Horizontal velocity structure functions in the upper troposphere and lower stratosphere: 2. Theoretical considerations

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 106, NO. D10, PAGES 10,233–10,241, MAY 27, 2001

Horizontal velocity structure functions in the upper troposphere and lower stratosphere 2. Theoretical considerations

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Abstract. The Kolmogorov equation for the third-order velocity structure function is derived for atmospheric mesoscale motions on an f plane. A possible solution is a negative third-order structure function, varying linearly with separation distance and mean dissipation, just as in three-dimensional turbulence, but with another scaling constant. On the basis of the analysis and the observed stratospheric third-order structure function, it is argued that there is a forward energy cascade in the mesoscale range of atmospheric motions. The off-diagonal part of the general tensor equation is also studied. In this equation there is an explicit Coriolis term that may be crucial for the understanding of the kinetic energy spectrum at scales larger than 100 km.

1. Introduction

The kinetic energy spectrum of atmospheric mesoscale motions has been the object of scientific debate for over two decades. A key issue in this debate is the direction of the kinetic energy flux, Π_u, from small to large wave numbers, i.e., the rate of kinetic energy that is transferred from large-scale motions to small-scale motions. Two opposite hypotheses have been put forward regarding the sign, or the direction, of the flux of kinetic energy.

First, there is the hypothesis [*Dewan*, 1979, 1997] that the kinetic energy flux is in the direction from small to large wave numbers. According to this hypothesis, long gravity waves break down to shorter waves in a cascade process, which is similar to three-dimensional (3-D) Kolmogorov turbulence, resulting in a positive kinetic energy flux. The only parameter that can determine the spectrum is the energy flux, and from dimensional considerations we obtain a $k^{-5/3}$ spectrum, where k is the wave number. At the shortest wavelengths, 100–1000 m say, a more violent instability occasionally sets in; the wave energy is broken down in intermittent spots of three-dimensional turbulence and finally dissipated at scales of the order of 1 cm.

Second, there is the hypothesis [*Gage*, 1979; *Lilly*, 1983] that the $k^{-5/3}$ spectrum is the spectrum of two-dimensional (2-D) turbulence with a negative energy flux, i.e., a flux from small to large scales, in accordance with Kraichnan's theory of 2-D turbulence [*Kraichnan*, 1967, 1970]. Such a range naturally emerges in 2-D direct numerical simulations (DNS) with forcing at large wave numbers [*Smith and Yakhot*, 1994; *Maltrud and Vallis*, 1991].

Lindborg [1999], hereafter L99, suggested that the classical third-order structure function relation of *Kolmogorov* [1941],

$$\langle \delta u_L \delta u_L \delta u_L \rangle = -\frac{4}{5}\epsilon r\,, \tag{1}$$

could be used to determine the direction of the spectral flux of kinetic energy, from large to small scales. Here ϵ is the mean dissipation, which is equal to Π_u, $\delta \mathbf{u} = \mathbf{u}' - \mathbf{u}$ is the difference of the velocities at two points $\boldsymbol{\xi}'$ and $\boldsymbol{\xi}$ with separation vector $\mathbf{r} = \boldsymbol{\xi}' - \boldsymbol{\xi}$ and the subscript L indicates the component in the same direction as $\mathbf{r}$. The brackets denote statistical averaging, often interpreted as ensemble averaging. Here we let it denote time averaging.

Generally it can be argued that a negative linear range of the third-order structure function,

$$\langle \delta u_L \delta \mathbf{u} \cdot \delta \mathbf{u} \rangle\,, \tag{2}$$

should be interpreted as a sign of a forward cascade, and a positive linear range should be interpreted as a sign of an inverse cascade. In three dimensions the third-order structure function is related to the spectral energy flux through wave number k through the relation (see L99 or *Frisch* [1995], section 6.2)

Paper number 2000JD900815.
0148-0227/01/2000JD900815\$09.00

$$\Pi_u(k) = -\frac{1}{4}\int_{|\mathbf{k}|<k}\frac{1}{(2\pi)^3}\int \nabla\cdot\langle\boldsymbol{\delta}\mathbf{u}\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle \times \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})\, d^3r\, d^3k\,, \quad (3)$$

with a corresponding relation in two dimensions. The third-order structure function corresponding to a constant energy flux, $\Pi_u(k) = F$, is a solution to the equation

$$\nabla\cdot\langle\boldsymbol{\delta}\mathbf{u}\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle = -4F\,. \quad (4)$$

This can be seen by substituting (4) into (3),

$$\begin{aligned}\Pi_u(k) &= -\frac{1}{4}\int_{|\mathbf{k}|<k}\frac{1}{(2\pi)^3}\int -4F\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})\, d^3r\, d^3k \\ &= \int_{|\mathbf{k}|<k} F\delta(\mathbf{k})\, d^3k \\ &= F\,, \end{aligned} \quad (5)$$

with a corresponding calculation in two dimensions. More rigorously, one should, of course, take into account that $\Pi_u(k)$ is constant in a finite range of wave numbers, $k_1 < k < k_2$ and that (4) can be true only in a corresponding finite range of separation lengths, $r_1 < r < r_2$. Provided that these ranges are sufficiently broad and that $\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle$ monotonically goes to zero as $r \to 0$, it can be argued (see L99) that the contribution to the flux integral from regions outside the range where (4) holds are negligible and that (5) is a good approximation.

By using the divergence theorem we can integrate (4) to

$$\int\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle\, d\Omega = -\frac{16\pi Fr}{3}\,, \quad (6)$$

where $d\Omega$ is the element of solid angle and the integral is over all angles. In two dimensions we instead find

$$\int_0^{2\pi}\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle\, d\theta = -4\pi Fr\,, \quad (7)$$

where $d\theta$ is the element of azimuthal angle. The relations (6) and (7) quite generally show that in both two and three dimensions, a negative linear range of $\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle$ implies a positive F, i.e., a positive and constant energy flux, and a positive linear range of the same function implies a negative and constant energy flux. Note that this argument is not dependent on any strong assumption of isotropy. Isotropy means that $\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle$ is independent of angle, and the integrals of (6) and (7) can be computed to give

$$\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle = -\frac{4}{3}Fr \quad (8)$$

in three dimensions and

$$\langle\delta u_L\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}\rangle = -2Fr \quad (9)$$

in two dimensions. By assuming that the third-order structure function tensor is isotropic and that the flow is incompressible one can further derive the relation (1) and the corresponding two-dimensional relation (L99)

$$\langle\delta u_L\delta u_L\delta u_L\rangle = -\frac{3}{2}Fr\,. \quad (10)$$

However, even for an anisotropic field, the relations (6) and (7) clearly suggest that a negative linear range of the third-order structure function is a sign of a positive energy flux. In this study we shall approximate $\boldsymbol{\delta}\mathbf{u}\cdot\boldsymbol{\delta}\mathbf{u}$ with the expression including only the horizontal velocity increment. Moreover, the separation vector is confined to the horizontal plane. As we shall see from a detailed analysis of the governing equations, these limitations will not prevent us from estimating Π_u, using the two-dimensional relation (9), rather than the three-dimensional relation (8).

