.40	0.33	0.99	10.42	0.75	2.70	0.20
3	3	1	7	2.40	0.30	0.99	10.16	0.73	2.74	0.20
3	3	1	8	2.40	0.32	1.01	10.21	0.75	2.90	0.21
3	3	1	9	2.40	0.35	0.99	10.85	0.80	2.12	0.23
3	3	1	10	2.40	0.36	0.99	11.06	0.82	3.37	0.25
3	3	1	11	2.40	0.37	0.99	11.21	0.83	3.59	0.27
3	3	1	12	2.40	0.38	0.99	11.35	0.84	3.73	0.28
3	3	1	13	2.40	0.38	0.99	11.50	0.86	3.97	0.30
3	3	1	14	2.40	0.39	0.99	11.61	0.88	4.40	0.33
3	3	1	15	2.40	0.40	1.00	11.68	0.89	4.63	0.35
3	3	2	1	2.40	0.25	0.63	13.71	0.61	2.96	0.13
3	3	2	2	2.40	0.28	0.78	11.38	0.64	2.96	0.17
3	3	2	3	2.40	0.32	0.83	11.88	0.72	2.96	0.18
3	3	2	4	2.40	0.34	0.87	11.84	0.76	2.96	0.19
3	3	2	5	2.40	0.35	0.92	11.41	0.78	2.96	0.20
3	3	2	6	2.40	0.34	1.01	10.57	0.79	2.96	0.22
3	3	2	7	2.40	0.34	1.06	10.16	0.80	2.96	0.23
3	3	2	8	2.40	0.33	1.16	9.66	0.83	2.95	0.25
3	3	2	9	2.40	0.34	1.28	9.25	0.88	2.95	0.28
3	3	2	10	2.39	0.36	1.38	8.80	0.92	2.95	0.31
3	3	2	11	2.39	0.37	1.50	8.21	0.94	2.96	0.34
3	3	2	12	2.39	0.38	1.59	7.93	0.97	2.96	0.36
3	3	2	13	2.39	0.40	1.64	7.87	1.00	2.96	0.38
3	3	2	14	2.39	0.42	1.70	7.73	1.03	2.95	0.39
3	3	2	15	2.39	0.43	1.77	7.55	1.06	2.95	0.42
3	3	2	16	2.38	0.45	1.86	7.30	1.09	2.95	0.44

Appendix B

Mie diagnostic system

This appendix contains a description of the Mie diagnostic system. A brief description of the Mie technique is given first followed by a discussion of the image correction procedures.

B.1 Description of technique

Planar Mie scattering imaging from methanol condensate was used to obtain information about the structure of the mixing layer and to determine the passive scalar fields. A passive scalar seeding technique was utilized [7, 8]. The supersonic stream was seeded with methanol liquid upstream of the plenum chamber. The liquid evaporated and then recondensed as the flow expanded through the nozzle. The liquid methanol droplets in the high speed stream served as scattering sites for incident laser light. The methanol to seeded stream mass flow ratio was roughly 0.005.

Laser illumination was provided by a frequency doubled Nd:YAG laser. The laser was pulsed at both 10 and 20 Hz for both single and multiple pulsed images. The pulse width was 10 ns, which was sufficiently short to freeze the flow. The laser was focused into a sheet roughly 5 cm wide and 1mm thick. This sheet was oriented in both cross-flow(flow out of the image) and spanwise(flow parallel to the image) planes during these experiments. The images were obtained with a Photometrics Star I CCD camera with an array of 576×384 pixels. The light intensities recorded by each pixel are stored in matrices of the CCD array dimensions. The intensity values range between 0 and 16,383.

Both time-mean and instantaneous images were obtained. The instantaneous images were taken by holding the shutter of the camera open for a single pulse of the laser. In order to obtain the time-mean images, the camera shutter was opened for 30 seconds and the laser pulsed at 20 Hz—yielding 600 pulses per image. The images presented were averaged over three of these multiply exposed images for a total of 1800 pulses. These raw images were then corrected for background scattering, laser sheet intensity variations, and perspective distortions using the procedures described in the next section.

B.2 Description of image correction procedures

This section contains descriptions of the three image correction procedures that were performed on each flow image to correct them for background scattered light and variations in incident laser sheet intensity; to remove any perspective distortions; and to scale them between the minimum seeded/unmixed intensity and the maximum unseeded/unmixed intensity.

B.2.1 Background and Response Transformations

In addition to the flow images, two additional sets of images were taken in each plane in order to correct the flow images for background scattered light and variations in incident laser sheet intensity and CCD array sensitivity.

Typical background, response and uncorrected flow images are shown in Figure B-1. The flow images are corrected using the background and response images by Eqn. B.1 [18]:

$$I_C = M_R \frac{I_U - I_B}{I_R - I_B} \quad (\text{B.1})$$

I_C is the corrected image matrix. I_U , I_B , and I_R are the uncorrected flow, background and response image matrices. M_R is the mean intensity of the response matrix.

B.2.2 Perspective Transformatation

Due to facility constraints, cross-flow images within several wavelengths of the mixer were taken at skewed angles, resulting in large image perspective distortions. A corrective procedure was developed in which the distortions are removed. In each plane, an image of a square grid was taken as shown in Figure B.2.2a. Using geometrical optics, a transformation procedure was derived in which the grid is undistorted. An unskewed grid results as shown in Figure B.2.2b.

In the transformation, the camera lens system is modelled as a single convex lens. A schematic of the (x,z) plane of the modelled object-lens-image system is shown in Figure B-3. The object and image are indicated by the bold solid lines. The subscripts i and o indicate image and object coordinates respectively. The lens axis is indicated by the dashed line between the image and object planes passing through the lens center. The origin of the two coordinate systems is defined as the intersection of the axis with their respective planes. f is the focal length of the lens. z_o and z_i are the distances between the two focal points of the lens and the object and image planes. θ is the angle between the x_o and lens axes; ϕ (not shown) is the angle between the y_o and lens axes. δx_i and δy_i are the x_i and y_i distances from the lens axis to image(detector) center.

Using geometrical optics, a relation between the object(x_o, y_o) and image(x_i, y_i) coordinates in

(a) Background

(b) Response

(c) Uncorrected Flow

(d) Corrected Flow

Figure B-1: Background, response, uncorrected flow, and corrected flow images 1λ downstream of the 15° forced mixer with $M_1 = 2.4$ and $M_2 = 1.6$.

(a) Skewed Grid

(b) Unskewed Grid

Figure B-2: Untransformed and transformed grids 1λ downstream of the 15° forced mixer. The 2 is on the tunnel centerline.

Figure B-3: Object-lens-image system.

Figure B-4: Illustration of image transformation process.

terms of the ϕ , θ , δx_i , δy_i , z_i , z_o , and f was derived

$$\begin{bmatrix} \cos(\theta) - \frac{x_i}{z_i+f} \sin(\theta) & -\left(\frac{x_i}{z_i+f}\right) \sin(\phi) \\ -\left(\frac{y_i}{z_i+f}\right) \sin(\theta) & \cos(\phi) - \left(\frac{y_i}{z_i+f}\right) \sin(\phi) \end{bmatrix} \begin{bmatrix} \frac{x_o}{z_o+f} \\ \frac{y_o}{z_o+f} \end{bmatrix} = \begin{bmatrix} \frac{x_i}{z_i+f} \\ \frac{y_i}{z_i+f} \end{bmatrix} \quad (\text{B.2})$$

Furthermore, z'_o is known in terms of z_i from the Gaussian lens formula:

$$\frac{1}{z'_o} + \frac{1}{z_i} = \frac{1}{f} \quad (\text{B.3})$$

Four points which form a square of dimensions $L \times L$ on the object are selected on the grid image. The four independent transformation parameters(ϕ , θ , δx_i , δy_i) are then iterated upon until Eqns. B.4-B.7 are satisfied on the transformed image. This transformation process is illustrated in Figure B-4.

$$|\vec{AB}| = L \quad (\text{B.4})$$

$$|\vec{AC}| = L \quad (\text{B.5})$$

$$\vec{AB} \cdot \vec{AC} = 0 \quad (\text{B.6})$$

$$\vec{BD} \cdot \vec{CD} = 0 \quad (\text{B.7})$$

B.2.3 Scaling transformation

The corrected flow images were intensity scaled between the maximum unseeded stream intensity and the minimum seeded stream intensity. The intensity field was then normalized and a passive scalar(η) defined which varies between 0 in the unseeded stream and 1 in the seeded stream. In Figure B-5, a mean corrected flow image and a mean passive scalar image are shown for comparison.

(a) Corrected mean Mie

(b) Passive scalar field

Figure B-5: Corrected mean mie and passive scalar fields 1λ down stream of the 15° forced mixer with $M_1 = 2.4$ and $M_2 = 1.6$.

Appendix C

Pressure measurement: diagnostic system and data analysis

This appendix contains a description of the pressure measurement diagnostic system, an error analysis, and descriptions of the procedures for calculating the derived quantities.

C.1 Instrumentation

All pressure measurements were made with Scanivalve ZOC16 temperature corrected electronic pressure scanners. All static pressure measurements were made with ± 5 psid transducers. Pitot pressures were measured with ± 50 psid transducers. Plenum total pressure measurements were made with ± 5 transducers for subsonic tests or ± 50 psid transducers for supersonic tests.

The transducers were calibrated twice weekly using either a Paroscientific 0 – 100 psia or a 0 – 25 psia transducer as a reference. The reference transducers are accurate to $\pm 0.01\%$ of their full scale readings. The ± 5 psid transducers are calibrated to be accurate to ± 0.004 psi, and the ± 50 psid transducers are calibrated to be accurate to ± 0.04 psi.

C.2 Wall static pressure measurements

An array of 50 wall static pressure taps were utilized at various times during the experiments. These taps were located in the two plenum chambers, at the leading edge of the mixer, along the centerline of the mixing duct subsonic and supersonic walls, and stretched spanwise across the duct at the axial locations of the Pitot surveys.

C.2.1 Calculated quantities

From the wall static pressure measurements, a number of derived quantities were calculated. The procedure for calculating each of these quantities is outlined below.

