

MODELLING CHOICE SET FORMATION IN DISCRETE CHOICE MODELS

by

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submitted to the Department of Civil Engineering
in partial fulfillment of the requirements of
the degree of

Doctor of Philosophy

ABSTRACT

This research aims at extending discrete choice modelling by adding an explicit representation of individuals' access to various alternatives.

Previous work in probabilistic choice set models have addressed this issue by focusing on the existence of constraints that imply the unavailability of certain alternatives. This approach finds its origins in earlier work in which logical rules were used to define the individual's choice set.

The motivation of this work is to incorporate into a single framework of choice set generation modelling the effects of stochastic constraints and the influence of perceptions and attitudes on the choice set generation process. The contribution of this thesis is methodological. The basic idea being to estimate a choice set formation model by using information contained in responses to alternative availability questions. The estimation approach developed by this thesis uses jointly the information on the individual's perceived choice set and the revealed preference information corresponding to the observed choice. The proposed methodology is empirically tested with mode to work choice data from Baltimore, Maryland.

The constraint based choice set formation models developed and estimated in this thesis are shown to be appropriate in a short-run setting for which the constraints are fixed. Thus, the previous framework is contrasted with a more dynamic approach which is appropriate in the long-run. For this purpose, a habit model, based on the recognition of the routine nature of certain choices is derived and empirically tested with mode to work choice data from Maceio, Brazil.

The estimation results show the potential gains, in terms of increased efficiency of the parameter estimates, of using, in addition to the information provided by the observed choice, information on perceived choice sets to estimate the models. The estimation results for the habit model show that it fits the data better than a logit or a logit captivity model thus confirming the routine nature of mode to work choices.

Thesis Supervisor: Dr Moshe Ben-Akiva
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CHAPTER 1

INTRODUCTION AND STATEMENT OF OBJECTIVES

1-1 Purpose and Motivation of the Thesis

This research aims at extending discrete choice modelling by adding an explicit representation of individuals' access to various alternatives. The characterization of the access to the different alternatives is naturally important since someone without access to a certain alternative cannot choose it.

Previous work in probabilistic choice set models (Swait, 1984; Swait and Ben-Akiva, 1987a and 1987b; Kitamura and Lam, 1984) have addressed the issue by focusing on the existence of operative probabilistic constraints that imply the unavailability of certain alternatives. This approach finds its origins in even earlier work (see, for example, Ben-Akiva and Lerman, 1974) where logical rules (i.e. deterministic constraints) were used to impute the individual's choice set.

An individual's choice set depends on the individual's specific environment, and the derivation of the choice set faced by an individual requires a characterization of that environment. This is a difficult task since that environment reflects not only objective constraints (e.g individual's socio-economic characteristics or attributes of the alternatives) but also subjective constraints that are related to the individual's attitudes and perceptions.

The state of the art of choice set generation modelling is limited in its behavioral interpretation of the choice set formation stage of the choice process by the fact that it has excluded attitudes and perceptions variables from the analysis, thus ignoring the subjective constraints. This is not to say that the influences of perceptions and attitudes on choice have not been researched. (See, for example, Benjamin and Sen, 1982.) However, the two behavioral aspects of the choice process, namely

- i) the influence of constraints on the choice set generation process and
 - ii) the influence of perceptions and attitudes on choices,
- have mostly been addressed separately by the literature.

Thus the motivation of our work is to integrate both of these aspects into a single framework of choice set generation modelling. The framework aims at incorporating the influence of individual's perceptions on the choice set formation. The contribution of this thesis is methodological. The basic idea is to estimate a choice set formation model by using information contained in responses to alternative availabilities questions. The rationale for this approach is described in what follows: The availability of each alternative is dependent upon the existence of unobservable (i.e. latent) constraints that determine an individual's exact choice set. The exact choice set is by definition the set of all alternatives to which, given the above constraints, the individual would have access. Therefore it is proposed to estimate a constraint based choice set formation model by making use of the information that is contained in alternative availabilities survey questions. The estimation approach developed by the thesis jointly uses this information on the individual's perceived choice set and the revealed preference information corresponding to the observed choice.

The thesis also aims at testing the proposed framework, and empirical applications are shown. Furthermore, the constraint based choice set formation model developed and estimated in the thesis is shown to be appropriate in a short-run setting in which the constraints are fixed.

Temporal aspects of choice set formation are addressed by the thesis which analyzes issues involved in differentiating short-run and long-run choice by developing and empirically testing a model based on the notion of habitual choice.

1-2 Scope of Thesis

This research sets for itself three objectives.

The first objective is to develop a probabilistic treatment of choice set formation that explicitly considers the influence of constraints, both objective and subjective, on the availability of alternatives. The proposed framework is based on a methodology that permits, through the exploitation of information on perceived access to various alternatives contained in survey responses to alternatives availabilities questions, the estimation of a latent variable probabilistic choice set model.

The second objective is to illustrate with an empirical application the theoretical developments of the thesis. The data used is on transportation mode to work choice for the city of Baltimore, Maryland. An effort is made to analyze the technical aspects of the estimation of the models that involve both observable and latent variables. The estimation results are useful in assessing the potential gains, in terms of increased efficiency of the parameters estimates, of using information provided by the responses to survey questions on perceived alternative availabilities in addition to the information provided by the observed choice.

The third objective is to address issues related to the temporal aspects of choice behavior. A choice model based on the notion of habit formation is developed and empirically tested with

mode to work choice data from the city of Maceio, Brazil. This enables us to show the limitations of the models developed and estimated earlier in the thesis by contrasting a dynamic approach to choice set formation with the more static view that is implicit in our work.

The thesis essentially focuses on the models of choice set generation based on the identification of constraints and on the utilization of information on perceived choice sets to estimate these models.

1-3 Outline of Thesis

Chapter 2 serves as a background chapter. It reviews the literature on probabilistic choice set generation models. This is essentially a summary of the recent contributions by Swait (1984) and Swait and Ben-Akiva (1987a and 1987b). Most of the work on choice set formation has resulted in model specifications that essentially ignore the subjective aspects that influence the determination of the individual's choice sets. However, as stated earlier, there has been research work on the influence of perceptions, feelings and preferences on individual's choice behavior. Chapter 2 also reviews this research effort on models of attitudes and choice.

The purpose of Chapter 3 is to address some of the issues raised in Chapter 2 by introducing our theoretical approach for the analysis of choice set formation. The proposed framework is based upon the description of the interactions of an individual and his or her environment resulting in the identification of random constraints that explain choice set formation. As we explained it in Section 1-1 above, the special feature of our approach is the use of responses to survey questions on perceived alternative availabilities which we relate through measurement equations to the

latent variables of the problems such as the latent constraints or the latent utilities. Appendix 1, which supplements this chapter, shows that our methodology is a special case of a more general integrated framework.

In Chapters 4 and 5, we present the calibration results for a standard discrete choice model, for probabilistic choice set models and for applications of the integrated framework. Estimation results are used to address a model specification issue on the appropriateness of the random constraint formulation (as opposed to a simpler multinomial logit) and to test the feasibility of using, for estimation purposes, information on perceived choice sets.

Chapter 4 provides a detailed description of the Baltimore data set and reviews the Baltimore data collection effort which aimed at providing an empirical derivation of individuals' choice sets. The calibration results given in this chapter are to serve as a benchmark for the extensions of the same models that are presented in Chapter 5. A comparison of the prediction results of the multinomial logit model and of the selected probabilistic choice set model is also provided.

Chapter 5 explores different specifications of the measurement equations of the models. The structural equations (utilities, alternative availabilities and choice set structure) are common to all models and are the same as in Chapter 4. Each model specification corresponds to a different way of modelling the influence of desirability on the responses to the alternative availabilities questions. Chapter 5 also shows the derivation of the log-likelihoods associated with each of the suggested model extensions. It is supplemented by Appendix 2 that provides the details of these derivations.

The purpose of Chapter 6 is to derive a model based on the observation that choices are often of a routine nature. Chapter 6 allows us to differentiate short-run choice models from long-run choice models. Static views of choice set generation that are implicit in most empirical work

(including in our models of Chapters 4 and 5) and dynamic approaches introduced in this chapter, are not necessarily in contradiction, and the two approaches are in fact complementary. Technical derivations of the choice probabilities of the resulting habit model are shown in Appendix 3 which supplements Chapter 6. Model estimation results are given for a logit, a logit captivity and a habit model with mode to work choice data from Maceio, Brazil. A comparison of the model prediction results is also shown. The results show that the habit model fits the data best.

Essentially, the methodological development of the thesis, which is its most important contribution, is an estimation approach of behavioral probabilistic choice set models that uses simultaneously revealed preference information and information contained in survey questions to perceived alternative availabilities. The empirical results of Chapter 4 confirm that probabilistic choice set models constitute a significant improvement over the multinomial logit. The empirical results of Chapter 5 show that the use of the survey responses on perceived choice sets improves significantly the precision of the parameters estimates.

Recommendations for further research are given in Chapter 7, which also presents an overview of the dissertation and summarizes its principal contributions.

CHAPTER 2

LITERATURE REVIEW

2-1 Introduction

The purpose of this chapter is to review and summarize the existing approaches to modelling individuals' access to various alternatives in a choice situation. It first reviews the existing literature on choice set formation models. This literature has focused on the existence of constraints that affect alternative availability but has for the most part ignored individuals' perceptions and attitudes toward transportation modes. However, recent developments in the transportation literature have resulted in models that take into account the influence of attitudes on choices.

This chapter is organized as follows:

Section 2-2 reviews models which examine the existence of situational constraints. These models address the issue of choice set formation. Section 2-3 reviews models which examine attitudinal differences that are intermediate elements in the decision making process. These models address the issue of incorporating psychometric data into choice theory. Section 2-4 concludes the chapter. It indicates the need of incorporating constraints and attitudes in a single framework that must be operationalized by integrating behavioral choice models with market research developments.

2-2 Influence of Constraints on the Choice Set Generation Process

Traditionally, choice models of travel mode have been based on the analysis of relationships between observed travel behavior and measures of system performance (engineering characteristics) and demographic characteristics. (See, for example, Ben-Akiva, 1973). As a result, the models have provided a limited understanding of the complexity of the individual's decision process. Recently approaches have appeared in the literature emphasizing the need to have a greater insight into the travel choice process. (See, for example, Kitamura and Lam, 1984; Koppelman and Lyon, 1981; Koppelman and Pas, 1980 and Swait and Ben-Akiva, 1987a).

As formulated by Manski (1977), choice theory postulates a two-stage choice process:

- i) generation of the set of alternatives from which the individual will make his or her selection;
- ii) selection of an alternative from the previously defined choice set.

Choice set generation is a process whereby feasible alternatives for the individual are identified. Understanding this process must start with the identification of the factors which establish the feasibility or infeasibility of each alternative. In order to do this, it is necessary to characterize the interconnections between the individual and his or her environment.

Clearly, an individual faces only limited options in making his travel decisions since many of these decisions are subject to some type of constraints. Observed travel behavior is the result of the choices made by many individuals under the influence of such constraints. These constraints are the results of personal circumstances, the characteristics of the existing transportation system and social considerations. Swait (1984) proposes a typology of constraints to individual urban travel that encompasses

- (a) household,

(b) societal, and

(c) personal constraints.

This typology is helpful in helping us understand the issues involved.

The personal constraints are primarily determined by the individual's affiliation within his or her household. This suggests that it is impossible to consider an individual decision maker in isolation. Instead, individual activities are primarily determined by the other members of the household (Brog and Erl, 1979).

To our view, there are two major categories of household constraints. The first of these are physical constraints. Examples in this category are residential location and resource availability (e.g. household income, automobile ownership). The second category of constraints is related to the individual's status within the household. For example, a secondary worker may not actually have access to a car. The differences in access to automobile within the same household are described in Supernak and Schoendorfer (1986). This last category of household constraints is related to household structure. Dynamic changes in the family modify this structure and therefore each individual's role within the family. By differentiating along dimensions such as lifecycle or lifestyle, Salomon (1980) shows that the latter can help identify relevant constraints on individual behavior.

Constraints resulting from the limitations imposed by the existing physical transportation system are part of what Swait considers societal constraints. These constraints are naturally the easiest to incorporate in choice set models since they can often be treated in a deterministic way.

Ben-Akiva and Lerman (1974), for instance, model the joint choice of auto-ownership level and mode to work by using deterministic alternative availability rules such as lack of walk access to transit for households situated in suburban areas.

The last group of constraints are personal constraints. These include objective restrictions to mode access such as possession of a driver's license and subjective restrictions that are related to the individual's attitudes and perceptions. For example, the head of the household, because of his perception molded by society, may be the only member to have use of an automobile. Closely related to the latter is the individual's access to information (and his attitude toward new information). Meyer (1979)'s work deals with constraints related to the availability of information by analyzing how individuals perceive new opportunities when subject to spatial and time constraints.

Having reviewed the concept of constraint, we can now look at the approaches that have been taken to incorporate these constraints in the modelling of choice set generation.

Constraints have been incorporated into disaggregate choice models by treating them as limitations imposed on the choice set.

Kitamura and Lam (1984) adopt a constraint-based approach in formulating binary choice models which they test using data generated by Monte Carlo simulation. Their example, using time and monetary budget limitations, shows that if the choices are constrained, there are significant differences between the constrained model results and the ordinary logit model results. This shows the inadequacy of model specifications that do not incorporate these constraints.

The limitations of estimation work that basically ignored the problem of choice set specification by assuming that everybody chose from the same choice set (see, for example, Domencich and MacFadden, 1975) was recognized early. For instance, Williams and Ortuzar, (1982) and Stopher (1980) give empirical verification of the inconsistency of estimates of parameters that can arise when the choice sets faced by individual decision makers are misspecified. Swait

(1984) provides a theoretical backing to these empirical findings by presenting a specification error analysis for a binary choice situation in which the analyst ignores the fact that some individuals are captive to one alternative. Swait is able to conclude that the misspecification of the choice sets will result in biased parameters (and more precisely, in the specific case under study, in an upward biased constant for the mode that has captive individuals).

The literature first dealt with the issue of correctly modelling the first stage of the choice process by introducing models with deterministic choice sets. (See, for example, Ben-Akiva and Lerman 1974; Train 1974). These models can be thought of as deterministic constraint models.

The probabilistic choice set models are based on Manski's (1977) formulation who suggested that the entire choice problem be expressed probabilistically as

$$P_n(i) = \sum_{C \in G_n(i)} P_n(i | C)P_n(C | G_n) \quad (1)$$

where $P_n(i)$ is the probability of individual n choosing alternative i;

$P_n(i | C)$ is the probability of individual n choosing alternative i given that his or her choice set is C;

$P_n(C | G_n)$ is the probability of choice set C being the set of alternatives available to individual n;

M_n is the set of all deterministically feasible alternatives for n ($M_n \subset M$)

M is the universal choice set;

G_n is the set of all non-empty subsets of M_n ; and

$G_n(j)$ is the set of all elements of G_n that contain alternative j.

The choice set generation process is defined by the set of choice sets, G_n , and the corresponding probabilities. Expression (1) reflects a two-stage choice paradigm:

- (i) a probabilistic choice conditioned on the choice set $P_n(i | C)$; and
- (ii) a probabilistic choice set generation model $P_n(c | G_n)$.

A high degree of complexity is implied by (1) since the number of possible choice sets is very large. The number of elements in G_n is $2^{m_n} - 1$ where m_n is the number of elements in M_n . As a result, researchers have developed approaches to reduce the dimensionality of the choice set generation problem by placing *a priori* restrictions on the possible choice sets.

An example is the formulation of models that explicitly incorporate a parametrized probabilistic component for generating the alternatives to be included in the choice sets. Gaudry and Dagenais (1979), for instance, presented a model specification that avoids the IIA (Independence of Irrelevant Alternatives) property of the logit model. This logit captivity model was derived by Ben-Akiva (1977) by assuming that an individual is either captive to an alternative or is free to choose from the full choice set according to a multinomial logit. Ben-Akiva (1977) also considered situations in which individuals have the full choice set or the full choice set less one unavailable alternative.

For the captivity model referred to earlier, in term of the notation introduced above, we have:

$$G_n = \{ i_n \} \text{ or } M$$

where i_n is the alternative to which individual n is captive if he or she has a restricted choice set. MacFadden (1976) investigated the properties of the log-likelihood of the logit captivity model and showed that in general it is not concave. Swait (1984) gives a detailed development of the model.

Another approach achieving a manageable model specification is the independent availability model (Ben-Akiva, 1977) with no *a priori* restrictions on the set of possible choice sets. It is based on the assumption that the probability of availability of one alternative is independent of the availability of the other alternatives. Such an assumption is analytically and computationally convenient, but it might not be justified in certain choice situations. For example, in a destination choice model, the availability of two destinations very close to each other are unlikely to be independent of each other. However, despite its limitations, this strong assumption leads to computationally feasible and interesting model specifications. The independent availability logit model is one example for which calibration results on work mode choice for the city of Maceio, Brazil are presented in Swait and Ben-Akiva (1986).

Probabilistic choice set models have been extended by Swait and Ben-Akiva (1987a) who suggest a behavioral theory of random constraints which provides an explanation for the probabilistic nature of choice sets and an approach to parametrizing choice set models. The theory of random constraints is thoroughly reviewed and extended in Chapter 3. The idea of random constraints is based on the fact that different individuals are expected to have varying perceptions of the degree to which an operative constraint limits their access to certain alternatives. For example, the maximum acceptable walking distance to a subway stop is likely to vary across individuals. Another justification for the introduction of random constraints is the fact that the underlying choice set generation process is unknown. In a detailed analysis of the transport survey data from the city of Maceio, Brazil, Kozel (1981) reports that she was unable to characterize groups whose access to a specific alternative was deterministically constrained.

Swait and Ben-Akiva (1987b) 's empirical work (the parametrized logit captivity model calibrated for work mode choice in Sao-Paulo, Brazil) is limited to a description of the overall impact of the constraints. Models with specification of individual constraints are not estimated.

Their parametrization of the captivity functions is similar to what is proposed by Ben-Akiva (1977) and is given by:

$$P(\text{alternative } i \text{ available to individual } n) = \frac{1}{1 + \alpha \exp(DX_{in})} \quad (2)$$

where

α is a constant;

D is a vector of parameters; and

X_{in} is a vector of characteristics of individual n and of attributes of alternative i .

Note that this approach allows the impact of constraints to vary by person and alternative.

In practical terms, Swait and Ben-Akiva (1987b) allow for a mixture of deterministic and probabilistic constraints since incorporation of any information that is known with certainty will necessarily result in a more efficient estimation.

This contrasts with Wermuth's (1978) approach, in a binary transportation mode choice, for which every constraint is dealt in a stochastic manner. For a binary mode choice between auto and public transit, he models each of the following elements

- 1) socioeconomic factors
- 2) auto ownership
- 3) car availability (permanent and trip specific)
- 4) captivity to one or the other mode, or freedom to choose from both

in a stochastic manner to derive the choice probabilities.

Other models of choice set formation have also been formulated. The limited ability of human beings in processing information results in individuals adopting screening procedure in their

selection process in order to avoid the search costs associated with the gathering of the necessary information. This aspect of the decision making process is well known and has led to choice models such as the "satisficing" model of Simon (1955) or the elimination-by-aspects of Tverski (1972). Swait (1984) considers the same issue and formulates a choice set generation model based on the assumed existence of a maximum choice set size. It is also possible to consider the acquisition of information about alternatives in a dynamic setting. Information gathering involves search costs that the individual is willing to bear if he or she has high enough expectations about the outcome of the search. This type of consideration is taken into account in the work of Richardson (1982), who views choice set generation as a search process and in the work of Meyer (1979), who incorporates information gathering in the context of destination choice. When the cost of incorporating an additional alternative is unacceptably high, the search is terminated (thus defining the choice set).

Finally, by analogy with the random utility models, Pitschke (1980) assumes that there is some stochastic utility accruing to an individual from a specific choice set. He derives a logit choice set formation model which he tests empirically with data from Minneapolis-St. Paul. Swait (1984) reviews Pitschke (1980)s'work and shows that it implies that the disutility of not having available any group of alternatives must be constant which is a very strong assumption.

