

**Growing Pains in Financial Development: Institutional Weakness  
and Investment Efficiency**

by  
**Daniel Green**

Submitted to the Sloan School of Management  
in partial fulfillment of the requirements for the degree of  
Master of Science in Management Science Research

and  
**Ernest Siyuan Liu**

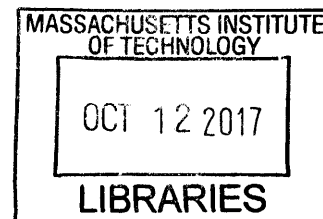
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## Abstract

There is little evidence that the expansion of microfinance has reduced poverty, but is instead increasingly associated with problematic multiple borrowing at high interest rates and high levels of debt and default. We develop a model that rationalizes these outcomes—if entrepreneurs cannot commit to exclusive borrowing from a single lender, expanding financial access by introducing multiple lenders may severely backfire. Capital allocation is distorted away from the most productive uses. Entrepreneurs choose inefficient and limited-growth endeavors. These problems are exacerbated when borrowers have access to more lenders, explaining why increased access to finance does not always improve outcomes.

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# Contents

1	Introduction . . . . .	9
2	Model . . . . .	14
3	Equilibrium . . . . .	16
4	Comparative Statics . . . . .	23
5	Endogenous Project Choice . . . . .	28
6	Policy Implications . . . . .	30
7	Conclusion . . . . .	33
	References . . . . .	51



# List of Figures

1	Timing of the Model . . . . .	14
2	The Lending Market Game at $s = 1$ . . . . .	15
3	Static Representation of Lending Market Equilibrium . . . . .	22
4	Expected Investment in the Dynamic Lending Game . . . . .	27
5	Expected Investment and Expected Debt . . . . .	28
6	Endogenous Project Choice . . . . .	30



# Growing Pains in Financial Development: Institutional Weakness and Investment Efficiency

## 1 Introduction

The innovation of microfinance has long been regarded as the singular promising approach to large-scale poverty reduction, based on the belief that expanding credit among the poor would promote entrepreneurial activity and in turn lead to growth. However, enthusiasm for micro lending has begun to erode. While it has spread to over 100 countries and now serves more than 130 million borrowers (Microcredit Summit Campaign), there has been little systematic evidence that microfinance has a significant impact on poverty alleviation. Randomized control trials of microcredit expansions find little impact on borrower income and business size (Banerjee, Karlan, and Zinman, 2015). Perhaps more surprisingly, cash grant experiments show that, despite the existence of microfinance institutions, micro-entrepreneurs still *effectively* face severe credit constraints as demonstrated by very high marginal returns to capital. (de Mel, McKenzie, and Woodruff, 2008).

Why has microfinance failed to live up to its promise? In this paper, we shed light on this question by developing a model that captures two empirical regularities of the industry’s rapid expansion—an increasing number of lenders available in a given market, and that entry of these overlapping lenders largely preceded the development and strengthening of the institutions that are thought to facilitate effective financial contracting. While such lender saturation or competition is normally associated with well functioning credit markets, we find that in the presence of weak credit market institutions, expanding financial access by introducing multiple sources of credit may severely backfire. Capital allocation can be distorted away from the most productive use, which in turn generates perverse incentive for micro-entrepreneurs to choosing the least ambitious production technology.

Empirically, lender saturation and weak contracting environments are increasingly associated with the narrative of a *multiple borrowing crisis* (e.g. see Faruque and Khalily, 2011). The quintessential example of multiple borrowing is the spectacular boom and bust of the microfinance

industry the Indian state of Andhra Pradesh. A nascent industry in the 1990s, allegedly thousands of lenders entered the market to supply credit to nearly the entire state, much of it in the form of “overlapping” loans from many lenders to the same borrower.<sup>1</sup> Borrowers accumulated large debt balances from many lenders, who individually had no way of observing or controlling a borrower’s total indebtedness. This ultimately proved unsustainable and culminated in a default crisis and near-collapse of the industry in 2010. Fear and realization of such multiple borrowing crisis have also arisen in Peru, Guatemala, Bolivia (de Janvry, Sadoulet, McIntosh, Wydick, Luoto, Gordillo, and Schuetz 2003), Uganda (McIntosh, de Janvry, and Sadoulet 2005), and Bangladesh (Faruquee and Khalily 2011) following the rapid entry of new microfinance lenders.

In each of these cases, lenders, policymakers, and academics have all noted the importance of a specific implication of institutional weakness—the inability of borrowers to commit to an exclusive relationship with a single lender. Such commitment requires effective institutions that allow lenders can verify exclusivity (credit registries) and enforce punishments for any deviations (courts). If borrowers are unable to commit to a single lender *ex-ante*, then *ex-post* commitment induced by low marginal returns to further borrowing will be valuable in obtaining agreeable financing terms and facilitating investment. This logic suggests a nuanced relationship between financial development and resource allocation. Our model formalizes this insight. When borrowers cannot commit to exclusive contracts, better projects can receive less investment, borrowers may choose to forgo the most productive investments in favor of economically inferior alternatives, and that the severity of these distortions is increasing in the number of lenders available to borrowers. Together, these results outline a new explanation of why increased access to finance does not always improve aggregate outcomes.

In the model, an entrepreneur visits multiple lenders to obtain funds for a new investment opportunity. Crucially, the entrepreneur lacks the ability to commit to exclusive borrowing from a single lender and cannot write loan contracts that are contingent on the terms of contracts subsequently signed with other lenders. The entrepreneur faces an uncertain cost of default that is realized when loans come due, and defaults if the debt owed exceeds this cost. Thus, the more debt owed the less likely it is to be repaid. This gives rise to an externality between lenders. New lenders willingly provide additional investment that existing lenders would not, because new lenders do not internalize the decreased likelihood of repayment of *existing* debt when pricing new debt contracts. Rational lenders anticipate this additional borrowing and offer loan terms that compensate them for it, making multiple borrowing undesirable *ex-ante*, but without commitment unavoidable *ex-post*. Inability to commit to an exclusive lending relationship is thus a binding constraint and in equilibrium induces distortions in lending and investment outcomes.

The economics of the commitment externality highlighted above were first demonstrated in the

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<sup>1</sup>At the peak in 2009, surveys estimated that 84% of rural villagers in Andhra Pradesh had loans from multiple lenders, including 58% borrowing from at least four lenders. See Johnson and Meka (2010), Taylor (2011) and references therein for a collection of facts and anecdotes about multiple borrowing in Andrah Pradesh.

seminal work of Arnott and Stiglitz (1991, 1993) and applied to credit market settings by Bizer and DeMarzo (1992). To help build intuition for how the commitment externality operates in our setting, we show that their main findings, that lack of commitment increases interest rates and debt accumulation, also hold in the investment context of our dynamic model. These implications of lack of commitment echo the observed high levels of debt and high interest rates in microfinance markets where borrowers can obtain loans from many lenders simultaneously but contracting sophistication is limited.

We then show that distortions to investment induced by commitment problems depend crucially on the nature of the new investment opportunity the entrepreneur is attempting to finance, and find that more is not always better. More efficient opportunities may receive less investment and face higher interest rates, and the most promising investment opportunities may not be undertaken at all. These results together introduce a novel microfoundation for the relationship between credit market imperfections and aggregate resource misallocation.

At the heart of these results is that lower marginal returns create *endogenous commitment power* not to borrow (much) from additional lenders—the benefits of marginal investment return are not as attractive relative to the cost of higher debt obligations. Since entrepreneurs cannot commit *ex-ante* to limit inefficient future borrowing, properties of their future marginal returns to investment that induce them to limit such borrowing *ex-post* are especially valuable. This induces a trade-off between an investment technology’s efficiency and concavity. Low marginal (and hence average) returns are bad for output, but declining marginal returns are good for incentives. When marginal returns are high, borrowers who lack commitment not to continue borrowing may end up on the wrong side of the “debt Laffer curve,” meaning they are receiving an amount of investment that could have been supported by a lower quantity of debt, if only they could have committed to it. Financial constraints are thus endogenously more severe for better projects when they also have sufficiently higher marginal returns.

This relationship between financial constraints and productivity generates a particularly stark form of misallocation: a negative correlation between the *level* of investment in a project and its productivity. In the presence of commitment problems, projects that warrant the most investment end up receiving the least. This is consistent with a growing body of evidence from micro and experimental studies conducted in developing economies that productive firms may be especially credit constrained. McKenzie and Woodruff (2008) study microenterprise in Mexico and use a randomized experiment to estimate returns to investment capital. In addition to finding very high returns on average, they also find a positive relationship between the returns to investment capital of borrowers and the financial constraints they face, indicating that it is the projects with the best economic fundamentals that are most under-served. Similarly, Banerjee and Duflo (2014), using policy changes in credit access programs in India, find similar evidence of a negative selection effect. In their paper, the ordinary least squares relationship between loan growth and profit growth has a lower magnitude than the causally estimated specification, which they conclude implies that it is the

least productive firms that are acquiring the most credit. Our paper shows that such misallocation can be explained by commitment problems in credit markets.

Next, we endogenize project choice by the entrepreneur to reveal the full extent of the misallocative forces of multiple borrowing and lack of commitment. Not only can lack of commitment cause better projects to receive less investment, but it can also cause entrepreneurs to reject the most profitable projects entirely in favor of less efficient endeavors. When commitment problems are severe, constrained entrepreneurs choose business plans that have low prospects for expansion precisely because these are the easiest to finance. Those who instead choose better projects will receive low levels of funding at high interest rates, reducing their ability to accumulate equity to finance expansion.

Turning to policy, we show that optimal policy in the face of commitment problems strongly mirrors India's regulatory response to the Andhra Pradesh crisis. In 2011, the Reserve Bank of India imposed new regulations on the banking industry and stated that they were in part meant to address multiple borrowing. A debt limit (USD 790) and interest rate cap (26%) were imposed.<sup>2</sup> The Malegam Report (2011) also recommended borrowers could only have loans from at most two microfinance lenders simultaneously, but this was not implemented in the final regulations. Each of these policies works by explicitly limiting the impact of lack of commitment in lending markets. However, these policies are all imperfect. If borrowers could commit, or incentivize themselves to commit through contingent contracting, the externalities that arise between lenders would disappear and outcomes would improve, including the efficiency of investment allocation across projects.

The emergence of exactly this sort of contingent contracting in sophisticated financial markets thus further substantiates the idea that commitment distortions are important. In the syndicated loan market in the United States and Europe, widely employed "performance pricing" covenants allow interest rates to vary based on changes in observable firm characteristics that occur *after* the loan has been issued. Making interest rates increasing in a firm's total amount of debt, as the majority of these covenants dictate, internalizes the spillovers between lenders and restores the full-commitment lending outcomes, even when borrowers cannot explicitly agree to exclusive borrowing.<sup>3</sup> Of course, implementing such contracts requires lenders can observe violations (for example through credit registries) and contract on them (for example through courts).

Theoretically, our model is based on the theme of common agency, which describes environments in which multiple principals with possibly conflicting interests act to influence the behavior of a single agent. Bernheim and Whinston (1986) formalize the notion of common agency and obtain results on the efficiency of the incentive structures required to implement the equilibrium action and the optimality of the action that prevails in equilibrium. Segal (1999) generalizes many common agency models in a unifying framework and shows that when contracting is not publicly observable

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<sup>2</sup>It was also stated that at least 75% of the total loans originated by any regulated MFI should be for the purpose of income generation. (Reserve Bank of India (2011))

<sup>3</sup>See Asquith, Beatty, and Weber (2005) for a detailed description of performance based pricing covenants.

inefficiencies occur when there are externalities at the efficient outcomes because the agent cannot compensate the principals for the externalities they impose on each other. In our setup, the principals can be thought of as the multiple lenders, each with a limited ability to control total borrowing of the borrower, the agent.

Early applied work on common agency is a collection of papers by Arnott and Stiglitz (1991, 1993), which study common agency in insurance markets. These papers recognize that additional insurance providers can impose externalities on each other through moral hazard of the buyer. Bizer and DeMarzo (1992) bring this idea to credit markets.<sup>4</sup> They study a model in which a borrower with a desire to smooth consumption between two periods can sequentially visit lenders to obtain loans backed by the borrower’s stochastic next-period income, which is influenced by non-contractible effort choice. As in the Arnott-Stiglitz series of papers, moral hazard generates an externality that new lenders offer loans that harm previous lenders by decreasing their expected profits. More recently, the “maturity rat race” of Brunnermeier and Oehmke (2013) highlights that commitment externalities can also arise in the context of maturity structure choice. There, it is the inability to commit to maturity structure, not the level of debt, that generates problems. Our main contribution to this literature is that we are the first to link commitment problems in credit markets to investment decisions. We show that commitment externalities cause better projects to receive less investment and encourage entrepreneurs to choose projects with low growth potential and rapidly diminishing returns to scale. Thus, our model provides a rationale for the failure of microfinance to make a significant impact on poverty and suggests that microfinance can only be successful if it is backed by the high quality institutions that support intermediation in the developed world.

It is also important to emphasize that our paper relates to competition in credit markets only through competition’s exacerbation of commitment externalities. We focus on the fact that easier access to additional lenders makes it harder to enforce exclusive borrowing and thus ex-post commitment mechanisms, such as selecting projects with limited returns to scale, are valuable. This channel is independent of the relationship between competition and market power in the context of imperfect credit markets, which has been studied in other papers. Of this work, the most related to our paper is Parlour and Rajan (2001), which shows that commitment problems prevent lender entry from competing away all lender profits. Also related is the theory of relationship lending developed by Petersen and Rajan (1994, 1995). This theory suggests that when borrowers cannot commit to long relationships with a single lender, limited market power of lenders erodes their ability to subsidize lending to early stage firms and recoup expenses later through rent extraction.

