A Simple Rule for Pricing with Limited Knowledge of Demand

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How should a firm price a new product for which little is known about demand? We propose a simple and practical pricing rule for new products where demand information is limited. The rule is simple: Set price as though the demand curve were linear. Our pricing rule can be used if three conditions hold: the firm can estimate the maximum price it can charge and still expect to sell some units, the firm need not plan in advance the quantity it will sell, and marginal cost is known and constant. We show that if the true demand curve is one of many commonly used demand functions, or even a more complex (randomly generated) function, the firm can expect its profit to be close to what it would earn if it knew the true demand curve. We derive analytical performance bounds for a variety of demand functions, calculate expected profit performance for randomly generated demand curves, and evaluate the welfare implications of our pricing rule. We show that with limited demand information (maximum price and marginal cost), our simple pricing rule can be used for new products while often achieving a near-optimal performance. We also discuss the limitations of our method by identifying cases where our pricing rule does not perform well.

Key words: Pricing, new products, unknown demand, pricing heuristics, linear demand approximation

1. Introduction

Firms that introduce new products must often set a price with little or no knowledge of demand, and no data from which to estimate elasticities. How should firms set prices in such settings? This problem has been the subject of a variety of studies, most of which focus on experimentation and learning, e.g., setting different prices and observing the outcomes (we discuss this literature below). Experimenting with price, however, is often not feasible or desirable; it is often common for firms to choose an introductory price and maintain that price for a year or more. We examine a much simpler approach to this pricing problem that does not involve any price experimentation.

We show that under certain conditions, the firm can use a very simple pricing rule. The conditions are that (i) the firm's marginal cost, c, is known and constant, (ii) the firm can estimate the *maximum price* P_m it can charge and still expect to sell some units (more precisely, P_m is defined as the price in which consumers won't buy, but if the price is slightly reduced, some consumers will buy), and (iii) the firm need not know or plan in advance the quantity it will sell. (We partially relax the second assumption in Section 3.) These conditions will not always hold—but as we will explain, they do for many new products, particularly those that involve new technologies.

Examples of new product introductions for which these conditions hold include new types of biochemical drugs introduced by pharmaceutical companies (such as Lilly's Prozac in 1987, Astra-Merck's Prilosec in 1995, AstraZeneca's Crestor in 2003, or Merck's Vytorin in 2004), software products introduced by technology companies (such as Adobe's Acrobat Distiller in 1993 and Intuit's TurboTax in 2001), and digital content delivered via downloads or streaming (such as Apple setting the price of music downloads when launching its iTunes store in 2002, or Netflix pricing subscriptions when it launched its movie streaming service in 2007). These conditions can also hold for companies introducing an existing product in a new and emerging market (such as P&G launching Pampers in China in 1998). In all of these examples, marginal cost is known and constant. (It is zero for software downloads and music or video downloads or streaming, close to zero for most biochemical drugs, and known from experience for diapers.) However, the firms in these examples knew very little about the demand curves they faced and had no data to estimate elasticities. They could, however, roughly estimate the maximum price they could charge, and with the possible exception of Pampers in China, there was no need for them to know in advance how much they could sell.

Because our pricing rule depends on these three conditions, it is important to be clear about what they mean, when they will or will not hold, and what happens to our results if they do not hold. Here are the conditions again, in more detail.

1.1. The Firm's Marginal Cost is Known and Constant

This condition, is straightforward. It is necessary because if marginal cost varies with respect to output, the optimal price depends on the slope of the demand curve, which we assume is unknown. When will this condition hold? For many new technology products, marginal cost is both known and constant, and often close to zero. While this is obvious for software downloads, music or video downloads or streaming (where marginal cost is zero), it might be less obvious for pharmaceuticals, which is an important category of new products.

We said that marginal cost is close to zero (in fact to a first approximation equal to zero) for biochemical drugs. To clarify, biochemical drugs are those that are essentially made by mixing chemicals together (sometimes at precise temperatures) and have a simple molecular structure that is easy to specify (and thus patent). An example is Pfizer's Lipitor, a statin-type anti-cholesterol drug, the molecule for which has 160 atoms and is made by mixing chemicals at a low temperature.[1]

[1] https://www.wsj.com/articles/SB10001424127887324338604578328623588135326

The chemicals that are combined usually cost very little, and the process of mixing them is easy, so that biochemical drugs are very cheap to manufacture. Until around 2000, the vast majority of drugs were biochemical in nature.[2]

1.2. The Firm Can Estimate the Maximum Price It Can Charge

By "maximum price" we do not mean the price at which the firm can sell *any units* (that price might be extraordinarily high), but rather a price at which the firm can still expect to serve a few percent of its potential market. Determining the maximum price P_m might not be easy but it is a much less difficult task than estimating the entire demand curve.

In practice, there are several ways of estimating P_m. For some products and services, the firm can intentionally create a scarcity situation (when a product is first introduced) and then observe the highest prices paid for the product or service through secondary channels such as eBay (for products) or StubHub (for events). Alternatively, the firm could first introduce a product or service by selling it in a highest bid auction format, as is done for prototypes of luxury items. Finally, it is common for firms to hire focus groups (marketing specialists or loyal/passionate customers), where one of the main topics is the maximal price that can be charged for the upcoming product. This approach is commonly used for the introduction of new software and digital services (e.g., paid subscriptions for online dating services).

A pharmaceutical company might estimate P_m by comparing a new drug to existing therapies (including non-drug therapies). For example, when pricing Prilosec, the first proton-pump inhibitor anti-ulcer drug, Astra-Merck could expect P_m to be two or three times higher than the price of Zantac, an older generation anti-ulcer drug. And when it planned to sell music through iTunes, Apple might have estimated P_m to be around \$2 or \$3 per song, as a multiple of the per-song price of compact discs. (A CD with 12 songs might cost \$12 to \$15 but most consumers would want only a few of those songs.) Likewise, Intuit might have used a simple survey to learn how much at least some consumers would pay for software to prepare their tax returns.

One might argue that it is unlikely that a firm will know its maximum price exactly. The firm can estimate the maximum price, but that estimate will be subject to error. What does that do to our results? We explore this question in Section 3 of the paper and show that while uncertainty over the maximum price can reduce the performance of our pricing rule, unless the uncertainty is very large the firm will still do well.

[2] Biotech drugs, based on recombinant DNA technologies, are quite different. The molecules are extremely complex (often too complex to even specify precisely, so the production process is patented rather than the molecule), and production can be expensive. Many of the new cancer therapies are biotech drugs.

1.3. The Firm Need Not Know the Quantity It Will Sell

How can a firm set price without also having an estimate of the quantity it will sell? For new (biochemical) drugs, marginal cost is near zero, so the firm can produce a large amount of pills and discard whatever is not sold. (We are assuming there is no capacity limitation.) Of course selling more is better than selling less, but the only decision the firm must make is what price to charge. For music or video downloads and streaming services, as well as software, no factories have to be built and marginal production cost (net of royalties) is zero. This assumption is also satisfied in settings where production lead times are short relative to product life. An example is Intel's production of processors for personal computers. Building the fabrication facility involves a large sunk cost, and a major modification of the facility is needed for each new generation of processor. But for each generation, the firm moves down its learning curve rapidly (in about six months), and from then on marginal production cost is very low. Thus, the firm can produce more chips than it expects to sell, and discard whatever it doesn't sell.

1.4. The Pricing Rule

We propose that if the three conditions discussed above hold, the firm can use the following rule: Given the maximum price P_m, set price as though the actual demand curve were linear, i.e.,

$$P(Q) = P_m - bQ, \tag{1}$$

With constant marginal cost c, the firm's profit-maximizing price is $P^* = (P_m + c)/2$, which we refer to as the "linear price." This price is independent of the slope b of the linear demand curve, although the resulting quantity, $Q_L^* = (P_m - c)/2b$, is not. But as long as the firm does not need to invest in production capacity or plan on a particular sales level, knowledge of b, and thus the ability to predict its sales, is immaterial. For any price strictly above c, more sales are better than less, but the only problem at hand is to set the price. We denote the resulting price and profit from using eqn. (1) by P^* and Π^* respectively.

How well can the firm expect to do if it sets P^*? Suppose that with precise knowledge of its true demand curve, the firm would set a different price P^{**} and earn a (maximum) profit Π^{**}. The question we address is simple: How close can we expect Π^* to be relative to Π^{**}, i.e., how well is the firm likely to do using this simple pricing rule? As one would expect, the answer depends on the true demand function. In this paper, we derive closed form bounds on the profit performance for several common demand functions and compute numerically the performance for randomly generated demands. We will show that in many cases this simple pricing rule performs well, i.e., Π^* is close to Π^{**}. We will also identify cases where the rule does not perform well.

