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Ultimate capacity of linear time-invariant bosonic channels with additive Gaussian noise

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ABSTRACT

Fiber-optic communications are moving to coherent detection in order to increase their spectral efficiency, i.e., their channel capacity per unit bandwidth. At power levels below the threshold for significant nonlinear effects, the channel model for such operation—a linear time-invariant filter followed by additive Gaussian noise—is one whose channel capacity is well known from Shannon’s noisy channel coding theorem. The fiber channel, however, is really a bosonic channel, meaning that its ultimate classical information capacity must be determined from quantum-mechanical analysis, viz. from the Holevo-Schumacher-Westmoreland (HSW) theorem. Based on recent results establishing the HSW capacity of a linear (lossy or amplifying) channel with additive Gaussian noise, we provide a general continuous-time result, namely the HSW capacity of a linear time-invariant (LTI) bosonic channel with additive Gaussian noise arising from a thermal environment. In particular, we treat quasimonochromatic communication under an average power constraint through a channel comprised of a stable LTI filter that may be attenuating at all frequencies or amplifying at some frequencies and attenuating at others. Phase-insensitive additive Gaussian noise—associated with the continuous-time Langevin noise operator needed to preserve free-field commutator brackets—is included at the filter output. We compare the resulting spectral efficiencies with corresponding results for heterodyne and homodyne detection over the same channel to assess the increased spectral efficiency that might be realized with optimum quantum reception.

Keywords: Ultimate capacity, spectral efficiency, linear time-invariant bosonic channel, additive Gaussian noise

1. INTRODUCTION

Determining the ultimate limits on reliable data-transmission rates for noisy communication channels is one of the most fundamental tasks in information theory. The mathematical foundation for determining such limits was set by Shannon in his seminal work, which introduced the notion of channel capacity as the maximum rate at which error-free communication is possible over a communication channel.¹ However, information is necessarily encoded in a physical system, and at optical frequencies noise sources of quantum mechanical origin impact the ultimate rate of reliable communication. It is therefore important to determine the information-carrying capacities of noisy quantum communication channels and also to establish means by which these limits can be approached in practical systems.

For optical communication systems—for instance, ones based on fiber or free-space propagation—bosonic channels provide a quantum model of communication in which the modes of the electromagnetic field are used as physical information carriers.^{2,3} The rate of reliable information transmission through such communication channels is significantly impacted by measurement statistics of the receiver used to extract the encoded information. In particular, conventional optical communication receivers, i.e., those that employ direct, homodyne, or heterodyne detection, have different capacities, owing to their different measurement statistics. Direct detection has superior photon efficiency (many bits/photon),⁴ hence it is the preferred choice for photon-starved applications like the Lunar Laser Communication Demonstration.⁵ Homodyne and heterodyne detection, however, offer better spectral efficiency (many bits/sec-Hz),⁶ so they are employed to maximize throughput in the

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Internet's fiber backbone. In fact, fiber-optic communications are moving to coherent detection for increased spectral efficiency,⁷ and it is germane to explore whether it is possible to approach the ultimate limits of reliable transmission with these standard detection techniques.

The ultimate capacity of a bosonic channel is its Holevo-Schumacher-Westmoreland (HSW) capacity,⁸ which will equal or exceed those of conventional systems. Until recently, the only bosonic channel whose HSW capacity was known was the pure loss channel, in which any attenuation between the transmitter and the receiver was accompanied by the minimum (vacuum-state) noise level needed to preserve the Heisenberg uncertainty principle.⁹ Now, however, with the proof of the minimum output entropy conjecture,^{10,11} the HSW capacities are known for the single-mode phase-insensitive Gaussian channels such as the thermal noise, additive noise, and quantum amplifier channels. These results were later extended to obtain the capacities of physical communication channels that are affected by nonzero memory and Gaussian noise.¹² However, the memory model used in Ref. 12—a cascade of identical discrete beam-splitters or amplifiers—is insufficiently general to include the quantum model of the archetypal communication channel from classical information theory, viz., transmission through a general continuous-time, linear-time invariant (LTI) filter followed by additive statistically-stationary Gaussian noise.