The rest of the paper is organized as follows. Section 2 describes our model and section 3 defines and solves for the equilibrium. In section 4 we show the implications of lack of commitment in credit markets with multiple lenders on interest rates and default rates, and derive our main

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<sup>4</sup>Other papers studying lack of commitment in credit markets include Bisin and Rampini (2006), Kahn and Mookherjee (1998), Parlour and Rajan (2001), Petersen and Rajan (1995) and Boot and Thakor (1994).

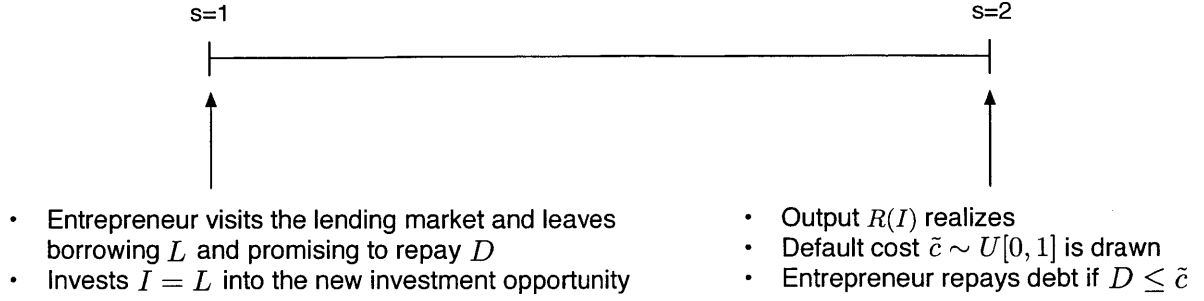


Figure 1: Timing of the Model

result that entrepreneurs with more attractive investment opportunities can endogenously face more severe credit constraints and thus invest less than those with inferior projects. We illustrate the implication on inefficient project choice in section 5. We next examine, in section 6, policies adopted by regulatory agencies to address the multiple borrowing problem through the lens of our model. Section 7 concludes. Proofs of all propositions and lemmas are relegated to the Appendix.

## 2 Model

### Overview and Timing

An entrepreneur attempts to finance a new investment opportunity with the constraints that it cannot commit to exclusive borrowing from a single lender and that there is limited enforcement of debt repayment, ie. the borrower only repays if the costs of default are high enough. The model has two stages ( $s = 1, 2$ ) and contains two sets of agents: a single entrepreneur and an infinite sequence of potential lenders. All parties are risk neutral and have no discounting. The entrepreneur is endowed with a *variable-scale* investment opportunity that returns  $R(I)$  deterministically at  $s = 2$  for any  $I \geq 0$  invested at  $s = 1$ . While the returns to this investment opportunity are observable, they are not perfectly pledgeable. After the projects returns are realized, the entrepreneur learns of its cost of defaulting on its debt, and only repays if the default cost exceeds the amount of debt owed. Specifically, we assume default costs  $\tilde{c}$  are drawn from a distribution  $F_{\tilde{c}}$ . Debt  $D$  is repaid if  $\tilde{c} > D$ , and we denote the probability of repayment as  $p(D) \equiv 1 - F_{\tilde{c}}(D)$ , which is of course a decreasing function of  $D$ . Figure 1 summarizes the timing of the model.

We model the entrepreneur raising capital from the lending market at  $s = 1$  as an infinite horizon dynamic game of complete information in which the entrepreneur sequentially visits a potentially infinite number of lenders. All lenders are risk neutral and do not discount between  $s = 1$  and  $s = 2$ . Their opportunity cost of funds is the risk-free rate, which we normalize to one throughout the paper. Upon meeting a lender, the entrepreneur makes a take-it-or-leave-it offer for a simple

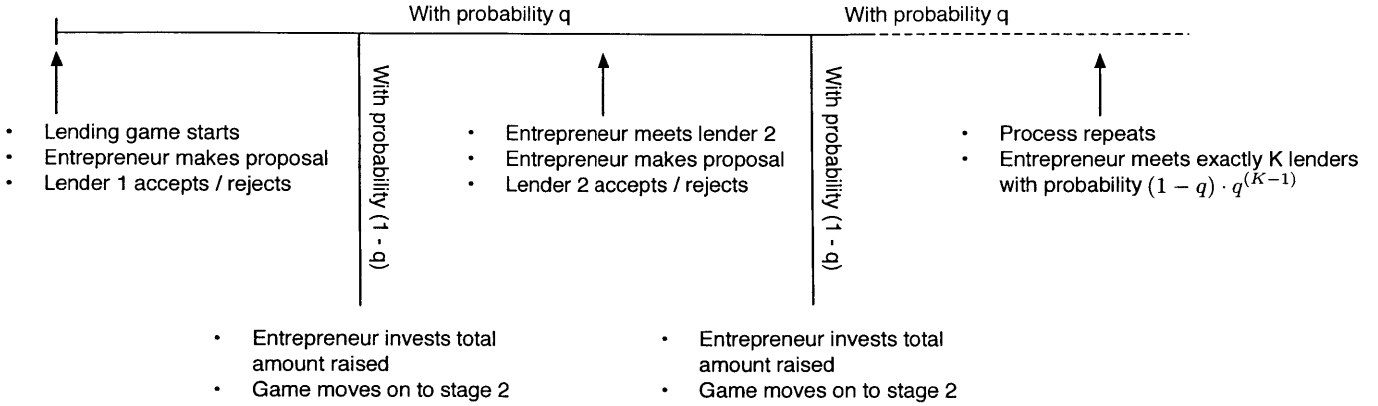


Figure 2: The Lending Market Game at  $s = 1$

debt contract  $(L_i, D_i)$  specifying the amount borrowed  $L_i$  and the promised repayment amount  $D_i$ . The lender, who can observe the history of the entrepreneur's borrowing from previous lenders, chooses whether to accept or reject this offer, and if accepted the funds are exchanged.

At this point, the entrepreneur loses access to the lending market with probability  $1 - q$ , with  $q \in [0, 1)$ . Otherwise, the entrepreneur meets a new lender and the process described above is repeated until eventually access to the lending market is lost or the last lender is visited. By assumption, no lender is visited more than once. After losing access to the lending market, the entrepreneur invests the aggregate financing it raised in the new project, and the model progresses to stage  $s = 2$ . Figure 2 provides a graphical exposition of the lending market game of stage  $s = 1$ .

A brief discussion of the important aspects of the model is in order. The crucial feature of the model is that the entrepreneur cannot credibly commit ex-ante to avoid borrowing from subsequent lenders it meets. More broadly, debt contracts cannot be contingent on the terms of other debt contracts written with subsequent lenders. This form of contractual incompleteness precisely reflects, in our view, important features of the institutional and contracting environments in which we think misallocation and suboptimal technology choice are most salient.

The assumption of sequential borrowing from a possibly large number of banks need not be taken literally. What drives our results is that lack of commitment to an exclusive lending relationship leaves room for the externalities between lenders and perversely affects equilibrium outcomes. Our results also hold when the entrepreneur borrows simultaneously, rather than sequentially, from a finite number of lenders where loan terms cannot be made contingent on the other loans taken by the borrower. However, such a model is analytically less tractable and subject to the standard critique in the simultaneous contracting literature that the results are sensitive to the specification of agents' off-equilibrium beliefs (Segal and Whinston (2003)).

We formulate the endogenous choice of default via a random default cost that is realized ex-

post. The formulation is similar to a random shock to an outside option, as in Aguiar, Amador, Hopenhayn, and Werning (2016), and reflects the fact that forces outside of the model generate ex-ante indeterminacy in the ex-post costs of defaulting on debt. Weak institutions are a salient driver of such indeterminacy; in the Andra Pradesh default crisis, repayment rates plummeted from near-perfect to near-zero almost overnight when grandstanding local politicians urged borrowers to stop repaying their debts. In the appendix, we provide a microfoundation of the random default through moral hazard.

The parameter  $q$  exogenously limits the amount of commitment power borrowers can obtain in the lending market. The closer  $q$  is to zero, the less likely it is that contingencies arise in which early lenders can be exploited by further borrowing. There are several appealing interpretations of this parameter. First,  $q$  can be thought of as inversely related to the difficulty or cost of subverting commitment, and reflects the quality of the contracting environment. As  $q \rightarrow 0$  the contracting environment is able to perfectly enforce contingency in loan terms. When  $q \rightarrow 1$  borrowers can costlessly find new lenders to provide marginal lending. A second view is that  $q$  reflects the composition of search frictions in the lending market and the limited time an entrepreneur has to raise money for an investment opportunity. Under this interpretation one would assume a complete inability to conduct contingent contracting or exclusive borrowing—the severity of the commitment problem is determined by the market structure of lending and time preference for funding.

### 3 Equilibrium

To begin solving the model, we introduce two simplifying assumptions which aide greatly in the exposition of the model and in highlighting the underlying economic forces generating our results.

**Assumption 3.1.** *The return on the new investment opportunity is linear and productive:*

$$R(I) = \alpha I \text{ and } \alpha > 1.$$

**Assumption 3.2.** *Default costs for the entrepreneur are uniformly distributed between zero and one.*

$$\bar{c} \sim U[0, 1].$$

Linearity in returns to the investment opportunity means that the level of investment financing obtained from prior lenders is not directly relevant for the objective of the borrowers or lenders at any stage of the lending game, greatly simplifying the model. We restrict to *efficient* linear projects ( $\alpha > 1$ ) to rule out trivial cases in which there is no demand for borrowing or investment. The uniform distribution of default costs between 0 and 1 normalizes the borrower's maximum debt capacity to one and generates simple expressions for properties of debt repayment. The probability of repayment is now simply  $p(D) = 1 - D$  and the expected debt servicing cost (either repayment

or strategic default) is  $\mathbb{E} [\min (\tilde{c}, D)] = D - D^2/2$ .

## The Full Commitment Case

We now characterize the solution to the model when borrower can commit to visiting a single lender and use this as a benchmark to illustrate how lack of commitment affects outcomes in the model. The ability of the entrepreneur to commit to borrowing from a single lender can be captured by setting  $q = 0$ , since this ensures it is common knowledge that the borrower will visit exactly one lender.

The above model now simplifies substantially. The entrepreneur makes a take-it-or-leave-it offer  $(D, L)$  to the lender and invests  $I = L$  in the new investment opportunity. The lender accepts any loan offer that is weakly profitable in expectation. The optimal lending contract is found by maximizing the entrepreneur's welfare subject to the lender participation constraint. Denote this problem  $P^{SL}$  for the "single lender" problem:

$$\begin{aligned} (P^{SL}) \quad & \max_{D, I} R(I) - \mathbb{E} [\min (D, \tilde{c})] \\ \text{s.t.} \quad & I \leq p(D) D \quad (\text{lender's participation constraint}) \end{aligned}$$

**Proposition 3.1.** *Under Assumptions 1 and 2, the solution to the single lender problem  $P^{SL}$  is characterized by the condition*

$$\alpha \times \left[ p(D^{SL}) + p'(D^{SL}) D^{SL} \right] = p(D^{SL}) \quad (1)$$

At the optimum, the costs and benefits of pledging an additional dollar of face value of debt must be equalized. The marginal cost is the increase in expected debt repayment costs associated with the additional borrowing, which is simply the probability of actually repaying the marginal dollar pledged.<sup>5</sup> The gain from pledging an additional dollar of face value of debt is the marginal return to investment times the marginal investment that can be raised from pledging an additional dollar of face value of debt. Because extra repayment reduces the value of all debt claims (through limited enforcement), the value of *existing* debt falls when new debt is issued. Because in this example there is only a single lender, the lender internalizes the change in value of all existing debt coming from the marginal issuance of new debt, and thus can make zero profits by supplying  $p(D) + p'(D) D$  of investment for a marginal dollar of promised repayment. We now highlight the externality that arises when the entrepreneur can obtain loans from more than one lender.

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<sup>5</sup>One must also consider the decrease in expected repayment probability and the increase in expected default costs conditional on default. However, these terms exactly offset each other—at the margin the borrower is simply replacing some physical repayment with increased default costs.

## Illustrating the Commitment Problem

When  $q > 0$  the borrower cannot obtain the same loan contracts from the first lender that it could when  $q = 0$ . The first lender now anticipates the possibility that the borrower will obtain additional debt from future lenders that dilutes the value of its original debt claims and thus cannot profitably accept the same loan terms as in the full commitment case. To illustrate why this problem arises, imagine the borrower can borrow sequentially from two lenders, but cannot commit to deal exclusively with one lender or to make terms of either loan conditional on those of the other. Therefore, loan terms with the second lender are arranged to maximize the surplus to the borrower and the second lender taking the terms of the first loan as fixed.