In the companion paper [*Cho and Lindborg*, this issue], hereafter Part 1, a plot (Figure 6) of the stratospheric third-order structure function (9) displays a negative linear range for 10 km $< r <$ 200 km. Thus this plot gives us an answer in favor of the forward cascade hypothesis. However, to put this interpretation on somewhat firmer ground and to quantitatively determine the kinetic energy flux, or the mean dissipation, a renewed analysis of the governing equations is required. In the case of atmospheric motions, possible effects from rotation and stratification should be carefully investigated.

The main objective of this paper is to derive the flux relation in the case of atmospheric mesoscale motions, using the weakest possible assumptions. This will be done by first deriving the governing equation for the second-order structure function tensor $\langle\delta u_i\delta u_j\rangle$ and then taking the trace of this equation. In addition to the flux relation, we will also study the off-diagonal part of the tensor equation, in which an explicit Coriolis term appears.

Many terms will be neglected in our analysis, which, inevitably, means that some uncertainty may adhere to the quantitative estimate of the kinetic energy flux or the mean dissipation. If all these approximations are justified, then we believe that our estimate of the mean dissipation in the lower stratosphere is one the most accurate that has been made so far. Future measurements and numerical simulations will tell us whether we are right.

2. Derivation of the Kolmogorov Equation on an f Plane

The general Kolmogorov equation is the dynamical equation for the second-order structure function tensor $\langle\delta u_i\delta u_j\rangle$. We use Cartesian tensor notation since it is very convenient. To derive the dynamical equation for $\langle\delta u_i\delta u_j\rangle$ on an f plane, we let x, y, and z be the longitudinal, latitudinal, and vertical length coordinates, and u, v, and w be the corresponding velocity components. To start, we make the following assumptions: (1) Statistical homogeneity in the longitudinal direction. (2) Quasi two dimensionality, by which we mean that

$$|w| \ll |u| \sim |v| , \tag{11}$$

without assuming that the z derivatives of w, u, or v are small. We allow the possibility that

$$\left|\frac{\partial w}{\partial z}\right| \sim \left|\frac{\partial u}{\partial x}\right| \tag{12}$$

and the corresponding relation for v. (3) Structure functions are invariant under rotation around a vertical axis, or in other words, they are axisymmetric. (4) Let $\rho = \rho_o + \tilde{\rho}$, where ρ is the mass density, $\rho_o = \langle\rho\rangle$, and tilde denotes the fluctuation around the mean. Then we assume that $|\tilde{\rho}| \ll \rho_o$. (5) Let $u_i = U_i + \tilde{u_i}$, where $U_i = \langle u_i \rangle$. Other capitalized quantities will also correspond to mean values. We assume that the fluctuating field is imcompressible,

$$\frac{\partial \tilde{u_i}}{\partial \xi_i} = 0 \text{ and } \frac{\partial \tilde{\rho}}{\partial t} = 0 , \tag{13}$$

which is justified if the Mach number is small. The convention of contraction over repeated indices is used. (6) The vertical mean velocity W is zero. (7) From mass conservation and the previous assumptions it follows that

$$\frac{\partial u_i}{\partial \xi_i} = -\frac{1}{\rho_0}\frac{\partial \rho}{\partial y} V . \tag{14}$$

(8) From the thermal wind equation we estimate

$$\left|\frac{1}{\rho_o}\frac{\partial \rho}{\partial y}\right| \sim \left|\frac{\partial U}{\partial z}\frac{f}{g}\right| , \tag{15}$$

where g is the gravitational acceleration, $f = 2|\boldsymbol{\Omega}|\sin\phi$, $\boldsymbol{\Omega}$ is the Earth's angular velocity vector, and ϕ is the latitude, and further assume that

$$\left|\frac{\partial U}{\partial z}\frac{f}{g}\right| r \ll 1 . \tag{16}$$

(9) The separation r is sufficiently small to make the approximations

$$\rho_o' = \rho_o \text{ and } \hat{\mathrm{e}} = \hat{\mathrm{e}}' , \tag{17}$$

where $\hat{\mathrm{e}}$ is the upward pointing vertical unit vector. The primed quantities are measured at $\boldsymbol{\xi}'$, and the unprimed quantities are measured at $\boldsymbol{\xi}$.

Now we write down the Navier-Stokes equations in a rotating frame of reference,

$$\begin{aligned} \frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial \xi_k} &= -\frac{1}{\rho}\frac{\partial p}{\partial \xi_i} - 2\epsilon_{ikl}\Omega_k u_l \\ &\quad - e_i g + \nu \frac{\partial^2 u_i}{\partial \xi_k \partial \xi_k} , \end{aligned} \tag{18}$$

where p is the pressure and ν is the kinematic viscosity. For simplicity, we have used the incompressible form of the viscous term. Note that the subscripted ϵ here is the alternating symbol used in standard tensor notation not the dissipation rate. The second-order structure function tensor divides itself into four terms,

$$\langle \delta u_i \delta u_j \rangle = \langle u_i' u_j' + u_i u_j - u_i' u_j - u_i u_j' \rangle . \tag{19}$$

To derive the equation for $\langle \delta u_i \delta u_j \rangle$, we derive the equations for the four separate terms within the averaging operator $\langle\rangle$ on the right-hand side of (19) and then add these together. Then we make the variable transformation

$$\begin{gathered} \boldsymbol{\xi} = \mathbf{x} , \quad \boldsymbol{\xi}' = \mathbf{x} + \mathbf{r} , \\ \left(\frac{\partial}{\partial \xi_i}, \frac{\partial}{\partial \xi_i'}\right) \to \left(-\frac{\partial}{\partial r_i} + \frac{\partial}{\partial x_i}, \frac{\partial}{\partial r_i}\right) \end{gathered} \tag{20}$$

average, and use the fact that differentiation commutes with the averaging. Essentially, this is the method developed by *von Kármán and Howarth* [1938], with the difference that they did not derive the equation for the structure function but for the equation for the correlation function $\langle u_i' u_j \rangle$, whose isotropic form is known as the Kármán-Howarth equation.