In this paper, we remedy the preceding omission by determining the HSW limit on spectral efficiency of an average-power constrained, quasimonochromatic bosonic channel comprised of a stable LTI filter—which, at any particular frequency, may be attenuating or amplifying—followed by additive phase-insensitive Gaussian noise arising from a thermal environment. We evaluate the HSW capacities of such channels by decomposing them into a set of parallel channels with additive Gaussian noise, and we compare the results so obtained with the corresponding capacities when coherent (homodyne or heterodyne) detection is used in lieu of optimal quantum reception.

2. LINEAR, TIME-INVARIANT BOSONIC CHANNEL

2.1 Bosonic channels

Bosonic channels play a very significant role in modeling optical communication channels that rely on fiber or free space propagation. In general, a K mode bosonic channel can be represented by K quantized modes of the electromagnetic field in a tensor-product Hilbert space $\mathcal{H}^{\otimes K} = \otimes_{k=1}^K \mathcal{H}_k$ with K pairs of input and output bosonic field operators $\{\hat{a}_k^{\text{in}}, \hat{a}_k^{\text{out}} : k = 1, 2, \dots, K\}$. For instance, at the single-mode level, attenuating and amplifying channels can be represented by bosonic channels whose channel input is an electromagnetic field mode with photon annihilation operator \hat{a}_{in} , and the resulting channel output is another field mode whose photon annihilation operator \hat{a}_{out} is given by the commutator-preserving transformations

$$\hat{a}_{\text{out}} = \begin{cases} \sqrt{\eta} \hat{a}_{\text{in}} + \sqrt{1-\eta} \hat{a}_{\text{env}}, & \text{attenuating channel} \\ \sqrt{\kappa} \hat{a}_{\text{in}} + \sqrt{\kappa-1} \hat{a}_{\text{env}}^\dagger, & \text{amplifying channel,} \end{cases} \quad (1)$$

where $0 < \eta \leq 1$ is the attenuating channel's transmissivity, $1 < \kappa < \infty$ is the amplifying channel's gain, and \hat{a}_{env} is the photon annihilation operator corresponding to an environmental-noise mode. The thermal-noise channel is an attenuating channel whose \hat{a}_{env} mode is in a thermal state, i.e., an isotropic Gaussian mixture of coherent states with average photon number $N_{\text{env}} > 0$:

$$\hat{\rho}_{\text{env}} = \int d^2\alpha \frac{\exp(-|\alpha|^2/N_{\text{env}})}{\pi N_{\text{env}}} |\alpha\rangle\langle\alpha|. \quad (2)$$

The amplifying channel's \hat{a}_{env} mode injects minimal quantum noise when it is in its vacuum state, but, more generally, it too could be in a thermal state given by (2).

2.2 LTI channel model with additive, phase-insensitive Gaussian noise

At power levels below the threshold for significant nonlinear effects, the fiber-optic communication channel is a bosonic channel whose ultimate classical information capacity must be determined from quantum-mechanical analysis, viz. from the HSW theorem.^{13,14}

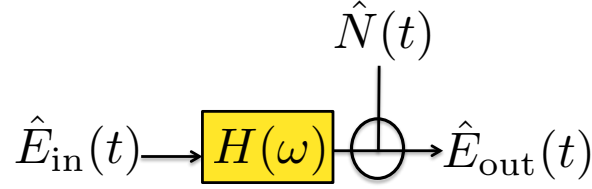


Figure 1. Schematic diagram of the transmission of a baseband field operator through an LTI filter—with frequency response $H(\omega)$ at detuning ω from the optical carrier frequency ω_0 —and additive, statistically-stationary, phase-insensitive Gaussian noise $\hat{N}(t)$ with noise spectrum $S_N(\omega) = \int d\tau \langle \hat{N}^\dagger(t + \tau) \hat{N}(t) \rangle e^{-i\omega\tau}$.

