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Coalitional Game Theory Based Local Power Exchange Algorithm for Networked Microgrids

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HIGHLIGHTS

- An auction-theory-based method is proposed for calculating the total utility of every valid microgrid coalition.
 - Coalition merging and splitting techniques are introduced to find the optimal partition among all possible coalition combinations.
 - The economy benefits of networked microgrids cooperation are simulated and analyzed.
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ABSTRACT

The future distribution network may encompass a large number of microgrids, in which case local networked microgrids can be formed and all the microgrids in the network be connected to the main grid through a distribution sub-station. To improve the efficiency of the entire network rather than focusing on improving the efficiency and reliability of each microgrid, this paper proposes a coalitional-game-theory-based local power exchange algorithm to identify incentives for coalitional operation and help microgrids in the network trade power locally with neighboring microgrids, so as to meet their own power requirements while achieving higher expected individual utility. Compared with the traditional operation situation, simulation results show that the proposed coalitional-game-theory-based local power exchange algorithm can help increase individual microgrid utility in the network. When there are 30 microgrids in the network, for example, each microgrid is expected to have a 16% increment of individual utility on average.

1. Introduction

Renewable energy applications will be a feasible solution for energy scarcity and environmental problems as a result of the rapid advancement of global economic growth [1]. However, the distribution characteristics of renewable energy make it difficult to integrate effectively into the traditional power grid. As power networks increase in scale, the drawbacks of the conventional grid, such as high cost and difficult operation, will become more apparent, and it will no longer meet increasing safety, reliability, and diversity needs [2,3].

In order to integrate renewable energy sources effectively and improve the quality of electric power on the demand side, microgrid (MG), which can be defined as a group of interconnected loads and distributed energy resources within clearly defined electrical boundaries that acts as a single controllable entity with respect to the grid, have been studied extensively. Generally, there are two operation modes for a MG: In grid-connected mode, MG exchanges power with the main grid through a distribution sub-station (DS) to reduce the variability of renewable energy and load. In isolated mode, the MG is disconnected from the main grid and works autonomously, balancing its own generation and load [4,5].

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Nomenclature			
k	Index for members in coalition C	$P_{ij, L}$	Line losses for <i>seller</i> MG j to meet the power demand of <i>buyer</i> MG i
i	<i>Buyer</i> MG	l_j	Lowest acceptable bidding price per unit power for <i>seller</i> MG j
j	<i>Seller</i> MG	h_i	The highest price per unit power that <i>buyer</i> MG i is willing to bid in trading with <i>seller</i> MG j
φ_k	Allocated coalition utility for k	h^F	Highest bid price
d_i	Demand power of <i>buyer</i> MG i	h^S	Second highest bid price
e_j	Excess power of <i>seller</i> MG j	R_{ij}	Resistance between <i>seller</i> MG j and <i>buyer</i> MG i
$P_{oi, S}$	Supplied power from DS to meet the power demand d_i of <i>buyer</i> MG i	R_{oj}	Resistance between <i>seller</i> MG j and DS
$P_{oj, R}$	Received power for DS if <i>seller</i> MG j sells e_j	R_{oi}	Resistance between <i>buyer</i> MG i and DS
$P_{ij, S}$	Supplied power from <i>seller</i> MG j to meet the power demand of <i>buyer</i> MG i	β	Fraction of loss in transformer at DS
$P_{ij, Smax}$	Amount of supplied power from <i>seller</i> MG j to <i>buyer</i> MG i	U_D	Voltage level at DS
$P_{ij, Rmax}$	Received power for <i>buyer</i> MG i if $P_{ij, Smax}$ is supplied from <i>seller</i> MG j	U_M	Voltage level at MG
$P_{oi, Smax}$	Power supplied from DS to make <i>buyer</i> MG i receive $P_{ij, Rmax}$	ω_g	Unit power price at DS
$P_{oj, Rmax}$	Received power for DS if <i>seller</i> MG j sells $P_{ij, Smax}$	$ C $	Number of MGs in coalition C
$P_{oi, L}$	Line losses for supplying $P_{oi, S}$ from DS to <i>buyer</i> MG i	$ C_b $	Number of MGs in coalition C_b
$P_{oj, L}$	Line losses for <i>seller</i> MG j to sell e_j to DS	$\mathcal{D}$	<i>Buyer</i> MG set in a coalition
		$\mathcal{S}$	<i>Seller</i> MG set in a coalition
		$\mathcal{N}$	Set of MGs with N members
		C	Coalition
		C_b	Sub-sets of a coalition C
		ζ	A random order of the members in $\mathcal{S}$
		Λ	Set of all random orders for the members in $\mathcal{S}$
		α	Line construction range

In traditional power system, loads are serviced by the distribution grid: The required power is delivered from the main grid through distribution lines. The addition of MGs helps relieve the burden on the main grid and improves reliability in certain areas. Even though large-scale renewable energy can be integrated into electric power systems, the intermittent nature of wind and solar power poses new challenges to the operation and control of autonomous MGs, especially with high penetration levels [6,7].