2-3 Models of Attitudes and Choice

The process by which consumers select means of travel is based on perceptions, feelings, and preferences. Practical travel demand models do not explicitly take into account the influence of attitudinal perceptions on the choice. However, there have been a few research studies that have investigated the effect of attitudinal factors. Nelson and O' Neil (1977) note that the alternatives

faced by an individual depend on the location of the individual relative to the total set of alternatives at a given time of the day, his socioeconomic class, the individual's attitudes and beliefs, and his familiarity with each alternative. In deterministic choice set models based on utility maximization, it is assumed that all individuals in a given market segment (defined by socioeconomic characteristics such as driver's license, automobile ownership, etc.) have the same choice set. In reality individuals differ in their access to information and in their attitudes. This leads Supernak (1986) to question the rationality of travel choices, thus proposing that travel demand modelling evolve to account for imperfect information and irrational behavior. Koppelman (1980) also considers that disaggregate choice models, as they are used now, are unsatisfactory since they exclude consumer's perceptions.

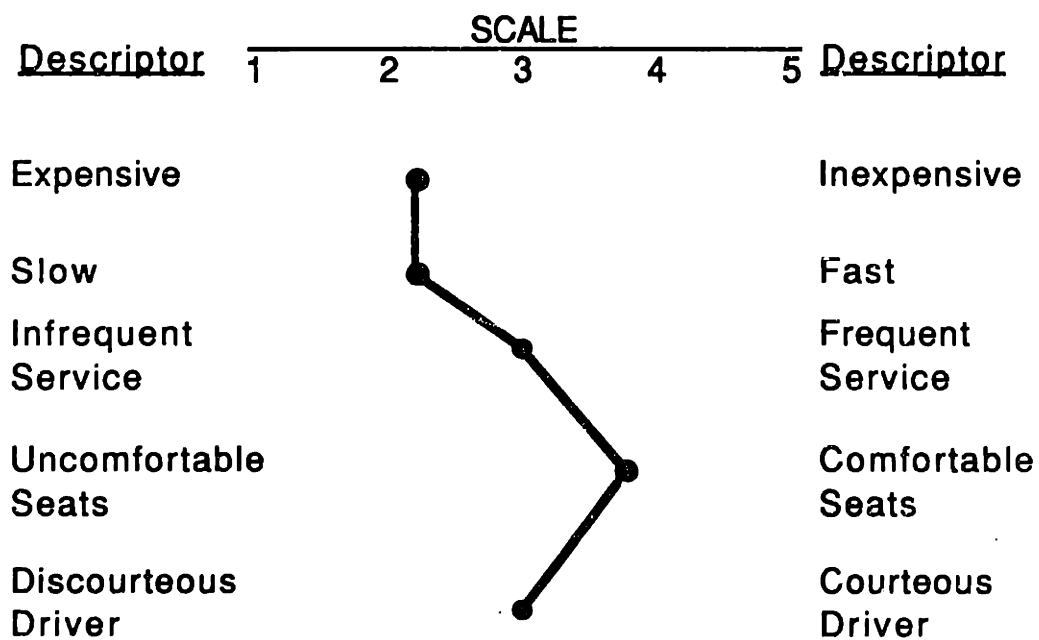
Research has been undertaken in transportation demand modelling to provide alternative explanations for understanding travel behavior. Some of the recent work in the area has been in the introduction of the household, rather than the individual, as the decision making unit, and the introduction of concepts such as lifestyle and lifecycle. See, for example, Havens (1979). As we mentioned earlier, Salomon (1980) has shown that lifecycle is a useful segmentation concept and identifies marked differences in household trip making patterns. Previous studies have adequately addressed the strength of the relationship between attitudinal variables and the actual behavior to which they are supposed to relate. (See, for example, Stopher, Meyburg, and Brog, 1979.) Recent contributions such as Hills and Mitchell (1979) have identified directions for future research, thus establishing an agenda, but the efforts have focused on exploratory and qualitative methods and not on the formalization of modelling methodologies.

The focus of the literature has been on the determination of attitudes and preferences of individuals with respect to various attributes of transportation modes. The literature has been concerned primarily with private automobile and local surface transit. Benjamin and Sen (1982), in a survey of the literature on passenger transportation attributes influencing modal

choice report that, out of 73 articles surveyed; 56 dealt with private auto; 60 with local bus transit; and 51 of these treated the two together. Benjamin and Sen (1982) also give a relative importance (according to transportation planners) of transportation attributes. The most important appears to be time spent travelling (and in particular in-vehicle travel time). Second in importance is cost, especially transit fares. Following time and cost importance are schedule (reliability) and in-vehicle comfort (crowding, temperature, availability of seating). Also of importance are out-of-vehicle comfort and safety.

There have been numerous studies of attitudinal aspects of transportation modes. See, for example, Sen and Benjamin (1979) and the various articles surveyed in Benjamin and Sen (1982). As a rule, the survey conducted in each of these studies include a set of questions where respondents are asked about their feelings regarding certain characteristics of transportation modes. For example, respondents in Richmond, Virginia, were asked to characterize on a scale from 1 to 5 (excellent to bad) several characteristics of transit including fare; availability of schedule information; nearness of stop to respondent's home and courtesy of driver. However, this data was not used in a statistical demand model and was only used to obtain an impression of the overall satisfaction of the transit riders in that city.

Often the respondents are asked to rate the importance of characteristics, such as waiting time or cost, in selecting a travel mode. The resulting data is then analyzed according to techniques, known as Thurstone scales, described in Torgerson (1958). Thurstone scales compare intensity of opinion on a particular scale (e.g. a 5 or 7 points scale) and are usually analyzed graphically by presenting median responses, attribute by attribute, as it is done in Figure 2-1 with a 5 points scale. Direct application of this technique can be found in Golob (1970) and in Sen and Benjamin (1979). An example, adapted from Benjamin and Sen (1982), is shown on Figure 2-1. Similar figures are often found in many other marketing studies. (See, for example, Hauser and Koppelman, 1979 and Koppelman, Hirsch and Schofer, 1986). The purpose of these marketing



(From Benjamin and Sen, (1982) p. 36)

FIGURE 2-1

EXAMPLES OF GRAPHICAL ANALYSIS OF PERCEPTION RESPONSES

studies is to develop a marketing campaign that will cause users and non-users to shift some of their perceptions toward, for example, a transit company. However, until a link between perceptions and mode choice is identified, it is unclear to what extent an improvement in some of the characteristics of the transportation system will translate into a change in ridership. Modelling must proceed beyond the level of simply identifying relevant attributes.

Clearly, a lot of information is contained in the survey questions on perceptions of various transportation alternatives. This information relates to the mechanisms that govern the individual's behavior at the different stage of the choice process including at the choice set formation stage. Instead of being used in a purely descriptive manner, this information could be used to fit a model which explicitly captures the attitudinal processes that govern the individual's choice behavior. Methodological developments that permit the use of psychometric data in econometric choice modelling have not been made operational. Chapter 3 develops an approach that attempts to use jointly, in the fitting of a probabilistic choice set model, the information contained in responses to survey questions relating to alternative availability and the information provided by the observation of the choice.

Recently, researchers have quantified some of the perceptions variables by use of attitudinal measurements techniques and shown their influence on travel behavior. (See, for example, Koppelman and Lyon, 1981; Koppelman and Pas, 1980). The approach consists of linking attitudes and travel behavior through an intermediate preference construct. Again the data most commonly used comes from categorical scale agreement ratings of perceptions or attitudes statements. For example, in an attitudinal analysis of work/school travel in Evanston, Illinois by Koppelman and Lyon (1981), the respondents were asked to rate on a scale each relevant travel mode by indicating their agreement with 22 attribute statements such as "*The bus is available when I need to go to work or school*". Then, factor analysis solutions were developed (with the three factors of convenience, general service, and psychological stress) in order to understand

individuals' perceptions of different modes. Ratings on feelings statements were also used. The next step was to use factor scores of attribute perceptions and feelings as explanatory variables in a logit model of individual's preferences. Then a "preference index" (sum over perceptions and feelings variables of the product of each variable and its coefficient estimate from the previous preference model) was constructed and used as one of the explanatory variables in a multinomial logit choice model.

These researchers have been seeking ways to improve our understanding of transport behavior by attempting to use the psychometric data in the fitting of choice models. However, at this stage, the specification of models that incorporate the information contained in these data is not derived from a sound econometric framework. There is no guarantee that coefficients of perceptions in a preference model constitute a good measure of attribute importance, which can be used in choice models. Note also that none of the models in the existing literature reports on a framework that would simultaneously use stated preference and revealed preference data.

Furthermore, as MacFadden (1986) points out, estimation of a choice or preference model treating the factor analysis constructs as non-stochastic may introduce a degree of statistical inconsistency. Another drawback of the approach is that the estimates of the importance weights for the perceptions measures may be unreliable if the stated preference experimental survey is not adequately designed.

2-4 Conclusions

As stated earlier, one of the goals of this chapter is to provide a background for the developments that are presented in Chapter 3. In doing so, Chapter 2 has summarized the current approaches toward analyzing travel choice behavior in the presence of constraints and under the influences of the individual's attitudes and perceptions.

We have seen that the state of the art of choice set generation modelling is limited in its behavioral interpretation of the choice set formation stage of the choice process by the fact that it has excluded attitudes and perceptions variables from the analysis, thus ignoring the subjective constraints. Although, as we have shown, the influences of perceptions and attitudes on choice has been researched by the literature, the two behavioral aspects of the choice process, namely the influence of constraints on the choice set generation process and the influence of perceptions and attitudes on choices have mostly been treated as separate issues by the literature.

Clearly the existing literature, by extending previous choice models of transportation mode, shows the potential benefits of including attitudinal measures in travel choice analysis since this leads to an improved understanding of individual behavior. In addition, most of the attitudinal models of Section 2-3 do not take into account the existing situational constraints. One exception is Koppelman and Lyon (1981), where the existence of constraints that limit access to various modes is touched upon by including a car availability measure in their analysis.

Similarly, the choice set formation models of Section 2-2 do not explicitly take into account information, attitudes and perceptions. As a rule, the effects of attitudes and perceptions on choice and on choice set formation are not explicitly accounted for. A constraint based approach to modelling choice set formation that considers explicitly the heterogeneity of individual's preferences and their environment is developed in the next chapter.

CHAPTER 3

FRAMEWORK FOR DISCRETE CHOICE MODELS WITH LATENT CHOICE SETS

3-1 Introduction

This chapter describes our theoretical approach to the analysis of choice set formation. It presents a general framework which must be adapted to fit specific situations of interest. The approach attempts to explain individuals' access to different alternatives by describing the relationships between an individual and his or her environment.

Clearly, the choices faced by an individual are constrained. For example, one of these constraints is travel time allocation. It is highly unlikely that an individual would decide to travel 50 miles during a lunch break of one hour. This time constraint eliminates certain destinations or certain modes for some destinations from the set of choices. The existence of such constraints must be considered in analyzing urban travel behavior since they are limitations imposed on the choice set faced by an individual.

It is difficult to identify constraints to individuals' access to various alternatives without considering a time frame of reference. What is a constraint at one time becomes a decision variable in another time since choice sets evolve over time. The issue of dynamic changes in the choice set itself is addressed in Chapter 6 where long-run choice models are introduced. This chapter is limited to short-run demand models. The framework developed in this chapter corresponds to a static approach.

The task of defining an individual choice set at any point in time is far from trivial since this choice set depends on the travel environment that is unique to that individual. This environment is difficult to characterize since it not only reflects objective characteristics, for example the existence of a subway system in a city or whether or not the individual has a driver's license, but it also reflects subjective characteristics. This means that the individual's (or the household's as a whole) attitudes also influence choice set formation. This implies that psychological variables such as perceived availability must be introduced into the analysis. For example, any level of household car ownership will seldom imply equal access to a car for all the members of the household.

The differences across individuals in the availability of information about alternatives further complicate the task of defining choice sets. For example, the level of awareness of transit schedules or routes will certainly imply variations in choice sets across individuals which might otherwise be considered as identical. This example reinforces the idea that choice set formation is influenced by individual attitudes.

These three aspects - operative constraints, perceptions and information - affect the individual's access to the different alternatives. Whether an alternative (other than the chosen) is in fact included in an individual's choice set is not observable. As a consequence, our framework is based on a probabilistic treatment of choice set formation and has the following characteristics:

- the travel behavior analysis is carried out at the level of the individual decision maker and explicitly considers heterogeneous situational constraints and preferences; and
- the choice models are specified according to an explanation of observed travel behavior that considers both latent factors, representing attitudes and perceptions, and socioeconomic characteristics.

Section 3-2 operationalizes the notion of random operative constraints in the form of probabilistic choice set formation models. Section 3-3 takes into account the unobservability of the constraints and their relationship to attitudes and perceptions by extending the approach presented in Section 3-2. It stresses a development that permits use of psychometric data for predictive demand analysis. Section 3-4 concludes the chapter.

3-2 Probabilistic Constraints and Availability

This section introduces two basic concepts; the first of these is the idea of random constraints and the second that of probabilistic alternative availability.

As we have noted earlier, choice set generation is related to the existence of constraints that are imposed by personal considerations and circumstances, social considerations, and by the nature of the existing transportation system. These constraints imply limitations on the choice set.

The behavioral interpretation given to the choice set formation process is adapted from Swait and Ben-Akiva (1987a). The choice set generation stage of the choice process can be considered as the process in which the availability of each alternative is established based on the relevant relationships between an individual and his or her environment.

As we mentioned in Section 2-2, the probability of observing the choice of some discrete alternative is expressed by

$$P_n(i) = \sum_{C \in G_n(i)} P_n(i | C) P_n(C | G_n) \quad (1)$$

where

$P_n(i)$ is the probability of observing the choice by an individual n of an alternative i ;

$P_n(i | C)$ is the probability of individual n choosing alternative i given that his or her choice set is C ;

$P_n(C | G_n)$ is the probability that C is the choice set faced by individual n ;

M_n is the set of all deterministically feasible alternatives for n ;

M is the universal choice set;

G_n is the set of all non-empty subsets of M_n ; and

$G_n(i)$ is the set of all elements of G_n that contain alternative i .

The probabilistic choice set generation model corresponds to the term

$P_n(c | G_n)$.

Let $A_{i,n}$ denote the availability of alternative i to individual n . $A_{i,n}$ is a latent binary variable defined by

$$A_{i,n} = \begin{cases} 1, & \text{if } i \text{ is available to } n; \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Our constraint based approach of choice set generation allows us to consider that what determines the availability of each alternative is whether or not some relevant constraints, specific to that alternative, are met. Defining the constraint index set for alternative i as K_i (meaning that each of the constraints is indexed by $k \in K_i$) and denoting each constraint as H_{kn} , we have:

$$A_{i,n} = 1, \text{ if and only if } H_{kn} \geq 0, \forall k \in K_i. \quad (3)$$

This last equation means that an alternative i will be available to individual n if all the corresponding constraints are met. For example, a decision maker might consider transit to be available if the walk access distance is less than a certain threshold. Assuming, for ease of exposition, this to be the only relevant constraint, we will have:

$$A_{1n} = 1, \text{ if and only if, } W_{1n} \leq D_{1n} \quad (4)$$

where

subscript 1 designates the transit alternative;

W_{1n} designates the walk access distance to transit for individual n ; and

D_{1n} designates the threshold walking distance considered by individual n when determining whether or not transit is available.

The walking distance threshold is unobservable and there is no reason to expect the transit walk access distance to be the same across individuals, thus the variable D_{1n} should be viewed as random. This suggests that the constraints should be considered as random variable which leads us to introduce the concept of random constraint, and equation (3) is replaced by:

$$Prob(A_{in} = 1) = Prob(H_{kn} \geq 0, \forall k \in K_i) \quad (5)$$

For ease of exposition, we will assume that each constraint H_{kn} can be decomposed additively as

$$H_{kn} = h_{kn} - v_{kn} \quad (6)$$

where h_{kn} is deterministic and v_{kn} is random.

The example that follows illustrates how a probabilistic model of choice set formation is derived from an assumption on the distribution of the v_{kn} 's.

Consider a choice of travel mode with two alternatives that are indexed by $i=1,2$. Suppose that the constraint below affects the availability of each of the alternatives:

$$c_{i_n}/Y_n \leq D 1_{i_n} \quad i=1,2 \quad (7a)$$

where c_{i_n} is the cost of travel by alternative i for individual n , Y_n is individual n 's disposable income and $D 1_{i_n}$ is an alternative specific threshold that varies across individuals. The existence of this constraint means that alternative i will be available to individual n provided that the cost of travelling relative to his or her disposable income does not exceed a certain threshold. The idea of random constraint is that the threshold is an unknown random variable. In addition, suppose that the second alternative is available only if the individual's income is greater than a certain threshold $D2$ that again is a random variable that varies across individuals. We have:

$$Y_n \geq D 2_n \quad (7b)$$

With our assumption of additive decomposition of each of the constraint, we write:

$$D 1_{i_n} = D 1_i - v_{1i_n}, i=1,1; \text{ and} \quad (8a)$$

$$D 2_n = D 2 + v_{2n} \quad (8b)$$

where $v_{1i_n}, i=1,1$ and v_{2n} are random variables with zero mean.

For that specific example, we have for alternative 1:

$$H_{1n}(c_{1n}, Y_n, D 1_1, v_{11n}) = h_{1n}(c_{1n}, Y_n, D 1_1) - v_{11n} \quad (9a)$$

where H is the random constraint and h is the deterministic constraint with

$$h_{1n}(c_{1n}, Y_n, D 1_1) = D 1_1 - (c_{1n}/Y_n) \quad (9b)$$

The two events

$$c_{1n}/Y_n \leq D 1_{1n}$$

and

$$H_{1n} \geq 0$$

are equivalent. For alternative 1, the constraint index set is reduced to 1 element (i.e. only one

constraint).

For alternative 2, we have:

$$H_{1n}(c_{2n}, Y_n, D_{12n}, v_{12n}) = h_1(c_{2n}, Y_n, D_{12}) - v_{12n} \quad (10a)$$

with

$$h_1(c_{1n}, Y_n, D_{12}) = D_{12} - (c_{1n}/Y_n) \quad (10b)$$

and for the second constraint,

$$H_2(Y_n, D_{2n}) = h_2(Y_n, D_2) - v_{2n} \quad (11a)$$

with

$$h_2(Y_n, D_2) = Y_n - D_2 \quad (11b)$$

For alternative 2, the constraint set has two elements.

Now, assume v_{11n} , v_{12n} and v_{2n} to be independent normal variables with zero means and respective variances σ_1^2 , σ_2^2 and σ_3^2 . Based on this assumption, we can define the probabilities of the availability of each alternative as follows:

$$Prob(A_{1n} = 1) = \Phi\left(\frac{1}{\sigma_1} h_1(c_{1n}, Y_n, D_{11})\right) \quad (12a)$$

$$Prob(A_{2n} = 1) = \Phi\left(\frac{1}{\sigma_2} h_1(c_{2n}, Y_n, D_{12})\right) \Phi\left(\frac{1}{\sigma_3} h_2(Y_n, D_2)\right) \quad (12b)$$

where Φ is the standard normal cumulative distribution function. Based upon these expressions, it is possible to translate these probabilities into choice set probabilities once assumptions on the relationships between choice sets and alternative availabilities (e.g, independent availability) are made.

The assumptions on the relationships between choice set probabilities and alternative availabilities are in fact assumptions about the joint distribution of the random component of the

constraints that define alternative availabilities. This was illustrated in the previous example where an the assumed independence of v_{11n} , v_{12n} and of v_{2n} enabled us to derive choice set probabilities in equations (12a) and (12b). For example, it is often assumed (independent availability) that the error component are independently distributed across alternatives, but not within.

The derivation of the choice set probabilities is based on equation (5). The probability that C , where $C \in G_n$, is individual n 's choice set is calculated as follows:

$Prob(C \text{ is the choice set} \mid G_n \text{ is not empty}) =$

$$\frac{Prob(A_{i_n} = 1, \forall i \in C \text{ and } A_{j_n} = 0, \forall j \in M_n - C)}{Prob(\text{not all } A_{l_n} = 0, l \in M_n)} \quad (13)$$

$M_n - C$ designates the complement of $M_n \cap C$. The above probability is normalized and is conditioned on the event that the possibility of an empty choice set is excluded. Substituting (5) into (13) yields for $C \in g_n$:

$Prob(C \text{ is the choice set} \mid G_n \text{ is not empty}) =$

$$\frac{Prob(\{ H_{kn} \geq 0, \forall k \in K_i, \forall i \in C \} \text{ and } \{ \forall j \in M_n - C, \exists k \in K_j, H_{kn} < 0 \})}{1 - Prob(\forall l \in M_n, \exists k \in K_l, H_{kn} < 0)} \quad (14a)$$

and finally,

$Prob(c \text{ is the choice set} \mid G_n \text{ is not empty}) =$

$$\begin{aligned}
& \text{Prob}(\{ h_{kn} \geq v_{kn}, \forall k \in K_i, \forall i \in C \}) \text{ and} \\
& \frac{\{ \forall j \in M_n - C, \exists k \in K_j, h_{kn} < v_{kn} \}}{1 - \text{Prob}(\forall l \in M_n, \exists k \in K_l, h_{kn} < v_{kn})}
\end{aligned} \tag{14b}$$

The equation above summarizes the random constraint approach to choice set generation. This approach is made operational, as it has been shown in the illustrative example, by assuming a joint distribution of the v's and a probabilistic structure of the choice set generation model.

