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A Physical Interpretation of Stagnation Pressure and Enthalpy Changes in Unsteady Flow

This paper provides a physical interpretation of the mechanism of stagnation enthalpy and stagnation pressure changes in turbomachines due to unsteady flow, the agency for all work transfer between a turbomachine and an inviscid fluid. Examples are first given to illustrate the direct link between the time variation of static pressure seen by a given fluid particle and the rate of change of stagnation enthalpy for that particle. These include absolute stagnation temperature rises in turbine rotor tip leakage flow, wake transport through downstream blade rows, and effects of wake phasing on compressor work input. Fluid dynamic situations are then constructed to explain the effect of unsteadiness, including a physical interpretation of how stagnation pressure variations are created by temporal variations in static pressure; in this it is shown that the unsteady static pressure plays the role of a time-dependent body force potential. It is further shown that when the unsteadiness is due to a spatial nonuniformity translating at constant speed, as in a turbomachine, the unsteady pressure variation can be viewed as a local power input per unit mass from this body force to the fluid particle instantaneously at that point. [DOI: 10.1115/1.4007208]

1 Introduction and Scope of the Paper

Unsteady flows are fundamentally different than steady flows. As stated succinctly by Kerrebrock [1]: "...in an unsteady flow there is a mechanism for moving energy around in the gas which is quite distinct and qualitatively different than steady flow. In steady flow, by and large, energy is carried along stream tubes.... This is not the case in unsteady flows...." This paper describes concepts associated with, and applications of, the mechanisms by which unsteady flows create this energy exchange.

Many turbomachinery texts include a demonstration that, for an ideal fluid, the flow must be unsteady for a turbine to produce shaft work or for a compressor to absorb shaft work [2–4]. The demonstration has two conceptual parts. The first is the relation between changes in the stagnation enthalpy of a fluid particle and the local time variation of static pressure,

$$\frac{Dh_t}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} \quad (1)$$

or the incompressible form [5–8]

$$\frac{Dp_t}{Dt} = \frac{\partial p}{\partial t} \quad (2)$$

The second is the illustration that fluid particles passing through a moving turbine or compressor blade row see a nonzero value of $\partial p / \partial t$.

The connection between unsteady flow and work exchange was made explicit by Dean [5], using arguments similar to those sketched in Fig. 1, which gives a representation of the static pressure in an axial flow turbine rotor passage. For a flow that is steady in the *relative* (blade fixed) frame of reference the pressure falls from pressure side to suction side and a static pressure varia-

tion exists in the *y*-direction. Because the blade moves at a speed Ωr , a temporal variation in static pressure is seen in the *absolute*, or stationary, frame of reference,

$$\left. \frac{1}{\rho} \frac{\partial p}{\partial t} \right|_{\text{abs}} = -\Omega r \left. \frac{1}{\rho} \frac{\partial p}{\partial y} \right|_{\text{rel}} \quad (3)$$

In the stationary frame, for fluid particles passing through the rotor blade row, $\partial p / \partial t < 0$, and, from Eq. (1), the stagnation enthalpy of a fluid particle falls. This is (as it must be) consistent with the energy extraction calculated from combining the expression for conservation of angular momentum and the steady flow energy equation, both applied in the stationary frame.

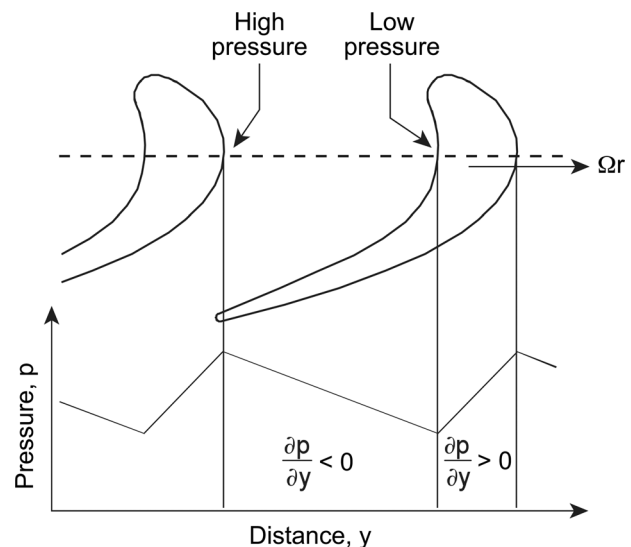


Fig. 1 Relative frame view of the static pressure distribution in an axial flow turbine rotor (after Dean [4])

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Turbomachinery aerodynamicists have not generally approached blade design from the perspective of unsteady flow. The methodology has relied on transforming the entering flow to a frame of reference in which the flow is (assumed) steady, stagnation quantities are conserved, and analysis is on more familiar ground, with subsequent transformation at the rotor exit, allowing a return to the stationary system. This approach has led to a long history of successful and sophisticated turbomachines, but two drivers increasingly push towards explicit inclusion of unsteady flows in the design process. One is the availability of computations that resolve unsteady features. A second, and more important, trend is that current machines have high levels of efficiency and there is incentive to grapple with unsteady flow issues as a route to possible performance increases.

