Information Disclosure and Network Formation in News Subscription Services

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Chin-Chia Hsu†, Amir Ajorlou†, Muhamet Yildiz‡, Ali Jadbabaie†

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Abstract

We study the formation of a subscription network where a continuum of strategic, Bayesian subscribers decide to subscribe to one of two sources (leaders) for news that is informative about an underlying state of the world. The leaders, aiming to maximize the welfare of all subscribers, have a motive to persuade the subscribers to take the optimal binary action against the state according to their own perspectives. With this persuasion motive, each leader decides whether to disclose the news to her own subscribers when there is news. When the subscribers receive the news, they update their beliefs; more importantly, even when no news is disclosed, the subscribers update their beliefs, speculating that there may be news that was concealed due to the leader's strategic disclosure decision. We prove that at any equilibrium, the set of news signals that are concealed by the leaders takes the form of an interval. We further show that when two leaders represent polarized and opposing perspectives, anti-homophily emerges among the subscribers whose perspectives are in the middle. For any subscriber with a perspective on the extremes, and for any leader, there exists an equilibrium at which the subscriber would follow the leader. Our results shed light on how individuals would seek information when information is private or costly to obtain, while considering the strategic disclosure by the news providers who are partisan and have a hidden motive to persuade their followers.

Keywords

Network Formation, Persuasion, Strategic Information Disclosure

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I. INTRODUCTION

Decision makers often rely on a few selected intermediaries, such as news media and opinion leaders, as their information sources. This has lead to a fragmented society in which different segments of the society receive quite distinct pieces of news [1], [2]. The problem has become especially acute with the rise of online social media: While in the past the news business was confined to a limited number of trusted agencies, most Americans today consume their news from a wide range of sources on social media [3]. Given the large number of intermediaries, however, news consumers can attend to a few of them due to limited cognitive resources [4]. Moreover, these information sources are biased and are motivated to influence the public opinion. Consequently they choose to promote news that can sway or persuade their subscribers towards their own political agenda [5] and conceal the unfavorable one. The political divide in our society is being intensified, without a commonly accepted set of facts.

In this paper, we take a first step towards understanding the structure of endogenous networks that emerge when biased intermediaries strategically choose what information to disclose and the subscribers choose their sources in order to make a more informed decision. We consider two intermediaries, which we call leaders, and a continuum of potential subscribers. Each subscriber is to decide between two actions, say, left and right, and the payoff from the actions depends on an unknown real-valued state. Everybody has the same preference on actions, but they have heterogeneous priors about the unknown state. In particular, ex-ante each leader has a known biased perspective: one is biased towards left, and the other to the right. Both leaders want to maximize the common social welfare function based on their own biased beliefs.

There are three stages of our subscription-disclosure game. At the beginning each individual subscribes to exactly one of the leaders. They all know that each leader may then have access to private and noisy information, which is referred to as news, about the state. At the second stage, if a leader receives news, then she decides whether to disclose the news at zero cost. Lastly, each subscriber only sees whether her leader disclosed news and its content if it was disclosed (without seeing whether any leader received news or what the other leader did). She then updates her belief and chooses her action.

When a subscriber receives news from her information source, her task is straightforward: She updates her prior belief according to Bayes' rule. Her task is more complicated if she does not hear from the leader. There are now two possibilities: the leader may not have received the news, or she may be concealing it. Since the leader strategically decides what to reveal, the subscriber's belief depends on the equilibrium strategy of her leader. Of course, the leader's optimal strategies may depend on who subscribes to whom and how their subscribers would update their beliefs in response to each piece of news or no news. This, in turn, complicates the subscribers' decisions of choosing one of the leaders as their information source.

We show that, at any equilibrium, each leader conceals the information when the news is in some interval she chooses. For example, a leader biased towards right would disclose the information that is neutral or favorable to right. She would not disclose the news that is moderately favorable to left, as such news would sway some moderate subscribers to the left without changing the leader's preferred action; she otherwise reveals the news that is strongly favorable to left, as she is now also swayed to the left. Based on the intervals of the news concealed by the two leaders, the individuals choose which leader to subscribe to at the beginning of the game.

We demonstrate that the resulting equilibrium network exhibits *anti-homophily* among subscribers with perspectives around the middle. The individuals with extreme bias are indifferent between the information sources at any equilibrium. For these extremist individuals, the information that may have been concealed by either leader is irrelevant. They would change their action only if they receive an extreme news contrary to their prior belief, and such news is disclosed by both leaders. Moderates are not indifferent however. An individual with a moderate perspective towards right would strictly prefer to subscribe to the leader with bias to *left*. When she receives news, she would choose left action only if the news is sufficiently favorable to left. Since the left-winged leader always discloses the news that is favorable to left, she does not lose relevant information when she subscribes to that leader. In contrast, she could lose that relevant information if she subscribes to the right-winged leader. When she does not receive any news, it matters whether the leader did not receive any news or is concealing: She would have chosen right if it is that the leader did not receive news, and she may have chosen left if the news is concealed (depending on the content of the news). As a result, the moderates subscribe to the leader with opposite bias to their own.

We do not mean to suggest that there should be anti-homophily on subscription networks. Indeed, casual observations indicate that social networks often exhibit homophily [1], [2], and our result relies on our assumptions that there are only two actions and that the leaders can only conceal or disclose the news. We do present this as a counterexample to the idea that the endogenous social networks should simply exhibit homophily. As this example suggests, the structure of the network depends highly on the motivations of the players (e.g. whether the individuals are motivated by information acquisition or advocacy), the way the information is transmitted (e.g., whether the leaders make recommendations or directly reveal their information), and the decision for which the information will be used (e.g. whether there is a continuum of actions or binary actions).

