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Economically motivated adulteration (EMA) is a serious threat to public health. In this paper, we develop a modeling framework to examine farms' strategic adulteration behavior and the resulting EMA risk in farming supply chains. We study both "preemptive EMA," where farms engage in adulteration to decrease the likelihood of producing low-quality output, and "reactive EMA," where adulteration is done to increase the perceived quality of the output. We fully characterize the farms' equilibrium adulteration behavior in both types of EMA and analyze how quality uncertainty, supply chain dispersion, traceability, and testing sensitivity (in detecting adulteration) jointly impact the equilibrium adulteration behavior. We determine when greater supply chain dispersion leads to a higher EMA risk and how this result depends on traceability and testing sensitivity. Furthermore, we caution that investing in quality without also enhancing testing capabilities may inadvertently increase EMA risk. Our results highlight the limitation of only relying on end product inspection to deter EMA. We leverage our analyses to offer tangible insights that can help companies and regulators to more proactively address EMA risk in food products.

Keywords: Economically motivated adulteration, farming supply chains, supply chain dispersion, traceability, testing sensitivity, quality risk, food safety, socially responsible operations

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1. Introduction

Food adulteration is a serious threat to public health and a major concern to most governments in both developed and developing countries. Food adulteration can occur in a broad range of scenarios. Unintentional adulteration often occurs as a result of negligence or incompetence, for example, bacterial contamination due to bad hygiene practices. In some scenarios, food adulteration is intentional, motivated by malicious intent to harm the public food system (e.g., bioterrorism). In many other scenarios, intentional adulteration is driven by economic motives and often referred to as *economically motivated adulteration* (EMA). The U.S. Food and Drug Administration defines EMA as the "fraudulent, intentional substitution or addition of a substance in a product for the purpose of increasing the apparent value of the product or reducing the cost of its production, i.e., for economic gain" (Johnson 2014).

We particularly focus on EMA that causes harm to human health. Over the last several decades there were many publicly-known EMA incidents of food products around the world, and a majority of them originated from developing countries. For example, consumption of melamine-tainted infant formula and milk led to six infant deaths and nearly 300,000 young children severely sickened in China in 2008 (Everstine et al. 2013). Melamine was added to the milk by farmers and collectors to increase the perceived protein content of the milk. In another example, outbreaks of avian flu led to extensive use of antibiotics and other illegal drugs in poultry farming in Asia. In particular, the 2012 KFC "instant chicken scandal" in China revealed that the chickens used by KFC were treated with as many as 18 illegal antibiotics on the farms (Pi et al. 2014). In both of these examples, the source of adulteration was in the upstream parts of the supply chains, specifically farms and collectors.

In this paper, we develop a new modeling framework to examine strategic adulteration behaviors of farms (and/or collectors) and the resulting EMA risk in a farming supply chain. Our models simultaneously capture various major drivers for EMA, including the uncertainty or variability of the quality of the farms' output, the dispersion and traceability of the supply chain, and the testing capabilities present in the supply chain. We address two main research questions: (i) What are the farms' optimal or equilibrium adulteration strategies under different EMA scenarios? (ii) How do the above drivers jointly impact the farms' adulteration behaviors? We validate the models with real cases and field data to ensure that the models are grounded in practice and consistent with empirical evidence. In addition, we analyze a few managerial levers, such as investing in traceability and testing capabilities, that a manufacturer can use to mitigate EMA risk in the supply chain. We leverage the analysis of our models to derive important and unique insights that can be used to help both regulators and commercial entities to better prioritize and address EMA risk more proactively. We next elaborate on the major EMA risk drivers captured in our models.

The first factor is the uncertainty or variability of the quality of a farm's output. Quality uncertainty can result from issues inherent to the production process; e.g., the quality of milk produced from a cow, typically measured by its compositional characteristics such as protein and fat content, depends on the health of the cow. Quality uncertainty can also be the result of external factors; e.g., epidemics like avian flu affect the quality, captured by the health and weight, of chickens raised in a farm. Quality uncertainty can be a major cause of EMA in markets with quality-based pricing, i.e., where farms receive a better selling price if the products appear to have higher quality. We divide food adulteration driven by quality uncertainty into two distinct scenarios. The first scenario is called "preemptive EMA" where adulteration occurs *before* the uncertain quality of the products is realized. The primary goal

of preemptive EMA is to decrease the likelihood of producing low-quality output. For example, farms overuse antibiotics to prevent producing sick or underweight (i.e., low-quality) animals. This is a serious concern in pork, poultry, and seafood farming in various countries including Bangladesh, China, India, and Vietnam (Doyle et al. 2013). The second scenario is called "reactive EMA" where adulteration occurs *after* the uncertain quality of the products is realized. The primary goal of reactive EMA is to increase the perceived quality of low-quality products and create fake high-quality ones. For example, the intentional adulteration of raw milk with melamine emerged due to price pressure for low-protein milk in China (Sharma and Rou 2014). Similarly, farms in India adulterated milk with urea to increase its perceived solids-not-fat (SNF) content and attract higher prices (Tanzina and Shoeb 2016).

Another factor that may contribute to EMA is the dispersion of a farming supply chain. We define supply chain dispersion as the extent to which agricultural products are sourced from a distributed network of farms, each producing a small fraction of the total quantity (Huang et al. 2017b). Dispersed farming supply chains with hundreds or thousands of smallholder farms are prevalent in many developing countries (Narrod et al. 2008, Chen et al. 2014). Supply chain dispersion may increase EMA risk for at least two possible reasons. First, with a dispersed network of farms, it is difficult (if not impossible) to inspect every farm or to trace every unit of supply back to the producing farm. Instead, manufacturers (in the best case) only inspect the aggregated supply after the products from all farms are mixed. Due to limited traceability, even if the manufacturer detects adulteration in the mixed supply, it often cannot identify the problematic farms nor effectively impose penalties to deter the farms from adulterating. In practice, some firms try to improve traceability by storing samples from (some of) the farms before mixing the supply (Flynn and Zhao 2014). We model traceability in light of such practices. Second, smallholder farms rely on the revenue from selling their products to sustain their families. Hence, when they face quality uncertainty and the associated price pressure, they are likely to become aggressive and engage in adulteration to ensure their only means of income.[1]

Adding to the complexity of a dispersed farming supply chain, the manufacturer's testing capability in terms of test sensitivity is another factor affecting EMA risk in food products. We model both perfect and imperfect testing scenarios. Perfect testing corresponds to scenarios where a known adulterant is being tested and accurate methods exist to examine whether the residue amount (if any) exceeds the maximum allowable limit defined by food safety standards. For example, there are highly sensitive methods to detect certain antibiotics in food products (Pikkemaat 2009, Mungroo and Neethirajan

[1] In Appendix O.1.1, we show that farms' aversion to quality uncertainty motivates them to adulterate even more.

2014). However, since the space of possible adulterants is practically unlimited, it is difficult (if at all possible) to develop sensitive tests for every adulterant. We thus model imperfect testing to capture scenarios where the adulterant is less studied or even unknown. In these cases, the ability to detect adulteration often depends on the relative amount of adulterants being used. For example, the presence of a large quantity of adulterants may change the characteristics of the product (e.g., its texture, smell, or color) or may lead to adverse symptoms when people consume the product, signaling adulteration. In fact, the melamine-tainted infant formula scandal broke out precisely because the quantity of melamine added to the milk became so high that many children got ill, thus alerting the authorities.

Related literature and contributions: This paper addresses a timely and crucial topic on food safety and makes important contributions to both research and practice. From a research standpoint, we develop a new modeling framework that realistically captures various major risk drivers of EMA and validate the model predictions with field data. Our work is related to three streams of literatures: supply chain risk management, quality management, and socially responsible operations. The supply chain risk management literature has mainly focused on the topic of supply *disruption* risks, typically modeled as exogenous shocks to the system (e.g., Sheffi 2005, Dada et al. 2007, Van Mieghem 2007, Tomlin 2009, Babich 2010, Wang et al. 2010, Simchi-Levi et al. 2014). A common conclusion in this literature is that dual or multi-sourcing helps to mitigate disruption risks and establish supply chain resilience. We instead focus on *quality* risks that stem from both exogenous uncertainty and endogenous actions within the supply chain. In this regard, we are closely related to works in the quality management and socially responsible operations literatures that examine opportunistic or unethical supplier behavior. We refer readers to Nagurney and Li (2016), Atasu (2016), and Bouchery et al. (2017) for comprehensive reviews on these two streams of literatures. We discuss here a subgroup of papers most relevant to ours. For example, Babich and Tang (2012) analyze deferred payment and inspection mechanisms as tools for a manufacturer to deter product adulteration by its supplier. Cho et al. (2015) study how a brand-name company may use quality and price levers to combat counterfeiters and show that either strategy could be ineffective when facing a deceptive counterfeiter. Mu et al. (2014, 2016) focus on milk supply chains with monopolistic or competing collection intermediaries and examine incentive schemes to induce better milk quality with minimal testing.

Our models differentiate from this group of papers in two key aspects. First, we explicitly model exogenous quality uncertainty and distinguish it from endogenous adulteration decisions, whereas prior works consider adulteration as equivalent to producing low-quality products at a lower cost. Our approach is essential to adequately capture how quality uncertainty combined with quality-based

pricing motivates adulteration. Second, while prior works treat detection of low-quality products as exogenous, we model imperfect testing where the probability of detecting adulteration is endogenously influenced by the farms' adulteration decisions. This endogeneity substantially complicates our analysis because we must consider the strategic interactions among farms and the interdependency of their adulteration decisions under quality uncertainty. In the case of reactive EMA with imperfect testing, we must solve for the farms' equilibrium adulteration strategies in a game of asymmetric information (as the number of realized low-quality units at each farm is the farm's private information).

The socially responsible operations literature studies supplier behaviors that are socially undesirable but do not directly impact the product's functional quality. For example, Wang et al. (2016) studies a regulator's optimal reward and inspection policy to motivate a company to voluntarily disclose a privately-observed, stochastically-occurring environmental hazard. Chen and Lee (2017) analyze how a company can employ contingency payment, supplier certification, and process audit to prevent unethical actions by a supplier. Letizia and Hendrikse (2016) and Orsdemir et al. (2016) examine how horizontal or vertical integration of supply chain parties may enhance responsible supplier practices. Plambeck and Taylor (2016) analyze a "backfiring condition" under which a supplier can effectively evade a buyer's audit and discuss its implications for motivating supplier responsibility. Fang and Cho (2016) and Caro et al. (2018) study joint or shared audits with a collective penalty and analyze when such audits lead to better supplier compliance compared to independent audits. Cho et al. (2018) examine a company's choice of inspection policy and wholesale price to combat a supplier's use of child labor. We differ from this body of research in the following ways. This literature mainly considers settings of a supply chain with one or two suppliers where the action of the supplier(s) may or may not be endogenized.[2] We instead focus on a network of suppliers each simultaneously making strategic decisions on adulteration. In addition, the two-step testing process and imperfect testing scenario in our context further complicate our analysis of strategic interactions among the suppliers, whereas audits of environmental and social issues typically act upon each individual supplier separately.

Due to these distinctive modeling features, our analysis yields new practical insights for addressing EMA risk in food more proactively. For example, we show that supply chain dispersion plays a critical role in affecting EMA risk. We determine when greater supply chain dispersion leads to a higher risk and how this result depends on traceability and testing capabilities. We demonstrate that supply chain

[2] Two recent exceptions are Huang et al. (2017a) and Zhang et al. (2017). The former studies a three-tier supply chain with one player in each tier and only the most upstream player could incur a responsibility risk. The latter examines the problem of curbing conflict minerals in a three-tier supply network consisting of manufacturers, smelters, and mines. They analyze the manufacturers' decisions to be compliant or not and the smelters' decisions to be certified or not in a deterministic setting.

dispersion could increase EMA risk beyond its impact on decreased traceability. In particular, even in a fully traceable supply chain, both preemptive and reactive EMA risk are higher in a more dispersed supply chain when testing is imperfect. Furthermore, increasing testing frequency has a limited effect on reducing EMA risk in a dispersed supply chain. Our results highlight the limitation of only relying on end product inspection to deter EMA unless highly robust testing methods can be developed (e.g., methods that can detect adulteration even without knowing what the adulterant is). Another new insight emerged from our analysis is that merely investing in quality improvement without enhancing testing capabilities at the same time may backfire and inadvertently increase EMA risk. This can occur when testing is imperfect, and hence, farms feel "safe" to adulterate to a moderate level without worrying about being caught later. Taken together, our results underscore the importance and necessity of applying a systemic, supply chain perspective to enable proactive management of EMA risk in food products. Such a perspective is missing in most current practices, which mainly rely on inspection and information at the product and individual company level to manage risk.

2. Modeling the Farming Supply Chain

We consider a farming supply chain in which a manufacturer (she) procures a total supply chain output of k units of an agricultural product from n homogeneous farms (he). Based on farming supply chain data we collected for multiple industries in China, we observe that when a manufacturer sources from multiple farms, the output level of each farm is very similar. We thus focus on homogeneous farms. Let m denote the number of output units from each farm, i.e., $m = k/n$. The quality of a farm's output is uncertain ex ante. Specifically, the quality of each unit is low with probability p_L and high with probability $(1 - p_L)$.[3] Hence, the total number of low-quality units at a farm follows a binomial distribution with parameters m (number of units) and p_L (probability of each unit being low-quality)[4].

We study the effect of supply chain dispersion by keeping the total supply chain output k constant and changing n. This effectively varies the fraction of the total supply chain output supplied by each farm. In particular, a larger n (i.e., a smaller m) represents higher supply chain dispersion, i.e., a supply chain with a larger number of farms each supplying a smaller fraction of the total output. Another parameter of interest is p_L, the probability of a unit being low-quality. Changing p_L allows us to study how a farm's adulteration behavior is affected by an increased or a decreased chance of producing

[3] In Appendix O.1.2, we analyze a case where the quality of all units from the same farm is perfectly correlated and obtain similar insights.

[4] In examples such as milking farms where the output is fluid, one unit of output corresponds to all the milk produced by one cow; i.e., quality uncertainty acts at the cow level. We normalize the milk volume produced by each cow to 1.

low-quality output. A high p_L means there is a greater chance for the farm to produce low-quality output (e.g., during epidemics).