The arguments concerning unsteady flow that were presented above are well recognized, but directly related aspects appear to be (at least from our observations at IGTI meetings) much less appreciated. First is that thinking in terms of Eqs. (1) or (2) provides insight into a number of manifestations of unsteady flow in turbomachines; for example the capability to define how unsteady effects scale with different parameters. Second is that the concepts presented provide a route to enhanced interpretation, and thus increased capability to make use of, computational simulations of unsteady flow. Third is the lack of a physical explanation for *why* the stagnation pressure changes if the flow is unsteady. While one can simply state that the equations lead to this consequence, the many discussions on the topic the authors have had with others (and between themselves) strongly suggest that such explanations would be useful to workers in the field.

An initial document in which the two threads—turbomachinery design and unsteady flow phenomena—were brought together in a clear and explanatory manner is a note by Roy Smith [9] on wake attenuation (see Sec. 2.3). With this as context, and perhaps as model, for the present discussion it is a pleasure to have the paper appear in a session dedicated to Roy and the insight he has brought to many different aspects of turbomachinery fluid dynamics.

To frame the issues, we illustrate some additional implications of Eq. (1) in the turbomachinery example of Fig. 1. Figure 2(a) gives a path line, as seen in the stationary system, for a “typical” fluid particle (i.e., defined by a velocity and pressure averaged across the passage) in the turbine. If the flow in the relative system is taken as steady, relative system streamlines and path lines coincide, with the average streamline closely following the blade passage. Path lines (particle trajectories) in the stationary system, however, can be almost normal to the blades. Figure 2(b) shows the link between the time derivative of the static pressure, and the variation in stagnation enthalpy, respectively, for the typical particle.

On one level, the behavior of the typical particle gives a useful picture of turbomachinery stagnation enthalpy and pressure rises; these quantities decrease in a turbine and increase in a compressor

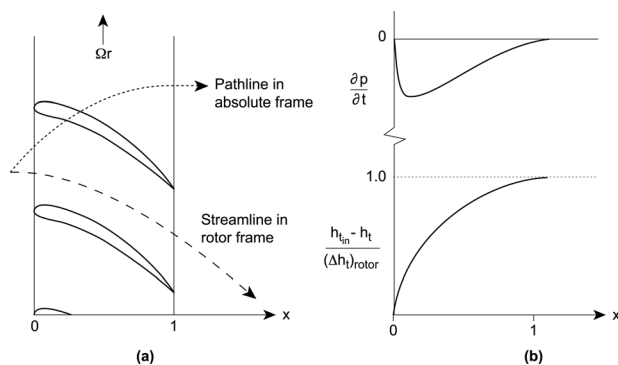


Fig. 2 (a) “Typical” (average) fluid particle motions in absolute and relative (rotor) frames; (b) time rate of change of static pressure and stagnation enthalpy for the typical particle

in accord with the unsteady pressure field associated with the blade forces. Within the passage (along the dashed line in Fig. 1, for example) blade forces generally point from pressure to suction surface in accord with the decrease in stagnation pressure.

On another level, however, the pressure upstream of the blade row varies about the average in the pitchwise direction so $\partial p/\partial t$ has both negative and positive values. Some particles thus have instantaneous *increases* in stagnation enthalpy, and their change along a path line is not monotonic. The association of increases in stagnation enthalpy with the predominant direction of the blade forces is less evident for these particles, as is the concept of blade forces in the upstream and downstream regions. The route to providing an appropriate interpretation of $\partial p/\partial t$ for these regions, as well as more generally throughout unsteady turbomachinery flow, is therefore not through direction considerations of blade forces, as is often implied with reference to Eq. (1). In this context the aim of the paper is to provide two items: (i) clear illustrations of the effects of flow unsteadiness on time-mean turbomachinery performance, and (ii) a description of the physical mechanism that underpins this alteration.

2 Stagnation Enthalpy and Stagnation Pressure Changes Due to Unsteady Flow

In the next sections we examine four examples of turbomachinery flow to illustrate the effects of unsteadiness in determining the time mean flow: (i) work input in tip clearance flow, (ii) wake interactions with blade surfaces, (iii) the effect on losses of wake behavior in downstream blade rows, and (iv) the effect of wake phasing on compressor work input. The phenomena are different, but it will be seen that the underlying ideas about stagnation enthalpy change provide a powerful framework for understanding and estimation of the physical mechanisms and magnitudes of the effects.

In the discussions below, we emphasize that the features encountered can be described in terms of inviscid fluid mechanics, although they are modified in practice (mainly damped) by viscous effects and heat transfer. Further, the essential behavior can be seen from the incompressible, constant density, inviscid flow arguments that are presented. Viscous stresses, heat transfer, and compressibility change the quantitative magnitudes, but not the central physical features.