The basic idea behind this paper is quite simple and is illustrated in Figure 1. The demand curve labeled "Actual Demand" was drawn so it might apply to a new drug, or to music downloads

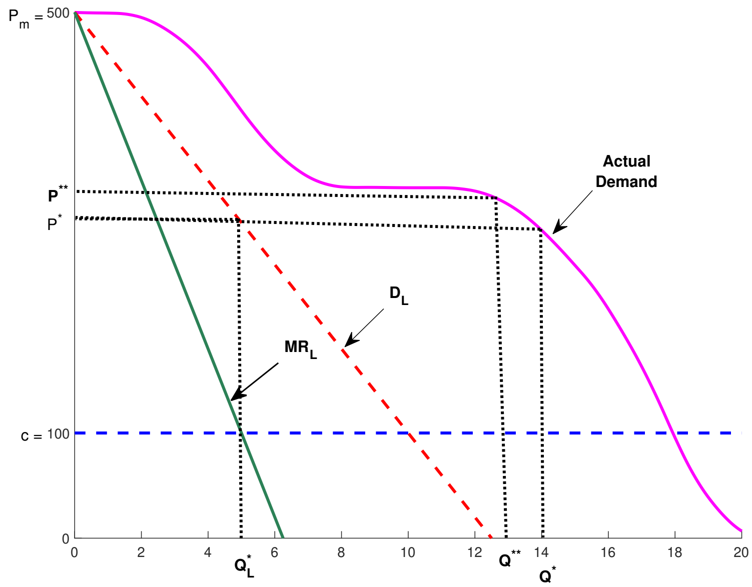


Figure 1 **Illustration for a representative demand curve**

in the early years of the iTunes store. A pharmaceutical company might estimate a price P_m at which some doctors will prescribe and some consumers will buy its new drug, even if insurance companies refuse to reimburse it. As the price is lowered and the drug receives insurance coverage, the quantity demanded expands considerably. At some point the market saturates so that even if the price is reduced to zero there will be no further increase in sales. For music downloads, at prices above P_m it is more economical to buy the CD and "rip" the desired songs to one's computer. At lower prices demand expands rapidly, and at some point the market saturates.

If the firm knew this curve, it would set the profit-maximizing price P^{**} and expect to sell the quantity Q^{**}. (In the figure, P^{**} and Q^{**} are computed numerically.) But the firm does not know the actual demand curve. A linear demand curve that starts at P_m has also been drawn and labeled D_L. This linear demand curve implies a profit-maximizing price P^* and quantity Q_L^*, where the subscript L refers to the quantity sold if D_L were the true demand curve. Note that P^* does not depend on the slope of the demand curve, b; *any linear demand curve that begins at P_m will yield the same profit-maximizing price P^*.*

How badly would the firm do by pricing at P^* instead of P^{**}? For the demand curve and marginal cost ($c=100$) shown in Figure 1, the profit and price ratios (determined numerically) are $\Pi^{**}/\Pi^* = 1.023$ and $P^{**}/P^* = 1.069$, i.e., the resulting profit is within a few percent of what the firm could earn if it knew the actual demand curve and used it to set price. (The firm would do a bit worse if $c=0$, in which case $\Pi^{**}/\Pi^* = 1.084$.)

There are certainly demand curves for which this pricing rule will perform poorly. For example, suppose the true demand curve is a rectangle, i.e., $P = P_m$ for $0 \leq Q \leq Q_{max}$ and $P = 0$ for $Q > Q_{max}$. Then, the profit-maximizing price is clearly P_m and the resulting profit is $\Pi^{**} = (P_m - c)Q_{max}$. Setting a price $P^* = (P_m + c)/2$ yields a much lower profit; in fact $\Pi^{**}/\Pi^* = 2.0$. We want to know how well our pricing rule will perform—i.e., what is Π^{**}/Π^*—for alternative "true" demand curves.

1.5. Related Literature

There is a large literature on optimal pricing with limited knowledge of demand, much of which deals with experimentation and learning. An early example is Rothschild (1974), who assumes that a firm chooses from a finite set of prices (exploration phase), observes outcomes, and because each trial is costly, eventually settles on the price that it thinks (perhaps incorrectly) is optimal (exploitation phase). The firm's choice is then the solution of a multi-armed bandit problem. (In the simplest version of the model, the firm prices "high" or "low.") The solution does not involve estimating a demand curve.

The marketing literature has also considered the problem of pricing for new products. Examples include classical works such as Urban et al. (1996) that consider pre-market forecasting, as well as Krishnan et al. (1999) that consider a variation of the generalized Bass model that yields optimal pricing policies that are consistent with empirical data. More recently, Handel and Misra (2015) introduce a dynamic non-Bayesian framework for robust pricing of new products.

Other studies focus on learning in a parametric or non-parametric context. Several papers address the use of learning to update estimates of parameters of a known demand function; see, e.g., Aviv and Pazgal (2005), Bertsimas and Perakis (2006), Lin (2006), and Farias and Van Roy (2010). A second stream examines the interplay between learning demand and optimizing revenues over time without imposing a parametric form. Following Rothschild (1974), several authors assume the seller first sets a price to learn about demand, and then adjusts the price to optimize revenues (see, e.g., Besbes and Zeevi 2009, Araman and Caldentey 2011, Balvers and Cosimano 1990).

The operations research literature examines dynamic pricing using robust optimization, where the functional form of the demand curve is known but one or more parameters are only known to lie in an "uncertainty set." For example, demand might depend on two unknown parameters α_1 and α_2, so the profit function is $\Pi(\alpha_1, \alpha_2, p)$. The price p is chosen to maximize the worst possible outcome over the uncertainty set, i.e., $\max_p \min_{\alpha_1,\alpha_2} \Pi(\alpha_1, \alpha_2, p)$.[3] In related work, Bergemann and Schlag (2011) consider a single consumer's valuation, with a distribution that is unknown but

[3] See, for example, Adida and Perakis (2006) and Thiele (2009). An alternative is the "distributionally robust" approach, where price is robust with respect to a class of demand distributions with similar parameters such as mean and variance (see, e.g., Lim and Shanthikumar 2007, Ball and Queyranne 2009).

assumed to be in a neighborhood of a given model distribution. The authors characterize robust pricing policies that maximize the seller's minimum profit (maximin), or that minimize worst-case regret (difference between the true valuation and the realized profit). Although robust optimization incorporates uncertainty, its focus on worst-case scenarios may yield conservative pricing strategies.

There is an extensive literature on mechanism design and auctions that is tangentially related to our paper. This literature considers simple mechanisms and demonstrates their performance relative to the optimal (often very complicated) mechanism. For example, Segal (2003) examines the profitability of bidding mechanisms relative to posted pricing. The author considers the case where the bidders' valuations are drawn from an unknown distribution and shows that the deterministic optimal price auction is asymptoticly optimal when the valuations come from distributions with bounded support. Hartline and Roughgarden (2009) show that simple approximation mechanisms remain almost optimal in general single-parameter agent settings.

Our paper is also related to studies of model misspecification. In particular, we study the performance of a simple linear demand model even if the true demand curve is far from linear. Others have shown that linear models can perform well (e.g., Dawes (1979) in clinical prediction and Carroll (2015) in contract theory). Besbes and Zeevi (2015) study the "price of misspecification" for dynamic pricing with demand learning. The authors propose a dynamic pricing algorithm in which the seller assumes demand is linear, and chooses a price to maximize revenue based on this linear demand function. They show that although the model is misspecified, one can achieve a good asymptotic regret performance. In our setting, however, the firm chooses a price and does not have the option to experiment over time. In addition, our paper investigates how consumer welfare is affected by demand misspecification.

1.6. What This Paper Does

Our approach to pricing is quite different from the studies cited above, and is related to the prescriptive rules of thumb found, for example, in Shy (2006). Managers often seek simple and robust rules for pricing (and other decisions such as levels of advertising or R&D), and other studies have shown that simple rules can be very effective.[4] The pricing rule we suggest is certainly simple; the extent to which it is effective is the focus of this paper.

[4] In related work, Chu et al. (2011) show how "Bundle Size Pricing" (BSP) provides a close approximation to optimal mixed bundling. In BSP, a price is set for each good, for any bundle of two, for any bundle of three, etc., up to a bundle of all the goods produced. Profits are close to what would be obtained from mixed bundling. Also Carroll (2015) examines a principal who has only limited knowledge of what an agent can do, and wants to write a contract robust to this uncertainty. He shows that the most robust contract is a linear one—e.g., the agent is paid a fixed fraction of output. Hansen and Sargent (2008) provide a general treatment of robust control, i.e., optimal control with model uncertainty.

The pricing rule $P^* = (P_m + c)/2$ follows from a linear approximation to the true demand curve. Note, however, that the same pricing rule can be obtained from a different set of modeling assumptions. Suppose the firm plans to sell the product to a representative consumer with a random valuation that is uniformly distributed $U[c, P_m]$. In this case, to maximize expected profits, the optimal price is also $P^* = (P_m + c)/2$. (The proof is presented in the Appendix.) The equivalence of the two models (linear demand curve and uniform consumer valuation) is well known, and provides an alternative way of justifying the pricing rule studied in this paper.

In some cases, estimating the maximum price P_m is difficult or impractical. However, one can extend the results and analysis of this paper to situations where the firm can determine a price $\bar{P} < P_m$, such that at $\bar{P}$ the firm can still sell to a small set of customers who are not very price sensitive. The pricing rule then becomes $P^* = (\bar{P} + c)/2$. The basic insights of this paper will still hold, although at the expense of a pricing rule that does not perform quite as well. Obviously, the performance deteriorates when the gap between $\bar{P}$ and P_m increases.