The equation for $u_i u_j$ is derived by multiplying the u_i equation by u_j, multiplying the u_j equation by u_i, and adding the two resulting equations. The equations for the other three terms in (19) are derived in a similar way. Thus the equation for $\langle \delta u_i \delta u_j \rangle$ can be written as a sum of eight equations. We analyze the resulting equation term by term. (1) The time derivative

$$\frac{\partial}{\partial t}\langle \delta u_i \delta u_j \rangle \tag{21}$$

is equal to zero by the definition of the average as a time average. (2) By treating $\boldsymbol{\xi}'$ and $\boldsymbol{\xi}$ as independent variables the advection terms can be written as

$$\begin{aligned} &\left\langle \frac{\partial}{\partial \xi_k}(u_k \delta u_i \delta u_j)\right\rangle + \left\langle \frac{\partial}{\partial \xi_k'}(u_k' \delta u_i \delta u_j)\right\rangle \\ &\quad - \left\langle \frac{\partial u_k}{\partial \xi_k}\delta u_i \delta u_j \right\rangle - \left\langle \frac{\partial u_k'}{\partial \xi_k'}\delta u_i \delta u_j \right\rangle . \end{aligned} \tag{22}$$

From assumptions 5–7 it is easily argued that the last two terms are negligible. By introducing the transformation (20) the first two terms can be written as

$$\frac{\partial}{\partial r_k}\langle \delta u_k \delta u_i \delta u_j \rangle + \frac{\partial}{\partial x_k}\langle u_k \delta u_i \delta u_j \rangle . \tag{23}$$

By the assumption of statistical homogeneity in the longitudinal direction, we can write

$$\frac{\partial}{\partial x_k}\langle u_k \delta u_i \delta u_j \rangle = \frac{\partial}{\partial y}\langle v \delta u_i \delta u_j \rangle + \frac{\partial}{\partial z}\langle w \delta u_i \delta u_j \rangle . \tag{24}$$

An order of magnitude estimate of these two terms can be obtained by replacing v and w with the corresponding mean velocities. Since we have assumed that the vertical mean velocity is zero we thus obtain the order of magnitude estimate

$$\frac{\partial}{\partial x_k}\langle u_k \delta u_i \delta u_j \rangle \sim V \frac{\partial}{\partial y}\langle \delta u_i \delta u_j \rangle , \tag{25}$$

where V is the meridional mean velocity. From the airplane data we calculated V in different latitude bands, and we found that $V \sim 1$ m s^{-1}, at most. From the latitudinal variation of the second-order structure functions (Part 1, Figures 2 and 3), the term (25) can then be estimated. A safe conclusion is that it is negligible compared to the other terms in the final equation, at least for $r < 100$ km. Actually, we have also calculated the term $\langle v\delta u_i \delta u_i\rangle$ directly, in different latitude bands. From this calculation we have come to the same conclusion, that the inhomogeneous part of the advection term is negligible for $r < 100$ km. (3) Using assumptions 3 and 9 the pressure terms can be written as

$$\begin{aligned} &-\tfrac{1}{\rho_o}\left\langle\left(\tfrac{\partial p'}{\partial \xi_i'}-\tfrac{\partial p}{\partial \xi_i}\right)(u_j'-u_j)\right\rangle \\ &-\tfrac{1}{\rho_o}\left\langle\left(\tfrac{\partial p'}{\partial \xi_j'}-\tfrac{\partial p}{\partial \xi_j}\right)(u_i'-u_i)\right\rangle . \end{aligned} \tag{26}$$

By introducing the transformation (20) the pressure terms can be rewritten as

$$-\frac{1}{\rho_o^2}\frac{\partial}{\partial x_i}\langle \delta p \rho_o \delta u_j\rangle - \frac{1}{\rho_o^2}\frac{\partial}{\partial x_j}\langle \delta p \rho_o \delta u_i\rangle + D_{ij}\,, \tag{27}$$

where we define the tensor D_{ij} by

$$\begin{aligned} D_{ij} \;=\; &\frac{1}{\rho_o^2}\left\langle \delta p\left(\frac{\partial(\rho_o u_j')}{\partial \xi_i'} - \frac{\partial(\rho_o u_j)}{\partial \xi_i}\right)\right\rangle \\ &+ \frac{1}{\rho_o^2}\left\langle \delta p\left(\frac{\partial(\rho_o u_i')}{\partial \xi_j'} - \frac{\partial(\rho_o u_i)}{\partial \xi_j}\right)\right\rangle . \end{aligned} \tag{28}$$

The first two terms in (27) can be neglected by the following arguments. We first assume that the inhomogeneities emanate from the mean pressure and mean velocity fields, which is very reasonable. For simplicity we study the trace of the relevant term. This can now be estimated as

$$\begin{aligned} \frac{2}{\rho_o}\frac{\partial}{\partial y}\langle \delta p \delta v\rangle \;\sim\; &\frac{2}{\rho_o}\left(\frac{\partial P}{\partial x}\frac{\partial V}{\partial y} r_x r_y \right.\\ &\left. + \frac{\partial P}{\partial y}\frac{\partial V}{\partial y} r_y r_y\right) . \end{aligned} \tag{29}$$