There is a large literature on strategic information transmission, stemming from the seminal works on disclosure games [6], [7] and cheap talk games [8]. The papers as follows are closet to our work in the regard of studying the behavior of decision makers seeking for information. [9] also considers a model with two information sources and two alternatives as in ours and suggests that a decision maker with a bias in preference on the alternatives would prefer the information source in favor of her bias rather than the unbiased one. [10] show that the experts may conceal their information if the experts and the decision maker have differing prior beliefs. [11] suggests that there will be *homophily* when there are binary actions but the leaders recommend what action to take rather than disclose the information behind their recommendation. Similarly, [12] obtain homophily when the information sources disclose their posteriors non-strategically, and the bias of the sources are unknown. In the work of [13], a one-leader-one-follower model is introduced with a continuum of actions. After the subscription decision, our model becomes similar to [13], and we use some of their insights in our paper. The role of voting rules is also studied by [14] in a similar model of strategic information transmission with binary signals. They do observe anti-homophily in that model using the same intuition above. We obtain the anti-homophily in a slightly richer model (with continuum of signals) and use it to analyze the network formation. The network formation adds substantial complexity to our problem, as the leader does not know the subscribers' perspectives (but the distribution of them), and their distribution is endogenously determined at equilibria.

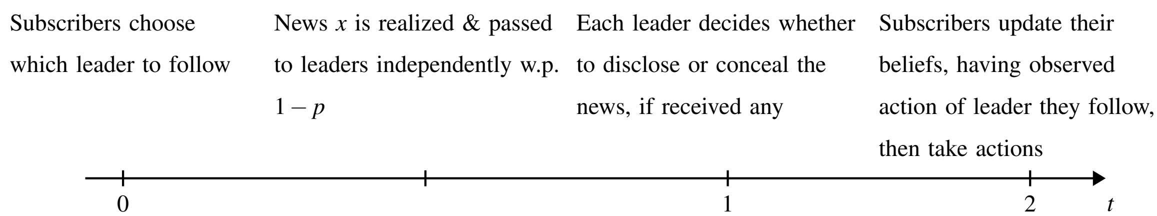

Fig. 1. Timeline of the subscription-disclosure game described in Section II.

II. MODEL

Our model consists of an unobservable state $\theta \in \mathbb{R}$, two leaders $i \in \{1,2\}$, and a unit-mass continuum of subscribers/followers of which the set is denoted as Λ.[1] Following standard Bayesian models, agents agree to disagree [15]; they have heterogeneous prior beliefs on the distribution from which the state is drawn. The leaders have prior beliefs

$$\theta \sim_1 N(\mu_1, \sigma_1^2) \text{ and } \theta \sim_2 N(\mu_2, \sigma_2^2),$$

where we assume that $\mu_1 > 0$ and $\mu_2 < 0$ indicating the leaders' ex-ante biased perspective on the ideology spectrum associated with the right and left wings, respectively. Each subscriber $j \in \Lambda$ possesses a prior belief

$$\theta \sim_j N(\mu_j, 1), \quad j \in \Lambda.$$

We refer to the mean of each subscriber's belief as her *perspective* and to its variance as the uncertainty level of her prior belief. The perspectives are distributed according to a CDF function $F(\cdot)$ with full support on the real line.

The news x is a noisy observation of the state, where the noise is assumed to be additive, normally distributed, and independent from the state. Specifically,

$$x = \theta + \varepsilon,$$

$$\varepsilon \sim N(0, \sigma_\varepsilon^2),$$

for some $\sigma_\varepsilon > 0$. Any agent (of both leaders and subscribers) who receives news x would update her belief on the state using Bayes' rule. The posterior beliefs are still normally distributed, with

[1] We may interchangeably refer to a subscriber (subscribe) as a follower (follow).

an updated mean and variance:

$$\theta|x \sim_i N\Big((1-\beta_i)\mu_i + \beta_i x, \beta_i \sigma_\varepsilon^2\Big), \quad i \in \{1,2\},$$
$$\theta|x \sim_j N\Big((1-\beta)\mu_j + \beta x, \beta \sigma_\varepsilon^2\Big), \quad j \in \Lambda, \tag{1}$$

where the weights agents place on the news are

$$\beta_i \triangleq \frac{\sigma_i^2}{\sigma_i^2 + \sigma_\varepsilon^2} \text{ for } i \in \{1,2\}, \quad \beta = \frac{1}{1+\sigma_\varepsilon^2}.$$

At the onset, subscriber j with prior perspective μ_j subscribes to exactly one of the two leaders, which we denote with $\lambda_j \in \{1,2\}$. After subscribing, subscriber j could receive the news that leader λ_j decides to share with her subscribers, as will be elaborated later in this section. We denote with $I_j \subset \mathbb{R}$ the information set available to subscriber j when taking her action. Particularly, $I_j = \{x\}$ if leader λ_j shares news x with her subscribers and $I_j = \emptyset$ if no news is shared. Incorporating the news shared by leader λ_j, if any, subscriber j then takes an action $a_j \in \{-1,+1\}$ that maximizes her expected utility, where the utility yielded from taking action a_j is given by

$$u_j(\theta, a_j) = a_j \theta. \tag{2}$$

Let $\Lambda_i \triangleq \{j|\lambda_j = i\}$ be the set of subscribers to leader i resulting from the subscriptions $\{\lambda_j\}_{j\in\Lambda}$ and F_i be the induced conditional distribution of perspectives of her subscribers Λ_i.[2] After a news item x is realized, nature passes the news to each leader independently with probability $1-p \in (0,1)$. The information set available to leader i is denoted with I_i, similarly following the notation used for followers. It is then clear that $I_i = \emptyset$ or $\{x\}$.

If receiving the news x, leader i has to decide whether to disclose it to her subscribers or not. Denote this binary action with $s_i(x) \in \{0,1\}$, where $s_i = 1$ implies disclosing the news while $s_i = 0$ indicates concealing the news.

