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TYPE SYNTHESIS PRINCIPLE AND PRACTICE OF FLEXURE SYSTEMS IN THE FRAMEWORK
OF SCREW THEORY
PART III: NUMERATIONS AND TYPE SYNTHESIS OF FLEXURE MECHANISMS

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ABSTRACT

In recent years, the increasing of application requirements call for development of a variety of flexure mechanisms with high precision or large motion and both. Therefore, in Part III of this series of papers we demonstrate how to use the methodology addressed in Part I to synthesize concepts for two kinds of flexure mechanisms, i.e. kinematics-type flexure mechanisms (KFM) and constraint-type flexure mechanisms (CFM) with the specified-DOF (Degree of Freedom) characteristics. Although most of them utilize parallel configurations and flexure elements, there is a clear difference in the behavior of flexures between KFM and CFM, The resultant type synthesis approaches fall into two distinct categories i.e. freedom-based and constraint-based one, both of which have presented in Part I. In order to derive useful flexure mechanism concepts available for different applications, a general design philosophy and rules are summarized firstly. As the main content of this part, the classifications, numerations, and synthesis for KFM and CFM are made in a systematic way. As a result, a majority of new precision flexure mechanisms are developed. In addition, qualitative comparisons are provided to demonstrate the performance and application differences between kinematic-type and constraint-type flexure mechanisms with the same DOF.

1 INTRODUCTION

In many applications, such as manufacturing equipment and MEMS/MOEMS devices and cell manipulation in biotechnology, etc., there is an urgent need for three-dimensional positioning and manipulation mechanisms (often called micro/nano-positioners or micromanipulators) to perform an ultra high-precision motion. One of the most essential parts in the micromanipulator or micro/nano-positioners is the mechanism that transfers the actuator motions to the end-effector motion. At the meantime, it is also the main error source that may be detrimental to the high accuracy of the system. Conventional mechanisms with assembled joints and rigid links cannot meet this demand due to their coarse precision caused by friction, backlash, etc. Generally, the resultant errors

are also hardly traded off by controllers and sensors. **Flexure mechanisms** [1] of miniature and monolithic workpieces, instead, can provide highly accurate motion because they have less wear, no backlash and no friction.

A flexure mechanism is a kind of precision compliant mechanisms [2] depending on lumped-compliance or distributed-compliance characteristics of flexures among the mechanism. The motion of the flexure mechanisms is achieved via the deflection of these flexures in the mechanism. This kind of mechanism can be designed as a complex but monolithic structure, which makes it possible to achieve a single-DOF or multi-DOF, a very high positioning accuracy as well as to decrease the manufacturing and assembly costs. While parallel architecture is preferably employed in that it just can reinforce the advantages and make up for the deficiencies of general compliant mechanisms due to its high structural stiffness, the possibility of compensating errors for symmetric structure design, the improvement of its loading status as a consequence of the existence of passive joints in the mechanism.

There are a majority of proposed studies and design examples on flexure mechanisms in the existing literature in recent years. However, most of them deal with the precision mechanisms that use flexures as replacements for conventional joints, sharing the same topology as their rigid-body counterparts. In this paper we call them *kinematics-type flexure mechanisms (KFM)*. KFMs have several familiar structures such as 6-DOF, 3-DOF (XYZ , $XY\theta_z$, and $\theta_x\theta_yZ$), 2-DOF (XY), and 1-DOF (X). Hudgens and Tesar [3], Bi and Zong [4], McInroy and Hamann [5], Hesselbach *et al.* [6], Culpepper *et al.* [7], etc., have extensively studied 6-DOF motion stages. Chang *et al.* [8], Zhang *et al.* [9], Ryu *et al.* [10], etc., focused the study on $XY\theta_z$ stages. Lee [11], Canfield [12], Clavel *et al.* [13] etc. have paid more attention to $\theta_x\theta_yZ$ stage. Nevertheless, more research focuses on XYZ or XY stages. For example, Arai *et al.* [14] designed two type of XYZ stage with RPPR structure or RPRP structure, respectively. Others include the work of Clavel *et al.* [15], Yu *et al.* [16], Chen *et al.* [17], Awtar *et al.* [18], etc. Notice that a motion-

decoupled KFM seems advantageous over other types because of satisfactory stiffness and decoupled characteristics.

In some sense, type synthesis of the KFMs can be regarded as simply an extension of the theory that has already been developed for rigid link mechanisms, such as displacement group approach [19-21], and screw theory approach [22-24], etc., except that in the case of particular structural design of flexures as the replacement of conventional rigid joints and consideration of symmetric design. Type synthesis of these mechanisms, therefore, becomes comparatively easy.

In fact, almost all flexure elements, whether a thin strip or a wild plate, are essentially constraint elements as well as kinematic elements from the perspective of precision machine design [1]. In this case, the other kind of flexure mechanism, i.e. *constraint-type flexure mechanisms (CFMs)* can be obtained correspondingly based on the constraint-based approach. The foundations of the constraint-based method were developed by Maxwell [25] in the 1880s. It was recently revisited by Blanding [26] and several researchers at the MIT [27-30]. The fundamental premise of the constraint-based method is that all motions of a rigid body are determined by the position and orientation of the constraints (constraint topology) which are placed upon the body. The resultant flexure mechanisms have very different configurations with conventional KFMs although both of them are usually parallel in architecture. In particular, a Freedom and Constraint Topology (FACT) approach [28], founded on constraint-based design theory, paves the way for combining geometric computational techniques with a systematic methodology for the synthesis and analysis of flexure systems. The proposed method is very attractive because it is based upon motion visualization and is therefore well-suited to conceptual development. What is more interesting, the constraint-based method can be also resorted into the framework of screw theory [28, 31, 32], which makes a unified type synthesis for both KFMs and CFMs feasible.

