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Robust Optimization: Lessons Learned from Aircraft Routing

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Abstract

Building robust airline scheduling models involves constructing schedules and routes with reduced levels of flight delays as well as fewer passenger and crew disruptions. In this paper, we study different classes of models to achieve robust airline scheduling solutions, with a focus on the aircraft routing problem. In particular, we compare one domain-specific approach and two general paradigms of robust models, namely, (i) an extreme-value based or robust optimization-based approach, and (ii) a chance-constrained or stochastic optimization-based approach. Our modeling and solution approach demonstrates the creation of data-driven uncertainty sets for aircraft routing using domain-specific knowledge and develops a completely data-driven simulation-based validation and testing approach. We first demonstrate that additional modeling, capturing domain knowledge, is required to adapt these general robust modeling paradigms to the aircraft routing problem, in order to meaningfully add robustness features specific to aircraft routing. However, we show that these models in their naive forms, still face issues of tractability and solution quality for the large-scale networks which are representative of real-world airline scheduling problems. Therefore, we develop and present advanced models that address these shortcomings. Our advanced models can be applied to aircraft routing in multiple ways, through varied descriptions of the uncertainty sets; and moreover,

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are generally applicable to linear and binary integer programming problems. Through our detailed computational results, we compare the performance of solutions arising from these different robust modeling paradigms and discuss the underlying reasons for their performance differences from a data-driven perspective.

Keywords: robust airline scheduling, aircraft routing, robust optimization, chance-constrained programming

1. Introduction

Historically, airline schedule planners modeled the air transportation system assuming that flights can depart and arrive as scheduled. Factors such as weather, air traffic delays, mechanical problems and security issues, however, regularly affect the Department of Transportation’s (DoT) (US DoT, 2009) on-time performance metric, as evidenced by the 15-minute flight on-time arrival rate of 77.87% over the last 10 years (from Jan 2006 to Dec 2015) (Bureau of Transportation Statistics, 2016). In other words, over 22% of all flights during this period were either delayed upon arrival by at least 15 minutes (20.14%), or cancelled (1.75%) or diverted (0.24%). Of the 20.14% flights delayed upon arrival by at least 15 minutes, more than a third (7.12%) were due to aircraft arriving late from the previous flight in their itinerary (Bureau of Transportation Statistics, 2016). Similarly, several of the cancelled flights are likely to be so due to the aircraft arriving late from their previous flights, though such data is not publicly reported. These delays and disruptions (like cancellations and diversions) are highly detrimental in an industry where the profit margins are typically small, with delay and fuel costs forming a major component of operating costs (Air Traffic Association, 2008). As a case in point, the total costs of U.S. domestic air traffic delays and disruptions in year 2007 were estimated at approximately \$31.2 billion (Ball et al., 2010). Of these, \$8.3 billion represents incremental operating costs due to delays to the airlines (including additional fuel, labor and material costs), \$16.7 billion represents the estimated costs to

the passengers, and the remaining \$6.2 billion represents indirect costs in the form of lost demand and impact on GDP (Ball et al., 2010). Note that, while
25 the delays have decreased slightly since they reached their highest levels in 2007, large delays are still prevalent. For example, across the last 10 years, the US flight cancellation rate in the year 2014 was the highest (even higher than that in 2007), while the on-time performance in 2014 was the worst since 2008. Thus, it is evident that delays and disruptions continue to be a major concern in the
30 US aviation system and if delays can be reduced, the airline industry and our society would benefit tremendously.

Robust airline scheduling is a way of *pro-actively* considering such delays and disruptions and creating schedules that are less susceptible to disturbances, or, easier to repair once disrupted. This is in contrast to prior practice, where the
35 delay mitigation efforts are often *reactive*, that is, after an event occurs. These reactive efforts, called schedule recovery actions, can be costly and complex to implement. While schedule recovery may not be completely avoided even with robust solutions, the extent of recovery and its costs can be mitigated. Robust airline scheduling, then, is a proactive planning technique aimed at reducing
40 total *realized* costs, including both planned and recovery costs.

We add a note here on the schedule recovery process at airlines. Usually recovery measures are undertaken when either a major weather event is anticipated in the near future, or the delay incurred till then exceeds a ‘large enough’ value that requires intervention through recovery measures. The def-
45 inition of ‘large enough’ depends on the operations management team’s policies, and thus can differ significantly between airlines. Moreover, since recovery actions involve multiple resources, airlines may choose to undertake recovery either sequentially or simultaneously. For example, they could sequentially recover aircraft, followed by crew (which would be constrained by aircraft recovery
50 decisions), followed by passengers. Alternatively, it is possible to integrate the decisions across the multiple resources which would have greater flexibility but also greater computational complexity. Moreover, these processes differ across airlines because different airlines may have somewhat different sets of priorities

and may assess their available tradeoffs differently. Therefore, in this paper we
55 focus on a version of robustness that makes schedules less susceptible to disruptions, rather than relying on explicitly evaluating the costs of recovery actions. We aim to generate robust schedules that are less likely to be inoperable as planned due to disruptions during the course of operations.

Ours is a data-driven study wherein we use simulation driven by empirical
60 data to evaluate the true level of robustness of the obtained solutions, because the objective function values of the planning optimization models do not necessarily reflect the realized costs or the actual robustness level of the solution. Instead, we evaluate the solutions through a Monte-Carlo simulation using data-driven distributions and measure their performance with respect to a wide range
65 of relevant robustness metrics.

1.1. Motivation

In this paper, our focus is on the aircraft routing step of the airline scheduling process. The aircraft routing problem is to find a feasible sequence of flight legs, called an aircraft routing or rotation, to be operated by each aircraft so that
70 maintenance restrictions on aircraft are satisfied. Each flight is required to be assigned to (or covered by) exactly one aircraft, using no more than the available number of aircraft and meeting all maintenance requirements. Though robust planning adds value at every step of the airline scheduling process, we choose aircraft routing because of its high impact on schedule reliability and relatively
75 low impact on flight operating costs, crew costs, and passenger revenues (Lan et al., 2006). We define robust airline schedules as those schedules that are less likely to be inoperable as planned due to disruptions occurring during the course of operations.

Let us demonstrate how aircraft routings differ in robustness and what we
80 mean by robust aircraft routings, with an example. In Table 1, we report performance of seven aircraft routing solutions as measured by the percentage of flights in the routing solution that arrive within 15 minutes, 30 minutes, 60 minutes, 120 minutes and 180 minutes of their respective scheduled arrival times.

Note that these percentages, for this illustrative example, were calculated over
85 22 selected days of operations of a major U.S. airline. For the instances under
consideration in this paper, all of which are drawn from actual airline opera-
tions, we compare various metrics of interest, as detailed in §3. The reported
variability in flight delays is significant, as even small differences of around 1%
can increase/decrease the airline’s ranking with respect to the DoT’s 15-minute
90 on-time performance metric (Bureau of Transportation Statistics, 2009b). If
the airline does not explicitly consider delays in selecting aircraft routings, the
airline may end up choosing any of these routings, and thus, can potentially
incur high delays. To illustrate, for this instance, the aircraft routing actually
operated by the airline is Routing Solution 5. Incidentally, the DoT on-time
95 performance ranking for this airline during this time period was third from the
bottom. Moreover, in addition to aircraft delay disparities, different routings
can lead to different levels of passenger disruptions and delays. A passenger
is considered to be disrupted if one or more flight legs in his/her itinerary are
canceled, or if delays leave insufficient connection time to the next flight leg
100 in his/her itinerary causing a misconnection. The second from last column of
Table 1 shows the number of disrupted passengers for each routing solution
and the last column shows the percentage reduction in the number of disrupted
passengers when compared with the airlines actual routing solution (Routing
Solution 5). Routing Solutions 1 and 2 can vastly improve upon the airline’s so-
lution on each of these metrics without requiring any additional resources, while
105 Routing Solution 7 can further deteriorate the airline’s performance on these
metrics greatly. Therefore, it is clear that considering robustness explicitly is
important.

1.2. *Propagated Delays and Robustness Metrics*

110 The relationship between aircraft routings and delays and disruptions can
be explained by the phenomenon of *propagated delays*. In terms of causes, flight
delays can be divided into two components (Lan et al., 2006): *independent*
delays that originate at the flight’s origin or during the flight, and *propagated*

Routing Solution	% Flights with Lower or Equal Delays					Pax Disruptions	
	≤ 15 min	≤ 30 min	≤ 60 min	≤ 120 min	≤ 180 min	#D-pax	% Reduction in D-pax
Routing Solution 1	79.1	86.7	93.4	98.0	99.1	988	10.1
Routing Solution 2	78.8	86.8	93.2	98.2	99.2	986	10.3
Routing Solution 3	78.3	86.2	92.9	98.1	99.0	1028	6.5
Routing Solution 4	78.3	86.0	92.9	97.5	98.6	1047	4.8
Routing Solution 5	77.7	85.8	92.8	97.7	98.9	1100	0
Routing Solution 6	77.6	85.7	92.4	97.4	98.6	1057	3.9
Routing Solution 7	76.5	84.7	92.0	97.2	98.5	1223	-11.2

Table 1: Flight Delay Distributions and Passenger Disruptions for Different Routing Solutions

115 *delays* resulting from delays in upstream flights that are not absorbed by slack
 time between flight legs. Note that the term *independent delays* is meant to
 convey the fact that these delays are assumed to be independent of the aircraft
 routings being used and are not assumed to be (statistically) independent of
 each other. In fact, as will be seen in our computational experiments, we do
 consider situations where the independent flights delays are both correlated and
 120 uncorrelated with each other. Sometimes, these independent delays are also
 called as the *root delays* or as *primary delays*. However, we will use the phrase
independent delays to remain consistent with the terminology used by Lan et al.
 (2006).

Delay propagation is illustrated in Fig. 1. The solid arrows show the planned
 125 schedule for flights f_1 and f_2 ; and the dotted arrows the operated schedule.
PDT, *ADT*, *PAT* and *AAT* are the planned departure time, actual departure
 time, planned arrival time and actual arrival time respectively, of flight f_2 . In
 Figure 1, flight f_1 is delayed, and its delay causes the remaining time between
 flight f_1 's actual arrival and f_2 's scheduled departure to be less than the mini-
 130 mum connection time required for the same aircraft to fly both flight legs. This
 causes propagated delay *PD* for flight leg f_2 . In addition, independent delay
 is incurred by f_2 , both at its departure (*IDD*) and its arrival (*IAD*), resulting
 in total departure delay (*TDD*) and total arrival delay (*TAD*). However, by
 changing the sequence of flights operated by each aircraft, propagated delay
 135 can be reduced (Figure 2). This involves changes in the routings of aircraft.

Figure 1: Delay Propagation along an Aircraft Route

Figure 2: Robust Routing with Optimal Slack Allocation

Because no new aircraft are being added and the flight schedule remains unchanged, the total slack in the system is not altered, instead only the positioning of the slack is changed. Also note that through this change in routing, we are able to influence the propagated delay and total delay, while independent delays, by definition remain unchanged. Thus, for a given routing of aircraft, the total propagated delay will differ from the total delay by a constant value equal to the total independent delay.

For the airline studied in this paper, propagated delay typically represents 20% to 30% of total flight delay. Because total independent delay is a constant for the flight schedule, reducing delay propagation by choosing Routing Solution 1 instead of, for example, Routing Solution 7 (see Table 1), has the effect of

Figure 3: Propagated Delays of Feasible Aircraft Routings, N_2

reducing total flight delay. The differences in propagated delays for Routing Solutions 1-7 are shown in Fig 3. Each scenario here represents a different possible combination of independent delays. Different aircraft routing solutions are not very different for ‘good’ scenarios like Scenario 10, but differences become
150 apparent for ‘bad’ scenarios like Scenario 5.

Metrics of Robustness: Good solutions to airline scheduling problems will have lower levels of flight delays, and good travel experience due to lower passenger delays and fewer passenger disruptions. Metrics such as total flight delay
155 minutes, total cost of delay, 15-minute on-time performance, 30-min on-time performance, and 60-minute on-time performance, are all examples of measures that reflect airline schedule reliability and robustness. One problem, however, is that these metrics are not always aligned with each other. For example, the 15-minute on-time performance metric does not get affected by delays greater
160 than 15 minutes, and therefore maximizing 15-minute on-time performance is not the same as minimizing total delay minutes. Similarly, minimizing aircraft delay minutes is different from minimizing passenger delay minutes or minimizing the number of passenger disruptions. In this paper, we will evaluate

robustness via aircraft delay-based metrics using different delay thresholds as
165 well as passenger disruption metrics.

1.3. Existing Work

Literature on building robust solutions to optimization problems falls into
three major categories: domain-specific approaches, probability distribution-
based approaches, and probability distribution-free approaches. These different
170 approaches attempt to capture uncertainty in differing ways.

1.3.1. Domain-Specific Approaches to Robustness

There is a rich set of literature on domain-specific approaches to robustness
in airline schedule planning. These approaches identify specific *features* of the
problem of interest that affect solution robustness. Eggenberg et al. (2008)
175 discuss a number of these features. In this section, we only present a summary
of some of the representative studies in this area. But, we discuss in more detail
the models that involve robust aircraft routing.