In this example the second lender imposes an externality on the first whenever the first lender has issued a positive amount of debt. To see this, denote the face value of debt promised to the first lender by  $d_1$  and the face value of debt promised to the second lender by  $d_2$  and define  $D \equiv d_1 + d_2$  to be the total face value of debt. Subject to the same regularity conditions as above, taking aggregate debt  $D$  as given the following first order equation characterizes  $d_2$ , the amount of debt issued to the second lender.

$$\alpha \cdot [p(D) + p'(D) \cdot d_2] = p(D). \quad (2)$$

This is identical to Equation 1 except that the term in square brackets is different. Here, the second lender only internalizes the change in the value of its own debt due to its marginal debt issuance at the optimum, not the change in the total value of debt issued. Thus the second lender imposes an externality on the first. When borrowers lack commitment lenders must anticipate future borrowing and charge higher interest rates that compensate them for value they expect to lose. This is exactly why the commitment problem changes lending outcomes in our model and is the key intuition behind the results in Bizer and DeMarzo (1992) that multiple lenders lead to higher interest rates and higher borrowing than would be obtained if commitment were possible. Now that we have clearly illustrated why lack of commitment changes outcomes we turn to solving the model and showing exactly how lack of commitment affects investment outcomes and technology choice.

## Equilibrium without Commitment

We now turn to studying the full model without commitment. Our solution concept is Markov Perfect Equilibrium (MPE) in which the borrower's strategy is a function only of a single state variable  $D$ , the cumulative face value of debt the issued so far in the game, and the lender's strategy is a function of  $D$  as well as the current loan proposed by the borrower. Modeling borrowing as sequential, rather than simultaneous, allows us to avoid issues of the sensitivity of our results to off-equilibrium beliefs. As we show at the end of this section, the equilibrium we identify captures the limiting outcome of the unique subgame perfect equilibria of finite-lender versions of the model

as the number of lenders grows large. We now provide a formal definition of a Markov Perfect Equilibrium in the lending market game.

**Definition 0.1.** *A Markov Perfect Equilibrium (MPE) of the infinite lender lending market game is a set of borrower and lender strategies that are mutual best responses at every subgame when subject to the following constraints.*

- a) The entrepreneur's strategy when encountering any lender is a mapping from how much it has already pledged to repay,  $D$ , to a simple debt contract  $(L_i, D_i)$ .
- b) All lenders' strategies are represented by functions mapping from the state variable  $D$  and the loan contract proposed by the entrepreneur to the lender's decision to either accept or reject the proposal.

We begin characterizing the equilibrium of this game by exploring the best responses of each player.

### Lender Best Responses

Lenders have rational expectations of future borrowing and thus only accept contracts that yield non-negative expected returns taking the strategies of the borrower and other lenders as given. A lender receiving a loan proposal must, given the state of the game, evaluate the expected profit from the loan taking into account the strategies of the borrower and future lenders and the likelihood that the borrower will be able to meet these lenders at all. Specifically, define  $D' \equiv D + D_i$ , and denote  $\tilde{p}_i(D')$  to be lender  $i$ 's perceived probability that the entrepreneur will not default on a loan  $(L_i, D_i)$  to this lender, given the borrower has previously obtained total face value of debt  $D$ . Such a loan is weakly profitable in expectation if  $L_i / D_i \leq \tilde{p}_i(D')$ . Since lenders are risk neutral and do not discount stage 2 cash flows, lender  $i$ 's optimal strategy is to accept the loan offer if and only if it is weakly profitable. We focus on a symmetric equilibrium in which lenders' best response can be characterized by a function  $\tilde{p}(\cdot)$  such that  $\tilde{p}_i(\cdot) = \tilde{p}(\cdot)$  for all lenders  $i$ .

### Borrower Best Response

Conditional on meeting a lender, and given lender strategies represented by  $\tilde{p}(\cdot)$ , the borrower makes a loan offer that maximizes its expected continuation utility, taking into account the chances of being able to meet more lenders and obtaining further marginal borrowing. Solving for the entrepreneur's best response function thus involves a dynamic optimization problem. Taking lender pricing as given the entrepreneur forms strategies that maximize its continuation utility at each value of the state variable  $D$ .

The borrower knows that lenders will accept any loan they expect to be profitable, so to maximize its own utility it will only offer loans that lenders expect to make exactly zero profits. Thus given the total face value of debt borrowed so far  $D$ , the borrower only proposes loans  $(L_i, D_i)$  that

satisfy  $L_i = \tilde{p}(D') D_i$ , where again  $D' \equiv D + D_i$ . Upon meeting any lender, if the entrepreneur has cumulative debt  $D$  and leaves the lender with cumulative debt  $D'$  then the maximum amount of new investment the lender would provide is given by  $\tilde{p}(D') [D' - D]$ . The lender expects the loan to be repaid with probability  $\tilde{p}(D')$ , in which case it will be return  $D' - D$ . Thus, taking loan pricing as given the entrepreneur solves:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) - (1 - q) \mathbb{E} [\min(D', \tilde{c})] + qV(D') \quad (3)$$

Conditional on arriving at a new lender having already issued face value of debt  $D$  the entrepreneur optimally chooses  $D'$ , the new total face value of debt it will have issued after contracting with this lender. The first term on the right hand side of Equation 3 is the marginal payoff from the new investment opportunity associated with obtaining additional investment  $\tilde{p}(D') (D' - D)$ . With probability  $1 - q$  the entrepreneur loses access to the lending market and either repays debt  $D'$  or defaults and pays the default cost  $\tilde{c}$  if it is lower than the cost of repayment. Finally, with probability  $q$  the entrepreneur does not lose access to the lending market and will receive continuation utility  $V(D')$  from future borrowing. Plugging in for  $\mathbb{E} [\min(D', \tilde{c})]$  given default costs  $\tilde{c}$  are uniform from zero to one gives:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) - (1 - q) \left( D' - \frac{D'^2}{2} \right) + qV(D') \quad (4)$$

Denote a policy function that solves the dynamic programming problem in Equation 4 by  $g(D)$ . Conditional on arriving to a new lender with aggregate face value of debt  $D$ , the borrower will leave with aggregate face value of debt  $D' = g(D)$ , having proposed a new additional loan  $(\Delta D, \Delta L)$  with  $\Delta D \equiv g(D) - D$  and  $\Delta L \equiv \tilde{p}(g(D)) (g(D) - D)$ . On path, following this strategy generates a sequence of total aggregate face values of debt that have been accumulated up to a given lender, conditional on the lending market progressing that far:  $\{g(0), g(g(0)), g^3(0), \dots\}$ . The aggregate face value of debt obtained in the lending market is thus a random variable  $D^{agg}$  that realizes a particular value of this sequence depending on how many lenders the borrower is able to visit before the lending game ends.

## Equilibrium Characterization

Given the discussion of borrower and lender best responses above, it is clear that a MPE of the model is equivalent to a solution a fixed point dynamic programming problem. Given lender strategies (captured by  $\tilde{p}(\cdot)$ ), the borrower's optimal strategy is represented by a policy function  $g(\cdot)$  that solves the dynamic programming problem in Equation 4. Given the distribution of total aggregate face value of debt  $D^{agg}$  induced by  $g(\cdot)$ , each lender forms rational expectations over the probability the borrower will repay, denoted by  $\tilde{p}(D') = \mathbb{E} [p(D^{agg}) | D']$ . A set of strategies forms

a MPE if lender strategies given by  $\tilde{p}(\cdot)$  are rational given  $g(\cdot)$ , and these borrower strategies are optimal given lender strategies embodied in  $\tilde{p}(\cdot)$ . We now summarize this characterization of an MPE in the following Theorem.

**Proposition 3.2.** *A symmetric Markov Perfect Equilibrium of the lending game is characterized by functions  $\tilde{p}(\cdot)$  and  $g(\cdot)$  that map from cumulative debt level  $D \in [0, 1]$  to the interval  $[0, 1]$  such that:*

- a)  $g(\cdot)$  is the policy function in the solution to the dynamic programming problem in Equation 4 taking  $\tilde{p}(\cdot)$  as given.
- b) Lenders' perceived expected repayment probabilities  $\tilde{p}(\cdot)$  used to form accept/reject strategies are correct taking  $g(\cdot)$  as given.

$$\tilde{p}(D) = \mathbb{E}[p(D^{agg}) | D] = 1 - \mathbb{E}[D^{agg} | D]$$

### Closed Form Solution

We now solve for the unique linear symmetric MPE in closed form. Notice that if  $\tilde{p}(D)$  were linear then the dynamic programming problem would have a linear-quadratic form, so we look for an equilibrium where  $\tilde{p}(D)$  and  $g(D)$  are linear functions of  $D$ . The following lemma characterizes the form of the solution to this problem.

**Lemma 3.1.** *For  $\alpha > 1$  and  $1 > q \geq 0$ , There exists  $\ell^*(\alpha, q) \geq 1$  and  $b^*(\alpha, q) \geq 1$  such that the unique linear MPE takes the following form:*

$$\begin{aligned}\tilde{p}(D) &= (1 - D) \cdot \frac{1}{\ell^*} \\ 1 - g(D) &= (1 - D) \cdot \frac{1}{b^*}\end{aligned}$$

where  $\ell^*$  and  $b^*$  respectively parameterize lender and borrower strategy in equilibrium.

There is an intuitive interpretation for both  $\ell^*$  and  $b^*$ . First, if the borrower arrives to a lender with current debt  $D$ , its remaining debt capacity is  $1 - D$ .<sup>6</sup> When leaving this lender the borrower will have pledged a total of  $g(D)$  and thus the borrower will have remaining debt capacity  $1 - g(D)$ . Therefore  $\frac{1}{\ell^*}$  is the fraction of current borrowing capacity that remains after visiting a lender. A higher  $b^*$  (or lower  $\frac{1}{b^*}$ ) corresponds to more aggressive borrowing by the borrower; it will deplete its available debt capacity more rapidly. Given  $g(D) > D$ , in equilibrium the entrepreneur borrows a positive amount from each lender it gets to visit, no matter how much it has already borrowed.

Second, recall that with commitment, the repayment probability is exactly  $\tilde{p}(D) = p(D) =$

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<sup>6</sup>If the borrower acquired additional debt of more than  $1 - D$  it would default on all debt for sure. Recall default costs  $\tilde{c} \sim U[0, 1]$ , so if the aggregate face value of debt equals or exceeds one the debt will never be repaid, as it will be less costly to default on it no matter what realization of default costs occur.

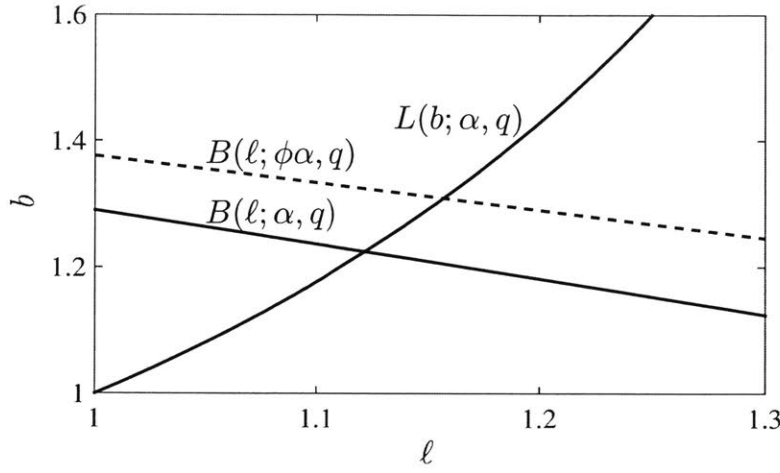


Figure 3: Static Representation of Lending Market Equilibrium

$\phi > 1$  indicates an increase in marginal returns to new investment. This shifts the borrower's loan aggressiveness best response curve upward but does not change the lender best response curve, which is only a function of  $q$ .

$1 - D$ , which corresponds to  $\ell^* = 1$ . Thus  $\ell^* > 1$  corresponds to lower expected repayment probability and higher interest rates. In other words, when  $\ell^*$  is high the lenders makes pricing decisions *as if* they expect the borrower to accumulate substantially more debt from future lenders, which deteriorates the value of the current lender's own claims, and set interest rates to reflect this.

Finally, there is also a static interpretation for the dynamic equilibrium. Taking lender's strategy parameterized by  $\ell$  as given (which determines  $\tilde{p}(D)$  and the interest rates), the entrepreneur chooses how much debt to issue when meeting each lender. This specifies a best response  $b = B(\ell)$  for the entrepreneur, which can be thought of as representing a loan demand schedule. Since higher interest rates induce the borrower to take out debt less aggressively, the loan demand schedule is downward-sloping ( $B'(\ell) < 0$ ). On the other hand, given the aggressiveness of entrepreneur's borrowing behavior parameterized by  $b$ , the lenders are able to determine the distribution of the face value of total borrowing. This maps into the distribution of the value of their own debt claims, and lenders set their decision rule such that they only accept loans they expect to be weakly profitable, and thus specifies a best response  $\ell = L(b)$  as the loan supply schedule. The interest rates lenders have to charge in order to break-even increases as the entrepreneur takes out loans more aggressively, hence the loan supply schedule is upward-sloping ( $L'(b) > 0$ ). The unique equilibrium is then the unique intersection of the "loan demand" equation  $B(\ell)$  and the "loan supply" equation  $L(b)$ , which is depicted in Figure 3.

## Equilibrium Choice

We choose to focus on the unique linear symmetric MPE in our infinite lender model because it is closely related to the unique SPE of finite lender version of our game. Because the number of lenders in any real world credit market is finite, our infinite lender model and the linear MPE solution concept is merely an abstraction that comes with the benefits of algebraic tractability.

Formally, we define the finite  $N$ -lender game by modifying our infinite lending game as follows. After meeting the  $i$ -th lender, the borrower gets to meet  $(i + 1)$ -th lender with probability  $q$  if and only if  $i + 1 \leq N$ , and with probability zero otherwise. That is, we truncate the game at a maximum of  $N$  lenders, while keeping the stochastic nature of the lending game unchanged for the first  $N$  lenders. Note that the finite  $N$ -lender game admits a unique SPE which can be solved by backward induction. The following proposition demonstrates that the unique linear symmetric MPE in the infinite lender game can be obtained as the limit of the sequence of the unique SPEs of the finite lender games as we take the number of lenders to infinity.