Therefore, the objective of this research is to provide an effective means of realizing systematic even unified type synthesis for both KFMs and CFMs in the framework of screw theory. The whole research is built upon the proposed approach presented in Part I [32] and the flexures given in Part II [33]. The purpose is to find as many KFMs and CFMs concepts as possible, without concerning the detailed size parameters and performance specifications of each mechanism.

The rest of this paper is organized as follows. Firstly, a basic design philosophy and general rules for flexure mechanisms are presented. Section 3 discusses the duality and symmetry on kinematics and constraints from the view of FACT, which is build upon the equivalence among different kinematic or constraint topology. After that, the classifications, numerations and type synthesis of multi-DOF KFMs and CFMs are investigated in a systematic way, respectively. At last, qualitative comparisons are provided to demonstrate the performance and application differences between kinematic-type and constraint-type flexure mechanisms with the same DOF.

2 GENERAL PHILOSOPHY AND RULES FOR CONCEPTUAL DESIGN

2.1 Classifications of Flexure Mechanisms

Configuration of any a flexure mechanism can be formed by connecting rigid links and flexures. Depending on the function

of these flexures in the mechanism is kinematic element or constraint one, the flexure mechanisms can be divided into KFMs and CFMs, as mentioned above. Depending on the design and the overall motion of the mechanism in different dimensional space, the flexure mechanisms can formally be divided into two main categories: planar (or in-plane) and spatial (or out-of-plane) one. X -direction and 2DOF XY translational flexure mechanisms, Z -direction rotational or helical ones, and 3DOF $XY\theta_Z$ ones are included in planar mechanisms, whereas the typical spatial flexure mechanisms include 2DOF $\theta_X\theta_Y$ rotational ones, 3DOF $\theta_X\theta_Y\theta_Z$ rotational ones, 3DOF XYZ translational ones, 3DOF $\theta_X\theta_YZ$ stages, 6DOF ones, etc. Each category can be further subdivided into serial, parallel, and hybrid (serial-parallel) type, thus the classification of flexure mechanisms can be illustrated in table 1 and table 2.

Table 1. Classifications of flexure mechanisms



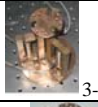
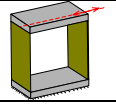

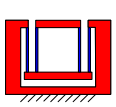
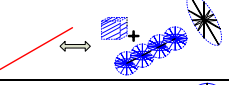
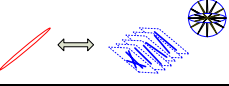
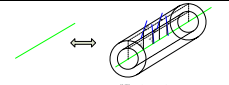
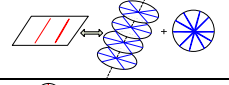
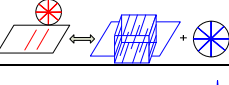
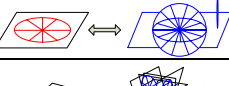
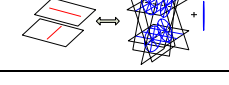
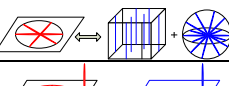
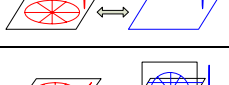
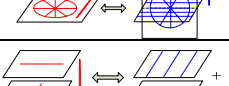

	KFMs	CFMs
Serial	 1RPS	 (Hale, 1999[27])
Parallel	 3-RPS	
Hybrid	 3-RPS&3-RRR	

Table 2. Multi-DOF flexure mechanisms and corresponding FACT

Dim	Class	FS Set	FACT
1	1R	$\mathcal{R}(N, u)$	
	1T	$\mathcal{P}(n)$	
	1H	$\mathcal{H}(N, u)$	
2	1R1T	$\mathcal{C}(N, s)$	
		$\mathcal{F}_2(N, u, n)$	
	2R	$\mathcal{U}(N, n)$	
		$\mathcal{N}_2(u, v)$	
	2T	$\mathcal{T}_2(n)$	
		$\mathcal{U}(N, n) \cup \mathcal{P}(n)$	
		$\mathcal{U}(N, n) \cup \mathcal{P}(u)$	
		$\mathcal{N}_2(u, v) \cup \mathcal{P}(n)$	

3	2R1T	$\mathcal{N}_2(u, v) \cup \mathcal{P}(u)$	
	2T1R	$\mathcal{F}(u)$	
		$\mathcal{R}(N, u) \cup \mathcal{T}_2(n)$	
	3R	$\mathcal{S}(N)$	
		$\mathcal{U}(N, n) \cup \mathcal{F}_2(N', u', n')$	
		$\mathcal{U}(N, n) \cup \mathcal{U}(N', n')$	
		$\mathcal{N}(u, v, w)$	
	3T	\mathcal{T}	
4	3R1T	$\mathcal{S}(N) \cup \mathcal{P}(n)$	
		$\mathcal{N}(u, v, w) \cup \mathcal{P}(n)$	
	3T1R	$\mathcal{R}(N, u) \cup \mathcal{T}$	
	2R2T	$\mathcal{F}(u) \cup \mathcal{R}(N, v)$	
5	3R2T	$\mathcal{S}(N) \cup \mathcal{T}_2(n)$	
	3T2R	$\mathcal{U}(N, n) \cup \mathcal{T}$	
6	3R3T	$\mathcal{S}(N) \cup \mathcal{T}$	
	2R1T + 2T1R	$\mathcal{F}(n) \cup \mathcal{U}(N, n) \cup \mathcal{P}(n)$	