The specification of the choice set structure must be made according to the context of the choice problem under study. For example, an independent availability assumption naturally requires that constraints for one alternative to the other be independent, which is a very strong assumption. If this assumption is justified, the choice set probabilities are given by :

$$P_n(C) = \frac{\prod_{i \in C} P(A_{in} = 1) \prod_{j \in M_n - C} P(A_{jn} = 0)}{1 - \prod_{j \in M_n} P(A_{jn} = 0)} \tag{15}$$

An ordered availability assumption is useful when there is a natural ordering of alternatives such as :

$$\forall n ; \forall i ; A_{in} = 1 \text{ implies } A_{jn} = 1 \text{ for } 1 \leq j < i \tag{16a}$$

For example, this might be appropriate in a car ownership level study. In this case, the restrictions on the possible choice sets are such that :

$$G_n = \{ C_1, C_2, \dots, C_j, \dots, C_J \} \tag{16b}$$

with

$$C_j = \{ 1, 2, \dots, j \}; 1 \leq j \leq J \tag{16c}$$

For the ordered availability assumption, the choice set probabilities are given in a recursive manner by :

$$P_n(C_j) = P_n(A_{jn} = 1 | C_{j-1})P_n(C_{j-1}) \quad (17)$$

Note that in this case, the specification of the alternative availabilities must be made conditional on the fact that other alternatives are available. Of course, the choice of a choice set structure is not limited to (15) or (17) but can also include, for example, a combination of the two assumptions, captivity and unavailability of one alternative.

Clearly, not all constraints are based on objective characteristics. For example, attribute perceptions for transit affect the perceived availability of that mode. If an individual trip-maker feels that it is unsafe to travel by bus at night, it is likely that this individual will exclude transit from his choice set when selecting his travel mode for evening trips. Naturally, whether or not the individual excludes transit at the first stage of the choice process depends on his subjective perception of the level of danger for a particular trip (which, among other things, is affected by the time of day and the transit route) and on his feelings toward the importance of safety. This implies that the derivation of constraint based choice set generation model can better be accomplished through a study of attitudinal and behavioral responses in the spirit of the models referred to in Section 2-3. This requires an extension of the random constraint framework that we just derived to allow for the incorporation of psychometric data in the analysis. This is the subject of the Section 3-3.

Note that the constraint approach to choice set formation is behaviorally quite different from the utility maximizing approach of the second stage of the choice process. The example above helps us illustrate this point in the following manner:

The constraint based approach to choice set generation postulates that at the first stage of the choice process, the individual excludes from any further consideration all alternatives that do not meet certain criteria. In the example above, the criteria is based on the requirement that a minimum perceived level of safety must exist for each alternative. The fact that this process occurs at the first stage of the choice process has important consequences. It implies that if the corresponding constraint is not met, the alternative will be rejected no matter what are the values of the other attributes. This shows that, at the model specification level, the inclusion of a safety variable in one of the constraints is not equivalent to the inclusion of that same variable in the utilities. In the latter case, the choice is made by comparing all other relevant alternatives including the one viewed as unsafe while in the former case, the other relevant alternatives are not compared to the excluded alternative. This has important policy implications since any policy action, such as a cost reduction or a frequency improvement, concerning an alternative which has been excluded on the basis of safety (or more generally on the basis of an attribute for which no policy action is undertaken), will have zero impact on the behavior of individuals for whom the safety constraint is not satisfied. This naturally would not be the case if the effect of safety was only felt at the second stage of the choice process, i.e. at the utility level, since one could easily conceive transit service level improvements that would result in some individuals switching to that mode. The difference is of course due to the fact that there exist trade-offs among variables that appear jointly in the utility function, whereas this is not the case for variables that appear in different constraints.

The impact of the change in one attribute of an alternative can be thought of as consisting of two separate effects:

- an availability effect, felt at the choice set generation stage of the choice process, due to the impact of the change on the constraints; and
- a substitution effect, felt at the second stage of the choice process, whose existence, for each alternative, is conditional on the availability of that alternative.

3-3 Extension of the Previous Framework

Conventional transportation demand forecasts often analyze individuals' behavior toward alternatives using statistical models that do not model explicitly the inner mechanisms that govern their behavior. As we pointed out in the introduction, psychological variables, accounting for heterogeneities across individuals and for the presence of subjective limitations to access to different alternatives, must be incorporated in the random constraint approach to choice set generation. This section proposes to extend the previous framework by incorporating psychometric data in the analysis. The developments described in what follows are based on MacFadden (1986) and on Ben-Akiva and Boccara (1987).

The proposed methodology is based on the paradigm of individual behavior depicted in Figure 3-1. The figure follows the convention of depicting a path diagram where the terms in ovals are latent variables and those in rectangular boxes are observed directly or measurable by suitable experiments. The empirical implementation of this behavioral paradigm involves both the observable and the latent variables as shown in Figure 3-1. In a conventional econometric modelling approach the latent factors are implicit in predictive equations that relate external factors and situational constraints to actual behavior. Our approach aims at explicitly analyzing these latent psychological factors in order to gain information on aspects of individual behavior that cannot be inferred from observed behavior alone.

Most of the relationships in Figure 3-1 are self-explanatory. The most important determinants of behavior are external factors (e.g. socioeconomic characteristics) and latent psychological factors representing attitudes and perceptions. The attitudinal factors are affected by social norms that represent the influences of the environment on individual behavior. These attitudes are formed over time and are affected by past experiences (i.e. previous choices) and external factors that

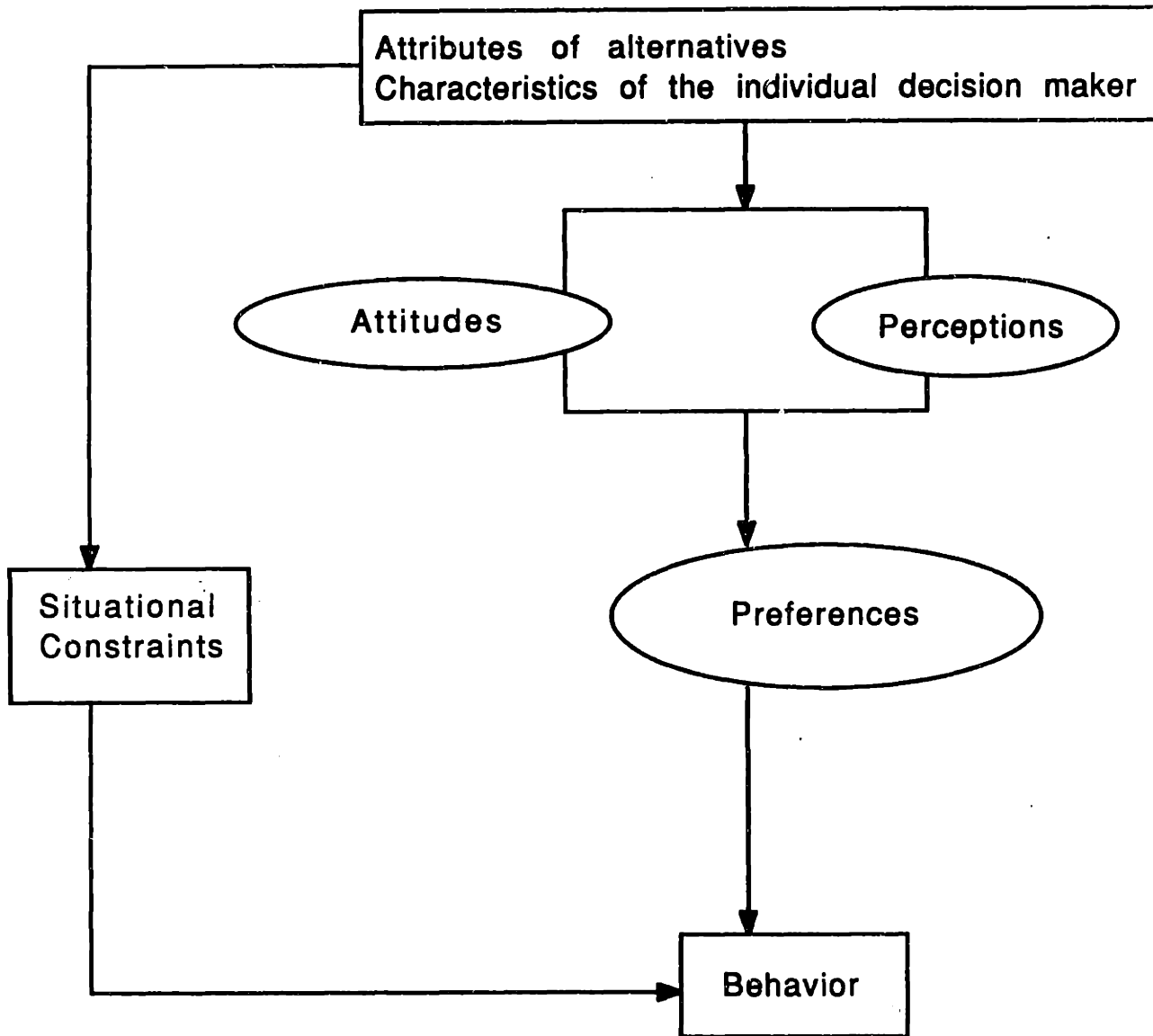


FIGURE 3-1

BASIC PARADIGM OF INDIVIDUAL BEHAVIOR

include socioeconomic characteristics. The perceptions are the individual's beliefs about the attributes of each alternative. These perceived attributes are clearly influenced by a variety of external factors including the information available to the individual, his or her socioeconomic characteristics, and the true values of the measurable attributes. The external factors and the attitudinal factors influence the preferences which are represented by a utility function (which is itself a latent variable).

Consider, for example, a survey that includes statements relating to individuals' perceptions of the level of service offered by transit. Example of such survey attitudinal statements include (Koppelman and Lyon, 1981):

"If I had to be somewhere on time, I would not take the bus",

"I worry of being mugged or assaulted when I travel by bus to work",

"The bus is available when I need to go to work",

"At night, it is safe to travel by bus",

"I can easily walk to the bus from my home or from work".

Respondents could be asked to indicate their agreement with these statements by rating them (for example, on a one to five scale). The resulting data could then be used to identify underlying factors used by trip makers to form service perceptions. The factor structure might, for example, identify reliability, safety and comfort. Similarly, individuals' feelings toward travel modes could be assessed by including in a survey statements that deal with personal beliefs. The attitudinal data could then be used by incorporating the results of the factor analysis into the discrete choice model of travel behavior. For example, one would expect that respondents with high factor scores for reliability would tend to feel more constrained by travel time uncertainty, thus excluding the corresponding alternatives from their choice set, than would respondents with

low factor scores. Therefore the factor analysis results should add explanatory power to the discrete choice model. This is the approach that was taken in the previous studies which were described in Chapter 2.

Alternatively, one can consider that the observed responses to such survey questions are behavioral indicators in the same way as the observed choice is. This suggests that the additional information provided by the indicators can be used directly. In addition to the choice model, the estimated equations also include expressions of the relationships between the latent variables and these indicators. This procedure avoids the use of fitted values from a factor analysis type model (which might introduce statistical inconsistency, see MacFadden, 1986) into the discrete choice model and instead estimates jointly all the equations derived from all the indicators. Essentially, our methodology can be viewed as an approach to estimate choice behavior using simultaneously several types of indicators, as opposed to econometric discrete choice modelling, for which estimation is based only on observed choice (i.e. the choice indicator only).

Naturally, each of the different ways in which the indicators can be used to specify the model requires a combination of market research techniques to form a coherent approach. The use of the market research techniques will obviously vary from study to study. The path diagram depicted in Figure 3-1 is a simpler version of a more general paradigm which we call integrated framework for which attitudes, perceptions, and constraints at the individual level are fully accounted for. Our methodology is an adaptation of this integrated framework to our specific modelling concern, namely the individual's choice set generation process in the first stage of his or her choice process. Appendix 1 supplements this section by sketching the development of the integrated framework and by providing a brief overview of market research techniques that can be used to implement the framework. The remainder of the section describes the adaptation of the integrated framework to the modelling of choice set generation with explicit constraints.

In order to develop an approach based on the developments of Section 3-2 and on the idea of incorporating psychometric data by using additional information provided by attitudinal indicators, we need to assume the existence of choice set formation response indicators r_{in} 's that are related to the latent variables of the problems (including the latent constraints). An example of such an indicator is the categorical variable constructed from the answer to the following question:

Is auto available for that trip? {

- always
- often
- sometimes
- never

The indicators that are suggested for use are derived mostly from answers given to survey questions concerning alternative availabilities. It is likely that such responses would not only relate to availability (i.e. to what extent are the relevant constraints satisfied) but also to desirability of unchosen alternatives. For example, an individual might indicate unavailability of an unchosen alternative for which he has a low utility. As a result, to better account for these biases in the responses, it is suggested that the choice set formation indicators be considered, not only as indicators of the latent constraints H_n (or of the latent availabilities) but as indicators of the latent utilities U_n as well. Therefore the indicators/latent variables relationships should be of the type:

$$r_n = f(H_n, U_n) + \text{error term} \tag{18a}$$

This general relationship, by relating the indicators to all the latent variables of the problem, incorporates the influences of availability and desirability on the responses. A simultaneous type equation between the latent variables would unnecessarily complicate the model specification.

Finally, note that indicators will often be of a polytomous, or even binary, nature. Since the latent variables are continuous, it will often be necessary to assume the existence of intermediate latent response variables r_{in}^* 's related to the discrete observed responses (i.e. the indicators) by a relationship, given here for a binary response indicator, such as :

$$r_{in} = 1 \text{ if } \lambda < r_{in}^* \tag{19}$$

$$r_{in} = 0 \text{ if } \lambda \geq r_{in}^*$$

In this case, equation (18a) must naturally be replaced by :

$$r_n^* = f(H_n, U_n) + \text{error term} \tag{18b}$$

All the models belonging to the class of probabilistic choice set models defined above and considered later on in this study are differentiated by the relationship describing the link between alternative availabilities and the choice set and by the relationship between the alternative availability indicators and the latent variables.

The proposed model is a special case of the integrated framework derived in Appendix 1 and will be referred to as integrated framework model, denoted in all tables as IFM, (and this in spite of the fact that it is only a submodel of the integrated framework since it constitutes an example of illustration of that framework). The same model specification without the use of additional indicators is a probabilistic choice model, denoted in all tables as PCS, based on the random constraint approach of Section 3-2.

The integrated framework model is schematically represented in Figure 3-2. This figure depicts the constraint approach to choice set generation by showing the following variables:

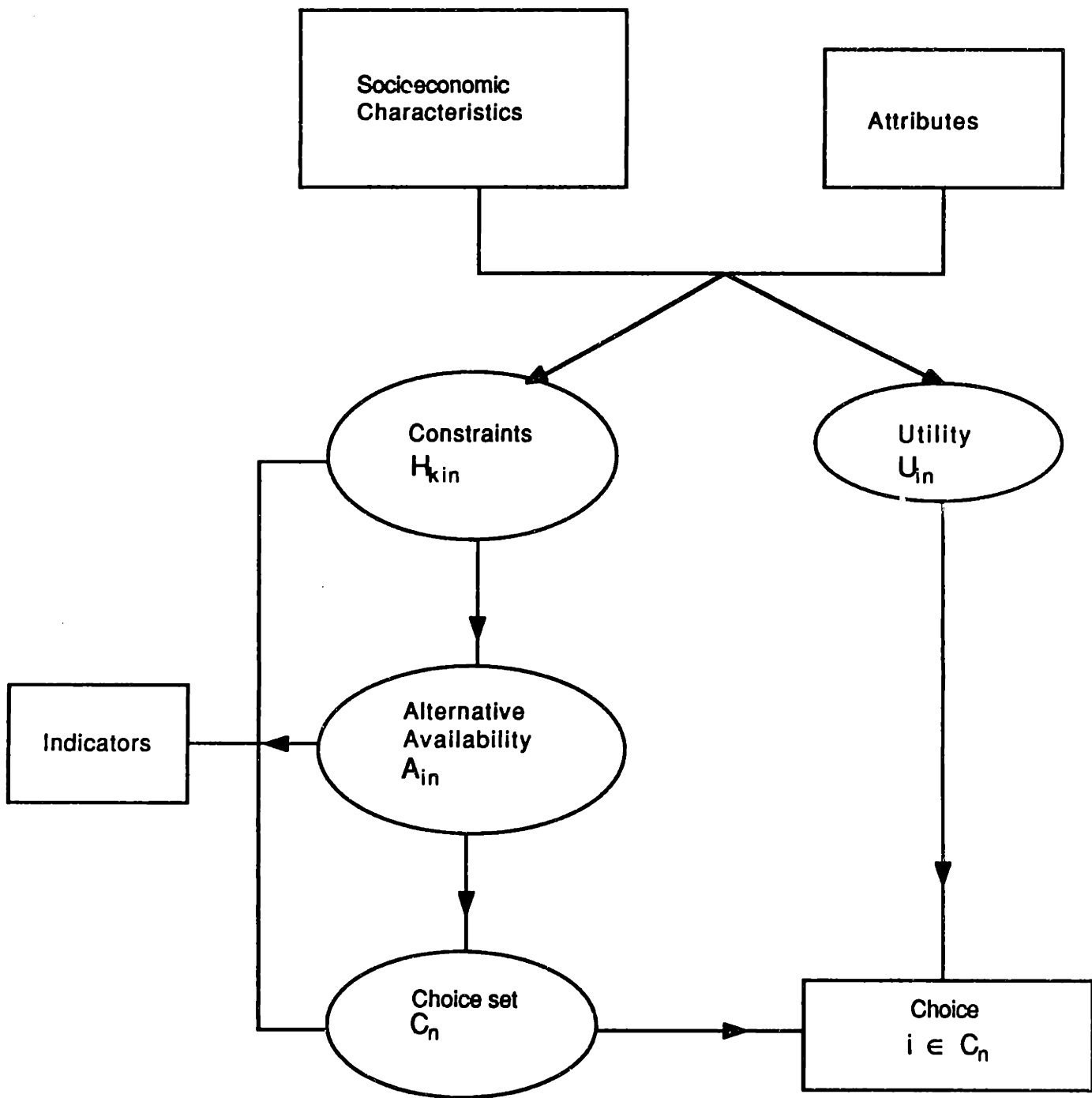


FIGURE 3-2.

FRAMEWORK FOR INTEGRATED MODEL

- the latent constraints;
- the latent availabilities;
- the latent choice sets that are related to the latent availabilities; and
- the choice set formation indicators which are related to the latent variables of the problem by the indicators/latent variables relationships.

This paradigm differs from previous applications of the analysis of travel behavior through the study of attitudinal responses where behavioral indicators have been used as predictor variables (see, for example, Koppelman and Hauser, 1979). This important aspect of our approach is best illustrated by the fact that in Figure 3-2, the arrow corresponding to the indicator/latent variable relationships points toward the indicators as opposed to pointing from the indicators, which would necessarily be the case if the indicators were used instead as predictors.

Figure 3-3 is similar to Figure 3-2 except that the choice set indicators are missing. Naturally, it corresponds to the probabilistic choice set model alone and is shown here to emphasize the difference between the model of Section 3-2 and the model of this section.

The use of the indicators in the estimation of a probabilistic choice set model could be done in two ways. In a joint estimation, the probabilistic choice set model is estimated together with the measurement equations that express the relationships between the latent variables and the indicators. A sequential estimation procedure would first estimate the latent constraints parameters using the information provided by the choice set (or constraints) indicators (this first step would only use the indicators/latent variables relationships and none of the other equations of the model). The estimated latent constraints would then be used to obtain a fitted value of $P_n(C | G_n)$, and finally a probabilistic choice set model would be estimated.

