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Estimation of Aircraft Taxi-out Fuel Burn using Flight Data Recorder Archives

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The taxi-out phase of a flight accounts for a significant fraction of total fuel burn for aircraft. In addition, surface fuel burn is also a major contributor to CO₂ emissions in the vicinity of airports. It is therefore desirable to have accurate estimates of fuel consumption on the ground. This paper builds a model for estimation of on-ground fuel consumption of an aircraft, given its surface trajectory. Flight Data Recorder archives are used for this purpose.

The taxi-out fuel burn is modeled as a linear function of several factors including the taxi-out time, number of stops, number of turns, and number of acceleration events. The statistical significance of each potential factor is investigated. The parameters of the model are estimated using least-squares regression. Since these parameters are estimated using data from operational aircraft, they provide more accurate estimates of fuel burn than methods that use idealized physical models of fuel consumption based on aircraft velocity profiles, or the baseline fuel consumption estimates provided by the International Civil Aviation Organization. Our analysis shows that in addition to the total taxi time, the number of acceleration events is a significant factor in determining taxi fuel consumption.

Nomenclature

ICAO	International Civil Aviation Organization
FDR	Flight Data Recorder
MTOW	Maximum TakeOff Weight
T_{amb}	Ambient absolute temperature
f	Total fuel consumed during taxi-out
t	Taxi-out time
n_s	Number of stops
n_t	Number of turns
n_a	Number of acceleration events

I. Introduction

Estimation of aircraft fuel burn plays an important role in determining the impact of air traffic operations as well as in estimating the benefits of efficiency-enhancing procedures, and has been a topic of interest to the research community for several years.¹ Taxi-out fuel consumption is most often determined using the fuel burn indices presented in the International Civil Aviation Organization (ICAO) engine emissions databank.² The ICAO fuel burn indices provide fuel burn rates for only four engine power settings (corresponding to 7% or taxi/idle, 30% or approach, 85% or climb-out, and 100% or takeoff), and are based on estimates provided

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by engine manufacturers.³ Recently published studies^{4,5} have shown that the ICAO estimates can be quite different from the actual fuel burn, when considering the departure flight phase in the terminal area. The terminal area fuel burn considered in these studies includes the fuel consumed during taxi-out as well as the initial part of the climb. In contrast, in order to estimate the benefits of surface traffic management strategies,⁶ it is necessary to have accurate estimates of the taxi-out (on-surface) fuel burn. Since fuel flow rates in the airborne flight phase are much higher than during taxi-out, total departure fuel burn may not be a good indicator of surface fuel burn. On the other hand, a large part of total flight delay is absorbed on the ground, before departure. To quantify the impact of this delay, it is again necessary to have an estimate of the fuel burn associated with surface trajectories of departing aircraft. Previous studies on this topic⁷ have used the ICAO fuel burn indices (augmented with physical models) to translate the surface trajectories into fuel burn estimates, and therefore may not be representative of operational aircraft. To the best of our knowledge, this paper is the first attempt to develop models of surface fuel burn using Flight Data Recorder archives from an actual, operational fleet.

A. Problem Description

The high-level objective of this work is build a model that, when given the surface taxi trajectory of a flight (for example, from a surface surveillance system such as the Airport Surface Detection Equipment - Model X, or ASDE-X), can produce an accurate estimate of its fuel burn. In order to build such a model, we need to estimate fuel consumption from estimates of aircraft position, velocity and acceleration. One method for doing so is to divide the surface trajectory into different taxi phases (for example, stops, turns, constant velocity taxi, etc.), to estimate the engine power settings for each of these phases (using physics-based models and pilot surveys), interpolate/extrapolate ICAO fuel burn indices to these power settings, and to use these estimated fuel burn indices to determine the fuel consumption of the trajectory.⁷

However, several factors confound such an estimation using ICAO data alone. Firstly, the ICAO fuel burn estimates provided by engine manufacturers may not reflect the characteristics of the engines in the operational fleet which are subject to frequent use. Secondly, the engines are staged and tested only at four power settings, and in particular, 7% may not be representative of the typical power setting during taxi. Thirdly, pilot behavior is a critical factor in determining the power settings during the different taxi modes: for example, some pilots may not change their power settings when they stop, or when they accelerate from a stop, choosing instead to “ride the brakes”. Idealized models of taxiing cannot capture the effect of these intricacies. Finally, several studies have shown a nonlinear dependence of fuel flow rate on engine power settings, and the relationship at low power settings is not well-understood. Therefore, caution must be extended in interpolating or extrapolating fuel burn indices at low power settings (such as near ground idle). In order to overcome these challenges, we adopt a data-driven approach to estimating the fuel burn of surface trajectories, using Flight Data Recorder (FDR) archives.