As given in Table 1, there are three well-known configurations employed in the current design of multi-DOF KFMs, i.e. serial, parallel, and hybrid ones. Each configuration of KFMs may stem from its rigid-body counterpart and has its own set of cons and pros. Compared to serial KFMs, parallel and hybrid KFMs have the advantages of (i) low inertia, (ii) high structural stiffness, (iii) balanced mechanical structures without large overhanging masses, and (iv) compactness. On the other hand, these advantages come at a price as parallel KFMs have (i) small workspaces, (ii) poor dexterity, (iii) and non-linear kinematics and dynamics. One of the major challenges in the realization of parallel and hybrid KFMs is the type synthesis of a mechanism with the appropriate DOF and characteristics. While a serial KFM is readily realized by stacking appropriate flexible joints and link pairs together, a parallel KFM requires the intersection of the DOFs of its all kinematic chains to produce the desired DOF at its end-effector.

In addition, serial KFMs are relatively simple to design, but at the same time incorporate moving actuators and cables that

deteriorate their dynamic performance. Moving cables are sources of disturbance, which is detrimental for nanometric positioning. Moving actuators, especially when large range of motion is required, are bulky and reduce the bandwidth of the axes that carry them. Parallel designs are free of these problems due to ground mounted actuators, and are also usually more compact, but on the other hand, provide smaller ranges of motion. In spite of the difficulties and challenge associated with parallel schemes, they are well-suited for micro/nano-positioning applications by perfectly merging it with flexures and actuators. In particular, parallel CFMs can make up for some shortcoming existing in parallel KFMs such as smaller motion, and more error resources because of their more compact and simple flexures.

2.2 General Rules for Conceptual Design of Flexure Mechanisms

This section concerns with a summarization of a general rules for selecting and designing a flexure mechanism from the viewpoints of geometry and motion.

(1). *Mechanical structure should be simple and compact enough to shorten the structural loops.* In order to decrease the errors and the calibration times, the mechanical structure of the flexure mechanism should be compact and simple, and each part of the mechanism should be connected rigidly. It is also necessary to shorten the structural loops to reduce vibration, to minimize heat deformation, and to maximize structural stiffness to realize a high-precision machine. On the other hand, in order to be free from backlash and Coulomb friction, the monolithic structure is the primary design.

(2). *In order to improve the characteristics of the flexure mechanism, both a rational arrangement for each of its chain structure and selection of an optimized structure from multiple concepts are necessary.*

(3). *Trade off between the workspace and accuracy of the flexure mechanism.* One of the current concerns on flexure mechanisms is the reconciliation of submicronic (nanometric) repeatability with relatively large workspace. One is often obtained to the detriment to the other. Therefore, it is necessary to trade off between large workspace and high accuracy under the condition of enough large workspace.

In the flexure geometry, when flexures are connected in series, it prefers to adding DOFs, and when flexures are connected in parallel, it prefers to adding degrees of constraints (DOCs), whereas a hybrid system with serial-parallel structure may exhibit a compromised DOFs or DOCs. Therefore, when synthesizing a flexure mechanism with large than 4DOF, it is generally a serial or hybrid structural topology; on the contrary, a flexure mechanism with less than 4DOF may exhibit a parallel, serial, or hybrid topology.

3 DUALITY AND SYMMETRY ON FREEDOM SPACES AND CONSTRAINT SPACES

3.1 Equivalence of freedom (constraint) spaces, subspaces and complete spaces

At Part I, we have introduced the concept of both freedom spaces (FSs) and constraint spaces (CSs). Now we use the set operation to derive the equivalence of freedom (constraint) spaces obtained by different combinations of fundamental building blocks (FBBs) by taking a concrete example.

■ Three-dimensional mixed space spanned by a 2D couple subspace and an orthogonal line

The space is denoted as combination of two FBBs, that is

$$\mathcal{R}(N, s) \cup \mathcal{T}_2(s) \quad (1)$$

and a basis for this space satisfies the condition that

$$\begin{cases} \mathcal{S}_1 = (s; r \times s) \\ \mathcal{S}_2 = (0; s_2) \quad (s \cdot s_i = 0, i = 2, 3) \\ \mathcal{S}_3 = (0; s_3) \end{cases} \quad (2)$$

Through a linear transformation $(\mathcal{S}_1 + \mathcal{S}_2, \mathcal{S}_1 + \mathcal{S}_3)$, a equivalent basis can be obtained as

$$\begin{cases} \mathcal{S}_{e1} = (s; r \times s) \\ \mathcal{S}_{e2} = (s; r_2 \times s) \\ \mathcal{S}_{e3} = (s; r_3 \times s) \end{cases} \quad (3)$$

Eq. (3) just corresponds to a basis of line space $\mathcal{F}(s)$. Thus we have

$$\mathcal{R}(N, s) \cup \mathcal{T}_2(s) = \mathcal{F}(s) \quad (4)$$

■ Three-dimensional mixed space spanned by a 2D parallel-line subspace and an orthogonal couple

The space is denoted as combination of two FBBs, that is

$$\mathcal{F}_2(N, s, n) \cup \mathcal{P}(n) \quad (5)$$

and a basis for this space satisfies the condition that

$$\begin{cases} \mathcal{S}_1 = (s; r_1 \times s) \\ \mathcal{S}_2 = (s; r_2 \times s) \quad (s \cdot s_3 = 0) \\ \mathcal{S}_3 = (0; s_3) \end{cases} \quad (6)$$