The two leading edge Mach numbers and the secondary to primary mass flow ratio were calculated from the plenum and leading edge pressure measurements and the leading edge area ratio.

$$M_p = \sqrt{\frac{2}{\gamma - 1} \left(\left(\frac{Pt_p}{p_p} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right)} \quad (C.1)$$

$$M_s = \sqrt{\frac{2}{\gamma - 1} \left(\left(\frac{Pt_s}{p_s} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right)} \quad (C.2)$$

$$\frac{\dot{m}_s}{\dot{m}_p} = \frac{p_s}{p_p} \frac{M_s}{M_p} \frac{A_s}{A_p} \sqrt{\frac{1 + \frac{\gamma - 1}{2} M_s^2}{1 + \frac{\gamma - 1}{2} M_p^2}} \quad (C.3)$$

The primary and secondary nozzle pressure ratios are calculated from the respective plenum total pressures and the duct exit static pressure.

$$NPR = \frac{Pt_p}{p_{exit}} \quad (C.4)$$

$$SNPR = \frac{Pt_s}{p_{exit}} \quad (C.5)$$

The local duct supersonic pressure coefficient is calculated from a local duct subsonic wall static pressure measurement, a supersonic leading edge static pressure measurement, and a supersonic plenum total pressure measurement.

$$Cp_{super}(x^*) = \frac{p(x^*) - p_p}{Pt_p - p_p} \quad (C.6)$$

The momentum mixing parameter at a point in the duct is calculated from a local static pressure measurement, the area-average inflow static pressure, and the fully-mixed static pressure as determined by a fully-mixed control volume model.

$$\Phi(x^*) = \frac{p(x^*) - \bar{p}_{in}}{p_{mix} - \bar{p}_{in}} \quad (C.7)$$

C.2.2 Error Analysis

A quantitative error analysis of all the derived quantities was performed using the procedure described in Beckwith *et al.* [4]. Estimates of all the known bias errors in the measurements were made and a statistical analysis was performed to estimate the random uncertainty in each of the measurements. These errors were then combined to estimate the total uncertainty in each of the

Figure C-1: Inflow quantities with typical error bars for supersonic-subsonic inflow conditions.

measurements. These uncertainties were then propagated through the calculation of the derived quantities to estimate their uncertainties.

In Figure C-1, the inflow conditions with error bars are shown for a range of static pressure measurements taken downstream of the flat plate with $M_p = 1.6$. In Figure C-2, the axial Cp_{super} distributions are plotted with error bars for the inflow conditions in Figure C-1. The momentum mixing parameter distributions are plotted with error bars in Figure C-3. These errors are typical of all the wall static pressure measurements.

C.3 Pitot/static pressure surveys

Pitot/static pressure surveys were also taken at up to four axial locations downstream of the mixers with a rake of fourteen Pitot probes. For the surveys conducted within 10 wavelengths of the mixer trailing edge, measurements were taken at 40 points spaced over 75% of the mixing duct for a density of ≈ 10 points per wavelength. For the surveys conducted downstream of 10λ , the survey density was reduced to ≈ 5 points per wavelength. Typical high density and low density survey grids are shown in Figure C-4.

Figure C-2: Sample axial Cp_{super} distributions with typical error bars for supersonic-subsonic inflow conditions.

Figure C-3: Sample axial Φ distributions with typical error bars for supersonic-subsonic inflow conditions.

(a) High density grid

(b) Low density grid

Figure C-4: Near field(high density) and far field(low density) Pitot survey grids.

C.3.1 Calculated quantities

Two mixing measures were calculated from the Pitot/static pressure surveys—a normalized Pitot mixedness and a normalized mass-average total pressure. The Pitot mixedness is the area-averaged Pitot mixedness and is analogous to the Pitot thickness utilized in many planar shear layer studies. For the matched total temperature streams of interest in these experiments, the normalized mass-averaged total pressure is a direct measure of the entropy rise associated with the mixing process.

The Pitot mixedness is calculated by area-averaging the Pitot pressures measured across the plane. In order to isolate the change in Pitot pressure associated with the mixing of the two stream from that associated with skin friction on the walls of the test section, the integration was performed only over the center wavelength span and was conducted outside the subsonic and supersonic wall boundary layers.

In order to compare the mixing performance of a given mixer at different inflow conditions, the area-averaged Pitot pressures were normalized using the mixer trailing edge area-average Pitot pressure and the fully-mixed Pitot pressure determined by a mixing out the two uniform inflow streams in a constant area control volume analysis. This normalized mixing parameter is defined by

$$\mathcal{M}_{pitot}(x) = \frac{\bar{P}_{pit}^A|_{TE} - \bar{P}_{pit}^A(x)}{\bar{P}_{pit}^A|_{TE} - \bar{P}_{pit}^A|_{mixed}}. \quad (C.8)$$

$\mathcal{M}_{pitot}$ varies between 0 at the trailing edge(TE) and 1 at the fully-mixed state.

It is important note that the normalized Pitot mixedness is not a monotonically increasing function for all possible combinations of inflow Mach numbers and static pressure ratios. However, for all inflow conditions of interest to this investigation, $\mathcal{M}_{pit}$ monotonically increases from 0 at the unmixed trailing edge to 1 at the fully-mixed conditions.

The mass-averaged total pressure was also calculated by integrating the Mach number and total pressure distributions across the plane. This integration was also performed only over the center wavelength span and outside of the wall boundary layers.

In order to compare the mixing performance of a given mixer at different inflow conditions, the mass-averaged total pressures were normalized using the mixer trailing edge mass-average total pressure and the fully-mixed mass-averaged total pressure determined by a mixing out the two uniform inflow streams in a constant area control volume analysis. This normalized mixing parameter is defined by

$$\Psi(x) = \frac{\bar{P}_t^m|_{TE} - \bar{P}_t^m(x)}{\bar{P}_t^m|_{TE} - \bar{P}_t^m|_{mixed}}. \quad (C.9)$$

Ψ varied between 0 at the trailing edge(TE) and 1 at the fully-mixed state.

Figure C-5: Normalized Pitot pressure distribution 4 duct heights downstream of the flat plate with $M_p = 1.6$ and $M_s = 0.3$.

C.3.2 Error analysis

A quantitative error analysis similar to that performed for the static pressure measurement was performed for all the quantities derived from the Pitot surveys. (The uncertainties in the measurements were propagated through the calculation of the derived quantities in order to estimate their uncertainties.) In Figure C-5, the normalized Pitot pressure distribution obtained 4 duct heights downstream of the flat plate with $M_p = 1.6$ is plotted with error bars.

Appendix D

Derivation of Vortex Model

This appendix contains a detailed derivation of the vortex model used to simulate the development of the cross-flow interface with distance from the trailing edge of the mixer.

D.1 Description of model

The evolution of the cross-flow interface is modelled using a series of viscous vortices. The vortices are initially arranged and their strengths specified to approximate the streamwise vorticity distribution shed from a lobed mixer. The vortices are then convected with their induced velocities to model the development of the interface.

Two additional models have been developed to adjust the vorticity distribution in order account for compressibility effects:

1. The movement of the vortex sheets associated with the adjustment of stream areas due to a mismatch in the stream static pressures at the trailing edge.
2. The generation of streamwise vorticity downstream of the mixer trailing edge associated with baroclinic torques in the mixing layer.

In order to calculate the effective cross-flow interface length for use in the differential control volume model, a passive scalar is tracked. After the position of the interface is determined in a number of planes in the mixing duct, the passive scalar field is then calculated at each of those planes. From the scalar fields, the effective interface length for the transport of momentum and energy is calculated(L_{eff}). The axial distribution of L_{eff} is then saved for use in the differential control volume model.

D.2 Inputs/Outputs

The inputs required by the model are

- The initial positions of each of the vortices ($\zeta_j^*|_{x^*=0}$);
- The normalized circulations of each of the vortices (Γ_j^{**});
- The streamwise circulation normalized by the mean inflow velocity and the lobe wavelength ($\Gamma^* h^*$);
- The growth rate of the velocity thickness of a planar shear layer (δ'_{vel});
- The initial thickness of the vorticity distribution (ϵ^*).

The outputs from the model are as functions of distance from the trailing edge of the mixer

- The locations of each of the vortices (ζ_j^*);
- The length of the effective cross-flow interface (L_{eff}^*);
- The scalar mixedness ($\mathcal{M}_{sc}$).

D.3 Formulation of vortex model

The key assumptions made in the development of this model are as follows.

- A slender body approximation is made in which a three-dimensional steady process is modelled as a two-dimensional unsteady process.
- This slender body approximation is supplemented by two additional compressibility effect models.
 1. An analytical model to estimate the vorticity generation associated with baroclinic torques due to shocks in the vicinity of the mixer trailing edge.
 2. A pressure matching control volume model to estimate the displacement of the vortex sheets due to the matching of the trough static pressures downstream of the mixers.
- In the calculation of the induced velocity field, the cross-flow is assumed to be incompressible.
- The superposition of the velocity fields associated with a number of viscous vortices is used to calculate the induced velocity with which the interface moves.
- An empirical turbulent diffusion model is utilized to estimate the rate at which the streamwise vorticity diffuses.

Figure D-1: Cartoon illustrating relationship between distance and time in the slender body approximation for lobed mixer flows[33].

The slender body approximation that is made is similar to that made by Qiu[23, 33] in which a steady three-dimensional flow field is approximated by a two-dimensional one as illustrated in Figure D-1. In this approximation, changes in the axial direction are modelled as changes with time as seen by an observer travelling at an appropriate convection velocity.

Fung[12, 33] conducted a detailed assessment of slender body theory in incompressible lobed mixer flows from which he identified sufficient conditions for the validity of the approximation. Fung's criteria suggests that the slender body approximation is valid if

$$\Gamma^* h^* \ll 1 \quad (D.1)$$

$$\frac{\Delta U}{\bar{U}} \ll 1. \quad (D.2)$$

Neither of these two requirements are satisfied downstream of lobed mixers of practical interest.