II. FDR Database

The Flight Data Recorder (FDR) is a device onboard commercial aircraft that stores the history of several parameters such as aircraft position, velocity, fuel flow rate, ambient and engine temperature, and so on. FDR archives from over 2300 flights in the year 2004, belonging to an international airline, were used for the purposes of modeling and validation in this study. The dataset included flights originating from the US, Europe, Asia and Africa. Aircraft types included in the dataset were the Airbus A320 family (A319, A320, A321), the A330 family (with Rolls Royce and General Electric engines) and the A340, the Avro RJ85 and Boeing’s B757, B767 and B777. A full list of aircraft and engine types, along with their Maximum Takeoff Weight (MTOW), is shown in Table 1.

A total of 105 parameters were available in the dataset, of which the ones of primary interest to us were the fuel flow rate, throttle setting, velocity, position (latitude/longitude), ambient temperature, thrust and engine fan speed (N1). It is believed that some of these quantities, such as the thrust, were derived estimates and not actual measurements.

A. Data Processing

Raw data was run through multiple preprocessing algorithms for the purposes of analysis. These steps included sorting of flights, removal of the post-takeoff phase, filtering of the estimated velocity, and the

Table 1. Aircraft types and Engines

Type	Engine	Number of Engines	Number of Flights	MTOW (kg)
A319	CFM56-5B5-2	2	140	64,000
A320	CFM56-5B4-2	2	238	73,500
A321	CFM56-5B1-2	2	174	83,000
A330-202	CF6-80E1A4	2	224	230,000
A330-243	RR Trent 772B-60	2	237	230,000
A340-500	RR Trent 553-61	4	260	372,000
ARJ85	LF507-1F	4	263	44,000
B757	RR RB211-535E4	2	178	106,600
B767	P&W 4060	2	285	186,900
B777	GE90	2	364	344,550

extraction of events of interest such as stops and turns. The velocity as derived from position required filtering due to the relatively low update rate of aircraft position during taxi-out (Table 2).

Table 2. Dataset update rates

Flight Phase	Update Rate
Taxi	5 sec
Takeoff/Landing Roll	1 sec
Climb/Descent	10 sec
Cruise	150 sec

Finally, the taxi-out phase was separated by extracting the portion of the surface trajectory after pushback from the gate and before commencement of the takeoff roll. Identification of pushback was carried out using a combination of fuel-flow rate and speed conditions, and the start of the takeoff roll was determined using a speed cut-off.

III. Taxi-out Process Characterization

A. Baseline Fuel Consumption

The ICAO procedure for estimation of taxi-out fuel burn assumes that taxi operations occur entirely at 7% thrust (power setting), and thus proposes the use of this constant rated thrust fuel flow for all calculations.² It defines the fuel burn index to be the fuel flow rate per engine at 7% thrust. The version of the ICAO database used for this study was from December 2010.⁸ To compare these numbers to actual data, we calculated the average fuel flow rate for each available aircraft type in the dataset, by dividing the taxi-out fuel burn by the taxi-out time and the number of engines, and averaging this over all aircraft of a given type. As seen from Figure 1, the ICAO fuel burn index is not necessarily a reflection of the true per-engine fuel burn rate. The comparison shown in Figure 1 is in agreement with the results from a previous study,⁵ where a comparison of *total* average fuel flow rate is available. It can be seen that in several cases, the ICAO method produces an overestimate of fuel burn. This result is important from the point of view of estimates that drive environmental policy, such as the quantification of total CO₂ emissions at airports.

B. Events of Interest during Taxi-out

The surface trajectory is composed of periods of constant velocity taxi in a straight line, interspersed by events such as stops and turns. The results of any estimation procedure that incorporates these events will likely be influenced by their exact definitions. Therefore, the algorithms used to detect the number of turns and stops during taxi-out, as used in this study, are discussed below.