In the following sections, we characterize the performance of our pricing rule by deriving analytical bounds for the profit ratio Π^{**}/Π^* for several classes of demand curves. We also find bounds for Π^{**}/Π^* for a general concave demand, and treat the case of a maximum price that is not known exactly. We then examine randomly generated "true" demand curves and determine computationally the expected profit ratio Π^{**}/Π^* and confidence bounds for the ratio. Finally, we examine the welfare implications of our pricing rule.

2. Common Demand Functions

Here we examine several demand models—quadratic, monomial, semi-log, and log-log. We also consider the case of a general concave demand. These demand models are used in many operations management and economics applications.[5] For each we compare the profits from our pricing rule to the profits that would result if the actual demand function were known. We will see that the profit ratio is often close to one.

Before proceeding, note that the relationship between the linear price P^* and the optimal price P^{**} depends on the convexity properties of the actual demand function. In the Appendix we show that if the actual inverse demand curve is convex (concave) with respect to Q,[6] the linear price is greater (smaller) than the optimal price:

[5] The log-log model is widely used in many retail applications (see, e.g., Montgomery 1997, Mulugeta et al. 2013, Cohen et al. 2017). The quadratic and semi-log functions are often used in the context of hedonic pricing (see, e.g., Wilman 1981, Milon et al. 1984).

[6] Assuming that the inverse demand is a continuous decreasing convex (resp. concave), then the demand is also convex (resp. concave).

THEOREM 1. *If the actual inverse demand curve $P_A(Q)$ is convex with respect to Q, then $P^{**} \leq P^*$, and if $P_A(Q)$ is concave, $P^{**} \geq P^*$.*

Note that we only need $P_A(Q)$ to be convex (or concave) in the range $[0, Q^{**}]$ and not everywhere. The value of Q^{**} might not be known but this result can still be useful in that it tells us whether our simple rule will over- or under-price relative to the optimal price, and it might be possible to correct for this error by adjusting the price up or down.

2.1. Quadratic Demand

Suppose the actual inverse demand function is quadratic:

$$P_A(Q) = P_m - b_1 Q + b_2 Q^2, \tag{2}$$

where, as before, P_m is the maximum price. Equivalently, the actual demand function is given by: $Q_A(P) = 0.5[b_1 - \sqrt{b_1^2 - 4b_2(P_m - P)}]/b_2$. We want analytical bounds for the profit ratio Π^{**}/Π^* and price ratio P^{**}/P^*. The bounds depend on the convexity properties of the function in (2) and are summarized in the following result. (Proofs are in the Appendix.)

PROPOSITION 1. *For the quadratic demand curve of eqn. (2), for any marginal cost $c \geq 0$, the profit and price ratios satisfy:*

- ***Convex case:*** $b_1, b_2 \geq 0$ *and* $b_2 \leq b_1^2/4P_m$

$$1 \leq \frac{\Pi^{**}}{\Pi^*} \leq \frac{8\sqrt{2}}{27(\sqrt{2}-1)} = 1.0116,$$
$$\frac{8}{9} \leq \frac{P^{**}}{P^*} \leq 1.$$

- ***Concave case:*** $b_1 \geq 0$ *and* $b_2 \leq 0$

$$1 \leq \frac{\Pi^{**}}{\Pi^*} \leq \frac{4\sqrt{2}}{3\sqrt{3}} = 1.0887,$$
$$1 \leq \frac{P^{**}}{P^*} \leq \frac{2}{3}\left(\frac{2P_m + c}{P_m + c}\right) \leq \frac{4}{3} = 1.33.$$

Note that the restrictions on the values of b_1 and b_2 are necessary and sufficient conditions to guarantee that the inverse demand curve is non-negative and non-increasing everywhere.

If demand is convex, the simple pricing rule yields a profit that is only about 1% less than what the firm could achieve if it knew the true demand curve. Also, this is a "worst case" result that applies when $c = 0$; if $c > 0$, the ratio Π^{**}/Π^* is even closer to 1. The price P^* can be as much as 12% lower than the optimal price P^{**}, but the concern of the firm is (or should be) its profit. (Also, P^{**}/P^* deviates the most from 1 when $c = 0$.)

If demand is concave, the resulting profit Π^* is within 8.87% of the optimal profit, irrespective of the parameters b_1 and b_2. In the proof of Proposition 1 in the Appendix, we show that the largest value of Π^{**}/Π^{*} (1.0887) occurs when $b_1 = 0$; for positive values of b_1, the profit ratio is closer to 1. The reason is that when b_1 increases, the curve becomes closer to a linear function. In addition, one can show that the profit ratio becomes closer to 1 for the concave case when either c or b_2 increase (recall than $b_2 \leq 0$).

2.2. Monomial Demand

Now suppose the actual inverse demand curve is a monomial of order n:

$$P_A(Q) = P_m - \gamma Q^n \ , \quad \gamma > 0. \tag{3}$$

Equivalently, the actual demand is given by: $Q_A(P) = [(P_m - P)/\gamma]^{1/n}$. Note that all functions of the form (3) are concave and decreasing, given that $\gamma > 0$. The Appendix shows that the profit and price ratios are now:

PROPOSITION 2. *For the inverse demand curve of eqn.* (3)*, the profit and price ratios satisfy:*

$$1 \leq \frac{\Pi^{**}}{\Pi^{*}} = \frac{2^{\frac{1}{n}+1} n}{(n+1)^{\frac{1}{n}+1}} \leq 2,$$
$$1 \leq \frac{P^{**}}{P^{*}} = \frac{2(nP_m + c)}{(n+1)(P_m + c)} \leq 2.$$

Thus for any monomial demand curve, the profit ratio only depends on the order of the monomial n; it does not depend on the values of P_m, c or γ. (The price ratio does depend on P_m, c and n, but not on γ.) Both ratios are monotonically increasing with the degree of the monomial n and converge to 2 and $2P_m/(P_m + c) \leq 2$ respectively, as $n \to \infty$. For monomials of order 3 and 4, the profit ratios are 1.19 and 1.27 respectively.

2.3. Semi-Log Demand

Now consider the semi-log inverse demand curve:

$$P_A(Q) = P_m e^{-\alpha Q} \ , \quad \alpha > 0 \ , \tag{4}$$

Or equivalently, $Q(P) = \frac{1}{\alpha}\log(P_m/P)$. The following result (proof in Appendix) bounds the profit and price ratios when the marginal cost $c = 0$ and when $c > 0$.

PROPOSITION 3. *For the semi-log inverse demand curve of eqn.* (4)*,*

- *When $c = 0$, the profit and price ratios are:*

$$\Pi^{**}/\Pi^{*} = 2e^{-1}/\log(2) = 1.0615,$$
$$P^{**}/P^{*} = 2e^{-1} = 0.7357.$$

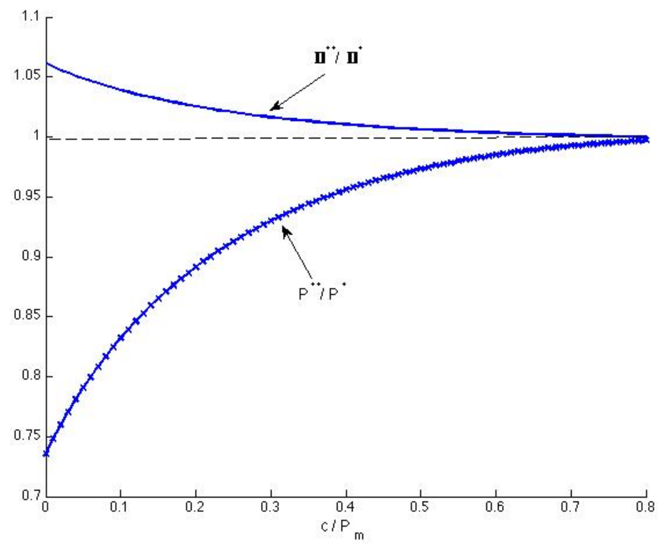


Figure 2 **Profit and price ratios for the semi-log inverse demand curve as a function of** c/P_m

- *When* $c > 0$*, the ratios are closer to 1:*

$$1 \le \Pi^{**}/\Pi^{*} < 1.0615,$$
$$0.7357 < P^{**}/P^{*} \le 1.$$

When $c = 0$ both ratios can be computed exactly and do not depend on α or P_m; in this worst case, the simple pricing rule yields a profit that differs from the optimal profit by only 6.15%, even though the prices differ by 26.5%. When $c > 0$, one cannot compute the ratios in closed form. Instead, we solve numerically for Π^{**} and P^{**} and present the results in Figure 2, where we plot the ratios as a function of c/P_m. (The ratios are independent of α.) Note that as c increases both ratios approach 1.

