

## MIT Open Access Articles

*Entrainment of Ankle-Actuated Walking Model to Periodic Perturbations via Leading Leg Angle Control*

The MIT Faculty has made this article openly available. **Please share** how this access benefits you. Your story matters.

**Citation:** Rigobon, Daniel, Julieth Ochoa, and Neville Hogan. "Entrainment of Ankle-Actuated Walking Model to Periodic Perturbations via Leading Leg Angle Control." Volume 1: Aerospace Applications; Advances in Control Design Methods; Bio Engineering Applications; Advances in Non-Linear Control; Adaptive and Intelligent Systems Control; Advances in Wind Energy Systems; Advances in Robotics; Assistive and Rehabilitation Robotics; Biomedical and Neural Systems Modeling, Diagnostics, and Control; Bio-Mechatronics and Physical Human Robot; Advanced Driver Assistance Systems and Autonomous Vehicles; Automotive Systems (October 11, 2017).

**As Published:** <http://dx.doi.org/10.1115/DSCC2017-5132>

**Publisher:** ASME International

**Persistent URL:** <http://hdl.handle.net/1721.1/119393>

**Version:** Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

**Terms of Use:** Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.



## ENTRAINMENT OF ANKLE-ACTUATED WALKING MODEL TO PERIODIC PERTURBATIONS VIA LEADING LEG ANGLE CONTROL

**Daniel Rigobon**

Massachusetts Institute of Technology,  
Dept. of Mechanical Engineering,  
Cambridge, MA, US.

**Julieth Ochoa**

Massachusetts Institute of Technology,  
Dept. of Mechanical Engineering,  
Cambridge, MA, US.

**Neville Hogan**

Massachusetts Institute of Technology,  
Dept. of Mechanical Engineering,  
Dept. Of Brain and Cognitive Science,  
Cambridge, MA, US.

### ABSTRACT

In prior work, a minimal mathematical model of bipedal walking was developed to investigate the experimentally observed entrainment behavior of human locomotion. While that model reproduced several salient properties of human walking, it failed to entrain to periodic perturbations with period longer than preferred walking period. To overcome that limitation, we introduced afferent feedback in the form of leading leg angle control that depended on the energetics of previous steps. The model response to periodic perturbations was again studied in simulation, testing several perturbation periods and initial perturbation phases. This revised model captured important aspects of human locomotion that had been previously observed experimentally: a finite basin of entrainment to both shorter and longer perturbation periods. Regardless of the (random) phases of the step cycle at which perturbations were initiated, all entrained simulations phase-locked with the torque pulses at the end of double stance. However, more than twice as many steps were required to entrain to longer perturbations. The results achieved with this revised walking model emphasize the importance of the oscillatory dynamics of bipedal locomotion and highlight possible applications of gait entrainment as a method for *permissive* motor guidance in the field of assistive and rehabilitation robotics.

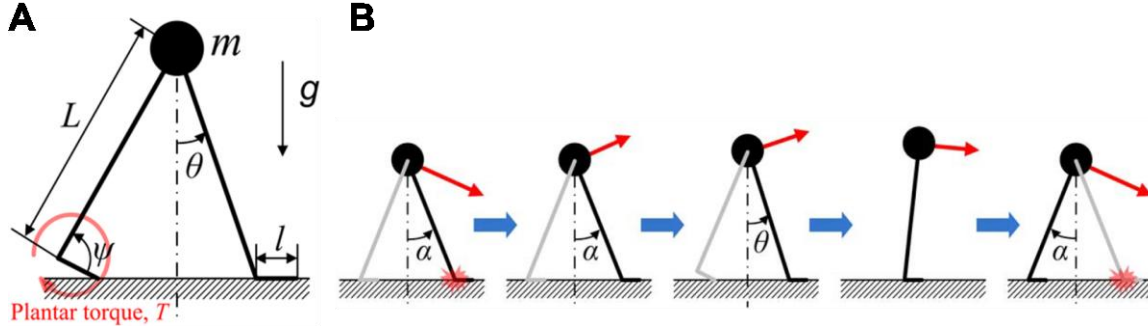
### INTRODUCTION

Even with a vast and growing literature, understanding the dominant control scheme of human bipedal locomotion remains challenging. Bipedal locomotion is characterized by foot-ground interactions with energy dissipation; positive work is performed by the trailing leg during ankle push-off while negative work is performed by the leading leg upon heel strike. This behavior has been thoroughly studied using dynamic walking models. Even though humans exhibit locomotor behavior that is much more sophisticated than that of typical dynamic walking models, these models provide insightful

information about the response of the locomotor system to physical interaction.