2.1 Turbine Casing Stagnation Temperature Variation.

The first example is given in Fig. 3, which shows contours of

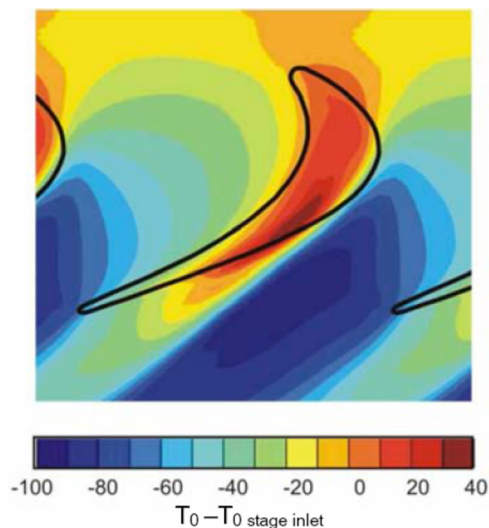


Fig. 3 Computed casing stagnation temperature in an HP turbine (Thorpe et al. [10])

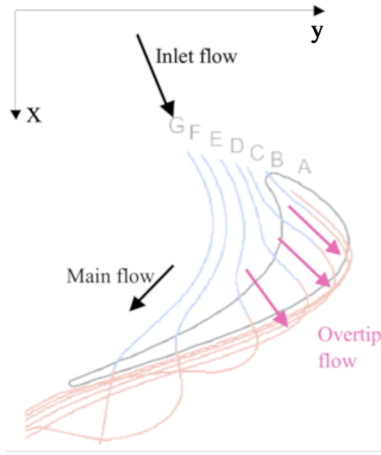


Fig. 4 Tip leakage flow trajectories in an HP turbine in the relative frame (adapted from Thorpe et al. [10])

the computed casing stagnation temperatures in a high-pressure turbine [10]. The feature of interest here is the stagnation temperature within the tip gap, which is higher than that at the inlet to the stage. The reason can be seen with reference to Fig. 1. The static pressure in the tip clearance region increases from suction side to pressure side, so fluid particles in the gap see a static pressure field in the stationary frame with $\partial p / \partial t > 0$. Figure 4 shows the trajectory of the leakage flow which is subjected to such a time variation. This rationale, used by Thorpe et al. [10] to explain the results of Fig. 3, can be regarded as an extension of the two-dimensional description of pressure and stagnation enthalpy changes in [5].

The casing temperature example involves a single blade row, so we can also motivate the result from steady flow considerations in the relative system. If the radius change of the tip clearance streamlines can be neglected, the relative stagnation enthalpy, h_{rel} , is constant for a fluid particle, and, from velocity triangles,

$$h_{abs} - (h_{abs})_{inlet} = \Omega r [u_{\theta rel} - (u_{\theta rel})_{inlet}] \quad (4)$$

The relative tangential velocity increases within the tip gap, where flow accelerates in leaking from the pressure to the suction side. The absolute stagnation enthalpy thus also increases within the tip gap before falling again as the tip leakage is turned back towards the passage from which it originally came.

2.2 Freestream Stagnation Pressure Variation in Turbine Blade Passages. The interaction of wakes with a downstream blade row is a situation that cannot be made steady by choice of

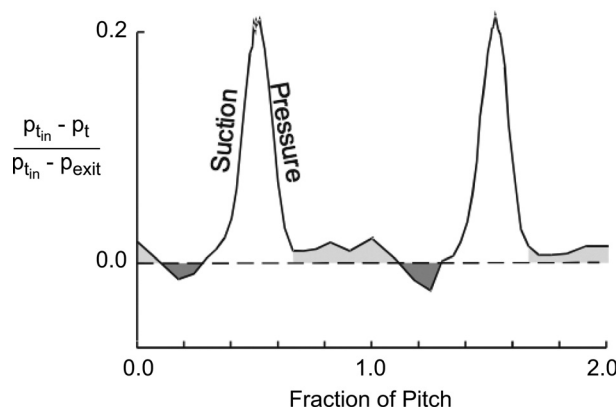


Fig. 5 Measured time-mean absolute stagnation pressure [1/4] axial chord downstream of LP turbine blades [11]

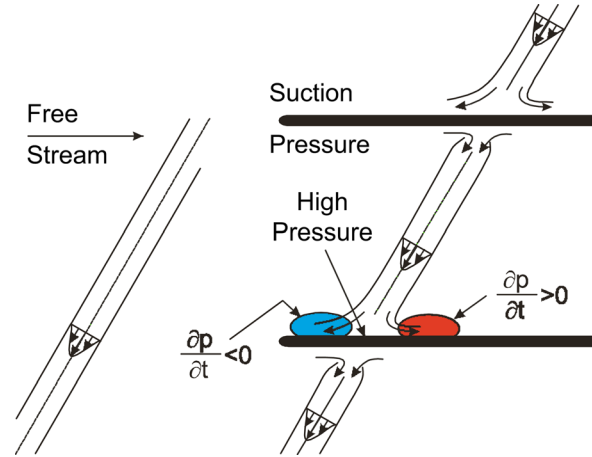


Fig. 6 Schematic of wake-blade interactions [11]

coordinate system. In the freestream between wakes from an upstream row the blade suction side stagnation pressure can be higher than the mean inlet level [11]. Measurements of this effect, for a cascade of low pressure (LP) turbine blades with wakes impinging, are given in Fig. 5, which shows the downstream time-mean stagnation pressure, referenced to the upstream value, as a function of the location across the blade pitch.