Through a linear transformation $(\mathcal{S}_2 + \mathcal{S}_3)$, a equivalent basis can be obtained as

$$\begin{cases} \mathcal{S}_{e1} = (s; r_1 \times s) \\ \mathcal{S}_{e2} = (s; r_2 \times s) \\ \mathcal{S}_{e3} = (s; r_3 \times s) \end{cases} \quad (7)$$

Eq. (7) is also identical to Eq. (3) corresponding to a basis of line space $\mathcal{F}(s)$. Thus we have

$$\mathcal{F}_2(N, s, n) \cup \mathcal{P}(n) = \mathcal{F}(s) \quad (8)$$

Using the set operation, we have

$$\begin{aligned} \mathcal{F}(s) &= \mathcal{R}(L, s) \cup \mathcal{R}(M, s) \cup \mathcal{R}(N, s) \\ &= \mathcal{F}_2(M, s, n) \cup \mathcal{R}(N, s) \\ &= \mathcal{R}(M, s) \cup \mathcal{P}(n) \cup \mathcal{R}(N, s) \\ &= \mathcal{R}(M, s) \cup \mathcal{R}(N, s) \cup \mathcal{P}(n) \\ &= \mathcal{F}_2(N, s, n) \cup \mathcal{P}(n) \\ &= \mathcal{R}(N, s) \cup \mathcal{P}(n) \cup \mathcal{P}(n) \\ &= \mathcal{R}(N, s) \cup \mathcal{T}_2(n) \end{aligned} \quad (9)$$

The equivalence of apparently different freedom (constraint) spaces may also lead to an equivalent pattern representation corresponding to a specified space, and here we denote it as an equivalent freedom (constraint) space (EFS or ECS). In a general case, the space spanned by all the highest dimensional FBBs belonging to this space may form a *complete space*. Whilst other equivalent spaces are viewed as the *subspaces* of the complete space with the same dimension.

For instance, the complete space equivalent to $\mathcal{F}(s)$ is $\mathcal{F}(s) \cup \mathcal{T}_2(s)$, as shown in Fig 1.

■ Three-dimensional space spanned by a 2D parallel-line

subspace and a out-of-plane parallel line

The space is denoted as a combination of two FBBs, that is

$$\mathcal{F}_2(N, s, n) \cup \mathcal{F}_2(N', s, n) \quad (10)$$

Because two subsets of the compound space meet

$$\mathcal{F}_2(N, s, n) \subset \mathcal{F}(s), \mathcal{F}_2(N', s, n) \subset \mathcal{F}(s) \quad (11)$$

Hence, any element \mathcal{S}_i in the space $\mathcal{F}_2(N, s, n) \cup \mathcal{F}_2(N', s, n)$ is included in $\mathcal{F}(s)$, that is

$$\mathcal{S}_i \in \mathcal{F}(s) \quad (12)$$

On the other hand, according to the dimension law, we have

$$\dim(\mathcal{F}_2(N, s, n) \cup \mathcal{F}_2(N', s, n)) = 3 \quad (13)$$

Therefore, it can be conclude that

$$\mathcal{F}_2(N, s, n) \cup \mathcal{F}_2(N', s, n) \subseteq \mathcal{F}(s) \quad (14)$$

Therefore, $\mathcal{F}_2(N, s, n) \cup \mathcal{F}_2(N', s, n)$ is also a subspace of $\mathcal{F}(s) \cup \mathcal{T}_2(s)$ with the same dimension, also a space equivalent to $\mathcal{F}(s)$.

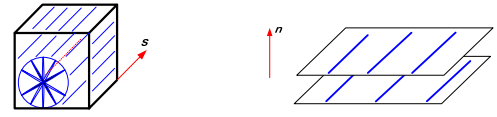


Fig.1 A space equivalent to $\mathcal{F}(s)$

Fig.2 A subspace of $\mathcal{F}(s)$

In this way, we may derive all *subspaces* of the complete space with the same dimension once a complete space is given.

3.2 Reciprocity and Symmetry on Freedom Spaces and Constraint Spaces

As described in the FACT method, a collection of commonly-used freedom and constraint line sets are denoted as FSs and CSs respectively. What is more important, luxuriant patterns representing a reciprocal or complementary mapping between topologies of FSs and CSs can be established in the framework of screw theory. Note that both FSs and constraint ones are essentially reciprocal screw systems or screw spaces. As such, the mapping can be obtained correspondingly based on screw theory. At Part I we have presented the detailed derivation on this reciprocity between FSs and CSs.

In fact, the reciprocity of FS and CS also leads to the uniqueness of their mapping from one to another. In other words, the FS of any given n -dimension kinematic system is unique to its $(6-n)$ -dimension CS, and the CS of any given kinematic system is unique to its FS. Screw system theory can prove this proposition. Table 2 numerates the patterns representing the uniquely mapping between topologies of FSs and their reciprocal or complementary CSs of typical flexure mechanisms, which can be obtained by Screw theory [29, 31]. Note that all FSs in this research are depicted as red color and all CSs are depicted as blue color. Also, a zero-pitch screw is represented by a line and an infinite-pitch screw is represented by a thin ellipse, while green line represents a general screw.