Human walking displays fundamental features that are characteristic of nonlinear limit-cycle oscillators. In particular, prior experimental work demonstrated entrainment of human walking to a periodic series of torque pulses delivered to the ankle joint at periods slightly shorter or longer than preferred stride cadence, accompanied by phase-locking at ankle push-off [2,1]. Additional simulation work showed that a simple one-degree-of-freedom walking model could reproduce experimentally observed behavior, but only when the perturbation periods were shorter than the model's unperturbed gait period [3]. Entrainment of the model to longer perturbation periods required the walker to decrease its cadence accordingly, which could only occur if the perturbations decreased its kinetic energy. Yet torque pulses phase-locked at the end of double stance could only increase the energy and velocity of the walker as step length was set to remain constant.

In this paper, we present a modified version of the model in [3]. The revised model includes afferent feedback not only in the form of trailing leg ankle actuation via torsional spring release based on the system's state, but also in the form of leading leg angle proportional control. Specifically, the leading leg angle at heel strike was adjusted based on the difference in work done by the perturbation over the previous steps. Simulations were conducted to test whether limitations of the previous model were overcome by incorporating step-to-step transitions based on the energetics of walking, such as adjustments of the leading leg angle upon collision. For these simulations, entrainment of the model was assessed in the same fashion as before, testing perturbation periods up to 140 ms shorter and 70 ms longer than preferred cadence. For each perturbation period tested, 200 simulations were conducted as the model walked a total 150 steps under perturbation. As in [3], perturbations consisted of ankle plantar-flexion torque pulses and were initiated at random phases of the step cycle.



**Figure 1: Schematic representation of the walking model [3]. (A)** Point mass ( $m$ ) is attached to two massless, rigid legs of length ( $L$ ). Plantar-flexion actuation of the rear ankle is provided via a pre-loaded torsional spring, released at the beginning of double stance. **(B)** Model behavior over one step cycle, which begins and ends when the leading leg strikes the ground. Throughout the step cycle, the model behaves as a four-bar linkage actuated at the rear ankle during double stance, or as an inverted pendulum during single stance.

## MODEL DESCRIPTION

A schematic of the simple state-determined walking model used in this study along with its defining variables and parameters is shown in Figure 1. The model, previously presented in [3], consists of a point mass supported by two massless legs of equal length moving along a vertical plane under the influence of gravity. Each leg has an ankle and a hip joint (a frictionless pivot that does not apply any torque). The model's behavior depends on whether one or both legs are in contact with the ground (i.e. single or double stance, respectively). During double stance, which begins after the leading leg strikes the ground, the model behaves as a four-bar linkage actuated at the rear ankle. The rear ankle joint is modeled as a linear torsional spring, released at the beginning of double stance, thus providing propulsion via plantar-flexion ankle torque as described below by Eq. 1. Conversely, the leading ankle joint behaves as a hinged joint during double and single stance as the rear leg advances forward and strikes the ground (i.e. until the next double stance phase where it becomes the rear ankle joint). During single stance, which begins after the rear ankle angle ( $\psi$ ) reaches its maximum plantar-flexion ( $\mu$ ), the model behaves as an inverted pendulum; single stance ends when the hip angle ( $\theta$ ) reaches a predefined critical angle ( $\alpha$ ).

$$T = k(\mu - \psi), \quad \left(\frac{\pi}{2} - \alpha \leq \psi \leq \mu\right) \quad \text{Eq. 1}$$

The model exhibits unperturbed asymptotically stable behavior since constant energy is added by the rear ankle torsional spring on each step, and energy is then dissipated by the leading leg's collision with the ground ('heel strike'). Specifically, the energy dissipated at heel strike is proportional to the speed of the model at impact,  $\frac{d\theta}{dt}$ , and the angle between legs at collision,  $2\alpha$ . Upon collision, the walker's kinetic energy is reduced by a factor of  $\cos^2(2\alpha)$ . In previous work by Ahn and Hogan, the ratio between pre- and post-collision angular velocity was derived as  $\cos(2\alpha)$ , which stayed constant throughout steps [3]. Therefore, if the model was walking faster than its preferred period, it would dissipate more energy upon

collision and slow down, while if it was too slow, less energy would be lost. Combined with the constant work done by the ankle spring, orbital stability was achieved.