The quantum model from which the fiber channel's HSW capacity can be derived is schematically shown in Fig. 1 for quasimonochromatic operation that is subject to an average power constraint. Here, $\hat{E}_{\text{in}}(t)$ and $\hat{E}_{\text{out}}(t)$ are the baseband $\sqrt{\text{photons/s}}$ field operators at the channel's input and output, respectively, both having the commutator relation $[\hat{E}_J(t), \hat{E}_J^\dagger(u)] = \delta(t - u)$, for $J = \text{in, out}$. The positive-frequency input and output field operators are thus $\hat{E}_{\text{in}}(t)e^{-i\omega_0 t}$ and $\hat{E}_{\text{out}}(t)e^{-i\omega_0 t}$, where ω_0 is the optical carrier frequency. The input-output relation can be written as $\hat{E}_{\text{out}}(t) = \int d\tau \hat{E}_{\text{in}}(\tau)h(t - \tau) + \hat{N}(t)$, where $h(t)$ is the baseband channel's impulse response—assumed to be causal and stable—and $\hat{N}(t)$ is a baseband noise operator needed to ensure the appropriate field-commutator relations. The frequency response $H(\omega)$ of the LTI filter $H(\omega) = \int dt h(t)e^{i\omega t}$ provides the Fourier-domain version of the above input-output relation

$$\hat{\mathcal{E}}_{\text{out}}(\omega) = \hat{\mathcal{E}}_{\text{in}}(\omega)H(\omega) + \hat{\mathcal{N}}(\omega), \text{ where } \hat{\mathcal{E}}_J(\omega) = \int dt \hat{E}_J(t)e^{i\omega t}, \text{ for } J = \text{in, out.} \quad (3)$$

Here, $\hat{\mathcal{N}}(\omega)$ is the Fourier transform of the noise operator $\hat{N}(t)$ satisfying the commutator relation

$$[\hat{\mathcal{N}}(\omega), \hat{\mathcal{N}}^\dagger(\omega')] = 2\pi\delta(\omega - \omega') (1 - |H(\omega)|^2). \quad (4)$$

2.3 Amplification and attenuation in LTI bosonic channels

The LTI filter described above can represent, depending on the magnitude of the frequency response $H(\omega)$, both the attenuation and amplification processes for optical communication. Let us define two regimes for the detuning frequencies: $\omega \in \Omega_{\text{att}}$, for which the filter is attenuating ($|H(\omega)| \leq 1$); and $\omega \in \Omega_{\text{amp}}$, for which the filter is amplifying ($|H(\omega)| > 1$). Note that Eq. (3) takes forms similar to those in Eq. (1) for both of these cases. We shall assume that the channel represented by such an LTI filter has the *minimum-noise* associated with the quasimonochromatic operation at temperature TK , in which case $\hat{N}(t)$ can be taken to be in a zero-mean, stationary Gaussian state with the phase-insensitive correlation function

$$R_N(\tau) = \langle \hat{N}^\dagger(t + \tau) \hat{N}(t) \rangle = \int \frac{d\omega}{2\pi} S_N(\omega) e^{i\omega\tau}, \quad (5)$$

where

$$S_N(\omega) = \begin{cases} \frac{1 - |H(\omega)|^2}{e^{\hbar\omega_0/kT} - 1} & \text{for } \omega \in \Omega_{\text{att}} \\ \frac{|H(\omega)|^2 - 1}{1 - e^{-\hbar\omega_0/kT}}, & \text{for } \omega \in \Omega_{\text{amp}}, \end{cases} \quad (6)$$

with k being Boltzmann's constant, follows from the Planck law for thermally-distributed photons in the quasimonochromatic limit wherein $e^{\hbar(\omega_0 + \omega)/kT} \approx e^{\hbar\omega_0/kT}$. The quasimonochromatic condition will apply when $H(\omega)$ is narrowband, in comparison with ω_0 , such as would be the case for a dense wavelength-division multiplexing (DWDM).^{16–18} Thus, in our numerical work we will consider the fourth-order Butterworth filter, for which $|H(\omega)| = H_0/[1 + (\omega/\omega_c)^8]$, where $0 < H_0 \leq 1$ is an attenuating filter, $H_0 > 1$ is an amplifying filter, and $\omega_c \ll \omega_0$ enforces the quasimonochromatic condition on the channel filter that will imply a similar quasimonochromatic constraint on $\hat{E}_{\text{in}}(t)$'s capacity-achieving excitation spectrum.