Another challenge is that the relatively small power scale of MGs makes it difficult to accurately forecast future net output power. These two problems mean that at times MGs will need extra power from the main grid in grid-connected mode and establish a need for certain reserves on the main grid side to guarantee reliability. To address these problems, researchers have proposed many ideas to improve reliability of operation and achieve more accurate renewable energy and load forecasting for MGs in traditional operation situations [8-10].

Given the fact that future distribution networks may encompass a large number of local MGs, instead of trying to solve the above mentioned problems from the perspective of a single MG in traditional operation situation, authors in [11] proposed coalitional-game-based networked MG local power flow exchange scheduling, which facilitates MGs' cooperation from a new perspective. It is beneficial for the MGs with a power deficit to exchange power locally with MGs with an

excess of power, instead of getting the power from the main electricity grid [12,13].

The advantages of such a local exchange for line losses and reserve saving are fourfold: 1) Because the distance between neighboring MGs is usually much shorter than between MGs and the specific DS, line losses for the local power exchange between MGs are usually much smaller. 2) Power losses on main grid transmission lines are also reduced because MGs in cooperation need less power from the main grid. 3) Power losses at the DS transformer level can be reduced. Finally, 4) coalitional MGs can behave as an entity, thus increasing the power scale and achieving larger geographical coverage, which in turn helps improve the forecasting accuracy of integrated renewable energy and the serviced load of those MGs. As a result, the reserve needed from the main grid to ensure reliability can be reduced, which can in turn reduce the corresponding costs on the main grid side. Considering these benefits, an algorithm that can model and promote local power exchanges between *buyer* MGs (those in need of power) and *seller* MGs (those with surplus power) is worth studying.

Based on [11], more researchers have proposed algorithms to facilitate cooperative local power exchange for networked MGs [14-17]. However, there are three main problems with those proposed algorithms. The first is that geographically closer MGs are usually designed to exchange power first. But in many real cases, this may not yield optimal individual utility,

since in some cases trading with more distant MGs may save even more power, thus increasing the expected individual utility. Second, different *seller* MG and *buyer* MG trading pairs are usually designed to have the same trading price when exchange their power and calculate coalition utility. However, coalition utility calculation based on global optimization that assumes the same price for all trading and treat the minimum total line losses as the objective may result in lower individual utility for some MGs, which would then lose their incentives to participate the power flow scheme. This is because all the units in those MGs are owned by different entities. If cooperation is possible, each MG will definitely prefer to trade with the MGs that can help generate the highest expected individual utility. Third, previous algorithms assume electric line has to be built between any two MGs. However, such complex networked structure will influence the safe operation of MGs and also has low economic benefits.

This paper proposes a coalitional-game-based local power exchange algorithm for networked MGs, created in three steps: 1) an auction-theory-based method is firstly proposed for calculating the maximum utility of every valid coalition and deciding the corresponding trading order. 2) The Shapley value method is applied to fairly allocate the utility of every coalition among its members. 3) Coalition merging and splitting techniques are introduced to find the optimal partition among all possible coalition combinations.

Compared with the existing algorithms for networked MGs cooperation, the merits of the proposed algorithm are threefold: 1) by adopting auction theory, MG's incentive can be reflected intuitively. 2) Reasonable electric lines connection between MGs are considered in this work, such that the time efficiency of the algorithm can be improved greatly and more cost can be saved. Finally, 3) with coalition formation techniques and based on the principle of incentive and fairness, stable partition of the networked MGs can be guaranteed.

The rest of this paper is organized as follows. Section 2 describes the structure of local networked MGs and the traditional MG operation situation. Section 3 introduces the auction-theory-based method for calculating the utility of every valid coalition. Section 4 focuses on the Shapley value method for allocating total coalition utility among its members and the method for selecting the optimal partition of coalitions. The results of the simulation test are presented in Section 5, and a conclusion in Section 6.

2. Networked MGs model

This section explains the structure of local networked MGs and traditional MG operation situation. Consider the scenario of a distribution network in which MGs are connected to a DS that is connected to the main grid through distribution lines. The MGs can have many kinds of main power sources (e.g., solar farms and wind farms) in different power scales, as shown in Fig. 1 below.