When a series of periodic mechanical perturbations in the form of plantar-flexion torque pulses was applied, Ahn and Hogan's model entrained to perturbation periods ( $\tau_p$ ) shorter than preferred step period ( $\tau_{step}$ ). When entrained, the model phase-locked with the torque pulses at the end of double stance—where propulsion was assisted [3]. Entrainment to longer  $\tau_p$  required the walker to increase its  $\tau_{step}$  until it matched the (longer)  $\tau_p$ , which could only occur if the perturbations decreased the kinetic energy of the walker. However, torque pulses occurring during double stance could only increase the energy and velocity of the walker. An increase in velocity decreased step period, as the walker's step length remained constant. Accordingly, the model failed to entrain to  $\tau_p$  longer than preferred  $\tau_{step}$ .

In this paper, we present a modified version of Ahn and Hogan's walking model, which uses the leading leg heel strike angle as a governing parameter that can be adjusted in response to the imposed perturbations to allow more dissipation of energy on collision. Selinger and colleagues showed that when perturbed via resistive knee torques that added an energetic penalty, healthy subjects could continuously modify their preferred motor programs towards new energetically optimal ones at higher/lower step frequencies [4]. In relation to the adaptation of our model to the torque pulses, entrainment only requires the model's  $\tau_{step}$  to be identical to  $\tau_p$  so that there is a constant amount of energy added to the system each step, which is then lost upon heel strike [5]. Inherently, higher steady-state energy levels correspond to faster walking and shorter  $\tau_{step}$ . Likewise, for the walker to slow down sufficiently to allow its  $\tau_{step}$  to match the longer (than preferred)  $\tau_p$ , it must dissipate more energy than it gains.

Since the energy lost on heel strike is proportional to  $\cos^2(2\alpha)$ , adjusting the leading leg angle continuously based on current and previous (historical) step data could result in energetically optimal behavior, lead to steady state, and

**Table 1: Parameter values for ankle-actuated walking model.**

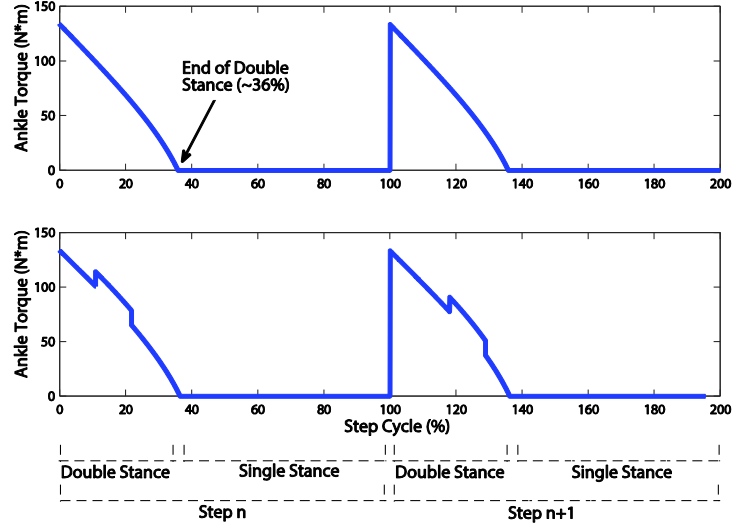
Parameter	Meaning	Value
$m$	mass	80 kg
$L$	leg length	1 m
$l$	foot length	0.2 m
$\alpha$	leading leg angle at heel strike (unperturbed walker only)	0.524 rad
$\mu$	Maximal plantar-flexion extension of the ankle	2.58 rad
$k$	ankle actuation stiffness constant	87.3 N·m/rad

possibly facilitate entrainment of the walker to  $\tau_p$  longer than preferred  $\tau_{step}$ . Specifically, continuously adjusting the energy cost of walking, the model may increase its  $\tau_{step}$  to match  $\tau_p$  while still allowing the torque pulses to assist propulsion when phase-locked at the end of double stance—as previously reported by Ahn and Hogan in the case of shorter  $\tau_p$  [3]. Furthermore, this modification of the model may reproduce the behavior reported in [1,6] regarding unimpaired gait entrainment to  $\tau_p$  both shorter and longer than subjects' preferred stride period.