Ehrgott and Ryan (2002) provide a bicriteria optimization between planned
cost and robustness, seeking to maximize robustness within allowable budgets
180 of deviation from the minimum cost solution. Robustness is defined in terms of
having the feature of sufficient ground times if crews have to change aircraft,
or as keeping the crew with the aircraft in case of tight ground times. Ageeva
(2000) creates robust aircraft routing solutions by building routings that contain
more ‘swap’ opportunities. The author examines multiple optimal solutions to
185 the aircraft routing problem and ranks them by their robustness, as measured
by the number of swaps. However, these two studies do not evaluate the result-
ing robust solutions through simulation. Kang and Clarke (2002) present an
approach to improve operational robustness using *degradable airline scheduling*.
Prioritizing on the basis of revenue, schedules are divided into layers that mini-
190 mize cross-layer interactions, so that any recovery actions taken in one layer will
not influence higher priority layers. Different models of incorporating degrad-
ability are presented, including degradable schedule partitioning, degradable

fleet assignment, and degradable aircraft routing. Rosenberger et al. (2004) develop a robust model for fleet assignment and aircraft routing that maximizes
195 the feature of ‘short cycles’. Under disruptions, when recovery is undertaken, airlines cancel not just one flight, but typically a cycle containing that flight, so as to reposition the aircraft to operate the rest of the schedule as planned. By increasing the number of *short* cycles, i.e., cycles containing a small number of flights, they can decrease the number of flights canceled. Evaluating the
200 solutions via simulation, the authors show that the incorporation of robustness in the form of short cycles produces fleet assignment solutions that decrease planned operating costs and passenger spill. Smith and Johnson (2006) solve the fleet assignment problem by limiting the number of aircraft types that are allowed to operate in each airport, and thus limit aircraft dispersion. They show
205 that this measure can help other aspects of airline schedule planning, such as maintenance routing, crew scheduling, etc., in making operations more robust.

Schaefer et al. (2005) introduce a measure to evaluate crew schedules, by incorporating planning costs and approximating the operational costs of crew schedules. They propose a combination of two methods: one based on expected
210 operating cost, in which disruptions are generated using SimAir, a Monte-Carlo simulation of airline operations; and another based on penalizing pairing features that may result in poor delay performance. Using simulation, they illustrate that solutions generated using these approaches perform better under uncertain conditions compared to deterministic approaches that do not take uncertainty into
215 account. Yen and Birge (2006) model crew scheduling as a two-stage stochastic program and show tradeoffs between planned crew costs and recourse costs. The first stage of the model minimizes expected crew costs. The second stage recourse model minimizes the expected costs of crews swapping between planes under scenarios of uncertainty. Shebalov and Klabjan (2004) propose the concept of move-up crews and improve costs by swapping crews, and show the
220 resulting benefits.

Jiang and Barnhart (2009) address demand uncertainty by introducing fleet re-timing as a dynamic scheduling mechanism and supplement re-fleeting with

re-timing. These features affect the number of connecting itineraries available
 225 to passengers and the number of seats available in the affected markets. They
 show that this can help to reduce passenger spill by better matching capacity to
 demand. Sohoni et al. (2011) define two service-level metrics, flight service level
 and network service level, to capture airline performance and passenger connec-
 tivity respectively. They develop stochastic integer programming formulations
 230 that perturb a schedule to maximize expected profit while ensuring the required
 minimum flight and network service levels, and thus achieve the tradeoff between
 service levels and profitability. Arikan et al. (2013) build stochastic models of
 airline networks to identify bottleneck airports and flights, identify metrics for
 measuring robustness and make schedules more robust. Their models help to
 235 develop three important robustness measures for airline networks, and enable
 policy recommendations to identify and manage bottleneck resources in the avi-
 ation system. Burke et al. (2010) build a memetic approach for multi-objective
 improvement of robustness-influencing-features (called robustness objectives) in
 airline schedules. They obtain improvement of the objectives by simultaneous
 240 flight retiming and aircraft rerouting, subject to a fixed fleet assignment, and
 also obtain approximations of the pareto-optimal front with respect to these
 multiple objectives. Significant improvements are shown using a large-scale
 simulation study.

Now we discuss some studies that are more relevant to the aircraft routing
 245 setting in our paper. Lan et al. (2006) introduce the term *delay propagation*
 and propose an aircraft routing model to reduce delay along the downstream
 flight legs. They show that aircraft routings can be made significantly more
 robust by re-arranging the slack in the schedule to place it where it is most
 needed. Thus robustness can be improved without changing the total slack and
 250 adding to planned costs. They also introduce a new approach to minimize the
 number of passenger misconnections by re-timing the departure times of flight
 legs within a small time window without significantly increasing costs. Their
 approach is very relevant to our work, and will be discussed in greater detail in
 later sections. AhmadBeygi et al. (2010) expand on this notion of propagated

255 delay using the concept of propagation trees. They measure delay propagation
 through aircraft, passengers and crew, and measure the extent to which each
 delay propagates. Using simulation, they show that airline schedules that are
 developed by considering delay propagation can significantly decrease opera-
 tional delays without increasing planned costs. Dunbar et al. (2012) present a
 260 way to minimize propagated delay by making both aircraft routing and crew
 pairing decisions in an integrated fashion capturing associated dependencies.
 The integrated models accurately capture the impact of the decisions on both
 aircraft and crew, and minimize the total delay to generate robust solutions. Yan
 and Kung (2015) discuss a framework to reduce propagated delay in the aircraft
 265 routing setting, when the flight delays lie in pre-specified and correlated uncer-
 tainty sets. They show that capturing this in an efficient new way is possible
 using the robust optimization method (the extreme-value approach that we also
 use in this paper). They show that in most cases this model outperforms Dun-
 bar et al. (2012), by increasing robustness and reducing both mean and extreme
 270 quantiles of delays. The work by Eggenberg et al. (2011), which is also relevant
 to our discussion, focuses on comparing domain-specific or feature-based mod-
 els with uncertainty-set based paradigms like robust or stochastic optimization.
 They introduce a framework that considers uncertain data by means of Uncer-
 tainty Features (UF). This framework is meant as an alternative to stochastic
 275 optimization and robust optimization frameworks that require the explicit de-
 scription of an uncertainty set, which may pose challenges. The authors present
 an Uncertainty Feature Optimization (UFO) framework as a multi-objective
 problem, and discuss the application of the framework to airline scheduling.

Table 2 summarizes the domain specific robust airline scheduling literature
 280 in terms of the steps of the airline schedule planning process that are addressed.

1.3.2. General Paradigms of Robust Models

Distribution-free and distribution-based approaches capture parameter un-
 certainty in a variety of ways. These approaches rely on two types of user
 inputs, namely, uncertainty sets and protection parameters. Uncertainty sets

	Schedule Design	Fleet Assignment	Aircraft Routing	Crew Scheduling
Ageeva (2000)			•	
Ehr Gott and Ryan (2002)				•
Rosenberger et al. (2004)		•	•	
Kang and Clarke (2002)	•	•	•	
Shebalov and Klabjan (2004)				•
Schaefer et al. (2005)				•
Lan et al. (2006)	•		•	
Smith and Johnson (2006)		•		
Yen and Birge (2006)				•
Jiang and Barnhart (2009)	•	•		
AhmadBeygi et al. (2010)			•	
Weide et al. (2010)			•	•
Burke et al. (2010)	•		•	
Sohoni et al. (2011)	•			
Dunbar et al. (2012)			•	
Eggenberg et al. (2011)	•			
Sohoni et al. (2011)	•			
Arikan et al. (2013)	•			
Froyland et al. (2014b)			•	
Froyland et al. (2014a)			•	
Yan and Kung (2015)			•	

Table 2: Summary of Domain-Specific Literature

285 can be thought of as a way of describing uncertainty, either in the form of distribution information or in the form of ranges of parameter variation. These can be obtained using any available knowledge of probability distributions or from historical datasets (Bertsimas and Thiele, 2006). Protection parameters specify the degree to which the uncertainty is protected against, within the
290 specified uncertainty sets. We provide the details of these distribution-free and distribution-based approaches as relevant to our problem in Sections 1.3.3 and 1.3.4 respectively.

The uncertainty set specifies the set of realizations of optimization model parameters or the set of scenarios that the robust formulation may protect
295 against. This is pre-defined by the user in both these general paradigms. The protection level parameters define, within this pre-defined uncertainty set, the fraction or the number of uncertain realizations to protect against. We describe this in detail in Sections 1.3.3 and 1.3.4.

1.3.3. *Extreme-Value (Distribution-Free) Paradigm*

300 Distribution-free approaches are designed to capture uncertainty in the parameters of a linear or integer program by considering uncertainty sets that describe ranges of variation of the optimization model parameters. These approaches make no assumptions about the distributions of the uncertain parameters within the specified uncertainty set, and as such assume that some or all the
305 parameters can take the worst-case or extreme values at the bounds of the user-defined uncertainty set. Soyster (1973) solves mixed integer-linear programs assuming that all parameters take on their worst-case values in a polyhedral uncertainty set. Bertsimas and co-authors (Bertsimas and Sim, 2004, 2003; Bertsimas and Thiele, 2003) argue that the approach of Soyster (1973) is too
310 conservative, and hence, expensive. In reality, not all uncertain parameters take on their worst-case values simultaneously. Therefore, they provide a mechanism to control the degree of conservatism through protection parameters Γ , which specify the number of uncertain parameters that can simultaneously take on their worst-case values. For integer values of Γ_i , the formulation protects

315 against the case when Γ_i parameters in constraint i assume their worst-case values and all other parameters assume their nominal values. Bertsimas and Sim (2004) use linear uncertainty sets in which each parameter varies in a fixed range around its nominal value. Other approaches have defined uncertainty sets differently; for example, some have defined ellipsoidal uncertainty sets. We refer 320 the reader to Ben-Tal and Nemirovski (1999, 2000, 1998) for further details on this topic.

1.3.4. *Chance-Constrained (Distribution-Based) Paradigm*

Distribution-based approaches are those that assume knowledge of a probability distribution for the uncertain parameters. Chance-constrained programming (CCP), specifically, admits random data variations and permits constraint 325 violations up to certain specified probability limits (Charnes and Cooper, 1963). Unlike in stochastic programming the decisions are not required to be feasible for every realization of the random parameters (as defined by the uncertainty set), but feasibility is ensured with at least some specified probability. The uncertainty set in this case is defined by the probability distribution or by a set of scenarios 330 that is representative of the probability distribution. When uncertainty in a constraint of a linear program is limited to the right-hand-side, Charnes and Cooper (1963) show that the randomness component can be modeled deterministically and the model can be converted into a linear program with very little increase in complexity. The model is intuitively appealing, but requires 335 at least partial knowledge of data distributions, compared to a distribution-free approach. The uncertainty set can then be described in terms of information about quantiles of the uncertain parameters (Marla, 2007). CCP models, however, face computational issues under uncertainty in the left-hand-side of the 340 constraint matrix, because the usage of multinomial distributions becomes necessary. This difficulty may be surmounted by describing the uncertainty set in terms of representative scenarios, either sampled from a distribution or obtained from historical data, which gives computationally tractable approximations of the CCP formulations (Nemirovski and Shapiro, 2006).

345 1.4. Contributions

Our work focuses on a fully data-driven approach to studying the three paradigms in the context of the robust aircraft routing problem. We study the performance of a domain-specific approach (Lan et al., 2006), and two general paradigms - the Extreme-Value(EV)-based distribution-free robust optimization
350 approach of Bertsimas and Sim (Bertsimas and Sim, 2004, 2003) and the probability distribution-based Chance-Constrained Programming approach (CCP) (Charnes and Cooper, 1959, 1963). A delay simulator is embedded within the modeling framework to parameterize the robust formulations. Additionally, the validation and testing of the solutions from the models is done via a data-driven,
355 simulation-based evaluator.

We focus on the most straightforward implementations of the extreme-value-based and chance-constrained paradigms to aircraft routing. While several other implementations under these same paradigms could exist, (especially for special cases such as pre-specified and correlated delay uncertainty sets, see Yan and
360 Kung (2015)), our goal is to understand the functioning of these general robust paradigms relative to each other and relative to domain-specific approaches. Therefore, we focus on comparing these general models without biasing them in favor of specific cases.

Our first contribution is to demonstrate how uncertainty is modeled in
365 composite variable-based formulations through the EV and CCP paradigms, and then show how to model the robust aircraft routing problem using the aforementioned three modeling approaches. Second, we demonstrate that existing protection-level based models, like CCP and EV, suffer from issues of non-monotonicity due to lack of pareto-optimality of their solutions. Third,
370 we develop new models in the extreme-value and chance-constrained modeling paradigms aimed at addressing the identified limitations related to monotonicity, ease of modeling and tractability. Fourth, we examine and compare the solution quality of the robust solutions from these different modeling approaches and provide insights in terms of the differences in solution quality, ease of implementation and tractability. Note that the findings and extensions from this work
375

are generally applicable to the broad class of network-based resource allocation problems (Marla, 2007).

1.4.1. Outline

In Section 2, we present the three classes of robust models studied in this paper. We present the basic aircraft routing model, and discuss the method to capture uncertainty in the composite variable setting. We then present Charnes and Cooper’s Chance-Constrained Programming approach, Bertsimas and Sim’s robust optimization approach, and Lan, Clarke and Barnhart’s robust aircraft routing approach. In Section 3 we present the experimental set-up for our computations, and in Section 4 we study the performance of the different models using the simulator presented in Section 3.4. In Section 5 we propose extensions and enhancements to these general classes of models that address the limitations of the models described in Section 2. We compare the performance of these enhanced robust aircraft routing models in Section 6, in terms of their complexity and run times (Section 6.1), model parameters (Section 6.2) and modeling paradigms (Section 6.3), after evaluating the performance of these solutions via simulation. We conclude with insights into the differences between modeling paradigms and their relative performances. In Section 7, we summarize our findings.

2. Robust Models

2.1. Deterministic Aircraft Routing Model

Following is the standard deterministic aircraft routing formulation, denoted by AR , which we attempt to make robust in the subsequent sections. We use this model as a benchmark model to describe the remaining models in the paper.