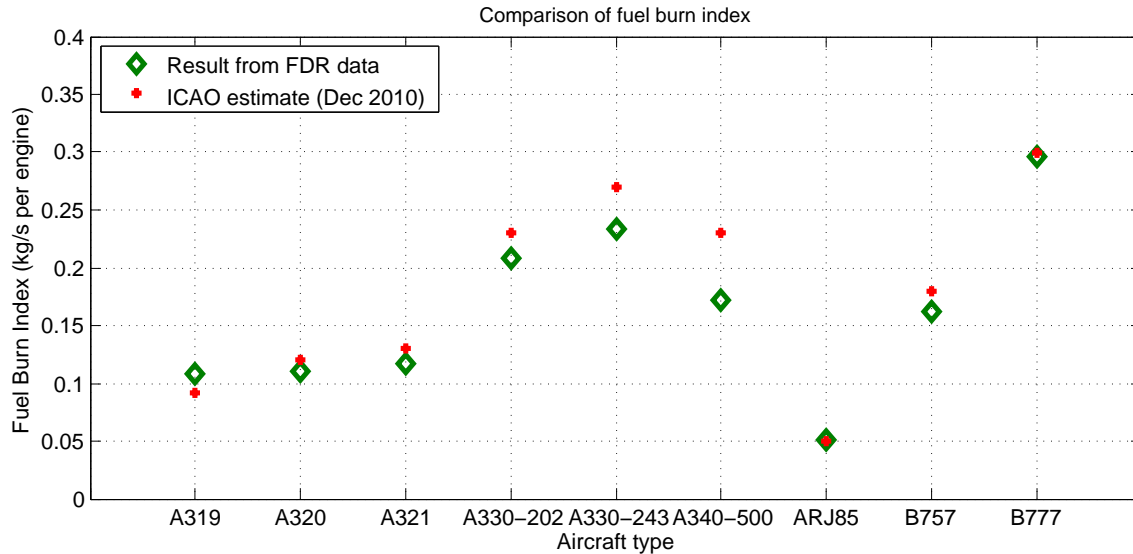


Figure 1. Comparison of fuel burn index as calculated from FDR data and that obtained from ICAO.

1. Detection of number of turns during taxi-out

The number of turns made by an aircraft taxiing on the ground was expected to affect the fuel burn. One reason is because the aircraft may slow down during its turn and have to speed up again after completing it, and the other reason might be the use of differential thrust for turning. A ‘turn’ was defined to be a heading change of at least 30 degrees, that was held over at least 30 seconds. Figure 2 shows the heading variation during taxi-out for a sample flight. Each time instant was tagged by a binary flag representing detection, or otherwise, of a turn in progress. Each set of contiguous non-zero flags was counted as one turn. The results from automatic detection were compared with visual inspection across several flights and were found to correctly count the number of turns in almost all cases.

2. Detection of number of stops during taxi-out

The number of stops made by aircraft was also expected to be a determinant of fuel burn, because of the throttle adjustments necessary during the stopping and restarting process. Usually, an aircraft has to stop during a handoff from one ground controller to another, because there is passing traffic on an intersecting taxiway/runway, or in the runway departure queue. There are two ways in which an aircraft can be brought to a halt: one way is to apply the brakes while reducing the thrust to idle, and the other is to apply the brakes while keeping the thrust constant. There are fuel burn tradeoffs involved with both methods. Reduction of the thrust while stopping reduces fuel consumption if the duration of the stop is long. However, thrust has to be increased to start taxiing again (*breakaway power*), and this is accompanied by a spike in the fuel consumption. Also, aircraft engines exhibit some time lag while spooling up, leading to slow response times when starting from a standstill. On the other hand, if the aircraft is stopped using only the brakes, fuel flow rate remains high, and can lead to significantly higher total fuel burn if the stop is prolonged. However, there is a performance benefit on restart as previously outlined. Consequently, pilots tend to use one of the two methods depending on personal preference and operational considerations. In this study, an aircraft was defined to have stopped during its taxi phase if its velocity dropped and stayed below a *stop threshold* of 2.25 m/s for at least 20 seconds, and then subsequently increased above a *start threshold* of 6.25 m/s. The results from the stop detection algorithm are shown in Figures 3 and 4 for two sample cases. Note that event logging takes place only if the aircraft is already taxiing, which means that the initial start is not counted.