## METHODS

Simulations were conducted to assess entrainment of the model to a periodic series of mechanical perturbations with periods that were shorter or longer than preferred step period. The model was modified to capture an aspect of natural human walking not previously incorporated—step length variability. The model was governed by parameters depicted in Table 1. In all simulations, the model walked a total of 150 steps. For each simulation, after the first two unperturbed steps, periodic perturbations in the form of ankle plantar-flexion torque pulses were superimposed in addition to the torsional spring ankle actuation. Figure 2 shows the walker's ankle torque profile for two consecutive step cycles, both unperturbed and perturbed. During swing phase, the contribution of the torque perturbations to the dynamics of the model was nullified given the model's massless legs. The magnitude (10% of maximum ankle torque of the walker,  $\sim 13$  N·m) and duration (100 ms) of these torque pulses were selected to be the same as in prior simulation work [3]. For comparison with related experimental work [1,6], the magnitude and duration of these perturbations approximated 10% of maximum ankle torque and 10% of one stride duration in normal adult walking, respectively [7–9]. Perturbations were initialized at random phases of the step cycle. Perturbation periods within the range

$$\tau_{step} - 0.14 \text{ s} = \tau_p = \tau_{step} + 0.07 \text{ s}$$

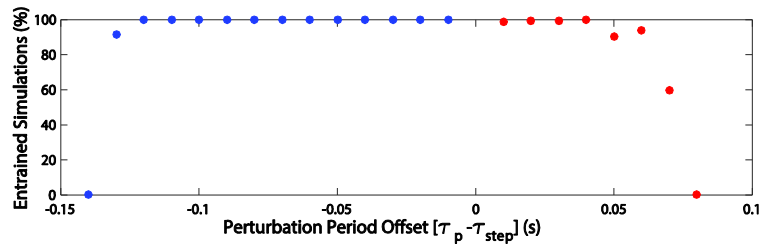


**Figure 2: Ankle torque profile of the walker for two consecutive unperturbed and perturbed step cycles.** After the end of double stance, which occurs at  $\sim 36\%$  of the step cycle, the walker's ankle torque is nullified since it behaves as an inverted pendulum.

with resolution of 0.01 s were evaluated; 200 independent simulations were conducted for each perturbation period.

## Experiment

The purpose of this experiment was to test whether adjusting the leading leg angle based on the difference in work done by the perturbation over the previous step allowed the model to entrain to periodic mechanical perturbations with periods that were not only shorter but also longer than preferred. Comparing the previous step to the current one, if work,  $W_p$ , was constant, the perturbation had to occur either entirely within the double stance phase, entirely within single stance, or in steady state (with the period of the walker matching the period of perturbation). If  $W_p$  decreased, the perturbation must have occurred at the boundary between single and double stance, and was drifting away from double stance so that less of the perturbation duration occurred within double stance and applied non-zero torque. If  $W_p$  increased, the perturbation was drifting into double stance and applying more torque.



**Figure 3: Percentage of entrained simulations vs. perturbation period offset (in seconds).** The success rate of entrained simulations was skewed towards shorter  $\tau_p$  (blue markers) compared to longer  $\tau_p$  (red markers). The basin of entrainment was  $\sim 9.3\%$  of  $\tau_{step}$ .

Thus, if work done by the perturbation increased, the model would increase its cadence. To entrain to the perturbations, the model had to be slowed down, which could be achieved by increasing  $\alpha$  to dissipate more energy and allow the perturbation to drift back towards double stance. Conversely, if work done by the perturbation decreased, the model would decrease its cadence. To entrain to the perturbations, the model had to be sped up by decreasing  $\alpha$  to lose less energy on collision. Scenarios of unequal work done by the perturbation could only occur when  $\tau_p$  did not match  $\tau_{step}$ , such that constant work done by the perturbation over infinitely many steps implied entrainment of the model. Therefore, the leading leg heel strike angle,  $\alpha$ , was multiplied by  $1 - K_p \cdot W_{diff}$ , where  $W_{diff} = W_p^n - W_p^{n-1}$ , was the difference in work done by the perturbation from the current step to the previous one, and  $K_p$  was a proportional gain constant. For sufficiently large offsets between  $\tau_p$  and  $\tau_{step}$ , the periodic series of perturbations could skip a step and a perturbation occurring at the end of double stance would not lead to entrainment of the model. To prevent the model from overcorrecting its heel strike ankle angle, the controller was set to leave the heel strike angle unchanged for 3 steps after skipping a step, thus preventing a sudden increase in work to speed up the walker unnecessarily. In addition, during these 3 steps without control, the heel strike angle was adjusted to a weighted average of the current angle and the initial unperturbed steady-state angle.

