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MODELING DYNAMIC ANKLE MECHANICAL IMPEDANCE IN RELAXED MUSCLE

Hyunglae Lee¹
hyunglae@mit.edu

Neville Hogan^{1,2}
neville@mit.edu

¹ Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge MA02139

² Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, Cambridge, MA02139

ABSTRACT

This paper presents identification and modeling of dynamic ankle mechanical impedance in relaxed muscles. A multi-variable estimation method using a wearable therapeutic robot enabled clear interpretation of dynamic ankle impedance both in the sagittal and frontal planes. Estimation results showed that dynamic ankle behavior apparently cannot be reconciled with a simple 2nd order model. Measurements in a seated and standing position verified that ankle impedance changes substantially depending on lower-limb posture. Identification results were fitted with a modified Hill model with a mass between the muscle and tendon elements. When coupled with foot inertia, either singly or antagonistically, this model successfully captured the dynamic behavior of the ankle impedance both in the seated and standing positions up to 20 Hz. At least a 4th order model having 2 complex zero and 1 complex pole pairs was required to describe relaxed ankle impedance either in the sagittal or frontal plane up to 20Hz. In the seated position, a 6th order model was slightly better than the 4th order model but with the expense of complexity, and a 8th order model might be used to describe dynamic ankle behavior up to 30~40Hz.

INTRODUCTION

The role of the ankle in lower extremity function is perhaps the most important of all joints in the kinematic chain between foot and trunk [1]. Thus a quantitative mathematical model of the ankle may provide valuable information to understand the contribution of the ankle to human locomotion. There has been a great deal of work on the ankle geometry and kinematics [2-5]. However, functions such as walking, balancing, and coordination are dynamic processes, thus an accurate dynamic model of the ankle, more specifically dynamic ankle mechanical impedance, will enable better interpretation of the roles of the ankle in its interaction with the environment. Ankle

mechanical impedance is an operator that maps a time history of displacement of the ankle joint onto a corresponding force time history, which is sometimes called dynamic stiffness or dynamic compliance. Gottlieb and Agarwal [6] measured the dynamic compliance of the normal human ankle joint by applying band-limited Gaussian torques. A similar system identification method was used by Kearney and Hunter [7, 8] and Weiss et al., [9, 10] to identify passive and active ankle dynamic stiffness. While most studies only focused on ankle motion in the sagittal plane, Zinder et al., [11] measured inversion/eversion ankle stiffness using a medial/lateral swaying cradle. All these studies modeled ankle mechanical impedance as a 2nd order underdamped system having inertial, viscous, and elastic terms. In contrast, Rastgaar et al., [12], showed that ankle behavior in the sagittal plane cannot be reconciled with a simple 2nd order model. In this study, we extended that work with a more accurate estimation procedure, and present a model of relaxed ankle mechanical impedance both in the sagittal plane (dorsiflexion- plantarflexion (DP) motion) and the frontal plane (inversion- eversion (IE) motion).

EXPERIMENTS

To characterize ankle mechanical impedance, we utilized a wearable therapeutic robot called Anklebot (Interactive Motion Technologies, Inc.) [13], which applies torque perturbations in 2 degrees of freedom (DOF), and measures displacements. Subjects wore a modified shoe and a knee brace, to which the robot was connected. To study the effect of lower limb posture on ankle impedance, it was measured in two different body postures with the sole at a right angle to the tibia in both cases (Fig.1).

Fig.4. Ankle impedance estimate in a seated position (Left: sagittal plane (DP), Right: frontal (IE), Blue: mean of all subjects, Light blue: mean $\pm$ 1 standard deviation (SD))

In both the seated and standing positions the frequency region over 9~10 Hz, response was more or less consistent with inertia-dominated behavior (magnitude increased at ~40dB/dec). In addition, resonant and anti-resonant behavior was observed in the high frequency region between 20 to 30Hz. In the low frequency region (<2Hz) stiffness was dominant with a magnitude slope close to zero.

However, in the seated position, the magnitude response in the mid frequency region, from about 2~4 Hz up to 9~10 Hz, was markedly different from a simple 2nd order system. Impedance magnitude in this mid frequency region was 2~3 times higher than in the low frequency region. The phase response in this mid frequency region showed a substantial phase lead followed by a subsequent phase lag.

Fig.5. Ankle impedance estimation in a standing position

Ankle impedance in the standing position, was significantly different from the seated position. In general it became closer to a 2nd order system although deviations in the mid frequency region still can be seen. Another interesting change was that the impedance magnitude in the low frequency region in the standing position was higher than in the seated. These two characteristics were consistent across all 6 subjects.

ANKLE IMPEDANCE MATHEMATICAL MODEL

A quantitative mathematical model describing the identified ankle impedance was developed. In this study, attention was focused on a frequency range less than 20 Hz, with special interest in the low and mid frequency region (up to 10Hz). We started modeling with the well known Hill muscle model which consists of a series elastic element (often associated with the muscle tendon) connected to a parallel combination of an ideal force source, a viscous damper and an elastic element. Force generators (f_{on} , f_{og}) are set as zero since this study was performed on the relaxed muscle conditions. However, a Hill model coupled with a rotational inertia was unable to represent the fast phase transition in the mid-frequency region.