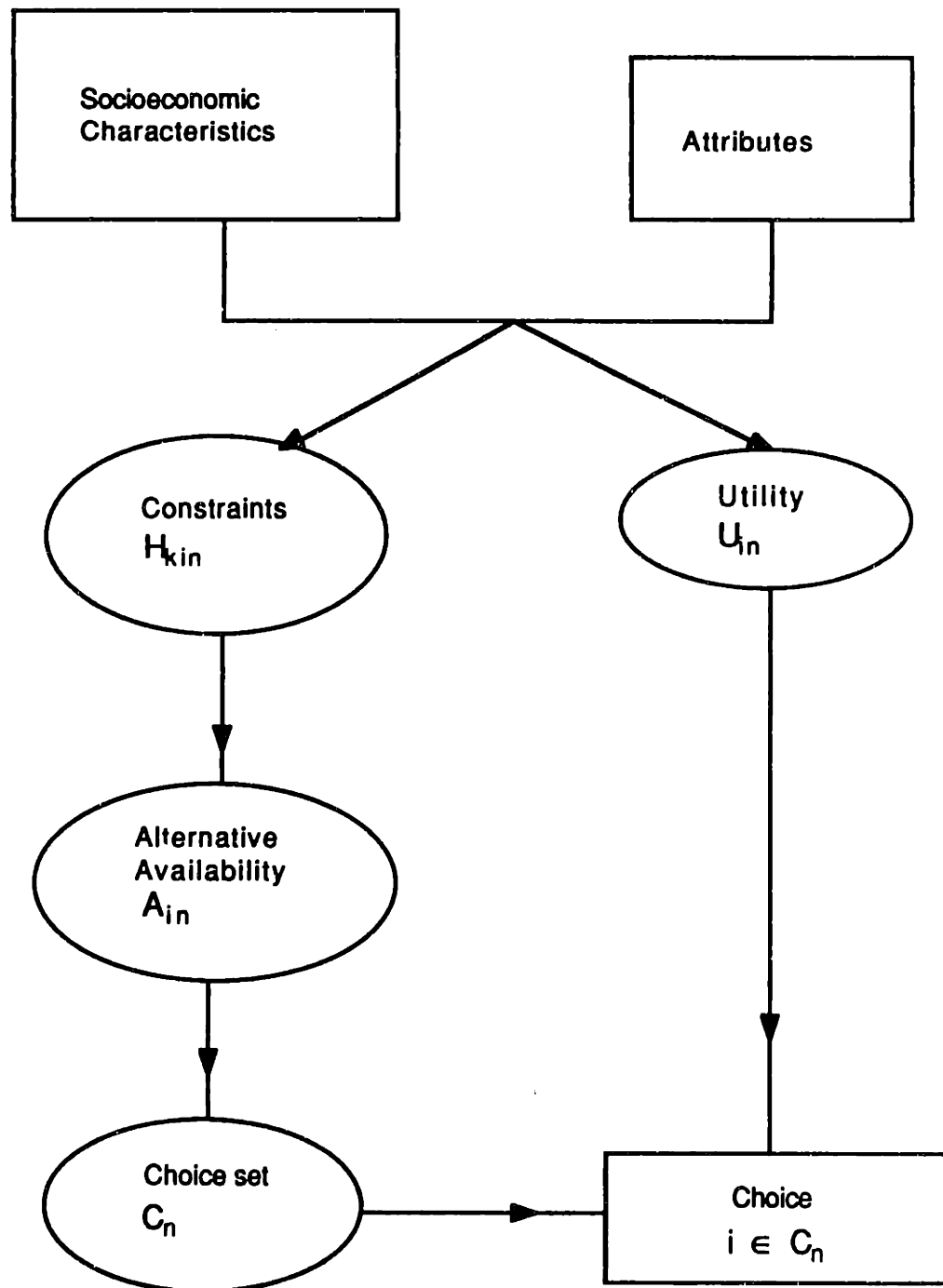


FIGURE 3-3.

FRAMEWORK FOR PROBABILISTIC CHOICE MODEL

If the choice set (or constraints) indicators are also indicators of the utilities (because availability responses are related to desirability of the corresponding alternatives), then the sequential estimation is not suggested because the set of equations for the model cannot be clearly separated into two parts (since the indicators/latent variables relationship contains not only the H 's but the U 's as well). The joint estimation of the probabilistic choice set model equations and of the indicators/latent variables relationship is more efficient but computationally more complex.

3-4 Conclusions

Chapter 3 has undertaken two principal tasks within the framework of this research: First, it has developed our framework of modelling choice set generation by identifying random operative constraints; second, it has extended this framework by proposing a methodology that permits, through the utilization of the information contained in response indicators, the incorporation of psychometric data in the analysis. To accomplish this second task, a development based on an application of an integrated framework to travel behavior analysis has been discussed.

The next two chapters of the dissertation aim at assessing, through an empirical application, the practicality of implementing the integrated framework. Chapter 4 lays out the preliminary work by presenting the results for a probabilistic choice set model based on the developments of Section 3-2. Chapter 5 illustrates the developments of Section 3-3 by comparing the model of Chapter 4 with various extensions based on the use of the additional data provided by choice set formation indicators.

CHAPTER 4

EMPIRICAL APPLICATION OF CHOICE SET FORMATION MODELLING

4-1 Introduction

This chapter presents and discusses the estimation of a probabilistic choice set formation model with data from the city of Baltimore, Maryland. The primary purpose of this chapter is to give estimation results for a probabilistic choice set model that will serve as a benchmark for the various extensions of that same model presented in Chapter 5.

Specifically, a probabilistic choice set model of work mode choice is estimated. The results indicate that the probabilistic choice set model constitutes a significant improvement over the multinomial logit. Models prediction results are also analyzed to predict changes in ridership associated with changes in the attributes.

However, as stated earlier, the purpose of this chapter is not to present an extensive modelling exercise at evaluating the statistical validity of modelling choice set generation with probabilistic choice set models. This was the focus of Swait (1984)'s doctoral dissertation. Therefore, Chapter 4 does not address issues such as comparing our specification with a multinomial logit with market segmentation. Furthermore, estimation of model extensions, in the spirit of what is developed in Section 3-3 of Chapter 3, as will be seen in Chapter 5, is extremely complicated. As a result, in order to achieve a manageable model specification in the next chapter, the specification of the systematic utilities and of the constraints is kept relatively simple.

Once again, the empirical work in this chapter corresponds to the theoretical developments of Section 3-2 of the previous chapter, while the empirical work of the next chapter corresponds to the theoretical developments of Section 3-3 of the previous chapter.

Section 4-2 presents the data used for the empirical work and details the strategy used to prepare the data. Section 4-3 reviews the validity of the Baltimore survey effort in developing a data set for derivation of empirical choice sets and shows the limited successes of the Baltimore experiment. Section 4-4 presents our specification for the multinomial logit model. Section 4-5 presents our specification for the probabilistic choice set models. Section 4-6 presents the estimation results. Section 4-7 looks at the models prediction results. Section 4-8 concludes the chapter.

4-2 Description of the Data Set

The selection of the data set to be used was based on the requirements of the extended models of Chapter 5, namely the fact that a suitable data set needed to be one for which the associated survey contained questions relating to choice set formation so that the corresponding responses could eventually be used as indicators of choice set formation. The Baltimore Disaggregate Data Set collected for the Federal Highway Administration was the only existing data set that we were able to locate that had the necessary information.

The Baltimore data set was collected with the purpose of developing empirical choice sets for disaggregate travel demand modelling. Unfortunately, the complexity of the task and the design of the survey, as will be shown in Section 4-3, were such that the Baltimore experience was not successful. The remaining part of this section describes the data set used for our empirical work.

Our data set comes from a comprehensive travel behavior survey of 831 individuals (each selected randomly as a primary respondent from their household) in Baltimore, Maryland. The data were collected in a home interview survey of a representative sample of households in May and June of 1977. For every respondent, a complete diary of the trips taken during the day was established. The complete diary eventually included several tours, with the first tour starting at home and the last tour ending at home. A negligible number of respondents for which the day did not start and end at the reported place of residence were eliminated (included in these were very unusual situations such as a respondent having been released from prison that day). Thus all the trips belong to one of the following four categories :

- 1) -simple tour home-work-home
- 2) -simple tour home-school-home
- 3) -simple tour home-other-home
- 4) -complex tour involving more than two trips.

A graphical representation of trips taken during the day by a survey respondent could look like the one represented in Figure 4-1. For this figure, we have two tours. Tour 6-7 is a simple tour of the third category while the remaining trips are part of a complex tour that includes work as one of the destinations.

The second and third category of tours were excluded because they do not include work trips. Dealing with complex tour is more delicate since our data requirements are as follows :

-exclude the minimum number of work trips to avoid selection effects problems due to reductions in the size of the data;

-include only work trips for which either the home-work or eventually the work-home portion of the complex tour does not include any other destination.

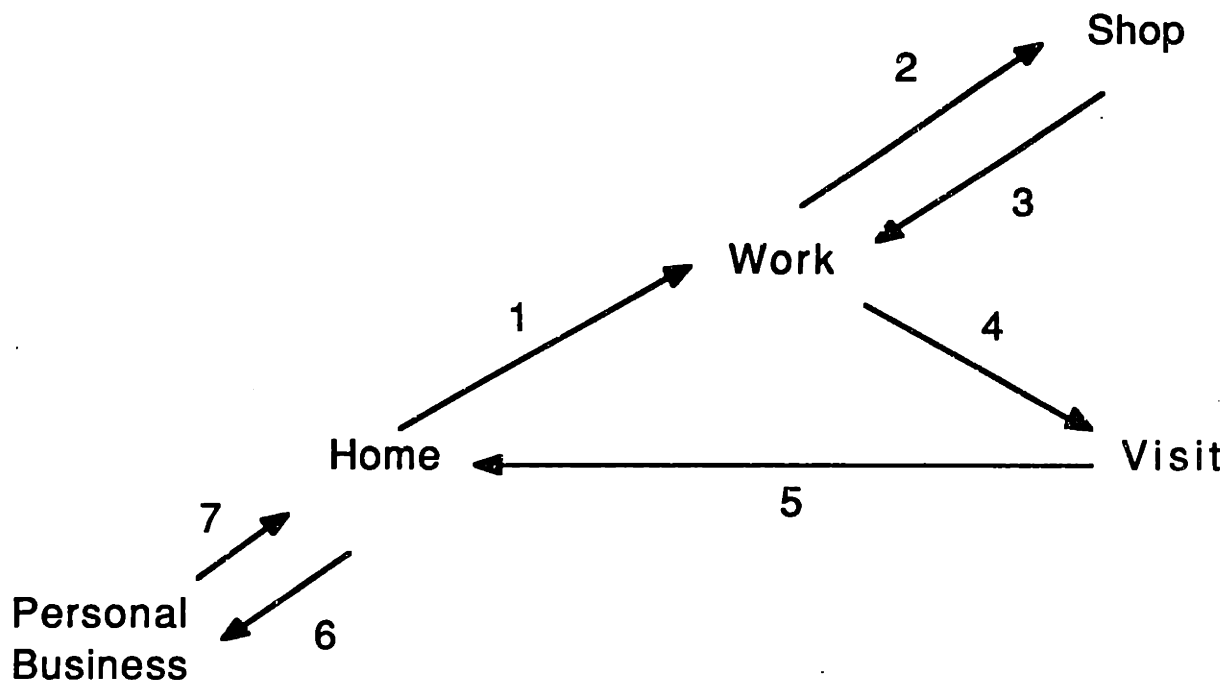


FIGURE 4-1

EXAMPLE OF TRIP DIARY REPORTED BY A RESPONDENT
(from Baltimore Survey Report, FHWA, 1980)

This last requirement comes from the fact that to calibrate a mode choice model, the observed choices must naturally correspond to trips taken under similar circumstances (otherwise the values of attributes such as time or costs are meaningless). Fortunately, all (except two) of the complex tours that included work as one of the destinations had a non-interrupted home-work trip (corresponding to Trip 1 in Figure 4-1) so that, except for these two trips, attrition of trips belonging to complex tours was not a problem. Therefore, the complex tours (involving work as one of the destinations) differ from the simple home-work-home tour in the sense that the return journey home involves at least one stop (for example, Trips 4 and 5 in Figure 4-1).

To summarize, the following observations were eliminated from the data set :

- i) individuals for which the day did not start and end at home;
- ii) all tours that do not include work as one of the destinations;
- iii) all return trips from work (for example, Trips 4 and 5 in Figure 4-1); and
- iv) work trips that did not start at home (for example Trip 3 in Figure 4-1).

Observations in i) or iv) are very few. Observations in ii) simply do not belong to the data set and observations in iii) would not add anything to the data set (no individuals are excluded).

Although an insignificant number of individuals (detailed count not available but conservatively less than ten) having at least one work trip have been eliminated so far, the percentage of original trips remaining in our data set is naturally very low. In terms of Figure 4-1, only one (Trip 1) out of seven trips is kept. Note also that not even all work trips will seem to be included in our data set since Trip 3 's destination is also reported as work, but this trip, which belongs to category iv defined earlier, is not included in our data set. The important aspect of the preparation of the data set is to be careful not to delete travellers to work from the set of survey respondents.

The final stage in preparing the data set involved looking at missing data and at mode choices. Detailed observation of the data revealed that a lot of variables had missing values. However, these variables are derived from survey questions that correspond to choice set formation indicators. We will see later that, for the models of Chapter 5 which use indicators in their specification, a joint estimation strategy was adopted and observations with missing values for the indicators were still used to calibrate the models instead of being eliminated from the data set. Therefore, no observations with missing data for that type of variables were deleted. Observations for which there was missing data for the mode attributes variables (such as in-vehicle travel time or cost) were deleted prior to estimation. Naturally, it is necessary to verify that no bias is introduced in the data. We address this issue after having taken into account observations deleted for their observed mode choice.

Most alternatives reported were of three types: drive alone, shared ride, and transit. The next alternative in terms of number of observations was walking with only 20 observations (some of which that would be eliminated anyway for missing data) which is far too low to permit accurate modelling. Furthermore, due to lack of information on travel distance, no time variable could be constructed for that mode. As a result, these observations were dropped from the data set. Note that due to the extreme difficulty (because of computational burden) of estimating the models of Chapter 5, it was certainly preferable to calibrate these models with only three alternatives (for which the number of choosers was sufficient) in the universal choice set.

A total of 79 out of 584 observations were dropped. These included the aforementioned walk trips and other observations with missing values for relevant mode attributes. A comparison of the data with and without the omitted observations does not lead one to suspect that bias had possibly been introduced since gender, driver's license, income and car availability distributions are very similar. Table 4-1 substantiates our claim by showing summary statistics for the data set with and without the deleted observations.

TABLE 4-1
STATISTICAL SUMMARY FOR THE DATA
(with and without deleted observations)

	deleted observations	all observations
<u>gender</u>		
male	60.1%	59.6%
female	39.9%	40.4%
<u>licensed drivers</u>		
	86.1%	85.9%
<u>annual household income(\$)</u>		
mean value	21,650	22,033
standard deviation	11,967	12,007
<u>car availability</u>		
mean value	0.51	0.53
standard deviation	0.33	0.34
<u>chosen mode</u>		
drive alone	62.4%	62.2%
shared ride	19.0%	20.5%
transit	18.6%	17.3%

The remaining data set includes 505 observations and the number of observed choices for the three alternatives are as follows:

Drive Alone, DA (315);

Shared Ride, SR (96); and

Transit, T (94).

This data set is similar to a work mode choice Baltimore data set used in previous disaggregate demand modelling (see, for example, Ting, 1987 who uses a data set adapted from Wilmot's work, 1982 for which the possible modes are also DA, SR, and T and for which the number of observations is 488).

All relevant aspects of the data preparation that are related to the selection of survey responses to be eventually used as indicators are presented in Chapter 5.

4-3 Assessment of the Empirical Approach to Choice Set Formation Used by the Baltimore Survey

The Baltimore survey included a number of questions that were designed to generate empirical choice sets for estimation of disaggregate demand models of mode and destination choice. In the trip report of the survey, the respondents were asked about alternative modes and destinations for a randomly selected trip. This effort is reviewed below.

The alternative identification process consisted of a hierarchy of questions about the detailed trip designed to identify alternative modes for all links. All alternatives used by the primary

respondent in the last six months (what is thought of as the evoked choice set in marketing studies) were selected for detailed reporting. This provided the basis for the choice set indicators that are used in the empirical work presented in Chapter 5.

However, for a number of reasons, this procedure did not provide a rich data set. First, commuters tend always to make their work trips by the same mode. Second, the random trip selection process gathered some data on many types of trips but too seldom on any particular trip to allow the development of statistical models. For example, despite an allowance for a large class of different modes (15, even including modes like horse), only four were represented with sufficient frequency. Finally, attrition from the data set, due to households either not travelling on the survey travel day (the day prior to the interview) or not reporting alternatives to the selected trip, was severe. In many cases, the primary respondent did not travel on the survey travel day; out of 966 households initially interviewed, 135 took no trips on the survey travel day. Out of the remaining 831 households, there were only 389 households that reported usage of alternative modes.

Finally, note that the trips by persons who made few trips on the survey travel day were more likely to be selected for detailed reporting than were trips by persons who made many trips. Therefore, indicators are more likely to be available for less frequent travellers (or low mobility individuals). This might have contributed to the lack of responses to the choice set formation questions since low mobility individuals possibly have lesser variability in their mode choices than high mobility individuals.

The Baltimore empirical approach to choice set formation does not appear to be an efficient way to collect data on individual choice sets. The data gathering may have been too complex of a

task, and attrition in the data suggests that questionnaires should have been simpler. The length and complexity of the alternative generation questions sequence probably added to confusion and misreporting.

4-4 The Multinomial Logit Model

To provide a basis for comparison, we first estimated a multinomial logit model. The following deterministic constraint was applied to specify the choice sets:

-no driver's license implies that drive alone is not available.

Shared ride and transit are assumed to always be available.

The linear in the parameters utility specification of the multinomial logit is as follows:

	b1	b2	b3	b4	b5	b6	b7	b8
DA	1	0	ivtt	aovtt	cost	carav	0	0
SR	0	1	ivtt	aovtt	cost	0	carav	0
T	0	0	ivtt	aovtt	cost	0	0	distr

where

ivtt = in-vehicle travel time;
aovtt = out-of-vehicle travel time divided by in vehicle travel time for auto;
cost = the cost of the trip;
carav = the number of cars owned or leased divided by the number of driving-age individuals in the household; and
distr = the walking distance to transit.

Therefore, the multinomial logit model has eight unknown utilities parameters.

Note that car availability really means car per driving-age individuals and that it does not correspond to a measure of deterministic availability of a car for a specific trip.

The behavioral justification for the aovtt variable is that the perception of the disutility associated with out-of-vehicle time is function of the trip length i.e the distance travelled or the total time involved. Since distance data was not available and since in-vehicle travel time for auto is calculated from engineering measures (not a variable reported by the respondent) in-vehicle travel time for auto is a good proxy for distance traveled.

The basic idea was to specify a simple multinomial logit model by constructing systematic utilities function of transportation level of service attributes and of alternate availability measures. However, an extensive specification search was conducted because the initial specifications resulted in an insignificant coefficient for the in-vehicle time variable. The strategy adopted for selecting a multinomial logit model is described in what follows.

The travel time variables tried included total time, in-vehicle travel time, out-of-vehicle travel time and their logarithms. Travel cost variables included travel cost, travel cost divided by income and their logarithms. The alternative availability variables tried included car availability

and distance to transit.

Numerous specifications corresponding to various combinations of the above variables were tried. The inclusion of income (as a market segmentation variable to account for taste variations), either as generic or alternative specific, or of travel cost divided by income did not give any improvement. Preferably, the specification search should be done on the probabilistic choice set model instead of the multinomial logit model. However, for computational reasons this was impractical, and it is felt that the utility equations as they now stand represent a reasonable specification of the utility functions for comparison of the multinomial logit, the probabilistic choice set model and the integrated framework models to be developed in the next chapter.

In addition, an outlier analysis was performed to check for the presence of observations with unusually low or high values for variables included in the systematic utilities. Observations were singled out as potential outliers by computing the estimated probability of the chosen alternative. For ten observations, this turned out to be lower than 0.1. However, analysis of attributes and socioeconomic characteristics for these observations did not reveal any abnormalities. Furthermore, results for multinomial logit models re-estimated without all or some of these observations did not show any significant differences. As a result, it was felt that poor performance of some of the models tried during our specification search could not be attributed to the presence of outliers and none of the ten observations above was removed from the data.

To maintain uniformity during the model comparisons, we will use this specification of the utilities for all the models.

4-5 The Probabilistic Choice Set Models

The specification of the models presented in this section corresponds to the theoretical approach developed in Section 3-2 of Chapter 3.