2.4. Log-Log Demand

We turn now to the commonly used log-log (isoelastic) demand model:

$$P_A(Q) = A_0 Q^{-1/\beta}; \qquad \beta > 1\,, \tag{5}$$

where $-\beta$ is the (constant) elasticity of demand. Because this demand curve has no maximum price, we truncate it so that $P(0) = P_m$. Setting $P_A(Q_0) = P_m$, the corresponding quantity is $Q_0 = (P_m/A_0)^{-\beta}$. We therefore work with the following modified version of eqn. (5):

$$P_A(Q) = \begin{cases} P_m\,; & \text{if } Q < Q_0 \\ P_m(Q/Q_0)^{-1/\beta}\,; & \text{if } Q \ge Q_0 \end{cases} \tag{6}$$

We require that $\beta > \beta_{min} = P_m/(P_m - c)$ for the optimal price P^{**} to be less than the maximum price P_m. In this case:

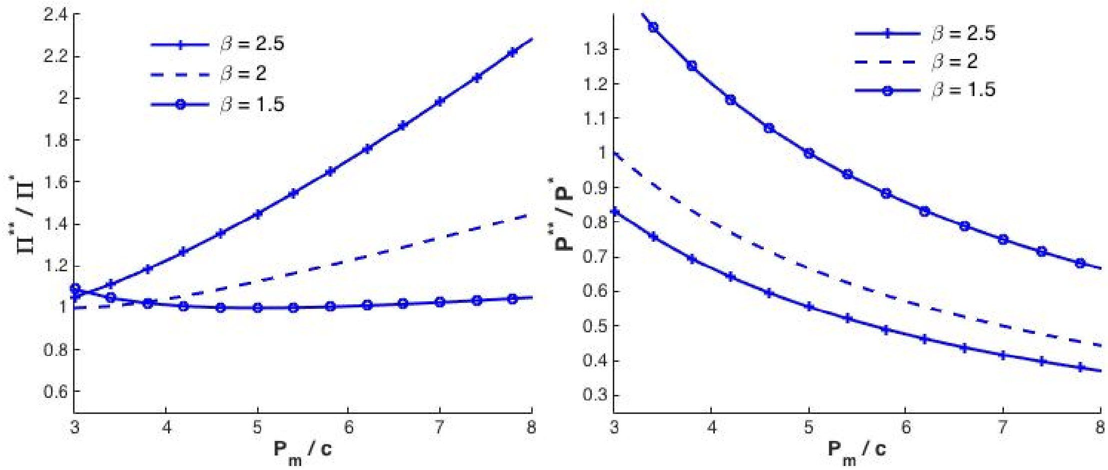


Figure 3 **Profit and price ratios as a function of P_m/c for the log-log demand curve**

PROPOSITION 4. *For the demand curve of eqn.* (6)*, the profit and price ratios are:*

$$\frac{\Pi^{**}}{\Pi^*} = \frac{2}{(P_m/c-1)(\beta-1)}\left[\frac{2\beta}{(P_m/c+1)(\beta-1)}\right]^{-\beta},$$

$$\frac{P^{**}}{P^*} = \frac{2\beta}{(P_m/c+1)(\beta-1)}.$$

Note that these ratios are exact and depend only on the elasticity β and P_m/c. Also, there is a unique value of $\beta^* = (P_m + c)/(P_m - c)$ for which both ratios equal 1.[7]

There are two limiting cases to note: c large and c very small. If c is large, i.e., $c \to P_m$, $\beta_{min} \to \infty$. If β_{min} is very large, β ($> \beta_{min}$) is also very large (i.e., demand is elastic), so both the profit and price ratios will be close to 1. At the other extreme, as $c \to 0$, $P^{**} \to 0$, whereas $P^* \to 0.5P_m$, and Π^{**}/Π^* is unbounded. But an isoelastic demand curve would then make little sense, because $Q^{**} \to \infty$.

The general case is illustrated in Figure 3, which shows the profit and price ratios as a function of P_m/c for $\beta = 1.5$, 2.0, and 2.5. If $\beta = 1.5$, Π^{**}/Π^* is always close to 1. But if $\beta = 2.5$, Π^{**}/Π^* can exceed 2 for large enough values of of P_m/c. Note that P^{**} can be larger or smaller than P^*.[8] As a result, if demand is very elastic (i.e., β is large) or the marginal cost c is very small, our pricing rule will not perform well. One limitation of our pricing rule for the log-log demand model is the fact that the performance crucially depends on the price elasticity, which is not known by firm.

[7] If $\beta = \beta^*$, the elasticity of the isoelastic demand equals the elasticity of the linear demand at the optimal price. The latter elasticity is $E_d = bP^*/Q_L^* = (P_m + c)/(P_m - c)$, so if $\beta = \beta^*$, both the linear and log-log demand curves have the same profit-maximizing price and output.

[8] The log-log demand curve is convex but truncating it modifies its convexity properties, which affects the relationship between P^{**} and P^* (see Theorem 1). If either β or P_m/c is small, the optimal quantity Q^{**} is small and can lie on the truncated—and non-convex—part of the curve.

Table 1 summarizes these results. It shows that our pricing rule works well for a variety of underlying demand functions—but not all. For example, if the true demand is a truncated log-log function, Π^{**}/Π^* can deviate substantially from 1 if demand is very elastic and/or the marginal cost is small. This follows from the convexity of this function and the fact that (unrealistically) the quantity demanded expands without limit as the price is reduced towards zero.

We conclude this section by deriving the performance of our pricing rule for a general concave demand function.

2.5. Concave Demand

PROPOSITION 5. *For any concave demand curve, we have:*

$$1 \leq \Pi^{**}/\Pi^* \leq 2\,, \qquad 1 \leq P^{**}/P^* \leq 2.$$

In the worst case, the profit and price ratios will equal 2 if the true demand curve is a rectangle. For other concave functions, $\Pi^{**}/\Pi^* < 2$, but except for specific functional forms, we cannot say how much less.

We might expect that in some cases the inverse demand curve will not be concave and may even have a flat area (plateau), as in Figure 1. In this case, Π^{**}/Π^* will be sensitive to whether the plateau is below or above P^*. If the plateau is below P^* and very long, Π^{**}/Π^* can be arbitrarily large; by pricing at P^*, the firm is missing a large mass of consumers. But if the plateau is above P^*, Π^{**}/Π^* will usually be close to 1. Thus if the firm believes there is such a plateau, it might set price below P^* in order to account for it.

3. Uncertain Maximum Price

So far, we have assumed that while the firm does not know its true demand curve, it *does* know the maximum price P_m it can charge and still expect to sell some units. Suppose instead that the firm only has an estimate of the maximum price:

$$\hat{P}_m = P_m(1+\epsilon), \tag{7}$$

where ϵ lies in some interval $[-B, B]$, with $0 \leq B \leq 1$. Our pricing rule is now $P^* = (\hat{P}_m + c)/2$, and suffers from two misspecifications: the form of the demand curve and the value of the intercept. To see how this second source of uncertainty affects the profit ratio Π^{**}/Π^*, we derive the profit ratios as closed-form functions of ϵ for the demand models we considered in Section 2. (Details are in the Appendix.) To simplify matters, we assume here that $c = 0$. (Recall from the previous section that Π^{**}/Π^* deviates from 1 the most when $c = 0$ for the demand curves we considered.)

Under a moderate misspecification of P_m (i.e., small values of ϵ), our pricing rule still performs well. However, depending on the "draw" for ϵ, the actual profit ratio could be farther from 1.

Inverse demand function	P^{**}/P^{*}	Π^{**}/Π^{*}
Quadratic convex: $P_A(Q) = P_m - b_1Q + b_2Q^2$ $b_1, b_2 \geq 0$ and $b_2 < b_1^2/4P_m$	$\frac{8}{9} \leq P^{**}/P^{*} \leq 1$	≤ 1.0116
Quadratic concave: $P_A(Q) = P_m - b_1Q + b_2Q^2$ $b_1 \geq 0$ and $b_2 \leq 0$	$1 \leq P^{**}/P^{*} \leq \frac{4P_m+2c}{3P_m+3c} \leq 1.33$	≤ 1.0887
Monomial: $P_A(Q) = P_m - \gamma Q^n$	$2(nP_m + c)/(n+1)(P_m + c)$	$2^{(n+1)/n} n/(n+1)^{(n+1)/n}$
$n = 3$	≤ 1.5	1.19
$n = 4$	≤ 1.6	1.27
Semi-log: $P_A(Q) = P_m e^{-\alpha Q}$		
$c = 0$	0.7357	1.0615
$c > 0$	< 0.7357	< 1.0615
Log-log (truncated): $P_A(Q) = \begin{cases} P_m; & \text{if } Q < Q_0 \\ P_m(Q/Q_0)^{-1/\beta}; & \text{if } Q \geq Q_0 \end{cases}$ $\beta \geq \beta_{min} = P_m/(P_m - c)$	$2\beta/(P_m/c+1)(\beta-1)$	$\frac{2}{(P_m/c-1)(\beta-1)}\left(\frac{2\beta}{(P_m/c+1)(\beta-1)}\right)^{-\beta}$

Table 1 Price and profit ratios for several "true" demand curves

To see how much farther, we use the closed-form expressions in the Appendix to plot the profit ratios as a function of ϵ for $-0.2 \leq \epsilon \leq 2$. As Figure 4 shows, the monomial demand (with $n = 3$) is most sensitive to the value of ϵ, with Π^{**}/Π^{*} reaching 1.5 when $\epsilon = -0.2$. For the other demand curves, $\Pi^{**}/\Pi^{*} < 1.25$ over the range of ϵ we consider. Thus a misspecification of the maximum price increases Π^{**}/Π^{*}, but only moderately.