From Table 2 we also notice such an interesting fact that there exists symmetry and identity on the FACT patterns between an n -dimension (n DOF) kinematic system and some $(6-n)$ -dimension flexure system. In other words, every n -dimension FS looks geometrically identical to some $(6-n)$ -dimension CS and every n -dimension CS looks geometrically identical to some $(6-n)$ -dimension FS. For example, the FACT pattern corresponding to 1R flexure mechanisms is identical to the

pattern of $3R2T$ flexure mechanism, while the FACT pattern corresponding to $1T$ flexure mechanisms is identical to the pattern of $3T2R$ flexure mechanism. This symmetry and identity is very important to achieve the unified type synthesis of CFMs and KFM. The following synthesis process will show this importance with more detail.

4 TYPE SYNTHESIS OF CFMS

Before making a formal type synthesis for flexure mechanisms, it is necessary to review the type synthesis process presented in Part I.

4.1 Review of Type Synthesis Process

Step 1: Denoting the specified freedom of the flexure mechanisms using a visualized freedom space.

Step 2: Determine all possible equivalent reciprocal spaces representing the constraints of the mechanism based on the approach given in section 3.

Step 3: Select the appropriate reciprocal space types from the constraint space obtained in Step 2. In other words, select those constraint spaces which can be realized physically. For example, the constraint FBBs $\mathcal{L}(N, n)$, $\mathcal{F}_2(N, u, n)$, $\mathcal{R}(N, u)$ and $\mathcal{T}(n)$ just corresponds to a wide sheet flexure, a thin sheet flexure, and a slender wire flexure and a short wire one, respectively. Some constraint FBBs such as $\mathcal{S}(N)$ and $\mathcal{U}(N, n)$ have no direct mapping into flexure primitive, but they are both achieved through combinations of slender wire flexures. Some constraint FBBs such as $\mathcal{C}(N, u)$ and $\mathcal{T}_2(n)$, However, can not be achieved through one single flexure primitive or combinations of primitives. Thus we can obtain a reduced version by eliminating a part of impractical constraint space types.

Step 4: Find the appropriate physical arrangement for each constraint space type obtained in Step 3. As one can see, the choice of the basis constraint modules is not unique. And even for the same constraint space, there may be also multiple physical arrangements. These multiple solutions provide the possibility of an optimal design at the level of topology.

Step 5: Find an optimal mechanism profile with high performances in terms of task requirements from the results in Step 4. This topic includes optimized constraint arrangement and optimized constraint geometry, even predetermination of geometrical parameters of each constraint, etc.

4.2 Type Synthesis

In this section, the detailed synthesis procedure is ignored, here only the results are numerated in tables according to the difference in DOF of CFMs. Note that R denotes rotational DOF, T denotes translational DOF, and H denotes helical DOF in this paper.

(1). $1R$ Case ($\mathcal{R}(N, u)$)

Table 3 $1R$ cases corresponding to constraint subspaces

ECS Set symbol	ECS Pattern	CFM case	KFM
$\mathcal{L}(N, n) \cup \mathcal{L}(N', n') \cup \dots$			
$\mathcal{S}(N) \cup \mathcal{S}(N') \cup \dots$ ($NN' \parallel N''N'' \dots$)			
$\mathcal{F}(u) \cup \mathcal{S}(N) \cup \mathcal{S}(N') \cup \dots$			

$\mathcal{L}(N, n) \cup \mathcal{L}(N', n)$			
$\mathcal{S}(N) \cup \mathcal{S}(N')$			
$\mathcal{F}(u) \cup \mathcal{S}(N)$			
$\mathcal{F}_2(N, u, n) \cup \mathcal{L}(N', n')$			
$\mathcal{T}_2(n) \cup \mathcal{S}(N')$			PPS
$\mathcal{U}(N, n) \cup \mathcal{F}(n)$			
$\mathcal{F}_2(N, u, n) \cup \mathcal{S}(N')$			[RPS] [RRS]

(2). $1T$ Case ($\mathcal{P}(n)$)

Table 4 $1T$ CFM cases corresponding to constraint subspaces

ECS Set symbol	ECS Pattern	CFM case
$\mathcal{L}(N, n) \cup \mathcal{L}(N', n)$		
$\mathcal{L}(N, n) \cup \mathcal{L}(N', n) \cup \mathcal{L}(N'', n) \cup \dots$		
$\mathcal{F}(u) \cup \mathcal{F}(v)$		
$\mathcal{U}(N, n) \cup \mathcal{T}$		

(3). $1H$ Case ($\mathcal{H}(N, u)$)

In this case, the CFM is a system with a freedom space that consists of a single screw line.

Table 5 $1H$ CFM case corresponding to constraint space

FS Set symbol	CS Pattern	CFM case
$\mathcal{H}(N, u)$		

(Hopkins, 2009[28])

(4). $2R$ Case

In this case, the CFM is a system with a freedom space that consists of two skew lines or a disk or pencil.

(a) $\mathcal{N}_2(u, v)$

Table 6 $2R$ CFM cases corresponding to constraint spaces (I)

ECS Pattern	CFM case

(b) $\mathcal{U}(N, n)$

Table 7 2R CFM cases corresponding to constraint spaces (II)

ECS Set symbol	ECS Pattern	CFM case
$\mathcal{S}(N) \cup \mathcal{F}_2(N', u, n)$		
$\mathcal{U}(N, n) \cup \mathcal{U}(N', n')$		
$\mathcal{U}(N, n) \cup \mathcal{F}_2(N', u', n')$		
$\mathcal{L}(N, n) \cup \mathcal{R}(N', u)$		
$\mathcal{S}(N) \cup \mathcal{R}(N, n)$		

(5). 2T Case ($\mathcal{I}_2(n)$)

Table 8 2T CFM and KFM cases

ECS (EFS) Pattern	CFM	KFM

(6). 1R1T Case

In this case, a typical CFM is probably a system with a freedom space that consists of parallel line plane or a line and couple parallel to each other.