However, in a series of numerical experiments, Krasnodebski[16] assessed the usefulness of the theory via comparisons between three-dimensional incompressible Reynolds-averaged, Navier-Stokes computations and two-dimensional spectral element slender body computations in regimes of practical interest where Fung's conditions are not strictly satisfied. The results of his comparisons demonstrate that the slender body approximation yields useful performance predictions over a much greater range of parameters than Fung's criteria suggest[33, 16].

In compressible lobed mixer flows, the slender body model is supplemented by two additional compressibility effect models. An analytical model, which is discussed in Chapter 4, has been developed to estimate the vorticity generation associated with interaction of the trailing edge shock with the density gradient across the mixing layer. For subsonic/supersonic stream density ratios(s) less

Figure D-2: Incompressible/compressible comparison[11, 33]

than 1, the vorticity generation is in the same sense as that shed by the mixer, and for density ratios greater than 1, the vorticity generated is in the opposite sense. The circulations about the vortices passing through this shock are adjusted accordingly. In order to model the displacement of the vortex sheets associated with a mismatch in the trailing edge static pressures, a pressure matching control volume model was developed; this model is described in Appendix E.

Once the vortex circulations and locations are adjusted for compressibility effects, the cross-flow is assumed incompressible in the calculation of the induced cross-flow velocity distribution. This approximation is supported by Euler computations performed by Elliott[11] for equal total pressure and Mach number inflow streams which are free of the above two compressibility effects. These computations indicate that the growth rate of the cross-flow interface, without trailing edge shock or mismatched steam static pressures, is unaffected by the compressibility of the axial flow. In Figure D-2, the length of the mean cross-flow interface is plotted downstream of two mixers ($\alpha = 5^\circ$, 22°) for inflow Mach numbers in both streams of 0.5 and 2.0. The axial distribution of the cross-flow interface length downstream of the two mixers varies only slightly between the cases, and thus indicating that incompressible means may be used to estimate the interface growth rate.

Figure D-3: Discrete vortex/spectral code streamwise circulation comparison for $\Gamma^* = 0.5$, $\delta'_{vel} = 0.04$, $\epsilon^* = 0.25$, and $h^* = 1.0$.

The superposition of the velocity fields associated with a number of viscous vortices is used to estimate the cross-flow velocity field. In order to assess this approximation, three comparisons were made to the spectral element slender body model developed by Qiu[23, 13]. These comparisons were made with a square lobe mixer with $h^* = 1.0$ and $\Gamma^* = 0.5$. (The h^* at which the comparisons were made was the maximum h^* at which the spectral element solver could be used.) The initial boundary layer thickness was 0.25λ , and comparisons were made at shear layer growth rates of 0.04, 0.08, and 0.20.

In Figures D-3 and D-4, the normalized streamwise circulation and scalar mixedness distributions are plotted for $\delta'_{vel} = 0.04$. The test case presented represents the worst comparison (of the three test cases) between the vortex and spectral codes.

An empirical turbulent diffusion model based on the planar shear layer growth rate correlation developed by Papamoschou[22] is utilized. The growth rate utilized in the turbulent diffusion model is a function of the inflow conditions, and the variation of the growth rate associated with the changing free-stream conditions through the duct is not modelled.

The primary advantages of the current discrete vortex model relative to the spectral element method utilized by Qiu are speed and flexibility. A vortex code run with $h^* = 1$, $\Gamma^* = 0.5$, $\delta'_{vel} = 0.08$,

Figure D-4: Discrete vortex/spectral code scalar mixedness comparison for $\Gamma^* = 0.5$, $\delta'_{vel} = 0.1$, $\epsilon^* = 0.25$, and $h^* = 1.0$.

$\epsilon^* = 0.25$, and a duct height of 2λ takes 46 minutes on an IBM RS6000, while the spectral element code requires 460 minutes. In comparison, a full three-dimensional Navier-Stokes code would require approximately 4500 minutes[12].

D.4 Derivation of model

Given the initial positions and circulations of N viscous vortices, the position of each vortex as a function of t^* is found by solving a system of N complex differential equations:

$$\frac{d\zeta_k^*}{dt^*} = v_k^*, k = 1, 2, 3, \dots, N \quad (D.3)$$

v_k^* is the cross-flow velocity of vortex k specified in Eqn. D.4.

The cross-flow velocity of each vortex is found by superimposing the induced velocities associated with the other vortices that compose the interface. The non-dimensional complex velocity at vortex k at time t^* associated with the other $N - 1$ viscous vortices of circulation Γ_j^{**} which form the

interface is given by

$$v_k^*(\zeta_k^*, t^*) = \left(\frac{i}{2\pi} \right) \sum_{j=1}^N \Gamma_j^{**} \frac{\zeta_k^* - \zeta_j^*}{|\zeta_k^* - \zeta_j^*|^2} \left(1 - e^{-\frac{Re_\Gamma}{2i^*} |\zeta_k^* - \zeta_j^*|^2} \right). \quad (D.4)$$

All quantities have been non-dimensionalized using the total circulation shed from a half-wavelength span of the mixer (Γ) and the lobe wavelength (λ). The non-dimensional complex velocity, v^* , is $\frac{v\lambda}{\Gamma}$; the non-dimensional complex position of the vortex at ζ_k with respect to the vortex at ζ_j is $\frac{\zeta_k - \zeta_j}{\lambda}$; The non-dimensional time t^* is $\frac{\Gamma}{\lambda^2}$; and the normalized single vortex circulation Γ_j^{**} is $\frac{\Gamma_j}{\Gamma}$.

Re_Γ is the Reynold's number based on the streamwise circulation and the turbulent viscosity and is given by

$$Re_\Gamma = \frac{\Gamma}{\nu_t}. \quad (D.5)$$

The turbulent viscosity ν_t is estimated using a method similar to that developed by Qiu [23]. However, Qiu's viscosity model was modified

- to give the mixing layer a finite thickness at the trailing edge(ϵ),
- to include the impact of the density ratio and the convective Mach number, and
- to use the 90% velocity thickness(δ_{vel}) in the estimation of the turbulent viscosity instead of the vorticity thickness.

The normalized incompressible velocity distribution across a shear layer as derived by Goertler[24] is given by

$$u' = \frac{u(y) - U_s}{U_p - U_s} = \frac{1}{2} [1 - erf(\xi)], \quad (D.6)$$

where ξ is given by

$$\xi = y \sqrt{\frac{\bar{U}}{2(x - x_0)\nu_t}}. \quad (D.7)$$

x is the distance from the mixer trailing edge. x_0 is the distance upstream of the trailing edge where a shear layer growing at δ'_{vel} with a thickness at the trailing edge of $\delta_{vel} = \epsilon$ would originate. As illustrated in Figure D-5,

$$x_0 = \frac{\epsilon}{\delta'_{vel}} \quad (D.8)$$

Figure D-5: Cartoon illustrating the specification of the initial thickness of the boundary layer

The 90% velocity thickness of the shear layer is defined by

$$\delta_{vel} \equiv y|_{u'=0.95} - y|_{u'=0.05}. \quad (D.9)$$

Using Eqn. D.7, δ_{vel} may be written as

$$\delta_{vel} = \sqrt{\frac{2(x - x_0)\nu_t}{\bar{U}}} (\xi|_{u'=0.95} - \xi|_{u'=0.05}). \quad (D.10)$$

Eqn. D.10 may then be solved for ν_t :

$$\nu_t = C_1 \delta_{vel}^2 \frac{\bar{U}}{x + \frac{\epsilon^*}{\delta'_{vel}}}, \quad (D.11)$$

where from Eqns. D.6 and D.10,

$$C_1 = \frac{1}{2(\xi|_{u'=0.95} - \xi|_{u'=0.05})^2} = 0.0921. \quad (D.12)$$

Using Eqns. D.5 and D.11, Re_Γ may be written as

$$Re_\Gamma = \frac{1}{C_1} \frac{\Gamma^* h^*}{\delta_{vel}^2} \frac{1}{x^* + \frac{\epsilon^*}{\delta'_{vel}}}, \quad (D.13)$$

where the shear layer velocity thickness (δ_{vel}) is

$$\delta_{vel} = \delta'_{vel} (x^* + \frac{\epsilon^*}{\delta'_{vel}}). \quad (D.14)$$

(δ'_{vel} is the growth rate of the velocity thickness of a planar shear layer and ϵ^* is the initial thickness

of the shear layer.) Re_Γ is expressed in terms of the non-dimensional time utilized in the vortex model, t^* , as

$$Re_\Gamma = \frac{1}{C_1} \left(\frac{\Gamma^* h^*}{\delta'_{vel}} \right)^2 \frac{1}{t^* + \epsilon^* \frac{\Gamma^* h^*}{\delta'_{vel}}} \quad (D.15)$$

In the calculation of $\left(\frac{\Gamma^* h^*}{\delta'_{vel}} \right)$, the Barber *et al.*[2] scaling was be used to estimate the streamwise circulation:

$$\Gamma^* \approx C_{shape} \tan(\alpha). \quad (D.16)$$

The compressible planar shear layer growth rate correlation developed by Papamoschou[22] was used to estimate the shear layer growth rate:

$$\delta'_{pit} = \frac{\delta}{x} = C_{pit} f(M_c) \frac{(1-r)(1+\sqrt{s})}{1+r\sqrt{s}} \quad (D.17)$$

where,

$$f(M_c) = -0.532M_c + 0.8693. \quad (D.18)$$

C_{pit} is the growth rate constant for the Pitot thickness; a value of 0.14 was used[22]. M_c , r , and s are functions of the two inflow Mach numbers and the stream inflow total temperature and static pressure ratios.