Computations of the incoming wake motion within the cascade indicate that, as first proposed by Meyer [12], the wakes can be viewed as a slip velocity superposed on an undisturbed freestream giving rise to a jet with velocity toward the source of the wake. As the wakes move through the passage, the impact of the jet on the suction surface creates a static pressure that varies with time in the relative (blade fixed) system. On the suction surface a region of high static pressure is created that moves with the wake.

Figure 6 portrays the moving high and low static pressure areas associated with the wake impingement, and the corresponding regions of high relative frame stagnation pressure ahead of the wake and low stagnation pressure behind the wake. For this case, the tighter streamline curvature on the downstream side of the wake, close to the suction surface, results in a higher spatial pressure gradient than on the upstream side. In the frame of the wake flow the spatial gradient is experienced as a higher temporal rate of change of static pressure, and hence stagnation pressure, on the downstream side as the wake is convected through the passage. The applicability of these basic ideas is well demonstrated although detailed experiments and unsteady computations for curved blade passages show additional features (and complexity).

2.3 Wake Attenuation in Compressors. The differences between steady and unsteady fluid motion are highlighted by examining the model problem of attenuation or amplification (in velocity difference and in width) of an inviscid wake as it moves through a downstream blade row, first for steady, then unsteady, flow. A wake in a *steady* flow with an increasing or decreasing static pressure in the freestream direction can be described as in Fig. 7, which shows the wake and the freestream in a diffuser. The changes in wake and freestream velocity are found by combining the one-dimensional momentum and continuity equations as

$$\frac{dA}{A} = -\frac{du}{u} = \frac{dp}{\rho u^2} \quad (5)$$

In Eq. (5), A and u are the area and velocity of either the wake or freestream, with the assumption that static pressure variation normal to streamlines can be neglected so the pressure change, dp , is the same for both. The wake has a lower dynamic pressure than the freestream, and Eq. (5) indicates that the fractional change in the velocity of the former is larger than in the latter. In steady

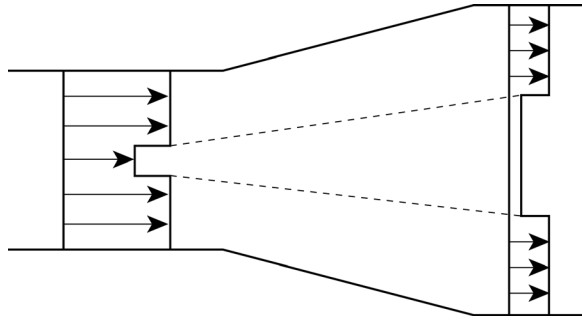


Fig. 7 Wake growth in a pressure gradient in steady flow

flow through a diffusing passage the fractional area occupied by a wake thus increases from inlet to exit, with the converse true for a nozzle. The mixing out of a wake at the state of the exit to the diffuser, which takes place between two streams of larger velocity difference than at the inlet, thus results in a larger mixing loss. Diffusing then mixing implies increased loss compared to mixing without diffusing.

The wake behavior is different, however, if the downstream blade row is in motion relative to the row in which the wake was created, so the flow is *unsteady*. A pioneering paper on this topic was that of Smith [9], whose treatment we follow in considering a stator wake, i.e., a region of low axial velocity, passing through a downstream rotor. If the static pressure can be considered uniform across the wake, particles in the wake see the same pressure variation, $\partial p / \partial t$, as the freestream. Because of the lower velocity and thus longer convection time, however, they are exposed to this unsteady pressure field for a greater time. From Eq. (2), for the motion over an incremental distance δx , the change in stagnation pressure can be approximately related to the axial velocity as

$$\delta p_t \approx \left(\frac{\partial p}{\partial t} \right) \delta t \approx \left(\frac{\partial p}{\partial t} \right) \frac{\delta x}{u_x} \quad (6)$$

Compressor stator wake fluid experiences a greater increase in stagnation pressure than does the freestream, so the difference in stagnation pressure at the rotor exit is less than at the inlet. This effect, known as “wake recovery” (Smith [9], see also [13,14]), has been addressed computationally in more detail by Tan and Valkov [15] and Van Zante et al. [16]. The reduction in the difference between the wake and free stream stagnation pressures, with a consequent reduction in the velocity difference and hence the mixing loss, is seen as an important factor in the efficiency increase occurring at subsonic Mach numbers when compressor axial spacing is decreased.

For turbine stator wakes passing through a rotor the wake behavior is less straightforward than that described above [17,18]. There is a need to address this aspect in turbines further, on both a quantitative and mechanistic basis.

2.4 Effects of Wake Phasing on Compressor Performance.

An unsteady flow encountered in turbomachinery and in bluff body flows is a row, or rows, of vortices, for example the double row of vortices in a wake. A moving vortex row in a stationary coordinate system will have, even in an isentropic flow, changes in time-mean stagnation temperature and pressure across it. For the basic example of a straight row of point vortices convecting along the x-axis, as in Fig. 8, the time-mean stagnation enthalpy and pressure change discontinuously across the x-axis. This can be seen from Crocco's Theorem, written below in a time averaged form

$$(\mathbf{u} \times \boldsymbol{\omega} = \nabla h_t)_{\text{time averaged}} \quad (7)$$

The velocity and vorticity fields are convected along with the vortices and are thus unsteady in the stationary coordinate system.