(a) $\mathcal{F}_2(N, u, n)$

Table 9 1R1T CFM cases corresponding to constraint spaces (I)

ECS Pattern	CFM case

(b) $\mathcal{L}(N, s)$

Table 10 1R1T CFM cases corresponding to constraint spaces (II)

ECS Pattern	CFM case

(7). 2R1T Case

In this case, there are four typical freedom spaces of the flexure systems, as shown in Table 2. Table 11 illustrates the synthesized CFM cases corresponding to these four freedom spaces.

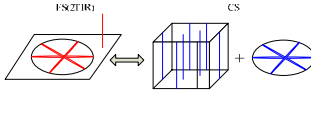
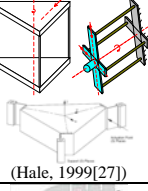
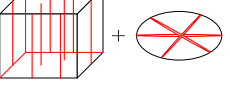
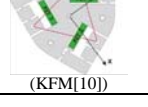
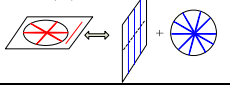

Table 11 2R1T CFM cases corresponding to FACT patterns

FS Set symbol	FACT Pattern	CFM case
$\mathcal{U}(N, n) \cup \mathcal{P}(n)$		
$\mathcal{U}(N, n) \cup \mathcal{P}(u)$		
$\mathcal{N}_2(u, v) \cup \mathcal{P}(n)$		
$\mathcal{N}_2(u, v) \cup \mathcal{P}(u)$		

(8). 2T1R Case

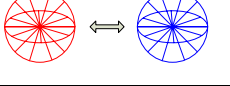
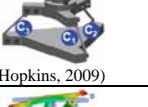
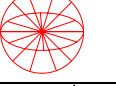

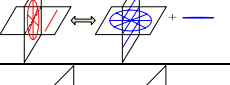
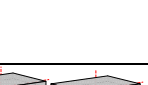
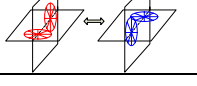
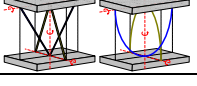
In this case, there are two typical freedom spaces of the flexure systems, as shown in Table 2. Table 12 illustrates the synthesized CFM cases corresponding to these two freedom spaces.

Table 12 2T1R CFM cases corresponding to FACT patterns

FS Set symbol	FACT Pattern	CFM/KFM case
$\mathcal{F}(u)$		 (Hale, 1999[27])
$\mathcal{F}(u)$		 (KFM[10])
$\mathcal{R}(N, u) \cup \mathcal{T}_2(n)$		

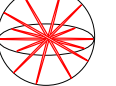
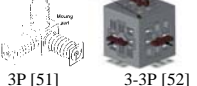
(9). 3R Case

Table 13 3R CFM or KFM cases corresponding to the FACT patterns

FS Set symbol	FACT Pattern	CFM/KFM case
$\mathcal{S}(N)$		 (Hopkins, 2009)
$\mathcal{S}(N)$		 (KFM)Callegari M, 2009[41]
$\mathcal{U}(N, n) \cup \mathcal{F}_2(N', u', n')$		
$\mathcal{U}(N, n) \cup \mathcal{U}(N', n')$		

(10). 3T Case

Table 14 3T KFM cases corresponding to the FACT patterns

EFS Set symbol	FS Pattern	KFM case
$\mathcal{P}(u) \cup \mathcal{P}(v) \cup \mathcal{P}(n)$		 3P [51] 3-3P [52]

5 NUMERATIONS AND TYPE SYNTHESIS OF KFMS

In addition to applying the rigid-body replacement method as mentioned above, Part I introduces other two approaches.

5.1 Review of Type Synthesis Process and Methods

Type synthesis of serial KFMs starts with specifying a freedom space but the objective is to find the desired serial kinematic chains with combinations of flexible joints.

In fact, there exist two methods for the type synthesis of a serial KFM. One is a freedom-based method, and the other is a constraint-based one.

❖ Freedom-based method

Step 1. Specify the desired freedom according to specifications of a serial KFM.

Step 2. Denote the specified freedom of the serial KFM using a visualized combined freedom space (like Table 14).

Step 3. Determine all possible subspaces by depositing the freedom space of the serial KFM in terms of principle of the dimension number identical with the mechanism.

Step 4. Make the appropriate physical arrangement for each freedom subspace type obtained in Step 3. For example, we may select an uncouple or decouple configuration.

❖ Constraint-based method

Step 1. Specify the desired freedom according to specifications of a serial KFM.

Step 2. Denote the specified constraint of the serial KFM using a visualized constraint space.

Step 3. Determine all possible equivalent reciprocal spaces representing the DOFs of the mechanism based on the approach given in section 3 or using the patterns shown in Table 2-14.

Step 4. Select the appropriate reciprocal space types from the freedom space obtained in Step 3.

Step 5. Select the lower-dimension subspace types from the freedom space, and then find the corresponding constraint subspace, but each of them corresponds to an available flexure.

Step 6. Arrange all flexures in series to form a serial flexure kinematic chain.

Type synthesis of parallel KFMs also starts with specifying a freedom space and the objective is to find all serial kinematic chains in parallel. The following will review the general type synthesis procedure presented in Part I.

Step 1. Denote the specified freedom of the parallel KFM using a visualized freedom space.

Step 2. Determine the reciprocal spaces representing the constraints of the mechanism.