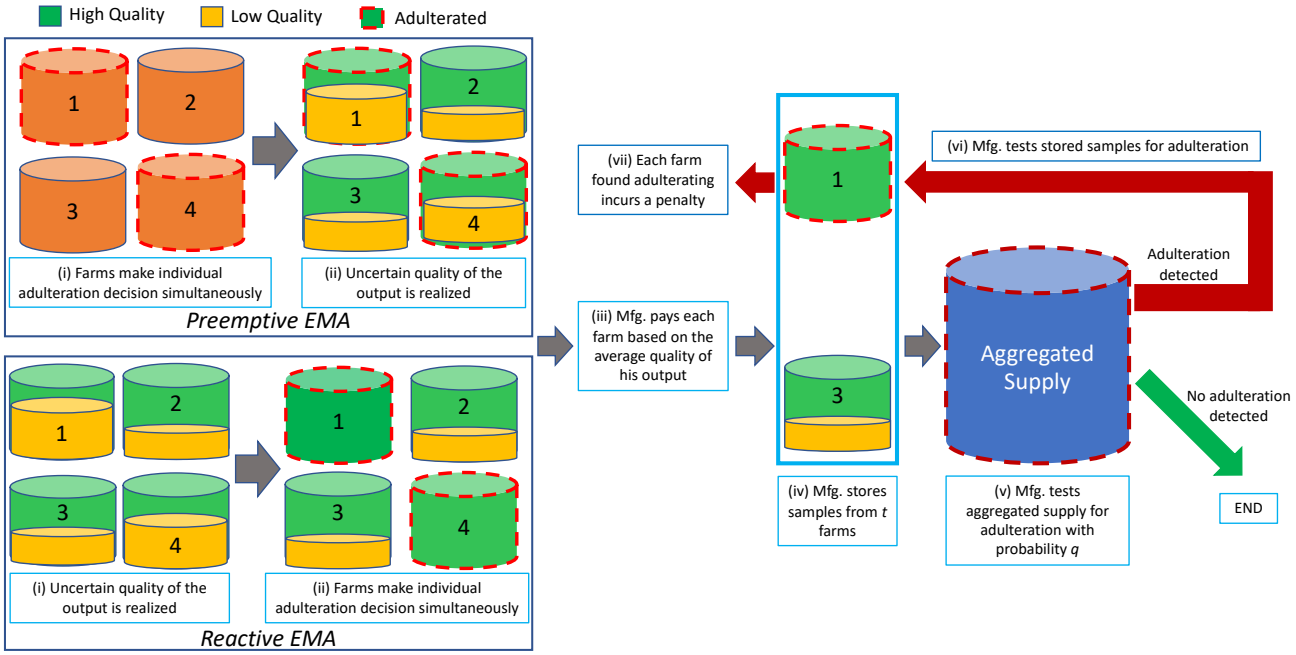
We study both preemptive and reactive EMA. Under preemptive EMA, farms make adulteration decisions *before* the uncertain quality of their output is realized. Preemptive EMA reduces the value of p_L, i.e., decreases the chance of producing low-quality output. We measure preemptive EMA risk in the supply chain based on the amount of adulterants added by the farms. Conversely, under reactive EMA, farms make adulteration decisions *after* the uncertain quality of their output is realized. Reactive EMA increases the perceived quality of the output and creates fake, high-quality units. Farms can condition their reactive EMA decisions on the realized number of low-quality units. Therefore, we measure reactive EMA risk in the supply chain by both the probability of an individual farm adulterating his output and the expected total number of adulterated units within the total supply chain output.

Figure 1 illustrates the sequence of events in our model for both preemptive and reactive EMA. The key differences between the two scenarios are in the first two steps. For preemptive EMA, the model dynamics are as follows: (i) Each farm simultaneously and individually decides the amount of adulterants to add to reduce p_L. (ii) The uncertain quality of the output is realized. (iii) The manufacturer purchases from all farms and pays each farm based on the average quality of his output. (iv) The manufacturer stores output samples from t randomly-chosen farms, where $t \in [0, n]$. (v) The manufacturer aggregates the output from all farms; with probability q, she tests the aggregated supply for adulteration. (vi) If adulteration is detected in the aggregated supply, then the manufacturer tests each of the stored samples. (vii) If a farm is found to have adulterated his output, then he is charged a penalty of cm, where c is the per-unit penalty. For reactive EMA, the first two steps in the model dynamics are instead as follows: (i) The uncertain quality of the output is realized. Each farm privately observes the number of low-quality units he produces. (ii) Each farm simultaneously and individually decides whether or not to adulterate all of the realized low-quality units to create fake high-quality ones. The remaining steps proceed exactly the same as in preemptive EMA.[5]

To map steps (i) and (ii) to practice, first consider for preemptive EMA the example of excessive or illegal use of antibiotics in poultry farming. Here, quality refers to the weight and health status of the grown-up chickens at the time of sale as these parameters largely determine the selling price. Step (i) in Figure 1 corresponds to a farm adding (or not) antibiotics to the feeds of all chickens, and step (ii)

[5] Mu et al. (2014, 2016) also consider adulteration in milk supply chains with individual and mixed testing. The major differences between our model and theirs are twofold: First, we distinguish between exogenous quality uncertainty and endogenous adulteration decisions, whereas they model adulteration as farms producing (deterministically) low-quality output. Second, we model both perfect and imperfect testing scenarios, whereas they only analyze perfect testing.

Figure 1 Sequence of Events for Preemptive and Reactive EMA



corresponds to some chickens growing healthily while others suffering from disease. For reactive EMA, consider the example of milking farms adding melamine to artificially increase the perceived protein content in milk. Here, quality refers to the compositional characteristics (e.g., protein and fat content) of the milk, which again substantially influence the selling price. Step (i) in Figure 1 corresponds to some cows producing milk with good protein content while others producing milk with lower protein level, and step (ii) corresponds to the farm adding (or not) melamine to the milk.[6] Note that the "farm" and "manufacturer" notation in our model represents more generally a player in the upstream (e.g., a farm or a collector) and downstream (e.g., a manufacturer or a wholesaler) of the supply chain.

We note a key difference in the farm's adulteration decision between preemptive and reactive EMA. In preemptive EMA, the farm chooses *how much* to adulterate, i.e., the amount of adulterants to add, and the adulterants are applied to all units. In reactive EMA, the farm chooses *whether or not* to adulterate; in the case of adulterating, he adulterates all of the realized low-quality units.[7]. In addition, we remark that we do not treat preemptive and reactive EMA as mutually exclusive choices by the farms. There could be situations when farms engage in both types of EMA. We consider the separate analysis of either scenario as a critical first step toward analyzing situations where both types of EMA

[6] Agricultural studies show that even in the absence of any equipment, milking farmers can predict the key compositional features (e.g., protein and fat content) of raw milk with high accuracy based on a number of features known to the farmers, e.g., the health of the cow, the quality of the feed, climate, the age and breed of the cow (Gale. and Hu 2009, Wongpom et al. 2017).

[7] In Appendix O.1.3, we allow farms to adulterate a fraction of their low-quality units under reactive EMA and obtain similar results.

could be relevant.[8] When we analyze the manufacturer's testing capability as a lever to deter EMA in §7, we allow the farms to engage in both types of EMA.

We next explain steps (iii)–(vii) in Figure 1 in more detail. First, in step (iii), we model quality-based pricing – a very common payment scheme in many agricultural industries (Bennett et al. 2001). In particular, the price for a high-quality (low-quality) unit is r_H (r_L) with $r_H > r_L$. This price difference is an important economic motive for farms to engage in adulteration. Note that the manufacturer does not need to test the quality of every unit to determine the price. Instead, our model captures linear pricing based on average quality.[9] In the example of preemptive EMA and poultry farming, our model corresponds to a process where all chickens from a farm are weighed and the farm gets paid based on the total weight. Similarly, in the example of reactive EMA and milk farming, a farm typically aggregates the milk from all the cows before bringing to the manufacturer. Hence, the manufacturer simply tests the (average) protein and fat content of the mixed milk and pays a price accordingly.

Second, in step (iv), we capture the *traceability* of the supply chain with the number of randomly-chosen farms, $t \in [0, n]$, from which the manufacturer stores samples. In a fully traceable supply chain ($t = n$), samples from all farms are stored and hence, can be tested for adulteration if needed. Conversely, in a partially traceable supply chain ($t < n$), only some of the farms' samples are stored.[10] Therefore, the traceability of the supply chain significantly impacts the manufacturer's ability to identify all of the adulterating farms. Only those farms that are traced could potentially incur a penalty from adulterating. In practice, some dairy (poultry) companies store samples of milk (meat) before aggregating the supply for processing. If a quality problem is detected in the output, the companies test the samples to identify problematic farms (Flynn and Zhao 2014, Zhang and Bhatt 2014).

Third, in steps (v) and (vi), we model the manufacturer's two-step process of testing for adulteration. Specifically, with some probability q (which captures inspection frequency), the manufacturer tests the aggregated supply for adulteration. Only if adulteration is detected in the aggregated supply will

[8] The two types of EMA could be linked in our analysis by allowing p_L to be dependent on the farms' preemptive EMA decision. Under the reasonable assumption of farmers being short-term oriented (e.g., Antle 1987, Chintapalli and Tang 2017, Hu et al. 2017), their preemptive and reactive EMA decisions are effectively decoupled. That is, when farmers are making preemptive EMA decisions (which would impact p_L), they are not likely to have the foresight to account for every possible realized number of low-quality units in the future and the corresponding reactive EMA decision. As such, the analyses in this paper can be directly applied to examine farms' adulteration behaviors and the resulting total EMA risk when both types of EMA occur.

[9] Mathematically, let n_H and n_L denote the number of high-quality and low-quality units from a farm, and recall that m is the total number of units at each farm. The price based on the average quality of the farm's output can be modeled as $r_H(n_H/m) + r_L(n_L/m)$. This formulation essentially linearizes the two extreme price points onto the average quality level. Therefore, the revenue to the farm from selling all m units is equal to $r_H n_H + r_L n_L$, equivalent to each high-quality (low-quality) unit selling at r_H (r_L).

[10] Storage of food/agricultural samples for an extensive period of time is difficult and costly due to perishability and the substantial facility investments needed (relative to the generally low margin of agricultural products). Furthermore, it is challenging (if not impractical) to build full traceability in a dispersed supply chain with a large number of small farms. These features partly explain why the traceability of agricultural supply chains is typically lower than that of pharmaceutical and electronics supply chains.

the manufacturer then test the stored samples. This two-step process is common in practice due to its cost effectiveness and potential resource constraints of the manufacturer (Draaiyer 2002, Mu et al. 2016). Note that tests for adulteration differ substantially from quality tests for pricing purposes. In particular, quality-based pricing typically considers a small number of quality parameters (e.g., weight of a chicken, or protein and fat content in milk) that can be measured with simple tests in real time. For example, checking the weight of chickens is simple and quick. Similarly, the Gerber butterfat test widely used in the dairy industry involves a simple procedure and takes about 10 minutes to complete (FAO 2009). In contrast, tests for adulteration often require more advanced technologies, take longer time, and are more expensive. For example, the HPLC-UV test for targeted detection of adulterants uses specialized equipment and is labor intensive and costly (Handford et al. 2016). In practice, many of these tests are done in formal testing labs. Thus, to do these tests requires the manufacturer to send samples to a lab and wait for up to a few weeks' time to get the results. However, the manufacturer often needs to pay the farms at the time of sale (or shortly after). Due to these differences, the predominant majority of food manufacturers do not perform adulteration tests all the time in current practices. Furthermore, the two-step process also captures cases where adulteration is detected more downstream in the supply chain when products are inspected by a third party (e.g., government agencies) or when consumers develop adverse symptoms due to consumption of adulterated products.

An important factor we capture related to testing is the sensitivity of the test in detecting adulteration. We model both perfect and imperfect testing. With perfect testing (e.g., advanced tests targeted for certain antibiotics), the test is very sensitive and can accurately detect whether the residue amount of an adulterant (if any) in the food exceeds the maximum limit allowed by food safety standards. With imperfect testing, the sensitivity of the test depends on the relative amount of adulterants in the total supply chain output. The larger this relative amount, the more likely that adulteration can be detected. We capture this dependency by modeling the detection probability to be linearly increasing in the relative amount of adulterants in the total output.[11]

Finally in step (vii), we model the penalty for an adulterating farm to be cm. This penalty structure is motivated by current food safety regulations in China regarding highly toxic adulterants (e.g., melamine, malachite green, or other legally controlled/banned compounds; Handford et al. 2016). For these adulterants, regulations and standards are defined based on whether or not the residue amount exceeds the maximum allowable limit (e.g., PRC-MIIT 2011, FAO 2017, CFDA 2018). As long as

[11] In Appendix O.1.4, we analyze settings where we relax the linearity assumption under imperfect testing.

Table 1 Notations

Variable	Definition
k	total supply chain output
n	total number of farms in the supply chain
m	Number of units supplied by each farm; $m = k/n$
p_L	Probability that a unit of output is of low quality
r_H	Per-unit price paid by the manufacturer to the farm for high-quality output
r_L	Per-unit price paid by the manufacturer to the farm for low-quality output
t	Number of farms whose samples are stored by the manufacturer; $t \in [0, n]$
q	Probability that the manufacturer tests the aggregated supply for adulteration
c	Per-unit penalty charged to a farm that is found adulterating

this limit is exceeded, monetary fines (or prison time) are primarily determined based on the sales revenue associated with the products (PRC-NPC 2015).[12] Furthermore, modeling the penalty to be proportional to the size of the farm can be interpreted as capturing the cost of lost future business if a farm is caught adulterating. This interpretation is particularly relevant in situations where smallholder farmers are rarely fined or legally prosecuted due to political sensitivity (e.g., in India).

Table 1 summarizes the key parameters in our model. In the next section, we examine the farms' adulteration behaviors and the resulting EMA risk in the supply chain in four different settings: preemptive or reactive EMA with perfect or imperfect testing. All proofs are deferred to the online appendix. Additional results presented in the online appendix are referenced as O.X.