**Proposition 3.3.** *Fix  $i$ . In the finite  $N$ -lender game with  $N > i$ , as  $N \rightarrow \infty$  the strategies at lender  $i$  in the unique SPE converge uniformly to the corresponding linear symmetric MPE strategy of the infinite-lender game.*

## 4 Comparative Statics

We now proceed by highlighting the role of commitment in equilibrium investment outcomes. First, because the outcome of the lending game depends on the random number of lenders the borrower meets, we will focus our attention on *expected* outcomes, such as the expected face value of debt, the expected level of investment, the expected aggregate interest rate, and expected welfare. Due to the linear strategies in the game these concepts have simple closed form expressions. We first explore comparative statics of these outcomes to the model parameters  $q$  and  $\alpha$ , and then turn to broader implications outside the model of how these forces can endogenously affect project choice by tightening financial constraints.

### 4.1 Comparative Statics of Commitment

How does the exogenous severity of the commitment problem  $q$  impact equilibrium outcomes? The expected number of lenders a borrower visits is  $\frac{1}{1-q}$  which increases with  $q$ . Therefore a higher  $q$  can be seen as a worsening of the commitment problem. The following result echoes what has been experienced in the microfinance industry by a broad range of developing markets: the availability of lenders is associated not only with multiple borrowing, but also higher levels of debt, higher interest rates, and lower repayment rates.

**Proposition 4.1.** *The following results describe the equilibrium effects of increasing access to additional lenders, as parameterized by  $q$ , for any  $\alpha > 1$  and  $q > 0$  :*

- a) Expected aggregate face value of debt ( $\mathbb{E}[D^{agg}]$ ) increases in  $q$ .*
- b) Expected investment ( $\mathbb{E}[I^{agg}]$ ) decreases in  $q$ .*
- c) Expected probability of default ( $\mathbb{E}[p(D^{agg})]$ ) increases in  $q$ .*
- d) The ex-ante expected interest rate ( $\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}]$ ) increases in  $q$ .*
- e) Ex-ante welfare of the entrepreneur decreases in  $q$ , and in the limit as  $q \rightarrow 1$  welfare converges to zero, the level that would be obtained if the entrepreneur does not have access to the lending market at all.*

As  $q$  increases the expected face value of debt raised in equilibrium increases. This is the net effect of two forces. First, for higher  $q$  the borrower's demand aggressiveness curve shifts down, meaning the borrower issues a smaller fraction of its debt capacity at each round of financing for a given interest rate strategy. This is because higher  $q$  means it is more likely that the borrower will be able to exploit the externality future lenders impose and obtain favorable pricing on subsequent loans. In effect, when  $q$  is higher the borrower wants to smooth borrowing over multiple lenders to obtain better aggregate financing terms. From the lender perspective, for a given borrowing strategy a higher  $q$  increases lenders' expectations of debt dilution, causing them to raise interest rates. This further reduces borrower demand aggressiveness. However, the net reduction in aggressiveness of debt accumulation is dominated by the increasing likelihood that more loans will be realized.

In other words, while lenders respond to concerns about debt dilution by raising interest rates, without commitment this cannot prevent increased expected borrowing. As summarized in Proposition 4.1, increasing  $q$  also increases the effective interest rate and probability of default, while expected investment falls. The possibility of multiple borrowing makes it harder to use available debt capacity to fund new investment.

Entrepreneur welfare (and hence total surplus) is also declining in  $q$ . In fact, as  $q \rightarrow 1$ , the the commitment problem becomes so severe that while there is positive borrowing and investment, all surplus from investment is offset by costly default. The entrepreneur would be just as well off not investing at all. This result is striking similar to the well-known Coase Conjecture in the industrial organization literature, that a durable good monopolist competing with itself intertemporally is unable to obtain any monopoly rents as consumers become very patient.<sup>7</sup> Both results illustrate the fact that dynamic commitment problems can completely unravel the ability of an agent to capture or generate surplus. In our model, welfare strictly decreases with  $q$  and at the limit the welfare level is as if the entrepreneur does not have access to a credit market at all. However,

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<sup>7</sup>Specifically, the Coase Conjecture states that as consumers become extremely patient, a durable goods monopolist who cannot commit to future prices would have to sell the its good at competitive prices instantaneously and is unable to raise any profits. See Fudenberg and Tirole (Chapter 10, 1991).

the welfare implications of lack of commitment for the monopolist (as in Coase Conjecture) are opposite—extreme patience drives monopolist rents to zero but maximizes social efficiency.

The predictions of Proposition 4.1 bear similarity to what is being observed in microfinance markets facing increases in competition among lenders. Most directly, McIntosh et al. (2005) find that increased competition between microfinance lenders in Uganda lead to increased debt accumulation and declining repayment rates of their borrowers. They cite conversations with local lenders that suggest that multiple borrowing is to blame:

“The chief executives of most of the [lending] institutions involved in this article were interviewed on the topic, and few were worried about competition insofar as it relates to the growth prospects of their institution. A common concern, however, was that, wherever two or more institutions are operating, many clients may be taking loans from several lenders simultaneously, or double-dipping...Nonetheless, they were unanimous in the opinion that this behavior does drive up default rates. ”

The most striking aspect of this anecdote is not that multiple borrowing occurs in the face of competition, but that lenders are very concerned about its effects on repayment and that its incidence cannot be controlled. Such anecdotes are abundant in the narrative surrounding multiple borrowing in microfinance.<sup>8</sup>

## 4.2 Comparative Statics of Project Returns

We now turn to the more surprising results about what happens when the marginal returns  $\alpha$  of the investment opportunity increase. When returns to the investment opportunity are higher the borrower has a greater incentive to exploit the externality associated with the commitment problem and take on more new financing at the expense of previous lenders. A central result of our paper is that for projects with higher returns, the effects of the commitment problem can be worsened to the extent that the equilibrium may involve lower levels of investment than would occur with less desirable projects.

To develop this result, first consider the effect of an increase in  $\alpha$  on the equilibrium face value of debt. As shown graphically in Figure 3, this increases borrowing aggressiveness—the borrower shifts debt accumulation toward earlier lenders, increases the expected face value of debt. Why? At any point in the game the borrower faces the following tradeoff. Taking interest rate strategies as given, it weighs the benefits of borrowing more from the current lender for sure, or taking the

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<sup>8</sup>A set of case studies of microcredit markets in Peru, Guatemala, and Bolivia (de Janvry et al. 2003) highlights the prevalence of multiple borrowing and high levels of debt that occurs following the rapid entry of lenders in each of these countries. Policy makers in Bangladesh are concerned that incidence of “overlapping borrowing” across multiple microfinance lenders is on the rise and incidence may be as high as 60% (Faruquee and Khalily 2011). A survey of microfinance usage in Andhra Pradesh analyzed by Johnson and Meka (2010) suggests that borrowers in Andhra Pradesh borrow from multiple lenders within the same month because they cannot obtain enough financing from individual lenders.

gamble that it will be able to meet a new lender from which to borrow marginally at better interest rates than the previous lender would not have offered. When the returns to investment are higher this tradeoff tilts away from a less aggressive borrowing strategy of waiting to try and exploit the externality on the current lender.

Of course, more aggressive borrowing means lenders have to charge higher interest rates in anticipation of higher aggregate borrowing. This result is summarized in Proposition 4.3.

**Proposition 4.2.** *Expected face value of debt  $\mathbb{E}[D]$  and the effective interest rate  $\mathbb{E}[D]/\mathbb{E}[I]$  are increasing in  $\alpha$ , the returns to scale of the investment technology.*

Now we turn to the main result on the relationship between the equilibrium level of investment  $\mathbb{E}[I]$  and the returns to scale of the new project.

**Proposition 4.3.** *Fix any  $q \in (0, 1)$*

- a) *The equilibrium level of investment  $\mathbb{E}[I]$  is non-monotone in  $\alpha$ . In particular, there exists a cutoff  $\bar{\alpha}(q)$  such that expected investment is increasing in  $\alpha$  below this cutoff, and decreasing in  $\alpha$  for  $\alpha > \bar{\alpha}(q)$ . Formally,*

$$\frac{d\mathbb{E}[I]}{d\alpha} \begin{cases} < 0 & \text{for } 1 \leq \alpha < \bar{\alpha}(q) \\ > 0 & \text{for } \alpha > \bar{\alpha}(q) \end{cases}$$

- b) *The stronger the commitment problem in the lending market, the lower is the cutoff level of marginal return  $\bar{\alpha}(q)$  for decreasing investment:*

$$\frac{d\bar{\alpha}(q)}{dq} < 0$$

These results can be visualized in Figure 4, which plots equilibrium expected investment as a function of  $\alpha$  for three different values of  $q$ . For all levels of  $q$ , the level of investment that the entrepreneur gets to raise in expectation first increases in  $\alpha$  and then decreases. Entrepreneurs with better opportunities could be facing tighter constraints. The second part of the proposition shows that such endogenous misallocation of resources is more severe in markets where the commitment problem is worse.

The seemingly perverse outcome of lower investments in better opportunities arises because when commitment problems are present, it is possible that the equilibrium involves using available debt capacity inefficiently. To better understand the intuition behind this result, recall that the present value of promised repayments to creditors is affected by the face value of these claims in two directions. Holding repayment fixed, higher face value translates into higher present value. However, a higher face value of debt also reduces the probability of repayment. This gives rise to an aggregate debt Laffer curve: the value of debt is initially increasing in the amount of debt pledged starting from zero, but for sufficiently high face values of debt, pledging more debt actually

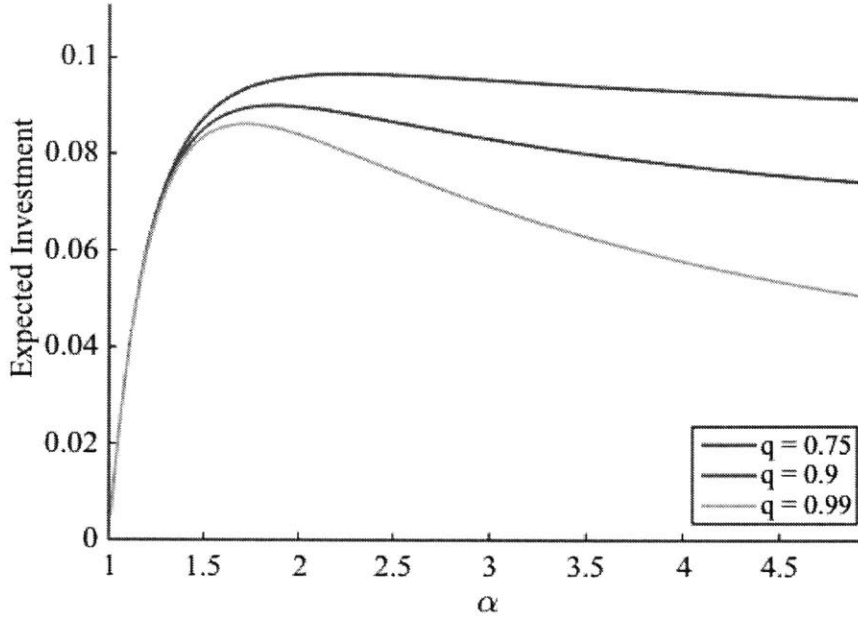


Figure 4: Expected Investment in the Dynamic Lending Game

results in a lower present value of debt because the negative force begins to dominate the positive one.

Figure 5 shows precisely how the debt Laffer curve arises in this model. This figure translates the best response curves of Figure 3 into the expected investment vs expected debt space by plotting the values of expected investment and debt that would result if the lenders were optimally responding to arbitrary strategies of the borrowers, and vice versa. This exercise generates two distinct curves, the (non-zero) intersection of which corresponds to the unique linear equilibrium allocation.<sup>9</sup> The hump-shaped curve embeds lender optimality over a range of responses to all possible borrower strategies  $b \geq 1$ . This is the debt Laffer curve introduced in the previous paragraph. Given a borrowing aggressiveness strategy of the entrepreneur, there is an associated zero-profit level of expected investment.

The upward sloping curves plot values of expected investment and debt if the borrower responds optimally to the range of all possible lender strategies  $\ell \geq 1$ . As illustrated in the graph, for higher  $\alpha$  this curve shifts to the right, representing more aggressive borrowing by the entrepreneur upon meeting each lender. As marginal returns increase, the entrepreneur has more to gain by increasing borrowing (and investment). Importantly but unfortunately this is also a curse. Without commitment, the entrepreneur will also want to increase borrowing from any future lenders it gets to meet.

<sup>9</sup>An intersection of the curves in Figure 3 implies an intersection in Figure 5, but the converse is not true. This is because the mapping between  $(b, \ell)$  and  $(\mathbb{E}[D^{agg}], \mathbb{E}[I^{agg}])$  is not one-to-one:  $(0, 0)$  in the  $(\mathbb{E}[D^{agg}], \mathbb{E}[I^{agg}])$  space is never an equilibrium of the model for  $\alpha > 1$ .