Both leaders are concerned with the aggregate welfare of the entire population of subscribers Λ. More precisely, the utility of leader i is the aggregation of the subscribers' utilities:

$$u_i(\theta, \{a_j\}_{j\in\Lambda}) = \int_{j\in\Lambda} a_j \theta.$$

[2]Note that Λ_1 and Λ_2 partition the set of all subscribers Λ since each agent is only allowed to subscribe to exactly one leader. As a result, $\Lambda_1 \cup \Lambda_2 = \Lambda$ and $\Lambda_1 \cap \Lambda_2 = \emptyset$.

Leader i chooses her disclosure/concealment strategy $s_i: \mathbb{R} \to \{0,1\}$ so as to maximize her expected utility. We alternatively represent a strategy s_i with its corresponding concealment set C_i defined as the set of news that leader i chooses to conceal from her subscribers if she receives the news:

$$C_i \triangleq \{x \in \mathbb{R} \mid s_i(x) = 0\}.$$

Similarly, we define $D_i = \mathbb{R} \setminus C_i$ as leader i's disclosure set.

The subscription-disclosure game described above proceeds at three stages $t = \{0,1,2\}$ as follows. At $t = 0$, for any $j \in \Lambda$, subscriber j subscribes to exactly one leader. Each subscriber is equally likely to miss the opportunity of making her subscription choice with small probability $\delta > 0$. In this case, nature assigns each of such subscribers to one of the two leaders uniformly at random. All of the random events are mutually independent. At the end of stage $t = 0$ the news item x is realized and each leader has independent access to it with probability $1 - p$. At $t = 1$, if leader i has observed the news x, she updates her belief on the state and decides to either disclose the news to her subscribers or conceal it from them according to her strategy s_i. At $t = 2$, only observing the action of the leader she follows, each subscriber updates her belief on the state and takes an action maximizing her expected utility, based on the utility function given in (2). We summarize this timeline in Figure 1.

III. Perfect Bayesian Equilibria

We solve for the Perfect Bayesian Equilibrium (PBE) of the game described in the previous section by proceeding backward, characterizing the equilibrium strategies $\{\lambda_j^*, a_j^*\}_{j \in \Lambda}$ and $\{C_i^*\}_{i=1,2}$.

The posterior means of subscribers' individual beliefs at $t = 2$ will appear often in our analysis. To this end, it is quite convenient to define

$$y(\mu_j, I_j) \triangleq \mathbb{E}_j\left[\theta \mid I_j\right],$$

which explicitly captures the dependence of the mean of subscriber j's posterior belief on her prior perspective μ_j as well as her information set I_j.

At stage $\mathbf{t = 2}$: Given her information set I_j, subscriber j takes an action a_j^* maximizing her expected utility. Recalling the form of the utility function in (2), it is easy to see that the optimal action of subscriber j is to match the sign of the mean of her posterior belief. That is, subscriber j takes action $a_j^*(\mu_j, I_j) = \text{sign}(y(\mu_j, I_j))$, yielding an expected utility equal to $|y(\mu_j, I_j)|$. In particular, a strategic subscriber updates her belief on the state even when hearing no news, speculating on the possibility of the leader concealing the news from subscribers. Using Bayes rule, we calculate the mean of a subscriber's posterior belief conditional on her information set, as formally stated in the following Lemma 1.

Lemma 1: For subscriber j who follows leader $\lambda_j \in \{1,2\}$ with concealment set C_{λ_j}, the mean of her posterior belief on the state is given by:

$$y(\mu_j, \{x\}) = (1-\beta)\mu_j + \beta x, \tag{3}$$

$$y(\mu_j, \emptyset) = \mu_j + \frac{(1-p)\mathbb{P}_j^0(C_{\lambda_j})}{p + (1-p)\mathbb{P}_j^0(C_{\lambda_j})}\mathbb{E}_j^0[\beta(x-\mu_j)|x \in C_{\lambda_j}]. \tag{4}$$

where superscript 0 and subscript j indicate that the probability measure and expectation are taken with respect to subscriber j's *prior* belief.

Proof. See the Appendix. ∎

<u>**At stage $\mathbf{t = 1}$:**</u> Leader i who received the news x decides whether to conceal or reveal it to her subscribers Λ_i, aiming to maximize her expected utility (i.e., the social welfare) by controlling the information sets of her subscribers. The utility-maximizing action of leader i is the solution to the following optimization problem:

$$s_i^*(x) = \underset{s \in \{0,1\}}{\text{argmax}}\, \mathbb{E}_i\left[\int_{j \in \Lambda} \theta a_j^*\big(\mu_j, I_j\big) | I_i = \{x\}\right],$$

where, however, only the information sets of leader i's own subscribers, i.e., $j \in \Lambda_i$, will be affected by her action. As a result, we can write

$$\begin{aligned} s_i^*(x) &= \underset{s \in \{0,1\}}{\text{argmax}}\, \mathbb{E}_i\left[\int_{j \in \Lambda_i} \theta a_j^*\big(\mu_j, I_j\big) | I_i = \{x\}\right] \\ &= \underset{s \in \{0,1\}}{\text{argmax}}\, \mathbb{E}_i[\theta | x]\, \mathbb{E}_{\substack{j \in \Lambda_i \\ \mu_j \sim F_i}}\left[a_j^*\big(\mu_j, I_j(x,s)\big)\right], \end{aligned}$$

where $I_j(x,0) = \emptyset$ and $I_j(x,1) = \{x\}$, for any subscriber $j \in \Lambda_i$. Recalling that $a_j^* \in \{-1, +1\}$, we see that the optimal action of leader i maximizes the number of her subscribers whose actions would match the sign of leader i's posterior mean.