AR :

$$\min 0 \tag{1}$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s = 1 \quad \forall i \in F \tag{2}$$

$$\sum_{s \in S_i^+} x_s - y_{i,d}^- + y_{i,d}^+ = 0 \quad \forall i \in F^+ \tag{3}$$

$$- \sum_{s \in S_i^-} x_s - y_{i,a}^- + y_{i,a}^+ = 0 \quad \forall i \in F^- \tag{4}$$

$$\sum_{s \in S} r_s x_s + \sum_{g \in G} p_g y_g \leq N \tag{5}$$

$$y_g \geq 0 \quad \forall g \in G \tag{6}$$

$$x_s \in \{0, 1\} \quad \forall s \in S \tag{7}$$

400 The decision variables in this formulation correspond to strings (sequences of flight legs) with each string starting and ending at a maintenance station and obeying FAA (Federal Aviation Administration) and other regulations regarding maximum time between maintenance checks. Strings capture multiple decisions simultaneously, and thus are what we refer to as *composite variables* (Armacost et al., 2002). These are typically used to yield strong formulations, and/or when they eliminate the need for complex, difficult to model, constraints, as in the case of aircraft routing.

We set up a time-space network representation of the network (Marla et al., 2016). A maintenance station is any airport where aircraft maintenance can be performed. Let F be the set of daily flight legs, F^+ be the set of flight legs
410 originating at a maintenance station, F^- be the set of flight legs ending at a maintenance station, and S be the set of possible strings. The set of ground arcs (including wraparound arcs which begin on one day and end on another day at the same location) is denoted by G , the set of strings ending with flight
415 leg i is S_i^- , and the set of strings beginning with flight leg i is S_i^+ . Ground variable $y_{i,d}^-$ represents the number of aircraft on the ground before flight leg i

420 departs and $y_{i,d}^+$ is the number of aircraft on the ground after flight leg i departs. Similarly $y_{i,a}^-$ is the number of aircraft on the ground before flight leg i arrives and $y_{i,a}^+$ is the number of aircraft on the ground after flight leg i arrives. a_{is} is 1 if flight leg $i \in F$ is contained in string $s \in S$ and 0 otherwise. r_s is the number of times string $s \in S$ crosses the count line, p_g is the number of times ground arc $g \in G$ crosses the count line, and N is the number of aircraft, of the fleet type under consideration, available to the airline. Constraints (2) are the cover constraints that require each flight leg to be covered by some aircraft exactly once. Constraints (3) and (4) balance the number of aircraft at each airport location, and constraints (5) count the number of aircraft. Decision variable x_s takes on value 1 if string $s \in S$ is selected to be operated by an aircraft, and 0 otherwise. Decision variable y_g is the number of aircraft on ground arc $g \in G$.

2.2. Modeling Uncertainty in Composite Variable Coefficients

430 The first set of challenges associated with achieving robust solutions are related to deciding what constitutes robustness and how to capture that in the optimization model. The sources of uncertainty, as discussed in Section 1.2 are independent delays of *flights*. However, the model formulation (1) - (7) has *string* composite variables and associated coefficients. Therefore, we need to be able to relate how uncertainty that is rooted at the flight-level translates to uncertainty at the string-level, and then capture that as uncertainty in the parameters of the optimization formulation (1) - (7). We do this by means of a data-driven simulator described below.

2.2.1. Simulator

440 Using schedule data and details of operated aircraft routings (available from the AOTP database (Bureau of Transportation Statistics, 2009a)), we compute the independent and propagated delay for each flight leg, in each of the simulation scenarios, using a method similar to the method used in Lan, Clarke and Barnhart (Lan et al., 2006). This is possible because independent delays are, by definition, independent of aircraft routings. The propagated delays in each
 445

scenario are computed as discussed in Section 1.2. Thus, by applying independent delays to *any* sequence of flights forming a string we estimate, for each scenario and for each flight leg i along that string, propagated delay (PD_s), total departure delay (TDD_i) and total arrival delay (TAD_i). With this simulator, it is also possible to compute the probability of a flight leg being delayed by at least a specified threshold, or the probability of a string experiencing at least a specified level of propagated delay, or the range of delays experienced by a flight leg or a string. These various characterizations of uncertainty are employed to compute the robust optimization model inputs. Please note that what we call as a simulator in this section does not conduct any Monte-Carlo or probabilistic simulations. It simply implements the algorithm proposed by Lan et al. (2006) to calculate the propagated delays for all strings under several scenarios in a deterministic manner. It is called a simulator because it helps simulate the propagated delays for even those strings which were never operated in the historical schedules, thus making them available for being chosen in a routing solution.

Next, we illustrate different modeling paradigms along with the associated computational challenges and their solutions.

2.3. Domain-Specific Approach

Lan, Clarke and Barnhart (Lan et al., 2006) describe an approach to develop robust aircraft routings by modifying the benchmark model AR to minimize the total expected string propagated delay. The benchmark model is modified to add an objective function that minimizes the expected value of the sum of the propagated delays across all strings in the solution. Let pd_{ij}^s be the propagated delay from flight leg i to flight leg j when i immediately precedes j in string s . This can be computed using the simulator (Section 2.2.1) for each scenario in the historical data. Using the notation introduced in Section 2.1, the expected propagated delay in the network is:

$$E \left[\sum_{s \in S} \left(\sum_{(i,j) \in s} pd_{ij}^s \right) x_s \right] = \sum_{s \in S} \left(x_s \times \sum_{(i,j) \in s} E[pd_{ij}^s] \right) = \sum_{s \in S} d_s x_s \quad (8)$$

where $d_s = \sum_{(i,j) \in s} E[pd_{ij}^s]$.

475 Recall from Section 1.1 that propagated delay differs from total flight delay by a constant value. Thus, minimizing propagated delay is equivalent to minimizing total flight delay.

In terms of problem structure and complexity, the domain-specific model by Lan, Clarke and Barnhart (*LCB*) is the same as *AR*, except that the feasibility
480 objective is replaced with the objective of minimizing expected propagated delay, specifically:

LCB :

$$\min \sum_{s \in S} d_s x_s \quad (9)$$

s.t. Cover, Balance, Count, Non-negativity, and Integrality (2) – (7) (10)

2.4. Probability Distribution-Based Chance-Constrained Programming Approach

In Chance-Constrained Programming, the chance or probability that a constraint of the model is satisfied is required to be at least equal to some specified
485 threshold probability level. One way to apply it to the original formulation of the aircraft routing problem (1) - (7), is by modifying the cover constraint (2). Relative to the benchmark model that has a cover constraint that ensures each flight is covered in exactly one string the solution, the chance-constrained version of (2) specifies that the probability that each flight is covered is at least
490 equal to a certain user-specified probability. The chance-constrained aircraft routing formulation is as follows:

$$\max 0 \tag{11}$$

$$\text{s.t. } \mathbf{P}(\sum_{s \in S} a_{is} x_s = 1) \geq \alpha_i \quad \forall i \in F \tag{12}$$

$$\text{Balance, Count, Non-negativity, and Integrality (3) -- (7),} \tag{13}$$

where $\mathbf{P}$ denotes probability, and α_i is a user-defined protection parameter indicating the minimum probability that the aircraft routing solution will satisfy constraint (2) for flight i . Charnes and Cooper (1959) describe how to model
495 the non-linear constraints (12) as linear constraints.

2.4.1. Uncertainty Set Construction

For this formulation, as discussed in Section 1.3.2, we are required to define the uncertainty sets and the protection parameters. An uncertainty set specifies the set of realizations of parameters that the formulation may protect against.
500 In chance-constrained programming, usually, probabilistic descriptions or their scenario-based approximations are used as uncertainty sets. In this paper, we utilize historical data to obtain empirical distribution information, and that combined with domain knowledge of airline industry helps us define the uncertainty sets. Because it is not possible to cover a flight (without delays) 100% of
505 the time, the protection level α specifies the frequency with which a given level of delays are protected against.

Our uncertainty set is constructed in a data-driven manner. From historical delay scenarios, we group the delay incurred by flight i in string s , into intervals of delays in minutes: $[0,15]$, $(15,30]$, $(30,60]$, $(60, 90]$, $(90, 120]$, $(120, 180]$ and
510 $(180, \infty)$. We define a flight as being covered, if it can be operated by a string s without a delay greater than a threshold delay t . For example, at delay levels of $t = 15$ minutes (beyond the $[0,15]$ minute interval), flights incur delays which are directly related to the DoTs 15-minute on-time performance metric; at $t = 30$ minutes (beyond the $[0,30]$ minute interval), connecting passengers typically
515 risk missing their connecting flight; and at $t = 90$ minutes (beyond the $[0,90]$

minute interval), flight cancellations may occur as airlines are likely to cancel flights that are expected to be over 90 minutes late. Note that, the thresholds for misconnections and cancelations are not exact, but are good approximations based on, respectively, typical passenger connection times and typical airline operational strategies. Thus our uncertainty set is defined in a data-driven manner which is informed by our domain knowledge. Specifically, empirical distributions are derived from historical data and domain knowledge related to flights' delays, passengers' misconnections and flight cancelations informs the choice of t . Our uncertainty set, therefore, is parameterized by t . The scenarios in the uncertainty set of interest have different interpretations depending on the choice of t .

2.4.2. Protection Parameters and Formulation

We model uncertainty *probabilistically* in the flight cover constraints (2) by defining p_{is} as the *probability* obtained from historical data, that flight leg i in string s is covered according to the required service level, that is, its arrival is delayed by less than or equal to t minutes when string s is operated. We obtain p_{is} from the simulator, for all possible flight-string combinations, over all scenarios.

Because each flight leg is present only in a single string in the solution, the probability of flight i being delayed less than or equal to t minutes is $p_i = \sum_{s \in S} p_{is} x_s$. Using this, we re-write (11) - (13) as:

CCP_f :

$$\max 0 \tag{14}$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s = 1 \quad \forall i \in F \tag{15}$$

$$\sum_{s \in S} p_{is} x_s \geq \alpha_i \quad \forall i \in F \tag{16}$$

$$\text{Balance, Count, Non-negativity, and Integrality (3) - (7)} \tag{17}$$

Constraints (16) are the robustness constraints that limit to $1 - \alpha_i$ the probability that flight leg i is delayed more than t minutes. In structure and complexity, CCP_f is similar to AR , though it adds constraints (16), one for each flight leg, and requires specification of the value of α_i for each constraint $i \in F$. A challenge associated with this model is the specification of α_i values. Values that are too high can lead to infeasibilities and values that are too low can lead to inadequate levels of robustness or protection. As a result, CCP_f might have to be solved repeatedly to find appropriate α -values by trial-and-error. Repeated solution of CCP_f , however, might be impractical in identifying the best set of α values.

2.5. Extreme-Value Robust Optimization Approach

Relative to the AR model, the EV model based on the Bertsimas and Sim robust optimization approach ensures that each flight is covered in the case when a subset of flights take on their worst-case delay values. We provide further details below. For applying the extreme-value (EV) robust optimization approach of Bertsimas and Sim to the aircraft routing problem, we define the uncertainty set in a manner similar to that described in Section 2.4. $\tilde{a}_{is}$ are the realizations of a_{is} parameters under uncertainty, with a_{is} itself being the nominal value without uncertainty. Under this approach, $\hat{a}_{is}$ is the width of the interval of uncertainty from the nominal, such that $a_{is} + \hat{a}_{is}$ is the worst-case realization of $\tilde{a}_{is}$. We set $\hat{a}_{is} = -1$ if flight $i \in F$ in string $s \in S$ has extreme value of delay exceeding t minutes in the historical data. Because the Bertsimas and Sim approach considers realizations of the uncertain parameters at their extreme (or worst-case) values (Bertsimas and Sim (2004, 2003)), if a flight i in string s has extreme delay exceeding t minutes using historical data, the extreme value $\hat{a}_{is} = -1$ results in $a_{is} + \hat{a}_{is} = 1 - 1 = 0$, reflecting the extreme occurrence that flight i is not covered according to the required service level. Similar to the CCP model, t can take on different values to define different uncertainty sets and associated worst-case scenarios. For example, as before, $t = 15$ min, $t = 30$ min and $t = 90$ min, could represent on-time performance, missed passenger

connections and flight cancellations respectively.

For each flight i , the Bertsimas and Sim robust optimization approach defines a ‘robustness’ or ‘protection’ parameter Γ_i taking on (possibly continuous) values in $[0, |S|]$, representing the number of coefficients in constraint i that can assume worst-case or extreme values and still satisfy feasibility of the constraint. The model ensures constraint feasibility in the case where up to $\lfloor \Gamma_i \rfloor$ coefficients take on extreme values, and one coefficient a_{it} changes by up to $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it}$. For our purposes, integer values of Γ_i are the most meaningful because of the uncertainty definition. For each flight $i \in F$, Γ_i represents the number of strings containing flight i , in which flight i does not experience worst-case delays greater than or equal to t minutes. The resulting extreme value formulation from the Bertsimas and Sim model is:

$$\min 0 \tag{18}$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s = 1 \tag{19}$$

$$\begin{aligned} & \sum_{s \in S} a_{is} x_s \\ & + \max_{\{S'_i \cup t_i | S'_i \subseteq S, |S'_i| = \lfloor \Gamma_i \rfloor, t_i \in S \setminus S'_i\}} \left\{ \sum_{s \in S'_i} \hat{a}_{is} w_s + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} w_{t_i} \right\} = 1 \quad \forall i \in F \end{aligned} \tag{20}$$

$$x_s \leq w_s \quad \forall s \in S \tag{21}$$

$$w_s \geq 0 \quad \forall s \in S \tag{22}$$

$$\text{Balance, Count, Non-negativity, and Integrality (3) – (7)} \tag{23}$$

If $\Gamma_i \geq 1$, the constraints (19) and (20) ensure that each flight $i \in F$ is covered by exactly one string s which contains flight i and has extreme value of delay less than t for flight i . Because the values of $\hat{a}_{is}$ are either 0 or -1, (18)-(23) simplifies to:

EV_f :

$$\min 0 \tag{24}$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s + \max \left\{ \sum_{s \in S} \hat{a}_{is} x_s, -\Gamma_i \right\} = 1 \tag{25}$$

$$\text{Balance, Count, Non-negativity, and Integrality(3) -- (7),} \tag{26}$$

with the second term in the left-hand side of (25) called the *protection function*, which incorporates the protection level parameters Γ_i . This formulation
585 can be linearized easily as described in (Bertsimas and Sim, 2004).