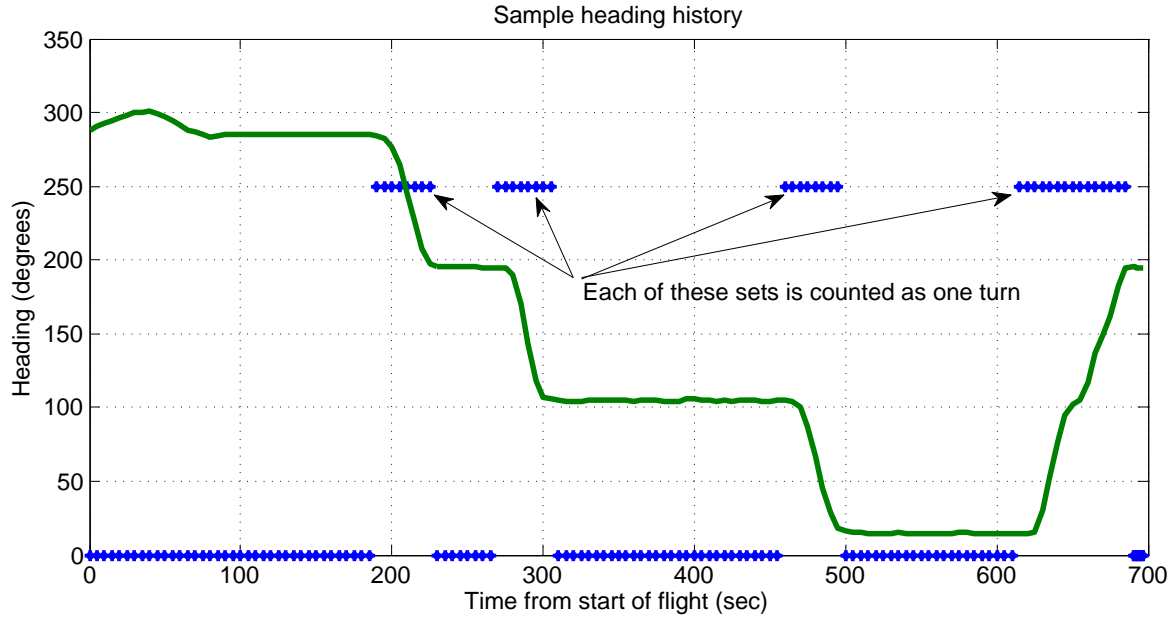


Figure 2. Plot of heading history for one flight

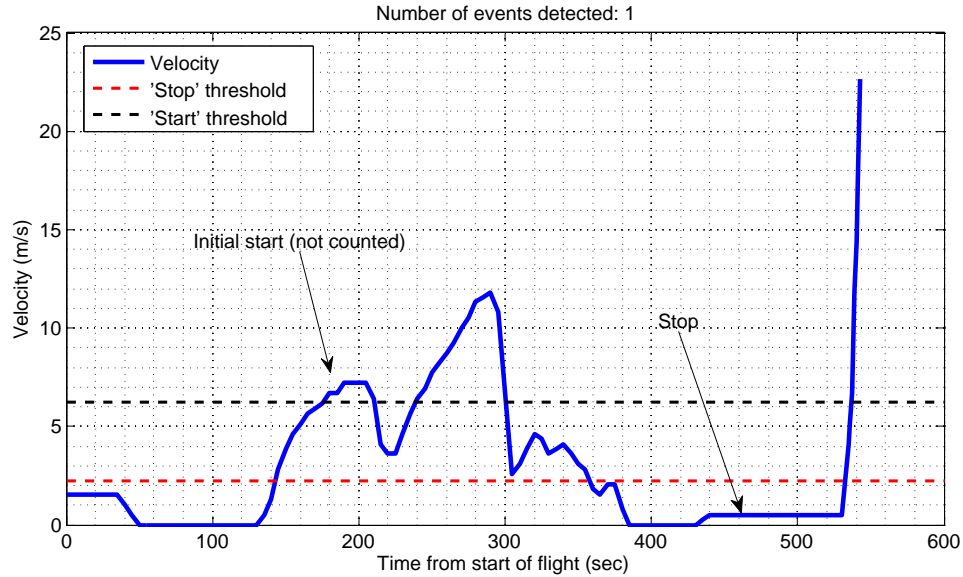


Figure 3. Flight with a single stop

IV. Estimation of Taxi-out Fuel

Having extracted the different taxi phases, we now investigate two possible linear regression models that estimate the fuel burn (normalized by square root of ambient absolute temperature) as a function of different

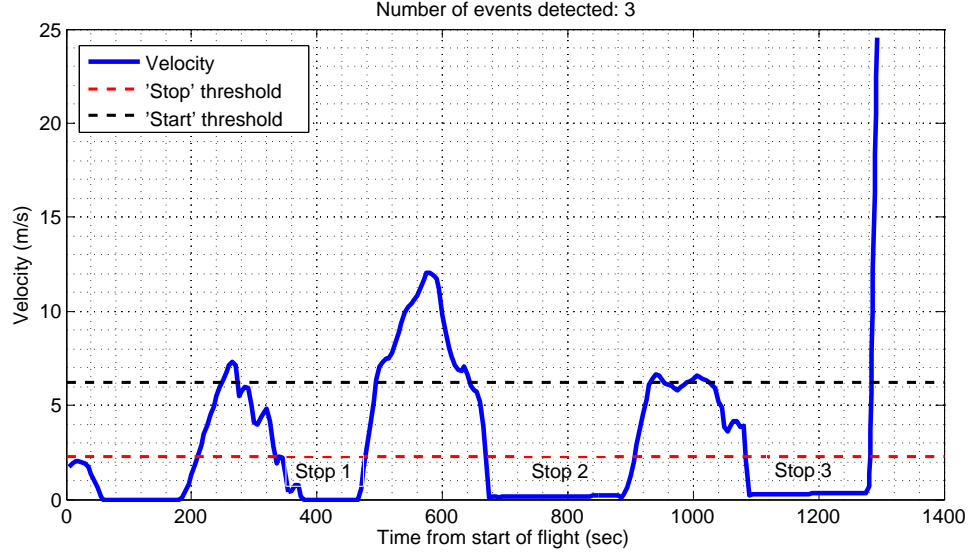


Figure 4. Flight with three stops

independent variables.