3. When Will Farms Adulterate?

3.1. Preemptive EMA

Let $x \in [0, 1]$ be a farm's adulteration decision, where $x = 0$ means the farm does not adulterate his output and $x = 1$ means the farm adulterates with the maximum dosage. In light of our earlier discussions, a decision of $x = 0$ represents the farm using the adulterant (e.g., veterinary drugs) within the legal limit. A decision of $x > 0$ represents using the adulterant beyond the legal limit (as opposed to using a positive quantity per se), and thus, it will cause harm to human health. We assume that adding more adulterants beyond the maximum dosage can no longer decrease the likelihood of producing low-quality output. Therefore, x can be interpreted as the relative quantity of adulterants used by the farm as compared to the maximum dosage. Let $h(x)$ denote the resulting probability that a unit of output is low-quality given x. We assume that $h(x)$ is convex decreasing in x with $h(0) = p_L^{\max}$, $h(1) = p_L^{\min}$, and $p_L^{\max} > p_L^{\min}$. The notations $p_L^{\max}$ and $p_L^{\min}$ represent the largest and smallest probability that a unit of output is low-quality given the farm's adulteration decision. With a large (small) $p_L^{\max}$, the farms

[12] For less toxic adulterants whose harm to human health gradually increases with quantity (e.g., using nonhygienic water to adulterate food; Handford et al. 2016), it may be reasonable to model the per-unit penalty to increase in the amount of adulterants used. In Appendix O.1.5, we show that all of our results remain the same with this alternative formulation.

face a higher (lower) risk of producing low-quality units without adulteration. During epidemics, for example, we can expect $p_L^{\max}$ to be large. The convexity of $h(x)$ implies that the effectiveness of the adulteration to reduce p_L is marginally decreasing. In what follows, we characterize the farms' optimal preemptive EMA strategies under perfect and imperfect testing separately.

3.1.1. Perfect testing. Given a farm's adulteration decision x, the probability that each unit of his output is of low quality is $h(x)$. Hence, the expected total number of low-quality units is $mh(x)$. If the farm chooses not to adulterate (i.e., $x = 0$), then he earns an expected payoff of $r_H m(1 - p_L^{\max}) + r_L m p_L^{\max}$. If instead the farm chooses to adulterate, then he earns an expected revenue of $r_H m(1 - h(x)) + r_L m h(x)$ but would incur a penalty of cm if he is caught by the manufacturer. We now analyze the probability that an adulterating farm will be caught by the manufacturer.

Recall from §2 that the manufacturer employs a two-step testing process. Suppose in the case that the manufacturer tests the aggregated supply (which happens with probability q), she randomly picks one unit of the aggregated supply (e.g., a chicken) to test for adulteration. With perfect testing, the manufacturer detects adulteration as long as any one farm has adulterated. Let n_a be the total number of farms that have adulterated their output. The chance that the manufacturer picks an adulterated unit is thus n_a/n. If the manufacturer detects adulteration in the first step, then she further tests the t samples. Since samples are taken from randomly-chosen farms, the chance that an adulterating farm's sample has been stored is t/n. Taken together, the probability that an adulterating farm will eventually be caught by the manufacturer is thus $q(n_a/n)(t/n)$.

We make two important observations from the analysis thus far. First, since testing is perfect, the farm faces the same level of penalty for any $x > 0$. In addition, his expected revenue without considering the potential penalty is increasing in x. Hence, if the farm decides to adulterate, it is in his best interest to adulterate with the maximum dosage, i.e., choosing $x = 1$. As a result, an adulterating farm's expected payoff is equal to $r_H m(1 - p_L^{\min}) + r_L m p_L^{\min} - q(n_a/n)(t/n)cm$. Second, a farm's adulteration decision depends on how many other farms are also adulterating (i.e., the value of n_a). Therefore, the farms' adulteration decisions impact each other's payoffs, and we solve for a Nash equilibrium (NE) in this static game of complete information (Fudenberg and Tirole 1991, Chapter I). Theorem 1 below characterizes the set of Nash equilibria in the game.

THEOREM 1. *For preemptive EMA with perfect testing, the total number of adulterating farms in any Nash equilibrium of the game is characterized by*

$$n_a^* = \max\left\{0, \min\left\{n, \left\lceil \frac{(r_H - r_L)(p_L^{\max} - p_L^{\min})n^2}{cqt} - 1 \right\rceil\right\}\right\}, \tag{1}$$

where $\lceil\cdot\rceil$ denotes the smallest integer greater than the argument. That is, any subset of n_a^* farms adulterating with the maximum dosage while the rest $(n-n_a^*)$ farms not adulterating constitutes a Nash equilibrium of the game.[13]

The farm's decision on whether or not to adulterate is driven by the tradeoff between the expected revenue gain and the potential penalty from adulteration. By adulterating, the farm increases his expected revenue by $(r_H - r_L)(p_L^{\max} - p_L^{\min})$ per unit of output, while in the meantime facing a penalty of $q(n_a/n)(t/n)c$ per unit. The equilibrium value n_a^* defines the threshold number such that none of the adulterating farms find it profitable to not adulterate, and none of the non-adulterating farms find it profitable to adulterate.

3.1.2. Imperfect testing. The key difference under imperfect versus perfect testing is that the detection of adulteration depends on the relative amount of adulterants in the output. Here, we model this dependency to be linearly increasing. Index the farms by $i = 1, \ldots, n$ and let $x_i \in [0,1]$ be farm i's adulteration decision. Due to imperfect testing, the amount of adulterants added (i.e., x_i) affects the chance that an adulterating farm will be caught. As a result, it is not necessarily optimal for the farm to adulterate to the maximum dosage (unlike in perfect testing). We first derive the chance of an adulterating farm eventually being caught by the manufacturer. Similar to §3.1.1, suppose with probability q, the manufacturer picks one unit (e.g., a chicken) from the aggregated supply to test for adulteration. In this case, the probability that the manufacturer detects adulteration in the aggregated supply can be derived as follows:

$$\begin{aligned}\mathbb{P}(\text{detection}) &= \sum_j \left[\mathbb{P}(\text{detection}|\text{farm } j\text{'s output is picked})\mathbb{P}(\text{farm } j\text{'s output is picked})\right] \\ &= \sum_j [x_j(1/n)] = \sum_j x_j/n.\end{aligned}$$

The conditional probability in the bracket is equal to x_j because (i) under preemptive EMA, the added adulterants affect all units of output at a farm, and (ii) the detection sensitivity is linearly increasing in the relative amount of adulterants added. If the manufacturer detects adulteration in the aggregated supply, then she will test the t samples. An adulterating farm's sample will be stored with probability t/n. Finally, the chance that the manufacturer detects adulteration in the sample again depends on the amount of adulterants added, x_i. To summarize, the chance that an adulterating farm i will eventually

[13] We assume that when a farm is indifferent between adulterating or not, he chooses not to adulterate.

be caught by the manufacturer is equal to $q(\sum_j x_j/n)(t/n)x_i$. Therefore, farm i's expected payoff given his adulteration decision x_i can be characterized as follows.

$$\pi^{PV}(x_i, x_{-i}) = r_H m(1-h(x_i)) + r_L m h(x_i) - q\left(\frac{x_i + \sum_{-i} x_{-i}}{n}\right)(t/n)x_i cm, \tag{2}$$

where $-i$ denotes all farms other than i. Since all the farms are homogeneous, we focus on analyzing symmetric NE of the game; i.e., NE in which the equilibrium strategy $x_i^{PV^*}$ has the same structure for all i. This approach is common in game-theoretic analysis with homogeneous players (e.g., Che 1993, Lee et al. 1997, Wang and Zender 2002, Golosov et al. 2014). The next theorem characterizes the unique symmetric NE of the game. We drop the subscript i to simplify notation.

THEOREM 2. *For preemptive EMA with imperfect testing, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.*

(a) If $c < \dfrac{-h'(1)(r_H - r_L)}{q(t/n)((n+1)/n)}$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq \dfrac{-h'(1)(r_H - r_L)}{q(t/n)((n+1)/n)}$, then the farms adulterate to some extent; i.e., $x^{PV^*} \in (0,1)$ and is the solution to the following equation: $-h'(x)/x = q(t/n)((n+1)/n)c/(r_H - r_L)$.

When the per-unit expected penalty is sufficiently small compared to the per-unit expected revenue gain, all farms adulterate to the maximum level as under perfect testing. However, when the per-unit expected penalty is large, farms in equilibrium adulterate to some extent but not to the maximum level (Theorem 2(b)). This is because both the expected revenue gain and the chance of being caught increase as farms increase the amount of adulterants.

3.2. Reactive EMA

Recall from §2 that under reactive EMA, the farms decide whether or not to adulterate his low-quality units to create fake high-quality ones *after* the uncertain quality of his output is realized. In addition, the total number of low-quality units at a farm follows a binomial distribution with parameters m (number of units) and p_L (probability of each unit being low-quality). This distribution can be well approximated by a normal distribution with mean mp_L and variance $mp_L(1-p_L)$ if $mp_L \geq 5$ and $m(1-p_L) \geq 5$ (Ross 2005, Stirzaker 1999, Clemens and Inderfurth 2015). To ensure tractability, we perform our analysis with this normal approximation. Let $f(x, m, p)$ denote the probability density function (PDF) of a normal distribution with mean mp and variance $mp(1-p)$ evaluated at x. We now characterize the farms' optimal reactive EMA strategies under perfect and imperfect testing separately.

3.2.1. Perfect testing. Let n_L be the realized number of low-quality units at a farm. If the farm does not adulterate, then he earns $r_H(m-n_L)+r_Ln_L$ based on the average quality of his output. If the farm decides to adulterate, then he earns a revenue of r_Hm but would incur a penalty of cm if he is caught by the manufacturer. Note that if a farm adulterates, then the adulterants will be present in his output, the manufacturer's aggregated supply, and the stored sample. For example, if a dairy farm adds melamine to his raw milk, then melamine will be present in the milk from this farm, the aggregated pool of milk at the manufacturer, and any sample stored from this farm. Under perfect testing, the manufacturer detects adulteration in the aggregated supply as long as any farm has adulterated and the manufacturer tests the aggregated supply (the latter occurs with probability q). If further the sample of an adulterating farm is stored (this occurs with probability t/n), then the farm will be caught and incur the penalty. Hence, the probability of a farm's adulteration being detected is $q(t/n)$, and the expected payoff of an adulterating farm is equal to $r_Hm-q(t/n)cm$. Observe that the farms' adulteration decisions do not affect each other's payoffs and can be solved independently. A farm chooses whether or not to adulterate to maximize his expected payoff. The following theorem describes the optimal adulteration strategy for a farm with n_L units of low-quality output.

THEOREM 3. *For reactive EMA with perfect testing, the optimal adulteration strategy for a farm is a threshold strategy: He does not adulterate if $n_L \in [0,\beta^{RP}]$, and he adulterates if $n_L \in (\beta^{RP},m]$. In addition, β^{RP} is decreasing in the price difference between high- and low-quality output (r_H-r_L) and increasing in testing frequency (q), the number of stored samples (t), and per-unit penalty (c).*

The threshold strategy in Theorem 3 follows from a tradeoff between revenue gain and the potential penalty from adulteration. Since the revenue gain increases with n_L whereas the expected penalty is independent of n_L, the farm finds it beneficial to adulterate when n_L is sufficiently large. The more frequent the manufacturer tests the aggregated supply or the better traceability in the supply chain (higher q and t), the higher chance that an adulterating farm will be caught, and hence, the less likely a farm is to adulterate. Similarly, a smaller per-unit penalty (c) and a greater price difference between high- and low-quality output (r_H-r_L) make the penalty from adulteration less severe and the gain more attractive, thus motivating a farm to adulterate more often. This last point is in line with qualitative evidence that many adulteration incidents occurred when there was external price pressure for low-quality products. For example, the Indian government found that milk adulteration was significantly more severe in Maharashtra where low-fat milk was rejected at collection centers versus in Gujarat where it was accepted at a lower price (Deshmukh 2011). We warrant that simply

reducing the price difference is not likely a viable way to deter adulteration, especially if farms rely on the price premium for higher quality to truly invest in quality. Instead, we advocate that fairer risk sharing between the manufacturer and the farms is essential (e.g., offering protective prices in light of quality uncertainty common in agricultural production).

3.2.2. Imperfect testing. In this case, the chance that the manufacturer detects adulteration when testing the aggregated supply is modeled as linearly increasing in the fraction of adulterated units within the total output. Therefore, the farms' (simultaneous) adulteration decisions affect each other's payoffs through this detection probability. Since the realized number of low-quality units at a farm is the farm's private information, we model the farms' strategic interactions as a static game of incomplete information and solve for the Bayesian Nash equilibrium (BNE) of the game (Fudenberg and Tirole 1991, Chapter III).

Formally, let $n_{L,i}$ denote the realized number of low-quality units at farm i ($i=1,\ldots,n$). Let $a_i(n_{L,i}):\{1,\ldots,m\}\to\{0,1\}$ be farm i's adulteration strategy, where a value of 1 (0) means adulterating (not adulterating). That is, farm i's adulteration strategy specifies for each realized number of low-quality units, whether or not the farm adulterates these low-quality output. From farm i's perspective, given all other farms' adulteration strategies $a_{-i}(n_{L,-i})$, the expected total number of adulterated units from these farms is equal to $\mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]$. The expectation is taken on the (uncertain) number of low-quality units at the other farms, which is not observable by farm i.

We next derive the chance of farm i being caught if he adulterates. First, the fraction of adulterated units within the total output (hence the chance that the manufacturer detects adulteration when testing the aggregated supply) is equal to $\dfrac{n_{L,i}+\mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]}{k}$. Conditional on adulteration being detected in the aggregated supply, if farm i's sample is stored (which occurs with probability t/n), then farm i will be caught with probability $n_{L,i}/m$. Since the manufacturer tests the aggregated supply with probability q, the ultimate probability for farm i to be caught if he adulterates is equal to

$$\gamma_i(n_{L,i},a_{-i}(n_{L,-i}))\equiv q\left(\frac{t}{n}\right)\left(\frac{n_{L,i}}{m}\right)\left(\frac{n_{L,i}+\mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]}{k}\right). \tag{3}$$

Thus, the final expected payoff if farm i chooses to adulterate is equal to $r_H m-\gamma_i(n_{L,i},a_{-i}(n_{L,-i}))cm$. Conversely, the final payoff if farm i chooses not to adulterate is equal to $r_H(m-n_{L,i})+r_L n_{L,i}$. Farm i decides whether or not to adulterate depending on which action yields a higher final payoff.

We again focus on analyzing symmetric BNE of this game; i.e., BNE in which the strategy $a_i^*(n_{L,i})$ has the same structure for all i. The next theorem characterizes the unique BNE of this game. We drop the subscript i to simplify notation.