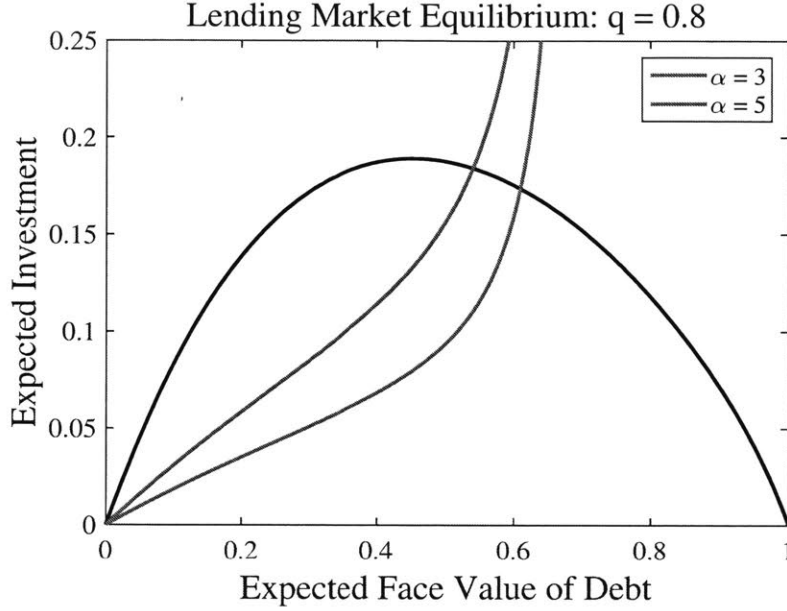


Figure 5: Expected Investment and Expected Debt

We know from Proposition 4.2 that  $\mathbb{E}[D]$  is monotonically increasing in  $\alpha$ . When marginal returns are high enough the equilibrium allocation will end up on the wrong side of the debt Laffer curve, where the entrepreneur is in equilibrium offering such high levels of repayment that it actually receives very little investment. Thus on the wrong side of the curve, increases in  $\alpha$  increase expected debt but decrease expected investment. With the ability to commit to one lender, the borrower would never choose loan terms in this inefficient region.

These results are surprising and run against common intuition that more productive projects induce higher levels of investment despite the presence of financial constraints. In many classical theories of inefficient investment choice, investment occurs if and only if the net present value of a project exceeds a certain threshold, which can be above or below zero. Such models do not generate the stark patterns of resource misallocation that plagues developing countries. The commitment friction we explore in this paper however can generate these distortions. In fact, this force is so strong that it can also distort the influence the choice of investment opportunity chosen by entrepreneurs. In the next section we illustrate this point.

## 5 Endogenous Project Choice

Given the role of high marginal returns in Proposition 4.3, it is natural to ask what happens when project returns are not linear, but instead concave. In such a setting, borrowers can obtain commitment power by obtaining sufficiently large amounts of capital from early lenders such that

the marginal returns to future borrowing, to which they cannot commit to avoid, are lower. Unfortunately, to study fully general production functions  $R(\cdot)$  we must rely on numerical methods. However, we are able to analytically characterize an extreme form of concavity to illustrate our point. Consider investment opportunities have linear returns up to a certain size of investment, but then deliver zero marginal return for any additional investment beyond that point. We call these “linear-flat” projects and we use them to show that even if one investment opportunity is strictly dominated by another in terms of having lower returns for any level of investment, the dominated project could yield higher welfare for the entrepreneur. This result is summarized in Proposition 5.1.

**Proposition 5.1.** *There exist pairs of projects  $R_1(\cdot)$  and  $R_2(\cdot)$  such that  $R_2(I) > R_1(I)$  for all  $I$ , but an entrepreneur would prefer to undertake the “worse” project  $R_1(\cdot)$  because doing so delivers higher welfare.*

Formally, a project is linear-flat if the return function can be parameterized by a slope parameter  $\alpha$  and a cutoff parameter  $\bar{L}$  such that  $R(I; \alpha, \bar{L}) = \alpha \min(I, \bar{L})$ . Define  $L^{SL}(\alpha)$  and  $D^{SL}(\alpha)$  respectively as the market and face value of debt that would be chosen by an entrepreneur who can commit to borrowing from a single lender when the investment opportunity is linear with marginal returns  $\alpha$ . Now consider the linear-then-flat project  $R(I; \alpha, L^{SL}(\alpha))$ , which is linear with slope  $\alpha$  and turns flat after raising an investment level of  $L^{SL}(\alpha)$ . It should be clear that even without commitment power this project achieves the same lending outcome as the linear project with slope  $\alpha$  under commitment: as there are no marginal returns from borrowing beyond  $(D^{SL}(\alpha), L^{SL}(\alpha))$ , the first lender that the entrepreneur visits would be willing to accept a loan at these terms since it will be assured that no future borrowing will occur to deteriorate the value of the claims. In this example the extreme concavity of investment opportunity effectively solves the commitment problem. We now use this result to illustrate Proposition 5.1.

Figure 6 shows the payoff functions of several projects. Consider Project 1 to be the linear-flat project with slope  $\alpha^L$  and a threshold for zero marginal return at  $L^{SL}(\alpha^L)$ , i.e.  $R_1(I) = \alpha^L \min(I, L^{SL}(\alpha^L))$ . For a given  $q > 0$ , we can find a fully linear Project 2 with slope  $\alpha^H$  such that the equilibrium welfare obtained by an entrepreneur endowed with either of these two projects would be equal. We must have  $\alpha^H > \alpha^L$  because the project needs to deliver higher output to compensate for the surplus loss due to the lack of commitment power.

Any point in the graph can be used to parameterize a project payoff function that is linear from the origin to that point, then turns flat after that. The dotted line plots the set of points that represent projects which give equivalent welfare to the entrepreneur as Project 1 and Project 2. Any project in the shaded region, such as Project 3, gives strictly higher welfare in equilibrium to the entrepreneur than Project 1 does. Any project outside the shaded region and below Project 2 gives strictly lower welfare.

The strength of this result can be seen by comparing Project 3 and Project 2. Despite the fact

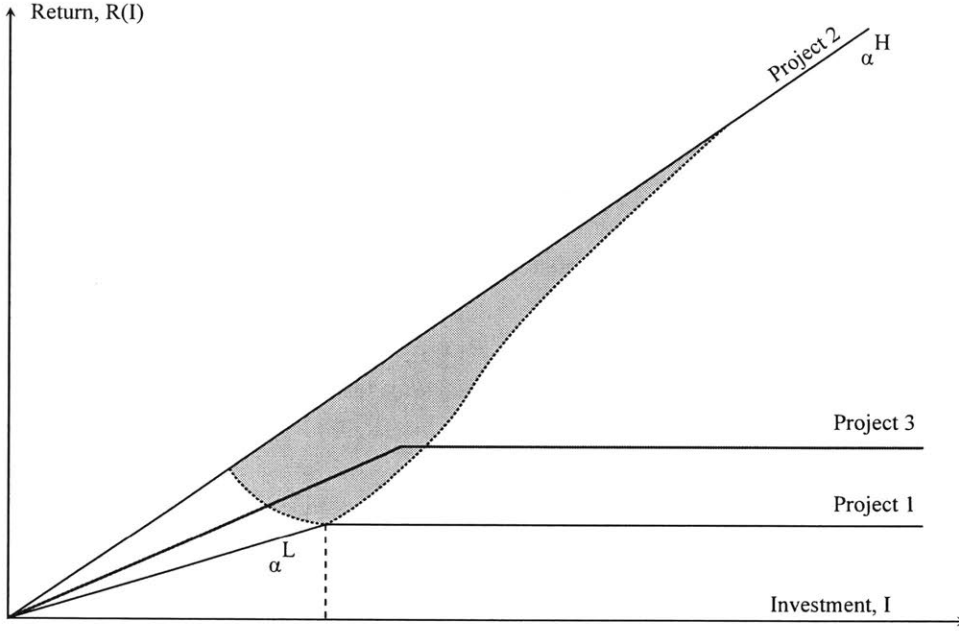


Figure 6: Endogenous Project Choice

that viewed in isolation, Project 3 yields lower return than Project 2 for any level of investment, it gives the entrepreneur strictly higher welfare in our model due to the commitment friction. By having zero marginal return beyond a certain level of investment, Project 3 endows the entrepreneur with some endogenous commitment power such that loans can be obtained under better interest rates. As a result, if the entrepreneur gets to choose ex-ante which investment projects to undertake, he may *endogenously* pursue an investment opportunity with lower returns, as long as it is sufficiently concave.

The result in this section highlights that in the presence of the commitment problem there is a powerful trade-off between the average and marginal returns of investment opportunities. Productivity is no longer the sole determinant of investment in economic activities. The commitment friction can be mitigated by undertaking instead in projects that have concave returns and thus embed some degree of commitment. Once these projects interior to the technology frontier are undertaken, they exhibit slower capital accumulation and growth potential.

## 6 Policy Implications

We now turn to studying simple regulatory policy tools that can improve outcomes in our model: limiting interest rates, imposing total borrowing limits, and limiting the number of lenders from which a borrower can obtain loans. Conventional arguments suggest that interest rate caps may

be helpful in improving allocations when there is a *lack* of competition among lenders as they limit monopoly power. Yet in some scenarios, it seems to have been the *entry* of new lenders into markets and the resulting increase in competition that has driven regulators to consider usury regulations. The microfinance crisis in Andhra Pradesh was precipitated by the rapid entry of thousands of new microfinance lenders and characterized by over indebtedness of borrowers from multiple lenders. A report commissioned by the Reserve Bank of India to study the causes and potential regulatory responses to the Andhra Pradesh crisis states:

“It has been suggested that with the development of active competition between MFIs there has been a deluge of loan funds available to borrowers which has fueled excessive borrowing and the emergence of undesirable practices ... Finally, it is believed that in consequence of over-borrowing, default rates have been climbing in some locations but these have not been disclosed because of ever-greening and multiple lending.” Malegam (2011).

Despite highlighting a high degree of competition, the report proceeds to propose regulation that limits interest rates charged to borrowers. The following proposition illustrates that, through the lens of our model of multiple borrowing and commitment problems, this type of regulatory response is rational and welfare improving.

**Proposition 6.1.** *Adding an upper bound on interest rates generates the following results:*

- a) *For any increasing and concave investment opportunity  $R(\cdot)$  with  $R'(0) > 1$ , there is an optimal interest rate cap  $\bar{r}^{SL}(R)$  that induces the full commitment allocation with the borrower obtaining funding from a single lender. When  $R(I) = \alpha I$ , the optimal interest rate cap is  $\bar{r}^{SL} \equiv 1 - \alpha^{-1}$ .*
- b) *Any interest rate cap  $\bar{r}$  has an associated debt limit  $\bar{D} = 1 - (1 + \bar{r})^{-1}$  that induces the same equilibrium.*
- c) *If  $\bar{r} < \bar{r}^{SL}$  then single lender borrowing prevails but debt is inefficiently low and the interest rate cap is too restrictive. Welfare will be lower than the unregulated equilibrium (i.e. with no interest rate cap) if the interest rate cap is sufficiently low.*
- d) *If  $\bar{r} > \bar{r}^{SL}$  then the interest rate cap is too loose. Imposing the cap increases expected investment and welfare while lowering expected debt and interest rates relative to the unregulated equilibrium. As  $\bar{r} \rightarrow \infty$  the interest rate cap becomes irrelevant and outcomes converge to the unregulated equilibrium.*

Interest rate caps have the potential to improve welfare because they embed commitment power. Recall that in the model the more debt the borrower has outstanding, the lower the probability of repayment, and thus the higher interest rates need to be on additional lending. Interest rate caps add commitment power because marginal borrowing at high enough levels of debt would need to violate the interest rate cap for these loans to break even and thus are never issued. Early lenders

can then be assured that such future borrowing, which increases the anticipated probability of default, will not occur, and can provide initial loans at interest rates that are closer to those that would prevail if borrowers could fully commit to exclusive borrowing. This better aligns borrower incentives, reducing face values of debt and interest rates in equilibrium. Importantly, interest rate caps also increase investment and improve welfare as long as they are not too severe that they prevent productive investment.

Further, for a given investment opportunity the interest rate cap can be set to fully overcome the inefficiencies induced by lack of commitment. By setting the interest rate cap at exactly the interest rate that would prevail in the full-commitment equilibrium, the borrower can credibly raise exactly the full commitment level of debt, and at the commitment-level interest rate, restoring the full commitment outcomes. By proposing such a loan to the first lender, the lender is assured that any future borrowing would necessarily need to be at interest rates above the allowed limit, and can thus be sure that no such additional borrowing would take place. Thus this loan proposal is expected to earn zero profits for the lender and is always accepted. By definition this allocation maximizes the entrepreneur's welfare given lenders at least break even, so any rational lender strategy induces this loan proposal as a best response.

In our model a limit on the total face value of debt a borrower can obtain is mechanically equivalent to a particular interest rate cap, and the equivalent debt limit is increasing in the interest rate cap, so all the results above also apply to total debt limits. Both total borrowing limits and interest rate caps were adopted for microfinance loans in India in 2011 (Dr. D. Subbarao (2011)). These policies are equivalent in the model because they both operate by shutting down contingencies of excessive debt accumulation: debt limits directly and interest rate limits indirectly through the fact that the lower bound on interest rates in any equilibrium is increasing in the total cumulative face value of debt. In the contingencies (potentially off-equilibrium) in which either the interest rate or debt limit is binding, the lender zero profit condition implies a unique relationship between interest rates and total face value of debt at this point:  $p(\bar{D}) = (1 + \bar{r})^{-1}$ , where  $\bar{D}$  is a borrowing limit and  $\bar{r}$  is the equivalent interest rate limit.