Because a fast phase increase followed by a phase lag can be obtained from a resonant behavior, we modified the Hill muscle model by adding a mass to the junction between muscle and tendon elements. An antagonistic pair of these model muscles coupled through a constant moment arm to a foot inertia (Fig.6) generated a 6th order system eq. (5).

Fig. 6. The ankle with modified Hill model muscles. Subscripts g and n denote agonist and antagonist muscles, respectively. (f_o : force source, k_m , b_m : elastic and viscous element of a muscle, k_t , b_t : elastic and viscous element of a tendon, m : inserted mass, r : moment arm, I : foot inertia)

$$\frac{\tau_{net}(s)}{\theta(s)} = r_g^2 \left\{ \frac{(m_g b_{tg} s^3 + (m_g k_{tg} + b_{mg} b_{tg}) s^2 + (b_{mg} k_{tg} + b_{tg} k_{mg}) s + k_{mg} k_{tg})}{m_g s^2 + (b_{mg} + b_{tg}) s + (k_{mg} + k_{tg})} \right\} + r_n^2 \left\{ \frac{(m_n b_{tn} s^3 + (m_n k_{tn} + b_{mn} b_{tn}) s^2 + (b_{mn} k_{tn} + b_{tn} k_{mn}) s + k_{mn} k_{tn})}{m_n s^2 + (b_{mn} + b_{tn}) s + (k_{mn} + k_{tn})} \right\} + I s^2 \quad (5)$$

When a single muscle model was used rather than an antagonistic pair, the model generated a simpler 4th order system eq. (6). Here subscript e denotes effective elements.

$$\frac{\tau_{net}(s)}{\theta(s)} = r^2 \left\{ \frac{(m_e b_{te} s^3 + (m_e k_{te} + b_{me} b_{te}) s^2 + (b_{me} k_{te} + b_{te} k_{me}) s + k_{me} k_{te})}{m_e s^2 + (b_{me} + b_{te}) s + (k_{me} + k_{te})} \right\} + I s^2 \quad (6)$$

The measured ankle impedance was fit to 6th, 4th and 2nd order models. For the 6th and 4th order models, an unconstrained nonlinear optimization method (Nelder-Mead simplex method: MATLAB's `fminsearch` function) [16] was used, and the mean of magnitude difference up to 20Hz between the model and the measurement was minimized. In current study, phase response was not considered for optimization process, since more accurate estimates of the magnitude response was our major interest. For the 2nd order model, its parameters were determined by fitting the low and high frequency regions, which correspond to stiffness and inertia, respectively.

Modeling error and goodness of fit were calculated to evaluate model performance. In detail, modeling error was defined as the mean magnitude difference between the model and the measurement in the low and mid frequency region (up to 10Hz). Percentage variance accounted for (%VAF) between the model and the identification result was computed to measure how well the model described the identified result (7).

$$\%VAF = 100 \times \left(1 - \frac{\text{var}(z - \hat{z})}{\text{var}(z)}\right) \quad (7)$$

where z is the frequency response magnitude of the measurement, $\hat{z}$ of the model, and $\text{var}(x)$ is an estimate of the variance of the random variable x .

Using these two performance measures, 4th and 6th order models were quantitatively compared with the 2nd order model.

ANKLE IMPEDANCE MODELING RESULTS

In the seated position (Fig.7) the 6th order model competently represented relaxed ankle impedance in both the DP and IE directions. Although its modeling error was slightly larger than the 6th order model (Table.1), a 4th order model competently represented important transitions in the response up to 20Hz. On the other hand, the 2nd order model resulted in substantial deviations in the mid frequency region for both magnitude and phase response.

Fig.7. Modeling relaxed ankle impedance in the seated position (Red: Model, Blue: Estimation, Pale lines: Mean $\pm$ 1 STD) – (a): 6th order model, (b): 4th order model, (c): 2nd order model

In the standing position, considering the slighter deviations from 2nd order behavior in the mid-frequency region, the identified impedance was fit with a 4th order model and compared with a 2nd order model (Fig.8).

In the standing position, the 4th order model competently represented the data with a modeling error comparable to the 6th order model in the seated position (Table 1). As in the case of the seated position, the 2nd order model was unable to represent the mid frequency region accurately even in the standing position.

Table.1. Quantification of modeling error

Seated Position	2 nd order Model	4 th order Model	6 th order Model
Z11 (DP)	11.26 (3.64)	2.32 (0.29)	2.03 (0.26)
Z22 (IE)	2.80 (0.62)	0.64 (0.19)	0.56 (0.19)
Standing Position	2 nd order Model	4 th order Model	—
Z11 (DP)	4.77 (1.57)	2.03 (0.38)	—
Z22 (IE)	2.58 (0.69)	0.72 (0.14)	—

(): Standard deviation, Unit: [Nm/rad]

Fig.8. Modeling relaxed ankle impedance in the standing position (Red: Model, Blue: Estimation, Pale lines: Mean $\pm$ 1 STD) – (a): 4th order model, (b): 2nd order model

In the seated position, the mean % VAF of all subjects was higher than 75% and 80% with the 4th order and 6th order models, respectively. In the standing position, this value was higher than 80% with the 4th order model (Table 2).