THEOREM 4. *For reactive EMA with imperfect testing, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy:* $a^*(n_L)=1$ *if* $n_L \in [0,\beta^{RV})$ *and* $a^*(n_L)=0$ *if* $n_L \in [\beta^{RV},m]$. *The threshold* β^{RV} *is unique and determined as follows:*

(a) If $n \le 2(1-p_L)\Big/\left(\sqrt{p_L^2+\frac{4(1-p_L)(r_H-r_L)}{cqt}}-p_L\right)$, *then* $\beta^{RV}\in(0,m)$ *and is the solution to the following equation:* $\beta=\frac{nk(r_H-r_L)}{cqt}-(n-1)\int_0^{\beta} x f(x,\frac{k}{n},p_L)\mathrm{d}x$.

(b) If $n > 2(1-p_L)\Big/\left(\sqrt{p_L^2+\frac{4(1-p_L)(r_H-r_L)}{cqt}}-p_L\right)$, *then* $\beta^{RV}=m$.

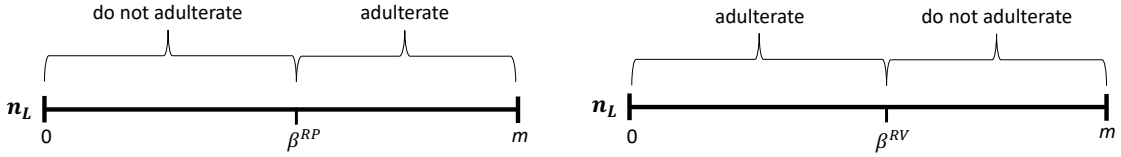
In addition, β^{RV} *is increasing in the price difference between high- and low-quality output* (r_H-r_L) *and decreasing in testing frequency* (q), *the number of stored samples* (t), *and per-unit penalty* (c).

Figure 2 contrasts the farms' optimal reactive EMA strategies under perfect testing (Theorem 3) versus imperfect testing (Theorem 4). Under perfect testing, farms adulterate when the realized number of low-quality units, n_L, is *greater* than the threshold β^{RP}. In contrast, under imperfect testing, farms adulterate when n_L is *smaller* than the threshold β^{RV}. This latter result is because under imperfect testing, the expected penalty from adulterating increases with n_L faster than the revenue gain does. Observe from Equation (3) that the chance of farm i being caught adulterating increases with $n_{L,i}$ quadratically, whereas the revenue gain, $(r_H-r_L)n_{L,i}$, increases with $n_{L,i}$ linearly. Therefore, farms find it more beneficial to adulterate when n_L is low. More intuitively, the key driver of this pattern is the farms' "free-riding" behavior; that is, when a farm believes that there is a sufficiently large number of high-quality units in the supply chain that can hide his adulteration (i.e., when n_L and the belief of other farms' n_L are small), the farm is more likely to adulterate. Evidence of such free-riding behavior exists in practice. For example, Gadzikwa et al. (2007) report that organic producers are more likely to adulterate (e.g., using pesticides) and fake organic products when the fake quantity is small relative to the total quantity. In an extension (§O.1.4), we allow the sensitivity of imperfect testing to increase faster than the linear model, and hence, become closer to the sensitivity of perfect testing. We demonstrate that the farms' equilibrium reactive EMA strategy becomes a combination of the two structures in Figure 2; i.e., farms adulterate when n_L is either small or large but do not adulterate in between.

Due to the aforementioned contrasting pattern under perfect versus imperfect testing, the effects of the price difference, testing frequency, traceability, and per-unit penalty on β^{RV} are opposite to those for β^{RP} (see Theorem 3). Comparing these two thresholds, we obtain the following result.

PROPOSITION 1. *(i) If* $\beta^{RP}\in(0,m)$, *then* $\beta^{RV}=m$. *(ii) If* $\beta^{RV}\in(0,m)$, *then* $\beta^{RP}=m$.

Figure 2 Reactive EMA: Farms' Optimal Adulteration Strategies under Perfect and Imperfect Testing



(a) Perfect testing (b) Imperfect testing

Proposition 1 shows that given the same model parameters, whenever farms would possibly adulterate under perfect testing (i.e., $\beta^{RP} \in (0, m)$), then they would always adulterate under imperfect testing. Similarly, whenever farms would possibly not adulterate under imperfect testing (i.e., $\beta^{RV} \in (0, m)$), then they would never adulterate under perfect testing. Consequently, the resulting reactive EMA risk is always higher under imperfect than perfect testing.

Corollary 1. *The reactive EMA risk in the supply chain is always higher under imperfect testing than under perfect testing.*

4. The Effect of Supply Chain Dispersion on EMA Risk

We now analyze how supply chain dispersion affects preemptive and reactive EMA risks in the supply chain. We measure preemptive EMA risk by the fraction of adulterating farms, n_a^*/n, under perfect testing, and the farms' adulteration decisions, x^{PV^*}, under imperfect testing. A larger (smaller) value of these terms indicates a larger (smaller) amount of adulterants being used in the total output, and hence, a higher (lower) risk of preemptive EMA. Similarly, we measure reactive EMA risk in two ways: P_n denotes the probability of an individual farm adulterating in a supply chain with n farms, and E_n denotes the expected total amount of adulterated output in the supply chain. Under perfect testing, $P_n = \text{Prob}\,(n_L > \beta^{RP})$, and $E_n = n\int_{\beta^{RP}}^{k/n} x f(x, k/n, p_L)\mathrm{d}x$, where β^{RP} is defined in Theorem 3. We can similarly define P_n and E_n under imperfect testing. For the analysis in this section, we treat n to be a continuous variable to ensure tractability. Our first result shows how supply chain dispersion (captured by n) impacts preemptive and reactive EMA risk under perfect and imperfect testing.

Proposition 2A. *Under perfect testing: When $t < n$ $(t = n)$, (i) preemptive EMA risk, measured by n_a^*/n, is increasing (constant) in n; (ii) reactive EMA risk, measured by P_n and E_n, is increasing in n if $qc \geq \dfrac{p_L(r_H - r_L)}{3(t/n)}$ $(qc \geq p_L(r_H - r_L))$.*

Proposition 2B. *Under imperfect testing: (i) Preemptive EMA risk, measured by x^{PV^*}, is increasing in n; (ii) reactive EMA risk, measured by P_n and E_n, is increasing in n.*

Proposition 2A considers perfect testing and characterizes conditions under which supply chain dispersion increases EMA risk (or not). First, greater dispersion leads to higher preemptive EMA risk under perfect testing only in partially traceable supply chains. That is, this higher risk is solely due to decreased traceability in a more dispersed supply chain (observe that n_a^*/n is constant in n if $t = n$). In Appendix O.1.1, we show that if farms are averse to quality uncertainty, then an additional reason for this higher risk is that a smaller farm faces larger uncertainty in the quality of his output. Second, regardless of traceability, dispersion increases reactive EMA risk under perfect testing if the per-unit expected penalty for adulteration (qc) is high. This result can be explained by the following dynamics. Imagine that the manufacturer diversifies her supply base by procuring one fewer unit from each of the existing farms and procuring the resulting gap from a new farm. When the per-unit expected penalty is *high*, these units originally procured from each existing farm would not be adulterated in expectation. However, when they are instead procured from a new farm, some of them would be adulterated at least sometimes. Therefore, reactive EMA risk *increases* in a more dispersed supply chain. Conversely, if the per-unit expected penalty is *low*, these units when procured from the original farms would all be adulterated in expectation, while only some of them would be adulterated when procured from the new farm. Hence, reactive EMA risk *decreases* in a more dispersed supply chain.[14]

Proposition 2B considers imperfect testing and shows that both preemptive and reactive EMA risk *always* increase in a more dispersed supply chain, regardless of traceability. With a large number of farms in the supply chain, each farm produces only a small fraction of the total output. As a result, each farm's adulteration has a limited impact on the total amount of adulterants and adulterated output in the aggregated supply, and hence, on the likelihood that the manufacturer would detect adulteration. Therefore, farms feel less risky to adulterate in a more dispersed supply chain, resulting in higher EMA risk. In sharp contrast, if the supply chain consists of only two farms each producing half of the total output, then the adulteration decision of each farm would have a much more prominent impact on the detection probability. Thus, these larger farms would be more cautious in their adulteration decisions.

Table 2 summarizes the key insights from Proposition 2. We observe that the effect of supply chain dispersion on EMA risk highly depends on the manufacturer's testing capability. When test is perfect, greater dispersion does not always result in higher EMA risk. However, when test is imperfect, increased dispersion is always harmful. Note that even if the manufacturer tests the aggregated supply with certainty (i.e., $q = 1$), these results remain. The insights from Table 2 thus highlight the limitation of

[14] We show analytically that when the condition in Proposition 2A part (ii) is not satisfied, P_n decreases with n (see the proof of Proposition 2A). For E_n, we observe from numerical simulation that it decreases with n when qc is sufficiently low.

Table 2 Effect of Increased Supply Chain Dispersion on EMA Risk

	Perfect testing	Imperfect testing
Preemptive EMA	Risk increases (is constant) under partial (full) traceability	Risk increases
Reactive EMA	Risk increases (decreases) if penalty is high (low)	Risk increases

relying on product inspection to deter EMA, particularly if the farming supply chain is highly dispersed and testing methods are imperfect (as is the case in most developing countries with a large variety of possibly unknown adulterants). Hence, to more proactively fight EMA, our results call for the need to incorporate a systemic, supply chain perspective to complement an inspection-centered approach adopted in current practices.

5. The Effect of Quality Uncertainty on EMA Risk

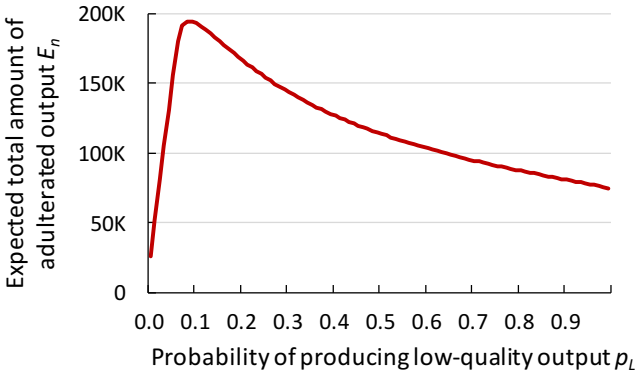
The following proposition demonstrates how the likelihood of producing low-quality output (p_L) affects both preemptive and reactive EMA risk.

PROPOSITION 3. *(i) For preemptive EMA, both n_a^*/n and x^{PV^*} are increasing in $p_L^{\max}$.*
(ii) For reactive EMA under perfect testing, both P_n and E_n are increasing in p_L.
(iii) For reactive EMA under imperfect testing, P_n is decreasing in p_L.

Proposition 3 parts (i) and (ii) show that as the likelihood of producing low-quality output ($p_L^{\max}$ or p_L) at a farm increases, the supply chain would face a higher risk of EMA for preemptive EMA under either testing sensitivity and reactive EMA under perfect testing. Specifically, a higher $p_L^{\max}$ means there is a greater chance of producing low-quality units, and hence, farms are more motivated to engage in preemptive EMA to reduce that chance. Similarly, a higher p_L means the realized number of low-quality units, n_L, is more likely to be large, as n_L follows a binomial distribution with parameters m and p_L. Therefore, farms are more likely to engage in reactive EMA (recall from Theorem 3 that under perfect testing, a farm adulterates if $n_L > \beta^{RP}$).

In sharp contrast, Proposition 3 part (iii) shows that under imperfect testing, the probability that an individual farm engages in reactive EMA *decreases* as p_L increases. This result is related to the farms' "free-riding" behavior discussed in §3.2.2. In particular, when p_L is low, an individual farm expects that the other farms would have many high-quality units. As a result, even if this farm chooses to adulterate, the fraction of adulterated units in the aggregated supply can be sufficiently low that the manufacturer would not be able to detect adulteration. Therefore, farms feel "safe" to adulterate and are more likely to do so. Finally, we observe that under reactive EMA with imperfect testing, the effect of quality uncertainty on E_n, the expected total amount of adulterated output in the supply chain,

Figure 3 Effect of Quality Uncertainty (p_L) on the Expected Total Amount of Adulterated Output (E_n)



is nonmonotone. Figure 3 illustrates the general pattern of this effect based on extensive numerical simulations. As p_L decreases (i.e., as overall quality becomes better), E_n first increases and then decreases. Furthermore, the decreasing region predominantly corresponds to situations when all farms adulterate all the time in equilibrium (which occurs when p_L is low). In this case, E_n is equal to the expected total number of low-quality units, and hence, it decreases as p_L decreases. These observations highlight that, if the current quality level is bad (i.e., when p_L is high), then a substantial quality investment (to decrease p_L significantly) is necessary to reduce reactive EMA risk with respect to E_n under imperfect testing. Nevertheless, if the focus is to mitigate P_n (e.g., when highly toxic adulterants are concerned), then investing in quality alone is not effective.

Investing in quality to reduce the probability of producing low-quality output is generally believed to be beneficial. Our analysis demonstrates that for reactive EMA with imperfect testing, this strategy may backfire if it is not accompanied by also improving the supply chain's capability to detect adulteration. This is because a lower chance of producing low-quality output can inadvertently motivate some parties in the supply chain to endogenously adulterate their output and create fake high-quality units. Without the capability to differentiate fake high-quality units from truly high-quality ones, consumers could suffer from consuming adulterated products. An example in China's dairy industry illustrates this point. After the 2008 melamine-tainted infant formula scandal, many dairy companies made substantial quality investments in their upstream supply chains. However, without similar improvement in testing methods, adulteration in raw milk escalated again in 2011 when a new type of protein-enhancing yet toxic substance was added by milk farms (Handford et al. 2016). We summarize the effect of quality uncertainty on EMA risk for the different modeling scenarios in Table 3.

Table 3 Effect of an Increased Probability of Producing Low-Quality Output ($p_L^{\max}$ or p_L) on EMA Risk

	Perfect testing	Imperfect testing
Preemptive EMA	Risk increases	Risk increases
Reactive EMA	Risk increases	Probability of adulteration decreases, while expected total amount of adulterated output first increases then decreases

Table 4 Parameter Values for Guangdong and Shandong Provinces

	n	m	r_H	r_L	c	q	t	$p_L^{\max}$	$p_L^{\min}$
Guangdong	8	84,000	19.07	0	19.07	$\{0.1,..,0.9\}$	$\{1,..,8\}$	$\{0.2,..,0.9\}$	0.1
Shandong	13	67,000	19.07	0	19.07	$\{0.1,..,0.9\}$	$\{1,..,13\}$	$\{0.2,..,0.9\}$	0.1

6. Relating Model Predictions to Empirical Observations

In this section, we calibrate our model parameters with field data to the extent possible to examine how well our models' predictions align with empirical evidence in two prominent EMA scenarios.