4. Random Demand Curves

In this section, we consider randomly generated demand functions. In practice, a firm introducing a new product may know little or nothing about the shape of the demand curve. Indeed, that is the motivation for this paper. The firm might have no reason to expect that demand is characterized by one of the commonly-used functions we examined earlier, or any other particular function. If the firm uses our pricing rule — with no knowledge at all of the true demand curve, other than the maximum price P_m — how well can it expect to do?

We address this question by randomly generating a set of "true" demand curves. For each randomly generated curve we compute (numerically) the profit-maximizing price and profit, P^{**} and Π^{**}, and compare Π^{**} to the profit Π^{*} the firm would earn by using our pricing rule, i.e., by setting $P^{*} = (P_m + c)/2$. We generate 100,000 such demand curves and examine the resulting distribution of Π^{**}/Π^{*}. The only restriction we impose on these demand curves is that they are non-increasing everywhere.

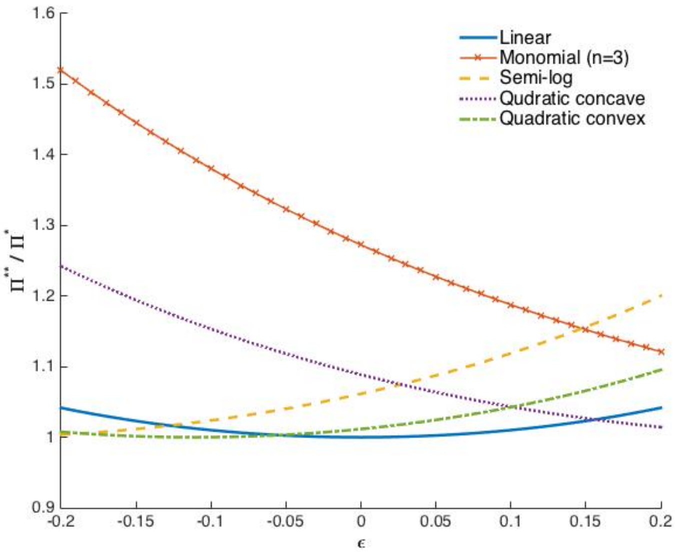


Figure 4 Profit ratios as a function of ϵ for different demand curves

We generate each demand curve as follows. We assume the maximum price P_m is known and so is the maximum quantity Q_{max} that can be sold at a price of zero (i.e., the maximum potential size of the market).[9] We divide the segment $[0, Q_{max}]$ into S equally-spaced intervals and generate a piecewise non-increasing demand curve by drawing random values for the different pieces. Since $P(0) = P_m$ and $P(Q_{max}) = 0$, there are $S-1$ breaking points between 0 and P_m. (One might interpret this partition of the market as representing customer segments, or simply an approximation to a continuous curve.) With this partition, we draw a random value for the end of the first segment from a distribution between 0 and P_m, which we will call P_1 (see one realization for P_1 in Figure 5). More precisely, we draw a random variable X_1 between 0 and 1 and $P_1 = P_m X_1$. Next, we independently draw a value for the end of the second segment, but now between 0 and P_1. Call this $P_2 = P_1 X_2$, where X_2 is drawn between 0 and 1. We repeat this process $S-1$ times, drawing a total of $S-1$ independent random variables $X_i;\ i = 1,\dots,S-1$ between 0 and 1, generating a random demand curve with S segments. Figure 5 shows an example of such a randomly generated demand curve that has 5 segments (for $P_m = 500$ and $Q_{max} = 5$). Given this demand curve, we numerically calculate P^{**}, Π^{**}, and the profit ratio Π^{**}/Π^* for $c = 0$ and $c = 0.5P_m$.

We draw the random variables $X_i;\ i = 1,\dots,S-1$ using a power distribution of the form $X^{1/\alpha}$, where $\alpha \geq 1$ is a skewing parameter and X is uniformly distributed between 0 and 1. Note that when $\alpha = 1$, this reduces to the uniform distribution. For simplicity, we present the results for the case of a uniform distribution (i.e., $\alpha = 1$); we obtained very similar results when $\alpha = 1.5$.

[9] To generate random demand curves, it is convenient to specify the maximum quantity Q_{max}. We tested different values of Q_{max} and found similar qualitative insights, so that our results are not sensitive to the value of Q_{max}.

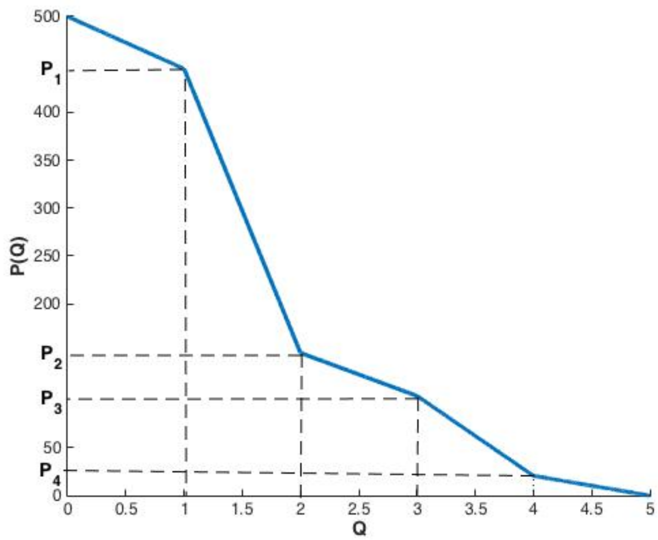


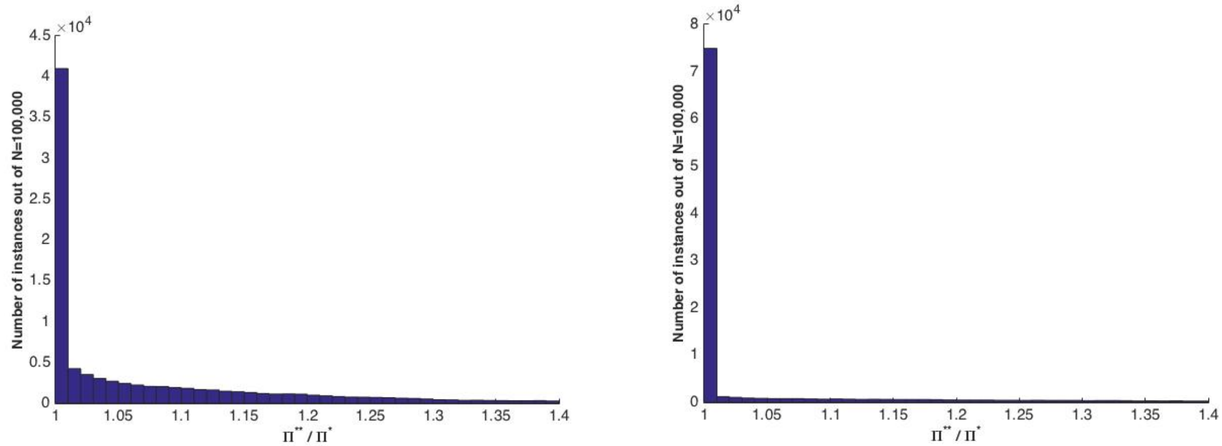
Figure 5 **Randomly generated inverse demand curve with $S=5$ pieces**

$c=0$				$c=0.5P_m$			
S	Mean	80%	90%	S	Mean	80%	90%
2	1.0672	1.1625	1.2442	2	1.0748	1.1255	1.3696
5	1.1332	1.2057	1.3926	5	1.0525	1.0645	1.2271
10	1.1351	1.2081	1.3979	10	1.0523	1.0647	1.2254
50	1.1379	1.2161	1.4071	50	1.0525	1.0621	1.2264
100	1.1344	1.2124	1.4045	100	1.0525	1.0628	1.2265

Table 2 **Profit ratios for randomly generated demands**

We generate 100,000 demand curves and compute 100,000 corresponding values for Π^{**}/Π^{*}. We calculate the mean value of Π^{**}/Π^{*}, as well as the 80% and 90% points (i.e., the value of Π^{**}/Π^{*}, such that 80% or 90% of the randomly generated ratios are below this number). The number of segments S can affect the resulting Π^{**}, so in Table 2 we show results for different values of S and for c/P_m equal to 0 and 0.5.

Observe that whatever the number of segments, S, the average profit ratio is less than 1.14 if $c=0$ and less than 1.08 if $c=0.5P_m$. Also, for 80% (90%) of the demand curves, the profit ratios are less than 1.22 (1.41) if $c=0$ and less than 1.13 (1.37) if $c=0.5P_m$. In Figure 6, we plot histograms of the 100,000 profit ratios for $S=5$ and both $c=0$ and $c=0.5P_m$. When $c=0$ ($c=0.5P_m$), more than 40% (75%)of the ratios are less than 1.01, and 54% (79%) are less than 1.05. Thus it is reasonable to expect our pricing rule to yield a profit close to what would result if the firm knew its actual demand curve.