Noting the importance of the sign of the posterior mean of the beliefs in optimal strategies, we introduce the notion of *flipping points* in the real line of news as the zeros to the posterior means in (1) when viewed as a function of perspectives. That is,

$$\bar{x}_{\mu_i} = -\frac{1-\beta_i}{\beta_i}\mu_i, \quad i=1,2; \quad \bar{x}_{\mu_j} = -\frac{1-\beta}{\beta}\mu_j, \quad j \in \Lambda.$$

Due to the linearity of the mean of a belief in news x, the sign of the posterior means is positive (negative) if and only if the news x is above (below) the flipping points. We assume that a leader would conceal the news when her posterior mean is zero (according to her belief, neither of the binary votes are strictly favorable to the society), or when she is indifferent between the the actions of concealment and disclosure.

At any equilibrium, leader i discloses the news x (i.e., $x \in D_i^*$) if and only if

$$x > \bar{x}_{\mu_i} \text{ and } \mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\{x\})>0\}\right] > \mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\emptyset)>0\}\right],$$

or,

$$x < \bar{x}_{\mu_i} \text{ and } \mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\{x\})<0\}\right] > \mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\emptyset)<0\}\right].$$

For the first case $x > \bar{x}_{\mu_i}$, i.e., when leader i believes action $+1$ is optimal, she would disclose the news x if and only if the fraction of her subscribers voting $+1$ when they see the news would be greater than when they don't hear any news from her. The value of the former is

$$\mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\{x\})>0\}\right] = \mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{\beta x+(1-\beta)\mu_j>0\}\right],$$

which is strictly increasing in the news x ($\beta > 0$ by assumption). Note that if the news is not disclosed, the fraction of the followers who vote $+1$ (i.e., $\mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\emptyset)>0\}\right]$) is a constant, regardless of the news, which is derived from followers' updated beliefs by inferring leader i's equilibrium concealment set. As a result, when $x > \bar{x}_{\mu_i}$, leader i would disclose the news x if and only if $x > x_i^+$ for some $x_i^+ \geq \bar{x}_{\mu_i}$.[3] Similarly, for $x < \bar{x}_{\mu_i}$, there exists $x_i^- \leq \bar{x}_{\mu_i}$ such that leader i discloses the news if and only if $x < x_i^-$. Putting these together, the concealment set is

[3]Specifically, there is the unique value $\tilde{x}_i^+ \in \mathbb{R}$ such that for all $x > \tilde{x}_i^+$, $\mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\{x\})>0\}\right] > \mathbb{E}_{\substack{j\in\Lambda_i \\ \mu_j\sim F_i}}\left[\mathbf{1}\{y(\mu_j,\emptyset)>0\}\right]$. Therefore, when $x > \bar{x}_{\mu_i}$, leader i would disclose the news x if and only if $x > \max\{\bar{x}_{\mu_i},\tilde{x}_i^+\} \triangleq x_i^+$.

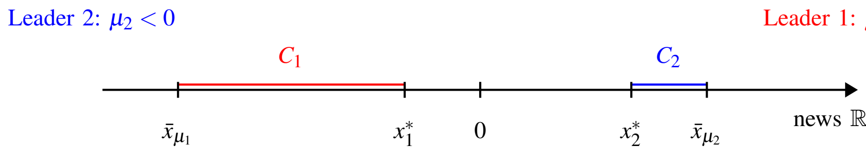

Fig. 2. Concealment strategy of the two leaders. The red line represents the concealment set C_1 while the blue line represents the concealment set C_2.

the interval $C_i^* = [x_i^-, x_i^+]$ where $x_i^- \leq \bar{x}_{\mu_i} \leq x_i^+$. Indeed, as we prove in Lemma 2, $\bar{x}_{\mu_i}$ has to be an endpoint of this interval at any equilibrium.

We characterize the leaders' equilibrium strategies in Lemma 2 below.

Lemma 2: At any equilibrium, the concealment sets corresponding to leaders' strategies are in the form of intervals. Given p, when $\mu_1 > 0$ ($\bar{x}_{\mu_1} < 0$) and $\mu_2 < 0$ ($\bar{x}_{\mu_2} > 0$), the concealment sets are

$$C_1^* = [\bar{x}_{\mu_1}, x_1^*], \text{ and } x_1^* \in (\bar{x}_{\mu_1}, 0);$$
$$C_2^* = [x_2^*, \bar{x}_{\mu_2}], \text{ and } x_2^* \in (0, \bar{x}_{\mu_2}),$$

where x_i^*, together with some subscriber k_i^* to leader i, is the unique solution to the following equations:

$$0 = y(\mu_{k_i^*}, \{x_i^*\}) = (1-\beta)\mu_{k_i^*} + \beta x_i^*, \tag{5}$$
$$0 = y(\mu_{k_i^*}, \emptyset) = \mu_{k_i^*} + \frac{\beta(1-p)\mathbb{P}^0_{k_i^*}(C_i^*)}{p + (1-p)\mathbb{P}^0_{k_i^*}(C_i^*)} \mathbb{E}^0_{k_i^*}[x - \mu_{k_i^*} | x \in C_i^*].$$

Moreover, $y(\mu_j, \{x\})$ for any given $x \in \mathbb{R}$ as well as $y(\mu_j, \emptyset)$ are strictly increasing in μ_j.

Proof. See the Appendix. ■

Figure 2 illustrates the equilibrium strategies described in Lemma 2. It is also worth noting that the equilibrium strategies of leaders do not depend on the distributions of subscribers F_i as long as Λ_i has a full support on the real line. This is fulfilled via the assumption on the existence of random subscriptions.

At stage $\mathbf{t = 0}$: Of the subscribers who can make their own subscription choices, given the strategies of leaders, subscriber j chooses her leader as follows:

$$\lambda_j(\mu_j) = \underset{i \in \{1,2\}}{\text{argmax}}\, \mathbb{E}^0_j[a_j^* \theta | \lambda_j = i].$$

When there is a tie, subscriber j follows either of the leader. Incorporating this into Lemma 2, we derive the following result.