Fig.1. Traditional MGs system.

In any given period of time, each MG in the local network can generate a certain amount of power by its micro-sources and needs a certain amount of power to meet its demand side power consumption. If the net output power of MG j is greater than zero, $e_j > 0$, it is defined as a *seller* MG. Similarly, if the net output power of MG i is less than zero, $d_i < 0$, it is defined as a *buyer* MG. Traditionally, the unbalanced power in MGs is evened out by the main grid in grid-connected mode, given the fact that not enough energy storage is available [18,19]. In other words, *seller* MGs will sell their excess power to the main grid through a DS, and *buyer* MGs will buy the demand power from the main grid, also through the DS. In this traditional operation situation, line losses can be calculated as (1) and (2) below for *seller* and *buyer* MG, respectively [20]:

$$P_{oj,L} = \frac{e_j^2 R_{oj}}{U_D^2} + \beta e_j \quad (1)$$

$$P_{oi,L} = \frac{P_{oi,S}^2 R_{oi}}{U_D^2} + \beta P_{oi,S} \quad (2)$$

where $P_{oi,S}$ can be solved with the equation below:

$$P_{oi,S} = \frac{P_{oi,S}^2 R_{oi}}{U_D^2} + \beta P_{oi,S} - d_i \quad (3)$$

For a power exchange between MGs to fulfill the power requirement of *buyer* MG i , the losses can be calculated as (4) and (5):

$$P_{ij,S} = \frac{P_{ij,S}^2 R_{ij}}{U_M^2} - d_i \quad (4)$$

$$P_{ij,L} = \frac{P_{ij,S}^2 R_{ij}}{U_M^2} \quad (5)$$

From the formulas above, it can be inferred that networked MGs have an incentive to cooperate in local power exchanges

with neighboring MGs. This is because in a local power exchange, β will be zero, as there is no loss in the transformer at the DS, and also the transmission distance is shorter. When MGs in a network choose to cooperate in order to get higher expected individual utility, they can coordinate their strategies to behave as coalitions as if there is a network virtual operator where they can share current power state information as Fig. 2.

A coalitional game with *transferable utility* can be denoted as $(\mathcal{N}, v)$, where the utility function $v(C)$ describes how much total utility the coalition C can achieve if some players in $\mathcal{N}$ decide to cooperate and form coalition C . To calculate the expected individual utility and facilitate this cooperation, a three-step algorithm needs to be executed for the virtual network operator: 1) Calculate the value of the utility function $v(C)$ for every valid coalition C . 2) Fairly allocate the calculated coalition utility among coalition members based on their contributions to coalition utility and coalition formation. And 3), according to the allocated individual utility for each valid coalition, determine the optimal coalition partition for the network. These steps will be explained in the following sections.

Fig.2. Illustration of local networked MGs system with virtual operator.

3. Coalition utility calculation

This section explains the auction-theory-based coalition utility calculation method. Firstly, we introduce the parameter α , which is the construction line range for the studied problem. Only the distance between two MGs is less than α , those MGs can be connected with electric lines and are able to trade power. With the restriction from α , total line length between all MGs will be a reasonable number and unrealistic coalitions can be avoided which could also help improve algorithm efficiency.

Given any valid coalition C that says there is at least one *buyer* MG and one *seller* MG in C and all members in C have electric lines connected between them, divide the coalition members into *buyer* MG set $\mathcal{D}$ and *seller* MG set $\mathcal{S}$. In the following short period of time, any MG $i \in \mathcal{D}$ is in need of power and is willing to buy power from other sources. Similarly,

any MG $j \in \mathcal{S}$ has excess power to sell. Given the fact that each different power trading pair $\{i, j\}$ has a different trading cost depending on the distance between them, for every possible coalition a fair and efficient method is needed to match the members in $\mathcal{S}$ and $\mathcal{D}$ so as to achieve efficient local power exchange with high coalition utility.

The matching problem was first introduced by Gale and Shapley [21] in 1962. After them, different matching models have been proposed and applied in many areas, such as biological computation [22,23], economics [24], computer vision [25], and climate [26,27]. However, to authors' knowledge, none of the famous existing matching models [28-30] are suitable for this networked MG coalitional local power exchange situation. The reasons are twofold: 1) Most existing models assume the traded goods are indivisible, which is not the case for electric power, and 2) as shown in formulas (1)-(3), there would be power losses in the process of trading that depend on the amount of traded power, thus the unit price of electric power will change for each different traded quantity. Actually, power trading among those MGs in the situation described is quite different from the power market of the transmission grid. In the transmission grid, line loss is not a key factor when calculating the clearing price in the market pool after power sellers and buyers submit their bids [31]. However, for networked MGs at the distribution level, line loss reduction should be considered for cost calculation.