Note that the robustness of the solution can also be varied by selecting the uncertainty set differently, through different values of the extreme delay parameter t . Reducing the value of t will have the effect of generating more conservative solutions, those that permit little delay, while increasing the value of t will have
590 the opposite effect. In this research, we ran computational experiments using various different values for the the uncertainty set, as parameterized by different t values. Our results and conclusions are based on these different experiments.

An alternative formulation of the extreme-value approach can be achieved by capturing uncertainty via the d_s coefficients in the *LCB* formulation, allowing
595 the uncertainty to be captured at the string level (instead of the flight-string combinations, as with EV_f). This requires a completely different definition and parameterization of the uncertainty set, and the protection parameters. This formulation, denoted by EV_s , is presented in Appendix A and its applications to the aircraft routing problem are discussed in Section 5.2.

600 3. Experimental Set-up

In this section, we introduce our experimental set-up and evaluation mechanism, through which we will demonstrate and evaluate the performance of our models.

3.1. Pre-Processing the Network Data

605 We conduct our experiments using data representing the flight networks of two major U.S. airlines that operate hub-and-spoke networks. Historical flight delay and cancelation data are obtained from the Airline On Time Performance (AOTP) database (Bureau of Transportation Statistics, 2009a). The fields in this database that are relevant to this work are: the basic flight information such
610 as date (as indicated by the fields ‘Year’, ‘Month’ and ‘Day of Month’), ‘Origin’, ‘Destination’, ‘Carrier’, and tail number (‘TailNum’), on-time performance information in the fields including planned departure time (‘CRSDepTime’), actual departure time (‘DepTime’), wheels off time (‘WheelsOff’), wheels on time (‘WheelsOn’), planned arrival time (‘CRSArrTime’), actual arrival time (‘ArrTime’),
615 and reason of delay if it exists (as indicated by the fields ‘CarrierDelay’, ‘WeatherDelay’, ‘NASDelay’, ‘SecurityDelay’ and ‘LateAircraftDelay’), as well as disruption information in the fields including ‘Cancelled’, ‘Diverted’, and reasons for cancelation or diversion (as indicated by the field ‘CancellationCode’).

First, for the airlines of interest, we infer operated routings based on tail
620 numbers. We choose the flights that were operated as part of a daily repeated schedule (i.e., the flights that were operated at least 5 days a week) for the year 2007. We separate these flights based on fleet type into different categories and identify sequences of flights operated by each tail number in the fleet. We fill in missing tail number fields for canceled flights using estimates of their planned
625 tail numbers, which are obtained assuming that the routings are repeated on a weekly basis excluding weekends (that is, the routing on each Monday is the same).

Second, we find independent delays for each flight for each day in year 2007. We divide the total delay for each flight into independent and propagated delay
630 as described in Lan et al. (2006). Note that for doing so, we do not need to know the planned aircraft routing but only need to know the *operated* aircraft routing, which is publicly available as described in the previous paragraph. Because independent delays are assumed to stay constant regardless of the routing being used, we can later find the propagated delay for any routings, even the ones not

635 used in the historical data.

After determining the independent delays of all flights that are operated on a daily basis (excluding weekends), we perform some further filtering by removing a handful of flights to ensure a balanced schedule, that is, the number of incoming flights is equal to the number of outgoing flights for each airport in the daily schedule. We select four such networks for our experiments, described in Section 3.2.

Finally, for the selected networks, we generate a timeline network representation (Barnhart et al., 2003), with a time horizon of 72 hours. This horizon indicates that aircraft have repeating rotations that cover flights in a way that every 72 hours the aircraft can be serviced at a maintenance station at an airport. Under this representation, each aircraft string begins at a maintenance station and follows multiple consecutive flights with ground times in between. Each string ends with the last flight in it arriving at a maintenance station followed by some specified hours of ground time that allows for maintenance checks. We build strings using a depth-first-search algorithm that enumerates all strings in each of the selected networks. For the purposes of this study, we have chosen networks that have less than 1,000,000 strings.

3.2. Description of Selected Networks

We denote the networks by N_1 , N_2 , N_3 and N_4 , each representing a different aircraft routing problem for a fleet type of an airline. Each of these networks in our dataset have daily repeating schedules, that is, each flight in these networks is operated every day by the exact same aircraft type. Various characteristics of N_1 , N_2 , N_3 and N_4 are shown in Table 3. One subset of the independent delay data is used as input to the aircraft routing models to generate robust solutions under each modeling paradigm. These solutions are then evaluated using another subset of the data, corresponding to a future time period, to detect and avoid overfitting.

Network	Number of Daily Flights	Airports	Strings	Scenarios	Sampling
N_1	50	16	9639	5000	Distribution-fitting
N_2	165	16	878207	200	Direct AOTP data
N_3	22	11	1959	102	Direct AOTP data
N_4	70	20	31800	120	Direct AOTP data

Table 3: Fleet Network Characteristics

3.3. Generating Scenario Data

We filtered our dataset to ensure that we used data from only those days
665 where all the flights in our dataset were operated. In our dataset, each flight
leg is operated several times, and we compute its independent delay value (as
described earlier in Section 3.1) for each time the flight leg is operated in our
filtered dataset. We regard each independent delay value (one for each day) as
one *scenario* for this flight leg. In our data-driven approach, we assume that the
670 independent delay of the flight legs has the same distribution as that given by
the historical independent delay values. For networks N_2 , N_3 and N_4 , we have
sufficient daily independent delay data in the year covered by our dataset. As
shown in Table 3, there were 200, 102 and 120 repetitions respectively, of these
flights available in our dataset. Therefore we use that data directly as scenarios
675 in our experiments. In other words, for networks N_2 , N_3 and N_4 , each day
in the historical data corresponds to a different delay scenario, which implies
that any possible correlation between independent delays of different flight legs is
automatically accounted for through the definition of our scenarios. For network
 N_1 , we have about 60 scenarios over three months where all flights are operated.
680 So, we fit a probability distribution to the historical independent delay values.
After fitting multiple distribution families including gamma, lognormal, Gumbel
and exponential distributions, lognormal distribution was found to have the best
fit to the independent delay data. We then sampled from this distribution to
generate 5000 scenarios as indicated in Table 3. We assume that the independent
685 delays of different flights are statistically independent of each other for network
 N_1 , though we are not required to make such an assumption for networks N_2 ,

N_3 and N_4 . Each of these two scenario generation approaches offers a different set of advantages and disadvantages; and together covers a wide spectrum of assumptions, thus leading to a more comprehensive evaluation of the models being considered in this paper.

3.4. Metrics and Evaluator

We compare the robustness of solutions from all the models in terms of three types of evaluation metrics: on-time performance, total propagated delay and passenger disruptions. For on-time performance, we look at 15-minute on-time performance (OTP), 30-minute OTP, 60-minute OTP, 90-minute OTP, 120-minute OTP and 180-minute OTP. 15-minute OTP relates to whether a flight is considered to be ‘late’ as per the DoT’s on-time performance measure. The 30-minute OTP is a good proxy for whether or not passengers can make it to their connecting flights. The 60-minute OTP is a good proxy for whether or not crew can make their connections. Finally, the 90-minute, 120-minute and 180-minute OTPs are good proxies for the potential of flight cancellations under different cancellation policies. Because total flight arrival delay is equal to propagated delay plus (constant) independent delay, comparison results based on propagated delay also apply to total delay. We also compare the number of disrupted passengers and the percentage of disrupted passengers.

We use Lan, Clarke and Barnhart’s (Lan et al., 2006) algorithm to compute from the airline’s solution, for each scenario in the test data set, the propagated delay ($PD(i)$), the independent departure delay ($IDD(i)$) and the independent arrival delay ($IAD(i)$) from the total departure and arrival delays ($TDD(i)$ and $TAD(i)$) for each flight i . Then, given independent delay for each scenario in the test data for each flight leg i , we re-employ Lan, Clarke and Barnhart’s (Lan et al., 2006) algorithm (in the reverse order of steps) to compute the delay and disruption metrics for each of the solutions we generate using our models. This last step includes computing total flight delay for each scenario, on-time performances with various thresholds, and the total number of passengers disrupted, for the aircraft routing solutions output by the different models. To

compute passenger disruptions, we enumerate all pairs of flights (f_1, f_2) between which passengers connect. Let $C(f_1, f_2)$ be the scheduled time available for the passenger to make the connection and let $m_{f_1 f_2}$ represent the minimum time
720 needed for a passenger to connect between flights f_1 and f_2 . If $C(f_1, f_2) - TAD(f_1) + TDD(f_2) \leq m_{f_1 f_2}$, the actual connection time between f_1 and f_2 is too short to connect and the passengers scheduled to make this connection are disrupted.

To ensure a fair comparison and also to avoid overfitting, we use a five-fold
725 cross-validation approach. The data is divided into five subsets, and each time one of the five subsets is tested on the solution obtained from solving the model over the other four subsets. This is repeated with each subset iteratively as the test subset. We find and report the average performance of the model over all subsets. Our models are implemented in Java and CPLEX v12.51 on a Dell
730 laptop with 16 GB RAM.

It is important to note that in evaluating robust routings, we assume that flight delays are allowed to propagate along the strings, without any recovery interventions such as cancelations or aircraft swaps. This allows us to estimate the levels of delay propagation and robustness of the strings *without* any inter-
735 vention. Because cancelation and swapping strategies are different for different airlines and they depend on a variety of tradeoffs that have to be carefully made by the airlines, assuming particular swapping and cancelation strategies could lead to bias.

To avoid such bias, we compare different robust aircraft routing methods
740 under the assumption that the airlines choose not to respond to large delays and disruptions, except in the most passive manner wherein the delays are allowed to propagate. On the one hand, this approach may be perceived as fair because it evaluates each robust aircraft routing method on its own merit, rather than having to rely on the assumption of a particular active recovery
745 method that may modify its performance. On the other hand, this approach may be perceived as favoring the robust aircraft routing methods that rely less on active recovery actions. We acknowledge this second way of thinking. In fact,

evaluating the combined effects of the various combinations of robust scheduling strategies and recovery strategies in a data-driven way would be the logical next step in this research. However, it is a substantially larger endeavor, and hence it is beyond the scope of this first paper. We identify that as an opportunity for future research.

4. Preliminary Results and Motivation for Advanced Models

The protection parameters α and Γ in the CCP_f , EV_f and EV_s (described in detail in Appendix A) models are designed to represent the desired extent of robustness, with larger values of α and Γ representing higher levels of solution robustness. However, note that choosing a protection level α_i or Γ_i for each constraint is a combinatorial decision, which is difficult given the large number of flights and associated constraints. This poses tractability questions around choosing the ‘right’ protection levels for the various flights. Because every possible combination of protection levels across flights cannot be tried for tractability reasons, we run our models with the same protection level parameters for all flights.

We now discuss the solution performance. While we discuss this with respect to networks N_1 and N_2 , similar phenomena are observed for other networks as well. Table B.13 (in Appendix B) and Figure 4 show that the CCP_f solutions’ robustness performance shows non-monotonicity. That is, for a fixed uncertainty set and with increases in α , the solutions to CCP_f can worsen with respect to flight on-time performance, passenger disruptions and total aircraft delay minutes. There are several reasons for this. First, we found that there exist multiple optimal solutions to this model, because of the large number of available strings. These solutions satisfy the same protection parameter values and yield the same objective function values, but use different sets of string to operate the flights. For example, optimal solutions to the CCP_f with $\alpha_i = 90$ for all $i \in F$ can include solutions that satisfy $\alpha_i = 90$ or $\alpha_i = 94$ or $\alpha_i = 96$ for all $i \in F$. All of these solutions are considered optimal by CCP_f even though,

Figure 4: CCP_f Model Solutions for Network N_2 ($t = 90$)

intuitively, the solution with the highest α value should be considered to be the most ‘robust’ among these. As a result, robustness metrics exhibit non-monotonicity property, as shown in Table B.13. In the case of $\alpha_i = 90$ for all $i \in F$, 95.63% of the flights are delayed less than or equal to $t = 90$ minutes and for $\alpha_i = 92$ for all $i \in F$, 95% of the flights are delayed less than or equal to $t = 90$ minutes. Similar results are observed for network N_1 , N_3 and N_4 , and for other choices of uncertainty sets represented by t .

Another reason for *not all* robustness metric values always increasing with increasing values of α is that the CCP_f model focuses on selecting routings that limit the likelihood of occurrence of ‘large’ flight delays as a proxy for robustness, and therefore we do not expect that the solution will simultaneously optimize the three different (and sometimes conflicting) robustness metrics that we evaluate, especially those related to passenger connections.

Similarly, as illustrated in Figure 5 and in Table B.14 (in Appendix B), for the same choice of uncertainty set t , higher values of Γ in EV_f do not always produce more robust solutions. The reasons for this occurrence are similar to that for CCP_f . *Multiple optimal solutions* to EV_f (and also EV_s) for given Γ

Figure 5: EV_f Model Solutions for Network N_2 ($t = 90$)

values satisfy the protection constraints defined by Γ , even though the solutions might have very different flight strings and actual levels of slack. Hence they exhibit very different performances with respect to our robustness metrics. In fact, some of the optimal EV_f solutions are even less robust with respect to *all* of our robustness metrics than the airline's routing itself, as is the case with $\Gamma = 3$ in Figure 5 and Table B.14. The second reason for higher levels of Γ in EV_f (and EV_s) sometimes leading to less robust solutions is the same as that for CCP_f . EV_f (and EV_s), like CCP_f , does not capture all the robustness metrics precisely, but rather builds robustness into the solution by ensuring that the flight legs are covered at the required service level, even in the extreme cases. Our results do, however, show slightly less variability in the alternate optima of the EV_s model, compared to the EV_f model, because it is based on extreme levels of propagated delay minutes for each string and accounts for every minute of delay, instead of only focusing on thresholds of t minutes.