The probabilistic choice models represent a choice context in which the decision maker's choice set is such that if DA is available then SR is also available. Therefore, we do not assume independent availability or ordered availability of alternatives. The rationale for our assumption is that anyone that drives alone always has the possibility of taking passengers in his or her car. Shared ride means more than single occupancy of the car and does not necessarily corresponds to a car-pool arrangement.

Our assumption naturally implies that $\text{Prob}(\text{SR available} | \text{DA available}) = 1$ and the possible choice sets are :

$\{\text{DA,SR,T}\} ; \{\text{DA,SR}\} ; \{\text{SR}\} ; \{\text{SR,T}\} ; \{\text{T}\}.$

In terms of the notation introduced in equation (1) of Section 3-2, the universal choice set M is $\{\text{DA,SR,T}\}$, the set of deterministically feasible alternatives M_n for individual n is also $\{\text{DA,SR,T}\}$ if the individual has a driver's license and is restricted to $\{\text{SR,T}\}$ if the individual does not have a driver's license. This is the only deterministic constraint that is imposed on the choice set structure. Therefore the set G_n of all the non-empty subsets of M_n includes the five possible choice sets for an individual with *a priori* unrestricted access to the three alternatives and is reduced to $\{ \{\text{SR}\}, \{\text{SR,T}\}, \{\text{T}\} \}$, a three element set, for individuals without driver's license.

Conditioned on his or her true choice set $C \in G_n$, an individual chooses an alternative i from the

choice set C according to a multinomial logit model.

For each alternative i , ($i=1,2,3$), the availability of that alternative is a latent binary variable $A_{i,n}$ for which we postulate a parametrization of $Prob(A_{i,n} = 1)$ or a parametrization of $Prob(A_{i,n} = 1 | A_{j,n} = 0)$. The random constraints example developed in Section 3-2 (equations 12a and 12b) shows that the parametrization of the alternative availabilities probabilities has a functional form that depends upon the assumption made about the distribution of the random elements of the constraints. Such parametrization will be called an inclusion function since it gives the probability, for an individual n , of an alternative being included in his or her set M_n . As it was shown in Section 3-2 of Chapter 3, the inclusion functions are derived as cumulative distribution functions of the disturbance term of the linear relationship defining the random constraints. For example, if the random constraints are normal, we have shown that we have a functional form of Φ , the standard normal cumulative distribution. These functions have a binary discrete choice interpretation, as will be shown below.

Our random constraint specification is the following:

- DA is available to individual n if his or her car availability is above a certain individual specific threshold.
- T is available to individual n if his or her distance to transit is below a certain individual specific threshold.

Note that the two variables, $carav$ and $distr$, that enter the functional form of the deterministic component of the random constraints are identical to the variables that entered the systematic utilities of the multinomial logit.

The car availability latent constraint H_1 can be decomposed additively as in equation (6) of Section 3-2 of Chapter 3 as follows:

$$H_{1n} = \alpha_1 + \alpha_2 carav_n - \epsilon_{1n}$$

where

H_{1n} is the random constraint;

$carav_n$ is a measure of car availability;

α_1 and α_2 are unknown parameters; and

ϵ_{1n} is a random variable.

For all the availability probabilities equations that follow, we will refer to DA as alternative 1, SR as alternative 2 and T as alternative 3.

Assume ϵ_{1n} to be normally distributed with mean 0 and unknown variance σ^2 , then

$$Prob(A_{1n} = 1) = Prob(H_{1n} \geq 0)$$

which is equal to

$$Prob(\epsilon_{1n} \leq \lambda + \mu \cdot carav_n) = \Phi(\lambda/\sigma + \mu/\sigma \cdot carav_n) \quad (1)$$

which is the same functional form obtained in section 3-2 of Chapter 3. As an identification restriction, σ is set equal to 1. This corresponds to a binary probit model with latent alternative availability as the dependent variable and with a constant and $carav$ as regressors.

If, instead, ϵ_{1n} is assumed to be a logistic random variable (with location parameter zero and a scale equal to 1), the model naturally becomes a binary logit model. In this case, equation (1) becomes

$$Prob(\epsilon_{1n} \leq \lambda + \mu \cdot carav_n) = 1/(1 + \exp(-\lambda - \mu \cdot carav_n)) \quad (1)$$

Note that two or more independent random constraints will give a product of cumulative

distribution functions. Note also that the random constraint interpretation for an inclusion function whose argument is a more complicated function of socioeconomic characteristics or mode attributes will likely not have a simple behavioral meaning. The parametrizations of the alternative availabilities probabilities can also be thought as describing, for each individual and for each alternative, the aggregate impact of the constraints (Ben-Akiva, 1977; Swait and Ben-Akiva, 1987).

Finally, a binary logit for the latent availability for DA and T is specified as follows :

$$Prob(A_{1n} = 1) = 1/(1 + \exp(\ln(b_9) + b_{10}.carav_n)) \quad (2a)$$

$$Prob(A_{3n} = 1) = 1/(1 + \exp(\ln(b_{12}) + b_{13}.distr_n)) \quad (2b)$$

The specification for the conditional availability of shared ride is

$$Prob(A_{2n} = 1 | A_{1n} = 0) = 1/(1 + \exp(\ln(b_{11}))) = 1/(1 + b_{11}) \quad (3)$$

It corresponds to a binary logit model with a single constant regressor $\ln(b_{11})$.

Note that the switch from probit to logit affects only marginally the values of the predicted probabilities (the parameters will, however, have a different scale). The justification for having $\ln(b_9)$ and $\ln(b_{12})$ instead of simply b_9 or b_{12} is the following:

Since the specification of the utilities is identical across all the models, the multinomial logit model is derivable from the probabilistic choice set model by setting at least one parameter in each argument of the inclusion function to minus infinity. The coefficient of the constant in each argument is written as a logarithm i.e. $\exp(\ln(b_9)+b_{10}.carav)$ instead of $\exp(b_9 +b_{10}.carav)$ so

that the null hypothesis to be tested when comparing the multinomial logit and the probabilistic choice set models is $b_9 = b_{11} = b_{12} = 0$ which avoids potential problems with coefficients becoming minus infinity (outside a compact space) when deriving the distribution of the likelihood ratio test statistic under H_0 .

Given the assumption on ordered availability made for DA and SR, we have:

$$Prob(A_{2n} = 1) = Prob(A_{2n} = 1 | A_{1n} = 1)Prob(A_{1n} = 1) + Prob(A_{2n} = 1 | A_{1n} = 0)Prob(A_{1n} = 0) \quad (4a)$$

which, after accounting for the fact that $Prob(SR \text{ available} | DA \text{ available}) = 1$, becomes

$$Prob(A_{2n} = 1) = Prob(A_{1n} = 1) + Prob(A_{2n} = 1 | A_{1n} = 0)(1 - Prob(A_{1n} = 1)) \quad (4b)$$

At this point, the deterministic component of the latent utilities and of the latent constraints have been specified. To render our model specification operational, it is still necessary to relate alternative availabilities and choice sets. The probabilities for each of the possible choice sets are defined with an implicit conditioning on the event that the empty choice set is excluded. This conditioning implies that the choice probabilities will automatically sum up to one. This is the type of approach adopted in previous empirical work on probabilistic choice sets (see, for example, Swait (1984) and Swait and Ben-Akiva (1987b)).

The choice set probabilities are defined by :

$$Prob\{DA, SR, T\} = P_0^{-1} Prob(A_{1n} = 1) Prob(A_{3n} = 1)$$

$$Prob\{DA, SR\} = P_0^{-1} Prob(A_{1n} = 1) Prob(A_{3n} = 0)$$

$$\text{Prob}\{\text{SR}\} = P_0^{-1} \text{Prob}(A_{2n} = 1 | A_{1n} = 0) \text{Prob}(A_{1n} = 0) \text{Prob}(A_{3n} = 0)$$

$$\text{Prob}\{\text{SR}, \text{T}\} = P_0^{-1} \text{Prob}(A_{2n} = 1 | A_{1n} = 0) \text{Prob}(A_{1n} = 0) \text{Prob}(A_{3n} = 1)$$

$$\text{Prob}\{\text{T}\} = P_0^{-1} \text{Prob}(A_{1n} = 0) \text{Prob}(A_{2n} = 0 | A_{1n} = 0) \text{Prob}(A_{3n} = 1)$$

with

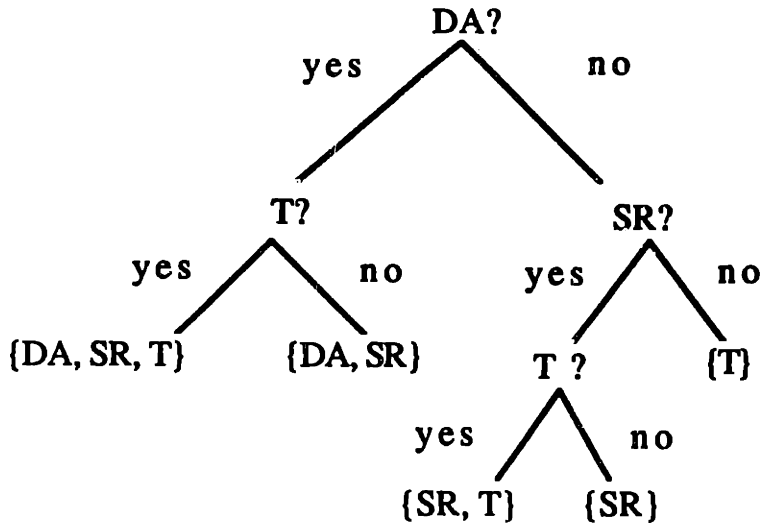
$$P_0 = \text{Prob}(A_{1n} = 1) \text{Prob}(A_{3n} = 1) + \text{Prob}(A_{1n} = 1) \text{Prob}(A_{3n} = 0) +$$

$$\text{Prob}(A_{2n} = 1 | A_{1n} = 0) \text{Prob}(A_{1n} = 0) \text{Prob}(A_{3n} = 0) +$$

$$\text{Prob}(A_{2n} = 1 | A_{1n} = 0) \text{Prob}(A_{1n} = 0) \text{Prob}(A_{3n} = 1) +$$

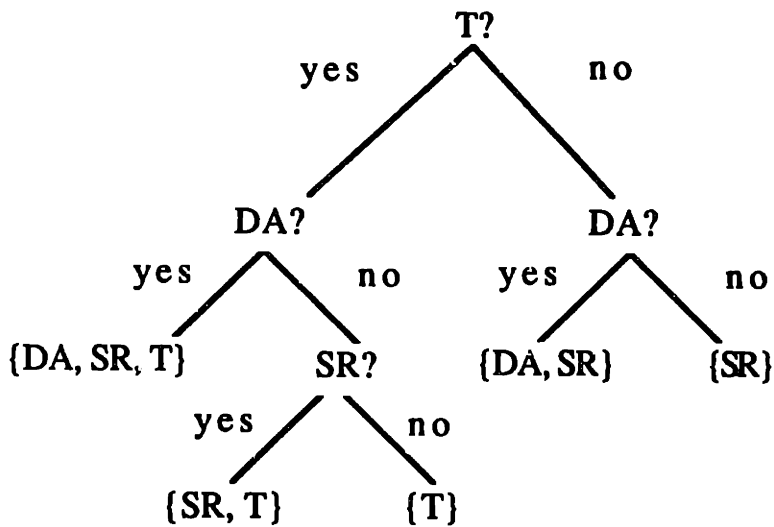
$$\text{Prob}(A_{1n} = 0) \text{Prob}(A_{2n} = 0 | A_{1n} = 0) \text{Prob}(A_{3n} = 1)$$

An alternative way of viewing the choice set generation process is to write down a plausible choice set tree structure, such as the one in the upper half of Figure 4-2, and to estimate the probabilities at each node. The two approaches are similar since they generate the same choice sets. With the second approach it is necessary to assume (in addition to the assumption common to both approaches that $P(\text{SR available} | \text{DA available}) = 1$) that the probability of transit being available given that none of the car modes (DA,SR) is available is 1. If, on the other hand, the choice set tree structure had been drawn with its first node at T instead of at DA (as in the lower half of Figure 4-2), the necessary assumption would have been that drive alone must be available if none of the other alternatives (SR,T) is available. The fact that the assumption required to exclude the empty choice set is specific to the order in which the alternatives appear in the choice set tree structure is a drawback of the second approach. Furthermore, the additional assumption, $\text{Prob}(\text{T available} | \text{SR and DA not available}) = 1$, implies that the availability of transit cannot be calculated independently of the availability of the two other modes. As a result, the expressions for the choice set probabilities become a lot more complicated. Therefore, this approach is eliminated from further consideration.



Assumption needed

$$P(T | \overline{SR}, \overline{DA}) = 1$$



Assumption needed

$$P(SR | \overline{DA}, \overline{T}) = 1$$

FIGURE 4-2

EXAMPLES OF CHOICE SET TREE STRUCTURE

The probabilistic choice set model, denoted as PCS, has 5 additional unknown parameters and a total of 13 parameters. The models maintain the deterministic hypothesis of DA unavailable for individuals without a driver's license. Other specifications involving income and/or carav in the argument of the inclusion functions for the three alternatives were also tried but did not give any improvement in the results.

4-6 Models Estimation Results

This section reports empirical experience with the multinomial logit model (MNL) of Section 4-4, with the choice set generation model PCS specified in the previous section, and with PCS1, a nested version of PCS.

Table 4-2 presents the results for MNL. All the coefficients show high level of significance (except for cost and distance to transit) and have the correct signs. Although the cost parameter is not statistically significant, we nevertheless calculate the implicit value of in-vehicle time implied by the model (calculated by the ratio of the in-vehicle time parameter and of the cost parameter) as \$1.60 per hour (or 2.67 cents per minute as the ratio of the parameters indicates). This is low, even for 1977.

Inclusion of the alternative specific car availability variable (for the first two alternatives, DA and SR) and of the alternative specific transit access variable (for the third alternative T only) can be interpreted as an ad hoc model of alternative availability (see, Swait, 1984). As car availability increases, it is expected that the probability of choosing either DA or SR should increase since the auto access constraint is decreased. This is exactly what the model says since the corresponding parameters are positive and significant. In addition, the car availability effect

TABLE 4-2
ESTIMATION RESULTS FOR MNL

variables	parameters	estimated values (t-statistic)
<i>utilities</i>		
constant for drive alone	b1	-1.61 (-3.63)
constant for shared ride	b2	-2.80 (-6.75)
in-vehicle travel time (min)	b3	-0.48 (-3.20)
out-of-vehicle travel time divided by distance	b4	-0.84 (-3.76)
cost (cents)	b5	-0.18 (-0.77)
car availability (specific to drive alone)	b6	4.25 (6.39)
car availability (specific to shared ride)	b7	3.86 (5.85)
distance to transit (miles) (specific to transit)	b8	-1.07 (-1.61)
<i>Summary Statistics</i>		
Number of observations		505
Number of parameters		8
Log-likelihood at zero		-526.01
Log-Likelihood at convergence		-335.77
ρ^2		0.361
$\bar{\rho}^2$		0.346

is not *a priori* expected to be identical for the two modes, and the effect should be stronger for DA. Again the model results confirm our intuition with the two car availability parameter estimates being respectively 4.25 for DA and 3.86 for SR. As the distance to transit increases, it is naturally expected, as predicted by the model that the transit choice probability should decrease. However, the fact that the parameter of distance in the transit utility is not very significant (as opposed to the car availability parameters, which are very significant) may indicate that the auto ownership constraint is a lot more binding than the transit accessibility constraint.

The comparison of the estimation results of the probabilistic choice set models and of the multinomial logit model will allow us to check for the consistency of the parameters estimates, thereby addressing the issue of the appropriateness of standard discrete choice models in explaining observed travel behavior. The results for the probabilistic choice set models are now described.

Table 4-3 presents the results for the two probabilistic choice set models, PCS (for which the choice set generation process requires conditioning on the event that the empty choice set is excluded) and PCS1 (a nested version of PCS). The results for PCS show that the estimated parameters for the car availability and the distance to transit variables in the systematic utilities are statistically insignificant. The nested version of PCS sets these parameters to zero. For the remaining parameters, the results for the two models are very similar and all significant coefficients have the expected sign.

Unlike for a linear in the parameter multinomial logit model, the log-likelihood of the probabilistic choice set models is not globally concave. Naturally this means that the set of coefficients for which the partial derivatives of the log-likelihood are equal to zero does not necessarily correspond to a set of coefficients that maximize the log-likelihood, and we may

TABLE 4-3
ESTIMATION RESULTS FOR PCS AND PCS1

Variables	Parameters	Estimated Values (t-statistic)	
		PCS	PCS1
<i>utilities</i>			
constant for drive alone	b1	-4.83 (-1.90)	-5.19 (-2.18)
constant for shared ride	b2	-4.83 (-2.15)	-5.57 (-2.85)
in-vehicle travel time (min)	b3	-0.23 (-2.17)	-0.22 (-2.16)
out-of-vehicle travel time divided by distance	b4	-5.66 (-3.18)	-5.59 (-3.05)
cost (cents)	b5	0.02 (0.29)	0.01 (0.30)
car availability (specific to drive alone)	b6	-0.44 (-0.24)	0
car availability (specific to shared ride)	b7	-1.06 (-0.57)	0
distance to transit (miles) (specific to transit)	b8	1.02 (0.18)	0
<i>availabilities</i>			
<i>drive alone</i>			
constant	b9	6.61 (1.72)	6.42 (1.67)
car availability	b10	-16.05 (-4.89)	-15.47 (-4.59)
<i>shared ride</i>			
constant	b11	7.11 (2.47)	6.56 (2.29)
<i>transit</i>			
constant	b12	1.19 (1.99)	1.19 (2.05)
distance to transit (miles)	b13	1.68 (1.84)	1.54 (2.16)

TABLE 4-3
(continued)
ESTIMATION RESULTS FOR PCS AND PCS1

	<i>Summary Statistics</i>	
Number of observations	505	505
Number of parameters	13	10
Log-likelihood at zero	-526.01	-526.01
Log-likelihood at convergence	-318.26	-319.18
ρ^2	0.395	0.393
$\bar{\rho}^2$	0.370	0.374

have a local extremum which is neither the overall maximum or minimum. This naturally makes the search process a lot more complex. In light of the limited knowledge that we have about the log-likelihood, an important aspect of the estimation strategy is to check that the algorithm converges to the same point if started at different initial values. This was the case, and we can expect that the log-likelihood is well behaved at least in the region where the coefficients take behaviorally plausible values. The same conclusion was reached by Kitamura and Lam (1984) and by Swait (1984) for similar models.

The Akaike Information Criteria for PCS and for PCS1 are respectively -331.26 and -329.18. Therefore, the nested version PCS1 is statistically superior to PCS. Note the fact that the systematic utilities car availability parameters are no longer significant but that the parameter for the same variable in the availability function for drive alone is very significant. This is precisely what was expected since our specification of the choice set formation should be such that the automobile access constraint effect appears in the alternative availability functional forms instead of in the utilities.

However, the purpose of the research is to compare a multinomial logit model, a probabilistic choice set model and several extensions of it. Therefore, it is necessary to have the same utilities equations for all the models. The fact that the systematic utilities are also a function of variables influencing alternative availabilities (distance to transit, car availability) is important since it enables us to compare the corresponding estimated parameters with and without alternative availability variables influencing the choice set formation.

This means that all integrated framework models whose estimation results are presented in the next chapter are specified according to the utility specification, the alternative availability specification and the choice set structure of PCS.

The following paragraphs first analyze the alternative availability parameters, which are specific to the probabilistic choice set model, and then the utilities parameters which are common to the probabilistic choice set model and to the multinomial logit model.

As far as alternative availabilities, the results are as follows. The mean value of the probability that drive alone is available is 0.86 with a minimum of 0.13. 72.7% of the sample has a drive alone access probability greater or equal to 0.9. The probability of shared ride being available given that drive alone is not available is a constant and is equal to 0.12. Finally, the mean value of the probability that transit is available is 0.36 with a minimum of 0.01. A large segment of the sample has little predicted access to transit since the transit access probability is less than 0.4 for 45.2% of the sample. Note that transit is actually used on the survey travel day by only 18.2% of the sample. Predicted availability, however, is not the same as predicted choice. This result highlights an important behavioral aspect of work trip mode choice in urban areas of the U.S. It is widely accepted that transit is dominated (in terms of preferences) by the two other modes for work trip. Therefore the choice of transit is more related to the unavailability of the two other modes rather than to the availability of transit itself. This means that distance to transit is not a major operative constraint for transit choosers since these individuals behave as if they were selecting transit as a last resort given that none of the two other auto modes could be chosen. This interpretation is certainly consistent with the low level of significance of the distance to transit variable and with the very high level of significance of the car availability variable. Thus, access to a car, which is equivalent to access to the dominant or most preferred mode, is the major operative constraint on choice set formation.