The typical matching method for this local power exchange setting is proposed in [11]: For every possible order of the *buyer* MGs in $\mathcal{D}$ in any given valid coalition, starting from the first *buyer* MG, the method ranks *seller* MGs in $\mathcal{S}$ based on the distance to the *buyer* MG, and trades according to this order. If any *buyer* MG (or *seller* MG) buys (or sells) all of its demand (or excess), it will be deleted from $\mathcal{D}$ (or $\mathcal{S}$) and the next one in line will have the opportunity to trade, in order, based on the new distance. Every trade in this method is designed to have the same unit power price. The buyer order and corresponding trading process that can yield maximum coalition utility will be selected. Other coalitional operation methods mentioned in Section 1 also assume the rules that the closest *buyer* and *seller* MGs should trade first, and the trading price should be the same within the network.

However, by assuming trade priority according to the shortest distance, MGs farther from other MGs usually will not have opportunity to trade cooperatively. This may not be the case in reality when considering their incentives, as those *buyer* (or *seller*) MGs far from other *seller* (or *buyer*) MGs may even prefer to pay more as compared with closer *buyer* (or *seller*) MGs so as to avoid trading with the main grid through a DS, simply because trading with the main grid in this situation might generate even higher costs. On the other hand, closer MGs might not lose too much utility while trading with the main grid. It is also unrealistic to assume that every MG in the network will have the same unit price for trading power, as all the units in those local MGs in the network are owned by different entities, and every entity pursues maximum utility. Therefore, auction theory is more suitable for this multi-agent problem than optimization in reflecting their incentives.

Another problem with these existing algorithms is that in any coalition, *buyer* (or *seller*) MGs are restricted to avoid trading with the main grid until no *seller* (or *buyer*) MGs are available. This may not be a good way to calculate coalition utility, because in some cases, trading with the main grid can save more losses if the MG is close enough. Considering all these facts, a second-price sealed-bid auction based algorithm for matching *seller* MGs and *buyer* MGs and calculating utility in a given coalition is proposed below. The trading pairs will be determined according to the bids that members in $\mathcal{D}$ are willing to provide to members in $\mathcal{S}$ and also the lowest bids that members in $\mathcal{S}$ are willing to accept.

Given any *seller* MG j and *buyer* MG i , the corresponding power $P_{ij,S}$ supplied from MG j so that MG i can receive d_i to meet its power requirement can be calculated according to formula (4). The amount of power that MG j is able to supply to MG i is $P_{ij,Smax} = \min\{P_{ij,S}, e_j\}$. Substituting $P_{ij,Smax}$ in equation (5), the maximum received power $P_{ij,Rmax}$ for MG i from MG j can be calculated as the difference between $P_{ij,Smax}$ and corresponding calculated line losses $P_{ij,L}$ during the power transmission. The highest price h_i per unit power that MG i is willing to bid in trading with MG j in order to receive corresponding $P_{ij,Rmax}$ can be calculated as shown in (6), which means the highest total cost for trading with MG j should be the same with the total cost for trading with DS for it to receive the same amount of power $P_{ij,Rmax}$. If the trading price is higher than h_i , MG i would rather trade with the main grid.

$$h_i = P_{oi,Smax} \cdot \omega_g / P_{ij,Smax} \quad (6)$$

In general, for any *seller* MG j , every *buyer* MG i in the same coalition can have a price-quantity bid $\{h_i, P_{ij,Smax}\}$. Similarly, the lowest price l_j that MG j is willing to accept from any *buyer* MG is calculated as follows:

$$l_j = P_{oj,R} \cdot \omega_g / e_j \quad (7)$$

In this case, the DS acts as a *buyer* MG with the price-quantity bid $\{l_j, e_j\}$ and MG j will be prefer to trade with DS if no *buyer* MG can offer higher bid.