For the matched total temperature streams of these experiments, the velocity thickness may be directly calculated from δ_{pit} . The normalized axial velocity of Eqn. D.6 may be written as a function of the local and two stream Mach numbers:

$$u' = \frac{M_1 \sqrt{1 + \frac{\gamma-1}{2} M_1^2} - M_2 \sqrt{1 + \frac{\gamma-1}{2} M_2^2}}{M_1 \sqrt{1 + \frac{\gamma-1}{2} M_1^2} - M_2 \sqrt{1 + \frac{\gamma-1}{2} M_2^2}}. \quad (D.19)$$

Eqn. D.19 may then be solved for the local Mach number as a function of u' and the two stream Mach numbers. The ratio of the local Pitot to static pressure is then calculated from the local Mach number. If the static pressure is assumed constant across the mixing layer, a local normalized Pitot pressure may be calculated

$$Pit' = \frac{\frac{P_{it}}{p} - \frac{P_{it2}}{p}}{\frac{P_{it1}}{p} - \frac{P_{it2}}{p}} \quad (D.20)$$

A typical normalized Pitot pressure distribution is plotted with the normalized velocity in Figure D-6.

Figure D-6: Normalized Pitot and velocity similarity profiles for $M_p = 1.6$, and $M_s = 0.3$.

The 90% Pitot thickness is defined as

$$\delta_{pit} \equiv y|_{Pit'=0.95} - y|_{Pit'=0.05}. \quad (D.21)$$

Using Eqn. D.7, δ_{pit} may be written as

$$\delta_{pit} = \sqrt{\frac{2x\nu_t}{\bar{U}}} (\xi|_{Pit'=0.95} - \xi|_{Pit'=0.05}). \quad (D.22)$$

The ratio of the 90% velocity to Pitot thickness of a planar shear layer may then be written as

$$\frac{\delta_{vel}}{\delta_{pit}} = \frac{\xi_{u'=0.95} - \xi_{u'=0.05}}{\xi_{Pit'=0.95} - \xi_{Pit'=0.05}} \quad (D.23)$$

$$= \frac{2.33}{\xi_{Pit'=0.95} - \xi_{Pit'=0.05}}. \quad (D.24)$$

In Figure D-7, contours of $\frac{\delta_{vel}}{\delta_{pit}}$ are plotted over the inflow Mach number ranges of interest to this investigation. The velocity to Pitot thickness ratio is seen to range between 1.0 and 1.3.

The initial thickness of the shear layer at the trailing edge of the mixers will be estimated using experimental Mie images, Pitot surveys, and Schlieren photographs. The average 90% boundary layer thickness at the leading edges of the the mixers is estimated to be $\approx 0.25\lambda$.

Figure D-7: Contours of the ratio of the velocity to Pitot thickness of a planar shear layer as functions of the stream Mach numbers for equal total temperature streams.

D.5 Modelling of lobed mixer flows

This section contains a discussion of three key issues concerning the modelling of lobed mixer flows with the discrete vortex model. First, the specification of the initial vorticity distribution to approximate that shed by forced mixers is discussed. Second, the satisfaction of the wall/symmetry boundary conditions is reviewed. Third, the method of implementation of the compressibility effect models in the vortex interface model is presented.

D.5.1 Specification of initial vorticity distribution

For the approximately square lobe mixers of interest in the current investigation, all the flow in the mixer troughs leaves the trailing edge of mixers at the lobe angle, and hence, the magnitude of the vorticity shed from the vertical walls of the mixer is constant. This continuous vorticity distribution is modelled using a number ($N_{vertical}$) of discrete vortices. The circulation about each vortex normalized by the total circulation shed from one vertical side wall is a function of the spacing. For evenly spaced vortices the normalized circulation Γ_j^{**} is given by-

$$\Gamma_j^{**} = \frac{1}{N_{vertical}}. \quad (D.25)$$

Figure D-8: Single square lobe vorticity distribution with evenly spaced vortices.

Additionally, in order to model the development of the entire interface. Vortices with no net circulation ($\Gamma_j^{**} = 0$) are initially distributed along the horizontal walls of the mixer. The vortex distribution used to model the mixers investigated in the current experiments is illustrated in Figure D-8.

For square lobe geometries with finite thickness boundary layers, the vortex circulations are tapered through the boundary layers, as illustrated in Figure D-9.

D.5.2 Boundary conditions

In order to model the mixer configurations investigated experimentally, it is only necessary to model a half-wavelength span of the mixer as the rest of the mixing layer may be constructed from symmetry. This half-wavelength span is modelled as though it were it an infinite array of lobes. Hence, symmetry boundary conditions are enforced on either side of the half-wavelength span using the method of images. (These symmetry boundary conditions are equivalent to wall boundary conditions, as illustrated in Figure D-10; hence, duct wall boundary conditions would also be satisfied using the method of images.)

If the vorticity distribution is in close proximity to the upper and lower duct walls, the upper and lower wall boundary conditions must also be satisfied using the method of images.

To strictly satisfy the wall/symmetry boundary conditions an infinite array of image lobes is required. However, for the sake of computational efficiency, it is also desirable to model the mixer with as few vortices as possible. Hence, the boundary conditions are only approximately enforced. The number of image lobes required to adequately satisfy the boundary conditions is a function of

1. the distance from the vorticity distribution to the symmetry plane/wall;
2. $\frac{\Gamma^* h^*}{\delta^*}$;
3. the distance from the trailing edge.

The importance of the image vortices decrease as the distance from the vorticity to the wall/symmetry plane increases. The higher $\frac{\Gamma^* h^*}{\delta^*}$ and the greater the maximum distance from the trailing edge of

Figure D-9: Normalized circulations along the vertical wall of a square lobe mixer with $h^* = 1.25$ and $\epsilon^* = 0.25$.

interest, the greater the number of image lobes required to satisfy the boundary conditions.

For the modeling of the mixer configurations investigated in the current experiments, between 5 and 21 image lobes were used to satisfy the symmetry/side wall boundary conditions. As the mixers were of relatively low penetration(25%), the induced velocities and the upper and lower walls were small, and no image vortices were used to satisfy the upper and lower wall boundary conditions.

D.5.3 Compressibility effects

Two additional models have been developed to adjust the vorticity distribution for the movement of the interface associated with the matching of the stream static pressures and for the generation of vorticity associated with baroclinic torques in the vicinity of the mixer trailing edge.

If the static pressures are not matched at the trailing edge of the mixers, the pressure matching control volume described in Appendix E is used to estimate the resulting displacement of the vortex sheets. This displacement is assumed to occur over an infinitesimally thin region just downstream of the mixer trailing edge.

If the two streams are of unequal densities and trailing edge shocks are present streamwise circulation is generated downstream of the mixer. A model to estimate the amount circulation of generated by the shock/density gradient interaction is described in Section 4.3.4. When this compressibility effect is included in the vortex code, the vortices are tracked and their circulations

Modelled half wavelength span

Region of + vorticity

Region of - vorticity

Lobe trailing edge

Figure D-10: Satisfaction of symmetry/sidewall boundary conditions via the method of images.

are augmented or reduced as they pass through the trailing edge shock.

D.6 Calculation of effective interface length

In order to calculate the effective interface length which is in turn used in the differential control volume to calculate the axial static and total pressure distributions, a passive scalar is tracked.

A series of axi-symmetric error function profiles centered on each of the vortices which compose the interface is used to model the passive field. The passive scalar field about a single vortex is given by

$$\eta = 1 - \operatorname{erf}(\xi^\dagger), \quad (\text{D.26})$$

where,

$$\xi^\dagger = r \sqrt{\frac{\bar{U}}{2(x - x_0)D_t}}. \quad (\text{D.27})$$

r is the distance to the vortex. η varies between a fully-mixed value of 1 and an unmixed value of 0. D_t is the scalar diffusivity; it is related to the turbulent viscosity by the Schmidt number–

$$D_t = \frac{\nu_t}{Sc}. \quad (\text{D.28})$$

The 90% scalar thickness of the shear layer is

$$\delta_{sc} = 2r|_{\eta=0.05}. \quad (\text{D.29})$$

Using Eqns. D.26 and D.27, δ_{sc} may be written as

$$\delta_{sc} = \sqrt{\frac{2(x - x_0)D_t}{\bar{U}}} \xi^\dagger|_{\eta=0.05}. \quad (\text{D.30})$$

Eqn. D.30 may be solved for $\sqrt{\frac{\bar{U}}{2(x - x_0)D_t}}$:

$$\sqrt{\frac{\bar{U}}{2(x - x_0)D_t}} = \frac{\xi^\dagger|_{\eta=0.05}}{r|_{\eta=0.05}} \quad (\text{D.31})$$

$$= \frac{1.3859}{\frac{\delta_{sc}}{2}}. \quad (\text{D.32})$$

Figure D-11: Scalar distribution(η) about a single vortex.

η may then be written as

$$\eta = 1 - \operatorname{erf} \left(\frac{r}{\delta_{sc}} 2.7686 \right). \quad (\text{D.33})$$

In Figure D-11, a two-dimensional slice of the the scalar field about a single vortex is drawn.

The ratio of the fully-mixed thickness(analogous to the displacement thickness) to the 90% thickness may be found by

$$\frac{\delta_{sc}|_{mix}}{\delta_{sc}} = \int_{-\infty}^{\infty} d \left(\frac{r}{\delta_{sc}} \right) \eta \quad (\text{D.34})$$

$$= 0.4072. \quad (\text{D.35})$$

The scalar field associated with N vortices is approximated by

$$\eta = \max [\eta_1, \eta_2, \eta_3, \dots, \eta_N], \quad (\text{D.36})$$

where η_n is the scalar field associated with vortex n . In Figure D-12, the approximate scalar field associated with two vortices is plotted.

The fully-mixed scalar field downstream of a lobed mixer with $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$ is shown at five axial locations in Figure D-13. The mean interface length and scalar

Figure D-12: Dual vortex scalar field.

mixedness are then calculated from the passive scalar field.

The scalar mixedness at an axial plane is defined as--

$$\mathcal{M}_{sc} = \frac{1}{A} \int \eta dA. \quad (\text{D.37})$$

The scalar mixedness varies between 0 at the unmixed trailing edge and 1 at the fully-mixed conditions. In Figure D-14, $\mathcal{M}_{sc}$ is plotted versus distance for the test case shown in Figure D-13.