The implicit value of in-vehicle travel time implied by the model is not calculated since the cost coefficient (insignificant) has the wrong sign. Most significant coefficients common to the two models are systematically larger for the probabilistic choice set model. Therefore, it is likely that

there has been a change of scale between the two models (in the sense that the error terms in the probabilistic choice set model have a smaller variance, see Ben-Akiva and Lerman (1985) for details).

In any case, whether or not a change of scale is suspected, a comparison of the models is not limited to a comparison of the coefficients since what matters is a comparison, for which scale is not an issue, of the predictive power of the models. This is done in the next section.

Given the log-likelihood of the multinomial logit model (-335.77), we utilize the calculated chi-square statistic of $-2(-335.77 + 318.26) = 35.02$, with 5 degrees of freedom, to test the hypothesis H_0 of no probabilistic choice set generation process. The critical value is 15.09 at the 99% confidence level, from which we conclude that H_0 can be rejected. Thus we find that the probabilistic choice set specification is statistically superior to the multinomial logit model despite the 5 additional parameters.

Other runs with differences in the utilities specification but with the same probabilistic choice sets equations showed a similar pattern in the results. As a rule, the probabilistic choice set models provide a significantly better fit to the data than the standard logit model. This improvement is explained by the models' more accurate representation of the choice set structure.

We shall now investigate further the differences between MNL and PCS by looking at model prediction results.

4-7 Models Prediction Results

It is necessary to go beyond measures of model fit to assess the differences between the multinomial logit and the probabilistic choice set models and to evaluate the gains provided by modelling choice set formation. We have shown that the probabilistic choice set model, PCS, was statistically superior to the multinomial logit, MNL. However, the additional difficulties in calibrating a probabilistic choice set model are significant when comparing to a multinomial logit. Therefore, the use of these more complicated models can only be justified if the benefits are substantial. If aggregate measures of behavioral responses differ significantly between the two models, we will consider this to be a justification of the worthiness of the extra effort, since policy decisions based on the two models could be different. It is implicitly assumed that the forecasts from the probabilistic choice set model are more accurate since our presumption is that the estimated parameters of the multinomial logit are inconsistent as a result of a specification error at the level of choice set formation. Inconsistency is problematic for the transportation planner only if it implies marked differences in the predicting accuracy of the models.

The approach taken is to compare, using sample enumeration, the models predictions (in terms of ridership changes) in the following three scenarios:

- i) a uniform 100% increase in in-vehicle travel time for the auto modes;
- ii) a uniform 100% increase in out-of-vehicle travel time for transit; and
- iii) a uniform increase in car availability by 1/3.

Several scenarios were tested, and the above three turned out to be the most revealing in terms of differences in behavioral responses. The reasons for selecting large changes (e.g. doubling of attributes as opposed, for example, to a 10% change which was also tested) is that the differences in predictions are naturally more apparent under conditions of extreme changes.

Table 4-4 shows the predicted impact for each of the scenarios for the multinomial logit, and Table 4-5 shows the corresponding results for the probabilistic choice set model.

The first scenario corresponds to an increase in highway congestion that results in major changes in travel time for auto users. For instance, this scenario could occur during a highway reconstruction project when the number of lanes is temporarily reduced while transit time, either because it is rail service or because special lanes are always reserved for buses, is unaffected. The multinomial logit shows a drastic loss in drive alone share (-34.4%) while the probabilistic choice set model shows a loss of only -7.1%.

The second scenario corresponds to a decreased frequency in transit, which translates into an increase in out-of-vehicle time for buses while having no impact on the attributes of the two other modes. For the multinomial logit, transit share decreases by 4.1%, while the response for the probabilistic choice set model is a transit share decrease of 3.7%.

The third scenario corresponds to a change in car ownership (car availability was uniformly increased by 1/3) level that corresponds approximately to an additional car in each household, since the average household size is 3. Surprisingly, the predictions of the models do not show very large changes in shares. The drive alone share increases by 7.1% for the probabilistic choice set model while it increases by only 2.7% for the multinomial logit.

An interesting pattern of differences between the two models is that the multinomial logit predicts larger changes in ridership than the probabilistic choice set model. The probabilistic choice set model displays less sensitivity to mode attributes since it gives fuller considerations to alternative availability. For example, any change in mode attributes for an alternative that is unavailable to an individual will not affect the behavior of that individual even if the corresponding systematic utilities (which are related to the preferences independently of the

TABLE 4-4

MODEL PREDICTION RESULTS FOR MNL

100% increase in ivtt for auto modes

	DA	SR	T
change in shares	-34.4%	-10.5%	+44.9%

100% increase in ovtt for transit

	DA	SR	T
change in shares	+2.3%	+1.8%	-4.1%

increase of 1/3 in car availability

	DA	SR	T
change in shares	+2.7%	+1.4%	-4.1%

TABLE 4-5**MODEL PREDICTION RESULTS FOR PCS****100% increase in ivtt for auto modes**

	DA	SR	T
change in shares	-7.1%	-10.2%	+17.3%

100% increase in ovtt for transit

	DA	SR	T
change in shares	+2.9%	+0.8%	-3.7%

increase of 1/3 in car availability

	DA	SR	T
change in shares	+7.1%	+1.7%	-8.8%

constraints) are affected. Therefore, a model that accounts for captivity effects or unavailability of a specific alternative to individuals will naturally have a tendency to predict a lessened reaction to attributes changes than a model that does not make these distinctions across individuals. The opposite is true for the change in car ownership scenario (a change in mode availability) for which the response (in terms of changes in share of the drive alone alternative) is stronger for the probabilistic choice set model.

Naturally, the above scenarios are not very realistic. Any major changes such as the ones evaluated will naturally affect the behavior of the individuals in a way that cannot be captured by the models since the estimation results are only valid under the environment in which the data was collected. A major change in the environment affects the decision making process of the individuals and thus would require models to be re-estimated. This critique of the uses of our models for forecasting is naturally related to the fact that the random constraint approach is valid for short-run predictions (short-run in the sense that the environment is not altered in a major way). Any significant change in the nature or the magnitude of the operating constraints affecting the data population at large means that the rules under which the alternative selection process have changed. This requires, as we just argued, either a re-estimation of the models or a modelling strategy that is based on a more dynamic approach, enabling the analyst to follow individuals through time as they experience changes in their environment. This point, which indicates some limitations of the models presented, will be discussed in greater detail in Chapter 6.

4-8 Conclusions

Chapter 4 has presented a modelling exercise aimed at providing an empirical application of the choice set formation modelling approach presented in Chapter 3. It provides a beginning of an effort to model the random constraints approach into discrete choice analysis. The estimated models show the validity of modelling choice set formation and confirms results obtained by various researchers in the field. See, for example, Swait (1984) and Ting (1987).

Again the focus of the dissertation is not to assess the validity of probabilistic choice set models. The purpose of this chapter has been the empirical application of probabilistic models based on the behavioral concept of random constraints. The models presented in this chapter constitute the preliminary work for the empirical work that follows in Chapter 5.

CHAPTER 5

USING CHOICE SET FORMATION INDICATORS IN THE ESTIMATION OF PROBABILISTIC CHOICE SET MODELS

5-1 Introduction

This chapter presents extensions of the probabilistic choice set model estimated in the previous chapter. These extensions of the model are all based on the use, in addition to the information provided by the revealed behavior (observed choice), of information obtained from survey responses to questions relating to the choice set formation process. This type of information is called choice set indicators.

The purpose of this chapter is to illustrate with an empirical application the theoretical developments presented in Section 3-3 of Chapter 3. Therefore the models calibrated in this chapter are all applications of the integrated framework introduced in Chapter 3. Specifically, three models are presented. Each one of these models correspond to a different specification of the indicators/latent variables relationship. The utilities, alternative availabilities and choice set formation equations are common to the three models and are identical to the equations defining the probabilistic choice set model of Chapter 4.

The results indicate that the use of indicators improves significantly the precision of the probabilistic choice set parameter estimates. However, this increased precision is obtained at a cost since the estimation of any of the integrated framework models is more complicated.

Section 5-2 presents the choice set indicators used. Section 5-3 introduces the different specifications and derives the log-likelihoods for each of the models. Section 5-4 presents the estimation results. Section 5-5 concludes the chapter.

5-2 The Selection of Choice Set Indicators

This section describes the choice set indicators used in the estimation of the integrated framework models of this chapter.

The choice set indicators are obtained from the responses to choice set related questions that are included in the home interview survey conducted for the collection of the Baltimore Disaggregate Data Set.

For each respondent, one trip was randomly selected, out of all the trips completed by the respondent on the survey day, for detailed trip reporting. A large set of questions, relating to reasons for observed mode selection and to consideration of other alternatives during the choice process, were then included in the survey for this particular trip. Therefore, in accordance with the purpose of the data collection effort in Baltimore, a detailed description of the choice set generation process and of the choice process itself were contained in the design of the survey questionnaires.

Unfortunately, as we have described in Section 4-3 of Chapter 4, the procedure did not result in a very comprehensive data set. Again one the principal reason for this seems to be that the survey questionnaires were too complicated. This problem was especially acute for the detailed trip reporting since this was the most complex part of the survey.

The choice set indicators selected correspond to the recorded responses to the three following alternative availability questions :

i) Is drive alone available for your trip ?

ii) Is shared ride available for your trip ?

iii) Is transit available for your trip ?

Note that, only two questions were actually asked since no availability question was asked for the chosen alternative. Therefore, any modelling attempt of the observed responses should meet the obvious requirement that the chosen alternative is available.

This choice of indicators is very simple, and there is clearly no doubt that the observed responses (which we will call availability responses) are related to the true availabilities, thus making them legitimate candidates for choice set formation indicators. However, if attrition had not been a serious problem, other indicators could have been extracted from the data. Some of these indicators are described below to illustrate what could possibly be included in a future survey.

A concept commonly used in market research studies is that of evoked choice set. See, for example, CRA (1986). This corresponds to the set of alternatives that have been used by the respondent in the past. Practically, as it is done in the Baltimore survey, the question is worded as follows :

What alternatives, besides the one you chose today, have you used in the last six months ?

There are two problems that are naturally associated with this type of question. First, the choice of a six month time period covered by the question is arbitrary as there are no criteria for relating availability to past uses in a given period of time. An individual who has routinely driven to work in the last six months would respond "none" to the above question, whereas he or she might consider transit to be available. However, that same individual would have included transit in

his or her response if the time period covered by the question had been longer such as to include an unusual situation (e.g. a car breakdown) for which the individual would have used transit to go to work. Also, if a respondent occasionally uses a bicycle during the summer months, this alternative will not be reported if the survey takes place, for example, around March. Clearly, the inclusion of an alternative in the evoked choice set is a sufficient condition for implying availability, but it is not necessary.

In addition, the evoked choice set response does not contain any information as to the degree of perceived availability. Questions that address directly the constraint aspect of choice set formation are needed for this.

An example of such a question included in the Baltimore survey for respondents that reported that transit was not available is the following :

Would you consider transit to be available if you could drive to transit ?

Other constraints related questions, although they are concerned more with choice than with choice set formation are the following :

Any items carried during the selected trip ?

Articles carried made a car necessary ?

Did the weather affect the mode of the trip ?

However, these last questions are related to constraints that will only occasionally influence the mode choice (especially for the first two questions) and should be of minor interest to the analyst, even if he or she is conducting behavioral research, since the constraints of interest are the ones that will be operative in most of the choice situations.

The examples of questions given above illustrate some of the issues raised in Section 4-3 of Chapter 4, during our assessment of the Baltimore experience, as they show how detailed the survey attempted to be. A question such as the first one on items carried actually differentiated between tools, packages and items (and there were three different questions asked).

Finally, since our modelling exercise is limited to work trips, all selected trips (for which we have indicators) that are not work trips are naturally eliminated. Out of the 505 observations used in the calibration of models in the previous Chapter, only 140 have choice set formation indicators. However, the estimation of the integrated framework models is carried out using the full data set with the additional data on the choice set indicators included only in 140 observations of the data set.

5-3 Specification of the Models

The specification of the models presented in this section corresponds to the theoretical approach presented in Section 3-3 of Chapter 3.

The specification of the utilities, of the alternative availabilities and of the choice set structure is identical to the specification of the probabilistic choice set model, PCS, estimated in the previous chapter. Therefore, the choice set structure is such that the possible choice sets are:

$\{DA,SR,T\}$; $\{DA,SR\}$; $\{SR\}$; $\{SR,T\}$; $\{T\}$,

based on the assumption that if DA is available then SR is also available.

All the models in this chapter utilize the information provided by choice set indicators. The models are all special cases of the integrated framework (IFM) developed in Chapter 3. There

are three models, denoted IFM1, IFM2, and IFM3. The models differ in their specification of the measurement equations that relate the choice set indicators and the latent variables of the problem.

Following Section 4-2, once the alternative chosen is observed, questions relating to the availability of the other two alternatives give two indicators r_{in}^- (dummy variable responses).

Since these indicators are binary, it is necessary (as we explained in equation (19) of Chapter 3) to relate these indicators to latent perceived availability responses r_{in}^* such that :

$$r_{in}^- = 1 \quad \text{if } r_{in}^* \geq 0 \quad (1)$$

$$r_{in}^- = 0 \quad \text{otherwise}$$

where i is one of the non-chosen alternatives for individual n .

These indicators are recoded into three indicators (two of which are random variables that will be considered independent of each other and a deterministic one):

$$r_{in} = 1_{Y_{in}=1} + 1_{Y_{in}=0} \cdot r_{in}^- \quad (2)$$

where $1_{Y_{in}=1}$ is equal to 1 if $Y_{in} = 1$ and is equal to 0 if $Y_{in} = 0$ (and similarly for $1_{Y_{in}=0}$).

Although the r 's are indicators of perceived availabilities (as opposed to the A 's that represent the true availabilities), we still need to ensure, as is indicated in the previous section, that the chosen alternative is reported as available. Naturally, this will always be the case given that the

indicators are only available for non-chosen alternatives (and given the way the indicators are recorded). Therefore cases of unavailable alternative being chosen are nonexistent, but it is important to keep in mind that observations of the choice set formation indicators are conditional on the choice being observed.

We argued in Section 3-3 of Chapter 3 (equation (18b)) that the latent responses should be related to the latent variables of the problems by the following relationship :

$$r_{in}^{\circ} = f(H_{in}, U_{in}) + \text{error term} \quad (3)$$

where

H_{in} are the latent constraints affecting individual n for alternative i; and

U_{in} is individual n's utility for alternative i.

This equation shows that the responses to perceived alternative availability questions are not only expected to be related to the true availability (i.e to the constraints) but also to the desirability (i.e the utilities) of that alternative. Therefore, the choice set formation indicators are not only indicators of the latent constraints but of the latent utilities as well.

In addition, the responses should be state dependent in the sense that the responses given to the alternative availability questions (for the non-chosen alternatives) should differ according to whether or not the alternative is truly available. The proposed relationship is as follows:

$$r_{in}^{\circ} = A_{in} \cdot g_1(\text{desirability}) + (1 - A_{in}) \cdot g_2(\text{desirability}) - \epsilon_{in} \quad (4)$$

where g_1 and g_2 are linear functions of a desirability variable and ϵ_{in} is a random component

(assumed to have a normal distribution).

Therefore, since the only difference between the various models of this chapter is the specification of the indicators/latent variables relationships, equation (4) implies that the models will be differentiated by the choice of the desirability variable which enters as argument of the functions g_1 and g_2 .

The most obvious candidates for desirability variables are the utilities (i.e. the desirability latent variables themselves) and the choice probabilities (i.e. a parametrization of the realization of a certain event connected with the desirability latent variables).

However, a high utility does not by itself indicate preference for a specific alternative since what matters is relative rather than absolute utilities. Thus the utility is considered to be an inadequate desirability variable.

The choice probabilities are not independent of the availabilities since they are the predictions from a constraint based probabilistic choice set mode. Thus it can be argued that they do not represent exclusively a desirability effect which itself is expressed by comparing alternative utilities independently of existing constraints. A choice probability calculated on the universal choice set (as if everyone faced the three alternatives) might be a better selection for a desirability variable since it naturally corresponds to an unconstrained choice, thus describing a long-run desirability effect.

Therefore the suggested desirability variables are :

- i) P_{in} , the true choice probability (given the constraints on the choice set formation); and
- ii) \bar{P}_{in} , a choice probability calculated for every individual on the universal choice set.

The definition of \bar{P}_{in} implies that

$$\bar{P}_{in} = \frac{\exp(V_{in})}{\sum_{j=1}^3 \exp(V_{jn})} \quad (5)$$

where the V's are the systematic utilities.

A special case of the integrated framework, denoted IFM1, is a model for which the desirability does not influence the responses on availability for non- chosen alternatives. For this special case, we have

$$g_{1i} = \alpha_i + \beta_i, \quad g_2 = \alpha_i \quad \text{and}$$

$$r_{in}^* = \alpha_i + \beta_i \cdot A_{in} - \epsilon_{in} \quad (6a)$$

Therefore, we have:

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0) = \Phi(\alpha_i) \quad (6b)$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1) = \Phi(\alpha_i + \beta_i)$$

For IFM1, each indicator introduces two additional unknown parameters to be estimated and there is a total of nineteen parameters to be estimated.

Integrated framework models for which the influence of the desirability on the responses is through \bar{P}_{in} and P_{in} are denoted IFM2 and IFM3.

For IFM2, we have :

$$r_{in}^{\rightarrow} = (\alpha_{1i} + \beta_{1i} \cdot P_{in}^-) \cdot A_{in} + (\alpha_{2i} + \beta_{2i} \cdot P_{in}^-) \cdot (1 - A_{in}) - \varepsilon_{in} \quad (7a)$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0) = \Phi(\alpha_{2i} + \beta_{2i} \tilde{P}_{in}) \quad (7b)$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1) = \Phi(\alpha_{1i} + \beta_{1i} \tilde{P}_{in})$$

and for IFM3 :

$$r_{in}^{\rightarrow} = (\alpha_{1i} + \beta_{1i} \cdot P_{in}) \cdot A_{in} + (\alpha_{2i} + \beta_{2i} \cdot P_{in}) \cdot (1 - A_{in}) - \varepsilon_{in} \quad (8a)$$

Note that β_{2i} is not identified because when conditioning on $A_{in} = 0$, the coefficient of P_{in} is equal to zero when $A_{in} = 0$. Thus we have:

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0) = \Phi(\alpha_{2i}) \quad (8b)$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1) = \Phi(\alpha_{1i} + \beta_{1i} P_{in})$$

For IFM2, each indicator introduces four (i.e. two for each of the function g's) additional, unknown parameters and there are a total of twenty-five parameters to be estimated. For IFM3, each indicator introduces three additional, unknown parameters, and there are a total of

twenty-two parameters to be estimated.

For the multinomial logit and for the probabilistic choice set model, the structure of the likelihood function is identical since the estimation of the model parameters rely only on the indicators of observed choice and we have:

$$LL = \sum_{n=1}^{N_1} \sum_{i=1}^3 Y_{in} \log P_{in} \quad (9)$$

where LL designates the log-likelihood and $P_{in} = Prob(Y_{in} = 1)$. Note that for each observation, only one of the terms in LL, which corresponds to the unique choice, is different from zero.

For the integrated framework models, the log-likelihood is:

$$\begin{aligned}
 LL = & \sum_{n=1}^{N_1} \sum_{i=1}^3 Y_{in} \log P_{in} \\
 & + \\
 & \sum_{n=N_1+1}^N \sum_{i=1}^3 \{r_{in} Y_{in} \log P_{in} \\
 & + \\
 & + r_{in}(1 - Y_{in}) \log Prob(r_{in} = 1, Y_{in} = 0) \\
 & + \\
 & + (1 - r_{in})(1 - Y_{in}) \log Prob(r_{in} = 0, Y_{in} = 0)
 \end{aligned} \quad (10)$$

where N_1 is the number of observations with only choice information and $N - N_1$ is the number of observations with choice and choice set indicators data.

We have:

$$\begin{aligned}
 & Prob(r_{in} = 1, Y_{in} = 0) = \\
 & Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1) \sum_{c \in C_i} Prob(Y_{in} = 0 | c) Prob(c) \\
 & \quad + \\
 & Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0) \sum_{c \in \bar{C}_i} Prob(Y_{in} = 0 | c) Prob(c)
 \end{aligned} \tag{12}$$

where

$C_i = \{\text{choice sets which include alternative } i\}$

$\bar{C}_i = \{\text{choice sets which do not include alternative } i\}$

and similarly for

$Prob(r_{in} = 0, Y_{in} = 0)$

All the relevant expressions are derived in Appendix 2.