Table.2. Quantification of % VAF

Seated Position	2 nd order Model	4 th order Model	6 th order Model
Z11 (DP)	52.02 (5.64)	79.85 (2.67)	87.03 (2.89)
Z22 (IE)	53.99 (9.15)	76.25 (4.81)	81.75 (4.26)
Standing Position	2 nd order Model	4 th order Model	–
Z11 (DP)	75.74 (4.12)	82.07 (3.40)	–
Z22 (IE)	65.15 (6.48)	81.67 (1.53)	–

(): Standard deviation

DISCUSSION

Dynamic ankle mechanical impedance with relaxed muscles was successfully identified using a multi-variable stochastic identification method. Unlike the previous work by Rastgaar et al. [12], the estimation method subtracted actuator impedance from the total impedance rather than actuator and shoe together. This yielded higher repeatability and higher

coherence from the lowest frequency region, less than 1Hz, to the high frequency region, up to 50Hz. As a result, subtle changes in IE impedance can be observed, which were unavailable with the previous method.

From the identification results, we found several interesting points. First, ankle impedance changes depending on lower limb posture. The mid frequency response in the seated position was clearly different from the standing position. This difference was observed across all 6 subjects. It can be explained with the property of gastrocnemius, which is a biarticular muscle, thus stretched more in the standing position than in the seated position.

This change of ankle impedance between the standing and seated positions may be important, because those two knee postures are common in our daily life such as level walking, stair ascent/descent, driving, etc. Second, the patterns of frequency response changes were qualitatively similar for both DP and IE impedance. Third, IE impedance was always lower than DP impedance, confirming previous work [12, 15].

Ankle impedance in the seated position showed clear deviations from the simple 2nd order model which has been assumed by many other research groups. The mid frequency impedance exceeded the low frequency impedance by more than a factor of 3 and 2 in the DP and IE directions, respectively. Thus, a 2nd order approximation was not competent to accurately describe the mid frequency dynamic behavior of the ankle. Although ankle impedance in the standing position more closely resembled a 2nd order system, both DP and IE impedance still exhibited a transition behavior in the mid frequency region. In average, the mid frequency impedance is higher than the low frequency impedance by about 25% and 50% in the DP and IE directions, respectively.

When coupled with the foot inertia, the modified Hill model having a mass between the junction of muscle and tendon elements successfully captured the dynamic behavior of ankle impedance up to 20Hz both in the seated and standing position. Constant moment arms were assumed based on the information from [17] that both dorsiflexor and plantarflexor moment arms change less than 10% in the experimental conditions used here (mean displacement amplitude due to the random perturbation was less than 7°). On average, the 6th order model derived from an antagonistic pair of modified Hill muscle models coupled with the foot inertia explained dynamic ankle impedance more accurately than the 4th order model. However, the modeling error of the 4th order was still good, comparable to the result of 6th order model. Impedance in the standing position was well described with a 4th order model. The modeling error in this case was not statistically significantly different ($p > 0.05$) from the result with a 6th order model in the seated position.

The model qualitatively represented the major phase changes up to 15Hz, but with some error. This error could be attributed to two reasons. First, the optimization method used

here minimized only magnitude errors between the model and the measurement. If the objective function was changed to the mean error between the complex-valued impedances of the model and the measurement, phase error might be improved. Second, a resonant behavior in the high frequency region was another source of phase error. In the current models, this behavior was ignored. However, large phase errors in the high frequency region still can affect the mid and low frequency region, since the phase effects of a pole or zero may be significant at frequencies a decade higher or lower. For example, an 8th order model may be justified by splitting the foot inertia in two and adding an elastic element between the two pieces. If this higher-order model is used to fit the observations, error may be improved in the low and mid frequency region as well as in the high frequency region over 20Hz. From the above two explanations, we can see that there are two distinct tradeoffs, between magnitude and phase error, and accuracy and complexity in modeling dynamic ankle impedance.

Currently we are working on mapping the optimized zero/pole pairs to the modified Hill model parameters. This will allow us to test whether these parameters are reasonably consistent with known biomechanical data. If successful, we may be able to use dynamic impedance measurements to identify abnormal muscle properties, which are common sequelae of neurological disorders such as stroke or Cerebral Palsy [18].

There is another issue of using vibratory perturbation in experiments. Although EMG amplitudes during perturbation were much less than the levels when muscles were active, they were still statistically significantly different from the ones during rest period ($p < 0.05$). As known from the previous study [19], vibratory motions at the ankle can evoke stretch reflex, which may affect ankle impedance. Further study is needed to identify the contribution of this reflex component to ankle impedance.

In future work, we plan to estimate dynamic ankle mechanical impedance with active muscles including antagonist muscle co-contraction.

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