Lemma 3: Consider the concealment sets $C_1^* = [\bar{x}_{\mu_1}, x_1^*]$ and $C_2^* = [x_2^*, \bar{x}_{\mu_2}]$ as characterized in Lemma 2. The following properties hold for the subscription network at any equilibrium:

1) A δ-sized population of subscribers who miss the time of choosing a leader are each assigned by nature to one of the leaders.

Of those who strategically subscribe,

2a) Anti-homophily: For any subscriber j with $\bar{x}_{\mu_j} \in (0, \bar{x}_{\mu_2})$, we have $\lambda_j(\mu_j) = 1$; similarly, if $\bar{x}_{\mu_j} \in (\bar{x}_{\mu_1}, 0)$, then $\lambda_j(\mu_j) = 2$.

2b) For any subscriber j with $\bar{x}_{\mu_j} \notin (\bar{x}_{\mu_1}, \bar{x}_{\mu_2})$ or $\bar{x}_{\mu_j} = 0$, she is indifferent between subscribing to either of the leaders.

Proof. See the Appendix. ■

When a subscriber does not hear the news, she expects that with some positive probability the news could be in the concealment set of the leader she follows. She therefore takes an action at $t = 2$ by partly taking into account the average of the news of that concealment set according to her prior belief. However, when such a concern about the concealed news is so strong to flip the sign of the mean of her belief, making her favor the opposite action against her prior perspective, the subscriber loses some utility that might have been obtained if the value of news could have been observed. For example, when the leader's concealment set contains the flipping point of the subscriber, some values of the concealment set would make the subscriber choose $+1$ while the others suggest the opposite if she could observe the news. The subscriber, nonetheless, cannot differentiate these values and can only take one action using the average of the set as a proxy for her subsequent action. It therefore leads to loss of the value for some news, which may be retrieved if the other leader would disclose such information.

This example explains the main force underlying the anti-homophily in the endogenous subscription network. Agents who are not extreme in their prior perspectives are more likely to change their polarity when they see the news favoring the opposite side. When they subscribe to the leader of the opposite wing, such information would not be concealed (since this information supports the leader's prior perspective and can help to pull more votes if it is disclosed) and these subscribers can hence attain the value of information for their decision making, reacting to the piece of news instead of averaging over the concealment set as a proxy. Furthermore, the opposite leader only conceals the news that has the same sign as these subscribers' prior perspectives: No matter whether such news is disclosed or not the subscribers maintain the polarity of their perspectives—the news is of no information value for their actions. Therefore, it suggests that these subscribers can achieve their maximal expected utility when they subscribe to the leader of the opposite wing.

On the other hand, any subscriber j having a prior perspective with large magnitude is indifferent between both leaders. The magnitude of the flipping points of these subscribers are so large that any news in the concealment sets of both leaders does not flip the polarity of their perspectives, indicating that the polarity will neither be altered even when no news is received. In addition, both leaders disclose the news that would change these subscribers' polarity and affect their optimal actions. As a result, the strategies of both leaders result in the same relevant information transmitted to these subscribers with more extreme prior perspectives, and make no differences in terms of their subscription choices. We summarize the PBEs in Proposition 1.

Proposition 1: Given p, $\mu_1 > 0$ and $\mu_2 < 0$, in any equilibrium the concealment sets are in the form of intervals,

$$C_1^* = [\bar{x}_{\mu_1}, x_1^*],$$

$$C_2^* = [x_2^*, \bar{x}_{\mu_2}].$$

where x_i^* is the unique solution to (5).

1) A δ-sized population of subscribers who miss the time of choosing a leader are each assigned by nature to one of the leaders.

Of those who strategically subscribe,

2a) Anti-homophily: For any subscriber j, if $\bar{x}_{\mu_j} \in (0, \bar{x}_{\mu_2})$, then $\lambda_j^*(\mu_j) = 1$; on the other hand, if $\bar{x}_{\mu_j} \in (\bar{x}_{\mu_1}, 0)$, then $\lambda_j^*(\mu_j) = 2$.

2b) Moreover, for any leader i and any subscriber j such that $\bar{x}_{\mu_j} \notin (\bar{x}_{\mu_1}, \bar{x}_{\mu_2})$ or $\mu_j = 0$, there exists an equilibrium in which $\lambda_j^*(\mu_j) = i$.

For any subscriber j, her equilibrium strategy given her information set I_j at $t = 2$ is $a^*(\mu_j, I_j) = \text{sign}(y(\mu_j, I_j))$, where the posterior mean $y(\mu_j, I_j)$ is computed according to (3) and (4).

Proof. The proof is immediate from Lemma 1 to Lemma 3. ■

IV. Conclusions

We provided a theory of network formation where the subscribers seek the leaders for information, taking into account the strategic disclosure due to the leaders' persuasion motives. Whether receiving the news or not, the subscribers rationally update their beliefs using Bayes' rule. Given the subscribers' updating rules, we showed that at any equilibrium the concealment set of any leader is in the form of an interval. Furthermore, when two leaders represent the two wings on the spectrum of perspectives, anti-homophily arises among the subscribers with perspectives close to the middle. Those extremist subscribers, on the other hand, feel indifferent between the leaders since both leaders disclose the news that would flip the signs of their perspectives. Our results apply to the scenario in which people seek for valuable information while information is private or costly to obtain. An example would be to study the news market where readers subscribe to media outlets who may intend to influence the public opinions.