The effective interface length is calculated from the scalar mixedness distribution by

$$L_{eff}^* = \frac{\frac{\partial \mathcal{M}_{sc}}{\partial x^*}}{\frac{\partial \mathcal{M}_{sc}}{\partial x^*}|_{FP}} \quad (\text{D.38})$$

$$= \frac{h_{duct}}{\delta'_{sc}|_{mixed}} \frac{\partial \mathcal{M}_{sc}}{\partial x^*} \quad (\text{D.39})$$

$$(\text{D.40})$$

In Figure D-15, L_{eff} is plotted versus distance from the trailing edge for the test case shown in Figure D-13.

Figure D-13: Fully-mixed scalar fields 1, 2, 3, 10, and 20λ downstream of a square lobe forced mixer with $h^* = 1.25$, $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.

Figure D-14: Scalar mixedness distribution downstream of a square lobe forced mixer with $h^* = 1.25$, $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.

Figure D-15: Mean interface length distribution downstream of a square lobe forced mixer with $h^* = 1.25$, $\Gamma^* = 0.75$, $\delta'_{vel} = 0.10$, and $\epsilon^* = 0.25$.

Figure D-16: Scalar fields 2λ downstream of a square lobe forced mixer with $h^* = 1.25$; $\epsilon^* = 0.25$; $\delta'_{vel} = 0.10$; and $\Gamma^* = 0.01, 0.25, 0.50, 0.75, 1.0$

D.7 Sensitivity studies

In this section the sensitivity of the mean interface length and scalar mixedness to Γ^* , δ'_{vel} and ϵ^* is assessed for the mixer trailing edge geometry (square lobe, $h^* = 1.25$) investigated experimentally in this study.

D.7.1 Sensitivity of M_{sc} and L_{eff} to Γ^*

In order to assess sensitivity of M_{sc} and L_{eff}^* to Γ^* , five runs of the vortex code were conducted at five different levels of Γ^* : 0.01, 0.25, 0.50, 0.75, 1.00. For all of these runs, the growth rate of the velocity thickness of the shear layer was 0.10 and the initial thickness of the shear layer was 0.25 wavelengths. (The growth rate and initial thickness utilized were typical of the experiments.) The shape of the mixer trailing edge and the vorticity distribution were constant ($h^* = 1.25$).

In Figure D-16, the fully-mixed scalar fields are drawn 2λ from the trailing edge of the mixers. Increasing levels of shed circulation are seen to produce more stretching of the interface.

In Figures D-17 and D-18, the scalar mixednesses at 20λ and the mean effective interface length over the first 20λ are plotted versus Γ^* . In these Figures, the mean $L_{effective}^*$ and M_{sc} at 20λ increase with Γ^* for $\Gamma^* \leq 0.75$. However, the predicted scalar mixedness and effective interface length reach a maximum between $\Gamma^* = 0.75$ and $\Gamma^* = 1.0$.

D.7.2 Sensitivity of M_{sc} and L_{eff} to δ'_{vel}

In order to assess the sensitivity of M_{sc} and L_{eff}^* to δ'_{vel} , thirty runs of the vortex code were conducted with ten different values of δ'_{vel} and three different levels of Γ^* . The range of δ'_{vel} was chosen to span the full range of growth rates investigated experimentally. The three levels of streamwise circulation

Figure D-17: Scalar mixedness at 20λ versus Γ^* for $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.

Figure D-18: Mean effective interface length versus Γ^* for $\delta'_{vel} = 0.10$ and $\epsilon^* = 0.25$.

Figure D-19: Scalar fields 2λ downstream of a square lobe forced mixer with $h^* = 1.25$; $\Gamma^* = 0.5$; $\epsilon^* = 0.25$; and $\delta'_{vel} = 0.02, 0.06, 0.10, 0.14, 0.20$.

are representative of that shed by the three lobed mixers utilized in these experiments.

In Figure D-19, the fully-mixed scalar fields 2λ from the trailing edge of the mixer with $\Gamma^* = 0.5$ are drawn for 5 different δ'_{vel} 's. As the growth rate increases, the thickness of the shear layer on the interface increases. Additionally, as the growth(diffusion) rate increases, the rate at which the streamwise vorticity is diffused increases as well, and less stretching of the interface occurs downstream of the trailing edge.

In Figures D-20 and D-21, the scalar mixednesses at 20λ and the mean effective interface lengths over the first 20λ are plotted versus δ'_{vel} for three different levels of Γ^* . In Figure D-20, the scalar mixedness at 20λ at all three levels of streamwise circulation increases with δ'_{vel} . Furthermore, the scalar mixing augmentation associated with the presence of streamwise vorticity is seen to decrease with an increasing diffusion rate. Likewise, in Figure D-21, the mean effective interface length decreases with increasing δ'_{vel} as the streamwise vorticity and increased initial interface length become less important. Additionally, the interface stretching associated with streamwise vorticity decreases as the shear layer growth rate increases.

D.7.3 Sensitivity of M_{sc} and L_{eff} to ϵ^*

In order to assess the sensitivity of M_{sc} and L_{eff} to the trailing edge boundary layer thickness(ϵ^*), six runs with the vortex code were conducted for six different ϵ^* 's with $\Gamma^* = 0.5$, $\delta'_{vel} = 0.1$, and $h^* = 1.25$.

In Figure D-22, the fully-mixed scalar fields are drawn 2λ from the trailing edge for four different ϵ^* 's. As ϵ increases, the thickness of the shear layer on the interface increases, and the level of interface stretching decreases.

In Figure D-23, the scalar mixedness at 20λ is plotted versus ϵ^* ; M_{sc} does not vary with in-

Figure D-20: Scalar mixedness at 20λ versus δ' for $\Gamma^* = 0.01, 0.50, 0.90$.

Figure D-21: Mean effective interface length versus δ'_{vel} for $\Gamma^* = 0.01, 0.50, 0.90$.

Figure D-22: Scalar fields 2λ downstream of a square lobe forced mixer with $h^* = 1.25$; $\Gamma^* = 0.5$; $\delta_{vel}^l = 0.10$; and $\epsilon^* = 0.10, 0.20, 0.30, 0.40$.

creasing ϵ^* . On the other hand in Figure D-24, the mean effective interface length decreases with increasing ϵ^* . The reason for this apparent discrepancy is due to two counter balancing effects. The scalar mixedness at the trailing edge of the mixer ($x^* = 0$) increases with increasing boundary layer thickness while the average rate of increase of the scalar mixedness drops with increasing ϵ^* since the vorticity distribution is becoming more diffuse and the transition to a planar mixing layer moves toward the trailing edge of the mixer.

Figure D-23: Scalar mixedness at 20λ versus ϵ^* for $\Gamma^* = 0.50$, $\delta'_{vel} = 0.1$, and $h^* = 1.25$.

Figure D-24: Mean effective interface length versus ϵ^* for $\Gamma^* = 0.50$, $\delta'_{vel} = 0.1$, and $h^* = 1.25$.

Appendix E

Derivation of static pressure matching control volume

This appendix contains the detailed derivation of the control volume analysis used to match the static pressures downstream of the trailing edge of the mixers and to estimate the displacement of the mixer vorticity associated with a trough static pressure mismatch. After the model is derived, a brief sensitivity study is conducted. Model/data comparisons are contained in Chapter 5.

E.1 Description of Model

In the supersonic/subsonic mixing environments of interest in this study, the static pressures in the two streams at the trailing edges of the mixers are not necessarily matched. If a difference does indeed exist, the two streams will adjust their areas downstream of the trailing edge of the mixer to match their static pressures. The situation downstream of the trailing edge is analogous to an under/over expanded supersonic nozzle in which either an oblique shock or expansion fan located at the nozzle lip decelerates or accelerates the nozzle flow to match the nozzle static pressure to ambient.

E.1.1 Input/Output

The inputs to the model are the trailing edge conditions:

- Mach numbers($M_{p_{te}}$ and $M_{s_{te}}$)
- Secondary to primary static pressure ratio(σ_{te})
- Secondary to primary area ratio(β_{te}).

The outputs from the model are the matched, or balance, point conditions:

- Mach numbers($M_{p_{mat}}$ and $M_{s_{mat}}$)
- Matched to primary trailing edge static pressure ratio(σ_p)
- Matched to primary trailing edge stream area ratios (β_s and β_p)

E.1.2 Assumptions

The assumptions made in the model are

- Pressure matching occurs in a constant area duct;
- No mixing occurs between streams while the pressures are matching;
- Secondary stream is isentropic;
- Stream static pressures are matched at exit of control volume.

E.2 Derivation of model

The conservation of mass for both streams, momentum for the entire control volume, conservation of energy for both streams, the equality of the stream static pressures at the control volume exit, the constant area condition, and the enforcement of an isentropic secondary stream yield a system of 8 equations and 8 unknowns:

$$\rho_{p_{te}} u_{p_{te}} A_{p_{te}} = \rho_{p_{mat}} u_{p_{mat}} A_{p_{mat}} \quad (E.1)$$

$$\rho_{s_{te}} u_{s_{te}} A_{s_{te}} = \rho_{s_{mat}} u_{s_{mat}} A_{s_{mat}} \quad (E.2)$$

$$p_{p_{te}} (1 + \gamma M_{p_{te}}^2) + p_{s_{te}} (1 + \gamma M_{s_{te}}^2) = p_{p_{mat}} (1 + \gamma M_{p_{mat}}^2) + p_{s_{mat}} (1 + \gamma M_{s_{mat}}^2) \quad (E.3)$$

$$\rho_{p_{te}} u_{p_{te}} A_{p_{te}} T t_{p_{te}} = \rho_{p_{mat}} u_{p_{mat}} A_{p_{mat}} T t_{p_{mat}} \quad (E.4)$$

$$\rho_{s_{te}} u_{s_{te}} A_{s_{te}} T t_{s_{te}} = \rho_{s_{mat}} u_{s_{mat}} A_{s_{mat}} T t_{s_{mat}} \quad (E.5)$$

$$p_{p_{mat}} = p_{s_{mat}} \quad (E.6)$$

$$A_{p_{te}} + A_{s_{te}} = A_{p_{mat}} + A_{s_{mat}} \quad (E.7)$$

$$p_{s_{te}} \left(1 + \frac{\gamma - 1}{2} M_{s_{te}}^2\right)^{\frac{\gamma}{\gamma - 1}} = p_{s_{mat}} \left(1 + \frac{\gamma - 1}{2} M_{s_{mat}}^2\right)^{\frac{\gamma}{\gamma - 1}} \quad (E.8)$$