In order to get trading pairs within the members of $\mathcal{D}$ and $\mathcal{S}$, the many-to-many matching situation is simplified to this many-to-one second-price sealed-bid auction scenario, given $\mathcal{D}$ and a *seller* MG j from $\mathcal{S}$. In this scenario, the *buyer* MG with the highest bid price h^F will have the opportunity to trade with MG j at the second highest bid price h^S if there are at least two bid prices higher than the corresponding lowest acceptable price l_j . If only one buyer has a bid higher than the lowest acceptable price, the clearing price would be the average of them. When there is no bid higher than l_j , no trading occurs between MGs, because the *seller* MG will sell all its excess power to the DS [32,33]. It is proven that every bidder in this second-price sealed-bid auction situation has the incentive to submit a real price-quantity bid [34,35]. Given a random order of the members in $\mathcal{S}$, $\zeta \in \Lambda$, the many-to-one auction will be repeated according to the order until no sellers or buyers are available to trade. Then the coalition utility for this given order of the members in $\mathcal{S}$ can be calculated as shown in (8) below,

which sums up all the individual incremental utility in every trading pair for trading the same amount of power, from traditional to coalitional operation.

$$\sigma(C, \zeta) = \sum_{i \in \mathcal{D}, j \in \mathcal{S}} \omega_g (P_{oi,Smax} - P_{oj,Rmax}) \quad (8)$$

Every possible member order in Λ will be tried and the corresponding utility as (8) is calculated. The order that can generate the maximum utility will be selected as the final order for $\mathcal{S}$ and the corresponding utility will be treated as the final coalition utility.

$$v(C) = \max_{\zeta \in \Lambda} \sigma(C, \zeta) \quad (9)$$

By applying this repeated second-price sealed-bid auction method, trading orders and amounts between *buyer* and *seller* MGs in a coalition can be determined based on their own incentives for trading cooperatively, and all the generated coalition utility values for different member orders in $\mathcal{S}$ will be used in calculating utility allocation by Shapley value (covered in the next section). Algorithm 1 below gives a detailed pseudo code for this coalition utility calculation method.

Algorithm 1. Pseudo code of auction theory based coalition utility calculation algorithm

1. **initialization**
2. Given a coalition C , divide C into $\mathcal{S}$ and $\mathcal{D}$.
3. **repeat**
4. Select a random order of members in $\mathcal{S}$, $\zeta \in \Lambda$.
5. **repeat**
6. (a) Denote the first member in current order of $\mathcal{S}$ as j .
7. (b) Calculate the price-quantity bid for every member in current $\mathcal{D}$ and the lowest acceptable price l_j for MG j .
8. (c) Denote the member in $\mathcal{D}$ with the highest bid as i .
9. **case 1:** At least two buyer MGs have bid price higher than l_j . Then MG i trades with MG j at h^S .
10. **if** $P_{ij,Smax} = e_j$
11. $\mathcal{S} = \mathcal{S} \setminus j$, $e_j = 0$, and $d_i = d_i - P_{ij,Rmax}$
12. **else**
13. $\mathcal{D} = \mathcal{D} \setminus i$, $d_i = 0$, and $e_j = e_j - P_{ij,Smax}$
14. **case 2:** One buyer MG has bid price higher than l_j . MG i trades with MG j at $(h^F + l_j)/2$.
15. **if** $P_{ij,Smax} = e_j$
16. $\mathcal{S} = \mathcal{S} \setminus j$, $e_j = 0$, and $d_i = d_i - P_{ij,Rmax}$
17. **else**
18. $\mathcal{D} = \mathcal{D} \setminus i$, $d_i = 0$, and $e_j = e_j - P_{ij,Smax}$
19. **case 3:** No buyer MGs have a bid price higher than l_j for MG j , $\mathcal{S} = \mathcal{S} \setminus j$.
20. **until** $\mathcal{S}$ or $\mathcal{D}$ is empty or no more trades can be made, calculate the coalition utility by (8)
21. **until** all possible orders in Λ have been tried
22. **output** coalition utility $v(C)$

4. Coalition utility allocation and optimal coalition partition

In this section, we will first apply the *Shapley value* method to allocate the coalition utility generated in Section 3 for all valid coalitions. In the second part, a method for selecting the optimal partition among all possible coalition combinations will be introduced.

4.1. Coalition utility allocation based on Shapley value

After the utility of each valid coalition is calculated, a way to allocate the coalition utility among its members is needed. The *Shapley value* method is adopted here for two reasons: 1) it emphasizes on *fairness*, that is, the MGs that contributing more coalition utility should be allocated more individual payoffs. 2) It can reflect the importance of certain MGs to the formation of the coalition. Without certain MGs, many coalitions are hard to form. The MGs that can help the formation of coalitions should be allocated more payoffs. The *Shapley value* method can consider the contribution to coalition formation by calculating the final coalition utilities with or without each of the MG in the coalition. The payoff of every member is calculated when it is the first to enter the game, the second, and the third. The average value of coalition incremental payoffs in every permutation order determines the payoff to each member [36]. Therefore, influence from entry order is eliminated in calculating the individual payoff. The Shapley Value is characterized by a collection of desirable properties as follows.