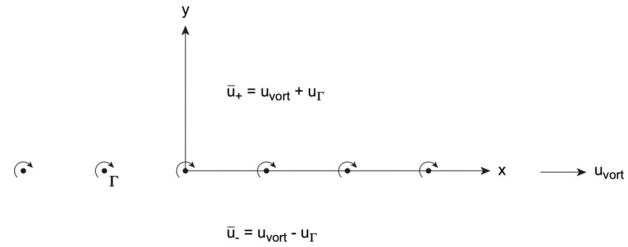


Fig. 8 Moving row of (point) vortices moving along the x-axis at velocity = u_{vort} . Time-mean velocity seen by stationary observer is \bar{u}_+ for $y > 0$ and \bar{u}_- for $y < 0$; $\bar{u}_+ > \bar{u}_-$.

For the row of point vortices, all the vorticities are confined to discrete locations on the x-axis so there is no change in time-mean stagnation enthalpy except for the step change across the x-axis [6].

A row of moving vortices is thus necessarily associated with a variation in stagnation temperature and stagnation pressure¹ [19]. An example in which this flow feature is of interest is the interaction of a transonic compressor with an inlet guide vane (IGV). The rotor shock waves impinge on the IGV, creating a time varying circulation and thus shedding of vorticity, resulting in a double row of counter-rotating vortices that convect downstream to the rotor. The shedding is locked to the rotor passing by the upstream pressure field of the rotors, so there are rows of vortices that enter each rotor passage at the same relative pitchwise orientation. If the axial spacing of the rotor and the guide vane is changed, the phasing of the shedding, and thus the pitchwise location of the vortices, is also changed.

Figure 10 [20] shows computed vortex paths for two different IGV-rotor spacings in a transonic compressor stage. In Fig. 9(a) the counterclockwise vortices (denoted by the pink line) are near the blade, whereas in Fig. 9(b), at a different IGV-rotor axial spacing, and thus a different convection time after the shock impinges, they are near the center of the passage.

The time-mean consequence of a row of vortices is, as described, a stagnation pressure nonuniformity. The difference in the location of the vortices implies a different stagnation pressure of the fluid surrounding the blade in the two cases. In one case the region of high stagnation pressure is near the center of the passage and the region of low stagnation pressure encompasses the blade and blade boundary layers; in the other case the location of the two regions is reversed.

Figure 10 depicts the vortex configuration corresponding to the vortices near the center of the passage and the corresponding stagnation pressure distribution entering the rotor blade passage, with high stagnation pressure near the center of the passage. In this situation the blade boundary layers have a lower “freestream velocity” than if the regions of high stagnation pressure were near the blades. They are less able to negotiate the rotor pressure rise, yielding a larger exit displacement thickness and a 3% decrease in work input. Further, the variation in performance with the phase of the wake also implies that the rotor performance can *either* increase or decrease as the IGV-rotor spacing is reduced, a behavior which is qualitatively different from the situation in subsonic flow.

Additional information about the effects of wake phasing and its influence on vortex position (and thus rotor performance) as well as the extension of these unsteady flow concepts to the modification of tip clearance flows can be found in [20].

¹The existence of a stagnation pressure nonuniformity can be inferred directly from Fig. 9. As seen in the stationary system, the velocity of convection of the row of vortices is the fluid velocity on the x-axis, which is u_{vort} . The velocity associated with the row of vortices, u_T , adds to this velocity for locations above the row and subtracts from it for locations below. The fluid velocity, and the stagnation pressure, is thus higher above the row than below it.

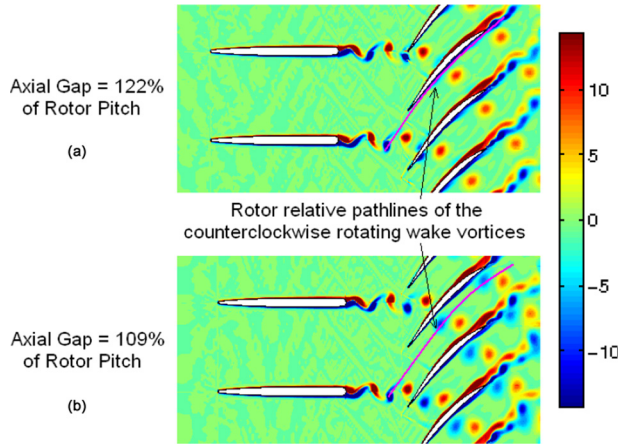


Fig. 9 Vorticity contours in a two-dimensional IGV-rotor configuration at two different interblade-row spacings. The pink line shows the path of the counter-rotating vortices in the rotor frame [19].

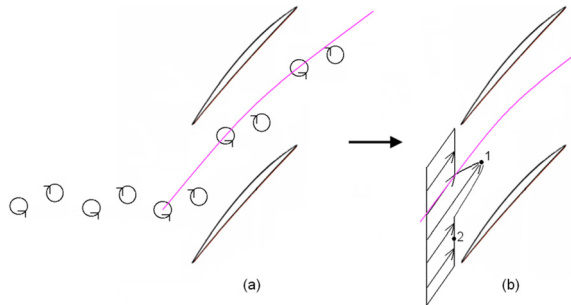


Fig. 10 Unsteady vortex flow field and time-mean representation. The pink line shows the path of the counterclockwise rotating vortices in the rotor frame. Label “1” denotes fluid with high relative stagnation pressure, while label “2” denotes fluid with low relative stagnation pressure.