This system of 8 equations is reduced to a system of four which may then be solved for the

quantities listed in Section E.1.1.

$$\sigma_p = \sigma_{te} \left(\frac{1 + \frac{\gamma-1}{2} M_{s_{te}}^2}{1 + \frac{\gamma-1}{2} M_{s_{mat}}^2} \right)^{\frac{\gamma}{\gamma-1}} \quad (E.9)$$

$$\beta_p = \frac{1}{\sigma_p} \frac{M_{p_{te}} \sqrt{1 + \frac{\gamma-1}{2} M_{p_{te}}^2}}{M_{p_{mat}} \sqrt{1 + \frac{\gamma-1}{2} M_{p_{mat}}^2}} \quad (E.10)$$

$$\sigma_{te} \beta_{te} M_{s_{te}} \sqrt{1 + \frac{\gamma-1}{2} M_{s_{te}}^2} = \sigma_p (1 + \beta_{te} - \beta_p) M_{s_{mat}} \sqrt{1 + \frac{\gamma-1}{2} M_{s_{mat}}^2} \quad (E.11)$$

$$(1 + \gamma M_{p_{te}}^2) + \sigma_{te} \beta_{te} (1 + \gamma M_{s_{te}}^2) = \sigma_p [\beta_p (1 + \gamma M_{p_{mat}}^2) + (1 + \beta_{te} - \beta_p) (1 + \gamma M_{s_{mat}}^2)] \quad (E.12)$$

This system of 4 equations is reduced to a system of 2 equations which is solved numerically to determine the matched points conditions.

E.3 Sensitivity Studies

Two sensitivity studies were conducted with the model to assess the sensitivity of $M_{p_{mat}}$, $M_{s_{mat}}$, $\frac{\bar{p}_{mat}}{\bar{p}_{te}}$, and $\frac{A_s}{A_p}|_{mat}$ to the inflow static pressure ratio and the inflow Mach numbers for $\frac{A_s}{A_p}|_{te} = 1$.

Sensitivity of matched point conditions to M_s and $\frac{p_s}{p_p}$ The system of equations listed in Eqns. E.10- E.12 was solved for the matched point conditions for $0.1 < M_s < 1.2$ and $0.8 < \frac{p_s}{p_p} < 1.2$. The results are plotted in Figures E-1 and E-2.

Two solutions are possible: a weak solution and a strong solution. In Figure E-1, the strong solution is indicated by a solid line, and the weak solution is indicated by a dotted line. The strong solution always satisfies the second law of thermodynamics, while the weak solution is only physically possible if both inflow Mach numbers are greater than 1.

In Figures E-1 and E-2, it may also be noted that there is a region of the $\frac{p_s}{p_p}$ and M_s parameter space investigated for which no real solution for the matched condition exists. In this region, the control volume compound chokes and can not pass the mass flow specified by the inflow conditions (M_p , M_s , and $\frac{p_s}{p_p}$)

From Figure E-1 and E-2 it can be seen that if $M_p > 1$, $M_s < 1$, and $\frac{p_s}{p_p} < 1$, the area-average static pressure and the secondary/primary area ratio drop through the control volume as both streams accelerate. On the other hand if $\frac{p_s}{p_p} > 1$, both streams decelerate and the secondary/primary area-average static pressure and secondary/primary static pressure ratio both increase.

Sensitivity of matched point conditions to M_s and M_p The system of equations listed in Eqns. E.10- E.12 also was solved for the matched point conditions for $0.1 < M_s < 1.2$ and $M_p = 1.3, 1.6, 2.4$. (The three primary Mach numbers selected are those for which experimental nozzles were

Figure E-1: Matched/inflow area averaged static pressure and matched secondary/primary area ratio vs. M_s for $\frac{p_s}{p_p} = 0.8, 0.9, 1.0, 1.1$. ($M_p = 1.6$ and $\frac{A_s}{A_p}|_{te} = 1.0$.)

(a) Matched M_p

(b) Matched M_s

Figure E-2: Matched Mach numbers vs. M_s for $\frac{p_t}{p_p} = 0.8, 0.9, 1.0, 1.1$. ($M_p = 1.6$ and $\frac{A_t}{A_p}|_{te} = 1.0$.)

designed.) The results are plotted in Figures E-3 and E-4.

Figure E-3: Matched/inflow area averaged static pressure and matched secondary/primary area ratio vs. M_s for $M_p = 1.3, 1.6, 2.4$ and $\frac{A_s}{A_p}|_{te} = 1.0$.

(a) Matched M_p

(b) Matched M_s

Figure E-4: Matched Mach numbers vs. M_s for $M_p = 1.3, 1.6, 2.4$ and $\frac{A_+}{A_p}|_{te} = 1.0$.

Appendix F

Derivation of baroclinic torque circulation model

An analytical model has been derived to estimate the streamwise circulation generated by the interaction of the primary trough trailing edge shock with a density gradient across the mixing layer. This shock may be seen in the Schlieren photograph in Figure 4-32 and in the schematic in Figure 4-33.

The rate of change of the streamwise circulation with distance from the trailing edge due to the generation of vorticity by the shock/mixing layer interaction is found from Kelvin's theorem[3]:

$$\frac{D\Gamma}{D\left(\frac{x}{U}\right)} = - \iint \left(\frac{\nabla \rho \times \nabla p}{\rho^2} \right) \cdot \hat{n} dA. \quad (\text{F.1})$$

If a density gradient exists across the mixing layer, streamwise circulation is generated at the intersection of the mixing layer and the trough shock as indicated in Figure F-1.

In the model, the trough flow is assumed to leave the mixer at the lobe angle and turn back into the axial direction through an oblique shock at an angle θ with respect to the axial direction.

Figure F-1: Angle definitions and region in which circulation is generated.

The static pressure and density are assumed to vary linearly across the shock and mixing layer respectively. Given these assumptions, Kelvin's theorem in Eqn. F.1 may be approximated by

$$\frac{D\Gamma}{Dx} \approx \frac{1}{\bar{\rho}^2 \bar{U}} \Delta \rho \Delta p \tan \theta. \quad (\text{F.2})$$

The total circulation generated by the shock is found by integrating Eqn. F.2 from the trailing edge to x_{shock} , where x_{shock} is the distance from the trailing edge at which the uppermost primary trough streamline is turned back into the axial direction,

$$\Gamma_{shock} = \int_0^{x_{shock}} \left(\frac{D\Gamma}{Dx} \right) dx. \quad (\text{F.3})$$

The ratio of the total shock to mixer circulation is determined from Eqns. F.3 and the Barber *et al.* [2] scaling law for square lobe mixers in Equ. 2.1–

$$\frac{\Gamma_{shock}}{\Gamma_{mixer}} = \frac{-8}{\gamma M_p^2} \frac{s-1}{(s+1)^2 (r+1)^2} \left(\frac{p_2}{p_1} - 1 \right) \frac{1}{\tan \alpha (\tan \alpha + \tan \theta)}. \quad (\text{F.4})$$

$\frac{p_2}{p_1}$ is the pressure ratio across an oblique shock of angle $(\theta + \alpha)$ [34]–

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_p^2 \sin^2 (\theta + \alpha) - \frac{\gamma-1}{\gamma+1} \quad (\text{F.5})$$

Appendix G

Derivation of differential control volume

This appendix contains a detailed description of the differential control volume model used to calculate the axial static and mass-averaged total pressure distributions given the inflow conditions, duct geometry, and effective cross-flow interface length distribution.

G.1 Description of model

In this model, two co-flowing streams of fluid are mixed in a duct via the exchange of momentum, energy, and a passive scalar. Analytical and numerical models are developed for the exchange rates of these quantities as functions of distance from the trailing edge of the mixer. The approach taken is similar to that adopted by Clark[6] and Teeple[29] in their compound flow analyses. The main differences between the current analysis and the two compound flow analyses are the addition of the passive scalar conservation equations for each stream and the development of a procedure to model the variation of interfacial length with associated with the convoluted mixer geometries and the presence of streamwise vorticity.

G.2 Inputs/Outputs

The inputs required by the model are the

- the inflow conditions(M_p , M_s , $\frac{T_{t_s}}{T_{t_p}}$);
- the duct geometry(inflow secondary/primary area ratio and the axial area distribution);
- the axial distribution of effective cross-flow interface length.

The outputs from the model are the axial distributions of the quasi-one-dimensional flow conditions.