Step 3. Select the appropriate but constraint lines from the CS of the system, and then determine the number of constraint lines corresponding to each chain in terms that whether the system is a proper-constraint one or an over-constraint one. That may be termed as the chain constraint subspace.

Step 4. Determine each flexure kinematic chain according to the chain constraint subspace. The solution can be found in above type synthesis of serial KFMs (two methods).

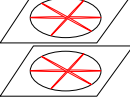
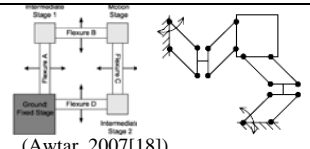
Type synthesis of hybrid KFMs essentially combines that of serial KFMs and parallel ones.

5.2 Numerations and Type Synthesis

(1). 2T Case

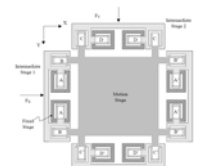
Table 15 illustrates two 2T KPM cases corresponding to the freedom space pattern.

Table 15 2T KFM cases corresponding to the FS patterns

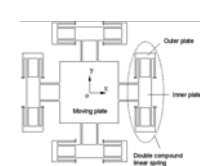
FS Set symbol	FS Pattern	KFM case
$\mathcal{P}(u) \cup \mathcal{P}(v)$		 (Awatar, 2007[18])



(a) (Awatar, 2007[18])



(b) (Awatar, 2007[18])



(c) (Choi, 2006[34])

Fig. 3 Four typical 2T KPMs

Figure 3 shows four KPMs for two axes translational motions. Each one is indeed an overconstrained mechanism due

to symmetric design. Here only take Fig. 3c as an example. This mechanism consists of a moving plate, quad-symmetric simple parallel linear springs, and quad-symmetric double compound linear springs, instead of quad-symmetric auxiliary moving plates. The double compound linear spring is composed of an inner plate, symmetric outer plates, and leaf type flexures. The flexures connect the inner plate to the outer plates, and the outer plates to the fixed frame. The inner plate of the double compound linear spring is allowed to move linearly between two outer plates by only the bending deflection of the flexures. Thus the double compound linear spring guides the motion of the moving plate transmitted through the simple parallel linear spring. In addition, the double compound linear spring compensates for the parasitic motion generated in the traverse direction of the simple parallel linear spring due to the pure bending deflection for the linear motion.

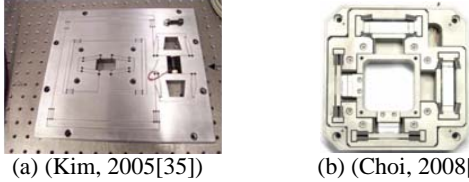


Fig. 4 Two typical 2T KPMs with large-deflection flexible joints

As shown in Fig. 4, each X and Y stage consists of an amplifying mechanism of motion and a guide mechanism of motion. The Y stage has the same structure except that it is inside the X stage. X and Y motion is decoupled. The double compound linear motion guide makes the motion straight. As a result, the XY stage of the single module parallel-kinematic flexure stage is made to have high orthogonality.

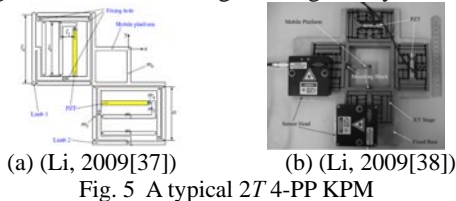


Fig. 5 A typical 2T 4-PP KPM

Another a decoupled XY 4-PP KPM is shown in Fig. 5. In addition, a serial but uncoupled XY KPM can be constructed by stacking an X stage and a Y one, as shown in Fig. 6.

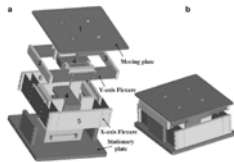


Fig. 6 A serial 2T 4-PP KPM (Kang, 2009[39])

(2). 2R1T Case

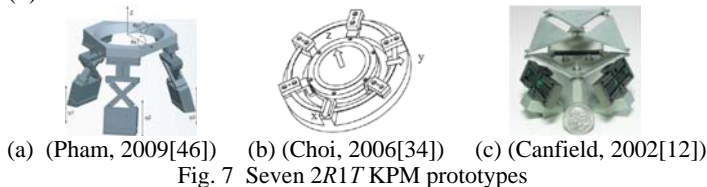


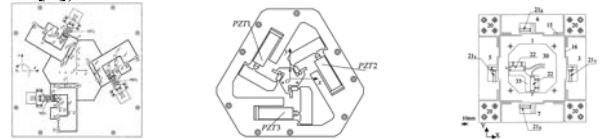
Fig. 7 Seven 2R1T KPM prototypes

As given in Part I, all possible kinematic chains of 2R1T KPMs can be synthesized by using freedom-based or constraint-based method. Generally, the possible 2R1T KPMs include 3-RPS type, 3-PRS type, 3-RRS type, etc. Fig. 7 illustrates three 2R1T KPM prototypes. As shown in Fig. 7b, the basic element of the flexure is a ring that is comprised of three fixed-fixed

beams. The mid-point of each of these beams provides a vertical deflection due to an applied vertical load. This vertical spring can be considered to be equivalent to a leg of the 3-RPS stage where the P joint is a passive linear spring. A modified version of this design yields the distributed flexure stage of Fig. 7b.

(3). 2T1R Case

Based on Table 12, we can make a systematic type synthesis for this class of planar mechanism. It can be designed as a parallel configuration (Fig. 8), or a hybrid one (Fig. 9). The hybrid 2T1R KFM is made up of a XY translational stage (Chang, 1999[8]).