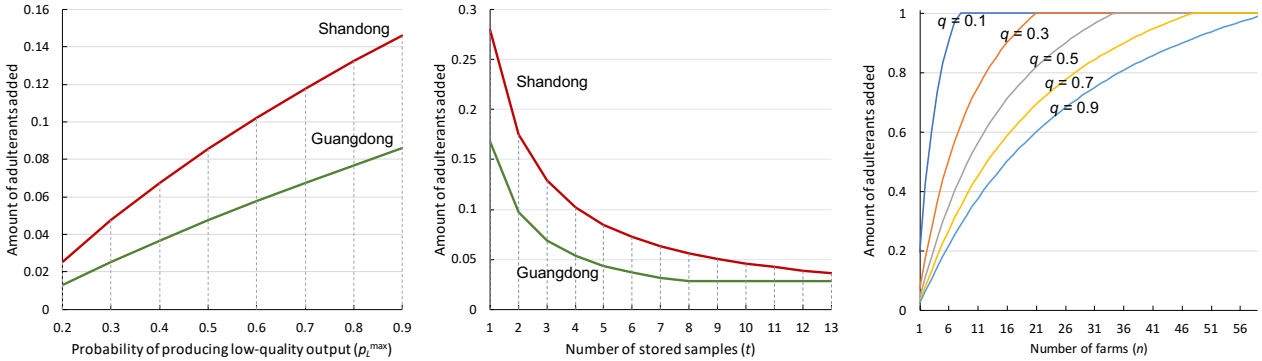
6.1. Preemptive EMA: Misuse of Antibiotics in Poultry Farming in China

As in many developing countries, China's poultry industry is subject to misuse of antibiotics, antivirals, and herbal medicines at poultry farms, especially since multiple outbreaks of avian flu (CCTV 2012, Huang et al. 2017b). We collected farming supply chain data published by China's General Administration of Quality Supervision, Inspection, and Quarantine (AQSIQ), a government agency responsible for entry-exit commodity inspection, certification, accreditation, and import-export food safety. By utilizing this data and various market data, we calibrate our model parameters and predict the risk levels of poultry manufacturers in two leading provinces of poultry production, Shandong and Guangdong, which account for 15% and 8% of the total production (Inouye 2017).

Table 4 summarizes the parameter values used in our analysis. The values of n and m are derived by averaging the number of farms and the size of farms (in the number of chickens produced annually) supplying to different poultry manufacturers in each province (18 manufacturers in Guangdong and 34 in Shandong). One immediate observation is that poultry supply chains in Shandong are more dispersed than those in Guangdong, as seen by a larger number of farms supplying to an average manufacturer (n) and the smaller size of an average farm (m). The value of r_H corresponds to the selling price of broiler chickens (in RMB per kilogram) in 2016 (Inouye 2017). Low-quality chickens mean sick chickens which cannot be sold; thus, $r_L = 0$. According to China's food safety law (PRC-NPC 2015), financial penalties on adulterating firms are determined based on the sales value of the products, with a larger marginal increase in penalty at a higher level of product value. We capture this structure by using $r_H m^2$ as the penalty for a farm of size m.[15] For the manufacturers' testing

[15] We verify that all of our results in §3 continue to hold with a convex (in m) penalty function (Proposition O.5 in Appendix O.1.5).

Figure 4 Effect of Traceability (t), Quality Uncertainty ($p_L^{\max}$), and Testing Frequency (q) on Preemptive EMA Risk



(a) Effect of quality uncertainty ($p_L^{\max}$) (b) Effect of traceability (t) (c) Effect of testing frequency (q)

Note. In Figures 4a and 4b, we use the average values of n and m presented in Table 4 for the two provinces. In Figure 4c, we use the whole range of n values in the data and do not distinguish between the two provinces.

frequency (q) and the traceability of their supply chains (t), we consider a wide range of values. We use $h(x) = (p_L^{\max} - p_L^{\min})x^2 - 2(p_L^{\max} - p_L^{\min})x + p_L^{\max}$ to ensure that $h(0) = p_L^{\max}$, $h(1) = p_L^{\min}$, and $h(x)$ is convex decreasing. We fix $p_L^{\min} = 0.1$ and vary $p_L^{\max}$ from 0.2 to 0.9 to capture different levels of quality uncertainty in the farms' output. Since poultry farms use a large variety of drugs in their practices, we consider the imperfect testing scenario. Given these parameter values, we calculate the risk levels of an average manufacturer in these two provinces with 936 different parameter combinations.

Our results show that an average poultry manufacturer in Shandong always faces higher preemptive EMA risk than an average manufacturer in Guangdong does. Averaging over all parameter instances, the risk in Shandong is twice of that in Guangdong. This prediction is consistent with empirical evidence; more poultry manufacturers in Shandong have been found to be involved in EMA incidents than those in Guangdong (Huang et al. 2017b). Among the manufacturers in our data, 4 out of 34 (11.8%) Shandong companies and 1 out of 18 (5.6%) Guangdong companies were caught in EMA incidents. Figures 4a and 4b further illustrate how preemptive EMA risk changes with $p_L^{\max}$ and t in these two provinces. We observe that adulteration increases at a faster rate for Shandong than for Guangdong as $p_L^{\max}$ increases or t decreases. Therefore, manufacturers in Shandong are at a greater risk of increased adulteration due to quality uncertainty, and they can gain more benefit in risk mitigation by improving the traceability of their supply chains.

Lastly in Figure 4c, we illustrate how preemptive EMA risk changes with supply chain dispersion (n) for different testing frequencies (q). The values of n are taken from our data. We observe that as the supply chain gets more dispersed, increasing q becomes much less effective in reducing preemptive EMA risk (e.g., risk > 0.6 even if $q = 0.9$ when $n > 20$). Note that we have kept the fraction of traced

farms (t/n) constant as we increase n in Figure 4c. The reason behind this observation is the following. When the supply chain is composed of a few large farms, each farm's adulteration has a big impact on the total amount of adulterants in the aggregated supply (and hence, the detection probability). Therefore, each farm is cautious about adulterating and sensitive to the testing frequency. However, when the supply chain is composed of many small farms, each farm produces only a small quantity, which, even adulterated, would not increase the detection probability noticeably. Hence, these small farms are not as concerned about the aggregated supply being tested frequently. As a result, increasing testing frequency has a limited effect on their adulteration behavior, and thus, the resulting preemptive EMA risk in the supply chain.

6.2. Reactive EMA: Melamine-Tainted Infant Formula Scandal

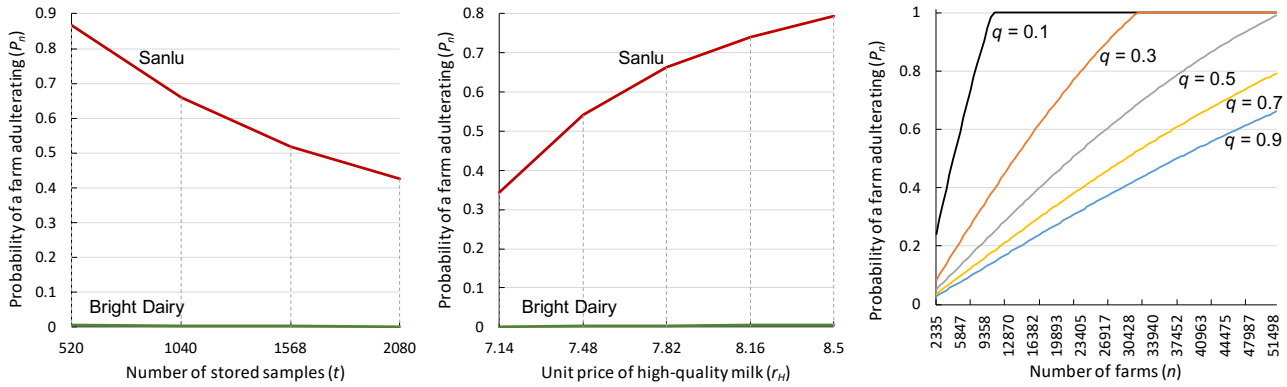
In the second empirical case, we examine the risk of two major Chinese dairy companies, Sanlu Group and Bright Dairy, operating at the time of the melamine-tainted infant formula scandal. Regarding Sanlu, the company sourced 2.6 million liters of raw milk from 52,000 small farms primarily through middlemen with little traceability (Chen et al. 2014). These small farms often produced low-quality milk due to poor diets and disease control of the cows (Gale. and Hu 2009). In addition, leveraging an exemption of certificate verification from the Chinese government, Sanlu rarely tested the raw milk before production (Flynn and Zhao 2014). All of these data suggest that p_L is high whereas t and q are low in Sanlu's supply chain. In sharp contrast, Bright Dairy sourced 1.625 million liters of raw milk from 2,335 corporate-owned or cooperative farms (Flynn and Zhao 2014). The company stored samples from each cow to ensure full traceability. In addition, it also made significant investments in feed and animal health in the farms and frequently conducted quality tests to uphold its high quality standards. Therefore, in Bright Dairy's supply chain, p_L is low while both t and q are high.

Based on the above discussion, we construct the parameter set as summarized in Table 5 for our analysis. In 2008, the average price of raw milk was 6.8 RMB per liter (Gale and Arnade 2015). We use this price as r_L and vary the premium for high-quality milk to be 5% to 25% above r_L. We again use $r_H m^2$ as the penalty function, and consider imperfect testing. Despite qualitative evidence that traceability in Sanlu's supply chain is almost absent, we allow for some traceability for Sanlu in our analysis. Hence, we likely underestimate its risk level. We analyze a total of 500 parameter combinations for each company and observe a stark contrast in the reactive EMA risk faced by the two companies. On average, the probability of an individual farm adulterating (P_n) is 0.617 for Sanlu versus 0.003 for Bright Dairy. Similarly, the expected fraction of adulterated supply in the total output (E_n/k) is 0.398

Table 5 Parameter Values for Sanlu Group and Bright Dairy

	n	m	p_L	t	q	r_L	r_H	c
Sanlu Group	52,000	50	$\{0.5,..,0.9\}$	$\{520,..,2080\}$	$\{0.1,..,0.5\}$	6.8	$\{1.05r_L,..,1.25r_L\}$	r_H
Bright Dairy	2,335	700	$\{0.1,..,0.5\}$	$\{520,..,2080\}$	$\{0.5,..,0.9\}$	6.8	$\{1.05r_L,..,1.25r_L\}$	r_H

Figure 5 Effect of Traceability (t), High-Quality Unit Price (r_H), and Testing Frequency (q) on Reactive EMA Risk



(a) Effect of traceability (t) (b) Effect of high-quality unit price (r_H) (c) Effect of testing frequency (q)

Note. We observe very similar patterns if we examine the expected fraction of adulterated output (E_n/k) as the y-axis.

for Sanlu versus 0.0005 for Bright Dairy. These predictions match empirical evidence. In particular, Sanlu's products were the most heavily adulterated in the scandal, whereas Bright Dairy passed all quality inspections by the authorities during the crackdown (Chen et al. 2014).

Figures 5a and 5b show, for each company, how P_n changes as t and r_H change. While Sanlu's risk increases quickly as t decreases or r_H increases, Bright Dairy's risk is almost 0 in all instances. Despite the lack of perfect testing for melamine (at that time), the less dispersed supply chain and better quality assurance established in Bright Dairy's supply chain have acted as important levers to protect the company from reactive EMA risk. Lastly in Figure 5c, we observe a very similar pattern as in Figure 4c. That is, increasing testing frequency q has a limited effect on reducing reactive EMA risk when the supply chain is highly dispersed.

7. Analyzing Managerial Levers to Mitigate EMA Risk

In this section, we take the manufacturer's perspective and examine a few managerial levers that could help to mitigate EMA risk in the supply chain. We first consider increasing supply chain traceability t and testing frequency q (Appendix O.2). Since strengthening these levers is costly, we develop an optimization model where the manufacturer's objective is to minimize total investment costs while satisfying a constraint that the resulting EMA risk in the supply chain cannot exceed a certain level. This modeling approach is common in the risk management literature (e.g., Federgruen and Yang

2008, Meena et al. 2011, Federgruen et al. 2015). We highlight two results. First, whenever a feasible solution exists (i.e., the risk constraint can be satisfied with full traceability and constant testing), the manufacturer optimally utilizes the two levers by prioritizing the more cost-effective one (Theorem O.8). Second, given a desirable risk constraint, it is always more difficult for a manufacturer with a more dispersed supply chain to satisfy the constraint. Conditional on being able to satisfy the risk constraint, it is always more costly for a manufacturer with a more dispersed supply chain to do so (Proposition O.6). Thus, higher supply chain dispersion results in a greater challenge for a manufacturer to manage and mitigate EMA risk, from both feasibility and financial standpoints. This result aligns with our earlier discussion regarding the adverse impact of supply chain dispersion on EMA risk.

We next consider the lever of testing capability. In particular, the manufacturer can choose to invest in perfect testing for either preemptive or reactive EMA or both. We consider the manufacturer's decision for the two types of EMA separately because in practice, companies contracting with testing labs (for perfect testing) typically can choose what adulterants to test and are priced accordingly. Our focus is to examine how the total EMA risk in the supply chain, accounting for both preemptive and reactive EMA, is affected by the testing capabilities of the manufacturer. To do so, we analyze a setting in which the manufacturer decides whether or not to adopt perfect testing for either type of EMA, and the farms strategically respond by making preemptive and reactive EMA decisions (Appendix O.3). The farms' reactive EMA decisions are conditioned upon their preemptive EMA decisions because the latter impact the likelihood of producing low-quality output, p_L, at the farms. Our main results are twofold. First, by Proposition 1, perfect testing always decreases reactive EMA risk as compared to imperfect testing. Thus, if the risk reduction outweighs the investment cost, then the manufacturer should invest in perfect testing to deter reactive EMA. In sharp contrast, perfect testing may lead to a higher preemptive EMA risk than imperfect testing, when the per-unit penalty c is not too low nor high enough.[16] This counterintuitive result can be explained as follows. Recall from Theorems 1 and 2 that for preemptive EMA under perfect testing, adulterating farms all adulterate with the maximum dosage, whereas under imperfect testing, they adulterate at a lower level that balances revenue gain and expected penalty. When the penalty is not high enough, many farms choose to adulterate with the maximum dosage under perfect testing. Collectively, they result in a larger amount of adulterants in the total supply chain output than under imperfect testing.