By assuming that the mean field is in geostrophic balance we can estimate this term as

$$\sim f\left(\frac{r}{L_y}\right)^2 V^2\,, \tag{30}$$

where L_y is the typical latitudinal length scale over which the mean field varies, which is at least 1000 km. As long as $r < L_y$, it is safe to neglect this term. We also observe that the tensor D_{ij} is traceless, which follows from mass conservation. (4) The Coriolis term can be written as

$$-2\epsilon_{ikl}\Omega_k\langle \delta u_l \delta u_j\rangle - 2\epsilon_{jkl}\Omega_k\langle \delta u_l \delta u_i\rangle\,, \tag{31}$$

which is straightforward to show. (5) The term associated with the gravitational force includes a factor $(e_i' - e_i)g$, and will thus be zero according to assumption 9. (6) By introducing the transformation (20) the viscous term can be written as

$$\begin{aligned} &-2\nu\left\langle \tfrac{\partial u_i'}{\partial \xi_k'}\tfrac{\partial u_j'}{\partial \xi_k'}\right\rangle - 2\nu\left\langle \tfrac{\partial u_i}{\partial \xi_k}\tfrac{\partial u_j}{\partial \xi_k}\right\rangle + 2\nu\tfrac{\partial^2}{\partial r_k \partial r_k}\langle \delta u_i \delta u_j\rangle \\ &\quad + \nu\left(\tfrac{\partial}{\partial x_k} - 2\tfrac{\partial}{\partial r_k}\right)\tfrac{\partial}{\partial x_k}\langle \delta u_i \delta u_j\rangle\,, \end{aligned} \tag{32}$$

of which we neglect the last term on the basis of homogeneity. Here we have not assumed that homogeneity can be generally applied. However, this term must be very small since the length scale over which $\langle \delta u_i \delta u_j\rangle$ varies is much larger than any viscous length scale. In fact, the penultimate term is also negligible for the case to which we will apply the equation. We will, however, keep this term for a while.

The resulting equation can now be written as

$$\begin{aligned} \frac{\partial}{\partial r_k}\langle \delta u_k \delta u_i \delta u_j\rangle = &D_{ij} - 2\epsilon_{ikl}\Omega_k\langle \delta u_l \delta u_j\rangle \\ &- 2\epsilon_{jkl}\Omega_k\langle \delta u_l \delta u_i\rangle - 2\nu\left\langle \frac{\partial u_i'}{\partial \xi_k'}\frac{\partial u_j'}{\partial \xi_k'}\right\rangle \\ &- 2\nu\left\langle \frac{\partial u_i}{\partial \xi_k}\frac{\partial u_j}{\partial \xi_k}\right\rangle + 2\nu\frac{\partial^2}{\partial r_k \partial r_k}\langle \delta u_i \delta u_j\rangle . \end{aligned} \tag{33}$$

One by one we have eliminated all terms describing the effects of inhomogeneities in the latitudinal direction as well as in the vertical direction. Quite generally, it can be argued that the conditions for neglecting inhomogeneous terms are

$$r_\rho \ll L_y \quad \text{and} \quad r_z \ll L_z\,, \tag{34}$$

where r_ρ and r_z are the projection of $\mathbf{r}$ onto the horizontal plane and the vertical axis and L_y and L_z are the typical latitudinal and vertical length scales of overall variation of the statistical properties of the atmospheric flow field. As for the airplane data the second condition is always fulfilled, since $\mathbf{r}$ can always be approximated as lying in a horizontal plane. In fact, in our structure function computations we only used data point pairs that were on the same standard flight level, i.e., $r_z < 300$ m (see Part 1). We find it safe to assume that the first condition is fulfilled if $r_\rho < 100$ km.

Now we define $\hat{\mathbf{n}}$ as the unit vector pointing in the same direction as the projection of the separation vector $\mathbf{r}$ on the horizontal plane. In our empirical investigation, $\mathbf{r}$ always lies in the horizontal plane, so in this case $\hat{\mathbf{n}}$ is the unit vector in the direction of $\mathbf{r}$. The unit vector $\hat{\mathbf{t}}$ we define as $\hat{\mathbf{t}} = \hat{\mathbf{n}} \times \hat{\mathbf{e}}$. The velocity increment $\boldsymbol{\delta}\mathbf{u}$ can be resolved into orthogonal components as

$$\boldsymbol{\delta}\mathbf{u} = \hat{\mathbf{n}}\delta u_L + \hat{\mathbf{t}}\delta u_T + \hat{\mathbf{e}}\delta w\,. \tag{35}$$

It can be noted that (35) is just the classical resolution of a vector into cylindrical components. The vector $\mathbf{r}$ can be resolved as

$$\mathbf{r} = \hat{\mathbf{n}} r_\rho + \hat{\mathbf{e}} r_z\,. \tag{36}$$

The two components r_ρ and r_z are very similar to the traditional cylindrical coordinates ρ and z, with the slight difference that they are the components of a true vector and not the coordinates of an arbitrary point. Since we have made assumption 3 of quasi two dimensionality, we will mainly be concerned with structure functions including the horizontal components δu_L and δu_T.

The horizontal parts of the second- and third-order structure functions can now be written as

$$\begin{aligned}\langle \delta u_i \delta u_j \rangle_H &= n_i n_j \langle \delta u_L \delta u_L \rangle + t_i t_j \langle \delta u_T \delta u_T \rangle \\ &\quad + (n_i t_j + n_i t_j) \langle \delta u_T \delta u_L \rangle \end{aligned} \qquad (37)$$

$$\begin{aligned}\langle \delta u_k \delta u_i \delta u_j \rangle_H &= n_k n_i n_j \langle \delta u_L \delta u_L \delta u_L \rangle + (n_k t_i t_j \\ &\quad + t_k n_i t_j + t_k t_i n_j) \langle \delta u_L \delta u_T \delta u_T \rangle \\ &\quad + t_k t_i t_j \langle \delta u_T \delta u_T \delta u_T \rangle + (n_k n_i t_j \\ &\quad + n_k t_i n_j + t_k n_i n_j) \langle \delta u_L \delta u_L \delta u_T \rangle . \end{aligned} \qquad (38)$$

The expressions (37) and (38) are complete representations of the horizontal parts of the structure functions and do not require any assumption of axisymmetry. The functions including an odd number of the transverse component δu_T are not invariant under reflections in vertical planes. The only physical mechanism that can break this kind of symmetry is rotation with respect to a vertical axis. Without system rotation we would therefore expect these functions to be zero. With system rotation, they are not generally zero. However, it was found from the data that the function $\langle \delta u_L \delta u_T \rangle$ was very small. Up to separations of 100 km the ratio $|\langle \delta u_L \delta u_T \rangle| / |\langle \delta u_T \delta u_T \rangle|$ was less than 2% and for larger separations it was less than 5%.