Therefore, it is not possible to compare the log-likelihood of an integrated framework model with the log-likelihood of either the multinomial logit or the probabilistic choice set model. However, the variance/covariance matrices for the parameters that are common to the integrated framework and the probabilistic choice set model can be compared.

These parameters correspond to the equations that define the choice probabilities which are the structural equations of the integrated framework model. These are the parameters which are of direct interest to the analyst since they are the only ones that influence the predictions of the model. On the other hand, the parameters that appear only in the integrated framework model

are not of direct interest since the associated indicators/latent variables relationships are measurement equations (for the choice set formation indicators) which are used only as an estimation tool to get a better inference on the parameters of the structural equations.

Essentially the integrated framework model is a system of four simultaneous equations (one for each indicator) while the probabilistic choice set model is a one equation system (we use the word system here because that one equation corresponding to the choice actually made is decomposed in a set of several equations). The difference between the estimation of the integrated framework model and the estimation of the probabilistic choice set model can be thought of as the estimation of a system of equations compared with the estimation of a single equation. Therefore, it is anticipated that if the choice set formation indicator equations are well specified, the estimation method with the integrated framework will be more efficient, and if they are misspecified, the parameters estimates from IFM will be inconsistent. This is analysed in the next section where the results are discussed.

5-4 Models Estimation Results

This section reports the estimation results of the three models IFM1, IFM2, and IFM3. Section 4-6 of Chapter 4 addressed a model specification issue related to the consistency of the parameter estimates by comparing the utilities coefficients of the multinomial logit model and of the probabilistic choice set model. This comparison was necessary to judge the appropriateness of standard discrete choice models to explain observed behavior. This section addresses a model estimation issue related to the efficiency of the parameters estimates by comparing the utilities and the alternative availabilities coefficients of the probabilistic choice set model and of

the integrated framework models. This comparison allows an assessment of the feasibility of using, for estimation purposes, the information provided by indicators instead of relying exclusively on the observed choice.

Table 5-1 presents the results for the 3 different integrated framework models, IFM1, IFM2, and IFM3. The log-likelihoods at convergence, the number of parameters of each of the models and the Akaike Information Criterion are given below:

model	IFM1	IFM2	IFM3
# parameters	19	25	22
log-likelihood	-439.59	-429.86	-433.51
Akaike Criterion	-458.59	-454.86	-455.51

We noted in Section 4-6 of Chapter 4 that the utilities coefficients varied substantially between the multinomial logit and the probabilistic choice set model. The utilities and the availabilities coefficients are now very similar between the probabilistic choice set model and the integrated framework models. This result was expected since all these models share the same structural equations, which we believe to be correctly specified. In this case, provided that the measurement equations are correctly specified, our estimation procedure will lead to an increase in the efficiency of the parameters estimates since we are jointly estimating a system of simultaneous equations (the structural and the measurement equations) as opposed to estimating

TABLE 5-1
ESTIMATION RESULTS FOR IFM1, IFM2, AND IFM3

Variables	Parameters	Estimates (t-statistic)		
		IFM1	IFM2	IFM3
<i>utilities</i>				
constant for drive alone	b1	-7.51 (-3.50)	-7.85 (-4.20)	-6.57 (-4.35)
constant for shared ride	b2	-7.24 (-3.98)	-7.40 (-4.53)	-6.42 (-4.14)
in-vehicle travel time (min)	b3	-0.29 (-3.10)	-0.25 (-5.07)	-0.24 (-4.59)
out-of-vehicle travel time divided by distance	b4	-7.14 (-4.49)	-7.29 (-5.85)	-5.90 (-5.35)
cost (cents)	b5	0.03 (0.70)	0.04 (1.02)	0.04 (0.96)
car availability (specific to drive alone)	b6	1.51 (1.08)	1.48 (1.19)	0.19 (0.33)
car availability (specific to shared ride)	b7	1.06 (0.73)	0.43 (0.51)	-0.87 (-1.61)
distance to transit (specific to transit)	b8	1.50 (0.28)	-2.90 (-1.58)	-4.81 (-4.47)
<i>availabilities</i>				
<i>drive alone</i>				
constant	b9	6.64 (4.11)	8.01 (5.36)	6.39 (4.21)
car availability	b10	-17.46 (-5.74)	-18.88 (-5.99)	-17.19 (-5.21)
<i>shared ride</i>				
constant	b11	4.56 (4.37)	5.08 (3.81)	6.02 (3.97)

TABLE 5-1
continued
ESTIMATION RESULTS FOR IFM1, IFM2, AND IFM3

Variables	Parameters	Estimates (t-statistic)		
		IFM1	IFM2	IFM3
<i>transit</i>				
constant	b12	0.79 (5.02)	1.05 (5.18)	2.36 (3.73)
distance to transit (specific to transit)	b13	1.63 (1.90)	0.75 (0.62)	-1.75 (-3.69)
<i>indicator 1</i>				
g1(1 avail)				
constant	b14	-0.72 (-2.77)	-0.28 (-0.61)	-0.51 (-3.33)
desirability	b15		0.83 (1.15)	0.98 (4.25)
g2(1 not avail)				
constant	b16	0.95 (2.66)	-8.19 (-5.84)	-6.74 (-5.75)
desirability	b17		12.73 (5.84)	
<i>indicator 2</i>				
g1(2 avail)				
constant	b18	0.13 (0.45)	-0.48 (-2.42)	-1.02 (-5.08)
desirability	b19		1.22 (1.35)	4.23 (4.25)
g2(2 not avail)				
constant	b20	-0.42 (-1.31)	0.62 (1.25)	0.34 (1.01)
desirability	b21		-2.28 (-1.27)	

TABLE 5-1
continued
ESTIMATION RESULTS FOR IFM1, IFM2, AND IFM3

Variables	Parameters	Estimates (t-statistic)		
		IFM1	IFM2	IFM3
<i>indicator 3</i>				
g1(3 avail)				
constant	b22	-0.35 (-2.07)	-4.34 (-3.47)	-0.54 (-3.79)
desirability	b23		-1.02 (-5.13)	-2.64 (-5.31)
g2(3 not avail)				
constant	b24	-3.23 (-1.10)	-0.62 (-2.94)	-0.55 (-2.67)
desirability	b25		1.13 (2.15)	

Summary Statistics

		IFM1	IFM2	IFM3
number of observations	505			
with indicators	140			
without indicators	365			
number of parameters		19	25	22
Log Likelihood at zero	-706.30			
Log Likelihood at convergence		-439.59	-429.86	-433.51
ρ^2		0.38	0.39	0.38
$\bar{\rho}^2$		0.35	0.36	0.35

the structural equations alone. However, if the measurement equations are misspecified, then the use of the choice set formation indicators will give inconsistent coefficients for the integrated framework models.

A Hausman specification test (Hausman, 1978) is performed for each of the integrated framework models to check for the presence of inconsistencies. The hypothesis H_0 to be tested is that the "distance" between the coefficients vectors of the probabilistic choice set model and of the integrated framework model is zero. The test statistic is equal to

$$(bPCS - bIFM)' \cdot (A - B)^{-1} \cdot (bPCS - bIFM)$$

where A and B are the following matrices:

A = variance/covariance matrix of the parameters estimates bPCS from PCS

B = upper 13x13 block of the variance/covariance matrix of the parameters estimates bIFM from IFM.

The test statistic is distributed as a chi-square with 13 degrees of freedom and its values are given below:

model	IFM1	IFM2	IFM3
test statistic	9.19	3.61	2.54

The critical value is 7.04 at 90% confidence level, from which we conclude that the coefficients for the integrated framework models (at least for IFM2 and for IFM3) are consistent (i.e. no rejection of H_0). The Hausman test is valid only if the matrix $A-B$ is positive definite and a GAUSS (1986) matrix procedure was used to check the positive definiteness of $A-B$. Therefore, the estimation procedure in the integrated framework models is more efficient than the estimation procedure in the probabilistic choice set model. Intuitively this can be seen by noting that for each statistically significant coefficient common to the two models, the t-statistic for any of the integrated framework model is greater in absolute value.

Another less formal test that can be performed is to compare the values of the log-likelihood of the probabilistic choice set model for the different sets of parameters. Naturally the log-likelihood will be the smaller (in absolute value) when evaluated for the probabilistic choice set model parameters since these values correspond to the minimum of the log-likelihood. Furthermore, it was argued in Section 4-6 of Chapter 4 that the log-likelihood must be reasonably well behaved around its minimum. Therefore, if the parameter estimates for the integrated framework model are also consistent, we should expect the values taken by the log-likelihood for these parameters to be fairly close to its minimum, and the difference in the log-likelihood when evaluated at the two set of parameters gives another measure of the "distance " between the two sets of coefficients. The values obtained are as follows:

model	PCS	IFM1	IFM2	IFM3
PCS log-lik.	-318.80	-321.47	-319.93	-319.49

This test follows very closely the results of the Hausman test since both measures of distance show that IFM2 performs a little bit worse than IFM3 without, however, clearly implying

inconsistency. This pattern is easily observed when comparing directly the coefficients of the models (shown in Table 5-1) with the corresponding coefficients of the probabilistic choice set model (shown in Table 4-3).

A misspecification of any of the integrated framework models' measurement equations is naturally equivalent, given equation (4), to an unsatisfactory choice of the desirability variable.

A comparison of the Akaike Information Criterion clearly demonstrates that desirability does influence the responses on availability since IFM2 and IFM3 give a much larger value than IFM1, for which responses are not influenced by desirability. The result for IFM2 seem to indicate that the best choice for a desirability explanatory variable is the long-run, unconstrained choice probability $P_{i,n}^-$ defined by (5). IFM3 is identical to IFM2 except that the variable $P_{i,n}^-$ is replaced by the choice probability $P_{i,n}$. The results are fairly similar, but the log-likelihood for IFM2 is smaller by 4.29.

In selecting one of the IFM models, the most important test is the consistency test. A model with a lower log-likelihood (in absolute value) but which would fail this test (or exhibit a significantly greater value for the consistency test) would not be acceptable since it would be misspecified, thus resulting in inconsistent coefficient estimates. IFM1 does not pass the consistency test (at the 90% confidence level) and gives a relatively high log-likelihood (in absolute value) since it ignores the influence of desirability on the responses. This specification is eliminated. The choice of our best specification is therefore limited to the selection of one of the two models IFM2 and IFM3. Unfortunately the two selection criteria are conflicting in the sense that none of the two models clearly dominate the other by having a lower log-likelihood (in absolute value) and a lower value for the test. A comparison of the coefficients for the two models shows that the results are similar. Since IFM3 has the highest Akaike Information Criterion, IFM2 is

selected as the best specification. Note that the desirability variable selected for IFM2 corresponds to the unconstrained choice probability which we argued was likely to better capture a desirability effect than a constrained choice probability.

The coefficient estimates for IFM2 are similar to the coefficient estimates for PCS. Unlike the case for the multinomial logit and the probabilistic choice set model, there does not seem to be a change of scale between the two models. The results of IFM2 are now compared with the results for PCS. Again the utilities car availability coefficients are insignificant while the car availability parameter in the drive alone alternative availability function is significant. The mean value of the probability that drive alone is available is 0.87, with a minimum value of 0.12, which corresponds as well to 10.9% of the sample for which car availability is zero. 78.8% of the sample has a drive alone access probability greater or equal to 0.9. The probability of shared ride being available given that drive alone is not available is 0.17. The mean value of the transit access probability is 0.44. The comments made in Section 4-6 of Chapter 4 also apply.

The non-structural coefficients show that as the alternative gets more desirable, the respondent is in general more likely to view that alternative as available. This can be derived from the observation that the desirability coefficients are always positive with two exceptions (the coefficient for g_2 for the shared ride alternative and the coefficient for g_1 for the transit alternative). For the drive alone alternative, the high coefficients of the availability variable for the function g_2 (meaning when that alternative is not available) show that respondents have a strong bias in favor of that mode in that they are likely to perceive it as available even if it is not. One example would be the case of a secondary user licensed driver who does not have access on a regular basis to the household car. The opposite is true for the transit alternative where the response is not affected by desirability for the function g_2 and is negatively affected for the function g_1 , which would imply that the more desirable transit is, the less likely it is to be perceived as available (when in fact it is available). Thus this parameter has the "wrong" sign.

This means that even if transit is available, the respondent will perceive it as such only in less than one half of the cases (the function g_1 being negative implies that the cumulative distribution function of the standard normal evaluated at g_1 will be less than one half). This shows a negative bias against transit which, by itself, is not counterintuitive.

An analysis of the predicted responses to the alternative availability questions shows this bias in that the predicted average transit availability probability (for individuals that did not choose transit) is 0.31 while the average predicted availability probabilities for shared ride and for drive alone (again for the individuals that did not choose the alternative) are respectively 0.58 and 0.61.

Given the similarity between the coefficients of the probabilistic choice set model and of the integrated framework model, we do not feel that it is necessary to have an entire section on model prediction results such as Section 4-7 of Chapter 4. The analysis of model prediction results was justified previously since Chapter 4 addressed a consistency issue while Chapter 5 addresses an efficiency issue. Table 5-2 is analogous to Table 4-5 and gives the model prediction results for IFM2. The results are similar to the results for PCS, and the same comments apply.

Finally, due to the lack of global concavity of the log-likelihood function, the additional difficulties of estimating the parameters of the integrated framework models are very significant. As the expressions given in Section 5-3 (equations (12) through (17)) show, the log-likelihood is extremely complicated. As a rule of thumb (with an impossibility of being more precise since it all depends on the "adequacy" of the starting values chosen), the estimation time for IFM1 is twice as long as the estimation time for PCS while the estimation time for IFM2 or IFM3 is about three times as long.

TABLE 5-2

MODEL PREDICTION RESULTS FOR IFM2

100% increase in in-vehicle travel for auto modes

	DA	SR	T
change in shares	-8.9%	-11.9%	+20.8%

100% increase in out-of-vehicle travel for transit

	DA	SR	T
change in shares	+3.2%	+1.2%	-4.4%

increase of 1/3 in car availability

	DA	SR	T
change in shares	+8.6%	+3.1%	-11.7%

5-5 Conclusions

Chapter 5 has presented an empirical application of the integrated framework presented in Chapter 3. The integrated approach involves both observable and latent variables. The latent variables are inferred from indicators and the approach suggests the use of the information provided by these indicators instead of relying exclusively on the use of the observed choice indicator. The introduction of latent variables and the use of indicators increases significantly the number of model parameters to be estimated but provides a more precise estimation of the structural choice equations.

The results of this chapter demonstrate the potential of the integrated framework approach. However, the gain in efficiency with the integrated framework models is very costly in terms of increased difficulties with estimation. This hinders the future use of similar models since a specification search is nearly infeasible.

Therefore, more experience needs to be gained with the application and practicality of the integrated framework, especially in the use of choice set generation models. In particular, similar specifications should be formulated and tested empirically in choice contexts other than the choice of travel mode for work trips, because these trips are of a routine nature for which most individuals rarely consider other alternatives than the one they usually utilize. The habitual nature of certain choice situations and the implications for choice modelling are explored in Chapter 6.

CHAPTER 6

MODEL OF HABITUAL CHOICE

6-1 Introduction

This chapter presents a simple model based on the observation that choices are often of a routine nature. The habit model that is derived is an attempt to take into account some of the dynamic aspects of choice behavior.

The models of Chapters 4 and 5 are essentially short-run choice models. For these models, the specification of the choice set faced by an individual and of the choice made by that individual is done in a static setting. This means that the set of constraints under which the observed choice is made is treated as if it were fixed in time. Therefore, the modelling exercise ignores all the temporal aspects of travel choices. The purpose of this chapter is to take into account some of these temporal aspects of travel choices by contrasting the issues involved in a dynamic setting with the issues involved in a static setting. By doing so, we are able to show the limitations of the models of Chapters 4 and 5. However, the model introduced in this chapter is not a fully dynamic choice model in the sense that habit changes throughout time are not modelled.

Estimation results for a logit, a captivity logit and a habit model for a binary choice of mode to work with cross-sectional data from Maceio, Brazil are discussed. Essentially the difference between the captivity logit and the habit model is that an individual captive to alternative i always has his or her choice set restricted to the single alternative i (regardless of his or her

utility for that alternative) whereas an individual whose habitual mode is alternative i has his or her choice set restricted to i conditional only on the fact that his or her utility for that mode is above a certain threshold. Otherwise the choice set of that individual is the full choice set. It is this possibility of facing the full choice set (conditionally on a certain event) that makes the habit model different from a captivity model.

Section 6-2 addresses issues related to the temporal aspects of the choice process by relating behavioral changes to changes in individuals' lifestyles. Section 6-3 motivates the habit model. Section 6-4 derives the habit model. Section 6-5 presents our specification for the three models estimated in the chapter. Section 6-6 presents the models estimation results. Section 6-7 analyzes the models prediction results. Section 6-8 concludes the chapter.

6-2 The Temporal Aspects of Travel Choices

This section describes the important dynamic aspects in choice behavior and establishes the need to develop model that appropriately recognize these aspects.

As we mentioned earlier, constraints can best be described in a dynamic framework since the individual's choice set itself evolves over time, as constraints in the short-run may become decision variables in the long run.

Thus, it becomes necessary to differentiate between short-run and long-run models. The concepts and examples that follow help us understand this differentiation.

Travel demand is derived from an associated pattern of activities. Therefore, changes in travel behavior are closely related to changes in the characteristics (e.g. locations) of these activities. For example, a study in Rennes, France (Bourgin and Godard, 1981) reports that changes from high school to university student status were often accompanied by a change in mode usage from motorbike to public transit. The same survey reports that increase in car usage was reported by men in their mid-thirties who often moved from the city to a suburban fringe. The study showed that changes in transportation behavior often take place in association with changes in educational, familial, or professional life. The evolutions in mode usage generally follow the fundamental changes in lifestyle or role of the individual whereas, in comparison for an individual, the changes in the mode attributes (for example, an improvement in transit) act only as secondary effects.

Based on the previous discussion, we will define the short-run as a time period in which travel decisions are assumed to be made separately from long-run activity location decisions. Short-run models are the models that are usually specified in most applications of transportation demand modelling. As we have seen, this includes the probabilistic choice set model and its extensions presented in Chapters 4 and 5. In this context, travel is modelled separately from activity location. In this partial equilibrium framework, the locations of the activities are fixed and the travel mode choice is modelled conditionally on the location of these activities. In contrast, a long-run model jointly analyzes activity location choices and the corresponding mode choices.

Short-run travel demand models describe decisions made at a certain point in time. These models are static and assume that the individual's environment is fixed. In particular it is assumed that the information available about travel alternatives is fixed. However, it is behaviorally more satisfactory to assume that one's information is imperfect and that it results from a dynamic learning process based on observations on the transportation system made over time. This learning process will often include activities specifically designed for acquiring

information when an individual experiences changes in his or her lifestyle. Since changes in travel mode choice are more likely to occur in conjunction with changes in the lifestyle, it is necessary to recognize the dynamic aspect and its relationship to the acquisition of information and travel behavior.

Thus travel demand models estimated with data collected at a single point in time ignore the behavioral effects of such dynamics and therefore may provide a poor prediction of travel behavior. The implication is that models should consider the dynamic evolution of individuals' behavior. As pointed out by Golob and Richardson (1981), models cannot be compared solely on the basis of their ability to reproduce cross-sectional data sets. Instead, tests of the predictive power of models should be based on time series data or, more precisely, on the use of panel data. Estimation methods for models using choice observations over time have been developed by Heckman (1981).

This section has argued that the relationship between lifestyle changes and changes in transportation inevitably requires that the analyst go beyond the current static approaches to travel demand modelling by viewing the transportation system users as making a sequence of choices over time.

6-3 The Concept of Habit Formation in Travel Choice

This section pursues the idea that the dynamic changes analyzed in the previous section can be captured by changes in information availability and defines the concept of habit formation.

The rationale for linking dynamic changes in transportation behavior with changes in information availability is based on the fact that individuals who experience a change in their lifestyle are the ones that are the most likely to seek and acquire information about the attributes of alternative transportation modes. As a result, these individuals might modify their mode utilization. In contrast, individuals whose environment remain stable are less inclined to acquire this information and will less likely modify their behavior even in the presence of a change in the transportation system. This does not say that a major improvement in service for one mode will not attract individuals whose lifestyle has not changed. Only major changes will be noticed by these individuals whereas individuals experiencing a transition in their lifestyle will most likely be aware of the present values of the service attributes.