It is important to re-emphasize that in general interest rate caps (and total borrowing limits) have ambiguous implications for welfare, because caps that are too low can restrict productive investment. While there are always welfare improving interest rate limits for a given project, imposing a market-wide policy can have ambiguous effects on welfare if the investment opportunities in the economy are sufficiently heterogeneous. From a utilitarian perspective, however, "reasonable" interest rate caps can be quite beneficial if many projects in the economy could benefit from them. Further, taking into account the possibility of endogenous project choice, interest rate caps have the potential to "unlock" the best projects available that were previously infeasible due to the endogenous credit constraint induced by lack of commitment.

Finally, a surprising and controversial policy recommendation of Malegam (2011) was to limit

borrowers to obtaining loans from at most two micofinance lenders. Limiting the number of lenders from which a borrower can obtain loans will mechanically increase commitment power by limiting the opportunities a borrower has not to commit. While this policy was not ultimately adopted, our model shows that in the face of commitment problems such a policy may actually be quite helpful.

## 7 Conclusion

This paper argues that commitment problems in lending markets can explain emerging empirical evidence that the rapid expansion of credit access can have perverse effects. When borrowers cannot commit to exclusive contracting, increasing the availability of lenders makes markets appear *less* competitive as interest rates rise and entrepreneur investment and welfare fall. More importantly, commitment problems can result in better projects receiving less investment than worse projects. This force can be so severe that what look like good opportunities are passed over for inferior investment technology. Finally, we show how simple regulatory tools such as interest rate ceilings and debt limits can improve outcomes and ameliorate the misallocative forces we highlight.

The intuition for these result is that the externalities the lenders impose on each other when commitment or contingent contracting is not possible can prevent the borrower from being able to use pledgeable cash flows efficiently. When explicit commitment is impossible, there is value in any implicitly commitment mechanisms that attenuate the demand for further borrowing. Thus, the return profile of an investment opportunities itself is an important driver in the severity of commitment distortions.

Since commitment is less of a problem for projects with lower marginal returns, when given the choice entrepreneurs will endogenously choose investment opportunities that are everywhere less productive than other available opportunities, as long as they are sufficiently more concave. Thus our model provides a new micro-foundation for the idea that commitment problems in lending markets can induce substantial misallocation in capital investment and can explain observations both of low growth and of economic activity below the technological frontier. While we have augmented our study with a sample of the growing anecdotal evidence that multiple borrowing is problematic, there is much more to learn. Formally testing the empirical validity the mechanisms we highlight in explaining the failure of increased access to finance to significantly improve outcomes is an important topic for future research.

## Appendix

### Microfounding Random Default with Moral Hazard

In this section we provide an alternative microfoundation of the random default through limited pledgeability and moral hazard. Instead of assuming the entrepreneur borrows against his default cost  $\tilde{c}$  as we do in the main text of the paper, we endow the entrepreneur with a pledgeable stochastic cashflows from assets-in-place that are realized at  $s = 2$ . The pledgeable cashflows realize one of two values: zero or one. The good payoff occurs with probability  $p$ , which is chosen by the entrepreneur at quadratic effort cost  $p^2$ . Moral hazard arises because effort is chosen after cashflows are (partially) pledged to lenders. Since there are only two realizations of these cashflows and the entrepreneur has no other pledgeable wealth, it is without loss of generality that claims issued to lenders take the form of debt contracts: they are repaid in full when the good realization occurs and the entrepreneur defaults when cashflows are zero. With this in mind, assume the entrepreneur has issued a total face value of debt  $D$  in the first stage. At  $s = 2$  the entrepreneur solves

$$p(D) \equiv \arg \max_p p[1 - D] - p^2$$

The entrepreneur expects the assets in place to pay out with probability  $p$ , and conditional on a positive payout the entrepreneur gets to keep  $1 - D$  of the cashflows as the residual claim. The cost of choosing the probability of positive cashflows to be  $p$  is  $p^2$ . Thus  $p(D)$  denotes the solution to the entrepreneur's choice of effort at stage  $s = 2$  conditional on having a total outstanding face value of debt  $D$ . The solution of entrepreneur's problem yields  $p(D) = \frac{1-D}{2}$ , and entrepreneur's expected payoff from the residual claim is  $\frac{(1-D)^2}{4}$ . The value function in (4) is modified to

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) - (1 - q) \frac{(1 - D)^2}{4} + qV(D')$$

and all of our results go through analogously.

### Proof of Lemma 3.1

Borrower's problem can be formulated recursively as:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) + (1 - q) \left[ \frac{(1 - D')^2}{2} - \frac{1}{2} \right] + qV(D')$$

where

$$\tilde{p}(D') \equiv \mathbb{E}[1 - D^{agg} | D']$$

We guess that borrower's policy function  $g(\cdot)$  and lenders' loan pricing function  $\tilde{p}(\cdot)$  both take a linear form and are each characterized by a single endogenous variable,  $b$  and  $\ell$ , respectively:

$$\tilde{p}(D) = \ell^{-1}(1 - D)$$

$$1 - g(D) = b^{-1}(1 - D)$$

To solve for borrower's policy function, we proceed to take first order condition and use the envelope condition for borrower's problem. The first order condition is:

$$-\alpha\ell^{-1}(g(D) - D) + \alpha\ell^{-1}(1 - g(D)) - (1 - q)(1 - g(D)) + qV'(g(D)) = 0$$

and the envelope condition is:

$$V'(D) = -\alpha\ell^{-1}(1 - g(D))$$

Plugging the envelope condition into the first-order condition and after simplifying, we can express the Euler condition as a quadratic function of  $b^{-1}$ :

$$q\alpha\ell^{-1}b^{-2} + (1 - q - 2\alpha\ell^{-1})b^{-1} + \alpha\ell^{-1} = 0$$

We thus solve for the endogenous parameter  $b^{-1}$  that governs the borrower's policy function as:<sup>10</sup>

$$b^{-1} = \frac{(2\alpha\ell^{-1} - (1 - q)) - \sqrt{(1 - q - 2\alpha\ell^{-1})^2 - 4q\alpha^2\ell^{-2}}}{2q\alpha\ell^{-1}} \quad (5)$$

To solve for the lender's loan pricing function, note

$$\begin{aligned} \tilde{p}(D) &= \mathbb{E}[1 - D^{agg}|D] \\ &= (1 - q) \left[ (1 - D) + q(1 - g(D)) + q^2(1 - g(g(D))) + \dots \right] \\ &= (1 - q) \left[ (1 - D) + qb^{-1}(1 - D) + q^2b^{-2}(1 - D) + \dots \right] \\ &= \frac{1 - q}{1 - qb^{-1}}(1 - D) \end{aligned}$$

Hence

$$\ell^{-1} = \frac{1 - q}{1 - qb^{-1}} \quad (6)$$

Equation (5) characterizes  $b^{-1}$  as a decreasing function of  $\ell^{-1}$ . On the other hand, equation (6) characterizes  $\ell^{-1}$  as an increasing function of  $b^{-1}$ . The two equations therefore yields a unique

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<sup>10</sup>There are two roots to the quadratic equation, one of which leads to explosive debt accumulation. We choose the other, stable root.

solution  $(b^*, \ell^*)$  for each  $q \in [0, 1)$  and  $\alpha \in (1, \infty)$ . In particular, we have

$$(b^*)^{-1} = \frac{(2\alpha - 1 - \sqrt{4(1-q)(\alpha^2 - \alpha) + 1})}{2q(\alpha - 1)}$$

□

**Lemma 7.1.** *The best response functions have the following properties:*

$$\begin{aligned} \frac{\partial \ell(b; \alpha, q)}{\partial b} &\geq 0; \quad \frac{\partial \ell(b; \alpha, q)}{\partial \alpha} = 0; \\ \frac{\partial b(\ell; \alpha, q)}{\partial \alpha} &\geq 0; \quad \frac{\partial b(\ell; \alpha, q)}{\partial \ell} \leq 0; \end{aligned}$$

where the inequalities are strict for  $q \in (0, 1)$  and  $\alpha > 1$ .

*Proof.* The results with respect to lender's best response immediately follow from equation (6).

We now with with equation (5) to derive the results respect to borrower's best response function. Let  $x \equiv 2\alpha\ell^{-1}$ , we have

$$\begin{aligned} b^{-1} &= \frac{(x - (1 - q)) - \sqrt{(1 - q - x)^2 - qx^2}}{qx} \\ &= \frac{1}{q} - \frac{1 - q + ((1 - q)(x - 1)^2 - q(1 - q))^{\frac{1}{2}}}{qx} \end{aligned}$$

Let  $\Delta \equiv ((1 - q)(x - 1)^2 - q(1 - q))$  and take derivative with respect to  $x$ , we have

$$\begin{aligned} \frac{\partial b^{-1}}{\partial x} &= -\frac{qx(1 - q)(x - 1) - q((1 - q)\Delta^{\frac{1}{2}} + (1 - q)(x - 1)^2 - q(1 - q))}{(qx)^2 \Delta^{\frac{1}{2}}} \\ &= -\frac{q(1 - q)}{(qx)^2 \Delta^{\frac{1}{2}}} (x^2 - x - \Delta^{\frac{1}{2}} - (x - 1)^2 + q) \\ &= -\frac{q(1 - q)}{(qx)^2 \Delta^{\frac{1}{2}}} (x + q - 1 - \Delta^{\frac{1}{2}}) \end{aligned}$$

Since  $\Delta^{\frac{1}{2}} = \sqrt{(x + q - 1)^2 - qx^2} \leq x + q - 1$ , we have that

$$\frac{\partial b^{-1}}{\partial x} \leq 0$$

and the inequality is strict for  $q \in (0, 1)$ . Given the definition of  $x$ , we have that  $\frac{\partial b(\ell; \alpha, q)}{\partial \alpha} \geq 0$  and  $\frac{\partial b(\ell; \alpha, q)}{\partial \ell} \leq 0$ . □

### Proof of Proposition 3.3

We begin with an outline of the proof. The proof will first show how to solve the finite-lender game by backwards induction, generating a recursive formulation for borrower and lender strategies. Next, we show that the fixed point of this recursion generates the strategies of the infinite-lender equilibrium defined in the main text. Finally, to demonstrate convergence, we show that this recursive formulation of strategies is characterized by a contraction mapping. This implies that, considering the strategies at a given lender, as the number of potential subsequent lenderse goes to infinity, the equilibrium strategies at this lender converge uniquely to the fixed point and thus to the strategies of the infinite-lender equilibrium.

#### Backward Induction in the Finite-Lender Game

Consider a finite version of the game with  $N$  lenders. For this proof, we abuse notation and index periods counting backwards from the end. Thus the last lender is indexed 1, and the first lender is indexed  $N$ . Therefore, after lender  $i$  there are at most  $i - 1$  more lenders for the borrower to visit. Let  $D_i$  denote the amount of cumulative debt the borrower accumulates from meeting lenders  $N$  through  $i + 1$ , thus  $D_0$  denotes the total amount of debt the borrower will accumulate if it gets to meet all  $N$  lenders. The probability of default from the last lender's perspective will be  $\tilde{p}_1(D_0) = 1 - D_0$  if the lender accepts a proposal that brings the borrower's cumulative debt to  $D_0$ . This defines the unique strategy of the final lender to accept only weakly profitable loans. Assume the borrower has any arbitrary face value of debt  $D_1$  upon meeting the final lender. The borrower solves

$$V_1(D_1) \equiv \max_{D_0} \alpha(D_0 - D_1)(1 - D_0) + \left( \frac{D_0^2}{2} - D_0 \right)$$

and the solution is

$$1 - D_0 = (1 - D_1) \underbrace{\frac{\alpha}{2\alpha - 1}}_{\equiv B_1}$$

with borrower's maximized value function being

$$V_1(D_1) = (1 - D_1)^2 \underbrace{\left[ \frac{2\alpha^3 - \alpha^2}{2(2\alpha - 1)^2} \right]}_{\equiv W_1} - \frac{1}{2}$$

We define

$$\begin{aligned} B_1 &\equiv \frac{\alpha}{2\alpha - 1} \\ L_1 &\equiv 1 \\ W_1 &\equiv \frac{2\alpha^3 - \alpha^2}{2(2\alpha - 1)^2} \end{aligned}$$

Thus the unique subgame perfect equilibrium strategies conditional on arriving to the last lender with some amount of debt  $D$  are:

$$\begin{aligned} 1 - g(D) &= B_1(1 - D) \\ \tilde{p}_1(D) &= L_1(1 - D) \end{aligned}$$

and any borrower considering leaving the second-to-last lender with a total face value of debt  $D$  realizes that continuation utility if it reaches the last lender is given by

$$V_1(D) = W_1(1 - D)^2 - \frac{1}{2}.$$

Thus we know that at lender  $i = 1$  players use strategies linear in  $1 - D$ . Now we show by induction that all lenders use such linear strategies. Assume for some  $n$  that players at all stages  $i < n$  use linear strategies and that the maximized value function at lender  $i$  is proportional to  $(1 - D_{i+1})^2$ . We will show that players at stage  $n$  also use linear strategies and that the maximized value function at lender  $n$  is proportional to  $(1 - D_{n+1})^2$ , and thus by induction prove that these claims do indeed hold for all  $n \in \mathbb{N}$ .