Axiom 1 (Efficiency): Total utility is allocated:

$$\sum_{k \in C} \phi_k^v = v(C) \quad (10)$$

Axiom 2 (Symmetry): If i and j are two members that are equivalent such that

$$v(C \cup i) = v(C \cup j) \quad (11)$$

for every C which contains neither i nor j , then

$$\phi_i^v = \phi_j^v \quad (12)$$

Axiom 3 (Linearity): If two coalitions with utilities u and w are combined, then

$$\phi_k^{u+w} = \phi_k^u + \phi_k^w \quad (13)$$

Axiom 4 (Null Player): Player k is *null* if for every coalition it makes no contribution:

$$v(C \cup k) = v(C) \quad (14)$$

The allocated utility for the null player is zero.

Given a coalition C , it is proven that the *Shapley value*, as calculated in (15), is the only map from the set of all games to payoff vectors that satisfies all four axioms above [37].

$$\phi_k^v = \sum_{C_b \subseteq C-k} \frac{|C_b|!(|C|-|C_b|-1)!}{|C|!} [v(C_b \cup k) - v(C_b)] \quad (15)$$

Note that in this networked MGs coalitional game, utility is divided among the members in any formed coalition C , and a merge and split technique will be used later to find a stable coalition partition in the network. Therefore, equation (15), which is usually used to find the utility division for the grand coalition by *Shapley value*, would be used on every valid coalition C in the game.

4.2. Optimal coalition partition of network

After the utility and corresponding allocation is calculated for each valid coalition, there would be many possible coalition combinations for network partition according to the setting. However, many of those coalition combinations are unstable, which means some MGs may want to jump out of their current coalitions and join other coalitions in order to get higher expected individual utility. Considering this fact, a method is needed to find the stable coalition partition of the network among all possible coalition combinations. To start with, the definition of *Pareto order* is given as following.

Definition 1: Given two collections of disjoint coalitions $Y = \{y_1, \dots, y_m\}$ and $Z = \{z_1, \dots, z_n\}$ formed from the same players in a grand coalition, collection Y is preferred over Z by *Pareto order* if and only if

$$Y \triangleright Z \Leftrightarrow \{\phi_k^Y \geq \phi_k^Z \forall k \in Y, Z\} \quad (16)$$

with at least one strict inequality ($>$) for a player k .

According to the definition of Pareto order, a group of MGs in the coalitional game prefers to become part of collection Y instead of Z if at least one member is able to improve its individual utility, without sacrificing the utility of any other members, if the partition of the network is changed from Z to Y . Then the following two operations of *merging* and *splitting* for coalition formation can be applied to find the stable partition of the networked MGs.

Definition 2 (Merging Operation): Merge any set of coalitions $\{C_1, \dots, C_l\}$ if $\{\cup_{j=1}^l C_j\} \triangleright \{C_1, \dots, C_l\}$; therefore, $\{C_1, \dots, C_l\} \rightarrow \{\cup_{j=1}^l C_j\}$

Definition 3 (Splitting Operation): Split any set of coalitions $\cup_{j=1}^l C_j$ where $\{C_1, \dots, C_l\} \triangleright \{\cup_{j=1}^l C_j\}$; therefore, $\{\cup_{j=1}^l C_j\} \rightarrow \{C_1, \dots, C_l\}$

The *merging* process can help small MG coalitions form bigger coalitions if greater utility can be achieved for some MGs among them and no MGs are sacrificed. Similarly, the *splitting* process can help big coalitions divide into smaller coalitions if no MGs are sacrificing their utility after the *splitting* process and some MGs among them can achieve higher expected individual utility. Iteratively applying the operations of *merging* and *splitting*, D_c -stable state, which is defined as a state in which no players have incentives to leave their current coalition and form a new partition for the network, can be achieved given any initial partition of the network [38].

5. Simulation Results and Analysis

In this simulation, the proposed algorithm will be applied to find out how the MGs in a local distribution network will operate cooperatively so as to improve their expected individual utility. Two examples with 12 MGs and 24 MGs each will be illustrated firstly to show the solution process of the proposed algorithm. In the second part of the simulation, analysis of the proposed algorithm from the perspectives of economic, incentive, and efficiency is conducted for different numbers of MGs in the network. Throughout the simulation, MGs are randomly generated in an area of 50 km x 50 km with the DS located at the center of the area. MGs within $\alpha = 14.5$ km are assumed to be connected with electric lines and are capable to trade power. Since renewable energy and load can be forecasted very accurately in minute-level, we assume each generated MG in the network has a certain amount of power to sell or to buy randomly, in the range of 0 MW to 100 MW and keep this output power for five minutes. The proposed algorithm and individual MG utility improvement will be simulated in this time period. The selected simulation parameters can be found in Table 1 below.