A. Model 1: Taxi time, number of stops and number of turns as independent variables

1. Formulation: Model 1

According to our initial hypothesis, total fuel burn on the ground would be a function of the taxi time, number of stops and number of turns made by the aircraft. It is easy to see that taxi time would be a determinant of fuel burn. In addition, given that the engines run at constant thrust for a large part of the taxi-out process, we would expect the effect of taxi time on fuel burn to be linear. Stops were expected to affect fuel burn because of the breakaway thrust required to start moving once an aircraft was stopped. This should add a relatively fixed fuel penalty per stop, resulting in a linear effect of the number of stops on fuel burn. Similarly, turns would require some adjustment of the power setting, but assuming that the adjustment would be similar for each turn, this effect should also be approximately linear. Finally, we know from available literature^{9,10} that engine sfc (specific fuel consumption) is proportional to the square root of ambient temperature. Therefore, we normalized for the effect of ambient temperature experienced by each flight, by formulating the regression as follows:

$$\frac{f}{\sqrt{T_{amb}}} = a_1 + b_1 \cdot t + c_1 \cdot n_s + d_1 \cdot n_t \quad (1)$$

Here, f is the total fuel consumed, t is the total taxi time, n_s is the number of stops, and n_t is the number of turns made by the aircraft during taxi. a_1 , b_1 , c_1 and d_1 are the parameters to be estimated. The coefficient b_1 will be the baseline fuel consumption rate of that aircraft type during taxi. Note that actual variation of temperature in the available data was only 17 K, which would not significantly impact the regression results, even if we had not normalized the fuel burn.

2. Results: Model 1

Table 3 lists the results of the parameter estimates calculated using least-squares regression. The estimates are accompanied by corresponding statistical p -values. A threshold of 0.1 was assumed for inferring statistical significance of each variable, *i.e.*, variables with p -values below 0.1 are assumed to be statistically significant. The aircraft types for which one or more of the variables are statistically insignificant have been highlighted.

Table 3. Regression Results: Number of Stops and Turns

Type	Constant		Taxi Time		# Stops		# Turns		Corr.	St. Dev.
	a_1 (kg/K ^{0.5})	p_{a1}	b_1 (kg/s-K ^{0.5})	p_{b1}	c_1 (kg/K ^{0.5})	p_{c1}	d_1 (kg/K ^{0.5})	p_{d1}	ρ	σ (kg)
A319	-0.01	0.94	0.0124	0	-0.01	0.82	-0.02	0.48	0.99	7.82
A320	-0.26	0.05	0.0125	0	0.1	0.1	-0.02	0.31	0.99	9.8
A321	-0.19	0.19	0.0133	0	0.15	0	-0.05	0.03	0.99	7.34
A330-202	0.98	0	0.0192	0	0.94	0	-0.02	0.8	0.97	19.29
A330-243	-1.6	0	0.0265	0	0.24	0	0.09	0.01	0.99	12.15
A340-500	-1.56	0	0.0371	0	0.25	0.27	0.07	0.46	0.99	38.31
ARJ85	-0.28	0	0.0103	0	0.08	0.02	0.01	0.28	0.99	4.67
B757	0.24	0.18	0.0175	0	0.19	0.06	-0.1	0.02	0.98	9.58
B767	-0.22	0.48	0.0178	0	0.73	0	0.15	0.01	0.95	23.32
B777	-1.71	0	0.0338	0	0.19	0	-0.01	0.8	.99	11.54

Note that some outliers with unusually long taxi times were ignored in the estimation procedure. There are no more than one or two of such points for any single type of aircraft, constituting approximately 0.5% of all data. Note also that the parameters a_1 , c_1 and d_1 have units of kg/K^{0.5}, while parameter b_1 has units of kg/s-K^{0.5}. In fact, b_1 should correspond to the (normalized) fuel burn index for the aircraft type, multiplied by the number of engines. A comparison with Figure 1 shows that this is indeed the case. Some important points to note, regarding Table 3, are listed below:

- p -values for the parameter b_1 are uniformly zero, which means that taxi time is certainly a determinant of fuel consumption. This was, of course, expected. It also contributes the most to total fuel consumed.
- The statistical significance of the number of stops depends on the aircraft type. Some types show a definite relationship, while others show almost none.
- The coefficients of the number of turns are very small, even in comparison with the coefficients of number of stops. Given the dominance of taxi time in the regression, the effect of turns on fuel burn can be said to be negligible.
- The statistical significance of the number of turns also depends on aircraft type. In addition, the significance of stops and turns does not appear to be related.
- ρ is the correlation coefficient between the estimated fuel burn and the actual fuel burn. These values are uniformly high.
- σ is the corresponding standard deviation of the residuals. Note that the reason that some of these values are larger than others is because the aircraft themselves are large (Table 1), which means that the total fuel consumed is more as well. The ratio of σ to average total fuel consumed is more or less the same for all types of aircraft.

3. Variability in statistical significance of stops and turns

The differences in p -values across aircraft types led us to investigate the stopping process in more detail. As seen from Figures 5 and 6, a start from having stopped was accompanied by a spike in fuel consumption in some cases, and by none in other cases. This variation was noticed between flights of the same aircraft type, across different aircraft types, and sometimes even between two different stop events on the same flight. No common characteristic was found to explain this difference across aircraft types. Possible factors considered were aircraft size, engine manufacturers, locations of operating airports, aircraft weight class and period of initial introduction of the aircraft. In addition, when considering the thrust setting profile, we find that acceleration events after stops that were not accompanied by an increase in the fuel burn rate were not accompanied by a change in thrust setting either. Therefore, we conclude that the difference in results was

due to differences in pilot behavior (whether reduction of thrust when stopping was more or less prevalent for the given aircraft type). A similar argument would hold true for thrust characterization during turns. Since our aim was to model the effect of stops as executed by the *average* pilot, we chose not to investigate the specific reasons for this variation.