2.4 Fourier mode-decomposition of the LTI bosonic channel

We now introduce a discretization based on transmitting a stream of T_s -sec long continuous-time symbols that are bracketed by ΔT_s -sec long guard bands. Specifically, we will assume that the input field $\hat{E}_{\text{in}}(t)$ is only in a non-vacuum state when $|t - n(T_s + \Delta T_s)| \leq T_s/2$, for integer n . Likewise, after offsetting the receiver's clock by the filter's group delay, we will assume that the receiver only measures the output field $\hat{E}_{\text{out}}(t)$ when $|t - n(T_s + \Delta T_s)| \leq T_s/2$, for integer n . By taking T_s to greatly exceed the filter's bandwidth $\omega_c/2\pi$, we can choose a fixed ΔT_s large enough to ignore intersymbol interference while maintaining $\Delta T_s \ll T_s$.¹⁹ It follows that we can focus our attention on a single n value in our discretization, so we will use the $n = 0$ operator-valued Fourier series representations,

$$\hat{E}_{\text{in}}(t) = \sum_k \hat{a}_k^{\text{in}} \frac{\exp(-i2\pi kt/T_s)}{\sqrt{T_s}}, \text{ for } |t| \leq T_s/2, \text{ and } \hat{E}_{\text{out}}(t) = \sum_k \hat{a}_k^{\text{out}} \frac{\exp(-i2\pi kt/T_s)}{\sqrt{T_s}}, \text{ for } |t| \leq T_s/2, \quad (7)$$

to obtain

$$\hat{a}_k^{\text{out}} = H(2\pi k/T_s) \hat{a}_k^{\text{in}} + \hat{n}_k, \text{ where } \hat{N}(t) = \sum_k \hat{n}_k \frac{\exp(-i2\pi kt/T_s)}{\sqrt{T_s}}, \text{ for } |t| \leq T_s/2. \quad (8)$$

The statistics for $\hat{N}(t)$ given in the previous section, together with the high time-bandwidth condition $T_s \omega_c/2\pi \gg 1$, imply that the noise operator's Fourier series is also its Karhunen-Loève series, so that the $\{\hat{n}_k\}$ are in a product state that is Gaussian, zero-mean, and completely characterized by $\langle \hat{n}_k^\dagger \hat{n}_j \rangle = S_N(\omega_k) \delta_{kj}$, where $\omega_k = 2\pi k/T$ and δ_{kj} is the Kronecker delta function.

The LTI bosonic channel we are considering can thus be decomposed into a set of parallel channels¹⁹ with operator-valued input-output pairs $\{(\hat{a}_k^{\text{in}}, \hat{a}_k^{\text{out}})\}$, where the output is related to the the input via Eq. (8). In the next section, we show that this decomposition in turn allows us evaluate the information capacities for optimum reception as well as coherent (homodyne or heterodyne) detection over LTI bosonic channels with Gaussian noise.

3. CAPACITIES OF LTI BOSONIC CHANNELS

3.1 Bosonic channel capacities

For the last few decades, extensive efforts have been put into determining the ultimate limits on the reliable communication through the bosonic channels. Shannon's noisy channel coding theorem showed that the classical capacity of a classical channel is the maximum mutual information between its input and output states over all encoding and decoding strategies. However, the quantum nature of single-mode attenuating and amplifying bosonic channel means that their classical information capacities must be found from the HSW theorem, by maximizing over both the transmitted quantum states and the receiver's quantum measurement. Consider a set of symbols $\{x\}$ that is represented by a collection of input states $\{\hat{\rho}_x\}$, and assume that these states are selected according to some prior distribution $\{p_x\}$. A quantum channel can, in general, be represented by a completely-positive-trace-preserving (CPTP) map (say \mathcal{M}), and the Holevo information $\chi(\mathcal{M})$ for this channel is given by

$$\chi(\mathcal{M}) = S\left(\sum_x p_x \hat{\rho}_x\right) - \sum_x p_x S(\hat{\rho}_x), \quad (9)$$

where $S(\hat{\rho})$ is the von Neumann entropy of a state $\hat{\rho}$. The classical capacity of this channel is

$$C = \sup_n \left(\max_{\{p_x, \hat{\rho}_x\}} [\chi(\mathcal{M}^{\otimes n})] / n \right). \quad (10)$$

The regularization step—the supremum over n channel uses—is necessary because it is not known, in general, whether the Holevo information is additive.