(a) $c = 0$ (b) $c = 0.5P_m$

Figure 6 Histogram of profit ratios when $S = 5$ for $c = 0$ and $c = 0.5P_m$

5. Welfare Implications

We now compare the total welfare (consumer plus producer surplus) obtained from our pricing rule $P^* = 0.5(P_m + c)$ to the welfare that would have resulted if the firm knew the true demand curve and set the price at P^{**}. We also look at consumer surplus separately to see how our pricing rule affects consumers. The total welfare, denoted by $W(P)$, is:

$$W(P) = \Pi(P) + CS(P) = (P - c)Q + \Big[\int_0^Q P_A(y)dy - PQ\Big]. \tag{8}$$

We are interested in $W(P^{**})/W(P^*) \equiv W^{**}/W^*$ and $CS(P^{**})/CS(P^*) \equiv CS^{**}/CS^*$. Note that these ratios can be less than one, i.e., our pricing rule can increase the total welfare and/or the consumer surplus relative to that when the profit maximizing price P^{**} is used. In particular, we have the following result.

Proposition 6. *If the actual demand is convex, then $W^{**} \geq W^*$ and $CS^{**} \geq CS^*$; whereas if it is concave, $W^{**} \leq W^*$ and $CS^{**} \leq CS^*$.*

Indeed, as long as $P \geq c$, $W(P)$ is non-increasing, so that the inequalities on W follow immediately from Theorem 1. The inequalities on CS also follow from Theorem 1 and from the fact that the consumer surplus is a non-increasing function of the price (for $P \leq P_m$). If the demand is concave, we know from Theorem 1 that $P^* \leq P^{**}$, so in this case using the "wrong" price P^* improves both the total welfare and consumer surplus (i.e., the benefit to consumers exceeds the loss to the firm). However, if the demand is convex, both the firm and consumers are worse off.

Next, we calculate the welfare and consumer surplus ratios (i) analytically for the demand models in Section 2 (see Proposition 7) and (ii) computationally for randomly generated demand curves following the approach of Section 4. To simplify matters, we assume that $c = 0$. We do not report the details of the derivations for conciseness. The closed form expressions are as follows:

PROPOSITION 7. *The welfare and consumer surplus ratios, W^{**}/W^{*} and CS^{**}/CS^{*}, for the different demand models are:*

- ***Quadratic convex:*** $P_A(Q) = P_m - b_1Q + b_2Q^2$; $b_1, b_2 \geq 0$ *and* $b_2 \leq b_1^2/4P_m$

$$\frac{W^{**}}{W^{*}} \leq 1.26045 \quad \text{and} \quad \frac{CS^{**}}{CS^{*}} \leq 1.5756$$

- ***Quadratic concave:*** $P_A(Q) = P_m - b_1Q + b_2Q^2$; $b_1 \geq 0$ *and* $b_2 \leq 0$

$$\frac{W^{**}}{W^{*}} \leq \frac{16\sqrt{2}}{15\sqrt{3}} = 0.8709 \quad \text{and} \quad \frac{CS^{**}}{CS^{*}} \leq 0.544$$

- ***Monomial:*** $P_A(Q) = P_m - \gamma Q^n$

$$W^{**}/W^{*} = \frac{2n+1-1/(n+1)}{3-1/(n+1)}\left(\frac{2}{n+1}\right)^{\frac{1}{n}+1} \quad \text{and} \quad \frac{CS^{**}}{CS^{*}} = \left(\frac{2}{n+1}\right)^{\frac{1}{n}+1}$$

- ***Semi-log:*** $P_A(Q) = P_m e^{-\alpha Q}$

$$W^{**}/W^{*} = 2(1-e^{-1}) = 1.2642 \quad \text{and} \quad \frac{CS^{**}}{CS^{*}} = 1.722$$

We omitted the truncated log-log demand as the welfare and consumer surplus ratios are complicated expressions that depend on P_m/c and β. For the exponential demand, $CS^{**}/CS^{*} \leq 1.6487$ (the welfare ratio is unbounded). For the monomial demand, when $n = 3$ and $n = 4$, W^{**}/W^{*} is 0.974 and 0.999 respectively, and CS^{**}/CS^{*} is 0.397 and 0.318 respectively. Also, as the order of the monomial n increases, W^{**}/W^{*} approaches 4/3 whereas CS^{**}/CS^{*} approaches 0. Indeed, as n increases, the inverse demand curves becomes more concave so there is a greater transfer of welfare from the firm to consumers. One can see that for these demand models, the loss in total welfare from using our approximation is at most 26%, but in some cases the loss (or gain) in consumer surplus can be quite large. (For example, for the semi-log demand, when $c = 0$ the loss in profit is 6.16% but the decrease in consumer surplus is 72%.)

We next calculate W^{**}/W^{*} and CS^{**}/CS^{*} for randomly generated demand curves following the approach of Section 4. As before, we fix P_m, c, and Q_{max} and compute the ratios for $c/P_m = 0$ and 0.5, using 100,000 randomly generated demand curves. (The results are very similar for different values of S.) The average welfare ratio for $c = 0$ and $c = 0.5P_m$ are 1.139 and 0.993 respectively. As for CS^{**}/CS^{*}, the average ratios for $c = 0$ and $c = 0.5P_m$ are 1.1885 and 0.9148 respectively (see Figure 7 in the Appendix). As one can see from the histograms, although CS^{**}/CS^{*} is close to 1 on average, for a significant fraction of demand curves, consumers will be either better off or worse off. Thus although our pricing rule often yields profits close to optimal, the misspecification of demand can have a significant (positive or negative) impact on consumer surplus.

6. Future Research and Related Open Questions

One might argue that because of the three conditions we imposed, our pricing rule is too narrow in terms of its range of applications. Here, we discuss several possible extensions of our work, which are left for future research. The first and most obvious extension is to relax each of the three conditions and determine whether an alternative pricing rule that is still relatively simple can be derived.

Second, our focus has been on a linear pricing rule (i.e., a price that is linear in the parameters P_m and c). It would be interesting to investigate to what extent a nonlinear rule (e.g., a quadratic function of P_m and c) yields a profit ratio closer to 1 relative to the linear rule.

Third, instead of assuming that the maximum price is known, it would be interesting to examine what happens if the firm knows a different point on the demand curve, that is, the firm knows a specific pair of points (P_0, Q_0), instead of $(P_m, 0)$. This is a relevant case in practice, as firms often have access to a single price-demand realization.

Perhaps one of the most important caveats is the fact that our analysis is entirely static. We assumed the true demand curve is fixed; it does not shift over time, potentially in response to network effects (which can be important for new products). We also assumed that the firm sets and maintains a single price; it does not change price over time to intertemporally price discriminate or to respond to changing market conditions, nor does it offer different prices to different groups of customers.

An additional limitation of our analysis is the fact that we assumed that capacity is not a relevant operational decision. (We provided several examples to justify this.) Extending the analysis for products that require capacity planning is an interesting avenue for future research. Finally, we have ruled out learning about demand, either passively or via experimentation, which has been the focus of the earlier literature on pricing with uncertain demand (learning and earning). To the extent that such dynamic considerations are important, our pricing rule can be viewed as a starting point. Managers often seek simple and robust rules for pricing; the rule we suggest is certainly simple, and we have seen that it is also often effective.

7. Conclusions

Setting price is one of the most basic economic decisions firms make. Introductory economics courses make this decision seem easy; just write down the demand curve and set marginal revenue equal to marginal cost. But of course firms rarely have precise knowledge of their demand curves. When introducing new products (or existing products in new markets), firms may know little or nothing about demand, but must still set a price. Price experimentation is often not feasible, and the price a firm sets is often the one it sticks with for some time.

We have shown that under certain conditions the firm can use a simple pricing rule. The conditions are: (i) marginal cost c is known and constant, (ii) the firm can estimate the maximum price P_m it can charge and still expect to sell some units, and (iii) the firm need not predict the quantity it will sell. These conditions hold for many new products and services, especially those introduced by technology companies. The firm then sets a price of $P^* = (P_m + c)/2$.

How well can the firm expect to do if it follows this pricing rule? We studied this question when the true demand curve is one of several commonly used demand functions, or even if it is a more complex function (e.g., randomly generated). Often, the firm will earn a profit reasonably close to the optimal profit it could earn if it knew the true demand curve. We also identified cases where the profit loss is significant. Our results can help us understand why linear demand functions are so popular in many applications (e.g., Koushik et al. 2012, Pekgün et al. 2013).

As mentioned in the Introduction, the results of this paper can be extended to cases where the firm does not know P_m, but it can estimate a price $\bar{P} < P_m$, such that at $\bar{P}$ the firm can sell to a small set of customers. However, as one would expect, the pricing rule $P^* = (\bar{P} + c)/2$ will not perform as well. For example, the profit ratio for the semi-log demand used in Section 2.3 will be now at most 1.181 for any value of $\bar{P}$ (the proof follows the same logic as the proof of Proposition 3 by translating the inverse demand curve).

The reader might be under the impression that the firms we are thinking about must be pure monopolists, but this is not the case. The firm cannot be a perfectly competitive one, because such a firm is not able to affect the price, which is determined as a market equilibrium. The firm must have some market power, so that it can set a price and expect to sell some quantity at that price. But we are not assuming that the firm is a monopolist.