3 A Physical Mechanism for Stagnation Pressure Changes in Unsteady Flow

Sections 2.1–2.4 illustrated different ways in which the stagnation pressure and temperature of a fluid particle can be altered in an unsteady flow. In sec. 3 we address the question of *why* (in terms of the physical process) the stagnation pressure and temperature change in the presence of a nonzero $\partial p/\partial t$. As mentioned earlier, we constrain the discussion mainly to incompressible inviscid flow, where the arguments can be made most cleanly.

It is useful at the start to clarify the difference between (i) increases in stagnation pressure in an unsteady flow and (ii) work transfer to the fluid. To do this we use the simple example of an incompressible liquid enclosed by a heavy piston in a cylinder. The stagnation and static pressures are the same and are determined by the weight of the piston. If additional weight is placed on the piston, static and stagnation pressure rise in step. In this unsteady process there is a stagnation pressure increase (which can be viewed as an increase in potential energy relative to the environment) even though no work is done on the fluid.

The above example is overly simplistic, but the same ideas apply to the fluid system in Fig. 11, in which a reservoir is fed by a pump at a constant flow rate, with an equal volume outflow through a nozzle. The pressure on the reservoir is p_0 and the reservoir discharges through the nozzle to atmospheric pressure, which is constant and equal to p_{atm} .

Suppose the pressure imposed on the reservoir is increased by an amount Δp during a time Δt . If Δt is small enough the frac-

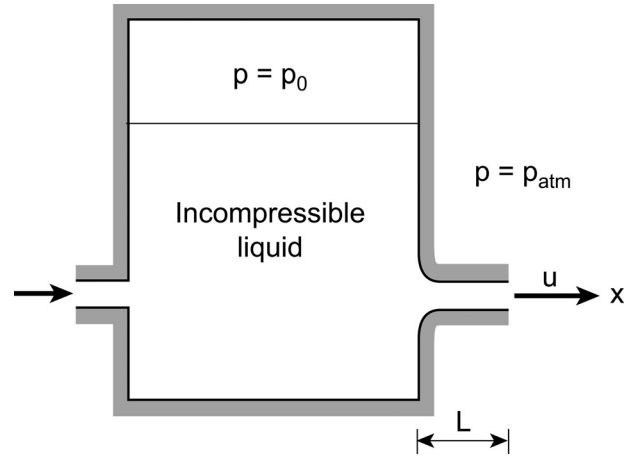


Fig. 11 Flow of an incompressible liquid out of a reservoir subjected to a time-dependent reservoir pressure

tional change in velocity in the nozzle will be negligible² and the process will take place with the velocity field nearly unchanged. If so, the reservoir fluid level will not change and, again, no work will be done on the fluid in the system by the increase in pressure during the time Δt . In spite of this the stagnation pressure throughout the reservoir (no matter what the size and shape of the reservoir and the amount of fluid included) will have increased by Δp , with an increase in the capability for work extraction from the stream.

Fluid in the nozzle, which is the only region with appreciable velocity, will experience a local acceleration proportional to the stagnation pressure gradient that exists along the nozzle as the result of the increase in reservoir pressure according to

$$\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) - \frac{1}{\rho} \frac{\partial p}{\partial x} = -\frac{1}{\rho} \frac{\partial p_t}{\partial x} \quad (8)$$

The acceleration will decrease as the nozzle velocity increases to the final value associated with the increased stagnation pressure [8]

$$u_{\text{final}} = \sqrt{\frac{2[(p_0 + \Delta p) - p_{\text{atm}}]}{\rho}} \quad (9)$$

The behavior of the fluid system just described casts the issues in a manner which we now recap: (i) no work is done in connection with the alteration in the stagnation pressure (ii) there is no change in the net force on a particle in the reservoir at any location away from the nozzle and (iii) the mechanical energy of the liquid, which resides essentially wholly in the fluid in the reservoir, has been changed relative to the ambient pressure level. Given that no work has been done, how can we characterize the process by which this has occurred? To answer this question we examine the analogy between pressure and force potentials. As will be seen the two play similar roles in altering the stagnation pressure.

3.1 Mechanical Energy Changes for a Fluid Particle. The equations of motion for incompressible, uniform density, inviscid flow are

$$\nabla \cdot \mathbf{u} = 0 \quad (10)$$

²For a nozzle of length L and a characteristic velocity in the nozzle of u , the fractional change in velocity during the time Δt is proportional to the quantity $[\Delta t \Delta p / \rho u L]$. If the latter is small the fractional change in velocity during Δt can be neglected.

$$\frac{D\mathbf{u}}{Dt} + \frac{\nabla p}{\rho} = \mathbf{F}_{\text{body}} \quad (11)$$

The term \mathbf{F}_{body} represents the possibility of body forces per unit mass that act on the fluid [21]. As is often the case (e.g., centrifugal force and gravity) we take the body force as the gradient of a potential, ψ , so that

$$\mathbf{F}_{\text{body}} = -\nabla\psi \quad (12)$$

Equation (11) then becomes

$$\frac{D\mathbf{u}}{Dt} + \nabla\left(\frac{p}{\rho} + \psi\right) = 0 \quad (13)$$

The force potential has been grouped with the pressure in Eq. (13) to introduce the idea that the two quantities play similar roles.