Table 1

Parameter selection for simulation.

Parameter	Value
R	0.2 Ω/km
β	0.03

ω_g	1/MW
α	14.5 km
U_D	50 kV
U_M	25 kV

5.1. Illustrations of a 12-MG network and a 24-MG network

Fig. 3 below shows a local distribution network with twelve randomly generated MGs. There are, in total, six *seller* MGs and six *buyer* MGs. The information of position and output power for each MG are listed below the MG number. Available electric lines between MGs are shown as light-blue lines. Assume all MGs are in their traditional operation mode initially. After the proposed algorithm is applied (with running time of 0.01s), a total of four coalitions are finalized, shown by the red dashed circles. Totally 7 *merging* operations in 2 rounds are needed to achieve the stable network partition for this 12-MG network. In round one, four coalitions are formed: coalition I consist of MG 8 and 7, coalition II consist of MG 2 and 10, coalition III consist of MG 5 and 9, and finally, coalition IV consist of MG 4 and 12. It shows that coalitions with the highest averaged individual utilities are formed firstly. In the second round, MG 11 is merged to coalition I, MG 6 is merged to coalition III, and MG 1 is merged to coalition IV. It shows that larger coalition can form if greater utility can be achieved for some MGs among them and no MGs are sacrificed. After this stage, no *merging* or *splitting* operations can be made anymore, the network partition is stable. In this scenario, no MG wants to leave its current coalition and join another coalition. In coalition I, *seller* MG 8 will first meet the extra power requirement of

Fig.3. Stable coalitions of the 12-MG network.

Fig.4. Stable coalitions of the 24-MG network.

buyer MG 7 and then the power requirement of buyer MG 11. The remaining excess power of MG 8 will be sold to the DS. Note that seller MG 8 will trade with buyer MG 7 first, because MG 7 has higher bid price than MG 11. The total utility for coalition I is $v(C_I) = 11.20 \times 10^6$ and the allocated individual utility for MG 7, 8, and 11 based on Shapley value would be $\phi^v = (4.15, 5.85, 1.20) \times 10^6$. In coalition II, buyer MG 2 will firstly buy all the excess power of seller MG 10 and then buy its remaining power requirement from the DS. The total utility for coalition II is $v(C_{II}) = 0.56 \times 10^6$ and the allocated individual utility for the members MG 2 and 10 would be $\phi^v = (0.28, 0.28) \times 10^6$. Similarly, in coalitions III, buyer MG 9 will first buy the excess power of seller MG 5 and MG 6, and then buy its remaining power demand from the DS. The total utility for coalition III is $v(C_{III}) = 11.02 \times 10^6$ and the allocated individual utility for MG 5, 6, and 9 would be $\phi^v = (2.25, 3.00, 5.77) \times 10^6$. In coalition IV, seller MG 12 will first meet the extra power requirements of buyer MG 4 and then the power requirement of buyer MG 1. The remaining excess power of MG 12 will be sold to the DS. Note that even though MG 4 has the longest distance to the DS, it will still trade with MG 12 before MG 1, because it has a higher bid and can thus create more coalition utility. The total coalition utility for coalition IV is $v(C_{IV}) = 12.92 \times 10^6$ and the allocated individual utility for MG 1, 4, and 12 would be $\phi^v = (5.20, 1.03, 6.69) \times 10^6$.

In this local distribution network with twelve randomly generated MGs, eleven MGs participated the proposed cooperation scheme, among them, seven MGs sell or buy all their excess or demand power with other MGs instead of with

the DS. For the power traded between MGs instead of with the DS, there is a 7.35% individual utility increment. Besides, the ratio of total length of the constructed lines between MGs over original total line length that connecting MGs and the DS is 42.5%, which is an acceptable number considering the high value of individual utility increment.