(a) (Ryu, 1996[10]) (b) (Chung, 2004[40])

Fig. 8 Two 2T1R KPMs

Fig. 9 A hybrid 2T1R KPMs

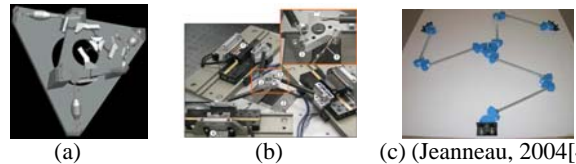
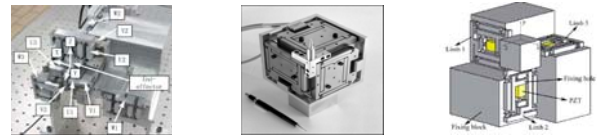


Fig. 10 Three typical 2T1R KPMs with large-deflection flexible joints

Also in designing a 2T1R KFM, a large-deformation flexible joint can be employed. For example, using the compliant rolling contact joint, a planar parallel mechanism was designed (Fig. 10c). This PPM was based on the 3-RRR structure, where all R-joints were replaced by the rolling joints.

(4). 3T Case

Based on Table 14, we can make a systematic type synthesis for this class of mechanism. It can be designed as a parallel configuration (Fig. 11-12), or a hybrid one (Fig. 13). The hybrid 2T1R KFM is made up of a XY translational stage and a Z translational one.



(a) (Chen, 2006[49])

(b) (Clavel, 2001[15])

(c) (Kong, 2009)

Fig. 11 Uncoupled 3T parallel KPMs

For example, by replacing the two passive P joints in each leg of the 3-PPP parallel manipulator with an inverted Awatar's mechanism, we obtain a novel 3-DOF CPM as shown in Fig. 11c. Each of three actuators can be separately taken as an active P joint, of which central part is connected with the central stage and of which other two side-parts are fixed

As shown in Fig. 12a, a parallel KFM is presented in which conventional bearings are replaced by pseudo-elastic flexure hinges. The mechanism consists of a spatial parallel structure with three translational degrees of freedom and is driven by three linear direct drives.



(a) (Yao, 2008[50]) (b) (Arai, 2000[48])

Fig. 12 Coupled 3T parallel KPMs Fig. 13 A hybrid 3T parallel KPM

(5). 3R 3T Case

6DOF KPMs may have two configurations, i.e. fully parallel one and hybrid one, as shown in Fig. 14 and Fig. 15, respectively. Generally, a 6DOF hybrid rigid parallel mechanism is made up of a 3DOF translational stage and a 3DOF rotational one, however, a 6DOF hybrid KPMs is usually made up of a 3DOF in-plane stage and a 3DOF out-of-plane one, which can be reduced to a more compact structure.

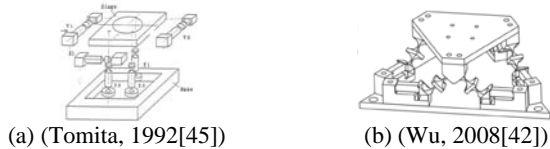


Fig. 14 Fully parallel 6-PSS KPMs

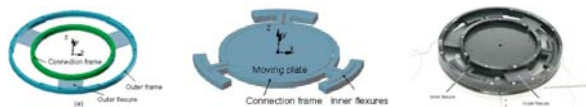


Fig. 15 Hybrid 6DOF parallel KPMs(Choi, 2005[43])

As shown in Fig. 15, the wafer stage has a 6DOF KFM for complete contact between the surface of the template and the surface of the wafer. The KFM consists of an inner mechanism for in-plane motion and an outer mechanism for out-of-plane motion. The inner and outer mechanisms have symmetric flexures, which were machined monolithically, onto each plane to cope with thermal deformation.

In addition, there are some special 6DOF KFM configurations, as shown in Fig. 16. including KFM with large-deformation flexible joints, planar orthogonal KFM, and dual-actuation one, etc.



Fig. 16 Three special 6DOF KPM cases

6 COMPARISONS

Compared with parallel KPMs, parallel CFMs have some merits of such as (i) exhibit comparably large-displacement characteristics, (ii) reduce some manufacturing and fabrication error resources due to simple and compact structural loops as well as simple flexure elements, (iii) easy to realize symmetrical and redundant design.

Compared with parallel CFMs, parallel KPMs have some merits of such as (i) possesses a greater load capacity and may achieve a greater range of motion in a smaller space with less susceptibility of buckling, sometimes even exhibits higher accuracy, (ii) exhibits higher structural stiffness and better dynamic performances, (iii) realize all possible DOF ranging from one to six.

7 SUMMARIES

In this paper we mainly demonstrates how to use the methodology addressed in Part I of the series papers to achieve a unified type synthesis for both two kinds of flexure mechanisms, i.e. KPMs and CFMs with the specified DOF characteristics. The classifications, numerations and type synthesis of multi-DOF KPMs and CFMs are investigated in a systematic way, respectively. As a result, not only a variety of known flexure mechanisms are completely classified, but also are some new

types of multi-DOF parallel CFMs developed. Qualitative comparisons in performances between kinematic-type and constraint-type flexure mechanisms with the same DOF show that parallel CFMs can exhibit comparably large-displacement characteristics and easy to manufacturing and fabrication, whilst parallel KPMs may possesses a greater load capacity and sometimes even exhibits higher accuracy and structural stiffness.

ACKNOWLEDGMENTS

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