Work done by the perturbation was calculated using the change in kinetic energy (KE) between the current and previous step, and the work done by the ankle spring ( $W_{ankle}^n$ ). For any step  $n$ , the speed at which collision will occur is known. Using the coefficient of energy dissipation  $\cos(2\alpha^n)$  and the work done by the ankle spring  $W_{ankle}^n = \frac{1}{2}k \cdot (\alpha^n + \mu - \frac{\pi}{2})^2$  [3], the work done by the perturbation is described by:

$$W_p^n = KE_c^n - (\cos 2\alpha^{n-1}) \cdot KE_c^{n-1} + W_{ankle}^n \quad \text{Eq. 2}$$

where  $KE_c$  is the kinetic energy of the model immediately before the leading leg collision occurs. Therefore, the model had to wait two steps to start changing the leading leg angle. Accordingly, the leading leg angle was adjusted as:

$$\alpha^n = \alpha^{n-1} \cdot (1 - (W_p^n - W_p^{n-1}) \cdot K_p) \quad \text{Eq. 3}$$

### Data analysis

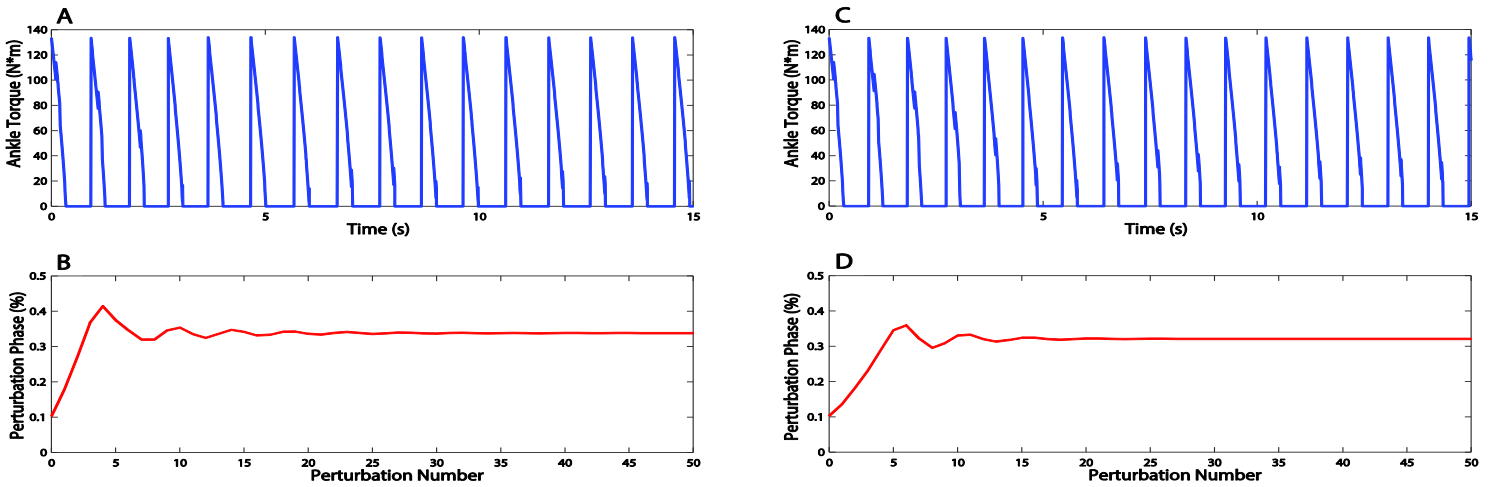
For each step, the perturbation phase was defined as the percentage of the step cycle at the onset of the torque pulse. The 150 perturbation phases were calculated in reverse order, starting from the 150<sup>th</sup> torque pulse. *Wraparounds* in the step cycle were allowed to avoid abrupt jumps in the perturbation phase when the onset of a torque pulse crossed the 0 or 100% boundaries. These wraparounds resulted in perturbation phase values less than 0% or greater than 100% of the step cycle.

A linear regression of perturbation phase onto perturbation number was applied to the last 10 torque pulses in each simulation. Entrainment of the walker's step cycle was concluded if the magnitude of the regression slope was smaller than 1% of  $\frac{|\tau_{step} - \tau_p|}{\tau_{step}}$ . (i.e. an entrained simulation corrected at least 99% of the initial slope of the phase-perturbation number trajectory)