The challenge of choosing solutions with the maximum protection parameter (α and Γ) values among multiple optimal solutions needs to be addressed by more advanced models, which we present in Section 5.

5. Advanced Modeling Approaches

5.1. Extended Chance-Constrained Programming Approach

To overcome the limitations of the CCP_f model discussed in Section 2.4, we propose the Extended Chance-Constrained Programming ($ECCP$) approach.

815 This method models all the protection levels α_i , one for each constraint, as variables and maximizes the overall sum of protection levels across all the constraints. For a general CCP formulation such as $\max \mathbf{c}^T \mathbf{x}$, s.t. $P(\sum_{j \in J} a_{ij} x_j \leq b_i) \geq \alpha_i \forall i \in I$, the $ECCP$ formulation would be: $\max \sum_{i \in I} \alpha_i$, s.t. $P(\sum_{j \in J} a_{ij} x_j \leq b_i) \geq \alpha_i \forall i \in I$, $\mathbf{c}^T \mathbf{x} \geq \mathbf{c}^T \mathbf{z}^* - \delta$, where δ is the maximum allowable loss in the

820 objective value (compared to the optimal cost $\mathbf{c}^T \mathbf{z}^*$ of the nominal formulation) to attain a more robust solution. Marla (2007) and Marla (2010) discuss how this can be linearized using partial knowledge of the uncertain parameters b_i , or using suitable scenario-based approximations. We refer interested readers to Appendix C for the detailed formulation.

825 For aircraft routing, the protection level for each flight in constraint (16) is not specified apriori in the $ECCP$ model. Instead it is treated as a decision variable in the model. This simultaneously achieves two purposes. First, it addresses the challenge of making the combinatorial choice of protection levels, which are difficult for the user to ascertain beforehand. Instead, the protection

830 levels in $ECCP$ are determined internally by the model. Second, it automatically guarantees that the chosen optimal solution always satisfies the chosen protection levels exactly. We call this formulation, given by (27) – (30), as the $\alpha - CCP_f$ model, which maximizes the sum of protection levels of all the constraints (flights) to determine the protection level for each flight. Therefore,

835 relative to the benchmark AR model, this model captures the probability of covering each flight by a variable α_i , and maximizes the total probability of coverage of all flights.

$\alpha - \mathbf{CCP}_f$:

$$\max \sum_{i \in F} \alpha_i \quad (27)$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s = 1 \quad \forall i \in F \quad (28)$$

$$\sum_{s \in S} p_{is} x_s \geq \alpha_i \quad \forall i \in F \quad (29)$$

$$\text{Balance, Count, Non-negativity, and Integrality (3) – (7)} \quad (30)$$

Note that maximizing the sum of protection levels is just one of the multiple possible approaches for addressing the shortcomings of the basic *CCP* model. Other possible approaches include: 1) maximizing a weighted sum of flight protection levels $\left(\sum_{i \in F} \alpha_i w_i \right)$ with weight w_i assigned to flight i ; or 2) maximizing the minimum protection level ($\max \alpha_{min}$) with constraints $\sum_{s \in S} p_{is} x_s \geq \alpha_{min}$, $\forall i \in F$ replacing constraints (29).

Yet another alternative formulation can also be obtained by minimizing the sum of expected string propagated delays for the best possible α_i values chosen by first solving the $\alpha - CCP_f$ model. This model is different from the other variants of CCP described above in that it has a string-based objective function instead of a flight-based one. We denote this formulation by $\alpha - CCP_s$ and is given by (31) – (34).

$\alpha - \mathbf{CCP}_s$:

$$\min \sum_{s \in S} d_s x_s \quad (31)$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s = 1 \quad \forall i \in F \quad (32)$$

$$\sum_{s \in S} p_{is} x_s \geq \alpha_i \quad \forall i \in F \quad (33)$$

$$\text{Balance, Count, Non-negativity, and Integrality (3) – (7)} \quad (34)$$

Here $d_s = \sum_{(i,j) \in s} E[pd_{ij}^s]$ is defined as before. We solve this model by first solving $\alpha - CCP_f$ and finding the ‘best’ values of protection parameters, and then setting those protection parameter values in (33) to minimize the sum of expected string propagated delays (31). This formulation accounts for the fact that the $\alpha - CCP_f$ model itself typically has multiple optimal solutions and ensures that it picks the one with the smallest value of the sum of expected string propagated delays.

5.2. Extended Extreme-value Robust Optimization Approach

As with the CCP model, the EV_f model requires the specification of a parameter value (in this case, Γ_i) for each cover constraint (25). To avoid the need to repeatedly solve EV_f to determine the best Γ values, which is computationally extremely expensive at best and practically impossible at worst, we propose an alternative model, denoted by $\Delta - EV$, in which the Γ parameters are modeled as decision variables. For a general 0-1 integer programming formulation such as $\max \mathbf{c}^T \mathbf{x}$, s.t. $\sum_{j \in J} a_{ij} x_j \leq b_i \forall i \in I, x_j \in \{0, 1\} \forall j \in J$, with realizations of uncertain parameters a_{ij} being $\tilde{a}_{ij}$, Marla (2010) formulates the robust problem by setting the protection parameters as variables, and maximizing the sum of protection parameters, within a cost budget, expressed relative to the optimal cost $\mathbf{c}^T \mathbf{z}^*$ of the nominal problem. Relative to the benchmark AR model, this model ensures that the maximum number of flights are covered when faced with extreme values of delay.

The general $\Delta - EV$ formulation of Marla (2010) is as follows: $\max \sum_i \Gamma_i$ subject to $\sum_j a_{ij} x_j + \max_{\{S'_i \cup \{t_i\} | S'_i \subseteq J, |S'_i| = \lfloor \Gamma_i \rfloor, t_i \in J \setminus S'_i\}} \{ \sum_{j \in S'_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_{t_i} \} \leq b_i \forall i \in I, \mathbf{c}^T \mathbf{x} \geq \mathbf{c}^T \mathbf{z}^* - \delta, -y_j \leq x_j \leq y_j \forall j, \Gamma_i \geq 0 \forall i, x_j \in \{0, 1\} \forall j \in J$. The author linearizes this by taking advantage of the 0-1 integer program setting and sorting the columns $j \in J$ associated with each constraint i , in increasing order of their $\hat{a}_{ij}$ values. Ties are broken arbitrarily. The formulation then selects the largest sized set of $\tilde{a}_{ij}$, in each constraint i , that cannot be protected against their worst-case values (that is, need to take on their nominal values for

feasibility), and seeks to minimize the sum of the sizes of these sets across all
880 constraints. We refer the reader to Appendix D for the detailed formulation.

For aircraft routing, the goal of this formulation is to minimize the number
of variables in the solution whose coefficients in constraint i are *not* allowed
to realize their extreme values (denoted by Δ_i) while ensuring feasibility of
constraints. Note that $\Delta_i = |S| - \Gamma_i$. We minimize sum of Δ_i across all flight
885 legs i , thus maximizing the protection level. However, aircraft routing problem
has a special structure where all the constraint coefficients a_{ij} equal 0 or 1 and
their extreme values differ from the nominal values by at most 1. Due to this
special structure, the $\Delta - EV$ model can be formulated without any Δ variables,
and also does not require any sorting of the columns. Also, the aircraft routing
890 problem is a feasibility problem in its nominal form and hence does not require
a budget constraint in the Δ formulation. The $\Delta - EV$ model for robust aircraft
routing is as follows:

$\Delta - EV_f$:

$$\max \sum_{i \in F} \sum_{s \in S} \hat{a}_{is} x_s \quad (35)$$

$$\text{s.t. } \sum_{s \in S} a_{is} x_s \geq 1 \quad \forall i \in F \quad (36)$$

$$\text{Balance, Count, Non-negativity, and Integrality (3) - (7)} \quad (37)$$

As defined in Section 2.5, $\hat{a}_{is} = -1$ if flight $i \in F$ in string $s \in S$ has
extreme value of delay exceeding t minutes, based on historical data; and $\hat{a}_{is} = 0$
895 otherwise. The objective (35) effectively maximizes $\sum_{i \in F} \Gamma_i$ or minimizes $\sum_{i \in F} \Delta_i$
by maximizing the total value of the protection function in (25) summed for all
flights $i \in F$.

Because the extreme value robust optimization framework also allows un-
certainty to be modeled in the objective function, an alternative to EV_f and
900 $\Delta - EV_f$ is to capture uncertainty in a manner similar to that of the domain-
specific *LCB* model by Lan et al. (2006), but within the extreme value frame-

work. In comparison to the benchmark AR model, this would involve protecting against the extreme values of string propagated delay while ensuring that the expected propagated delay value of the chosen strings is within a pre-specified budget. This requires a completely different definition of the uncertainty set, and its parameterization, and redefining the protection parameters, as we now describe. Capturing uncertainty via the objective can be formulated as minimizing the total amount of propagated delay when any Γ strings in the solution experience their respective extreme propagated delay values and all other strings experience their nominal propagated delay values (that is, zero). This results in a formulation we refer to as the EV_s model, with a complex protection function like (20). The detailed formulation is presented in the Appendix A. The difficulty with this model, again, is specifying the ‘best’ value of Γ . Because a value of Γ does not explicitly indicate the overall robustness of the solution, it is difficult to specify a priori. We therefore present the $\Delta - EV_s$ model that finds a solution such that the sum of the extreme delay values of any subset with Γ strings does not exceed the maximum allowable system propagated delay D . D is a user-specified threshold on total propagated delay, obtained by analyzing the historical occurrences of propagated delays. The $\Delta - EV_s$ solution, therefore, is an aircraft routing that allows the largest number of strings to realize their worst-case propagated delay values without exceeding the maximum allowable system propagated delay D . This model, like the $\Delta - EV_f$, is a simplified version of the general Δ model proposed by Marla (2010).

We set to zero the nominal propagated delay value for any string s , and let $\hat{d}_s$ represent the expected extreme or worst-case value of propagated delay for string s , as computed using historical data. Let $\bar{S}$ be the set of strings $s \in S$ with realizations of non-zero propagated delays in the historical data, that is, with non-zero $\hat{d}_s$. We set $v_s = 1$ if string $s \in S$ has to take on its nominal propagated delay value of 0 and not its worst-case (historical) value for the solution to be feasible. To maximize the size of the largest subset of strings that can realize their worst-case values and ensure feasibility, we sort the strings in increasing order of their $\hat{d}_s$ values such that $\hat{d}_1 \leq \hat{d}_2 \leq \dots \leq \hat{d}_{|S|}$. Δ is the

minimum number of strings with propagated delays subject to uncertainty that must assume nominal, not worst-case, values to be feasible.

$\Delta - \mathbf{EV}_s :$

$$\min \Delta \tag{38}$$

$$\text{s.t. } \sum_{s \in S} \hat{d}_s x_s - \sum_{s \in S} \hat{d}_s v_s \leq D \tag{39}$$

$$\Delta \geq \sum_{s \in S} v_s \quad \forall s \in \bar{S} \tag{40}$$

$$v_s \leq x_s \quad \forall s \in \bar{S} \tag{41}$$

$$v_s \leq w_s \quad \forall s \in \bar{S} \tag{42}$$

$$v_s \geq x_s + w_s - 1 \quad \forall s \in \bar{S} \tag{43}$$

$$w_s \geq w_{s+1} \quad \forall s \in |S| - |\bar{S}| + 1, \dots, |S| - 1 \tag{44}$$

$$\text{Cover, Balance, Count, Non-negativity, Integrality (2) - (7)} \tag{45}$$

$$v_s \in [0, 1] \quad \forall s \in S \tag{46}$$

$$w_s \in \{0, 1\} \quad \forall s \in S \tag{47}$$

935 Constraints (39) require that the total worst-case propagated delay be limited by D when any $|\bar{S}| - \Delta$ strings take on worst-case propagated delay values. $v_s = 1$ for all strings $s \in \bar{S}$ in the solution whose delay value must be set to 0 to achieve feasibility. $w_s = 1$ for all strings $s \in \bar{S}$ for which there exists a $k \geq s$ such that $v_k = 1$. Inequalities (41) force $v_s = 0$ unless string s is present
940 in the solution. Inequalities (42) allow v_s to be 1 only if w_s is 1. Inequalities (43) allow v_s to be 0 only if either $w_s = 0$ or $x_s = 0$. Constraints (40), in combination with the objective (38) count the minimum number of strings $s \in \bar{S}$ whose coefficients must take on the nominal propagated delay value of 0. The explanation for this constraint lies in the realization that when the strings in S
945 are sorted in increasing order of $\hat{d}_s$ values, the *minimum number* of coefficients that must assume nominal values to maintain feasibility is determined by forcing the strings with the smallest $\hat{d}_s$ values, for $s \in \bar{S}$, to have their associated v_s

values set to 1 if x_s is in the solution. Constraints (44) ensure that the values of w_s variables are non-decreasing in s . Constraints (46) and (47) restrict w and v variables to take on values of 0 or 1 only.

6. Results

So far in this paper we have presented three basic models and four advanced models. The basic models include the domain-specific approach (LCB), the chance-constrained approach (CCP_f) and the extreme-value approach (EV_f); while the advanced models include the flight-based and string-based variants of the $\alpha - CCP$ model, and the flight-based and string-based variants of the $\Delta - EV$ model. Each of these seven model formulations yields an aircraft routing solution (for each parameter setting) for each of our case study networks, N_1 , N_2 , N_3 and N_4 . In this section, we present the computational results of experiments on these seven models presented in this paper, studying similarities and differences in the obtained solutions in terms of complexity, run time and robustness as measured by our metrics. We compare these model solutions with each other and also with the routing solution actually used by the airline. We then discuss the similarities and differences between the solutions, caused by the differences between the modeling paradigms, and the manner in which they capture the underlying data distributions.