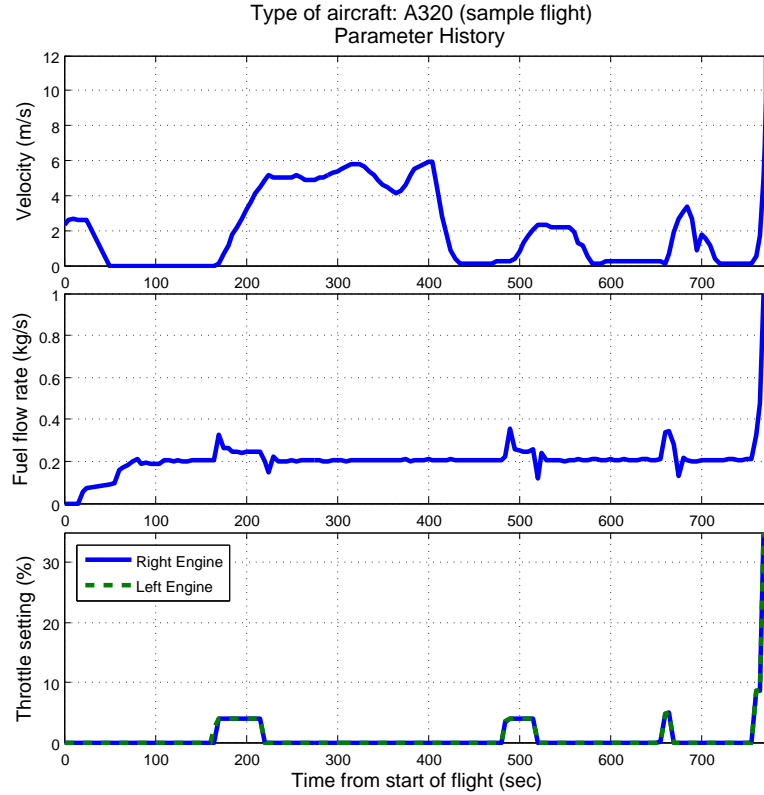


Figure 5. Simultaneous plot of velocity, fuel consumption rate and engine thrust settings for an Airbus A320: Increase in velocity (after stops) accompanied by spikes in fuel flow rate

B. Model 2: taxi time and number of acceleration events as independent variables

1. Formulation: Model 2

The model discussed previously produced good estimates of the total fuel burn, as seen from the values of the standard deviation of residuals. However, the differences in statistical significance of the explanatory variables suggested that other factors might be more important determinants of fuel burn. We therefore decided to drop the number of stops and number of turns from the regression, and instead add the number of acceleration events as an independent variable. The logic behind this decision was that fuel flow rates were seen to increase for aggressive starts from standstill, as opposed to gradual ones. An acceleration event was logged if the aircraft accelerated at more than 0.15 m/s^2 for at least 10 seconds. In Equation (2), n_a is the number of acceleration events. The other variables have the same definition as before.