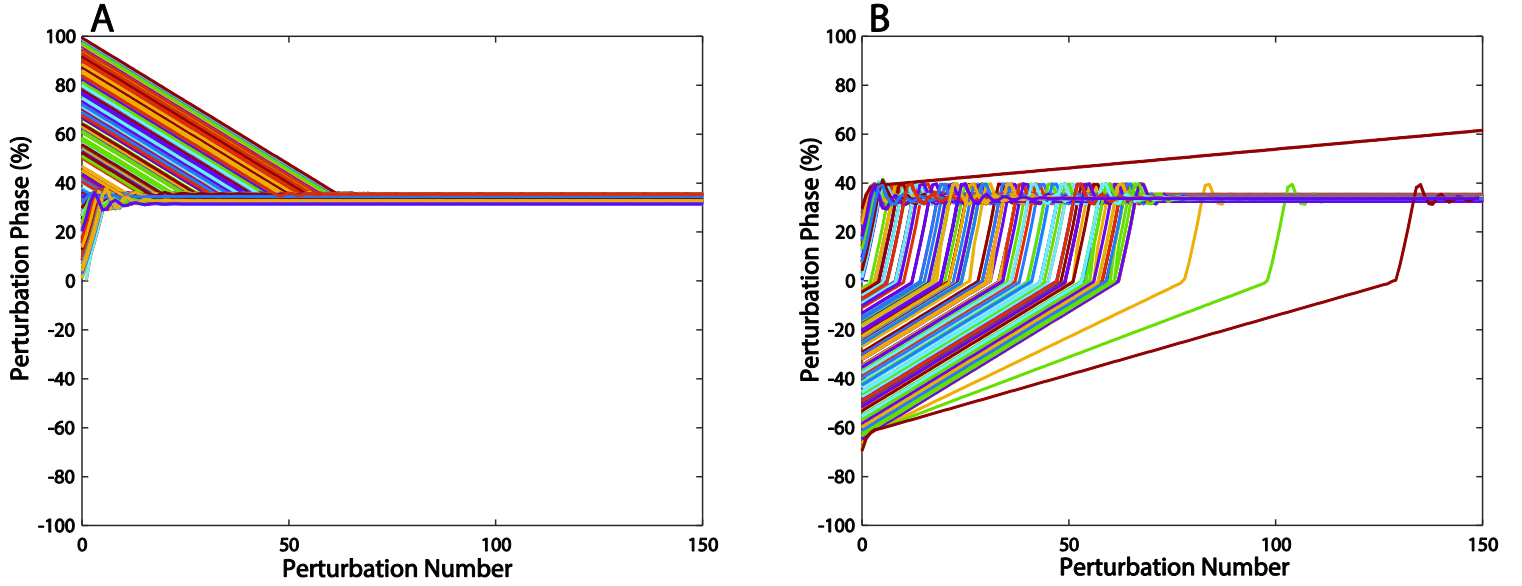
The phase converged ( $\phi_{conv}$ ) was determined as the average perturbation phase across the last 10 perturbation cycles. The number of steps required for entrainment were defined as the first perturbation phase within an interval of  $\phi_{conv} \pm 0.025 \cdot \tau_{step}$ .

### RESULTS

The updated model entrained to perturbation periods both shorter and longer than preferred, up to  $\tau_p = \tau_{step} - 0.12$  s and



**Figure 4: Representative behavior of the model in response to  $\tau_p = \tau_{step} \pm 0.02$  s. (A) Ankle Torque and (B) Perturbation Phase in response to longer  $\tau_p$ . (C) Ankle Torque and (D) Perturbation Phase in response to shorter  $\tau_p$ . In both cases, perturbations were initiated at the same phase of the step cycle (~10%); the final converged phase was also the same for both simulations (end of double stance). The oscillations in the control of the leading leg angle to synchronize the model's step cycle with the perturbation periods were more prolonged in the case of longer  $\tau_p$ .**