Now consider the subgame where the borrower meets lender  $n$  with cumulative debt  $D_n$  obtained from previous lenders. Since all future lenders and borrowers use linear strategies, we can compute lender  $n$ 's expected probability of repayment:

$$\begin{aligned} \tilde{p}_n(D_{n-1}) &= (1 - q)(1 - D_{n-1}) + q\tilde{p}_{n-1}(D_{n-2}) \\ &= (1 - q)(1 - D_{n-1}) + qL_{n-1}(1 - D_{n-2}) \\ &= (1 - q)(1 - D_{n-1}) + qL_{n-1}B_{n-1}(1 - D_{n-1}) \\ &= [(1 - q) + qB_{n-1}L_{n-1}](1 - D_{n-1}) \end{aligned}$$

where the first to second line follows from the assumption that lender  $n - 1$  is using a linear strategy  $\tilde{p}_{n-1}(D) = L_{n-1}(1 - D)$ , and moving from the second to the third line relies on the assumption that the borrower at  $n - 1$  is using a linear strategy  $1 - D_{n-2} = B_{n-1}(1 - D_{n-1})$ . Thus we know that lender  $n$  follows the strategy given by

$$L_n = 1 - q + qB_{n-1}L_{n-1}$$

Next, under our inductive hypothesis we can write the borrower's problem visiting lender  $n$  as:

$$\begin{aligned}
V_n(D_n) &= \max_{D_{n-1}} \alpha \tilde{p}_n(D_{n-1})(D_{n-1} - D_n) + (1 - q) \left( \frac{D_{n-1}^2}{2} - D_{n-1} \right) + q V_{n-1}(D_{n-1}) \\
&= \max_{D_{n-1}} \left\{ \alpha (D_{n-1} - D_n) (1 - D_{n-1}) L_n + (1 - q) \left( \frac{D_{n-1}^2}{2} - D_{n-1} \right) \right. \\
&\quad \left. + q \left( W_{n-1} (1 - D_{n-1})^2 - \frac{1}{2} \right) \right\}
\end{aligned}$$

Taking the first order condition and solving for  $D_{n-1}$  verifies that the borrower's strategy does indeed have the hypothesized linear form:

$$1 - D_{n-1} = (1 - D_n) \underbrace{\frac{\alpha L_n}{2\alpha L_n - (1 - q + 2qW_{n-1})}}_{\equiv B_n}$$

and maximized value function

$$V_n(D_n) = (1 - D_n)^2 \underbrace{\left( \alpha (1 - B_n) B_n L_n + (1 - q) \frac{B_n^2}{2} + q B_n^2 W_{n-1} \right)}_{\equiv W_n} - \frac{1}{2}$$

Thus the inductive proof is completed and all strategies satisfy the proposed form.

## Recursive Formulation of Strategies

It is clear from above that the vector  $(B_n, L_n, W_n)$  is generated by a system of 3 difference equations:

$$\begin{aligned}
L_n &= (1 - q) + q B_{n-1} L_{n-1} \\
B_n &= \frac{\alpha L_n}{2\alpha L_n - (1 - q + 2qW_{n-1})} \\
W_n &= \alpha (1 - B_n) B_n L_n + (1 - q) \frac{B_n^2}{2} + q B_n^2 W_{n-1}
\end{aligned}$$

rearranging so each of  $(L_n, B_n, W_n)$  is a function of only the lagged variables:

$$\begin{aligned}
L_n &= 1 - q + q B_{n-1} L_{n-1} \\
B_n &= \frac{\alpha (1 - q + q B_{n-1} L_{n-1})}{2\alpha (1 - q + q B_{n-1} L_{n-1}) - (1 - q + 2qW_{n-1})} \\
W_n &= \frac{\alpha^2 (1 - q + q B_{n-1} L_{n-1})^2}{4\alpha (1 - q + q B_{n-1} L_{n-1}) - 2(1 - q + 2qW_{n-1})}
\end{aligned}$$

where the last equation can be simplified to

$$W_n = \frac{\alpha}{2} B_n L_n.$$

### Convergence to Infinite-Lender Strategies

Defining a new variable  $x_n \equiv B_n L_n$ , the set of difference equations above can be rewritten as

$$\begin{aligned} L_n &= (1 - q) + qx_{n-1} \\ B_n &= \frac{\alpha((1 - q) + qx_{n-1})}{2\alpha((1 - q) + qx_{n-1}) - (1 - q + \alpha qx_{n-1})} \\ W_n &= \frac{\alpha}{2} x_n \end{aligned}$$

Hence the sequence  $\{x_n\}$  is defined by  $x_1 \equiv B_1 L_1 = \frac{\alpha}{2\alpha-1}$  and a continuous function  $f(\cdot)$  such that  $x_n = f(x_{n-1})$ , where

$$f(x) = \frac{\alpha((1 - q) + qx)^2}{(1 - q)(2\alpha - 1) + \alpha qx}$$

First note that  $f(x) > 0$  if  $x > 0$ , and since  $x_1 > 0$ , we have  $x_n > 0$  for all  $n$ . It is also easily verified that  $f(\cdot)$  has a unique fixed point  $x^*$ , which corresponds to the unique fixed point  $(B^*, L^*, W^*)$  of the system of difference equations above. Moreover, it is a matter of algebra to show that the fixed point coincides with our closed form solution of the infinite lender game, implying that  $b^{*-1} = B^*$  and  $\ell^{*-1} = L^*$ .

We next show that  $f(\cdot)$  defines a contraction mapping which, given the continuity of  $f(\cdot)$ , shows that  $x_n \rightarrow x^*$ . This in turn implies  $(B_n \rightarrow b^{*-1}, L_n \rightarrow \ell^{*-1})$  as  $n \rightarrow \infty$ . In words, this means that as the number of potential future lenders in the game after lender  $n$  grows towards infinity, the unique subgame perfect strategies the players at stage  $n$  converge to the stationary strategies employed by all players in the infinite-lender game.

To show  $f(\cdot)$  defines a contraction mapping, we show  $\|f'(x)\| < 1$ . Taking derivatives of  $f$  with respect to  $x$ , we have (after much simplification)

$$\begin{aligned} f'(x) &= q - \frac{q(\alpha - 1)^2(1 - q)^2}{(2\alpha + q - 2\alpha q + \alpha qx - 1)^2} \\ &= q \left( 1 - \left( \frac{(\alpha - 1)(1 - q)}{(2\alpha - 1)(1 - q) + \alpha qx} \right)^2 \right) \end{aligned}$$

Since  $x > 0$ ,  $1 > q \geq 0$ ,  $\alpha > 1$ , we have

$$0 < \frac{(\alpha - 1)(1 - q)}{(2\alpha - 1)(1 - q) + \alpha qx} < 1$$

$$0 < \frac{(\alpha - 1)(1 - q)}{(\alpha - 1)(1 - q) + \alpha(1 - q) + \alpha qx} < 1$$

Hence

$$0 < f'(x) < 1$$

which shows that  $f(\cdot)$  is a contraction mapping.

□

### Proof of Proposition 4.1

Before proving the comparative static propositions, it will be useful to derive some expressions for equilibrium objects of interest in terms of parameters  $\alpha$  and  $q$ . Let  $I^{agg}$  denote the aggregate investment and  $D^{agg}$  denote the aggregate debt that have been attained when the lending market game ends. In equilibrium, ex-ante, these are random variables with respect to the number of lenders the borrower will be able to visit.

**Lemma 7.2.**  $\mathbb{E}[I^{agg}] = \mathbb{E}[p(D^{agg})D^{agg}]$

*Proof.* Let  $N$  denote the random number of lenders the borrower gets to visit before losing access to the lending market game. The random aggregate face value of debt and aggregate investment can be expressed as:

$$D^{agg} = \sum_{j=1}^{\infty} D_j \mathbf{1}(N \geq j)$$

$$I^{agg} = \sum_{j=1}^{\infty} I_j \mathbf{1}(N \geq j)$$

where  $D_j$  is the amount of debt given by the  $j$ -th lender. Similarly denote  $I_j$  to be the amount of investment capital provided by the  $j$ -th lender. Pick any  $j > 0$ , the zero-profit condition for his loan and investment size is:

$$\mathbb{E}[p(D^{agg}) | N \geq j] D_j \mathbf{1}(N \geq j) = I_j \mathbf{1}(N \geq j)$$

Taking expectation over  $N$  on both sides and applying the law of iterated expectation, we get:

$$\mathbb{E}[p(D^{agg}) D_j \mathbf{1}(N \geq j)] = \mathbb{E}[I_j \mathbf{1}(N \geq j)]$$

We next sum the previous equation over all lenders. By the linearity of the expectations operator,

we can bring the sum inside:

$$\mathbb{E} \left[ p(D^{agg}) \sum_{j=1}^{\infty} D_j \mathbf{1}(N \geq j) \right] = \mathbb{E} \left[ \sum_{j=1}^{\infty} I_j \mathbf{1}(N \geq j) \right]$$

Substituting in the definitions of  $D^{agg}$  and  $I^{agg}$ :

$$\mathbb{E} [p(D^{agg}) D^{agg}] = \mathbb{E} [I^{agg}]$$

□

**Lemma 7.3.** *We can express the expected debt and investment as functions of  $b^{-1}$ :*

$$\mathbb{E} [D^{agg}] = \frac{1 - b^{-1}}{1 - qb^{-1}}$$

$$\mathbb{E} [I^{agg}] = \frac{b^{-1} (1 - b^{-1}) (1 - q)}{(1 - b^{-1}q) (1 - b^{-2}q)}$$

*Proof.* Denote the expected aggregate debt upon leaving a given lender with cumulative debt  $D$  as  $\mathbb{E} [D^{agg}|D]$ . From lender's zero-profit condition, we have

$$\mathbb{E} [D^{agg}|D] = 1 - \tilde{p}(D)$$

The ex-ante expected aggregate debt  $\mathbb{E} [D^{agg}]$  is simply the expected aggregate debt upon leaving a lender with zero outstanding debt, times  $\frac{1}{q}$  (since the borrower meets the first lender with certainty, not probability  $q$ ). Thus we have

$$\begin{aligned} \mathbb{E} [D^{agg}] &= \frac{1}{q} \mathbb{E} [D^{agg}|0] \\ &= \frac{1}{q} \left( 1 - \frac{1 - q}{1 - qb^{-1}} \right) \\ &= \frac{1 - b^{-1}}{1 - qb^{-1}} \end{aligned}$$

To get the expression for expected investment:

$$\begin{aligned}
\mathbb{E}[I] &= \tilde{p}(g(0))g(0) + q\tilde{p}(g^2(0))\left[g^2(0) - g(0)\right] + q^2\tilde{p}(g^3(0))\left[g^3(0) - g^2(0)\right] + \dots \\
&= \ell^{-1}\left[(1 - g(0))g(0) + q(1 - g^2(0))\left[(1 - g(0)) - (1 - g^2(0))\right] + \dots\right] \\
&= \ell^{-1}\left[b^{-1}(1 - b^{-1}) + qb^{-2}\left[b^{-1} - b^{-2}\right] + q^2b^{-3}\left[b^{-2} - b^{-3}\right] + \dots\right] \\
&= \ell^{-1}b^{-1}(1 - b^{-1})\left[1 + qb^{-2} + q^2b^{-4} + \dots\right] \\
&= \ell^{-1}b^{-1}\frac{1 - b^{-1}}{1 - qb^{-2}} \\
&= \frac{b^{-1}(1 - b^{-1})(1 - q)}{(1 - b^{-1}q)(1 - qb^{-2})}
\end{aligned}$$

□

**Lemma 7.4.** Let  $z \equiv \sqrt{4(1 - q)(\alpha^2 - \alpha) + 1}$ . The analytic solution of expected debt, investment, and welfare can be expressed as the following functions of parameters  $q$  and  $\alpha$ :

$$\mathbb{E}[D^{agg}] = \frac{2\alpha - 1 - z}{2q\alpha}$$

$$\mathbb{E}[I^{agg}] = \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)}$$

$$V(0) = \frac{1 - 2\alpha(1 - q) - 2q + z}{4q}$$

*Proof.* These expressions can be obtained by substituting the analytic solution of  $b^*$  from lemma 1 into the expressions in lemma 2. □

Now continuing on, we can express equilibrium  $b^*$  as

$$(b^*)^{-1} = \frac{2\alpha - 1 - z}{2q(\alpha - 1)}$$

Also note that

$$\begin{aligned}
\frac{\partial z}{\partial \alpha} &= z^{-1}(1 - q)(4\alpha - 2) \\
\frac{\partial z}{\partial q} &= -2z^{-1}(\alpha^2 - \alpha)
\end{aligned}$$

We now proceed to prove Proposition 4.1 claim by claim.

*Claim 1.*  $\mathbb{E}[D^{agg}]$  is decreasing in  $q$ .