6.1. Typical Computation Times

Table 4 reports average computation times for network N_2 with $t = 90$ and $D = 350$. That is, we define a flight as being ‘covered’ if it is delayed by less than $t = 90$ minutes representing cancelations, in all the models except $\Delta - EV_s$. In the $\Delta - EV_s$ model, we restrict total *propagated* delay levels in the worst case to be less than the 85th percentile propagated delay from historical data, which was 350 minutes for network N_2 . (For network N_1 , the 85th percentile propagated delay is 160 minutes, as seen in Tables 5 and 6.) For the CCP_f and EV_f models, multiple iterations are required to determine the appropriate α and Γ values,

Model	Uncertainty set Parameters	Protection Parameters	Iterations	Run time per iteration (sec)
AR	None	None	1	25.14
LCB	None	None	1	42.81
CCP_f	t	$\alpha_i \forall i$	1 for each α_i	52.30
$\alpha - CCP_f$	t	None	1	72.91
$\alpha - CCP_s$	t	None	1	60.61
EV_f	t	$\Gamma_i \forall i$	1 for each Γ_i	45.33
$\Delta - EV_f$	t	None	1	42.13
$\Delta - EV_s$	D	None	1	6018.39

Table 4: Complexity and Run Times for Network N_2 with $t = 90$, $D = 350$

respectively. Because there is no prior indication if a particular protection level α_i or Γ_i (for flight i) renders the model infeasible or improves upon the currently available routing, experimentation with different values is necessary. Therefore, for these two models, Table 4 reports the run times as the average of several runs with multiple protection parameter values. While these particular results are for network N_2 with $t = 90$ and $D = 350$, similar results are obtained for different choices of uncertainty sets and for network N_1 as well. Although they each had to be solved only once, the computation times of AR , LCB , $\alpha - CCP_f$, $\alpha - CCP_s$, and $\Delta - EV_f$ models are found to be comparable to the time required to solve a single iteration of CCP_f and EV_f . On the other hand, $\Delta - EV_s$ has a much larger run time due to the large number of routes/strings, because the model has the number of constraints equal to the number of strings. Therefore, we should be cautious while using it in an even larger network, as the number of constraints grows exponentially with the number of flights.

6.2. Solution Differences due to Modeling Paradigms

In this section, we compare the quality of solutions from all the models in terms of three evaluation metrics: on-time performance, total propagated delay

	Flight On-time Performance					
	≤ 15 min	≤ 30 min	≤ 60 min	≤ 90 min	≤ 120 min	≤ 180 min
Airline's Routing	82.61%	91.21%	96.37%	98.00%	98.80%	99.44%
LCB	89.63%	95.39%	98.36%	99.18%	99.55%	99.82%
CCP_f	84.36%	92.71%	97.24%	98.56%	99.16%	99.63%
$\alpha - CCP_f$	90.89%	95.87%	98.50%	99.24%	99.59%	99.83%
$\alpha - CCP_s$	89.60%	95.37%	98.35%	99.18%	99.55%	99.82%
EV_f	83.68%	93.04%	97.37%	98.61%	99.19%	99.63%
$\Delta - EV_f$	90.89%	95.87%	98.50%	99.24%	99.59%	99.83%
$\Delta - EV_s$	82.76%	91.42%	96.47%	98.06%	98.84%	99.46%

Table 5: On-time Performance Results for Network N_1 with $t = 90$, $D = 160$

and passenger disruptions. The definitions of these metrics and evaluation setup are as described in Section 3.4. We first discuss the results for networks N_1 and N_2 , for the sake of clarity. Because the results for N_3 and N_4 are similar, we present them after discussing N_1 and N_2 .

At this juncture, it is worth reviewing our first result. As noted in Section 4, the original CCP_f and EV_f models exhibit performance that does not improve monotonically with increasing values of the associated protection parameters. This is because these models do not choose solutions that are on the pareto-frontier with respect to the protection parameters. While a similar result has been discussed by Iancu and Trichakis (2014), this is the first data-driven evidence for large-scale network-based problems, in which the complexity is compounded by the user needing to choose among a combinatorially growing set of protection parameters.

Our second major result is that both the $\Delta - EV_f$ and $\alpha - CCP_f$ model solutions improve upon the airline's routing, unlike the solutions generated by the EV_f model which did not outperform the airline's routing with respect to some metrics. In general, the extended models (with the exception of $\Delta - EV_s$) perform better than the original ones. Table 5 and Table 6 list the on-time performance, total propagated delay and passenger disruption metrics for net-

	Propagated delay	Pax Disruptions
	Total average (min)	% Disrupted pax
Airline's Routing	95.90	6.50%
LCB	61.88	4.40%
CCP_f	81.35	5.74%
$\alpha - CCP_f$	62.35	3.94%
$\alpha - CCP_s$	61.87	4.40%
EV_f	107.06	4.68%
$\Delta - EV_f$	61.43	3.94%
$\Delta - EV_s$	93.85	6.46%

Table 6: Propagated Delay and Passenger Disruption Improvements in N_1 with $t = 90$, $D = 160$

work N_1 . Note that for CCP_f and EV_f , the tables report the 'best' solutions obtained over multiple parameter values, chosen using the five-fold cross validation approach. For N_1 , we notice that $\alpha - CCP_f$, and $\Delta - EV_f$ perform the best overall across the various robustness metrics. $\alpha - CCP_s$ and LCB perform second best in terms of our metrics of interest. CCP_f , EV_f and $\Delta - EV_s$ perform poorly compared to the other models, but still are somewhat better than the routing solution that was actually used by the airline. Table 7 and Table 8 present the same set of performance metrics for network N_2 . For N_2 , we notice that $\alpha - CCP_s$ performs the best overall across these different metrics. Also, LCB , CCP_f , $\alpha - CCP_f$ and $\Delta - EV_f$ continue to perform well. On the other hand, as in case of network N_1 , the EV_f and $\Delta - EV_s$ models perform poorly compared to other models, but nevertheless improve slightly upon the routing actually used by the airline. Thus, our third major result is that while $\Delta - EV_f$, $\alpha - CCP_f$, and $\alpha - CCP_s$ are the best performers across the different metrics and across different networks, the performance of the basic models CCP_f and LCB (with CCP_f 's best parameter settings) is only marginally worse. However, CCP_f model along with EV_f and $\Delta - EV_s$ models encounters issues of tractabil-

ity, and CCP_f and EV_f models also suffer from the difficulty in specifying the
1030 ‘right’ values of the protection parameters. Instead, the extended models allow
us to find the ‘best’ levels of robustness parameters. The solutions obtained by
these extended models are robust and consistently improve upon the airline’s
routing with respect to on-time performance metrics, total propagated delays
and passenger disruption metrics.

1035 We now compare the performance of the flight-based models $\alpha - CCP_f$ and
 $\Delta - EV_f$ with the string-based models $\alpha - CCP_s$ and $\Delta - EV_s$, to arrive at
our fourth major result. From this point onward, we compare also the results
from networks N_3 and N_4 as we are discussing the performance of the advanced
modeling approaches we propose in this paper. From Tables 5 - 12, we see that
1040 the models that capture delays at the flight level, by capturing uncertainty in the
parameters $a_{i,s}$, perform somewhat better than those that capture uncertainty at
an aggregate string-level through the d_s parameters. The only exception is that
for network N_2 , where $\alpha - CCP_f$ slightly outperforms $\alpha - CCP_s$. This is because
these flight-level models $\alpha - CCP_f$ and $\Delta - EV_f$ control delay at the individual
1045 flight level, compared to the $\alpha - CCP_s$ and $\Delta - EV_s$ models. Between two
strings with similar total propagated delay but different levels of flight delays,
the flight-based models choose the string with lower individual flight delays.
Therefore they improve flight on-time performance, allow better connections
for passengers, and hence perform better than the string-based models.

1050 Finally, we compare the performances of the flight-based and string-based
models for the four networks N_1 , N_2 , N_3 and N_4 , to arrive at our fifth ma-
jor result. As mentioned earlier, the overall performance of the results from
the flight-based models is better than those of the string-based models. This
phenomenon is very clear when the performance is evaluated on network N_1 ,
1055 which is based on distribution-fitting-approach and thus assumes that the in-
dependent delays of different flights are statistically uncorrelated. However,
our results show that the relative performance of both the string-based models
($\alpha - CCP_s$ and $\Delta - EV_s$) compared with those of the flight-based counterparts
($\alpha - CCP_f$ and $\Delta - EV_f$) improves for networks N_2 and N_3 . In fact, $\alpha - CCP_s$

	Flight On-time Performance					
	≤ 15 min	≤ 30 min	≤ 60 min	≤ 90 min	≤ 120 min	≤ 180 min
Airline's Routing	72.92%	81.40%	89.45%	93.66%	96.23%	98.58%
LCB	73.43%	81.92%	89.90%	93.95%	96.46%	98.63%
CCP_f	73.61%	82.01%	89.90%	93.91%	96.37%	98.60%
$\alpha - CCP_f$	73.79%	82.07%	89.96%	93.96%	96.44%	98.62%
$\alpha - CCP_s$	74.55%	82.82%	90.41%	94.27%	96.63%	98.68%
EV_f	72.73%	81.33%	89.48%	93.66%	96.25%	98.58%
$\Delta - EV_f$	73.54%	81.80%	89.75%	93.83%	96.31%	98.55%
$\Delta - EV_s$	73.55%	81.98%	89.90%	93.92%	96.43%	98.62%

Table 7: On-time Performance Results for Network N_2 with $t = 90$, $D = 350$

turns out to be the overall best model for networks N_2 and N_3 . This can be explained by the fact that there are significant correlations between the independent flight delays in these networks and capturing uncertainty at the string levels allows the string-models to account for such correlations between the independent flight delays. Flight-based models do not have this ability and hence perform slightly worse for network N_2 and N_3 than for network N_1 .

6.3. Influences of Underlying Data Distributions

We now discuss the influence of the underlying data distributions on the efficacy of each of the modeling paradigms. Because the $\Delta - EV_f$ and $\Delta - EV_s$ models are distribution-free while the $\alpha - CCP_f$ and $\alpha - CCP_s$ models are distribution-based, the differences in their efficacy are also expected to depend on the underlying data distributions.

To illustrate this point, consider Figure 6 in which coverage probabilities of two flight-string combinations i_1s_1 and i_2s_2 are shown. $\Delta - EV_f$ and EV_f do not distinguish between both these realizations, because both i_1s_1 and i_2s_2 have non-zero historical probability of realizing a delay value greater than t . On the other hand, the $\alpha - CCP_f$, $\alpha - CCP_s$ and CCP_f distinguish between these realizations in a probabilistic manner.

Related to the string-based models $\Delta - EV_s$ and $\alpha - CCP_s$, we find that the

	Propagated delay	Pax Disruptions
	Total average (min)	% Disrupted pax
Airline's Routing	458.59	6.35%
LCB	412.77	5.53%
CCP_f	342.26	4.89%
$\alpha - CCP_f$	394.61	5.01%
$\alpha - CCP_s$	342.56	5.19%
EV_f	443.18	6.45%
$\Delta - EV_f$	371.25	5.04%
$\Delta - EV_s$	516.15	6.39%

Table 8: Propagated Delay and Passenger Disruption Improvements in N_2 with $t = 90$, $D = 350$

	Flight On-time Performance					
	≤ 15 min	≤ 30 min	≤ 60 min	≤ 90 min	≤ 120 min	≤ 180 min
Airline's Routing	75.61%	82.90%	85.00%	91.72%	92.23%	94.33%
LCB	80.76%	86.20%	89.50%	94.00%	95.60%	97.44%
$\alpha - CCP_f$	84.20%	86.90%	88.45%	94.89%	98.48%	98.00%
$\alpha - CCP_s$	84.50%	87.24%	89.56%	95.23%	98.20%	98.50%
$\Delta - EV_f$	84.60%	86.92%	88.38%	94.70%	98.70%	98.90%
$\Delta - EV_s$	83.95%	84.70%	87.30%	93.40%	96.83%	98.20%

Table 9: On-time Performance Results for Network N_3 with $t = 60$, $D = 176$

	Propagated delay	Pax Disruptions
	Total average (min)	% Disrupted pax
Airlines Routing	195.29	5.20%
<i>LCB</i>	186.49	3.62%
$\alpha - CCP_f$	175.94	3.56%
$\alpha - CCP_s$	170.02	3.64%
$\Delta - EV_f$	173.89	3.57%
$\Delta - EV_s$	183.62	3.74%

Table 10: Propagated Delay and Passenger Disruption Improvements in N_3 with $t = 60$, $D = 176$

	Flight On-time Performance					
	≤ 15 min	≤ 30 min	≤ 60 min	≤ 90 min	≤ 120 min	≤ 180 min
Airline's Routing	78.76%	83.48%	86.00%	90.90%	95.40%	97.85%
<i>LCB</i>	82.28%	86.57%	88.00%	89.70%	90.76%	95.20%
$\alpha - CCP_f$	86.00%	89.50%	92.20%	94.50%	95.49%	98.35%
$\alpha - CCP_s$	85.71%	88.65%	91.43%	93.64%	94.31%	98.10%
$\Delta - EV_f$	87.00%	90.76%	93.75%	95.29%	96.20%	98.79%
$\Delta - EV_s$	84.80%	87.90%	90.92%	93.45%	95.37%	98.35%

Table 11: On-time Performance Results for Network N_4 with $t = 60$, $D = 210$

	Propagated delay	Pax Disruptions
	Total average (min)	% Disrupted pax
Airline's Routing	155.01	5.53%
<i>LCB</i>	136.51	4.92%
$\alpha - CCP_f$	131.30	4.56%
$\alpha - CCP_s$	126.89	4.60%
$\Delta - EV_f$	127.76	4.60%
$\Delta - EV_s$	137.02	4.78%

Table 12: Propagated Delay and Passenger Disruption Improvements in N_4 with $t = 60$, $D = 210$

Figure 6: The Realization Probabilities of a_{is} in Network N_2

string delays often follow a bimodal distribution. As an illustration, consider
 1080 Figure 7(a) in which the distributions of propagated delays for strings in network
 N_1 are shown. The propagated delay distribution is seen to be roughly bi-modal,
 with a large (88% to 95%) probability of the propagated string delay being on
 the lower end of the scale, and a small (5% to 12%) probability of the propagated
 delay being close to its worst-case value. The probability of occurrence of interim
 1085 values is relatively small. This causes the average value and worst-case value of
 string propagated delays to be as shown in Figure 7(b).