Using assumption 3 of axisymmetry, we will now take the trace of equation (33). With a slight change in notation this can be written as

$$\frac{1}{r_\rho} \frac{\partial}{\partial r_\rho} (r_\rho \langle \delta u_L \boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} \rangle) + \frac{\partial}{\partial r_z} \langle \delta w \boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} \rangle = -2\langle \epsilon' \rangle - 2\langle \epsilon \rangle + 2\nu \nabla_r^2 \langle \boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} \rangle , \qquad (39)$$

where $\boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} = \delta u_L \delta u_L + \delta u_T \delta u_T$ by the assumption of quasi two dimensionality, $\langle \epsilon' \rangle$ and $\langle \epsilon \rangle$ are the average rates of dissipation of kinetic energy per unit mass at the points $\boldsymbol{\xi}'$ and $\boldsymbol{\xi}$, and ∇_r^2 is the $\mathbf{r}$ space Laplace operator. The Coriolis terms disappear from (33) when we take the trace, which can easily be seen by writing it in vector notation as $-4\langle (\boldsymbol{\Omega} \times \boldsymbol{\delta}\mathbf{u}) \cdot \boldsymbol{\delta}\mathbf{u} \rangle = 0$. The second term on the left-hand side cannot be neglected only on the basis of quasi two dimensionality. It is true that it contains the vertical velocity increment δw, but it also contains a derivative with respect to the vertical direction. If the relevant vertical length scale is very small, then this term could, in principle, be important. Nevertheless, we think it is reasonable to neglect this term. To do this, it is sufficient to assume that for a given $\boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u}$, which is positive definite, there is equal probability for a negative δw as for a positive δw of a given magnitude. This would, for example, be the case if the joint probability distribution of δw and $\boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u}$ were Gaussian. This is a reasonable assumption for horizontal separations r_ρ, which are large compared to the typical length scale of vertical displacements. Thus we neglect the second term in (39).

Assuming that $\langle \epsilon' \rangle \approx \langle \epsilon \rangle$, (39) can now be written as

$$\frac{1}{r_\rho} \frac{\partial}{\partial r_\rho} (r_\rho \langle \delta u_L \boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} \rangle) = -4\langle \epsilon \rangle + 2\nu \nabla_r^2 \langle \boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} \rangle . \qquad (40)$$

Neglecting the last viscous term in (40), there are two terms left, and the equation can be integrated to yield

$$\langle \delta u_L \delta u_L \delta u_L \rangle + \langle \delta u_L \delta u_T \delta u_T \rangle = -2\langle \epsilon \rangle r , \qquad (41)$$

where we have written r instead of r_ρ since it is evident that we are now confined to the horizontal plane.

Comparing (41) with (9), we see that it can also be given the interpretation that the spectral energy flux, $\Pi_u(k)$, from small to large wave numbers in Fourier space, is positive, constant, and equal to $\langle \epsilon \rangle$. An inverse energy cascade, from large to small wave numbers, equivalently from small to large scales, would have given a positive linear range of the third-order structure function (41), and not a negative linear range, as observed in Part 1. Thus we can quite safely conclude that the energy flux is in the direction from large to small scales, just as in 3-D turbulence. For obvious reasons, we can, however, be certain that the underlying physical mechanism of this energy flux is not classical 3-D turbulence. So far we have made no assumption about what sort of mechanism this could be. The relation (41) is derived from the Navier-Stokes equations, using only very general assumptions. A reasonable guess [*Dewan*, 1979, 1997] is that the underlying mechanism is a cascade process of nonlinearly interacting gravity waves, in which energy is transferred from larger to smaller wave modes. In all likelihood the actual dissipation takes place in very localized and intermittently distributed "blobs" or "blini" of high intensity 3-D clear air turbulence (CAT). The typical dissipative length scale is of the order of 1 cm. It may seem strange that (41) relates the dissipation to the third-order structure function measured at separations of many orders of magnitude larger than the length scale at which the dissipation actually takes place. Indeed, this is a remarkable property of the Kolmogorov relation.

We shall also study the off-diagonal part of (33). Projecting this equation onto $n_i t_j$ we can, after some manipulation, obtain the equation

$$\frac{1}{r} \langle \delta u_T \delta u_T \delta u_T \rangle - \left(\frac{2}{r} + \frac{\partial}{\partial r} \right) \langle \delta u_L \delta u_L \delta u_T \rangle + n_i t_j D_{ij} = f(\langle \delta u_T \delta u_T \rangle - \langle \delta u_L \delta u_L \rangle) . \qquad (42)$$

Here we have assumed that all viscous terms in (33) are diagonal and that the term $(\partial / \partial r_z) \langle \delta w \delta u_L \delta u_T \rangle$ can be neglected by using the same type of argument when the corresponding term was neglected from (40). The subscript ρ has been dropped from r as in (41), since we are confined to the horizontal plane. All the terms in (42) clearly have opposite signs on opposite sides of the equator, since f, as we have defined it, is positive in the Northern Hemisphere and negative in the Southern Hemisphere. If mean values are formed from a data set including both the Northern and Southern Hemispheres, then two opposing orientations of the vector $\hat{\mathbf{t}}$ must be used on each side of the equator to avoid cancellation of the two hemispherical contributions. This applies to structure functions with terms that have an odd number of δu_T. Therefore we redefined $\hat{\mathbf{t}}$ in the Southern Hemisphere when we calculated the structure functions from the airplane data (see Part 1).

3. Comparison With Empirical Data

We shall now investigate to what extent the empirical data (see Part 1) can be interpreted in light of (41) and (42). The left-hand side of (41) is plotted for the troposphere (Part 1, Plate 1a) and for the stratosphere (Part 1, Plate 1b). For the troposphere the statistical scatter is very large up to $r = 100$ km, and we do not dare to interpret the points as evidence of a linear region. However, the left-hand side of (41) is negative, and it may very well be the case that a linear region would appear if the scatter could be removed by using even more data. As for the stratosphere, there is indeed evidence of a linear region in the range from 10 to 200 km (Part 1, Figure 6). By fitting the data to a straight line in this range and using (41), we can estimate the mean dissipation per unit mass to be

$$\langle \epsilon \rangle = 6 \times 10^{-5}\,\mathrm{m}^2\,\mathrm{s}^{-3}\,. \tag{43}$$

This is a very reasonable value that falls somewhere in the middle of the wide range of previous estimates from various types of measurements, which can be found in reviews of previous measurements [*Vinnichenko*, 1969; *Crane*, 1980; *Dewan*, 1997; *Hocking and Mu*, 1997].