*Proof.* We first express  $\mathbb{E}[D^{agg}]$  as a function of  $\alpha$ ,  $q$ , and  $z$ :

$$\begin{aligned}
\mathbb{E}[D^{agg}] &= \frac{1 - \frac{2\alpha-1-z}{2q(\alpha-1)}}{1 - \frac{2\alpha-1-z}{2(\alpha-1)}} \\
&= \frac{2q(\alpha-1) - 2\alpha + 1 + z}{2q(\alpha-1) - 2q\alpha + q + qz} \\
&= \frac{(2q\alpha - 2q - 2\alpha + 1 + z)(z+1)}{q(z-1)(z+1)} \\
&= \frac{(2\alpha - 1 - z)(1-q)(\alpha-1)}{2\alpha q(1-q)(\alpha-1)} \\
&= \frac{2\alpha - 1 - z}{2\alpha q}
\end{aligned}$$

Differentiating with respect to  $q$ , we get

$$\frac{d\mathbb{E}[D^{agg}]}{dq} = \frac{-2\alpha q \frac{dz}{dq} - (2\alpha - 1 - z)2\alpha}{(2q\alpha)^2}$$

which implies

$$\begin{aligned}
\text{sign}\left(\frac{\partial \mathbb{E}[D^{agg}]}{\partial q}\right) &= \text{sign}\left(2q(\alpha^2 - \alpha) - (2\alpha - 1 - z)z\right) \\
&= \text{sign}\left(2q(\alpha^2 - \alpha) + 4(1-q)(\alpha^2 - \alpha) + 1 - (2\alpha - 1)z\right) \\
&= \text{sign}\left((\alpha^2 - \alpha)(4 - 2q) + 1 - (2\alpha - 1)z\right)
\end{aligned}$$

Let  $RHS \equiv (\alpha^2 - \alpha)(4 - 2q) + 1 - (2\alpha - 1)z$ . The remaining proof consists of three steps: 1) show  $\frac{dRHS}{d\alpha} \geq 0$  for all  $\alpha \geq 1$ ,  $q \in [0, 1]$ ; 2) show  $\frac{dRHS}{dq} \geq 0$  for all  $\alpha \geq 1$ ,  $q \in [0, 1]$ , with equality holding only when  $\alpha = 1$  or  $q = 0$ ; 3) RHS evaluated at  $\alpha = 1, q = 0$  is zero, concluding that  $RHS > 0$  for  $\alpha > 1, q > 0$ .

Step 1: show  $\frac{dRHS}{d\alpha} > 0$  for all  $\alpha \geq 1, q \in [0, 1]$ . Differentiating  $RHS$  with respect to  $\alpha$ , we have

$$\begin{aligned}
\frac{dRHS}{d\alpha} &= (2\alpha - 1)(4 - 2q) - 2z - (2\alpha - 1) \frac{\partial z}{\partial \alpha} \\
&= (2\alpha - 1) \left( 4 - 2q - z^{-1} (1 - q) (4\alpha - 2) \right) - 2z \\
&> (2\alpha - 1) \left( 4 - 2q - z^{-1} (1 - q) (4\alpha - 2) \right) \\
&\geq (2\alpha - 1) \left( 4 - \max_{q \in [0, 1]} (2q) - \max_{q \in [0, 1]} z^{-1} (1 - q) (4\alpha - 2) \right) \\
&= (2\alpha - 1) (4 - 2 - 2) \\
&= 0
\end{aligned}$$

Step 2: show  $\frac{dRHS}{dq} > 0$  for all  $\alpha \geq 1, q \in [0, 1]$ , with equality holding only when  $\alpha = 1$  or  $q = 0$ . Differentiating  $RHS$  with respect to  $q$ , we have

$$\begin{aligned}
\frac{dRHS}{dq} &= -2(\alpha^2 - \alpha) - (2\alpha - 1) \frac{\partial z}{\partial q} \\
&= (\alpha^2 - \alpha) (2z^{-1} (2\alpha - 1) - 2) \\
&= \underbrace{2z^{-1}}_{>0} \underbrace{(\alpha^2 - \alpha)}_{\geq 0} (2\alpha - 1 - z)
\end{aligned}$$

The last term is non-negative and is zero only when  $q = 0$ . To see this, note

$$\begin{aligned}
z &= \sqrt{4(1 - q)(\alpha^2 - \alpha) + 1} \\
(\text{equal only if } q = 0) &\leq \sqrt{4\alpha^2 - 4\alpha + 1} \\
&= 2\alpha - 1
\end{aligned}$$

Hence we have  $\frac{dRHS}{dq} > 0$ .

Step 3: conclude the proof. Note

$$RHS|_{q=0, \alpha=1} = 0$$

Hence we have, for any  $\alpha > 1$  and  $q > 0$ ,  $RHS > 0$ . Thus  $\frac{\partial \mathbb{E}[D^{agg}]}{\partial q} > 0$ .  $\square$

*Claim 2.*  $\mathbb{E}[I^{agg}]$  is decreasing in  $q$ .

*Proof.* Given the result in lemma (7.4), we first show that investment is decreasing in  $q$  if and only

if the ex ante welfare for the borrower is decreasing in  $q$ . To see this, note

$$\begin{aligned}
\mathbb{E}[I^{agg}] &= \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)} \\
&= \frac{2(\alpha - 1)}{\alpha(2\alpha - 1)} \cdot \left[ \frac{(z + 1 - 2\alpha(1 - q) - 2q)}{4q} + \frac{1}{2} \right] \\
&= \frac{2(\alpha - 1)}{\alpha(2\alpha - 1)} \cdot \left[ V(0) + \frac{1}{2} \right]
\end{aligned}$$

To show  $V(0)$  is decreasing in  $q$ , first note

$$\begin{aligned}
V(0) &= \frac{1 - 2\alpha(1 - q) - 2q + z}{4q} \\
\frac{dV(0)}{dq} &= \frac{4q(2\alpha - 2 + z_q) - 4(1 + 2\alpha(q - 1) - 2q + z)}{16q^2} \\
&= \frac{1}{4q^2} [q(2\alpha - 2 + z_q) - 1 + 2\alpha(1 - q) + 2q - z] \\
&= \frac{1}{4q^2} \left[ -2\alpha \left[ (\alpha - 1) \left( \frac{q}{z} \right) - 1 \right] - (1 + z) \right]
\end{aligned}$$

Define  $\Omega \equiv -2\alpha \left[ (\alpha - 1) \left( \frac{q}{z} \right) - 1 \right] - (1 + z)$ , we then have  $\text{sign} \left( \frac{dV(0)}{dq} \right) = \text{sign}(\Omega)$ .

To compute the derivative of  $\Omega$  with respect to  $q$ :

$$\begin{aligned}
\frac{d\Omega}{dq} &= -2\alpha(\alpha - 1) \frac{z - qz_q}{z^2} - z_q \\
&= -\frac{1}{z^3} 4\alpha^2 (\alpha - 1)^2 q \leq 0
\end{aligned}$$

Therefore  $\Omega$  is declining in  $q$  for all  $\alpha > 1$ . This means to check that  $\frac{dV(0)}{dq} < 0$  it is sufficient to check that  $\Omega|_{q=0} \leq 0$ .

$$\begin{aligned}
\Omega(q = 0) &= (2\alpha - 1) - z \\
&= 0
\end{aligned}$$

Hence welfare is decreasing in  $q$ . □

*Claim 3. Default probability is increasing in  $q$ .*

*Proof.* Note  $\mathbb{E}[Pr(\text{Default})] = \mathbb{E}[D^{agg}]$  and the claim follows directly from claim 1. □

*Claim 3.*  $\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}]$  is increasing in  $q$ .

*Proof.* Follows directly from claims 1 and 2. □

*Claim 4.*  $\lim_{q \rightarrow 1} V(0) = 0$ .

*Proof.* We can write the ex-ante welfare as

$$V(0) = \frac{1 + 2\alpha(q-1) - 2q + z}{4q}$$

Using the fact that  $\lim_{q \rightarrow 1} z = 1$  and taking limit, the result is immediate. □

### Proof of Proposition 4.2

Take  $\alpha_1 > \alpha_2$ . From lemma 7.1, we have  $\ell(b, q, \alpha_1) = \ell(b, q, \alpha_2)$  and that  $b(\ell, q, \alpha)$  is increasing in  $\alpha$ . This means that the downward sloping borrower best response curve in Figure 3 shifts upwards as  $\alpha$  increases, as the figure illustrates. Since the lender's best response slopes upwards ( $\ell_b(b, q, \alpha) > 0$ , also from Lemma 7.1), it follows that  $b^*(q, \alpha_1) > b^*(q, \alpha_2)$  and  $\ell^*(q, \alpha_1) > \ell^*(q, \alpha_2)$ .

Next, from lemma (7.3) we have

$$\mathbb{E}[D^{agg}] = \frac{1 - b^{-1}}{1 - qb^{-1}}$$

$$\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}] = \frac{1 - b^{-2}q}{b^{-1}(1 - q)}$$

both of which are increasing in  $b$ , which establishes the result that they are also both increasing in  $\alpha$ . □

### Proof of Proposition 4.3

From lemma (7.4) we have

$$\mathbb{E}[I^{agg}] = \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)}$$

Differentiating with respect to  $\alpha$ , we get

$$\frac{d\mathbb{E}[I^{agg}]}{d\alpha} = \frac{q(2\alpha - 2\alpha^2 z - 10\alpha^2 + 8\alpha^3) - (z - 6\alpha + 4\alpha^2 z + 12\alpha^2 - 8\alpha^3 - 4\alpha z + 1)}{2\alpha^2 q(2\alpha - 1)^2 z}$$

From this expression, one can verify that for any given  $q \in (0, 1)$ , there exists a unique  $\bar{\alpha}(q) \in (1, \infty)$  such that

$$\frac{d\mathbb{E}[I^{agg}]}{d\alpha} \begin{cases} < 0 & \text{for } 1 \leq \alpha < \bar{\alpha}(q) \\ > 0 & \text{for } \alpha > \bar{\alpha}(q) \end{cases}$$

and

$$\frac{d\bar{\alpha}(q)}{dq} < 0.$$

□

### Proof of Proposition 5.1

Consider projects 1 and 2 as represented in figure 6. Project 2 would yield the borrower a value of  $V(0; \alpha^H, q)$ , whereas project 1 would yield him

$$\begin{aligned} U^{SL}(\alpha^L) &= \alpha^L L^{SL}(\alpha^L) + \mathbb{E}[\min(\tilde{c}, D^{SL})] \\ &= \frac{2(\alpha^L)^3 - (\alpha^L)^2}{2(2\alpha^L - 1)^2} - \frac{1}{2} \end{aligned}$$

By construction,  $\alpha^H$  were chosen such that  $V(0; \alpha^H, q) = U^{SL}(\alpha^L)$ . Since  $U^{SL}(\alpha)$  is increasing in  $\alpha$ , any linear-flat project of the form

$$R(I) = \alpha \min(I, L^{SL}(\alpha))$$

where  $\alpha^H > \alpha > \alpha^L$  would yield strictly higher utility to the borrower than project 2, despite the fact that for any given level of investment, the linear-flat project yields lower output than the project 2:

$$\alpha^H I > \alpha \min(I, L^{SL}(\alpha)) \quad \text{for all } I > 0$$

□

### Proof of Proposition 6.1

Proof of Claim 1. For an increasing and concave investment function  $R(\cdot)$  with  $R'(0) > 1$ , the first-order condition in equation (1) that characterizes the single-lender equilibrium can be re-written as

$$R'(p(D^*)D^*) \times [p(D^*) + p'(D^*)D^*] = p(D^*)$$

If an interest rate cap were set to be  $1 + \bar{r}^{SL} \equiv \frac{1}{p(D^*)}$ , the borrower could propose to pledge  $D^*$  and raise  $p(D^*) D^*$  from the very first lender. No future lender would be willing to provide additional investment to the borrower because doing so would require an interest rate higher than  $1 + \bar{r}^{SL}$  to break even, but such a rate is prohibited by the interest rate cap. Hence the full commitment allocation can be achieved under  $\bar{r}^{SL}$ .

When  $R(I) = \alpha I$ , the first-order condition simplifies to

$$\alpha(1 - 2D^*) = 1 - D^*$$

hence  $D^* = \frac{\alpha-1}{2\alpha-1}$ . The optimal interest cap is thus

$$\begin{aligned} 1 + \bar{r}^{SL} &= \frac{1}{1 - D^*} \\ &= 1 - \frac{1}{\alpha} \end{aligned}$$

**Proof of Claim 2.** Consider an interest rate cap of  $\bar{r}$ . As explained in Claim 1, the maximum debt that can be pledged is

**Proof of Claim 3.** When  $\bar{r} < \bar{r}^{SL}$ , the interest rate cap is inefficiently low and the borrower can pledge less debt facevalue than he would have done under full commitment. The unique equilibrium under the interest rate cap would involve the borrower pledging  $D = 1 - \frac{1}{1+\bar{r}}$  debt and raising  $(1 - D) D$  investment from the very first lender. In the extreme case where  $\bar{r} = 0$ , the borrower would be unable to raise any investment from the lenders, achieving an even lower level of welfare than under the unregulated equilibrium.

**Proof of Claim 4.** When  $\bar{r} > \bar{r}^{SL}$ , the full commitment allocation is unattainable as with probability  $q$  the borrower will meet the second lender and pledge a strictly positive amount of debt for any level of outstanding debt below one.

Next we show that for  $\bar{r} < \infty$  the cap unambiguously improves expected investment and welfare while lowering expected debt and interest rate relative to the unregulated equilibrium. Using the techniques created in the proof for proposition 3.3, for any game with finite lenders  $K$  we can find a sequence of aggregate debt  $\{D_1^K, \dots, D_K^K\}$  where  $D_i^K$  corresponds to the aggregate debt level had the borrower reach lender  $i$  in a game with total lender  $K$ , where lender indices start backwards with the last lender being lender 1. Using a simple perturbation argument, we know that for  $K$  such that  $D_1^K < \bar{D} \leq D_1^{K+1}$ , the infinite lender game with debt cap  $\bar{D}$  would have a unique SPE where the borrower reaches the debt cap when borrowing from  $(K + 1)$ -th lender. Furthermore, using the same recursive definition of lender and borrower strategies in equilibrium as we adopted in proposition 3.3, it is clear that borrower's ex-ante expected investment, aggregate debt level, and welfare with debt cap  $\bar{D}$  is in between the corresponding equilibrium quantities for the finite lender games with  $K$  and  $K + 1$  lenders.



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