In the second example with 24 MGs in the network as in Fig. 4, as the number of MGs increased, bigger coalition can form. There are totally twelve seller MGs and twelve buyer MGs that are randomly generated. Similarly, the information of position and output power for each MG are listed below the MG number, and electric lines between MGs are plotted as light-blue lines. After the proposed algorithm is applied (with running time of 0.33s), a total of six coalitions are finalized, shown by the red dashed circles. In this scenario, no MG wants to leave its current coalition and join another coalition. Totally 13 merging and splitting operations are needed in 5 rounds to achieve stable network partition. In this local distribution network with 24 randomly generated MGs, 22 MGs participate the cooperation scheme, among them, 16 MGs sell or buy all their excess or demand power with other MGs instead of with the DS. For power traded between MGs instead of with the DS, there is a 9.67% individual utility increment. Besides, the ratio of total length of the constructed lines between MGs over original total line length that connected MGs to the DS is 92.5%, which is an acceptable ratio considering the higher value of individual utility increment.

5.2. Analysis of the proposed algorithm with different number of MGs

In order to show the utility improvement in other cases with a different number of MGs in the network, 3 to 30 MGs were randomly generated with the same simulation parameters and setting as Table 1, and the proposed algorithm was applied plenty of times (1,000,000 for each number of MGs in the network) to get the average individual utility incremental switching from traditional to coalitional operation. Similarly, assume every MG in the network has certain power to sell or to buy randomly in the range of 0 MW to 100 MW in the following 5 minutes. The average individual utility incremental and the ratio of total lines length between MGs and between MGs and the DS are plotted as blue and purple lines, respectively, in Fig. 5. It is obvious that as the number of MGs in the network increases, applying the proposed method based on coalitional game theory can help MGs in the network get higher individual utility. When there are 30 MGs in the network, MGs that participate coalitions are expected to have a 16% increment of individual utility on average. This is mainly because as the number of MGs in the network increases, every MG will have a better chance to cooperate with neighboring MGs and less chance to exchange power with the DS. As a trade-off, MGs within the distance range α will be connected by new constructed lines to form valid coalitions. When there are 30 MGs in the network, the ratio of total line length between MGs over total line length between MGs and the DS in traditional operation situation is 1.28 which is an acceptable number considering the high averaged individual utility increment comparing to traditional operation.

Fig.5. Ratio of average individual incremental utility switching from traditional to coalitional operation and ratio of total line lengths between MGs and between MGs and DS with different numbers of MGs.

The proportion of MGs choose to participate cooperation and average proportion of power traded within MGs with different number s of MGs are plotted in Fig. 6 below. It can be seen in the figure, as the number of MGs increases, MGs are more willing to trade power with other MGs instead of with DS. When there are 30 MGs in the network, about 71% MGs choose

to cooperate with other MGs and almost 87% of their demand or excess power will be traded within local MGs instead of with the DS on average. On the other hand, it can be inferred that as the proportion of power traded within MGs increases with the number of MGs in the local network, reserve on the main grid side can be also greatly reduced.

Fig.6. Proportion of MGs choose to participate cooperation and average proportion of power traded within MGs with different numbers of MGs.

In the last part of simulation, we explored the efficiency of the proposed algorithm by showing the average number of *merging* and *splitting* operations and the algorithm running time with different number of MGs in the network. This part of the simulation was run in an Intel Processor 5Y70 CPU 1.3 GHz environment. All the random trials achieved stable network partition at the end. The average number of *merging* and *splitting* operations and the algorithm running time with increasing MG numbers are plotted as purple and blue lines, respectively, in Fig. 7. When there are 30 MGs in the network, about 18.3 merging-splitting operations are expected on average, and the running time is about 1.06 seconds, which is very efficient for minute-level power flow scheduling.

Fig.7. Average number of merging-splitting operations and running time of the proposed algorithm with different numbers of MGs.

6. Conclusion

In this paper, a coalitional-game-based local power exchange algorithm is proposed that can help networked microgrids trade power locally so as to meet their own power requirements with higher individual utility. An auction-based matching method is firstly proposed for calculating the total utility for every valid coalition, and the Shapley value method is applied to allocate the coalition utility fairly. Finally, a merging-and-splitting technique is included in the algorithm to find the stable and optimal partition of the network among all possible coalition combinations. Compared with the traditional operation situation, simulation results show that the proposed coalitional-game-based local power exchange algorithm can help increase individual utility for the networked microgrids. When there are 30 microgrids in the network, for example, each microgrid is expected to have a 16% increment of individual utility on average. For future work, economic benefits from the reduction of reserve and line losses on transmission side by adopting the proposed method will also be explored, which are expected to be additional increases to overall system utility. Besides, in this research, we focus more on economic benefits for coalitional microgrids in grid-connected mode. While in the islanded operation mode, microgrid need to disconnect from the main grid and thus would be more concerned with reliability issues. As one of our future work, coalitional game for local networked microgrids with objective of high reliability worth studying as well.

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