For a single-mode pure-loss bosonic channel, when the transmitter is constrained to use at most N_S photons on average per channel use, the capacity is given by $g(\eta N_S)$, where $g(x) \equiv (x+1) \log_2(x+1) - x \log_2(x)$ is the

von Neumann entropy of a bosonic thermal state with average photon number x , and η is the transmissivity of the channel.⁹ Moreover, this capacity has been shown to be achievable with an isotropic Gaussian encoding over coherent states, and it exceeds what is achievable with the coherent (homodyne and heterodyne) detection, namely, $C_{\text{hom}} = \frac{1}{2} \log_2(1 + 4\eta N_S)$, $C_{\text{het}} = \log_2(1 + \eta N_S)$. It is also known that heterodyne detection is asymptotically capacity-achieving, for the pure-loss channel, as $N_S \rightarrow \infty$. With the proof of the long-standing minimum output entropy conjecture last year,^{10,11} the following HSW capacities for the single-mode thermal noise and amplifying channels are now also known

$$C(\mathcal{M}_{\text{therm}}) = g(\eta N_S + (1 - \eta)N_{\text{env}}) - g((1 - \eta)N_{\text{env}}), \quad (11)$$

$$C(\mathcal{M}_{\text{ampl}}) = g(\kappa N_S + (\kappa - 1)(N_{\text{env}} + 1)) - g((\kappa - 1)(N_{\text{env}} + 1)), \quad (12)$$

where N_{env} is the average photon number of the environmental noise mode.

3.2 Capacity and spectral efficiency of LTI bosonic channels

To evaluate the HSW capacities and the spectral efficiencies, along with their homodyne and heterodyne counterparts, for the LTI thermal-noise attenuating and amplifying channels, we proceed by considering their parallel channel decomposition as described in Sec. 2. With this decomposition of the LTI bosonic channels with additive Gaussian noise, the discretized capacity problem is then reduced to maximizing the Holevo information subject to the average photon-flux constraint *

$$\frac{1}{T_s + \Delta T_s} \sum_k \bar{n}(\omega_k) \leq P, \quad (13)$$

where $\bar{n}(\omega_k) = \langle \hat{a}_k^{\text{in}\dagger} \hat{a}_k^{\text{in}} \rangle$. The results of Refs. 11,12 imply that the discretized-channel's HSW capacity is achieved by coherent-state encoding. For such encoding, the Holevo information rate (in bits/sec) is

$$\chi(P) = \frac{\sum_k \{g[|H(\omega_k)|^2 \bar{n}(\omega_k) + S_N(\omega_k)] - g[S_N(\omega_k)]\}}{T_s + \Delta T_s}, \quad (14)$$

and the constrained maximization of $\chi(P)$ can be accomplished by a Lagrange multiplier technique, as was done for the multiple-spatial-mode, broadband, pure-loss channel,¹⁵ and for the beam-splitter and amplifier cascade channels.¹² Passing to the limit $T_s \rightarrow \infty$, with ΔT_s fixed then yields the LTI channel's HSW capacity (in bits/sec):

$$C_{\text{HSW}}(P) = \int \frac{d\omega}{2\pi} \{g[|H(\omega)|^2 \bar{n}(\omega) + S_N(\omega)] - g[S_N(\omega)]\}, \quad (15)$$

where $S_N(\omega)$ is given by Eq. (6) and the average photon-number distribution is

$$\bar{n}(\omega) = \max \left\{ \left[(e^{\beta/|H(\omega)|^2} - 1)^{-1} - S_N(\omega) \right] / |H(\omega)|^2, 0 \right\}, \quad (16)$$

where the Lagrange multiplier β is chosen to saturate the photon-flux bound

$$\int \frac{d\omega}{2\pi} \bar{n}(\omega) \leq P. \quad (17)$$

The spectral efficiency (in bits/sec-Hz), i.e., the capacity per unit bandwidth ω_c , can be evaluated from the above HSW capacity,²⁰ and in our case we have,

$$C_{\text{HSW,SE}}(P) = \frac{2\pi}{\omega_c} \int \frac{d\omega}{2\pi} [g(|H(\omega)|^2 \bar{n}(\omega) + |1 - |H(\omega)|^2| S_N(\omega)) - g(|1 - |H(\omega)|^2| S_N(\omega))], \quad (18)$$

*Because $\hat{E}_{\text{in}}(t)$'s excitation is quasimonochromatic, this constraint limits the average power to at most $\hbar\omega_0 P$.

subject to the photon-flux constraint.