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Appendix

Equivalence with the valuation model

Consider a representative consumer with a random valuation for the product. The valuation is assumed to be between the cost c and the maximal price P_m. We assume that the seller knows the valuation distribution, represented by the cdf $\mathbb{F}(\cdot)$ and the pdf $f(\cdot)$. We also assume that the seller seeks to maximize its expected profit, given by:

$$\Pi(p) = (p-c)\mathbb{P}(p < v) = (p-c)[1-\mathbb{F}(p)], \tag{9}$$

where p denotes the price set by the seller and v denotes the valuation of the consumer (unknown to the seller). If the price is below the valuation, the consumer will purchase the item and the seller extracts a profit of $p-c$. Otherwise, there is no sale and 0 profit for the seller. One can take the first order condition and obtain:

$$P^* = c + \frac{1-\mathbb{F}(P^*)}{f(P^*)}. \tag{10}$$

Eqn. (10) is a well-known result called the virtual valuation. If we further assume that the valuation distribution is uniform in $[c, P_m]$, we have: $f(p) = 1/(P_m - c)$ and $\mathbb{F}(p) = (p-c)/(P_m-c)$. Therefore, eqn. (10) becomes: $P^* = (P_m + c)/2$.

Proof of Theorem 1

The actual inverse demand curve, $P_A(Q)$, satisfies $P_A(0) = P_m$. One can write: $P_A(Q) = P_m - bQ + f(Q)$, with $f(0) = 0$ and $P_A''(Q) = f''(Q)$. Equating marginal revenue with marginal cost:

$$Q^{**} = \frac{P_m - c + f(Q^{**}) + f'(Q^{**})Q^{**}}{2b}.$$

This yields an expression for the optimal price as a function of Q^{**}:

$$P^{**} = P_A(Q^{**}) = P_m - \frac{1}{2}\Big[P_m - c + f(Q^{**}) + f'(Q^{**})Q^{**}\Big] + f(Q^{**}).$$

Recall that $P^* = (P_m + c)/2$ and thus: $P^{**} = P^* + 0.5\big[f(Q^{**}) - f'(Q^{**})Q^{**}\big]$. From the first order Taylor expansion, we have for any differentiable function $f(\cdot)$: $f(x) = f(a) + f'(a)(x-a) + R_1$, where $R_1 = 0.5 f''(\zeta)(x-a)^2$, for some $\zeta \in [x, a]$. Then:

$$f(Q^{**}) - f'(Q^{**})Q^{**} = -R_1 = \frac{f''(\zeta)}{2}(Q^{**})^2 = \frac{P_A''(\zeta)}{2}(Q^{**})^2.$$

Consequently, $P^{**} - P^* = -P_A''(\zeta)(Q^{**})^2/4$, for some $\zeta \in [0, Q^{**}]$. Therefore, if $P_A(Q)$ is convex, $P_A''(\cdot) \geq 0$ so that $P^{**} \leq P^*$; and if $P_A(Q)$ is concave, $P_A''(\cdot) \leq 0$ so that $P^{**} \geq P^*$.

Proof of Proposition 1

Convex case: We have:

$$\Pi^* = \frac{P_m - c}{2}\Big[\frac{1}{2b_2}\Big(b_1 - \sqrt{b_1^2 - 2b_2(P_m - c)}\Big)\Big],$$
$$P^{**} = P_m - \frac{b_1}{3b_2}\Big[b_1 - \sqrt{b_1^2 - 3b_2(P_m - c)}\Big] + \frac{1}{9b_2}\Big[b_1 - \sqrt{b_1^2 - 3b_2(P_m - c)}\Big]^2,$$
$$Q^{**} = \frac{b_1 - \sqrt{b_1^2 - 3b_2(P_m - c)}}{3b_2}.$$

The optimal profit is $\Pi^{**} = (P^{**} - c)Q^{**}$. One can express the profit and price ratios as functions of c and b_2 and check the monotonicity to conclude that the profit and price ratios are largest when $c = 0$ and $b_2 = b_1^2/4P_m$, in which case $P^{**} = (4/9)P_m$ and $Q^{**} = (2P_m)/(3b_1)$. Finally, we can now compute both profits:

$$\Pi^* = \frac{b_1 P_m}{4b_2}\Big(1 - \frac{1}{\sqrt{2}}\Big) = \frac{P_m^2}{b_1}\Big(1 - \frac{1}{\sqrt{2}}\Big), \quad \Pi^{**} = \frac{2b_1 P_m}{27b_2} = \frac{8P_m^2}{27b_1}.$$

Then the profit and price ratios are: $\Pi^{**}/\Pi^* = 8\sqrt{2}/\big[27(\sqrt{2}-1)\big] = 1.0116, \quad P^{**}/P^* = 8/9$. These are the largest values for the ratios, so that the corresponding inequalities hold.

Concave case: The optimal Q^{**} is obtained by equating marginal revenue to marginal cost:

$$Q^{**} = \frac{-b_1 \pm \sqrt{b_1^2 - 3b_2(P_m - c)}}{-3b_2}.$$

Since $Q^{**} > 0$, the positive root applies. Then the optimal price is given by:

$$P^{**} = P_A(Q^{**}) = \frac{2P_m + c}{3} - \frac{b_1^2}{9b_2} + \frac{1}{9b_2} b_1 \sqrt{b_1^2 - 3b_2(P_m - c)}.$$

Finally, the optimal profit as a function of P_m, c, b_1, and b_2 follows from $\Pi^{**} = (P^{**} - c)Q^{**}$. Our pricing rule is $P^* = (P_m + c)/2$, so

$$Q_A(P^*) = \frac{-b_1 \pm \sqrt{b_1^2 - 2b_2(P_m - c)}}{-2b_2}.$$

We select the positive root so as to satisfy $Q^* > 0$. The profit is then:

$$\Pi^* = (P^* - c)Q_A(P^*) = \frac{P_m - c}{2}\Big[\frac{1}{-2b_2}\Big(\sqrt{b_1^2 - 2b_2(P_m - c)} - b_1\Big)\Big].$$

Expressing the profit and price ratios as functions of b_1 and checking the monotonicity, one can see that the worst case for both ratios occurs when $b_1 = 0$. Intuitively, the larger b_1 is, the more linear the function is, making the ratios closer to 1. If $b_1 = 0$, $P^{**} = (2P_m + c)/3$ and $P^* = (P_m + c)/2$, so

$$\Pi^{**} = \frac{2(P_m - c)}{3}\frac{\sqrt{-3b_2(P_m - c)}}{-3b_2}, \quad \Pi^* = \frac{P_m - c}{2}\frac{\sqrt{-2b_2(P_m - c)}}{-2b_2}.$$

Then, the profit and price ratios are:

$$\frac{\Pi^{**}}{\Pi^*} = \frac{4\sqrt{2}}{3\sqrt{3}} = 1.0887, \quad \frac{P^{**}}{P^*} = \frac{2}{3}\frac{2P_m + c}{P_m + c} \le \frac{4}{3} = 1.33.$$

For $b_1 > 0$, we have inequalities for both ratios.

Proof of Proposition 2

Equating marginal revenue and marginal cost, $MR_A(Q^{**}) = P_m - (n+1)\gamma(Q^{**})^n = c$. Thus: $Q^{**} = [(P_m - c)/(n+1)\gamma]^{1/n}$ and $P^{**} = P_A(Q^{**}) = (nP_m - c)/(n+1)$. Note that P^{**} is independent of γ. Next, the optimal profit is:

$$\Pi^{**} = (P^{**} - c)Q^{**} = \frac{n}{(n+1)^{\frac{1}{n}+1}\gamma^{1/n}}(P_m - c)^{\frac{1}{n}+1}.$$

Recall that $P^* = (P_m + c)/2$, so the corresponding quantity is $Q_A(P^*) = \left[(P_m - c)/(2\gamma)\right]^{1/n}$. Therefore, $\Pi^* = (P^* - c)Q_A(P^*) = \left[(P_m - c)^{\frac{1}{n}+1}\right]/\left[2^{\frac{1}{n}+1}\gamma^{1/n}\right]$. We can now compute both ratios:

$$\frac{\Pi^{**}}{\Pi^*} = \frac{2^{\frac{1}{n}+1}n}{(n+1)^{\frac{1}{n}+1}} \leq 2, \quad 1 \leq \frac{P^{**}}{P^*} = \frac{2(nP_m + c)}{(n+1)(P_m + c)} \leq 2.$$

Proof of Proposition 3

First, suppose $c = 0$. Equating marginal revenue and marginal cost, $MR_A(Q^{**}) = P_m e^{-\alpha Q^{**}} - \alpha P_m Q^{**} e^{-\alpha Q^{**}} = 0$, so $Q^{**} = 1/\alpha$. Then $P^{**} = P_m e^{-1}$ and $\Pi^{**} = P_m e^{-1}\alpha^{-1}$. If the firm prices at P^*, the profit is $\Pi^* = (P^* - c)Q_A(P^*) = 0.5P_m Q_A(P^*)$. Since $c = 0$ and $P^* = 0.5P_m$, we obtain: $Q_A(P^*) = -(1/\alpha)\log(0.5)$, and hence $\Pi^* = 0.5P_m \log(2)/\alpha$. We then have:

$$\frac{\Pi^{**}}{\Pi^*} = \frac{P_m e^{-1} 2\alpha}{\alpha P_m \log(2)} = \frac{2e^{-1}}{\log(2)} = 1.0615, \quad \frac{P^{**}}{P^*} = \frac{P_m e^{-1}}{P_m/2} = 2e^{-1} = 0.7357.$$