G.3 Derivation of model

A system of 10 ordinary differential equations was derived from the conservation of mass, momentum, energy, and a passive scalar for each stream in addition to the conservation of cross-flow area, the equality of static pressure in a cross-flow plane and the ideal gas law. This system of equation is written in terms of x^* in Eqns. G.2-G.3.

$$\frac{d}{dx^*} (\rho_p u_p A_p) = 0 \quad (G.1)$$

$$\frac{d}{dx^*} ((p_p + \rho_p u_p^2) A_p) - p_p \frac{dA_p}{dx^*} = \frac{1 + \alpha}{h_{duct}^*} \frac{L_{eff}}{2Re_\delta} (\rho_p + \rho_s) (u_p - u_s)^2 \quad (G.2)$$

$$+ C_f \rho_p U_p^2 \left(1 + \frac{2}{1 + \frac{A_s}{A_p}} \frac{h_{duct}^*}{w_{duct}^*} \right) \frac{1 + \alpha}{h_{duct}^*}$$

$$\frac{d}{dx^*} (\rho_p u_p T t_p A_p) = \frac{1 + \alpha}{h_{duct}^*} \frac{L_{eff}}{2Re_\delta} (\rho_p + \rho_s) (u_p - u_s) \left(\frac{1}{Pr} (T_p - T_s) + (u_p - u_s)^2 \right) \quad (G.3)$$

$$\frac{d}{dx^*} (\rho_p u_p \eta_p A_p) = \frac{1 + \alpha}{h_{duct}^*} \frac{L_{eff}}{2Re_\delta} (\rho_p + \rho_s) (u_p - u_s) \frac{1}{Sc} (\eta_p - \eta_s) \quad (G.4)$$

$$\frac{d}{dx^*} (\rho_s u_s A_s) = 0 \quad (G.5)$$

$$\frac{d}{dx^*} ((p_s + \rho_s u_s^2) A_s) - p_s \frac{dA_s}{dx^*} = -\frac{1 + \alpha}{h_{duct}^*} \frac{L_{eff}}{2Re_\delta} (\rho_p + \rho_s) (u_p - u_s)^2 \quad (G.6)$$

$$+ C_f \rho_s U_s^2 \left(1 + \frac{2}{1 + \frac{A_s}{A_p}} \frac{h_{duct}^*}{w_{duct}^*} \right) \frac{1 + \alpha}{h_{duct}^*}$$

$$\frac{d}{dx^*} (\rho_s u_s T t_s A_s) = -\frac{1 + \alpha}{h_{duct}^*} \frac{L_{eff}}{2Re_\delta} (\rho_p + \rho_s) (u_p - u_s) \left(\frac{1}{Pr} (T_p - T_s) + (u_p - u_s)^2 \right) \quad (G.7)$$

$$\frac{d}{dx^*} (\rho_s u_s \eta_s A_s) = -\frac{1 + \alpha}{h_{duct}^*} \frac{L_{eff}}{2Re_\delta} (\rho_p + \rho_s) (u_p - u_s) \frac{1}{Sc} (\eta_p - \eta_s) \quad (G.8)$$

$$\frac{d}{dx^*} (A_p + A_s) = \frac{dA}{dx^*} \quad (G.9)$$

$$p_p = p_s \quad (G.10)$$

$$p_s = \rho_s R_s T_s \quad (G.11)$$

L_{eff} is the local effective cross-flow interface length in wavelengths calculated by the vortex model. (For flat plate mixer geometries, $L_{eff} = 1$.) The width and height of the mixing duct in wavelengths are given by w_{duct} and h_{duct} respectively. Re_δ is the Reynold's number based on the velocity thickness,

$$Re_\delta = \frac{\Delta U \delta_{vel}}{\nu_t} \quad (G.12)$$

$$= \frac{2}{C_1} \frac{1}{\delta'_{vel}} \frac{U_p - U_s}{U_p + U_s} \quad (G.13)$$

Pr and Sc are the turbulent Prandtl and Schmidt numbers.

G.4 Sensitivity studies

Four sensitivity studies were conducted with the differential control volume model to assess the sensitivity of the mixing rate and the axial distributions of the normalized mixing measures to L_{eff} , Pr , the inflow Mach numbers, and the inflow area ratio.

G.4.1 Sensitivity of the mixing rate to L_{eff}

In order to assess the sensitivity of the mixing rate to the effective interface length, four test runs of the differential control volume were conducted for four different effective interface lengths. For each run, the effective interface length was held constant through the duct for each run. The test runs were conducted with inflow Mach numbers of 1.6 and 0.3 and at an inflow total temperature ratio of 1.0. The secondary to primary area ratio was 1.0 and the duct for all the cases was of constant length.

In Figure G-1, the momentum mixing parameter(Φ) is plotted versus distance from the trailing edge for $L_{eff} = 1.0, 2.0, 4.0, 8.0$. (Φ varies between 0 at the mixer trailing edge and 1 at the fully-mixed conditions.) The initial rate of rise of Φ is proportional to L_{eff} . For the tested duct geometry and inflow conditions, the two streams are not completely mixed out by the end of the duct for $L_{eff} = 1.0$ and 2.0. However, for effective interface lengths of 4.0 and 8.0, the two streams are mixed out before the end of the duct.

G.4.2 Sensitivity of the axial distributions of the mixing measures to Pr

In order to assess the sensitivity of the axial distributions of the mixing measures to the Prandtl number, a series of test runs with the differential control volume were conducted. Varying the Prandtl number will have no effect on the fully mixed conditions; rather, changing the Prandtl number simply changes the relative rates at which momentum and thermal energy are exchanged between the two streams.

In Figure G-2, the normalized mass-averaged entropy is plotted versus the momentum mixing parameter (a normalized static pressure) for several runs with $M_p = 1.6$, $M_s = 0.3$, $\frac{T_{t_s}}{T_{t_p}} = 1.0$ and turbulent Prandtl numbers ranging between 0.5 and 2.0. For these equal total temperature streams, the Prandtl number does not affect on the pressure-entropy path which the mixing process follows.

Figure G-1: Normalized axial pressure distribution for $L_{eff}^* = 1.0, 2.0, 4.0, 8.0$ with $M_p = 1.6$, $M_s = 0.3$, $\frac{A_s}{A_p} = 1.0$, and $\frac{Tt_s}{Tt_p} = 1.0$

Figure G-2: Entropy/pressure paths for $M_p = 1.6$, $M_s = 0.3$, $\frac{p_s}{p_p} = 1.0$, $\frac{Tt_s}{Tt_p} = 1.0$, $\frac{A_s}{A_p} = 1.0$ and $0.5 < Pr < 2.0$.

G.4.3 Sensitivity of the fully-mixed conditions and axial distributions of the mixing measures to M_p and M_s

In order to assess the sensitivity of the fully-mixed conditions and the axial distribution of the normalized mixing measures to the two inflow Mach numbers, two series of runs with the differential control volume were conducted. In the first series, the primary Mach number was held constant at 1.6 and the secondary Mach number was varied between 0.2 and 1.2. In the second, the secondary Mach number was held constant at 0.3 and the primary Mach number was varied between 0.8 and 2.4.

In Figure G-3, the ratio of the fully-mixed to inflow static and mass-averaged total pressure ratios are plotted for $M_p = 1.6$, $\frac{T_{t_s}}{T_{t_p}} = 1.0$, $\frac{A_s}{A_p} = 1.0$ versus the secondary Mach number. As indicated in the figure, two fully-mixed solutions are possible—one supersonic and one subsonic. (The two fully-mixed solutions (lines) were calculated in an algebraic control volume analysis—two streams in one stream out.) The subsonic solution always satisfies the second law. On the other hand for $M_p = 1.6$, the supersonic solution satisfies the second law only if $M_s > 0.3$. The supersonic solution is always characterized by a supersonic fully-mixed Mach number and a pressure drop through the mixing duct, and the subsonic solution is characterized by a subsonic fully-mixed Mach number and a pressure rise through the mixing duct.

Also noted in Figure G-3 is the fully-mixed solution predicted by the differential control volume calculated by mixing the two streams differentially until they are fully-mixed. The differential CV will only generate one of the two possible fully mixed solutions. In Figure G-3, the differential control volume is seen to generate the subsonic solution for $M_s \leq 0.7$, and the supersonic solution for $M_s > 0.7$. With the current mixing models, the sign of the compound flow indicator at the inflow of the differential control volume determines the root to which the differential solution proceeds. No compound throat may exist in the mixing duct. If the inflow is compound subsonic, the solution proceeds to the subsonic root, and if the inflow is supersonic, the solution proceeds to the supersonic root. (It is possible to specify a mixing model which will permit a compound throat to exist in the duct.)

In Figure G-4, the normalized entropy-pressure paths along which the mixing process proceeds are plotted for $M_s = 0.2, 0.3, \dots, 0.7$. Changing the subsonic Mach number is seen to alter the entropy-pressure path along which the process proceeds. For low secondary Mach numbers, the entropy rise occurs more rapidly than the static pressure rise. As the secondary Mach number increases, the initial rate of rise of the static pressure approaches that of the entropy. For $M_p = 1.6$ if the secondary Mach number increases beyond 0.7, the inflow becomes compound supersonic and the differential solution moves toward the supersonic solution.

In Figure G-5, the normalized entropy pressure paths are plotted for $M_s = 0.8, 0.9, \dots, 1.2$. In this figure, the entropy and static pressure have been normalized by the fully-mixed supersonic solution.

Figure G-3: Fully mixed/inflow pressure ratios versus the secondary Mach number for $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$.

For low secondary Mach numbers, unlike the subsonic solution, the normalized static pressure rise occurs faster than the entropy rise. As the secondary Mach number increases, the initial rate of rise of the entropy approaches that of the static pressure.

An additional series of runs were also conducted with a fixed subsonic secondary Mach number (0.3) and a varying primary Mach number. In Figure G-6, the fully-mixed subsonic and supersonic solutions are plotted versus M_p in addition to the fully-mixed differential solutions. For the inflow Mach numbers and area ratio of this series of the runs, the inflow is compound subsonic, and the differential control volume generates the subsonic solution.

In Figure G-6, the mixed/inflow static pressure ratio increases and the mixed/inflow total pressure ratio decreases with increasing secondary Mach number. In Figure G-7, the entropy-pressure paths are shown for a fixed secondary Mach number for a series of primary Mach numbers. As the primary Mach number increases, the initial rate of rise of entropy increases relative to the initial rate of rise of the static pressure.

Figure G-4: Normalized entropy-pressure paths for $M_s = 0.2, 0.3, \dots, 0.7$ with $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$. $M_s = 0.2$ for the upper curve and 0.7 for the lower curve. Ψ and Φ are normalized by the fully-mixed subsonic solution.

Figure G-5: Normalized entropy-pressure paths for $M_s = 0.8, 0.9, 1.0, 1.1, 1.2$ with $M_p = 1.6$, $\frac{A_s}{A_p} = 1.0$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$. $M_s = 0.8$ for the lower curve and 1.2 for the upper curve. Ψ and Φ are normalized by the fully-mixed supersonic solution.

Figure G-6: Fully-mixed/inflow pressure ratios vs M_p for $M_s = 0.3$, $\frac{A_s}{A_p} = 1.0$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$.

Figure G-7: Entropy/pressure paths for $M_p = 0.8, 1.0, \dots, 2.4$; $M_s = 0.3$; $\frac{T_{t_s}}{T_{t_p}} = 1.0$; and $\frac{A_s}{A_p} = 1.0$. $M_p = 2.4$ along the upper curve, $M_p = 0.8$ along the lower curve.