The *LCB* model uses average values of string propagated delay, shown in Fig.
 7(b). Because the average is typically at around the 85th or 90th percentile of
 string propagated delay, *LCB* effectively ignores the extreme value occurrences
 1090 forming the remaining 5-10% of the distribution. Its objective of minimizing the
 expected value of the total propagated delay seems to correlate well with our
 robustness metrics on average, with the *LCB* solutions performing well on all
 metrics. Although the *LCB* model does not capture the probability distribution
 of delays, by using average delay values, it effectively behaves like a special case
 1095 of the string-based $\alpha - CCP_s$ model where the quantiles of d_s in the objective

Figure 7: Propagated Delay Distributions of Strings

function of $\alpha - CCP_s$ model are set, for these problems, to approximately the 90th percentile level. The advantage of *LCB* over $\alpha - CCP_s$ is that it is able to capture uncertainty in a simpler way, resulting in a highly tractable model; while its disadvantage is that the resulting solutions are overall slightly less robust than those of the $\alpha - CCP_s$ model (as seen in Tables 5, 6, 7 and 8).

Because the $\alpha - CCP_f$ and $\alpha - CCP_s$ models capture the probabilistic nature of events, their emphasis on high-probability events results in optimal solutions which correlate, on an average, consistently well with improved values of the metrics under consideration (Tables 5, 6, 7 and 8). Multiple alternative optima to the $\alpha - CCP_f$ and $\alpha - CCP_s$ models exist, but there is not a lot of difference in their performance with respect to the values of our robustness metrics.

Further investigation into the solutions of the $\Delta - EV_f$ models shows that, like EV_f , they too can often have multiple optimal solutions, each with the same objection function values but differing significantly in the robustness metrics. We see that the performances of at least one optimal solution from the $\Delta - EV_f$ models is very similar to those of *LCB* and $\alpha - CCP_f$ models; but additionally there also exist multiple alternate optima for $\Delta - EV_f$ that perform very differently. Depending on the solution output provided by the optimization solver being used, these models can have significantly varying performances. Simi-

1115 larly, the $\Delta - EV_s$ model is based on the extreme values of string propagated
delays. This emphasis on extreme values of delays does not necessarily drive
the extreme-value models towards good values of the average-case robustness
metrics. Because the $\Delta - EV_f$ and $\Delta - EV_s$ models are formulated to avoid
low probability situations involving worst-case delays or non-coverage, optimal
1120 solutions can have widely varying values of average performance as measured by
average on-time performance, average propagated delays, and average number
of disrupted passengers.

We can apply the insights gained in solving the aircraft routing problem
with LCB , $\alpha - CCP_f$ and $\alpha - CCP_s$ models to improve the extreme value-
1125 based robust optimization models. Specifically, by adjusting the uncertainty set
definition or ‘extreme’ values in the $\Delta - EV_s$ model to, say, capture additional
quantiles of string propagated delay, we can generate solutions using these mod-
els that perform closer to those of the LCB , $\alpha - CCP_f$ and $\alpha - CCP_s$ models.
This, of course, underscores the difficulty of setting these uncertainty sets a
1130 priori, and the sensitivity of the solutions to these model inputs.

7. Conclusions

In this paper, we study the application of three types of models - domain-
specific, probability distribution-free and probability-distribution based - to the
problem of aircraft routing. These three approaches lead to different models with
1135 different solutions which have different robustness performances with respect to
various metrics of interest.

The probability distribution-free extreme-value models (EV_f and EV_s) were
based on the Bertsimas and Sim robust optimization approach (Bertsimas and
Sim, 2004, 2003). The probability distribution-based models are based on
1140 Charnes and Cooper’s chance-constrained programming approach (Charnes and
Cooper, 1959, 1963). Finally, the domain-specific model (LCB) was Lan, Clarke
and Barnhart’s (Lan et al., 2006) robust aircraft routing approach. Significantly
increased complexity and solution times are associated with the extreme-value

(*EV*) and chance-constrained programming (*CCP*) models, when compared
1145 to a deterministic model, because the models have to be solved several times
for different values of the robustness parameters Γ and α to identify the most
appropriate parameter values. To avoid iterative re-solving, we developed ad-
vanced models, namely, $\Delta - EV_f$, $\Delta - EV_s$ for the extreme-value approach, and
 $\alpha - CCP_f$ and $\alpha - CCP_s$ for the chance-constrained programming approach. All
1150 of our extended models can be solved in a single iteration, with runtimes of all
but the $\Delta - EV_s$ model equivalent to or lower than a single iteration of the basic
models. We evaluated solutions to the different models through a simulation-
based process on four airline fleet networks extracted from real-world schedules.
We then measured each solution’s performance in terms of total propagated
1155 delay, on-time performance metrics, and passenger disruption metrics. Our re-
sults showed that the extended models lead consistently to generation of more
robust solutions (compared to the basic models, and compared to the solution
that was operated by the airline), as defined by the metrics of interest. The
more general versions of our extended models, applicable to the broad class of
1160 problems involving network-based resource allocation, are presented in Marla
(2007) and Marla (2010).

The first take-away from these experiments is that the robust models, by
capturing delay propagation explicitly, provide solutions that improve signifi-
cantly upon the airline’s routing with respect to on-time performance metrics,
1165 total propagated delays, and passenger disruption metrics. Second, although
the basic *CCP* and *EV* models encounter issues of intractability and difficulty
in specifying the ‘right’ values of protection parameters, when the ‘best’ pa-
rameters are found, the solutions obtained are robust and improve upon the
airline’s routing. Third, in three of the four networks, the models that induce
1170 robustness at the flight level, such as $\alpha - CCP_f$ and $\Delta - EV_f$, result in superior
performance, on average, than string-level (route-level) robust models such as
 $\alpha - CCP_s$ and $\Delta - EV_s$. This is because the former two models allow for a
larger degree of control at the flight-level as opposed to the string-based models
that constrain the propagated delays only at the string-level. This ensures both

tractable formulations as well as superior solutions. String-level models (especially the $\alpha-CCP_s$ model) do a better job of capturing the correlations between independent flight delays and thus have an improved (robustness) performance when independent delays are correlated. Fourth, the extreme-value based models including the $\Delta-EV_f$ and $\Delta-EV_s$ models, in most cases, exhibit significant variability in performance due to their focus on the worst-case values of delays. Like the EV model, they can often have multiple optimal solutions. Although alternative optimal solutions have the same objective function value, they differ significantly in terms of the associated values of the various robustness metrics. Our findings show that extreme-value based models EV_f , $\Delta-EV_f$ and $\Delta-EV_s$ generate solutions with variable performance levels; varying from good improvement compared to the airline's routing, to no significant improvement or even deterioration. This behavior is because the mechanism used for capturing robustness (Γ or Δ) is driven by *extreme* values of delay. This dependence on extreme delay values, ignoring probabilistic information, leads in some cases to a large disparity in the performance of the alternative optimal solutions. In such cases, extra care should be taken in evaluating alternative optimal solutions to these approaches. Thus, we conclude that it is not effective to drive the solution process with extreme values when these extreme values are rare.

The domain-specific approach and the probability distribution-based approaches are very similar in that expected values of string propagated delay, used to drive the LCB model, are typically found to be at around the 85th or 90th percentile of the string propagated delay distribution; and hence cause the formulation to focus on higher-probability events. Similarly, probability distribution-based approaches also focus on higher-probability delay events, and produce improved routings according to our metrics. These approaches thus capture more information about the system and focus on more-likely delay events, and thus are more in line with our metrics of interest. Though the domain-specific approach in itself does not explicitly capture knowledge of probability distributions, by implicitly incorporating some of the relevant delay quantiles in its objective it can achieve improved results through a less complex

model. The probability distribution-based approaches allow more fine-tuning of robustness using the α and t parameters, but come at a cost of creating a larger, more complex formulation (albeit very tractable for this application) and the added implementation cost, compared to *LCB*, due to the requirement of
1210 knowing the delay distributions.

In conclusion, our paper highlights that the efficacy of any given robust approach is determined not by the approach or model alone, but by the interaction between the model, data (including network structure and delay patterns), and evaluation metrics. Thus, an opportunity for future extension of this analysis
1215 is to test our comparison framework through controlled experiments on several networks with varying characteristics, such as hub-and-spoke or point-to-point network structure, and specific delay patterns and (or) correlations, in order to see how each model would perform under each setting.

Our paper therefore, underscores the importance of choosing an approach
1220 that aligns itself well with the data distributions for the aircraft routing problem and metrics of interest to the DoT, industry and passengers. When applying general robust approaches to more specific problems, care should be taken to understand the nature of uncertainty and to choose robustness parameters in relation to the metrics, especially when the metrics affect multiple stakeholders.

1225 **Appendix A. EV_s Model Formulation**

The EV_s model plans for the case when a user-defined number of strings Γ in the formulation realize their *worst-case values* of propagated delay, while other strings do not vary from their assumed deterministic or nominal delay values (equal to zero).

1230 The protection parameter Γ is defined in the interval $[0, |S|]$. Let $\hat{d}_s$ be the worst-case (maximum or 100th percentile) value of propagated delay of string s observed in the historical data. Note that this is not the worst possible realization of propagated delay, but it is the worst-case actual realization in the selected period of historical data.

1235 The formulation minimizes the extreme (or worst-case) value of string propagated delay caused by *any* set of Γ strings in the solution realizing their worst-case values, and the other strings attaining their nominal values of propagated delay (equal to zero). That is, it minimizes the maximum propagated delay caused by any subset of Γ strings in the solution realizing their extreme propagated delay values, as shown in (A.1). Variable w_s for string $s \in S$ takes on value 1 if string s is present in the solution, and the model protects against its extreme value being realized. The extreme-value formulation, according to Bertsimas and Sim (Bertsimas and Sim, 2003), is as follows.

$$\min \left\{ \sum_{s \in S} 0x_s + \max_{\{S' \cup \{t\} | S' \subseteq S, |S'| = \lfloor \Gamma \rfloor, t \in S \setminus S'\}} \left\{ \sum_{s \in S'} \hat{d}_s w_s + (\Gamma - \lfloor \Gamma \rfloor) \hat{d}_t w_t \right\} \right\} \quad (\text{A.1})$$

$$\text{s.t. Cover, Balance, Count, Non-negativity, and Integrality (2) – (7)} \quad (\text{A.2})$$

$$x_s \leq w_s \quad \forall s \in S \quad (\text{A.3})$$

$$w_s \geq 0 \quad \forall s \in S \quad (\text{A.4})$$

This formulation can be linearized using the process described in Bertsimas and Sim (2004) and Marla (2010), and cast as a mixed integer program, as follows:

EV_s :

$$\min z\Gamma + \sum_{s \in S} p_s \quad (\text{A.5})$$

$$\text{s.t. } z + p_s \geq \hat{d}_s w_s \quad \forall s \in S \quad (\text{A.6})$$

$$\text{Cover, Balance, Count, Non-negativity, and Integrality (2) -- (7)} \quad (\text{A.7})$$

$$x_s \leq w_s \quad \forall s \in S \quad (\text{A.8})$$

$$w_s \geq 0 \quad \forall s \in S \quad (\text{A.9})$$

$$z \geq 0. \quad (\text{A.10})$$

These mixed integer programming formulations lose the special set partitioning structure of the original *AR* formulation (Marla, 2007) due to the type of constraints added to the formulation, and face computational challenges (see Section 6.1).

1250

Appendix B. Tables for Preliminary Results - Section 4

	% Flight Delay (min) improvements					Pax Disruptions	
	≤ 15	≤ 60	≤ 90	≤ 120	≤ 180	#D-pax	D-pax reduced
$\alpha_i = 90 \forall i$	78.54	93.10	95.63	97.82	98.91	1025	6.8
$\alpha_i = 92 \forall i$	77.54	92.54	95.00	97.36	98.54	1209	-9.9
$\alpha_i = 94 \forall i$	79.54	93.73	96.00	98.18	99.18	987	10.2
Airline's Routing	77.72	92.82	95.30	97.73	98.91	1100	0.0

Table B.13: Non-Monotonicity of Robustness Metrics for Network N_2 for the CCP_f Model ($t = 90$)

	Flight Delays				Pax Disruptions	
	≤ 15 min	≤ 60 min	≤ 120 min	≤ 180 min	#D-pax	D-pax reduced
$\Gamma_i = 1 \forall i$	78.63	93.00	97.36	98.54	1222	-11.0
$\Gamma_i = 2 \forall i$	79.54	93.54	98.09	99.18	987	10.3
$\Gamma_i = 3 \forall i$	74.18	91.00	96.54	97.90	1353	-23.0
Airline's Routing	77.72	92.82	97.73	98.91	1100	0.0

Table B.14: Non-Monotonicity of Robustness Metrics for Network N_2 for the EV_f Model ($t = 90$)

Appendix C. Extended Chance-Constrained Programming - General Formulation

The standard linear optimization problem that is required to be made robust

is:

$$\max \quad \mathbf{c}^T \mathbf{x} \quad (\text{C.1})$$

$$\text{s.t.} \quad \sum_{j \in J} a_{ij} x_j \leq b_i \quad \forall i \in I \quad (\text{C.2})$$

$$\mathbf{x} \geq 0, \quad (\text{C.3})$$

where I is the set of constraints, and J the set of variables, c_j is the profit coefficient for variable j and b_i is the right-hand side value for constraint i , $\forall i \in I$. a_{ij} for all $i \in I$, $j \in J$ is the coefficient of variable j in constraint i . Assume that the uncertainty is in the right hand side parameters b_i and in the objective function coefficients c_j . Charnes and Cooper (1963) write the general chance-constrained formulation as follows:

$$\max \quad f(\mathbf{c}, \mathbf{x}) \quad (\text{C.4})$$

$$\text{s.t.} \quad P\left(\sum_{j \in J} a_{ij} x_j \leq b_i\right) \geq \alpha_i \quad \forall i \in I \quad (\text{C.5})$$

$$\mathbf{x} \geq 0, \quad (\text{C.6})$$

where ‘P’ indicates ‘probability’, and f could be expected value-based, variance-based or probabilistic forms in the objective. The most common form for f is

the expected value. α_i ($0 \leq \alpha_i \leq 1$) for all i , is a user-specified *protection level* for constraint i , and $(1 - \alpha_i)$ specifies the maximum probability of violation of constraint i .