APPENDIX

Proof of Lemma 1. Equation (3) is the immediate result of (1). Given the concealment set C_i, (4) is derived using Bayes' rule as analyzed in [13]. ■

Proof of Lemma 2. As discussed, concealment set C_i must be an interval. First, we show that given her leader i and concealment set C_i, subscriber j's posterior mean $y(\mu_j, \{x\})$ for any news x and $y(\mu_j, \emptyset)$ are increasing in μ_j so that we then justify the fixed-point equations (5) that determine x_i^* at any equilibrium. It is obvious to see the result for the case $I_j = \{x\}$. For the case $I_j = \emptyset$, subscriber j updates her posterior perspective $y(\mu_j, \emptyset)$ based on her prior belief on the news distribution $N(\mu_j, 1+\sigma_\varepsilon^2)$ and the concealment set C_i. Take the partial derivative of (4) with respect to μ_j, we have

$$\frac{\partial}{\partial \mu_j} y(\mu_j, \emptyset) = 1 + \tilde{E}(\mu_j)\frac{\partial}{\partial \mu_j}\tilde{p}(\mu_j) + \tilde{p}(\mu_j)\frac{\partial}{\partial \mu_j}\tilde{E}(\mu_j),$$

where

$$\tilde{p}(\mu_j) \triangleq \frac{(1-p)\mathbb{P}_j^0(C_i)}{p+(1-p)\mathbb{P}_j^0(C_i)},$$

$$\tilde{E}(\mu_j) \triangleq \mathbb{E}_j^0[\beta(x-\mu_j)|x \in C_i].$$

It is easy to observe that $\mathbb{P}_j^0(C_i)$, the probability of C_i w.r.t $N(\mu_j, 1+\sigma_\varepsilon^2)$, is first increasing up to the midpoint of C_i, say $m(C_i)$, and then decreasing. Therefore, $\frac{\partial}{\partial \mu_j}\tilde{p}(\mu_j) > 0$ when $\mu_j < m(C_i)$ while $\frac{\partial}{\partial \mu_j}\tilde{p}(\mu_j) \le 0$ when $\mu_j \ge m(C_i)$. Similarly, we also observe that $\tilde{E}(\mu_j) > 0$ when $\mu_j < m(C_i)$ while $\tilde{E}(\mu_j) \le 0$ when $\mu_j \ge m(C_i)$. Together we know that $\tilde{E}(\mu_j)\frac{\partial}{\partial \mu_j}\tilde{p}(\mu_j) \ge 0$ for all $\mu_j \in \mathbb{R}$. To evaluate $\frac{\partial}{\partial \mu_j}\tilde{E}(\mu_j)$, we directly compute the term and have

$$\frac{\partial}{\partial \mu_j}\tilde{E}(\mu_j) = -\beta + \mathrm{Var}_j^0\left[\beta(x-\mu_j)|x \in C_i\right].$$

where the conditional variance is taken w.r.t. subscriber j's prior. Thus

$$\begin{aligned}\frac{\partial}{\partial \mu_j} y(\mu_j, \emptyset) &\ge 1 + \tilde{p}(\mu_j)\Big(-\beta + \mathrm{Var}_j^0\left[\beta(x-\mu_j)|x \in C_i\right]\Big), \\ &= (1-\tilde{p}(\mu_j)\beta) + \tilde{p}(\mu_j)\mathrm{Var}_j^0\left[\beta(x-\mu_j)|x \in C_i\right], \\ &> 0.\end{aligned}$$

In the following, we analyze the concealment strategy of leader 1 ($\bar{x}_{\mu_1} < 0$) and the analysis for leader 2 follows the same approach by symmetry. We first show that the endpoints of the conealmentset are $\bar{x}_{\mu_1}, x_1^*$ satisfying the relations $\bar{x}_{\mu_1} \le x_1^* \le 0$ if any solution exists.

Suppose the left endpoint is $x_1^- < \bar{x}_{\mu_1}$. Consider the news item x_1^-, which leader 1 would conceal. We have

$$\mathbb{E}_{\substack{j\in\Lambda_1\\ \mu_j\sim F_1}}\left[\mathbf{1}\{y(\mu_j, x_1^-) < 0\}\right] = \mathbb{P}_{\substack{j\in\Lambda_1\\ \mu_j\sim F_1}}[x_1^- < \bar{x}_{\mu_j}].$$

However, we can show that there exists $\hat{x} > x_1^-$ such that

$$\mathbb{E}_{\substack{j\in\Lambda_1\\ \mu_j\sim F_1}}\left[\mathbf{1}\{y(\mu_j,\emptyset) < 0\}\right] \leq \mathbb{E}_{\substack{j\in\Lambda_1\\ \mu_j\sim F_1}}\left[\mathbf{1}\{y(\mu_j,\emptyset) \leq 0\}\right] = \mathbb{P}_{\substack{j\in\Lambda_1\\ \mu_j\sim F_1}}[\hat{x} \leq \bar{x}_{\mu_j}] < \mathbb{P}_{\substack{j\in\Lambda_1\\ \mu_j\sim F_1}}[x_1^- < \bar{x}_{\mu_j}], \quad (6)$$

which indicates that concealing x_1^- is a worse action for leader 1 (contradiction) and the left endpoint is therefore $\bar{x}_{\mu_1}$. To see (6), first note that for a subscriber ℓ with the flipping point $\bar{x}_{\mu_\ell} = x_1^- < 0$ (and her prior perspective $\mu_\ell > 0$), she has $y(\mu_\ell,\emptyset) > 0$ when not hearing the news according to (4). By the continuity of function $y(\mu_j,\emptyset)$ in μ_j, there exists another subscriber m with $0 < \mu_m < \mu_\ell$ and $0 > \bar{x}_{\mu_m} > \bar{x}_{\mu_\ell}$ such that $y(\mu_m,\emptyset) = 0$. Then by taking $\hat{x} = \bar{x}_{\mu_m}$, (6) follows due to the monotonicity of $y(\mu_j,\emptyset)$ in μ_j and the continuity of CDF F_1.