Two different types of measurements of the mean dissipation in the stratosphere are reported in the literature. First, there are estimates made from radar observations using the scattering cross section per unit volume [*Crane*, 1980; *Hocking and Mu*, 1997] or the Doppler spectral width [*Nastrom and Eaton*, 1997]. To estimate the dissipation from the former quantity requires some rather elaborate model assumptions, while the latter is susceptible to contamination by beam, shear, and wave-broadening effects [*Hocking*, 1996]; both may be biased high because very weak turbulence may not be detected at all. Second, there are airplane [*Vinnichenko*, 1969; *Lilly et al.*, 1974] and balloon [*Cadet*, 1977] measurements of the 3-D turbulence inertial range energy spectra, alternatively second-order structure functions, performed in more or less localized turbulent patches. If it is assumed that the Kolmogorov constant is known, then the mean dissipation can be estimated, either from the measured spectrum or from the structure function.

There are important principal differences between our method of estimating the mean dissipation and these methods. To estimate an overall mean value of the dissipation either from radar measurements or from direct turbulence measurements, it is necessary to average over a wide range of subsets of data, for which the individual values of the dissipation can vary by over 2 orders of magnitude, owing to the very large spatial and temporal variation of this quantity. It is not easy to estimate the relative weights of these subsets.

Our estimate is based on a time and space average of a quantity describing properties of the flow field at scales from 10 to 200 km. Whatever the nature is of the underlying dynamical process of our data, the process is definitely not as rarely occurring as 3-D turbulence. The quantity that we actually measure is the kinetic energy flux at larger scales. We also use a huge data set, with samples from many locations. This should give us a more representative overall mean value.

As compared to radar measurements, our estimate relies on an analytical relation (41) with far more theoretical justification than the assumptions that are required to estimate the mean dissipation from radar data.

As compared to in situ measurements from airplanes or balloons, there is another important difference. Our estimate relies on a relation (41) with a linear dependence of the dissipation. This means that we will measure a true mean value, even though there may be a very large spatial and temporal variation in the set of individual samples on which the mean value is based. The measured mean value over two or more subsets of data, for which the mean dissipations are very different, will actually be the true mean value. This is not true if we rely on nonlinear relations such as (44) or (45) [see *Landau and Lifshitz*, 1987, p. 140]. A necessary condition to use such a relation for an estimate of the dissipation is that there is not too much variation of the turbulence intensity in each data set that is used.

We now investigate what value we obtain for the constant C, corresponding to the Kolmogorov constant of 3-D turbulence. We assume that we can write

$$\langle \boldsymbol{\delta}\mathbf{u} \cdot \boldsymbol{\delta}\mathbf{u} \rangle = C \langle \epsilon \rangle^{2/3} r^{2/3}\,, \tag{44}$$

with corresponding relations (see L99)

$$\begin{aligned} E(k) &= C_o \langle \epsilon \rangle^{2/3} k^{-5/3} \\ E_1(k_1) &= C_1 \langle \epsilon \rangle^{2/3} k_1^{-5/3} \end{aligned} \tag{45}$$

for the 2-D and one-dimensional (1-D) kinetic energy spectra. The relations between the three constants are $C_o \approx 0.17C$ and $C_1 \approx 0.12C$ (L99, p. 271). Using the values of the measured parameters a_1 and a_2 (Part 1, Table 2) based on all stratospheric data, we obtain

$$C = 5.5\,, \quad C_o = 0.95\,, \quad \text{and} \quad C_1 = 0.67\,. \tag{46}$$

Table 1. Mean Energy Dissipation Rates in the Lower Stratosphere

Latitude	$\langle\epsilon\rangle$ (10^{-5} m^2 s^{-3})
30°–40°	9.6
40°–50°	7.4
50°–60°	5.7
60°–70°	4.5

We do not claim that these values are very accurate nor that the constants necessarily are universal, since a very intermittent distribution of ϵ makes universality a disputable issue [see *Landau and Lifshitz*, 1987]. However, the fact that the measured values are of the order of unity strengthens the credibility of the measured kinetic energy dissipation, as well as the interpretation of the $k^{-5/3}$ range of the kinetic energy spectrum as originating from an energy cascade, similar to the cascade in 3-D turbulence. Using the value (46) of C, which was calculated using the total average of the second-order structure function, and using Table 2 from Part 1, we can actually estimate the latitudinal variation of the mean dissipation in the lower stratosphere. The resulting values are listed in Table 1. There is, of course, some uncertainty in these values, although we think that the trend is significant.

For separations of the order of 1000 km, the latitudinal inhomogeneities may become important in the governing equations for the structure functions. The plots of the second-order structure functions (Part 1, Figures 2 and 3) show a significant latitudinal variation, both in the troposphere and the stratosphere.

There is an established hypothesis [*Charney*, 1971] that the observed kinetic energy spectrum in this range should be the spectrum predicted by Kraichnan's 2-D turbulence theory [*Kraichnan*, 1967]. According to this theory, there is a range in Fourier space through which there is a constant spectral flux of enstrophy Π_ω, and in this range the kinetic energy spectrum should be of the form $E(k) = \Pi_\omega^{2/3} k^{-3}$. The very concept of a spectral flux of enstrophy in Fourier space rests on the assumption of homogeneity of (40). Moreover, a constant enstrophy flux, Π_ω, being equal to the mean dissipation of enstrophy, $\langle\epsilon_\omega\rangle$, requires that there is a very special balance in this equation. Applying the Laplace operator to (40), we must have

$$\nabla_r^2 \nabla_r \cdot \langle \delta\mathbf{u}\delta\mathbf{u}\cdot\delta\mathbf{u}\rangle = 4\langle\epsilon_\omega\rangle \qquad (47)$$

if there is a constant enstrophy flux range in Fourier space. The left-hand side is directly connected to the definition of the enstrophy flux. The right-hand side originates from the last, viscous, term in (40). The equation (47) can be integrated to obtain (see L99)

$$\langle \delta u_L \delta u_L \delta u_L\rangle + \langle \delta u_L \delta u_T \delta u_T\rangle = \frac{1}{4}\langle\epsilon_\omega\rangle r^3 . \qquad (48)$$

The cubic positive dependence of (48) is a direct consequence of a constant and positive spectral enstrophy flux. Recently, the relation (48) has been numerically verified [*Lindborg and Alvelius*, 2000] in a DNS of a constant enstrophy flux range in 2-D homogeneous turbulence.