[16] When the penalty is very low, all farms adulterate with the maximum dosage under either testing scenario, and hence, the resulting preemptive EMA risk is independent of the testing capability.

8. Conclusions

In this paper, we develop a new set of analytical models to investigate how exogenous quality uncertainty, supply chain dispersion, traceability, and testing sensitivity (with regard to detecting adulteration) jointly impact farms' strategic adulteration behavior in a farming supply chain consisting of a distributed network of farms. We focus on economically motivated adulteration and consider two distinct scenarios: "preemptive EMA" in which adulteration occurs *before* the uncertain quality of a farm's output is realized with the primary goal of reducing the probability of producing low-quality output; "reactive EMA" in which adulteration occurs *after* the uncertain quality of a farm's output is realized with the primary goal of increasing the perceived quality of the output and creating fake high-quality units. We fully characterize the farms' equilibrium adulteration strategies for both scenarios. We show how these strategies are impacted by quality uncertainty, the level of dispersion and traceability in the supply chain, and the sensitivity of the manufacturer's test for adulteration. We calibrate our model based on real cases and field data and demonstrate that the models' predictions are in line with empirical observations. Furthermore, we examine a number of model extensions to confirm that our main conclusions are robust to a few modeling assumptions. Finally, we analyze several managerial levers available to the manufacturer to mitigate EMA risk in the supply chain.

Our analysis offers important and unique insights for policy makers and commercial entities in food supply chains to more proactively address EMA risk. First, we demonstrate when supply chain dispersion increases EMA risk in farming supply chains and how this result depends on traceability and testing sensitivity. Our results complement the supply chain risk management literature in which multi-sourcing is found to be desirable for guarding against disruption risks (e.g., earthquake, fire, hurricane). We study, instead, quality risks related to both exogenous uncertainty and endogenous decisions within the supply chain (i.e., farms adulterating their output). A recent paper by Huang et al. (2017b) empirically shows the adverse effect of supply chain dispersion on EMA risk in China's farming supply chains, based on supply chain and quality data across five different industries. Second, by explicitly modeling exogenous quality uncertainty (common in agricultural production) and distinguishing it from endogenous adulteration decisions, we caution that quality investments need to be accompanied by improvement in testing capabilities. This is because a lower (exogenous) chance of producing low-quality output could inadvertently induce suppliers to endogenously adulterate the realized low-quality units, if the detection of adulteration relies on having a sufficient amount of adulterants in the total output. In such situations, the ability to differentiate fake high-quality products from truly high-quality ones is essential to combat EMA.

Our results have important practical implications and also open up a fruitful avenue for future research. We recommend that companies address risks resulting from supply chain dispersion by mitigating the potential underlying issues in a dispersed supply chain, for example, by enabling better traceability and risk sharing between farms and manufacturers. Some possible strategies include creating farming cooperatives that can allow for better traceability and knowledge transfer in terms of best practices, or developing fairer contracts and support systems such as protective prices and guaranteed distribution channels for farms (especially smallholder farms who face significant financial pressures). Analytical and empirical research is needed to better understand the effectiveness of these remedies in mitigating EMA risk under different market and socioeconomic environments. In addition, future research can build upon our models to further examine other relevant supply chain settings. For example, it would be valuable to jointly consider quality and disruption risks (or yield and demand uncertainty when dispersion may be beneficial) to develop further insights regarding the role of dispersion in a farming supply chain. Analyzing repeated interactions where the manufacturer adapts her testing strategies over time could also be helpful. For policy makers and regulators, we underscore the importance of collecting data and verifying a food manufacturer's sourcing supply chain to more proactively manage EMA risk in food products. Current practices primarily focus on sampling products and inspecting facilities, with limited attention to the upstream parts of the supply chains where agricultural production happens. As countries around the world scale up their food defense efforts, with prominent examples such as the recent enactment of the Food Safety Modernization Act in the U.S. and the new food safety law in China, our results offer timely and actionable insights on the relatively overlooked supply chain perspective in the defense of food safety.

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Online Appendix

O.1. Analytical Details on Model Extensions

O.1.1. Preemptive EMA When Farms are Averse to Quality Uncertainty

To model this extension, we subtract a penalty, $\lambda\mathbb{E}[\mathbb{I}\{n_L > \gamma m\}]$, from the farm's expected payoff function formulated in §3.1. This penalty captures the farm's aversion to producing too many low-quality units, which can threaten the livelihood of his family. The next two theorems characterize farms' equilibrium adulteration behavior under perfect and imperfect testing when they are averse to quality uncertainty.

THEOREM O.1. *For preemptive EMA with perfect testing and farms aversive to quality uncertainty, the total number of adulterating farms in any NE of the game is characterized by* $n_a^{RA^*} = \min\{n, \lceil T^{RA} - 1\rceil\}$, *where*

$$T^{RA} \equiv n^2 \frac{(r_H - r_L)(p_L^{\max} - p_L^{\min}) + (\lambda/m)\left(\overline{F}(\gamma, m, p_L^{\max}) - \overline{F}(\gamma, m, p_L^{\min})\right)}{cqt},$$

and $\overline{F}(\gamma, m, h(x)) \equiv 1 - \int_0^{\gamma m} f(x, m, h(x))\mathrm{d}x$. *We have the following three cases.*

(i) If $T^{RA} \le 1$, *then no farm adulterates.*

(ii) If $T^{RA} > n$, *then all farms adulterate.*

(iii) If $T^{RA} \in (1, n]$, *then any subset of* $n_a^{RA^*}$ *farms adulterating while the rest* $(n - n_a^{RA^*})$ *farms not adulterating constitutes a NE of the game.*

THEOREM O.2. *Define* $T(x) \equiv -h'(x)\dfrac{(r_H - r_L) + \lambda f(\gamma m, m, h(x))\left(\frac{\gamma(1-2h(x))+h(x)}{2h(x)(1-h(x))}\right)}{cq(t/n)((n+1)/n)}$. *For preemptive EMA with imperfect testing and farms aversive to quality uncertainty, if* $\gamma \le \dfrac{p_L^{\min}}{1 - 2p_L^{\min}(1 - p_L^{\min})}$ *, then there exists a unique symmetric NE of the game in which* $x_{RA}^{PV^*}$ *is determined as follows.*

(a) If $c < T(1)$, *then all farms adulterate to the maximum level; i.e.,* $x_{RA}^{PV^*} = 1$.

(b) If $c \ge T(1)$, *then the farms adulterate to some extent; i.e.,* $x_{RA}^{PV^*} \in (0, 1]$ *and is the solution to the equation:* $x = T(x)$.

We observe that the farms' equilibrium adulteration behavior under either testing scenario follows a very similar pattern as in Theorems 1 and 2 in §3.1. The condition on γ in Theorem O.2 means that a farm should begin to exhibit aversion when the number of his low-quality units is not too large. This condition is reasonable given our focus on smallholder farms. Our next result shows that the risk of preemptive EMA in the supply chain is higher when farms are averse to quality uncertainty than when they are expected-profit maximizers, under both perfect and imperfect testing.

PROPOSITION O.1. $n_a^{RA^*} \ge n_a^*$ *and* $x_{RA}^{PV^*} \ge x^{PV^*}$.

With respect to the effect of supply chain dispersion on preemptive EMA risk in the supply chain, we show as in Proposition 2 that greater dispersion leads to higher risk under imperfect testing.

PROPOSITION O.2. *If* $\gamma \le \dfrac{p_L^{\min}}{1 - 2p_L^{\min}(1 - p_L^{\min})}$, *then* $\dfrac{\partial x_{RA}^{PV^*}}{\partial n} \ge 0$.[17]

For the case of perfect testing, we cannot characterize the effect of dispersion on risk analytically. Therefore, we perform extensive numerical simulation and observe that in a total of 32,000,000 numerical instances we run, greater dispersion always leads to a higher risk.[18]

[17] We treat n as a continuous variable in this analysis for tractability.

[18] We use the following parameter values in the numerical simulation: $k = 100000$, 100 m values in $\{100, \ldots, 1000\}$, 20 $p_L^{\max}$ values in $\{0.1, \ldots, 0.9\}$, $p_L^{\min} = 0.1$, $c = 19.07$, 20 q values in $\{0.1, \ldots, 0.9\}$, $r_H = 19.07$, $r_L = 0$, $t = n$, $\lambda = \{1, 2, \ldots, 40\}$, and 20 γ values in $\{0.1, \ldots, 0.9\}$.

O.1.2. EMA Risk When the Quality of All Units from the Same Farm is Perfectly Correlated

In this extension we analyze a setting where *all* units of the same farm are of low (high) quality with probability $p_L^{max}(1-p_L^{max})$. This setup captures scenarios where quality of units of the same farm are highly correlated (in contrast to being independent as in our base model). We again analyze a setting where farms decide whether or not to adulterate low quality units. We first note that our results for preemptive EMA in §3.1 will not change in this setup. This is because farms' expected payoff does not change in either perfect or imperfect testing with this updated distrbution of low quality units. Next, we characterize farms' equilibrium adulteration behavior under both perfect and inperfect testing for reactive EMA.

THEOREM O.3. *(i) For reactive EMA with perfect testing,*

(a) if $r_H - r_L \le q(t/n)c$, then none of the farms adulterate low quality units.

(b) If $r_H - r_L > q(t/n)c$, then all the farms always adulterate low qualtiy units.

(ii) For reactive EMA with imperfect testing,

(a) if $q(t/n)c \le \dfrac{(r_H - r_L)n}{p_L^{max}(n-1)+1}$, then all the farms always adulterate low quality units.

(b) If $cq(t/n) \in (\dfrac{(r_H - r_L)n}{p_L^{max}(n-1)+1}, (r_H - r_L)n)$, then a mixed strategy Nash equilibrium exists where all farms adulterate low quality units with probability $p_{ad} \in (0,1)$ and $p_{ad} = \dfrac{(r_H - r_L)n^2}{cqt(n-1)p_L^{max}} - \dfrac{1}{(n-1)p_L^{max}}$.

(c) if $(r_H - r_L)n \le q(t/n)c$, then farms never adulterate low quality units.

Theorem O.3 shows that under perfect testing, farms always (never) adulterate low quality units if the expected per unit penalty is smaller (larger) than the expected per unit revenue gain from adulteration. In contrast, since expected penalty increases with total amount of adulterants under imperfect testing, a symmetric mixed strategy Nash equilbirum that randomizes between adulteration and non adulteration balances the payoffs. Our next proposition shows that similar to Proposition 2B, the risk of reactive EMA as measured by the total expected number of adulterated output in the supply chain is again increasing in supply chain dispersion.

PROPOSITION O.3. *For reactive EMA with imperfect testing where all units of the same farm are of low quality with probability p_L^{max}, $\dfrac{\partial E_n}{\partial n} \ge 0$.*

O.1.3. Reactive EMA with Decision on How Much to Adulterate

In §3.2.2, we focus on the setup where farms adulterate either all or none of the realized low-quality units. An alternative setup is for farms to decide how many of the realized low-quality units to adulterate. This setup can capture scenarios in which a farm adulterates to fake the overall quality of his output to a desirable level. We first note that our results for perfect testing will not change under this setup. This is because under perfect testing, any amount of adulteration induces the same level of expected penalty, whereas the revenue gain increases in the number of units being adulterated. Hence, a farm would always adulterate all of his low-quality units if he decides to adulterate. Theorem O.4 below characterizes the farms' equilibrium adulteration strategy under imperfect testing, where $a^*(n_{L,i})$ denotes the number of realized low-quality units that farm i adulterates in equilibrium.

THEOREM O.4. *For reactive EMA with imperfect testing where a farm can choose how many of the realized low-quality units to adulterate, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = n_{L,i}$ if $n_{L,i} \in [0, \beta_f^{RV}]$ and $a^*(n_{L,i}) = \beta_f^{RV}$ if $n_{L,i} \in (\beta_f^{RV}, m]$, for all i. The threshold β_f^{RV} is unique and determined as follows:*

(a) If $cq\left(\dfrac{t}{n}\right) \ge \dfrac{n(r_H - r_L)}{2+(n-1)p_L}$, then $\beta_f^{RV} \in (0,m)$ and is the solution to the equation:

$$2\beta = \frac{nk(r_H - r_L)}{cqt} - (n-1)\Big(\int_0^{\beta} x f(x, \tfrac{k}{n}, p_L)\mathrm{d}x + \int_{\beta}^{m} \beta f(x, \tfrac{k}{n}, p_L)\mathrm{d}x\Big).$$

(b) If $cq\left(\dfrac{t}{n}\right) < \dfrac{n(r_H - r_L)}{2+(n-1)p_L}$, then $\beta_f^{RV} = m$.

Theorem O.4 shows that when farms can choose to adulterate a fraction of his low-quality units, then they adulterate all low-quality units up to a threshold, after which they adulterate a constant number of low-quality units. This structure is very similar to that in Theorem 4. The only difference is that in the current setup, when a farm has many low-quality units, the farm would adulterate just enough to make the marginal revenue gain from adulteration equal to the marginal penalty, i.e., adulterating β_f^{RV} units (as opposed to not adulterating at all in Theorem 4). Similar to Proposition 2B, we show that the risk of reactive EMA as measured by the total expected number of adulterated output in the supply chain is increasing in supply chain dispersion.

PROPOSITION O.4. *For reactive EMA with imperfect testing where a farm can choose how many of the realized low-quality units to adulterate,* $\frac{\partial E_n}{\partial n} \geq 0$.

O.1.4. Alternative Models of Imperfect Testing Sensitivity

In this extension, we analyze settings of imperfect testing to model scenarios where detection probability is not linearly increasing in the relative amount of adulterated output in the total supply chain output (as in §3.1.2 and §3.2.2). In particular, we examine three alternative models where the detection probability is (i) convex increasing in the relative amount of adulterated output, (ii) convex increasing in the relative amount of adulterated output and reaches 1 in the interior of the (0, 1) interval, and (iii) linearly increasing in the relative amount of adulterated output and reaches 1 in the interior of the (0, 1) interval. The last two alternatives capture scenarios in which detection probability increases quickly as a small amount of adulterants are added.