To derive the equation that describes stagnation pressure changes of a fluid particle, we take the scalar product of Eq. (13) and the velocity, \mathbf{u} , to obtain,

$$\frac{D(u^2/2)}{Dt} + \frac{D[(p/\rho) + \psi]}{Dt} = \frac{D(E/\rho)}{Dt} = \frac{\partial(p/\rho)}{\partial t} + \frac{\partial\psi}{\partial t} \quad (14)$$

Equation (14) defines a quantity, E , the total mechanical energy per unit mass, which can be changed by time variations in either static pressure or force potential (or both). Equation (14) can be written in terms of stagnation pressure, p_t , ($p_t = p + u^2/2$) as

$$\frac{1}{\rho} \frac{D(p_t + \psi)}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} + \frac{\partial\psi}{\partial t} \quad (15)$$

Equations (14) and (15) describe variations in solely mechanical quantities.

3.2 The Analogy Between Pressure and Force Potentials.

If the flow is steady [$\partial(\cdot)/\partial t = 0$], Eqs. (14) and (15) are often presented as a statement of conservation of mechanical energy for a conservative system, expressed as

$$\frac{D[(u^2/2) + (p/\rho) + \psi]}{Dt} = \frac{D(E/\rho)}{Dt} = 0 \quad (16)$$

The term $(u^2/2)$ is the kinetic energy per unit mass. As described by Batchelor [22], the pressure field produces a force on a fluid element “which is the same as a body force per unit volume equal to $-\nabla p$. This suggests that under certain conditions the pressure might play the part of a potential energy...” In this context, Eq. (16) states that the sum of kinetic energy per unit mass (the first term in the square bracket) and potential energy per unit mass (the second and third terms in the square bracket) is constant along a streamline. The mechanical energy can shift between any of the three quantities in the square bracket as the particle moves. Finally, *the behavior and effects of the pressure and the force potential are the same.*

The thread of Eqs. (13)–(16) is that there is a formal analogy between the terms $\partial(p/\rho)/\partial t$ and $\partial\psi/\partial t$. The effect of a time variation in pressure is the same as that of a time variation in force potential, and both create changes in the total mechanical energy of a fluid particle.

3.3 Physical Content of the Terms $\partial\psi/\partial t$ and $\partial(p/\rho)/\partial t$: Motion of a Particle in a Time-Varying Force Potential. Given that an analogy exists, we now need to define the physical content of the terms $\partial\psi/\partial t$ and $\partial(p/\rho)/\partial t$ ³. This can be done through

³They are *not* force terms, because pressure forces and body forces are represented by *spatial* gradients.

examination of a single particle of mass m in a potential field, following the description in [23]. Confining the discussion to one-dimensional particle motion, suppose the force potential is a function of position, x , and time, t , so that $\psi = \psi(x, t)$. The force per unit mass is given by $F = -\partial\psi/\partial x$, so the rate of change of mechanical energy per unit mass for the particle is

$$\frac{dE}{dt} = \frac{d}{dt} \left[\frac{u^2}{2} + \psi(x, t) \right] = u \frac{du}{dt} + \frac{\partial\psi}{\partial x} \frac{dx}{dt} + \frac{\partial\psi}{\partial t} \quad (17)$$

The quantity dx/dt is the particle velocity, u , so

$$\frac{dE}{dt} = u \left[\frac{du}{dt} + \frac{\partial\psi}{\partial x} \right] + \frac{\partial\psi}{\partial t} \quad (18)$$

The two terms in the square bracket express Newton’s second law for a unit mass and thus sum to zero. The rate of change of mechanical energy is therefore

$$\frac{dE}{dt} = \frac{\partial\psi}{\partial t} \quad (19)$$

Equation (19) states that if the force potential varies with time the mechanical energy of the particle, E , also varies. It is only when the force potential is constant in time that the energy is a constant⁴. Further, the change in energy does not explicitly correspond to work done on the particle, in that one cannot point to a force acting through a distance to change the energy.

A final comment concerns general circumstances in which the mechanical energy of a particle can change [22]. Suppose the potential is independent of time but the conservative force it represents is only part of the force on the particle. The total force is

$$\frac{F}{m} = -\frac{\partial\psi}{\partial x} + \frac{F'}{m} \quad (20)$$

where F' is an additional (nonconservative) force, such as friction. The time rate of change in mechanical energy is equal to the power delivered by the additional force

$$\frac{d}{dt} \left[\frac{u^2}{2} + \psi \right] = uF' \quad (21)$$

Equation (21), in which there is an evident term representing rate of work done, describes a qualitatively different physical mechanism than the change in energy due to the time dependence of the force potential.

The two main points concerning changes in the mechanical energy of a fluid particle can be summarized as follows. First, the total mechanical energy of a particle in a potential field can change if the force potential changes with time. Second, with respect to mechanical energy changes, the static pressure field in a fluid behaves in the same way as a time-varying force potential; changes in the static pressure with time are equivalent to changes in the force potential with time and thus to changes in the mechanical energy.