Indeed, there is a rather narrow range, $r \approx$ 500–1000 km, with a positive cubic dependence of the third-order structure function in the lower stratosphere (Part 1, Figure 6). A realistic value of the enstrophy flux and enstrophy dissipation could be estimated from this curve. However, it can be questioned whether the effects from latitudinal inhomogeneities are so small that this interpretation is the whole truth.

We have no alternative theory that can explain the shape of the second-order structure functions, alternatively the kinetic energy spectrum, at scales of the order of 1000 km. We shall only point out another direction that may lead to an explanation, if it is further investigated. Instead of studying (40) at these scales, we shall study (42). In Figures 1 and 2 we have plotted the functions $\langle \delta u_T \delta u_T \delta u_T\rangle$ and $\langle \delta u_T \delta u_L \delta u_L\rangle$ for the tropospheric and stratospheric data. The most striking feature of the plots is that both these functions seem to grow approximately as r^2 in the range r = 100–1000 km. This behavior is most evident in the stratospheric plots. We shall not try to explain this behavior, although we find it quite remarkable. Instead we shall focus on the contribution of the terms

$$\frac{1}{r}\langle \delta u_T \delta u_T \delta u_T\rangle - \left(\frac{2}{r} + \frac{\partial}{\partial r}\right)\langle \delta u_L \delta u_L \delta u_T\rangle \qquad (49)$$

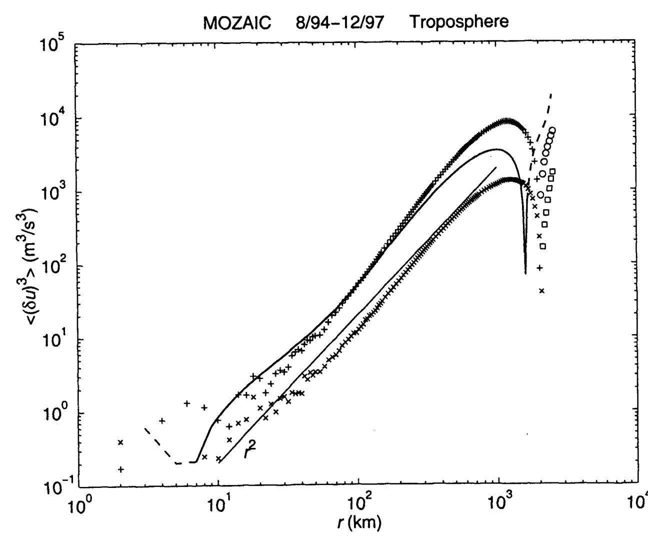

Figure 1. Sum of the off-diagonal third-order horizontal velocity structure functions for upper tropospheric data. The $\langle(\delta u_T)^3\rangle$ data are denoted by plusses (positive) and circles (negative). The $\langle\delta u_T(\delta u_L)^2\rangle$ data are denoted by crosses (positive) and squares (negative). The quantity $[2 + r(d/dr)]\langle\delta u_T(\delta u_L)^2\rangle$ is plotted as thick solid (positive) and dashed (negative) curves. The thin straight line depicts an r^2 dependency as a guide to the eye.

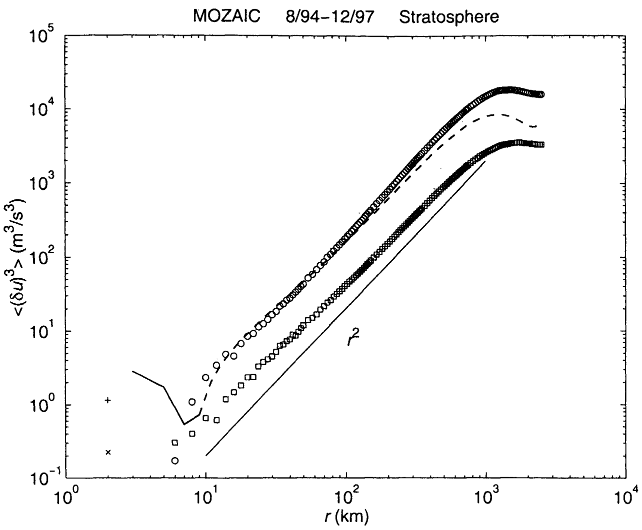

Figure 2. Same as Figure 1, except for lower stratospheric data.

to (42). We computed the second term by fitting the data points to an eighth-order polynomial and then performing the differentiation. The result was multiplied by r and plotted in Figures 1 and 2. It is a remarkable fact that these two terms tend to cancel each other in the region of $r = 100$ km. In the same range, the Coriolis term on the right-hand side grows with r. Hence the Coriolis term will be at least of the same order of magnitude as the advection term, from scales of the order of 100 km up to larger scales.

It may very well be the case that the main balance in (42) is between the Coriolis term and the pressure-related term, and thus

$$f(\langle \delta u_T \delta u_T \rangle - \langle \delta u_L \delta u_L \rangle) = n_i t_j D_{ij} \,. \qquad (50)$$

If this is the case, then the difference, $\langle \delta u_T \delta u_T \rangle - \langle \delta u_L \delta u_L \rangle$, between the transverse and longitudinal second-order structure functions, is determined by a kind of geostrophic balance equation at scales of the order of 100 km up to at least 1000 km, and probably at even larger scales. Since these functions have the same type of behavior, their sum is also determined by this equation. Therefore the kinetic energy spectrum would also be determined by the same equation in the corresponding wave number range. It may very well be the case that the origin of the k^{-3} spectrum [*Nastrom and Gage*, 1985] should not be explained by any direct effects of the nonlinear advection term, but rather as a result of a balance between the Coriolis and pressure terms. Geostrophic balance is generally believed to apply only for scales that are much larger than 100 km. However, here we have found evidence for such a balance at smaller scales, at least in a statistical sense. The strong latitudinal dependence of the second-order structure function in the troposphere also makes it very plausible that the Coriolis term is crucial for the understanding of the energy spectrum at scales of the order of 1000 km. However, since the pressure term is unknown, both from an analytical and experimental point of view, we have not been able to develop any theory that could predict the shape of the structure functions, or the kinetic energy spectrum, from the balance (50).