Formally, let the detection probability $S_1 : [0,1] \to [0,1]$ be a convex increasing function such that $S_1(0) = 0$ and $S_1(1) = 1$ under scenario (i). That is, the detection probability should be 0 (1) if none (all) of the output is adulterated. Under imperfect testing, if farm i adulterates with x_i, then the chance that the manufacturer detects adulteration when testing the aggregated supply is equal to $\frac{S_1(x_i) + \sum_{-i} S_1(x_{-i})}{n}$. If the manufacturer further tests the individual sample of farm i, then she detects adulteration in the sample with probability $S_1(x_i)$. Since the manufacturer tests the aggregated supply with probability q, the ultimate probability for farm i to be caught if he adulterates is equal to

$$\gamma_i(x_i, x_{-i})) \equiv q\left(\frac{t}{n}\right)\left(\frac{S_1(x_i) + \sum_{-i} S_1(x_{-i})}{n}\right) S_1(x_i).$$

Similarly, the probability for farm i to be caught if he adulterates under imperfect testing is equal to

$$\gamma_i(n_{L,i}, a_{-i}(n_{L,-i})) \equiv q\left(\frac{t}{n}\right) S_1\left(\frac{n_{L,i}}{m}\right) \mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{n_{L,i} + \sum_{-i} n_{L,-i} a_{-i}(n_{L,-i})}{k}\right)\right].$$

Under model (ii), let the detection probability $S_2 : [0,1] \to [0,1]$ be such that $S_2(0) = 0$, $S_2(a)$ is convex increasing in a for $a \in [0, \tau)$, and $S_2(a) = 1$ for $a \in [\tau, 1]$, for some $\tau \in (0,1)$. Lastly, let the detection probability $S_3 : [0,1] \to [0,1]$ under model (iii) be a piecewise increasing function such that $S_3(x) = min(\alpha x, 1)$ for some $\alpha \geq 1$. Note that α captures the detection level of testing. Higher the α, higher is the range in which detection of adulterants happens with perfect accuracy. Under preemptive case, if farm i adulterates with x_i, then the chance that the manufacturer detects the adulteration when testing the aggregated supply is equal to $min(\frac{\alpha(x_i + \sum_{-i} x_{-i})}{n}, 1)$. If the manufacturer further tests the individual sample of farm i, then she detects the adulterants in the sample with probability $min(\alpha x_i, 1)$.

We characterize farms equilibrium adulteration behavior under both preemptive and reactive EMA for the three models. Under preemptive EMA, the equilibrium adulteration behavior (Theorem O.5) again follows the same structure as in Theorem 2 with updated thresholds. In particular, if penalty is smaller than a threshold, then all farms adulterate upto the maximum level and if it is large enough then farms adulterate to some extent but not to the maximum level.

THEOREM O.5. *(i) For preemptive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_1(\cdot)$, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.*

(a) If $c < -h'(1)(r_H - r_L)/[S_1'(1)q(t/n)((n+1)/n)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq [-h'(1)(r_H - r_L)/[S_1'(1)q(t/n)((n+1)/n)]$, then the farms adulterate to some extent; i.e., $x^{PV^*} \in (0,1)$ and is the solution to the following equation: $-h'(x) = S_1(x)S_1'(x)q(t/n)((n+1)/n)c/(r_H - r_L)$.

(ii) For preemptive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_2(\cdot)$, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.

(a) If $c < -h'(\tau)(r_H - r_L)/[S_2(\tau)S_2'(\tau)q(t/n)((n+1)/n)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq -h'(\tau)(r_H - r_L)/[S_2(\tau)S_2'(\tau)q(t/n)((n+1)/n)]$, let $x^* \in (0,\tau)$ be the solution to the equation: $-h'(x) = S_2(x)S_2'(x)q(t/n)((n+1)/n)c/(r_H - r_L)$. There are two cases:

i. *If $c < h(x^*)/[q(t/n)(1-S_2(x^*)^2)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*}=1$.*

ii. *If $c \geq h(x^*)/[q(t/n)(1-S_2(x^*)^2)]$, then all farms adulterate to some extent; i.e., $x^{PV^*} = x^*$.*

(iii) For preemptive EMA with imperfect testing and testing sensitivity modeled by S_3, there exists a unique symmetric NE of the game in which x^{PV^*} is determined as follows.

(a) *If $c < -h'(1/\alpha)(r_H - r_L)/[\alpha q(t/n)((n+1)/n)]$, then all farms adulterate to the maximum level; i.e. $x^{PV^*} = 1$.*

(b) *If $c \leq -h'(1/\alpha)(r_H - r_L)/[\alpha q(t/n)((n+1)/n)]$, then let $x^* \in (0, 1/\alpha)$ be the solution to the following equation: $-h'(x)/x = q(t/n)((n+1)/n)c\alpha^2/(r_H - r_L)$.*

i. *If $c < (h(x^*) - h(1))(r_H - r_L)/[\alpha^2 q(t/n)(1-x^{*2})]$ then $x^{PV^*} = 1$*

ii. *If $c \geq (h(x^*) - h(1))(r_H - r_L)/[\alpha^2 q(t/n)(1-x^{*2})]$ then $x^{PV^*} = x^*$*

Under reactive EMA, we again analyze a setting where a farm adulterates either all or none of his realized low-quality units; i.e., the adulteration strategy can be characterized by the mapping $a_i(n_{L,i}) : \{1, \ldots, m\} \to \{0, 1\}$.

THEOREM O.6. *(i) For reactive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_1(\cdot)$, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^S)$ and $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^S, m]$, for all i. The threshold β^S is unique and determined as follows:*

(a) If $cq\left(\frac{t}{n}\right) \geq \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then $\beta^S \in (0, m)$ and is the solution to the equation:

$$\beta^S(r_H - r_L) = q\left(\tfrac{t}{n}\right) S_1\left(\tfrac{\beta^S}{m}\right) \mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{\beta^S + \sum_{-i} n_{L,-i}\mathbb{I}\{n_{L,-i} < \beta^S\}}{k}\right)\right] cm,$$

where $\mathbb{I}\{\cdot\}$ is an indicator function whose value is 1 if the argument is true and 0 otherwise.

(b) If $cq\left(\frac{t}{n}\right) < \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then $\beta^S = m$.

(ii) For reactive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_2(\cdot)$, in any BNE of the game, there exist thresholds β^l and β^u such that $\beta^l < \beta^u$ and the following must hold:

(a) If $cq\left(\frac{t}{n}\right) \geq \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then in equilibrium $a^*(n_{L,i}) = 1$ if $n_{L,i} < \beta^l$ or if $n_{L,i} > \beta^u$; $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^l, \beta^u]$. Furthermore, we must have $\beta^u \geq \tau m$.

(b) If $cq\left(\frac{t}{n}\right) < \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then all farms always adulterate.

(iii) For reactive EMA with imperfect testing and testing sensitivity modeled by S_3, there exists a symmetric BNE of the game in which a farm's adulteration strategy is a threhold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^S]$ or if $n_{L,i} \in [\beta^U, m]$.

(a) *If $cq(t/n) \leq max\Big(\frac{r_H - r_L}{\alpha}, \frac{k(r_H - r_L)}{\alpha(p\alpha(k-m)+m)}\Big)$ then $\beta^S = \beta^U = m$*

(b) *If $cq(t/n) > max\Big(\frac{r_H - r_L}{\alpha}, \frac{k(r_H - r_L)}{\alpha(p\alpha(k-m)+m)}\Big)$ then*

$$\beta^S = max\Big(0, \frac{k(r_H - r_L)}{cq\alpha^2} - (\frac{k}{m} - 1)(\textstyle\int_0^{\beta^S} xf(x,m,p)dx + \int_{\beta^U}^m xf(x,m,p)dx)\Big) \text{ and}$$

$$\beta^U = min\Big(m, \frac{cqm(t/n)}{r_H - r_L}, \frac{(\frac{k}{m} - 1)(\int_0^{\beta^S} xf(x,m,p)dx + \int_{\beta^U}^m xf(x,m,p)dx)}{\frac{k(r_H - r_L)}{cqm\alpha} - 1}\Big)$$

Theorems O.5 and O.6 below show that the farms' equilibrium adulteration behavior under scenario (i) follows a very similar structure as in Theorems 4 in and §3.2.2. Under scenario (ii) and (iii), the structure of the equilibrium strategy in Theorem O.6

can be viewed as a combination of the equilibrium strategies described in Theorems 3 and 4 in §3.2. In particular, the equilbrium strategy here combines the adulteration strategies identified in perfect and imperfect testing. If the number of low quality units are very low, then the testing sensitivity is similar to the imperfect testing case and farms adulterate when their low quality units are less than a threshold. In contrast, if low quality units are greater than a thereshold, then testing sensitivity is similar to perfect testing and farms adulterate when their low quality units are greater than a threshold.

O.1.5. Alternative Penalty Structures

In this extension, we first analyze cases in which per unit penalty is linearly increasing in the amount of adulterants. This section is valuable in capturing scenarios where harmful effects of adulterants are increasing in the amount of adulterants. We model such case by assuming that penalty is linearly increasing in the amount of adulterants. Thus for a farm that adds x_i adulterants per unit under preemptive testing and is caught, the penalty is $cm(mx)$. We again find that the equilibrium structure under all the scenarios is similar to the ones identified in Section§3.2 and §3.1 except in one case: For penalty alternative (i) and preemptive EMA with perfect testing, adulterating farms use an amount of adulterants that balances the revenue gain with the penalty from adulterating. The additional tradeoff arise because the penalty on being caught adulterating is no longer constabt but increaisng in the amount of adulterants.

THEOREM O.7. *(i) For preemptive EMA with perfect testing, there exists a unique symmetric NE in which x^{PP^*} is determined as follows.*

(a) If $c \geq -h'(0)(r_H - r_L)n/[mq(t/n)(n+1)]$, then

$$n_a^* = \left\lceil \frac{-h'(0)(r_H - r_L)n^2}{mcqt} - 1 \right\rceil \tag{O.1}$$

(b) If $c < -h'(0)(r_H - r_L)n/[mq(t/n)(n+1)]$, then $n_a^* = n$

Let x^* be the solution to the following equation:$-h'(x)(r_H - r_L) = mcq(t/n)(n_a^*/n)$. Then $x_a^{PP^*} = min(x^*, 1)$ and $x_a^{PP^*} \in (0,1)$.

(ii) For preemptive EMA with imperfect testing, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.

(a) If $c < -h'(1)(r_H - r_L)/[mq(t/n)((2n+1)/n)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq [-h'(1)(r_H - r_L)/[mq(t/n)((2n+1)/n)]$, then the farms adulterate to some extent; i.e., $x^{PV^*} \in (0,1)$ and is the solution to the following equation: $-h'(x)/x^2 = q(t/n)((2n+1)/n)c/(r_H - r_L)$.

(iii) For reactive EMA with perfect testing,

(a) if $r_H - r_L \leq q(t/n)cm$, then none of the farms adulterate; i.e. $\beta^{RP^*} = 0$.

(b) If $r_H - r_L > q(t/n)cm$, then all the farms adulterate to the maximum level; i.e., $\beta^{RP^*} = m$

(iv) For reactive EMA with imperfect testing, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^{RV})$ and $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^{RV}, m]$, for all i. The threshold β^{RV} is unique and determined as follows:

(a) If $cq\left(\frac{t}{n}\right) \geq \frac{k(r_H - r_L)}{m(m+(n-1)mp)}$ then $\beta^{RV} \in (0,m)$ and is the solution to the equation: $\beta^2 + \beta(n-1)\int_0^\beta x f(x, \frac{k}{n}, p_L)\mathrm{d}x = \frac{nk(r_H - r_L)}{cqt}$.

(b) If $cq\left(\frac{t}{n}\right) < \frac{k(r_H - r_L)}{m(m+(n-1)mp)}$, then $\beta^{RV} = m$.

Next, we find qualitative evidence that larger companies who are caught adulterating face more severe penalty than smaller ones. For example, they face longer jail terms and are fined more heavily (Yan 2017). This observation can be captured by modeling the total penalty from adulterating as convex increasing in m. We find that all of our results continue to hold in this alternative setup with updated threshold values.

PROPOSITION O.5. *All of our results in §3.1 and §3.2 continue to hold if the penalty that a farm incurred for adulterating is convex increasing in the total number of units, m, supplied by the farm.*

O.2. Investing in Traceability and Testing Frequency to Mitigate EMA Risk

Given a supply network of farms, the manufacturer has two levers to mitigate the risk of EMA in the supply chain: increasing supply chain traceability and the frequency of testing the aggregated supply. Developing these capabilities can be costly. For example, it is very difficult to trace and inspect every individual farm in a supply chain sourcing from thousands of farms (Nestlé 2015). Therefore, the manufacturer needs to balance between the cost of investing in these capabilities and the benefit of reducing EMA risk in the supply chain. To address this tradeoff, we develop an optimization model from the manufacturer's perspective, where the objective is to minimize total investment costs while satisfying a constraint that the resulting risk of EMA in the supply chain cannot exceed a certain level.

First consider preemptive EMA. In this setting, the overall risk of EMA in the supply chain is measured by n_a^*/n under perfect testing and x^{PV^*} under imperfect testing (see §4). Define $l(q)$ and $g(t)$ as the manufacturer's investment costs for increasing testing frequency and traceability, both of which are convex and increasing functions. The manufacturer's optimization problem under preemptive EMA can be characterized as follows.

$$\Pi^{PP}(q,t) \equiv \min_{q,t} \{l(q)+g(t) \,|n_a^*/n \le \alpha,\ q\in[0,1],\ t\in[0,n]\}, \tag{O.2}$$

$$\Pi^{PV}(q,t) \equiv \min_{q,t} \left\{l(q)+g(t) \,\middle|\, x^{PV^*} \le \alpha,\ q\in[0,1],\ t\in[0,n]\right\}, \tag{O.3}$$

where n_a^* and x^{PV^*} are defined in Theorems 1 and 2, and α is the maximum level of risk allowed. For reactive EMA, the manufacturer's optimization problem can be modeled similarly as follows.

$$\Pi^{RI}(q,t) \equiv \min_{q,t} \{l(q)+g(t) \,|P_n \le \alpha,\ q\in[0,1],\ t\in[0,n]\}, \tag{O.4}$$

$$\Pi^{RE}(q,t) \equiv \min_{q,t} \{l(q)+g(t) \,|E_n \le \alpha,\ q\in[0,1],\ t\in[0,n]\}, \tag{O.5}$$

where P_n and E_n are defined in §4 given the farms' optimal adulteration strategies under perfect and imperfect testing, characterized in Theorems 3 and 4. The key difference is that we measure the risk of reactive EMA in the supply chain in two ways: the probability of an individual farm adulterating (i.e., P_n as in Model (O.4)) and the expected total amount of adulterated output in the supply chain (i.e., E_n as in Model (O.5)).