The homodyne and heterodyne spectral efficiencies—to which we will compare the preceding HSW limit on the spectral efficiency—presume coherent-state encoding. Hence their capacities are well known, because homodyne and heterodyne measurements convert the Fig. 1 model into classical LTI channels with additive Gaussian noise. In particular, assuming unity homodyne and heterodyne efficiencies, the homodyne channel corresponding to Fig. 1 has an input that is a real-valued, classical, photon-units field $E_{\text{in}}^{\text{hom}}(t)$ and an output that is a real-valued, classical, photon-units field $E_{\text{out}}^{\text{hom}}(t)$. The homodyne channel's input-output relation is then

$$E_{\text{out}}^{\text{hom}}(t) = \int d\tau E_{\text{in}}^{\text{hom}}(\tau)h(t - \tau) + N_{\text{hom}}(t), \quad (19)$$

where $N_{\text{hom}}(t)$ is a stationary, zero-mean, real-valued Gaussian random process with spectral density $S_{N_{\text{hom}}}(\omega) = (2S_N(\omega) + 1)/4$ [†]. The corresponding channel model for heterodyne detection has complex-valued, classical, photon-units input and output fields that are related by

$$E_{\text{out}}^{\text{het}}(t) = \int d\tau E_{\text{in}}^{\text{het}}(\tau)h(t - \tau) + N_{\text{het}}(t), \quad (20)$$

where $N_{\text{het}}(t)$ is a stationary, zero-mean, isotropic, complex-valued Gaussian random process with spectral density $S_{N_{\text{het}}}(\omega) = (S_N(\omega) + 1)/2$. Standard Shannon theory results now lead to the following homodyne and heterodyne capacities (in bits/sec)¹⁹

$$C_{\text{hom}}(P) = \int \frac{d\omega}{2\pi} \frac{1}{2} \log_2 \left(1 + \frac{\bar{n}_{\text{hom}}(\omega)|H(\omega)|^2}{S_{N_{\text{hom}}}(\omega)} \right), \quad (21)$$

where

$$\bar{n}_{\text{hom}}(\omega) = \max(\beta_{\text{hom}}/2 - S_{N_{\text{hom}}}(\omega)/|H(\omega)|^2, 0), \quad (22)$$

with the Lagrange multiplier β_{hom} chosen to give

$$\int \frac{d\omega}{2\pi} \bar{n}_{\text{hom}}(\omega) = P, \quad (23)$$

and

$$C_{\text{het}}(P) = \int \frac{d\omega}{2\pi} \log_2 \left(1 + \frac{\bar{n}_{\text{het}}(\omega)|H(\omega)|^2}{2S_{N_{\text{het}}}(\omega)} \right), \quad (24)$$

where

$$\bar{n}_{\text{het}}(\omega) = \max(\beta_{\text{het}} - 2S_{N_{\text{het}}}(\omega)/|H(\omega)|^2, 0), \quad (25)$$

with the Lagrange multiplier β_{het} chosen to give

$$\int \frac{d\omega}{2\pi} \bar{n}_{\text{het}}(\omega) = P. \quad (26)$$

The corresponding spectral efficiencies for the homodyne and heterodyne detection are given by

$$C_{\text{hom,SE}}(P) = \frac{2\pi}{\omega_c} \int \frac{d\omega}{2\pi} \log_2 \left(1 + \frac{\bar{n}_{\text{het}}(\omega)|H(\omega)|^2}{2S_{N_{\text{het}}}(\omega)} \right), \quad (27)$$

$$C_{\text{het,SE}}(P) = \frac{2\pi}{\omega_c} \int \frac{d\omega}{2\pi} \log_2 \left(1 + \frac{\bar{n}_{\text{het}}(\omega)|H(\omega)|^2}{2S_{N_{\text{het}}}(\omega)} \right). \quad (28)$$

From Eqs. (22) and (25), it is apparent that the capacity achieving photon-flux spectra for homodyne and heterodyne detection have “water filling” interpretations, e.g., the capacity-achieving photon flux for homodyne

[†]Here, without appreciable loss of generality, we have assumed that the impulse response $h(t)$ is real valued.