Let's turn to the right endpoint $x_1^*(= x_1^+) \geq \bar{x}_{\mu_1}$. The reasoning behind the equilibrium equations (5) determining x_1^* is as follows. First, Λ_1 has a full support on the real line due to the random subscribers and the strong law of large numbers. Besides, we have shown that for any subscriber j who follows leader 1, $y(\mu_j,\emptyset)$ is strictly increasing in μ_j. Consequently, when the news is not disclosed, there exists a subscriber k_1^* to leader 1 such that the mean of her posterior is zero and the subscribers choose $+1$ iff their prior perspectives are greater than $\mu_{k_1^*}$. Similarly, thanks to the monotonicity of $y(\mu_j,\{x\})$ in μ_j, when news x is revealed, there also exists a subscriber with the zero posterior mean and the subscribers with prior perspectives larger than hers vote $+1$. Since at any equilibrium, leader 1 feels indifferent between disclosure and concealment for news x_1^*, the fraction of subscribers choosing $+1$ must be the same in both cases, which equivalently translates to the same subscriber k_1^* having zero posterior mean.

Now we show that $x_1^* \leq 0$: If $x_1^* > 0 > \bar{x}_{\mu_1}$, then there exists the type of subscriber k_1^* to leader 1 with $\mu_{k_1^*} < 0$ such that his posterior perspective is exact 0 when receiving the news $x_1^* > 0$. If subscriber k_1^* doesn't receive the news, her posterior perspective would be $y(\mu_{k_1^*},\emptyset)$. Since $\mathbb{E}_{k_1^*}[\beta(x-\mu_{k_1^*})|\bar{x}_{\mu_1} \leq x \leq x_1^*] < \beta(x_1^* - \mu_{k_1^*})$, we have

$$
\begin{aligned}
y(\mu_{k_1^*},\emptyset) =& \mu_{k_1^*} + \frac{(1-p)\mathbb{P}^0_{k_1^*}(C_1)}{p+(1-p)\mathbb{P}^0_{k_1^*}(C_1)}\mathbb{E}^0_{k_1^*}[\beta(x-\mu_{k_1^*})|\bar{x}_{\mu_1} \le x \le x_1^*],\\
<& \mu_{k_1^*} + \frac{(1-p)\mathbb{P}^0_{k_1^*}(C_1)}{p+(1-p)\mathbb{P}^0_{k_1^*}(C_1)}\beta(x_1^*-\mu_{k_1^*}),\\
<& \mu_{k_1^*} + \beta(x_1^*-\mu_{k_1^*}) \quad (\because \mu_{k_1^*} < 0 < x_1^*),\\
=& \beta x_1^* + (1-\beta)\mu_{k_1^*} = 0.
\end{aligned}
$$

Then it contradicts to (5). Hence, the concealment set is $C_1^* = [\bar{x}_{\mu_1}, x_1^*]$ where $\bar{x}_{\mu_1} \le x_1^* \le 0$.

Finally, we show the existence and uniqueness of the solution to (5). We can rewrite the equations as

$$
\begin{aligned}
&0 = (1-\beta)\mu_{k_1^*} + \beta x_1^*,\\
&\beta(x_1^*-\mu_{k_1^*}) = \frac{(1-p)\mathbb{P}^0_{k_1^*}([\bar{x}_{\mu_1},x_1^*])}{p+(1-p)\mathbb{P}^0_{k_1^*}([\bar{x}_{\mu_1},x_1^*])}\mathbb{E}^0_{k_1^*}[\beta(x-\mu_{k_1^*})|\bar{x}_{\mu_1} \le x \le x_1^*].
\end{aligned}
$$

We introduce $z_{k_1^*} = -\mu_{k_1^*} \le 0$ and $x_1^* = \frac{(1-\beta)z_{k_1^*}}{\beta} \in [\bar{x}_{\mu_1}, 0]$ (thus $z_{k_1^*} \in [\frac{\beta\bar{x}_{\mu_1}}{1-\beta}, 0]$), and reduce (5) to

$$
z_{k_1^*} = \tilde{p}(z_{k_1^*})\mathbb{E}_{z\sim N(0,\beta)}\left[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \le z \le z_{k_1^*}\right], \tag{7}
$$

where

$$
\tilde{p}(z_{k_1^*}) = \frac{(1-p)\mathbb{P}_{N(0,\beta)}([\beta(\bar{x}_{\mu_1}+z_{k_1^*}), z_{k_1^*}])}{p+(1-p)\mathbb{P}_{N(0,\beta)}([\beta(\bar{x}_{\mu_1}+z_{k_1^*}), z_{k_1^*}])}.
$$

To show that (7) has unique zero in the range $[\frac{\beta\bar{x}_{\mu_1}}{1-\beta}, 0]$, we define a continuous function h:

$$
h(z_{k_1^*}) = z_{k_1^*} - \tilde{p}(z_{k_1^*})\mathbb{E}_{N(0,\beta)}\left[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \le z \le z_{k_1^*}\right].
$$

It can be seen that $h(0) > 0$ and $h(\frac{\beta\bar{x}_{\mu_1}}{1-\beta}) < 0$, guaranteeing the existence of the solution and also furthur showing that $x_1^* \in (\bar{x}_{\mu_1}, 0)$. It suffices to show the uniqueness by proving the monotonicity of $h(\cdot)$ in the range of interest.

$$
\begin{aligned}
\frac{\partial}{\partial\bar{z}}h(\bar{z}) =& 1 - \mathbb{E}_{N(0,\beta)}\left[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \le z \le z_{k_1^*}\right]\frac{\partial}{\partial\bar{z}}\tilde{p}(z_{k_1^*})\\
&- \tilde{p}(z_{k_1^*})\frac{\partial}{\partial\bar{z}}\mathbb{E}_{N(0,\beta)}\left[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \le z \le z_{k_1^*}\right].
\end{aligned}
$$