$$\frac{f}{\sqrt{T_{amb}}} = a_2 + b_2 \cdot t + c_2 \cdot n_a \quad (2)$$

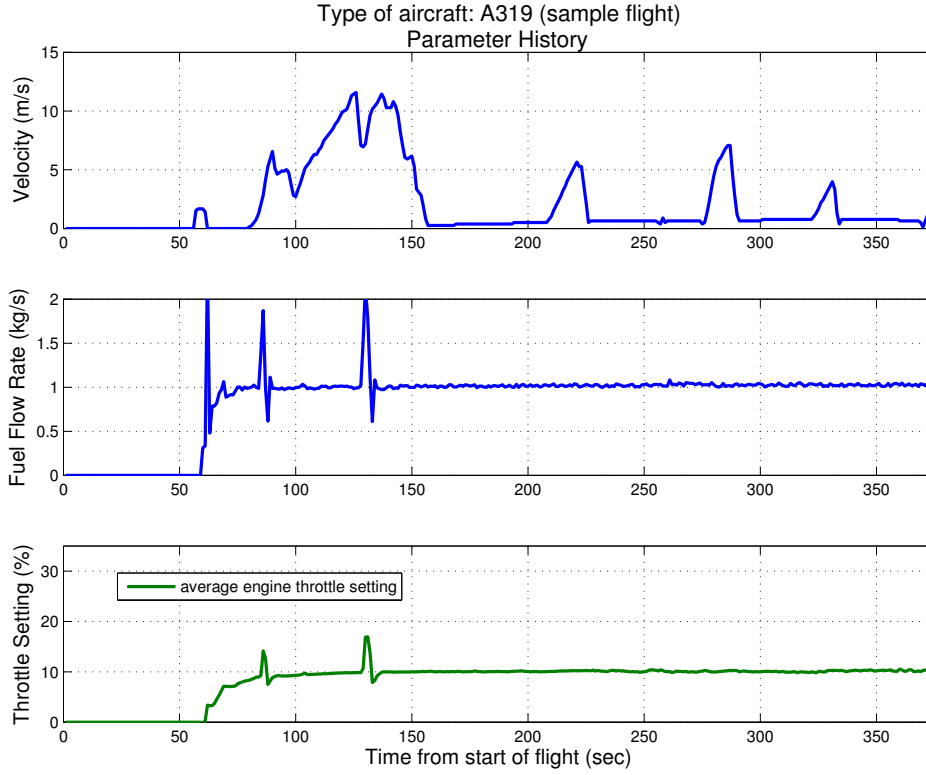


Figure 6. Simultaneous plot of velocity, fuel consumption rate and engine thrust settings for an Airbus 319: No change in fuel flow rate during acceleration from some stop events.

2. Results: Model 2

The results from parameter estimation for Model 2 have been listed in Table 4. It is seen that both independent variables are statistically significant for all aircraft types but one. Even for the Boeing 757, the p -value for the number of acceleration events is not very large. Comparing the results with those from Table 3, we can also see that the correlation coefficients are higher and the standard deviation of residuals is consistently lower for Model 2. Figures (7)-(8) compare the fuel burn estimates to actual burn for two aircraft types. It is clear that the prediction accuracy of Model 2 is very good. Figure 9 shows a scatter plot of the residuals for the Boeing 777. The whiteness of these residuals was tested by calculating the autocorrelation of the residuals vector within each aircraft type.^{11,12} A sample plot of the result is shown in Figure 10. It can be seen that the residuals are white, which means that optimal use of the information in the dataset has been made.

V. Results

The two regression models provide us with several interesting results. The first one is that the total taxi time is by far the main driver of fuel consumption. In other words, an accurate estimate of the fuel burn index along with the taxi time can provide a reasonably accurate estimate of the fuel consumption of a surface trajectory. The analysis also shows that for some of the aircraft types studied, the ICAO engine databank overestimates the fuel burn indices.

The FDR analysis also suggests that contrary to assumptions made in prior studies,⁷ stops or turns by themselves may not necessarily result in an increase in fuel burn rate, and therefore do not contribute much information beyond additional taxi time. We believe that this can be explained by the variability in pilot behavior (since there may not be significant thrust changes accompanying stops or turns), the

Table 4. Regression Results: Acceleration events

Type	Constant		Taxi Time		# Acc. Events		Corr.	Std. Dev.
	a_2 (kg/K ^{0.5})	p_{a2}	b_2 (kg/s-K ^{0.5})	p_{b2}	c_2 (kg/K ^{0.5})	p_{c2}	ρ	σ (kg)
A319	0.0811	0.31	0.0122	0.0	0.0965	0.0004	0.9938	6.85
A320	-0.0896	0.24	0.0124	0.0	0.1174	0.0000	0.9924	8.90
A321	0.0942	0.37	0.0129	0.0	0.0832	0.0184	0.9858	8.14
A330-202	0.2904	0.02	0.0217	0.0	0.3809	0.0001	0.9816	14.44
A330-243	-0.0903	0.25	0.0265	0.0	0.1007	0.0312	0.9965	9.12
A340-500	0.3626	0.10	0.0375	0.0	0.3984	0.0137	0.9918	30.59
ARJ85	0.0973	0.00	0.0102	0.0	0.0366	0.0203	0.9928	4.12
B757	0.2133	0.03	0.0173	0.0	0.0699	0.2007	0.9861	8.83
B767	0.1584	0.20	0.0202	0.0	0.1929	0.0012	0.9795	16.50
B777	-0.1223	0.02	0.0335	0.0	0.1385	0.0093	0.9985	8.75