3.4 Unsteadiness Due to a Moving Spatially Periodic Disturbance (Turbomachinery Rotor). For flows in which the unsteadiness is produced by a translating spatially periodic disturbance, such as a turbomachinery rotor, an interpretation can be given which does link to forces and work done. For simplicity, consider a periodic, two-dimensional flow which has an x -component velocity nonuniformity moving upwards at velocity Ωr . The

⁴“Succinctly, *invariance under time translation implies energy conservation.*” [23] (italics due to the original author).

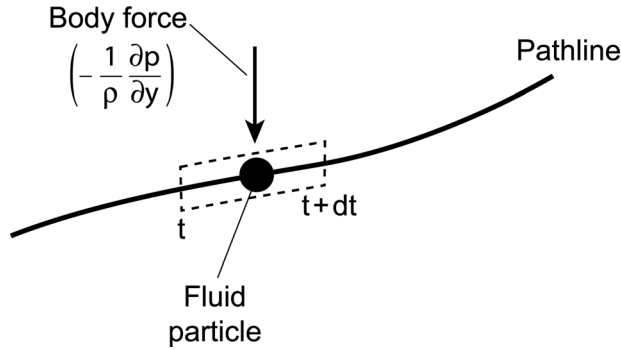


Fig. 12 Energy input to a fluid particle along a path line between times t and $t + dt$.

flow quantities at any x -location have a y and t dependence set by the moving disturbance and thus of the form $f[y - (\Omega r)t]$. Using the relation between the time and spatial dependencies given in Eq. (3),

$$\frac{1}{\rho} \frac{Dp_t}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} = - \left[\Omega r \left(\frac{1}{\rho} \frac{\partial p}{\partial y} \right) \right] \quad (22)$$

The term in Eq. (22), in square brackets is the product of the translational velocity of the disturbance (i.e., the rotor velocity for a turbomachine) and the component of the pressure force per unit mass in the direction of this translation. It thus represents a power input or extraction per unit mass at the location of a fluid particle. The situation is shown schematically in Fig. 12 for a fluid particle on a path line at times t and $t + dt$. The particle's change of stagnation pressure per unit mass during the interval dt is dp_t , representing work produced by a local power input of $-[(\Omega r/\rho)(\partial p/\partial y)]dt$.

Horlock and Daneshyar [7] have given a related expression for stagnation pressure change in terms of the circulation of moving blades. For a general unsteady flow, however, there appears to be no link that can be made with power input and thus the most appropriate view of the role of the $\partial p/\partial t$ term is as an unsteady force potential.

3.5 Stagnation Temperature Changes in a Compressible Flow. For a compressible flow thermal energy must also be considered, but we can still illustrate the ideas with recourse to the analogy between force potential and pressure. For a body force derivable from a potential, the energy equation is (e.g., [8])

$$\frac{D}{Dt} \left(h + \frac{u^2}{2} + \psi \right) = \frac{D}{Dt} (h_t + \psi) = \frac{1}{\rho} \frac{\partial p}{\partial t} + \frac{\partial \psi}{\partial t} \quad (23)$$

The correspondence between the pressure and the force potential term can again be seen, although for the compressible flow the density variation must be taken into account in the interpretation. With this caveat, however, the arguments carry over to compressible flow. The quantities of interest are either $h_t + \psi$ (if there is a body force) or h_t when (as in the more common situation) there is no body force. For a turbomachine where there is a periodic unsteadiness which moves at speed Ωr , we can again regard fluid particles to have a local power input or extraction of rate $[-(\Omega r/\rho)(\partial p/\partial y)]$.

4 Summary and Conclusions

Several examples, including both turbines and compressors, have been given of the way in which a time-varying static pressure can change the stagnation enthalpy and stagnation pressure. In some of these, the flow can be made steady by a change of reference frame, but in others, and as is often the case when two

or more turbomachine blade rows are involved, there is no coordinate system in which the flow can be viewed as steady. A physical interpretation has been given for the mechanism of stagnation pressure and temperature change in an unsteady isentropic flow. The time dependence of the static pressure, $\partial p/\partial t$, has been shown to play the same role as does a time-dependent body force potential in particle mechanics; a change in the force potential corresponds to a change in the system mechanical energy. When the unsteadiness is due to a spatial nonuniformity that translates at constant speed, as in a turbomachine, however, an interpretation can be made in terms of a local power input per unit mass to a fluid particle. Applications have been presented to illustrate the concepts.

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Nomenclature

Variables

- A = area
- b = blade axial chord
- F = blade force
- h_t = stagnation enthalpy
- p = static pressure
- p_t = stagnation pressure
- r = radius
- s = blade spacing
- t = time
- \mathbf{u} = velocity
- u = velocity magnitude
- x, y = spatial coordinates
- α = absolute flow angle
- ρ = density
- ω = vorticity
- Ω = angular rotation speed

Subscripts

- abs = evaluated in the absolute, or stationary, system
- E = freestream value
- rel = evaluated in the relative, or moving, system
- x = axial component
- 1, 2 = denote blade row inlet and outlet stations
- θ = circumferential component
- $\langle \rangle$ = denotes an average

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