Note that the expected value is non-positive and $\tilde{p}(z_{k_1^*})$ is increasing in $[\frac{\beta\bar{x}_{\mu_1}}{1-\beta},0]$, meaning that $\frac{\partial}{\partial\bar{z}}\tilde{p}(z_{k_1^*}) \geq 0$. For the derivative of the expectation, using the property of truncated normal, we obtain

$$
\begin{aligned}
&\frac{\partial}{\partial\bar{z}}\mathbb{E}_{N(0,\beta)}\left[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \leq z \leq z_{k_1^*}\right],\\
=&\frac{\partial}{\partial\bar{z}}\sqrt{\beta}\left[\frac{\phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))-\phi(\frac{z_{k_1^*}}{\sqrt{\beta}})}{\Phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\Phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))}\right],\\
=&\frac{\left(\phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\beta\phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))\right)\left(\phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))\right)}{\left(\Phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\Phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))\right)^2}\\
&+\frac{\frac{z_{k_1^*}}{\sqrt{\beta}}\phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\sqrt{\beta}\beta(\bar{x}_{\mu_1}+z_{k_1^*})\phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))}{\Phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\Phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))},\\
=&1-\frac{1}{\beta}\mathrm{Var}_{N(0,\beta)}[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \leq z \leq z_{k_1^*}]\\
&+\frac{(1-\beta)\phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))}{\Phi(\frac{z_{k_1^*}}{\sqrt{\beta}})-\Phi(\sqrt{\beta}(\bar{x}_{\mu_1}+z_{k_1^*}))}\times\frac{1}{\sqrt{\beta}}\Big(\beta(\bar{x}_{\mu_1}+z_{k_1^*})-\mathbb{E}_{N(0,\beta)}\left[z|\beta(\bar{x}_{\mu_1}+z_{k_1^*}) \leq z \leq z_{k_1^*}\right]\Big),\\
\leq&1.
\end{aligned}
$$

Using these results, $\frac{\partial\tilde{h}(\bar{z})}{\partial\bar{z}} \geq 1-\tilde{p}(z_{k_1^*}) > 0$ and the uniqueness is therefore proved. ■

Proof of Lemma 3. The ex-ante expected utility of subscriber j if she subscribes to leader i is

$$\mathbb{E}_j^0\Big[a_j^*\theta\Big|\lambda_j=i\Big]=\mathbb{E}_j^0\Big[\mathbb{E}_j\left[a_j^*\theta|I_j\right]\Big|\lambda_j=i\Big]=\mathbb{E}_j^0\Big[\left|y(\mu_j,I_j)\right|\Big|\lambda_j=i\Big].$$

Expanding the term further, we obtain

$$
\begin{aligned}
&\left|p\mu_j+(1-p)\mathbb{E}_j^0\left[(\beta x+(1-\beta)\mu_j)\mathbf{1}\{x\in C_i^*\}\right]\right|+(1-p)\mathbb{E}_j^0\left[\left|\beta x+(1-\beta)\mu_j\right|\mathbf{1}\{x\in D_i^*\}\right],\\
\leq&p|\mu_j|+(1-p)\mathbb{E}_j^0\left[\left|\beta x+(1-\beta)\mu_j\right|\mathbf{1}\{x\in C_i^*\}\right]+(1-p)\mathbb{E}_j^0\left[\left|\beta x+(1-\beta)\mu_j\right|\mathbf{1}\{x\in D_i^*\}\right].
\end{aligned}
$$

When $\mu_j \neq 0$, the equality holds iff there is no $x \in C_i^*$ such that $\mathrm{sign}(\mu_j) = -\mathrm{sign}(\beta x+(1-\beta)\mu_j)$; otherwise when $\mu_j = 0$ it holds iff $\mathrm{sign}(x)$ is the same for all $x \in C_i^*$.

If $\mu_j = \bar{x}_{\mu_j} = 0$, it is easy to see that for any leader i, $\mathrm{sign}(x)$ is the same for all $x \in C_i^*$ ($C_1^* \subset \mathbb{R}_-, C_2^* \subset \mathbb{R}_+$), meaning that subscriber j with zero prior perspective attains her maximal utility following either of the leaders. W.L.O.G. we study the case with $\mu_j > 0$ and her flipping point $\bar{x}_{\mu_j} < 0$. When subscriber j follows leader 2 with concealment set $C_2^* = [x_2^*, \bar{x}_{\mu_2}]$ such that $\beta x + (1-\beta)\mu_j > 0, \forall x \in C_2^*$, she maximizes her ex-ante expected utility. If she follows the leader 1 with concealment set $C_1^* = [\bar{x}_{\mu_1}, x_1^*]$, we have two cases:

1) $\bar{x}_{\mu_j} > \bar{x}_{\mu_1}$: For the news $x \in [\bar{x}_{\mu_1}, \min\{x_1^*, \bar{x}_{\mu_j}\})$, $\text{sign}(\beta x + (1-\beta)\mu_j) = -1 = -\text{sign}(\mu_j)$ and subscriber j does not attain the maximum.
2) $\bar{x}_{\mu_j} \leq \bar{x}_{\mu_1}$: It is obviously that no news $x \in C_1^*$ is such that $\text{sign}(\beta x + (1-\beta)\mu_j) = -1$ and subscriber j would attain her maximum.

As a result, any subscriber j with her flipping point $\bar{x}_{\mu_j} \in (\bar{x}_{\mu_1}, 0)$ would strictly favor leader 2 and, by symmetry, any subscriber j with $\bar{x}_{\mu_j} \in (0, \bar{x}_{\mu_2})$ would strictly favor leader 1. Otherwise, she feels indifferent between the leaders. ■

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