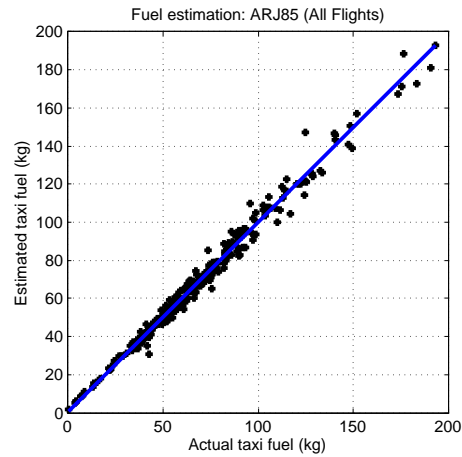


Figure 7. Regression for the Avro-RJ 85 using number of acceleration events

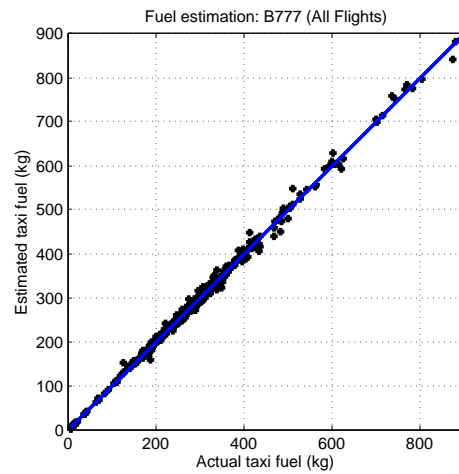


Figure 8. Regression for the Boeing 777 using number of acceleration events

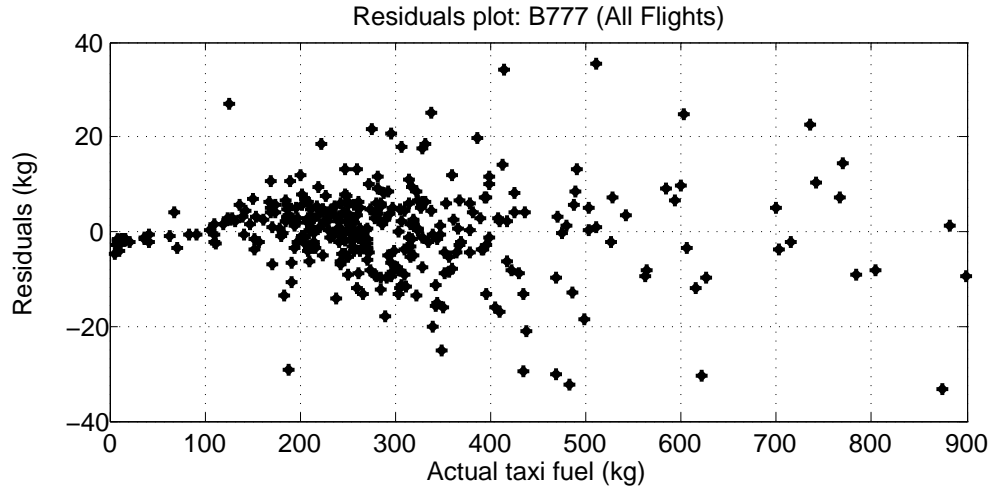


Figure 9. Residuals from the estimation of fuel consumption for the B777

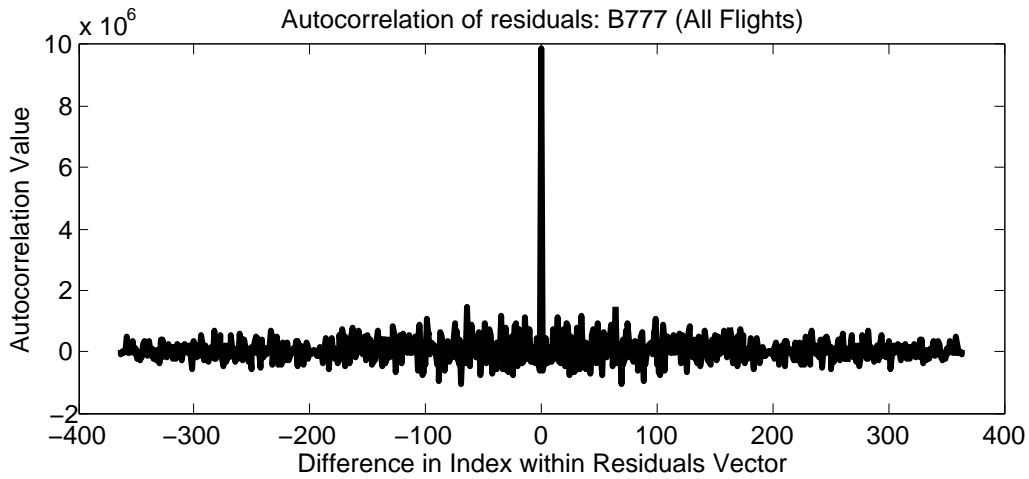


Figure 10. Autocorrelation of residuals for the B777

inherent variability in the thrust settings during taxi (which may be significantly different from the ICAO 7% assumption), and the significant dominance of the total taxi time as the driver of taxi fuel burn. However, acceleration events (defined as the aircraft accelerating at more than 0.15 m/s^2 for at least 10 seconds) have a small but statistically significant impact on the taxi fuel burn. The inclusion of these effects will provide a more accurate estimate of surface fuel consumption, and will also need to be considered in surface traffic optimization.

VI. Conclusions and Future Work

In this paper, we presented a procedure for the estimation of the taxi-out fuel consumption corresponding to an aircraft trajectory. We showed using FDR data that our model can provide more realistic estimates of fuel burn than the standard ICAO fuel burn indices, or other dynamics-based algorithms that use the ICAO fuel burn indices for conversion of surface trajectory to fuel burn. Therefore, our procedure can be used to better quantify the total fuel burn and CO_2 impact of surface operations at airports. Similar procedures may also be extended to estimate other emission impacts such as NO_x and unburned Hydro Carbons, which have a nonlinear dependence on fuel flow rates.

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