Figure G-8: Fully-mixed/inflow pressure ratios vs $\frac{A_s}{A_p}$ for $M_p = 1.6$, $M_s = 0.3$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$.

G.4.4 Sensitivity of the fully-mixed conditions and axial distributions of the mixing measures to $\frac{A_s}{A_p}$

In order to assess the sensitivity of the fully-mixed conditions and the axial distributions of the mixing measures to the inflow secondary to primary area ratio, a series of runs with the differential control volume were conducted with $M_p = 1.6$, $M_s = 0.3$, $\frac{T_{t_s}}{T_{t_p}} = 1.0$, and $0.5 \leq \frac{A_s}{A_p} \leq 2.0$.

In Figure G-8, the fully-mixed to inflow static and total pressure ratios are plotted versus $\frac{A_s}{A_p}$. Over the range of area ratios investigated, the subsonic static and total pressure ratios decrease with increasing $\frac{A_s}{A_p}$.

In Figure G-9, the normalized entropy-pressure paths are plotted. For $\frac{A_s}{A_p} = 2.0$, the initial rate of rise of entropy relative to the rate of rise of the static pressure is the highest of the test cases. As $\frac{A_s}{A_p} \rightarrow 0$, the initial rate of rise of pressure increases relative to the rate of rise of the entropy.

Figure G-9: Entropy/pressure paths for $\frac{A_s}{A_p} = 0.5, 0.75, \dots, 2.0$ with $M_p = 1.6$, $M_s = 0.3$, and $\frac{T_{t_s}}{T_{t_p}} = 1.0$.
 $\frac{A_s}{A_p} = 2.0$ along the upper curve and $\frac{A_s}{A_p} = 0.5$ along the lower curve.

Appendix H

Model/data static pressure comparisons

This appendix contains comparisons between the mixer-ejector model and experimental wall static pressure data. In this Appendix, comparisons are made for all four mixers at primary Mach numbers of ≈ 0.6 , 1.6, and 2.4.

H.1 Flat plate

In Figure H-4, both predicted and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the flat plate for $M_p = 1.6$. These same comparisons are made for $M_p = 2.4$ in Figure H-5.

H.2 Convoluted plate

In Figure H-3, both predicted and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the convoluted plate for $M_p \approx 0.6$. These same comparisons are made for $M_p = 1.6$ in Figure H-4 and for $M_p = 2.4$ in Figure H-5.

H.3 15° forced mixer

In Figure H-6, both predicted and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the 15° forced mixer for $M_p \approx 0.6$. These same comparisons are made for $M_p = 1.6$ in Figure H-7 and for $M_p = 2.4$ in Figure H-8.

Figure H-1: Flat plate subsonic wall $C_{p,super}$ model/data comparisons for $M_p = 1.6$.

Figure H-2: Flat plate subsonic wall $C_{p_{super}}$ model/data comparisons for $M_p = 2.4$.

Figure H-3: Convoluted plate subsonic wall $C_{p_{super}}$ model/data comparisons for $M_p = 1.6$.

Figure H-4: Convoluted plate subsonic wall $C_{p_{super}}$ model/data comparisons for $M_p = 1.6$.

Figure H-5: Convoluted plate subsonic wall $C_{p_{super}}$ model/data comparisons for $M_p = 2.4$.

Figure H-6: 15° forced mixer subsonic wall $C_{p,super}$ model/data comparisons for $M_p \approx 0.6$.

Figure H-7: 15° forced mixer subsonic wall $C_{p,super}$ model/data comparisons for $M_p = 1.6$.

Figure H-8: 15° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 2.4$.

H.4 25° forced mixer

In Figure H-9, both the predicted and measured duct exit Cp_{super} are plotted for all four primary Mach numbers versus the secondary to primary mass flow ratio for the 25° forced mixer.. In Figure H-10, both predicted and measured Cp_{super} distributions are plotted versus distance from the trailing edge of the 15° forced mixer for $M_p = 1.3$. These same comparisons are made for $M_p \approx 0.6$ in Figure H-11, for $M_p = 1.6$ in Figure H-12, and for $M_p = 2.4$ in Figure H-13.

Figure H-9: 25° forced mixer duct exit Cp_{super} model/data comparisons.

Figure H-10: 25° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 1.3$.

Figure H-11: 25° forced mixer subsonic wall $C_{p_{upper}}$ model/data comparisons for $M_p \approx 0.6$.

Figure H-12: 25° forced mixer subsonic wall $C_{p_{upper}}$ model/data comparisons for $M_p = 1.6$.

Figure H-13: 25° forced mixer subsonic wall Cp_{super} model/data comparisons for $M_p = 2.4$.

Bibliography

- [1] Boeing Commerical Airplanes. High speed civil transport study. Technical Report CR-4233, NASA, 1900.
- [2] T. Barber, R. W. Paterson, and S. A. Skebe. Turbofan forced mixer lobe flow modelling. Technical Report CR-4147, NASA, 1988.
- [3] G. Batchelor. *An Introduction to Fluid Mechanics*. Cambridge University Press, Cambridge, 1967.
- [4] T.G. Beckwith, R.D. Marangoni, and J.H. Lienhard. *Mechanical Measurements*. Addison-Wesley Publishing Company, 1993.
- [5] G. Brown and A. Roshko. On density effects and large structures in turbulent mixing layers. *Journal of Fluid Mechanics*, 64:775–816, 1974.
- [6] L. T. Clark. Application of compound flow analysis to supersonic ejector-mixer performance prediction. Technical Report 95-0645, AIAA, 1995.
- [7] N. T. Clemens and M. G. Mungal. Two- and three- dimensional effects in the supersonic mixing layer. Technical Report 90-1978, AIAA, 1990.
- [8] N. T. Clemens and M. G. Mungal. A planar mie scattering technique for visualizing supersonic mixing flows. *Experiments in Fluids*, 11:175–185, 1991.
- [9] Douglas Aircraft Company. Study of high speed civil transports. Technical Report CR-4235, NASA, 1900.
- [10] P. Dimotakis. Turbulent free shear layer mixing. Technical Report 89-0262, AIAA, 1989.
- [11] J. Elliott. A computational investigation of the fluid dynamics of a three dimensional compressible mixing layer with strong streamwise vorticity. Master’s thesis, Massachusetts Institute of Technology, 1990.
- [12] A. K. S. Fung. A synthesis of models for mixer-ejector nozzle flows. Master’s thesis, Massachusetts Institute of Technology, 1995.

- [13] A.K.S. Fung, 1996. Unpublished work.
- [14] J. E. Harris and D. K. Blanchard. Computer program for solving laminar, transitional, or turbulent compressible boundary-layer equations for two-dimensional and axisymmetric flow. Technical report, NASA TM-83207, 1982.
- [15] J. L. Kerrebrock. *Aircraft Engines and Gas Turbines*. The MIT Press, Cambridge, 1992.
- [16] J. Krasnodebski. Numerical investigations of lobed mixer flow fields. Master's thesis, Massachusetts Institute of Technology, 1995.
- [17] W. K. Lord, C. W. Jones, A. M. Stern, V. L. Head, and E. A. Krejsa. Mixer ejector nozzle for jet noise suppression. Technical Report 90-1909, AIAA, 1990.
- [18] Photometrics Ltd. *Series 200 CCD Camera System User's Manual*.
- [19] T. Manning. Experimental studies of mixing flows with streamwise vorticity. Master's thesis, Massachusetts Institute of Technology, 1991.
- [20] D. McCormick. *Vortical and Turbulent Structure of Planar Lobed Mixer Free-Shear Layers*. PhD thesis, The University of Connecticut, 1992.
- [21] M. N. O'Sullivan. A computational study of the effects of viscosity on lobed mixer flowfields. Master's thesis, Massachusetts Institute of Technology, 1993.
- [22] D. Papamoshchou and A. Roshko. The compressible turbulent shear layer: An experimental study. *Journal of Fluid Mechanics*, 197:453–477, 1988.
- [23] Y. Qiu. *A Study of Streamwise Vortex Enhanced Mixing in Lobed Mixer Devices*. PhD thesis, Massachusetts Institute of Technology, 1992.
- [24] H. Schlichting. *Boundary Layer Theory*. McGraw-Hill Book Company, 1960.
- [25] J. M. Seiner and E. A. Krejsa. Supersonic jet noise and the high speed civil transport. Technical Report 89-2358, AIAA, 1989.
- [26] Sivells. A computer program for the aerodynamic design of axisymmetric and planar nozzles for supersonic and hypersonic wind tunnels. Technical Report AEDC-TR-78-63, AEDC, 1978.
- [27] Michael J. T. Smith. *Aircraft Noise*. Cambridge University Press, 1989.
- [28] G. S. Stanislaw. Utility of the flow substitution principle for high speed propulsion applications. Master's thesis, Massachusetts Institute of Technology, 1996.
- [29] B.S. Teeple, 1996. Unpublished work.

- [30] D. E. Tew. A computational study of mixing downstream of a lobed mixer with a velocity difference between the co-flowing streams. Master's thesis, Massachusetts Institute of Technology, 1992.
- [31] T. G. Tillman, R. W. Paterson, and W. M. Presz. Supersonic nozzle mixer ejector. *Journal of Propulsion and Power*, 8:513–519, 1992.
- [32] T. G. Tillman, W. P. Patrick, and R. W. Paterson. Enhanced mixing of supersonic jets. *Journal of Propulsion and Power*, 7:1007–1014, 1991.
- [33] I.A. Waitz, Y.J. Qiu, J.K. Elliott, A.K.S. Fung, J.M. Kerwin, J.K. Krasnodebski, M.N. O'Sullivan, D.E. Tew, E.M. Greitzer, F.E. Marble, C.S. Tan, and T.G. Tillman. Streamwise vorticity enhanced mixing. To appear in *Aerospace Sciences*, 1997.
- [34] M.J. Zucrow and J.D. Hoffman. *Gas Dynamics*. John Wiley and Sons, 1976.