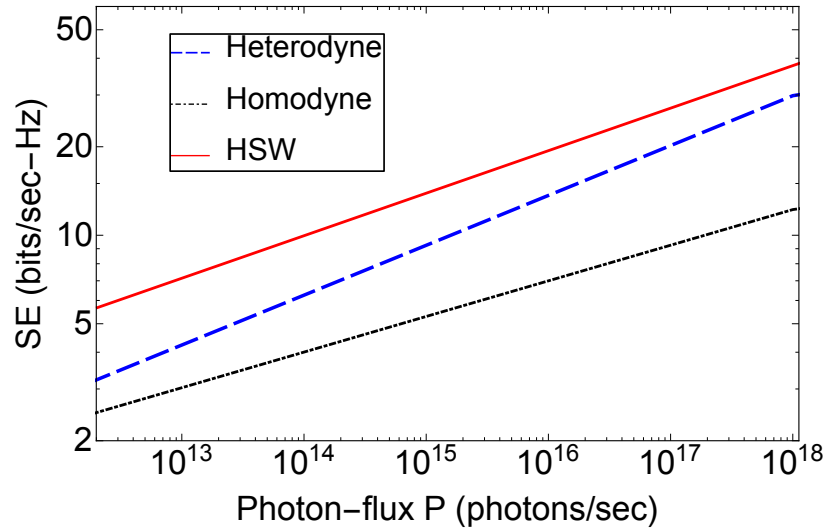


Figure 2. Spectral efficiencies (SEs) (in bits/sec-Hz) for optimum quantum reception, homodyne, and heterodyne detection plotted versus the transmitted photon flux for an attenuating DWDM filter modeled by a fourth-order Butterworth filter, for which $|H(\omega)| = H_0/[1 + (\omega/\omega_c)^8]$ with peak value $H_0 = 0.1$ and bandwidth $\omega_c/2\pi = 40$ GHz. These plots assume ω_0 corresponding to a $1.55 \mu\text{m}$ center wavelength and $T = 300$ K thermal environment.

detection is allocated across detuning frequencies keeping $\bar{n}(\omega) + S_{N_{\text{hom}}}(\omega)/|H(\omega)|^2$ constant while satisfying Eq. (23).^{19,21}

In Fig. 2, we compare the spectral efficiency for optimum quantum reception with those for homodyne and heterodyne detection as functions of the transmitted photon flux. These curves assume an attenuating DWDM filter modeled by a fourth-order Butterworth filter, for which $|H(\omega)| = H_0/[1 + (\omega/\omega_c)^8]$. We find that the heterodyne capacity exceeds the homodyne capacity over the entire a broad range of photon fluxes. This occurs because their capacities increase linearly with increasing bandwidth but increase only logarithmically with increasing signal-to-noise ratio, implying that heterodyne detection's factor-of-two bandwidth advantage over homodyne detection overwhelms the latter's factor-of-two maximum SNR advantage over the former. Furthermore, at high photon-flux levels, the spectral efficiency of heterodyne detection approaches the HSW limit.

4. CONCLUSIONS

We have presented a quantum-mechanical model for optical communication through LTI bosonic channels with additive Gaussian noise and have reported a framework for evaluating the ultimate limits on reliable communication over such channels. These bosonic channels, operating in the thermal environment, can represent the effects of quantum amplification or attenuation. Our work provides tools that enable comparison of optimum reception spectral efficiencies for representative DWDM filters—with or without amplification—with the corresponding results for homodyne and detection. Although motivated by the fiber application, these results are applicable to all LTI channels with additive, phase-insensitive Gaussian noise, so they can be applied to unguided channels as well.

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