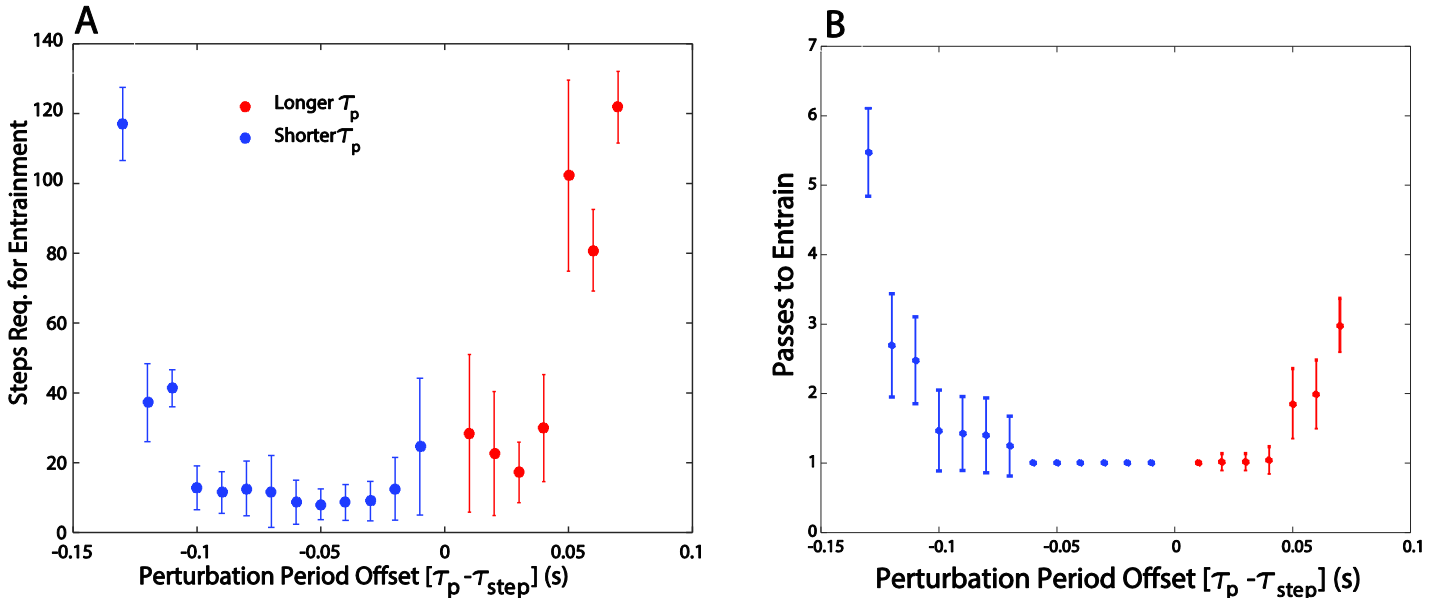


**Figure 5: Perturbation phase as a function of perturbation number for all 200 simulations in response to  $\tau_p = \tau_{step} \pm 0.01$  s. (A) Shorter  $\tau_p$ . (B) Longer  $\tau_p$ .** Torque pulse perturbations were initiated at random phases of the unperturbed walker's step cycle. For entrained simulations, the walker's step cycle synchronized with the torque pulses at the end of double stance.

$\tau_p = \tau_{step} + 0.06$  s, respectively. This resulted in a basin of entrainment of approximately 9.3% of  $\tau_{step}$ , which was skewed towards shorter  $\tau_p$ . Figure 3 shows the percentage of entrainment success across all 200 simulations for each of the various perturbation periods tested. A comparison of ankle torque and step cycle convergence to  $\tau_p = \tau_{step} \pm 0.02$  s can be seen in Figure 4. The model's response across all 200

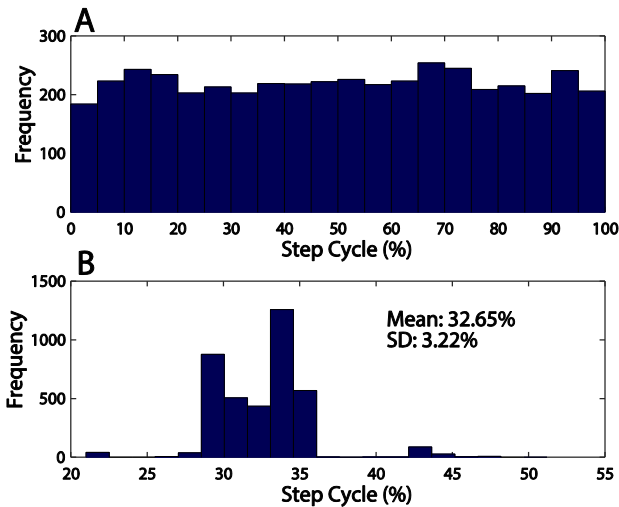
simulations for  $\tau_p = \tau_{step} - 0.01$  s and  $\tau_p = \tau_{step} + 0.01$  s (with perturbations initialized at random phases of the step cycle) is shown in Figure 5A and Figure 5B, respectively.

The basin of entrainment was defined by the gain constant and the weighted average used. Larger  $K_p$  values entrained the model to perturbation periods that were further from  $\tau_{step}$ , thus increasing the basin of entrainment. Conversely, smaller  $K_p$



**Figure 6: Requirements to entrain to both shorter (blue) and longer (red) periods  $\tau_p$ .** (A) Steps required for phase convergence across the different perturbation periods within the walker's basin of entrainment, and (B) Passes through the entrainment point (end of double stance) required for entrainment to occur. Shorter perturbation periods required an average of 21 steps and 1.71 passes before phase convergence, compared to 48 steps and 1.56 passes in the case of longer perturbation periods. When perturbations with shorter  $\tau_p$  were initialized near the end of double stance ( $\phi_{conv}$ ), fewer steps were generally required for entrainment.