4. Conclusions

We have derived the general Kolmogorov equation for atmospheric mesoscale motions on an f plane. We have shown that there are no explicit Coriolis terms or gravitational terms in the flux equation. We have argued that terms describing effects of statistical inhomogeneities can be neglected, at least for $r < 100$ km. By assuming that the joint probability distribution of vertical and horizontal velocity increments is Gaussian, the flux equation could be integrated and compared with the measured horizontal third-order structure function in the lower stratosphere. A reasonable value of the spectral flux of kinetic energy, and thereby the mean dissipation, could be estimated from this curve.

To derive the flux relation and to compare it with the experimental data, no assumption was required regarding the details of the physical mechanism by which kinetic energy is transferred from large to small scales. However, the gravity wave cascade hypothesis seems reasonable. Our results are compatible with the description by *Bretherton* [1969] of nonlinear interaction between gravity waves, which tend to transfer energy to shorter and shorter wavelengths, propagating more and more slowly relative to the mean wind. The cascade is limited by the appearance of regions where the local Richardson number is sufficiently small for nonwavelike instabilities to grow, leading to transient patches of true 3-D turbulence.

For separations of the order of 1000 km, one must be very cautious in making predictions based on equations in which the terms describing statistical inhomogeneities have been neglected. Nevertheless, we think that (42), which is the off-diagonal part of the general Kolmogorov tensor equation, gives us a hint that direct effects from the Coriolis force should be taken into consideration, when attempting to explain the shape of the kinetic energy spectrum for scales of the order of 1000 km.

Acknowledgments. Financial support from the Swedish Research Council for Engineering Sciences is gratefully acknowledged by E.L. J.Y.N.C. was supported by NASA grant NAG1-2306. We would also like to thank the MOZAIC program for use of their data, especially Valérie Thouret and Alain Marenco. MOZAIC was funded in part by the European Communities (DG XII-C, DG XII-D) with strong support from Airbus Industrie and its partners, Air France, Lufthansa, Austrian Airlines, and Sabena.

References

Bretherton, F. P., Waves and turbulence in stably stratified fluids, *Radio Sci.*, *4*, 1279–1287, 1969.

Cadet, D., Energy dissipation within intermittent clear air turbulent patches, *J. Atmos. Sci.*, *34*, 137–142, 1977.

Charney, J. G., Geostrophic turbulence, *J. Atmos. Sci.*, *28*, 1087–1095, 1971.

Cho, J. Y. N., and E. Lindborg, Horizontal velocity structure functions in the upper troposphere and lower stratosphere, 1, Observations, *J. Geophys. Res.*, this issue.

Crane, R., A review of radar observations of turbulence in the lower stratosphere, *Radio Sci.*, *15*, 177–193, 1980.

Dewan, E. M., Stratospheric wave spectra resembling turbulence, *Science*, *204*, 832–835, 1979.

Dewan, E., Saturated-cascade similitude theory of gravity wave spectra, *J. Geophys. Res.*, *102*, 29,799–29,817, 1997.

Frisch, U., *Turbulence: The Legacy of A. N. Kolmogorov*, 296 pp., Cambridge Univ. Press, New York, 1995.

Gage, K. S., Evidence for a $k^{-5/3}$ law inertial range in mesoscale two-dimensional turbulence, *J. Atmos. Sci.*, *36*, 1950–1954, 1979.

Hocking, W. K., An assessment of the capabilities and limitations of radars in measurements of upper atmospheric turbulence, *Adv. Space Res.*, *17*, 37–47, 1996.

Hocking, W. K., and P. K. L. Mu, Upper and middle tropospheric kinetic energy dissipation rates from measurements of $\overline{C_n^2}$—Review of theories, in-situ investigations, and experimental studies using the buckland park atmospheric radar in australia, *J. Atmos. Solar-Terr. Phys.*, *59*, 1779–1803, 1997.

Kolmogorov, A. N., Dissipation of energy in the locally isotropic turbulence, English translation in *Turbulence and Stochastic Processes: Kolmogorov's Ideas 50 Years On,* edited by J. C. Hunt, O. M. Phillips, and D. Williams, pp. 15–17, Royal Soc. of London, London, 1941.

Kraichnan, R. H., Inertial ranges in two-dimensional turbulence, *Phys. Fluids*, *10*, 1417–1423, 1967.

Kraichnan, R. H., Inertial-range transfer in two- and three-dimensional turbulence, *J. Fluid Mech.*, *47*, 525–535, 1970.

Landau, L. D., and E. M. Lifshitz, *Fluid Mechanics*, 539 pp., Pergamon Press, New York, 1987.

Lilly, D. K., Stratified turbulence and the mesoscale variability of the atmosphere, *J. Atmos. Sci.*, *40*, 749–761, 1983.

Lilly, D. K., D. E. Waco, and S. I. Adelfang, Stratospheric mixing estimated from high-altitude turbulence measurements, *J. Appl. Meteorol.*, *13*, 488–493, 1974.

Lindborg, E., Can the atmospheric kinetic energy spectrum be explained by two-dimensional turbulence?, *J. Fluid Mech.*, *388*, 259–288, 1999.

Lindborg, E., and K. Alvelius, The kinetic energy spectrum of the two-dimensional enstrophy turbulence cascade, *Phys. Fluids*, *12*, 945–947, 2000.

Maltrud, M. E., and G. K. Vallis, Energy spectra and coherent structures in forced two-dimensional and beta-plane turbulence, *J. Fluid Mech.*, *228*, 321–342, 1991.

Nastrom, G. D., and F. D. Eaton, Turbulence eddy dissipation rates from radar observations at 5-20 km at White Sands Missile Range, NM, *J. Geophys. Res.*, *102*, 19,495–19,505, 1997.

Nastrom, G. D., and K. S. Gage, A climatology of atmospheric wavenumber spectra of wind and temperature observed by commercial aircraft, *J. Atmos. Sci.*, *42*, 950–960, 1985.

Smith, L. M., and V. Yakhot, Finite-size effects in forced two-dimensional turbulence, *J. Fluid Mech.*, *274*, 115–138, 1994.

Vinnichenko, N. K., Empirical studies of atmospheric structure and spectra in the free atmosphere, *Radio Sci.*, *4*, 1115–1126, 1969.

von Kármán, T., and L. Howarth, On the statistical theory of isotropic turbulence, *Proc. R. Soc. London*, *A164*, 192–215, 1938.

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(Received April 7, 2000; revised October 17, 2000; accepted November 7, 2000.)