Before characterizing the manufacturer's optimal investment strategy under each of these model scenarios, we first define the following useful constants.

(i) For Model (O.2):

$$u^{PP} \equiv \frac{(r_H-r_L)(p_L^{\max}-p_L^{\min})n^2}{c(1+\lfloor n\alpha\rfloor)}. \tag{O.6}$$

(ii) For Model (O.3):

$$u^{PV} \equiv \frac{-h'(\alpha)(r_H-r_L)n}{\alpha c(n+1)/n}. \tag{O.7}$$

(iii) For Model (O.4) under perfect testing:

$$u^{RP} \equiv \left(\frac{n^2(r_H-r_L)}{ck}\right)\left(\frac{p_Lk}{n}+\phi^{-1}(1-\alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}\right). \tag{O.8}$$

(iv) For Model (O.4) under imperfect testing:

$$u^{RV} \equiv \left(\frac{kn(r_H-r_L)}{c}\right)\left(\frac{p_Lk}{n}+\phi^{-1}(\alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}+(n-1)\int_0^{\frac{p_Lk}{n}+\phi^{-1}(\alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}} xf(x,k/n,p_L)dx\right)^{-1}. \tag{O.9}$$

The notation ϕ represents the PDF of the standard normal distribution. These constants are the values of qt when the risk constraint in the corresponding models indicated is binding. Note that in the optimal solution to these models, the risk constraint must be binding because n_a^*, x^{PV^*}, and P_n are all decreasing in q and t, whereas the investment costs are increasing in q and t. The following theorem summarizes the manufacturer's optimal investment strategy for Models (O.2), (O.3), and (O.4) under perfect and imperfect testing.

THEOREM O.8. *Given the constants u^j with $j \in \{PP, PV, RP, RV\}$ defined in Equations* (O.6)–(O.9)*, we have the following results for $\alpha \leq 0.5$.*

(i) If $u^j > n$, then the corresponding manufacturer problem is infeasible.

(ii) If $u^j \leq n$, then the optimal solution to the corresponding manufacturer problem (q^*, t^*) can be characterized as follows.

(a) If $l'(1) \leq u^j g'(u^j)$, then $(q^*, t^*) = (1, u^j)$.

(b) If $g'(n) \leq \frac{u^j}{n^2} l'\left(\frac{u^j}{n}\right)$, then $(q^*, t^*) = \left(\frac{u^j}{n}, n\right)$.

(c) If $l'(1) > u^j g'(u^j)$ and $g'(n) > \frac{u^j}{n^2} l'\left(\frac{u^j}{n}\right)$, then $(q^*, t^*) \in (0,1) \times (0,n)$ and satisfy the following first-order conditions: $q^* = \sqrt{\frac{u^j g'(u^j/q^*)}{l'(q^*)}}$ and $t^* = \frac{u^j}{q^*}$.

Theorem O.8 part (i) suggests that if the manufacturer cannot satisfy the risk constraint even when the supply chain is fully traceable and she always tests the aggregated supply, then additional levers are necessary to meet the risk constraint. Given our earlier discussions in §4, one possible solution is to reduce supply chain dispersion. Theorem O.8 part (ii) shows that when a feasible solution exists, the manufacturer always chooses the solution with the best cost-effectiveness. If the marginal cost at maximum testing frequency is lower than the marginal cost at the minimum necessary traceability to satisfy the risk constraint (i.e., increasing testing frequency is in general more cost effective than increasing traceability; Theorem O.8 part (ii-a)), then it is optimal for the manufacturer to always test the aggregated supply and build just enough traceability given the risk constraint. Conversely, if increasing traceability is in general more cost effective than increasing testing frequency (Theorem O.8 part (ii-b))), then it is optimal for the manufacturer to build full traceability in the supply chain and test just enough given the risk constraint. If neither of the above is true (Theorem O.8 part (ii-c))), then the optimal investment is an interior solution that achieves the best cost balance between investing in the two levers.

Our next proposition characterizes how the optimal investment solution (q^*, t^*) described in Theorem O.8 and the resulting optimal cost change with supply chain dispersion.

PROPOSITION O.6. *Let SC_H and SC_L be two supply chains such that the supply chain dispersion in SC_H is greater than that in SC_L (i.e., $n_H > n_L$). Consider each of the manufacturer's optimization problems formulated in Models* (O.2)*,* (O.3)*, and* (O.4) *under perfect and imperfect testing. We have the following results for $\alpha \leq 0.5$.*

(i) If the manufacturer's problem is infeasible for SC_L, then it is also infeasible for SC_H.

(ii) If the manufacturer's problem is feasible for SC_H, then it is also feasible for SC_L.

(iii) Assume that the manufacturer's problem is feasible for SC_H. Let (q_H^*, t_H^*) and (q_L^*, t_L^*) be the optimal solution for SC_H and SC_L respectively. Then, $q_L^* \leq q_H^*$, $t_L^* \leq t_H^*$, and the resulting optimal cost for the manufacturer is lower in SC_L than in SC_H.

Proposition O.6 highlights two results. First, given a desirable risk constraint, it is always more difficult for a manufacturer with a more dispersed supply chain to satisfy the constraint (parts (i) and (ii)). Second, conditional on being able to satisfy the risk constraint, it is always more costly for a manufacturer with a more dispersed supply chain to do so (part (iii)). Therefore, higher supply chain dispersion results in greater challenges for a manufacturer to manage and mitigate the risk of individual farms adulterating, from both feasibility and financial standpoints.

Finally, we consider the manufacturer's problem formulated in Model (O.5). The key difference in this model versus the others is that the risk constraint is imposed on E_n, the expected total amount of adulterated output in the supply chain. Since E_n aggregates all farms' adulteration decisions, we cannot derive the manufacturer's optimal decisions analytically. Nevertheless, consistent with Proposition O.6, we show that higher supply chain dispersion again makes it more costly for the manufacturer to satisfy a desirable risk constraint, regardless of testing sensitivity (perfect or imperfect testing).

PROPOSITION O.7. *Let SC_H and SC_L be two supply chains such that the supply chain dispersion in SC_H is greater than that in SC_L (i.e., $n_H > n_L$). Consider the manufacturer's optimization problem formulated in Model* (O.5) *and assume that it is feasible for SC_H. If $\alpha \leq n \int_{mp/3} x f(x, m, p) \mathrm{d}x$, then for both perfect and imperfect testing, the optimal cost for the manufacturer is lower in SC_L than in SC_H.*

Table O.1 Total EMA Risk When Farms Engage in Both Preemptive and Reactive EMA

	Reactive EMA, perfect testing	Reactive EMA, imperfect testing
Preemptive EMA, perfect testing	$\lambda m n_a^* + (n-n_a^*)\int_{\beta^{RP}}^{m} x f(x,m,p_L^{\max})dx$	$\lambda m n_a^* + (n-n_a^*)\int_{0}^{\beta^{RV}} x f(x,m,p_L^{\max})dx$
Preemptive EMA, imperfect testing	$\lambda k x^{PV^*} + n\int_{\beta^{RP}}^{m} x f(x,m,h(x^{PV^*}))dx$	$\lambda k x^{PV^*} + n\int_{0}^{\beta^{RV}} x f(x,m,h(x^{PV^*}))dx$

O.3. Investing in Testing Capabilities to Mitigate EMA Risk

We examine the effect of the manufacturer investing in perfect testing on mitigating EMA risk in the supply chain. To this end, we allow farms to engage in both preemptive and reactive EMA in response to the manufacturer's testing capability. The model dynamics are similar to those described in §2 with the first two steps revised as follows. (i) The manufacturer chooses whether or not to adopt perfect testing for preemptive or reactive EMA respectively. The farms observe the manufacturer's choice. (ii) Each farm simultaneously and individually decides the amount of adulterants to add to reduce the likelihood of producing low-quality output from $p_L^{\max}$ to some $p_L \le p_L^{\max}$ (preemptive EMA). (iii) The uncertain quality of each unit of output is realized. (iv) Each farm simultaneously and individually decides whether or not to adulterate all of the realized low quality units n_L to create fake high-quality ones (reactive EMA). The remaining steps are exactly the same as in Figure 1. We are interested in analyzing how the total EMA risk, accounting for both preemptive and reactive EMA, is affected by the manufacturer's testing capability for either type of EMA. In particular, we analyze whether adopting perfect testing always reduces EMA risk in the supply chain. In this analysis, we take the farms to be short-term oriented (see footnote 8), and thus, they do not account for every possible realization of n_L and the corresponding reactive EMA decision when making their preemptive EMA decision. To simplify exposition, we also assume that when the farms adulterate preemptively with the maximum dosage, p_L becomes 0. Our results remain qualitatively the same without this simplifying assumption. Table O.1 summarizes the total EMA risk in the supply chain for the four different scenarios we analyze in §3. For example, the top left cell shows the total EMA risk if the manufacturer invests in perfect testing for both preemptive and reactive EMA. By Theorem 1 we know that under perfect testing, a subset of n_a^* farms adulterate preemptively with the maximum dosage while the remaining do not adulterate at all. Thus, the expected total amount of adulterants added preemptively is mn_a^*. In the reactive EMA stage, we again know from Theorem 3 that under perfect testing, farms adulterate when their low-quality units are greater than β^{RP}. Hence, the total EMA risk in the supply chain is equal to $\lambda m n_a^* + (n-n_a^*)\int_{\beta^{RP}}^{m} x f(x,m,p_L^{\max})dx$. Note that λ here measures the importance of preemptive EMA relative to reactive EMA when the manufacturer evaluates the total EMA risk. We can similarly characterize the total EMA risk in the supply chain for the other three scenarios.

By Proposition 1, we know that reactive EMA risk is always lower when the manufacturer adopts perfect testing. Thus, we only need to compare the total EMA risk in the left two cells in Table O.1. We first focus on comparing the preemptive EMA risk between these two scenarios. Our results are summarized in the next proposition.

PROPOSITION O.8. *Let $R_p^P \equiv mn_a^*$ and $R_{ip}^P \equiv kx^{PV^*}$ denote the preemptive EMA risk under perfect and imperfect testing respectively . Then,*

(i) If $c < -h'(1)(r_H - r_L)/[q(t/n)(n+1)/n]$, then $R_p^P = k$ and $R_{ip}^P = k$. Thus, all farms adulterate to the maximum level under both perfect and imperfect testing cases.

(ii) If $c \in [-h'(1)(r_H - r_L)/[q(t/n)(n+1)/n, (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)]]$, then $x^{PV^*} \in (0,1)$ and $n_a^* = n$. Thus, $R_p^P \ge R_{ip}^P$, i.e., preemptive EMA risk is higher under perfect testing than under imperfect testing.

(iii) If $c \ge (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)]]$, and

(a) *If $c < (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)h^{-1}((p_L^{min} - p_L^{max})(1+1/n))]$, then $R_p^P \ge R_{ip}^P$, i.e., preemptive EMA risk is higher under perfect testing than under imperfect testing.*

(b) *If $c \ge (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)h^{-1}((p_L^{min} - p_L^{max})(1+1/n))]$ then $R_{ip}^P \ge R_p^P$, i.e., preemptive EMA risk is higher under imperfect testing than under perfect testing.*

Proposition O.8 shows that when the per-unit penalty is not high enough, adopting perfect testing can in fact backfire and result in *higher* preemptive EMA risk. This is because under perfect testing, adulterating farms all adulterate with the maximum dosage, while under imperfect testing, they adulterate at a lower level to trade off revenue gain with the expected penalty. When the penalty is not high enough, more farms adulterate to the maximum dosage under perfect testing, therefore leading to a higher risk. Our next result shows that this observation remains true when considering the total EMA risk that also accounts for reactive EMA.

THEOREM O.9. *Let R_p^T denote the total EMA risk under perfect testing for both preemptive and reactive EMA, and R_{ip}^T the total EMA risk under imperfect testing for preemptive EMA and perfect testing for reactive EMA. We have the following results.*

(i) If $c < -h'(1)(r_H - r_L)/[q(t/n)(n+1)/n]$, then $R_p^T = R_{ip}^T$.

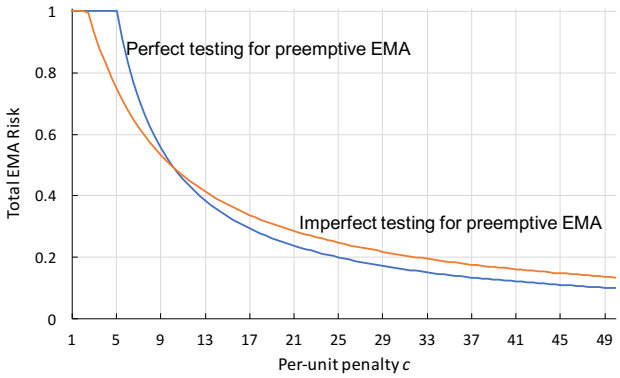
(ii) If $c \geq (r_H - r_L)/[q(t/n)]$, and

(a) *If $c < (r_H - r_L)(p_L^{max})/[q(t/n)h^{-1}(-p_L^{max}(1+1/n))]$), then $R_p^T \geq R_{ip}^T$.*

(b) *If $c \geq (r_H - r_L)(p_L^{max})/[q(t/n)h^{-1}(-p_L^{max}(1+1/n))]$), then $R_{ip}^T \geq R_p^T$.*

We complement Theorem O.9 with extensive numerical simulation for the range of c values that we cannot characterize the total risk analytically. Figure O.1 presents a representative pattern of how the total EMA risk changes with c under either perfect or imperfect testing for preemptive EMA and perfect testing for reactive EMA. Observe that adopting perfect testing for preemptive EMA in fact leads to higher total EMA risk inadvertently when c is not sufficiently high (for $c < r_H$ in this example).

Figure O.1 Total EMA Risk under Perfect or Imperfect Testing for Preemptive EMA and Perfect Testing for Reactive EMA



Note. We use the following parameters in this example: $k = 100,000$, $m = 1,000$, $q = 1$, $r_H = 10$, $r_L = 0$, $t = n$, $c \in \{1, 1.5, \ldots, 50\}$, $p_L^{\max} = 0.5$, and $p_L^{\min} = 0$.