We now show that when $c > 0$, both ratios are closer to 1. We start with the price ratio by showing that $\frac{\partial}{\partial c}[\frac{P^{**}}{P^*}] \geq 0, \ \forall \ 0 \leq c \leq P_m$. We have:

$$\frac{\partial}{\partial c}\left[\frac{P^{**}}{P^*}\right] = \frac{\frac{\partial P^{**}}{\partial c}P^* - \frac{\partial P^*}{\partial c}P^{**}}{(P^*)^2}. \tag{11}$$

For eqn. (11) to be nonnegative, we need: $\frac{\partial P^{**}}{\partial c}\frac{1}{P^{**}} \geq \frac{\partial P^*}{\partial c}\frac{1}{P^*}$. Recall that $P^* = (P_m + c)/2$ and therefore: $\partial P^*/\partial c = 0.5$. As a result, we need to show:

$$\frac{\partial P^{**}}{\partial c} \geq \frac{P^{**}}{P_m + c}. \tag{12}$$

From the first order condition: $MR_A(Q^{**}) = P_m e^{-\alpha Q^{**}} - \alpha P_m Q^{**} e^{-\alpha Q^{**}} = P^{**}(1 - \alpha Q^{**}) = c$. By differentiating both sides with respect to c and isolating $\partial P^{**}/\partial c$:

$$\frac{\partial P^{**}}{\partial c} = \frac{1 + \alpha P^{**}\frac{\partial Q^{**}}{\partial c}}{1 - \alpha Q^{**}}. \tag{13}$$

Recall that $P^{**} = P_m e^{-\alpha Q^{**}}$ and hence by differentiating with respect to c:

$$\frac{\partial P^{**}}{\partial c} = -\alpha P^{**}\frac{\partial Q^{**}}{\partial c}. \tag{14}$$

By combining (13) and (14), we obtain $\partial P^{**}/\partial c = 1/(2 - \alpha Q^{**})$. Since the demand curve is convex, from Theorem 1: $P^{**} \leq P^* = (P_m + c)/2$ and therefore: $P^{**}/(P_m + c) \leq 0.5$. From the first order condition, $0 \leq 1 - \alpha Q^{**} \leq 1$ (so that $P^{**} \geq c$). Thus $1 \leq 2 - \alpha Q^{**} \leq 2$, so $1/(2 - \alpha Q^{**}) \geq 0.5$, implying that (12) is satisfied. This concludes the proof for the price ratio.

The same logic applies to the profit ratio, i.e., $\partial[\frac{\Pi^{**}}{\Pi^*}]/\partial c \leq 0, \ \forall \ 0 \leq c \leq P_m$.

Proof of Proposition 4

Equating marginal revenue to marginal cost, $MR_A(Q^{**}) = P_m\Big(1-\frac{1}{\beta}\Big)(Q^{**}/Q_0)^{-1/\beta} = c$. Thus: $Q^{**} = Q_0\Big[\frac{\beta c}{(\beta-1)P_m}\Big]^{-\beta}$. Note that Q^{**} is larger than the truncation value Q_0. The optimal price and profit are: $P^{**} = \beta c/(\beta-1)$ and $\Pi^{**} = Q_0 c/(\beta-1)\Big[\frac{\beta c}{(\beta-1)P_m}\Big]^{-\beta}$. By requiring $\beta \geq P_m/(P_m-c)$ we ensure that $P^{**} \leq P_m$. We next compute the profit under P^*: $\Pi^* = (P^*-c)Q_A(P^*) = 0.5(P_m - c)Q_A(P^*)$. We have: $Q_A(P^*) = Q_0\Big(\frac{P_m+c}{2P_m}\Big)^{-\beta} \geq Q_0$. Then: $\Pi^* = 0.5Q_0(P_m-c)\Big(\frac{P_m+c}{2P_m}\Big)^{-\beta}$. We can now compute both ratios:

$$\frac{\Pi^{**}}{\Pi^*} = \frac{2}{(P_m/c-1)(\beta-1)}\Big(\frac{2\beta}{(P_m/c+1)(\beta-1)}\Big)^{-\beta}, \quad \frac{P^{**}}{P^*} = \frac{2\beta}{(P_m/c+1)(\beta-1)}.$$

Expressions for Section 3

Here are the closed-form expressions of Π^{**}/Π^* as a function of ϵ for the demand models we considered when $c=0$. (Setting $\epsilon=0$ yields the expressions in Section 2.)

- **Linear:** $P_A(Q) = P_m - bQ \quad \Pi^{**}/\Pi^*(\epsilon) = 1/(1-\epsilon^2)$
- **Quadratic convex:** $P_A(Q) = P_m - b_1Q + b_2Q^2; \quad b_1, b_2 \geq 0$ and $b_2 \leq b_1^2/4P_m$

$$\frac{\Pi^{**}}{\Pi^*}(\epsilon) \leq \frac{8\sqrt{2}}{27(1+\epsilon)}\frac{1}{\sqrt{2}-\sqrt{1+\epsilon}}$$

- **Quadratic concave:** $P_A(Q) = P_m - b_1Q + b_2Q^2; \quad b_1 \geq 0$ and $b_2 \leq 0$

$$\frac{\Pi^{**}}{\Pi^*}(\epsilon) \leq \frac{4\sqrt{2}}{3\sqrt{3}}\frac{1}{(1+\epsilon)\sqrt{(1-\epsilon)}}$$

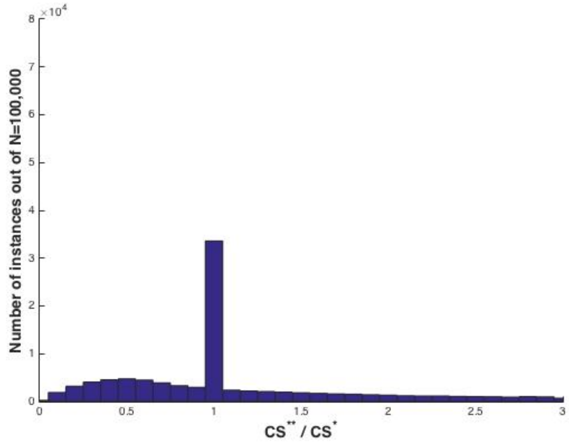
- **Monomial:** $P_A(Q) = P_m - \gamma Q^n \quad \Pi^{**}/\Pi^*(\epsilon) = \frac{2^{\frac{1}{n}+1}n}{(n+1)^{\frac{1}{n}+1}}\frac{1}{(1+\epsilon)(1-\epsilon)^{1/n}}$
- **Semi-log:** $P_A(Q) = P_m e^{-\alpha Q} \quad \Pi^{**}/\Pi^*(\epsilon) = \frac{2e^{-1}}{(1+\epsilon)\log(\frac{2}{1+\epsilon})}$
- **Log-log (truncated):** $P_A(Q) = \begin{cases} P_m\,; & \text{if } Q < Q_0 \\ P_m(Q/Q_0)^{-1/\beta}\,; & \text{if } Q \geq Q_0 \end{cases}$

$$\frac{\Pi^{**}}{\Pi^*}(\epsilon) = \frac{2}{\Big[\frac{P_m}{c}(1+\epsilon)-1\Big](\beta-1)}\left[\frac{2\beta}{(\beta-1)\Big[\frac{P_m}{c}(1+\epsilon)+1\Big]}\right]^{-\beta}$$

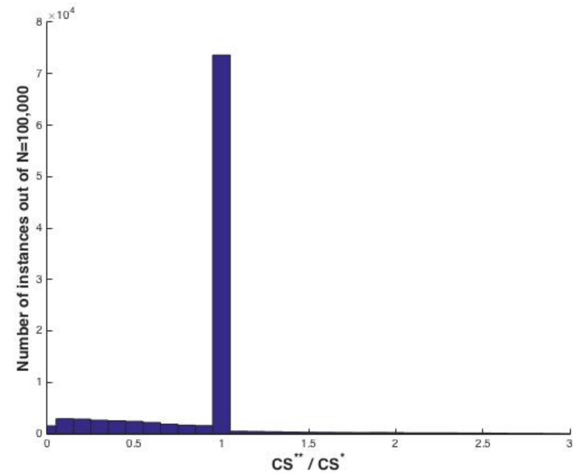
Proof of Proposition 5

Consider any non-increasing concave demand curve. We know from Theorem 1 that $P^* \leq P^{**}$. Recall that $P^* = 0.5(P_m+c)$ and therefore, $P^* \leq P^{**} \leq P_m = 2P^* - c \leq 2P^*$. We next show the inequality for the profit: $\Pi^{**} = (P^{**}-c)Q_A(P^{**}) \leq 2(P^*-c)Q_A(P^{**}) \leq 2(P^*-c)Q_A(P^*) = 2\Pi^*$, where the last inequality follows form the fact that $Q_A(\cdot)$ is non-increasing. In conclusion, we have $1 \leq \Pi^{**}/\Pi^* \leq 2$ and $1 \leq P^{**}/P^* \leq 2$.

Histograms for the consumer surplus



(a) $c = 0$



(b) $c = 0.5P_m$

Figure 7 **Histogram of consumer surplus ratios when** $S = 5$ **for** $c = 0$ **and** $c = 0.5P_m$