Ben-Israel (Ben-Israel, 1962) shows that the chance-constraint (C.5) can be linearized as follows:

$$P\left(\sum_{j \in J} a_{ij}x_j \leq b_i\right) \geq \alpha_i \Leftrightarrow \sum_{j \in J} a_{ij}x_j \leq F_{b_i}^{-1}(1 - \alpha_i), \quad (\text{C.7})$$

with $y = F_{b_i}^{-1}(1 - \alpha_i)$ equal to the *quantile* value in the cumulative distribution function (CDF), F_{b_i} , of b_i such that the probability that b_i takes on values less than or equal to y is $(1 - \alpha_i)$. That is, if $f(b_i)$ is the probability density function of b_i , $\int_{-\infty}^y f(b_i)db_i = 1 - \alpha_i$.

In Marla (2010), to avoid the need to specify the protection level for each constraint explicitly, the author includes a constraint on the overall expected profit of the robust solution (similar to D.4) and changes the objective to one of maximizing the sum of protection levels provided for all constraints, as follows:

$$(\text{C.8})$$

where $\mathbf{c}^T \mathbf{z}^*$ is the profit of the nominal optimal solution $\mathbf{z}^*$ to (C.1)-(C.3), found using nominal values of the $\mathbf{b}$ and $\mathbf{c}$ parameters.

To linearize (??) - (??), we require the knowledge of some quantiles of the right-hand-side for each constraint, and the expected values of the profit coefficients. Let b_i be the nominal value for the right-hand-side of the i th constraint, and c_j be the expected profit coefficient corresponding to the j th variable, for all $j \in J$. Let K_i represent the set of discretized protection levels known for b_i . We now set the protection levels α_i as variables, representing the quantile that is chosen from among the $\|K_i\|$ available quantiles. Let b_i^k be the k th quantile value of the right-hand-side (RHS) parameter of the i th constraint, for all $k \in K_i, i \in I$; and $z_j^* \forall j \in J$ be the nominal optimal solution to (C.1) - (C.3). p_i^k is the protection probability associated with quantile $k \in K_i$ for constraint

$i \in I$. The objective function equals the sum of protection levels achieved over
 1290 all constraints $i \in I$. To capture the tradeoff of robustness with profits, we as-
 sume that the planner is willing to forego a (user-specified) profit Δ to instead
 achieve a robust plan.

Decision variables y_i^k are binary variables that equal 1 if the protection level
 (expressed as a probability p_i^k , with $0 \leq p_i^k \leq 1$) represented by the k th quantile
 1295 ($k \in K_i$) is attained in constraint $i \in I$; and 0 otherwise. Note that if the k th
 quantile value is protected against, the $(k + 1)$ st quantile is also automatically
 protected against. This is ensured by the fact that constraints C.11 are “less
 than or equal to” inequalities. The y_i^k values for any constraint i , follow a
 step function. Variables α_i represent the protection level attained for the i th
 1300 constraint, for all $i \in I$.

The extended chance-constrained model (ECCP) is as follows:

$$\max \sum_{i \in I} \alpha_i \quad (\text{C.9})$$

$$\text{s.t. } \sum_{j \in J} c_j x_j \geq \sum_{j \in J} c_j z_j^* - \Delta \quad (\text{C.10})$$

$$\sum_{j \in J} a_{ij} x_j \leq \sum_{k=1}^K b_i^k (y_i^k - y_i^{k-1}) \quad \forall i \in I \quad (\text{C.11})$$

$$y_i^k \geq y_i^{k-1} \quad \forall k = 1, \dots, K_i, i \in I, \quad (\text{C.12})$$

$$y_i^0 = 0 \quad \forall i \in I \quad (\text{C.13})$$

$$y_i^K = 1 \quad \forall i \in I \quad (\text{C.14})$$

$$\alpha_i \leq \sum_{k=1}^{K_i} p_i^k (y_i^k - y_i^{k-1}) \quad \forall i \in I \quad (\text{C.15})$$

$$x_j \geq 0 \quad \forall j \in J \quad (\text{C.16})$$

$$y_i^k \in \{0, 1\} \quad \forall k \in K_i, i \in I \quad (\text{C.17})$$

$$0 \leq \alpha_i \leq 1 \quad \forall i \in I \quad (\text{C.18})$$

The objective function (C.9) maximizes the total probability that each con-
 straint $i \in I$ is feasible. Alternatively, it can also be re-written to maximize the
 minimum value of α_i across all constraints. (C.10) ensures that the solution's

expected profit is within Δ units of the expected profit associated with the nominal optimal solution (found by solving the problem using nominal values of the **b** and **c** parameters). For all $i \in I$, (C.11) forces the left-hand-side (LHS) to be less than or equal to b_i^k if y_i^k equals 1 and y_i^{k-1} equals 0, thereby ensuring constraint satisfaction with at least the probability associated with quantile k . For the smallest quantile k^* for which this condition can be satisfied, the $y_i^{k^*}$ value is 1, and quantiles $k < k^*$ have $y_i^k = 0$. Thus, the RHS value of this constraint is selected as the smallest one that can be satisfied by the solution. (C.12) ensures that the y_i^k s are non-decreasing and follow a step function, such that if a smaller quantile (higher protection) is achieved, the larger quantile (lower protection) is automatically achieved. (C.13) and (C.14) set the boundary values of the y_i^k step functions. Constraints (C.15) set α_i to be no greater than the highest protection level provided to constraint i by the solution. The x_j s are non-negative for all $j \in J$; y_i^k s are binary for all $k \in K_i$ and for all $i \in I$; and α_i s are non-negative and at most 1, for all $i \in I$ as required by (C.16), (C.17) and (C.18), respectively.

Appendix D. Δ Model - General Formulation

The standard binary integer program that is required to be made robust is:

$$\max \quad \sum_{j \in J} c_j x_j \quad (\text{D.1})$$

$$\text{s.t.} \quad \sum_{j \in J} a_{ij} x_j \leq b_i \quad \forall i \in I \quad (\text{D.2})$$

$$x_j \in \{0, 1\} \quad \forall j \in J, \quad (\text{D.3})$$

where the notation used is the same as that for (C.1)-(C.3). a_{ij} for all $i \in I, j \in \bar{J}_i$, is subject to uncertainty, with $\tilde{a}_{ij}$ as its realized value. a_{ij} is the nominal value of $\tilde{a}_{ij}$, and also the mean value of its symmetric range of variation. $\hat{a}_{ij}$ is the width of one half of the symmetric range of variation of $\tilde{a}_{ij}$, for all $i \in I, j \in J$. $\hat{a}_{ij} = 0$ for $j \in J \setminus \bar{J}_i$. x_j for all $j \in J$, is a binary decision variable that equals 1 if that variable is present in the solution and 0 otherwise.

To avoid the need to specify Γ values, Marla (2010) modifies the Bertsimas-Sim (Bertsimas and Sim, 2004), (Bertsimas and Sim, 2003) formulation to include a constraint requiring the total profit of the robust solution to be within a difference δ from the nominal optimal profit value. Additionally the objective is modified to one of minimizing the number of variables that *must* assume their nominal, rather than extreme, values to satisfy all constraints. Variable Δ_i equals the number of x_j variables in the solution (i.e., those with $x_j = 1$) whose coefficient values for constraint i are subject to uncertainty and must assume their nominal values to remain feasible. For each constraint i , we sort its associated columns $j \in J$, in increasing order of their $\hat{a}_{ij}$ values (ties are broken arbitrarily). After ordering, the rank of the j th column in the i th constraint is denoted by $l(i, j)$. Also, the original index (j) of the variable that takes the l th position in the sorted $\hat{a}_{ij}$ values for constraint i is denoted by $j(i, l)$. For example, the variable j in constraint i with the smallest $\hat{a}_{ij}$ value has $l(i, j) = 1$. The variable with the largest $\hat{a}_{ij}$ value has $l(i, j) = N$, with N equal to the number of binary variables in set J .

Let y_j^* be the optimal value of variable j for all $j \in J$ for the nominal problem (D.1)-(D.3). Since δ is the user-specified incremental cost that is acceptable for increased robustness, the profit of a robust solution from the Δ formulation is at least $\sum_{j \in J} c_j y_j^* - \delta$. Let variables v_{ij} equal 1 if the uncertain coefficient $\tilde{a}_{ij}$ is not allowed to take on its extreme value, and instead takes on its nominal value in the solution of the Δ model. Variables w_{il} equal 1 if there exists a $k \geq l$ with $v_{ik} = 1$, for $l = |J| - |\bar{J}_i| + 1, \dots, |J| + 1$. w_{il} variables for $l = |J| - |\bar{J}_i| + 1, \dots, |J| + 1$ in each constraint i follow a non-increasing step function.

This leads to the following $\Delta - EV$ formulation:

$$\min \sum_{i \in I} \Delta_i \quad (D.4)$$

$$\text{s.t. } \sum_{j \in J} c_j x_j \geq \sum_{j \in J} c_j y_j^* - \delta \quad (D.5)$$

$$\sum_{j \in J} (a_{ij} + \hat{a}_{ij}) x_j - \sum_{j \in J} \hat{a}_{ij} v_{ij} \leq b_i \quad \forall i \in I \quad (D.6)$$

$$\Delta_i \geq \sum_{l=|J|-|\bar{J}_i|+1}^{|J|} [(l - |J| + |\bar{J}_i|)(w_{i,l} - w_{i,l+1}) + v_{i,j(i,l)} - w_{i,l}] \quad \forall i \in I \quad (D.7)$$

$$\Delta_i \geq \sum_{j \in J} v_{ij} \quad \forall i \in I \quad (D.8)$$

$$v_{ij} \leq x_j \quad \forall j \in J, \forall i \in I \quad (D.9)$$

$$v_{i,j(i,l)} \leq w_{i,l} \quad \forall l \in J \quad (D.10)$$

$$v_{ij} \geq x_j + w_{i,l(i,j)} - 1 \quad \forall j \in J, \forall i \in I \quad (D.11)$$

$$w_{i,l+1} \leq w_{i,l} \quad \forall l = |J| - |\bar{J}_i| + 1, \dots, |J|, \forall i \in I \quad (D.12)$$

$$w_{i,|J|-|\bar{J}_i|+1} \geq 1 \quad \forall i \in I \quad (D.13)$$

$$w_{i,|J|+1} = 0 \quad \forall i \in I \quad (D.14)$$

$$w_{il} = 0 \quad \forall l = 1, \dots, |J| - |\bar{J}_i|, \forall i \in I \quad (D.15)$$

$$x_j \in \{0, 1\} \quad \forall j \in J \quad (D.16)$$

$$v_{ij} \in [0, 1] \quad \forall j \in 1, \dots, |J|, \forall i \in I \quad (D.17)$$

$$w_{il} \in \{0, 1\} \quad \forall l \in 1, \dots, |J| + 1, \forall i \in I \quad (D.18)$$

The formulation is described as follows: The objective (D.4) is to minimize the sum of the number of coefficients in each constraint that must assume their nominal values to satisfy all constraints. This serves the purpose of trying to maximize the number of coefficients that can take their extreme values and still maintain feasibility. Constraints (D.5) require that the profit from the ‘robust’ solution does not differ from the profit of the deterministic solution by more than a ‘robustness budget’ of δ . Feasibility is assured by constraints

(D.6). Variables v_{ij} are set equal to 1 in constraint i , for all coefficients j that
1360 *must* be set to their nominal values. Inequalities (D.9) prevent a coefficient
from taking on its nominal value unless the variable is present in the solution,
that is, v_{ij} is zero if x_j is zero in the solution. Constraints (D.7) and (D.8)
provide different mechanisms to count the maximum number of variables whose
coefficients in constraint $i \in I$ are subject to uncertainty and must take on
1365 nominal values. The explanation for these constraints lies in the realization
that when the columns are sorted in increasing order of $\hat{a}_{ij}$ values for each
row i , the *minimum number* of coefficients that must assume nominal values to
maintain feasibility is determined by forcing the smallest $\hat{a}_{ij}$ values (for which
 $\hat{a}_{ij} \neq 0$) in the solution to have their associated v_{ij} values set to 1, if variable
1370 j is in the solution, that is, if $x_j = 1$. Though constraints (D.7) and (D.8)
are essentially the same, both are retained in order to provide better bounds
on the solution. (D.10) requires the w variable to be at least as large as the
corresponding v variable for that j and i . Also, (D.11) forces v to be 1 if both
the corresponding x and w are 1. Constraints (D.12), (D.13) and (D.14) ensure
1375 that the w_{il} variables corresponding to uncertain coefficients in each i form
a step function. The w_{il} variables corresponding to coefficients that are not
subject to uncertainty are zero (D.15). The x and w variables are binary, as
required by (D.16) and (D.18) respectively. We thus force the v variables to
take on binary values as well. One can think of this model as maximizing the
1380 total number of coefficients summed over all constraints that can take on their
worst-case values, under budget limitations.

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