**Figure 7: Distribution of initial and final step cycle phases for all entrained simulations. (A)** Histograms of the (random) gait phases at which perturbations were initiated across all perturbation periods for which the model entrained. **(B)** Histogram of the gait phases in the last 10 torque pulses for all entrained simulations (i.e.  $\phi_{\text{conv}}$ ).

entrained the model to perturbation periods that were closer to  $\tau_{\text{step}}$ .

The number of steps required for entrainment of the model to different perturbation periods and randomly selected initial phases are shown in Figure 6A and Figure 6B, respectively. Entrainment to shorter  $\tau_p$  required an average of 21 steps. On the other hand, entrainment to longer  $\tau_p$  required an average of 48 steps. This observation was comparable to the experimental results reported in [1]; a greater number of perturbation cycles was required for healthy subjects to entrain their gaits to longer perturbation periods. Shorter perturbation periods, for which the initial phase of perturbation was near the end of double stance, required notably fewer steps to entrain. Longer perturbation periods did not necessarily follow this trend.

In all simulations displaying entrainment of the walker, the phase converged to  $32.7 \pm 2.4\%$  (i.e. the end of double stance). Importantly, this converged phase was independent of the phase within the step cycle at which torque pulses were initiated. Figure 7 shows a comparison of the initial perturbation phase and the (final) converged phase for all entrained simulations across the different perturbation periods tested.

## CONCLUSION

As with the previous model, our results indicate that simple afferent feedback processes, such as ankle actuation and leading leg angle control based on step-to-step energetics, may account for the limit-cycle behavior of human walking. Importantly, this model did not include a *central* pattern generator (CPG) independent of peripheral mechanics to account for its limit-cycle response to the imposed perturbations. Therefore, our results suggest higher-level

control of locomotion and the existence of a CPG as a self-sustaining rhythm generator may not be crucial for orbitally stable bipedal walking.

Our observations imply that therapeutic/assistive robots for locomotion should be developed with the aim of guiding the locomotor pattern by strongly emphasizing *non-encumbering* physical interactions. Specifically, excessive encumbrance might impose an energetic cost that would not be representative of walking without a therapeutic/assistive robot. In that case, any attempt by the locomotor controller to minimize energy expenditure would be rendered irrelevant. Alternatively, entrainment of the locomotor pattern to subtle mechanical perturbations at the ankle joint may serve as a promising method for *permissive* motor guidance in the field of assistive and rehabilitation robotics.

## REFERENCES

- [1] Ochoa, J., Sternad, D., and Hogan, N., 2016, "Entrainment of Overground Human Walking to Mechanical Perturbations at the Ankle Joint," *Proceedings of the 6th International Conference on Biomedical Robotics and Biomechanics*, IEEE, Singapore.
- [2] Ahn, J., Patterson, T., Lee, H., Klenk, D., Lo, A. C., Krebs, H. I., and Hogan, N., 2011, "Feasibility of Entrainment with Ankle Mechanical Perturbation to Treat Locomotor Deficit of Neurologically Impaired Patients," *Proceedings of the 33rd Annual International Conference of the Engineering in Medicine and Biology Society*, IEEE-EMBS, Boston, MA, pp. 7474–7477.
- [3] Ahn, J., and Hogan, N., 2012, "A Simple State-Determined Model Reproduces Entrainment and Phase-Locking of Human Walking," *PLoS One*, 7(11), p. e47963.
- [4] Selinger, J. C., O'Connor, S. M., Wong, J. D., and Donelan, J. M., 2015, "Humans Can Continuously Optimize Energetic Cost during Walking," *Curr. Biol.*, 25(18), pp. 2452–2456.
- [5] Soo, C. H., and Donelan, J. M., 2012, "Coordination of Push-off and Collision Determine the Mechanical Work of Step-to-Step Transitions When Isolated from Human Walking," *Gait Posture*, 35(2), pp. 292–297.
- [6] Ahn, J., and Hogan, N., 2012, "Walking Is Not like Reaching: Evidence from Periodic Mechanical Perturbations," *PLoS One*, 7(3), p. e31767.
- [7] Murray, M. P., Drought, A. B., and Kory, R. C., 1964, "Walking Patterns of Normal Men," *J. Bone Jt. Surg.*, 46(2), pp. 335–360.
- [8] Kadaba, M. P., Ramakrishnan, H. K., Wooten, M. E., Gainey, J., Gorton, G., and Cochran, G. V. B., 1989, "Repeatability of Kinematic, Kinetic, and EMG Data in Normal Adult Gait," *J. Orthop. Res.*, 7(6), pp. 849–860.
- [9] Perry, J., and Burnfield, J. M., 2010, *Gait Analysis: Normal and Pathological Function*, Slack Inc., Thorofare, NJ.