Elastic sheets: Cracks by design

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Cracks by design:
Stretching membranes over curved substrates can be used to guide and control crack paths

Different methods exist to control fracture in thin media in order to produce some desired shape or curved edge. As children, many of us have experience with perforated paper, which allows the easy removal of arbitrary shapes by punching them out of a larger sheet of material. Here, pre-existing perforations simplify the process, because stress concentration about each slit causes emanating cracks from neighboring slits to merge. However, as scales change from the micron to the geological realm, the luxury of being able to pre-crack a specimen in a desired repetitive manner becomes unrealistic and more subtle techniques are needed to guide fracture. The study by Mitchell et al, explores the possibility of guiding crack paths by draping sheet specimens on top of substrates with non-zero Gaussian curvature. The out-of-plane elastic deformation this imposes causes an inhomogeneous stress distribution within the sheet. By introducing a small crack, the pre-load in the membrane can cause the crack to spontaneously grow. Depending on how the substrate geometry is chosen, the crack growth can be made to conform to some curved path and/or arrest after a desired length has been reached. This opens up the possibility of a new methodology for incising 2D shapes from sheets by way cracking them over a reversed engineered bumpy substrate surface.

The true complexity of this idea lies within the complexity of fracture mechanics. Predicting the curved path a crack will take in an inhomogeneous environment is a difficult problem, and few closed-form solutions have been found by analytical means [cite: Ghelichi and Kamrin, Soft Matter 2015; B. Cotterell and J. R. Rice, Slightly curved or kinked cracks, Int. J. Fract., 1980, 16, 155–169.; Y. Sumi, S. Nemat-Nasser and L. M. Keer, On crack branching and curving in a finite body, Int. J. Fract., 1983, 21, 67–79.]. Fracture growth and crack turning are phenomena driven by elastically singular stress fields in the vicinity of a crack tip. These fields develop as part of a global stress field, which must be modified continuously as a crack grows. To consider crack guidance by way of stretching over a curved surface, one must determine how draping a cracked membrane over such a surface influences the stress intensity at the crack tips and how this influence varies as the crack grows.

Mitchell et al consider multiple means, both qualitative and quantitative, to address this challenge. In so doing, the work also provides a survey of current experimental, numerical, and analytical methods in applied fracture mechanics. The key questions being asked are: How can substrate curvature be used to arrest or spontaneously grow a crack, and how does curvature influence the direction in which a growing crack bends?

From a qualitative standpoint, the answer lies in the behavior of an elastic curvature potential, which can be computed from the substrate geometry. It determines the

pressure field corresponding to an uncracked sheet draped over the substrate. The potential can be used to understand several basic features of how cracks will behave when introduced. Backed by experiments with PDMS sheets conforming to positive and negative Gaussian curvature substrates, the authors confirm that positive Gaussian curvature tends to increase stretching, which increases the potential, and causes higher stress intensity when cracks are introduced. Generally speaking, the higher the Gaussian curvature, the smaller a crack needs to be to grow spontaneously when introduced. The gradient of the curvature potential also gives an indication of how cracks will turn as they grow.

For a more precise description of the influence of the substrate geometry on the crack path, the authors modified an analytical technique based on perturbation from a straight growth path [cite: Coterrell and Rice]. They were able to obtain predictions for kink angle and path curvature that match their experiments up to a certain distance of crack growth, beyond which the perturbation analysis loses validity.

To model longer crack paths, the authors consider a phase-field numerical approach as well as a discrete spring-lattice model. Phase field modeling in particular is a subject of much current interest in fracture mechanics [cite: Karma, A., Kessler, D. A. & Levine, H. Phase-field model of mode III dynamic fracture. Phys. Rev. Lett. 87, 045501 (2001); Spatschek, R., Brener, E. & Karma, A. Phase field modeling of crack propagation. Philos. Mag. 91, 75–95 (2011).] for its ability to simulate the growth of crack paths without the task of representing singular fields ahead of the crack tip. This is achieved by introducing a phase field, which acts like a damage variable that softens material continuously to a vacuum state depending on the local energy density and a penalty based on the phase field's own curvature. A thin domain of fully damaged media then represents the crack, and, moving away from the crack, the phase field variable smoothly ramps back to representing an undamaged elastic media over a distance regularized by the curvature penalty. Using the phase-field approach, the authors were able to simulate fracture in elastic sheets draped over various types of substrates, demonstrating the ability to "protect" zones of material by placing them over a bump or sequence of bumps, and to control the direction of a crack path by confining it between rows of bumps [show Figure 5cd from paper].

The authors have shown the promise of using substrate curvature as a novel means to affect the growth or directionality of cracks. The key next step would be to convert these ideas into a design algorithm, where one inputs a desired crack path and computes what substrate geometry is needed to achieve that path. It is a challenging but not insurmountable inverse problem, which could lead to new techniques for punching out arbitrary two-dimensional shapes over a wide range of scales.