It is often argued that individuals tend to minimize the number of decisions to be taken. For instance, the choice process is simplified when the number of alternatives is very large by considerations of satisfying levels (see, for example, Golob and Richardson, 1981). Furthermore, choice behavior is altered only if the stimulus is strong enough in the sense that choices may be based on thresholds rather than on continuous values of an attribute (see, for example, Foerster, 1981). This implies that there is a certain degree of choice inertia and that choice may be largely determined by habits.

The idea of habitual choices is closely related to a decision making process involving a sequential comparison of alternatives. Once a "satisfactory" (i.e an alternative that satisfies

certain individual specific criteria or that reaches a minimum acceptable level of utility) alternative is found, the sequential search process is terminated. A model developed on this idea of sequential search is the model of Golob and Richardson (1981). The sequential choice process can be justified by the existence of search costs. Under these circumstances, the decision maker will go through an additional search only if the expected gain in utility (which must exceed a certain threshold) is greater than the certain utility loss due to the search process itself. The likelihood of seeking to acquire new information (i.e. going through the search process) naturally decreases for individuals who have little knowledge about other alternatives since these individuals, given their poor information about the other alternatives, are more likely to underestimate their expected gain in utility from a change in their habitual mode. Thus once habits become established, they will have a tendency to be reinforced.

The above description of a choice process is a realistic representation of the context under which transportation mode choices are made (and especially the mode choice to work). The traveller little knowledge about the specific attributes of competing alternative. (Recall, for instance, that the results of Chapter 5 showed that individuals may not even know exactly which alternatives are in fact available to them.) The traveller's perceptions of the attributes of competing modes are based on information accumulated over time. For mode choice decisions that are routine choices it is reasonable to expect that the travellers are well informed about the attributes of the habitually used modes.

The justification of the habit formation hypothesis comes precisely from the routine nature of certain choices. Therefore, under normal circumstances, meaning that the expected utility of an individual's habitual mode does not change significantly, it is expected that an individual will not compare the utilities of all relevant alternatives. This behavioral hypothesis provides the basis for the derivation of the habit model in Section 5-4.

6-4 Derivation of the Habit Model

The hypothesis from which the habit model is derived is that transportation mode choice (especially for work trips) is a routine choice and that therefore under normal circumstances (meaning that the utility of an individual's habitual mode does not drop below a certain minimal acceptable level) an individual will not compare the utilities of other relevant alternatives with the utility of his or her habitual alternative.

For example, drivers are often unaware of the characteristics of the transit system of the city where they live. Our hypothesis would suggest that this is not linked to the lack of information available about transit but rather to the fact that there is no perceived need to search for that information given the nature of the choice process. The decision to buy a car is a major choice of the same type as the decision of where to work or of residential location. Often this decision is associated with the decision of who will have access to the car, when and for what purposes. This is when the choice of the habitual mode is made, and one can only expect minor variations from this behavioral decision on an everyday basis.

The approach taken by the habit model is to distinguish long-run and the short-run choices. The long-run choice outcome is the habitual mode which is not necessarily the chosen mode on the survey travel day since there can be daily deviations from that choice under the following circumstances:

- i) the utility of the habitual mode dropped under a certain threshold; and
- ii) an alternative other than the habitual alternative yields the highest utility.

The choice paradigm of the habit model is shown in Figure 6-1.

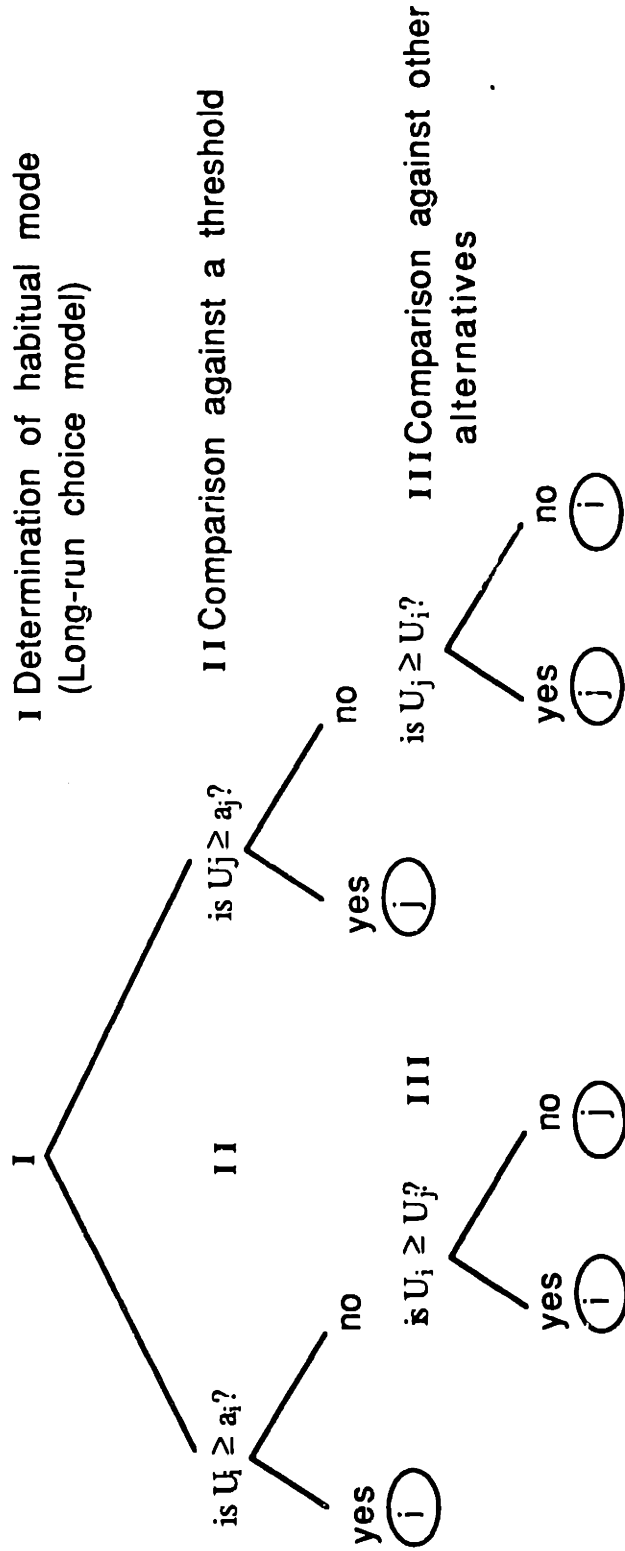


FIGURE 6-1
CHOICE PARADIGM FOR THE HABIT MODEL FOR A BINARY CHOICE CASE
(habitual mode's utility compared to a threshold)

Strictly speaking, the individual is not maximizing his or her utility since there might be some alternative that on the travel day would yield a greater utility than that of the habitual mode. However, the individual is initially only interested in fluctuations of his or her habitual mode's utility and will only reconsider his or her choice if the habitual mode becomes unsatisfactory in the sense that the corresponding utility is below a certain threshold.

Thus the behavioral idea of the habit model can be interpreted as a type of psychological captivity. The reluctance of individuals to consider alternatives other than their habitual mode implies a behavior that is similar to what would result from a population of captive decision makers. The choice inertia acts as a constraint that effectively limits individuals' choice sets. The analogy with our previous approach to modelling individuals' access to various alternatives can even be carried out further since the fact that the thresholds vary across individuals means that the above constraint is in fact a random constraint. However, in a captivity model, a captive individual is never able to choose an alternative other than the one to which he or she is captive whereas in the habit model, the short-run choice model allows deviation from the habitual choice. This constitutes an important difference between the habit and the captivity models.

An alternative way of defining the concept of habit formation is to relate the individual's informational inertia to search costs as in the sequential search model mentioned earlier. The hypothesis is that if the expected gains exceed the search costs, the individual will acquire information and reevaluate his or her choice. Note that this is the process that may also be used to describe an individual undergoing a lifestyle transition or experiencing major changes in the existing transportation system (i.e. there is a non-zero probability that the expected change in utility will be sufficient to cover the search costs). Under these circumstances an individual would seek new information and eventually may revise his or her long term choice. Therefore the individual is assumed to maximize his or her expected utility (with the utility having been redefined in such a way as to include the search costs).

The precise way in which the search costs are incorporated in the decision making process is described in what follows. Let S designate the individual specific search costs which, for ease of exposition, are assumed to be identical for all alternatives. The individual compares the utility of his habitual alternative, $U(\text{habitual alternative})$, with the expected utility of each of the searched alternatives j , $U(j)$ after having subtracted from each one of these the search cost S . Therefore the objective function is the following:

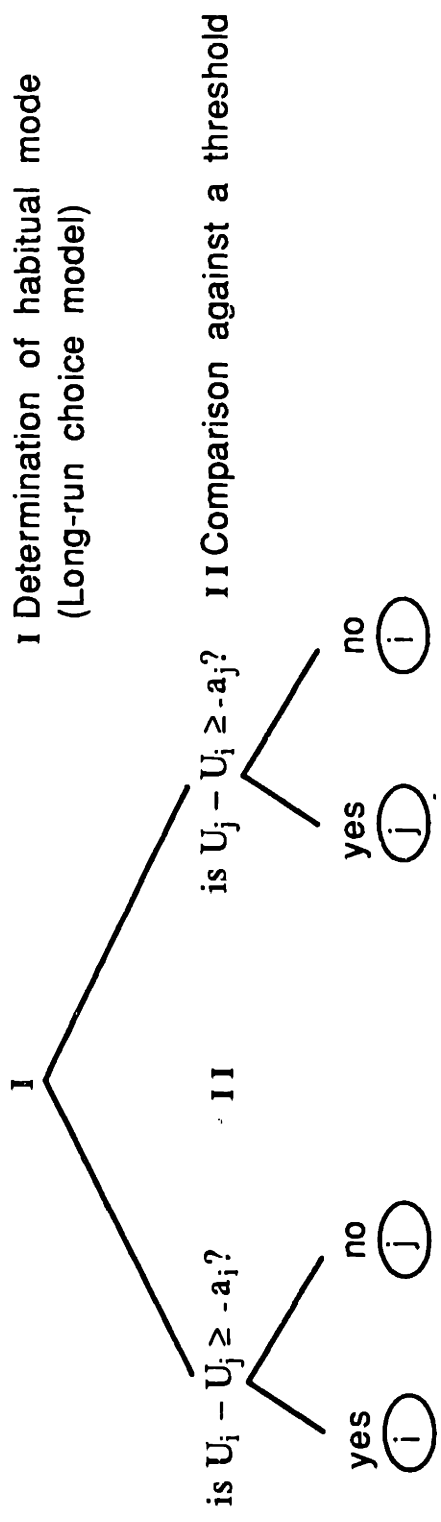
$$\text{Max}\{ U(\text{habitual alternative}) , U(j) - S , j \in C \}$$

where C is the individual's choice set.

Therefore a search will actually take place if and only if $\text{Max}\{ U(j) , j \in C \}$ exceeds $U(\text{habitual alternative}) + S$. The mode selected after the search becomes the habitual mode and, at the time of the search, has the highest expected utility. The magnitude of the search cost is itself relevant since it is what determines whether or not a search and therefore a comparison of utilities will in fact take place.

The choice paradigm of this search cost based habit model is shown in Figure 6-2. For this model, it is now the difference in the utilities of the two competing alternatives which is compared to a threshold (i.e a search cost in this case) as opposed to the previous model for which the habitual mode's utility alone was compared to a threshold.

Note also the possible asymmetries that might exist in such a model. Irreversibilities may exist in which, for example, changes in a mode attribute in one direction do not counterbalance equal changes in the same attribute in the reverse direction. This is an hysteresis effect which in our context would manifest itself in the following manner. Suppose that as a result of changes in their environment, individuals who were initially psychologically captive to a certain mode A switch to a mode B , which then becomes their new habitual mode. Suppose now that exactly the opposite change occurs in these individuals' environment. The hysteresis hypothesis is that



yes = habitual mode is used

FIGURE 6-2
CHOICE PARADIGM FOR THE SEARCH COST BASED HABIT MODEL FOR A BINARY CHOICE CASE
(difference of utilities compared to a threshold)

some of these individuals will not reverse their habit by switching back from B to A. For a description of these effects, see, for example, Goodwin, 1977. Such hysteresis effects have been supported by a study of weekend travel and gasoline prices in London, England (Blase, 1979).

For the model shown in Figure 6-1, the habitual mode remains the chosen one as long as it is satisfactory (in the sense defined earlier) and this regardless of what happens to the other modes. Thus the model corresponds to situations of stronger choice inertia (i.e habits that are more difficult to overcome) than the model shown in Figure 6-2.

The remainder of this section formulates a binary habit model based on the paradigm shown in Figure 6-1. For ease of exposition, we first formulate the short-run habit model for which the individuals' habitual modes are given (i.e individuals are classified according to their habitual alternative). This corresponds to stages II and III of the choice paradigm of Figure 6-1. Then the model is extended, by including a model for stage I, to a habit model for which the habitual modes are the results of a long-run choice.

Let i and j be the two alternatives and let j designates the habitual mode. Then

$U_j \geq a_j \Rightarrow$ choose the habitual mode j as long as U_j is greater than the threshold a_j

$U_j < a_j \Rightarrow$ compare the habitual mode with the other alternative.

There are no requirements that the thresholds be constant across observations and they can themselves be dependent upon individual characteristics.

Therefore, if j is the habitual mode (indicated by a conditioning event $h=j$), the probability that individual n will be observed to travel in mode j is

$$P_n(j | h = j) = P_n(U_j \geq a_j) + P_n(U_j \geq a_j)P_n(U_j \geq U_i | U_j \leq a_j) \quad (1)$$

Since

$$P_n(U_j \geq a_j)P_n(U_j \geq U_i | U_j \leq a_j) = P_n(U_i \leq U_j \leq a_j)$$

we have :

$$P_n(j | h = j) = P_n(U_j \geq a_j) + P_n(U_i \leq U_j \leq a_j) \quad (1')$$

This formulation is valid for individuals for which the habitual mode is alternative j . In order to develop this habit model, it is first necessary to classify the individuals in two groups

$J = \{n; \text{habitual mode is alternative } j\}$

$I = \{n; \text{habitual mode is alternative } i\}$

$\forall n \in N, n \in I \text{ or } n \in J \text{ o that } N = I \cup J; I \cap J = \emptyset$

and we will have

$$\forall n \in J, P_n(j) = P_n(j | h = j)$$

as given by (1')

$$P_n(i) = P_n(i | h = j) = 1 - P_n(j | h = j)$$

$$\forall n \in I, P_n(i) = P_n(i | h = i)$$

$$P_n(j) = P_n(j | h = i) = 1 - P_n(i | h = i)$$

with

$$P_n(i | h = i) = P_n(U_i \geq a_i) + P_n(U_j \leq U_i \leq a_i)$$

a_i and a_j and are unknown parameters (utility thresholds) to be determined along with the utilities parameters.

The habit model is now extended to a probabilistic habit model for which the classification of the individuals in two groups I,J corresponds to the long-run choice which determines the habitual mode.

Let

$$Q_n(j) = \text{Prob}(n \in J) = 1 - \text{Prob}(n \in I) = 1 - Q_n(i)$$

and the probabilistic habit model is defined by:

$$P_n(i) = Q_n(i)P_n(i | h = i) + Q_n(j)P_n(i | h = j)$$

or

$$P_n(i) = Q_n(i)P_n(i | h = i) + (1 - Q_n(i))(1 - P_n(j | h = j)) \quad (2)$$

The expression $Q_n(i)$ is the choice probability for a long-run choice model (e.g. binary logit).

The derivations above show that it is necessary to compute the following expression

$$P_n(U_i \leq U_j \leq a_j)$$

This is done in Appendix 3 where it is shown, assuming that the error terms of the utilities are iid Gumbel, that

$$P_n(U_i \leq U_j \leq a_j) = \frac{1}{1 + e^{-\mu(v_j - v_i)}} e^{-\mu(a_j - v_j)} [1 + e^{-\mu(v_j - v_i)}]$$

and by rearranging terms that

$$P_n(U_i \leq U_j \leq a_j) = P_n(U_j \geq U_i) \cdot P_n(U_j \leq a_j) \cdot P_n(U_i \leq a_j) \quad (3)$$

and we finally have :

$$P_n(j | h = j) = P_n(U_j \geq a_j) + P_n(U_j \geq U_i) \cdot (1 - P_n(U_j \geq a_j)) \cdot (1 - P_n(U_i \geq a_j)) \quad (4)$$

$P_j(i | h = i)$ is defined in a similar way.

Equations (2) and (4) define the habit model once a functional form for the long-run choice probability of the habitual mode (for example a binary logit with the same utility specification as the short-run choice model (up to a scale parameter) or with a completely different utility specification) is defined.

6-5 Specification of the Models

The data were obtained from a household survey in Maceio, Brazil during 1977. The two modes considered are private car and public transportation. The number of observations is 355 with 237 (66.76%) choosing public transportation and 118 (33.24%) choosing private car. This Maceio data set is a binary version of the data set used by Swait (1984). A good description of the data can be found in Swait and Ben-Akiva (1986).

Maceio is situated in Northeast Brazil and has experienced substantial population growth. It also experienced in the last few years a large increase in car-ownership with the rate increasing from 17.3 vehicles per 1,000 persons in 1967 to 44.4 in 1976. Car occupancy data shows that the car occupancy rate of 1.76 is higher in Maceio than in cities such as Porto-Alegre (1.38) or Sao-Paulo (1.45).

The specification of the utility functions for the logit model are as follows:

$$V(\text{transit}) = b_1 + b_2.\text{ovtt} + b_3.\text{ivtt} + b_4.(\text{cost}/\text{inc})$$

$$V(\text{auto}) = b_2.\text{ovtt} + b_3.\text{ivtt} + b_4.(\text{cost}/\text{inc}) + b_5.\text{carav}$$

where

ivtt = in-vehicle travel time;

ovtt = out-of-vehicle travel time;

cost = the cost of the trip;

carav = the number of cars owned or leased divided by the number of driving-age individuals in the household

An identical specification is used for the utility functions of all the models estimated in the chapter.

As it was mentioned in the derivation of the habit model, it is possible to model the utility thresholds as function of the socioeconomic characteristics of the individuals. As long as a_j varies across individuals in a deterministic way, nothing needs to be modified in the derivation of the habit model. To illustrate this point, a habit model for which the threshold for transit varies with car availability is specified. The specification adopted for the determination of the habitual mode is as follows:

$$\begin{aligned} \text{Prob (car is the habitual mode)} &= 1 - \text{Prob (bus is the habitual mode)} \\ &= 1 / (1 + \exp(b_6 + b_7 \cdot \text{carav})) \end{aligned}$$

This specification corresponds to a long-run choice model for which the difference in the long-run utility of car and transit is $\Delta V = \alpha + \beta \text{carav}$ where α and β are unknown parameters. This model specification is referred to in the section that follows as HAB1 (Habit Model 1). A more complicated model specification is one for which the long-run utility has an identical functional form as the short-run utility. This model specification is referred to as HAB2 (Habit Model 2). HAB1 is naturally a nested version of HAB2 for which the parameters corresponding to the time and cost variables (in-vehicle travel time, out-of vehicle travel time and cost divided by income) are set equal to zero in the long-run utility.

The specification for the thresholds is as follows:

$$\begin{aligned} \text{utility threshold} &= b_8 + b_9 \cdot \text{carav} \\ \text{(transit)} & \end{aligned}$$

$$\begin{aligned} \text{utility threshold} &= b_{10} \\ \text{(auto)} & \end{aligned}$$

The habit model is compared to a logit captivity model for which the functional forms of the captivity functions are identical to the parametrized probability that car is the habitual mode ("psychological captivity"). Therefore the specification of the logit captivity is as follows:

$$\text{Prob}(\text{captivity to transit}) = 1/(1+\exp(b6' + b7'.\text{carav}))$$

$$\text{Prob}(\text{captivity to auto}) = 1/(1+\exp(b8' + b9'.\text{carav}))$$

This is a very simple specification since the access to a car (through the car availability variable) is the only constraint affecting the choice set probabilities. A captive individual is restricted to choosing the alternative to which he or she is captive whereas an individual who is not captive to either transit or auto faces the two alternatives, transit and auto, and chooses one of the two alternatives according to a binary logit.

6-6 Models Estimation Results

Results for the two habit models, HAB1 and HAB2, are shown in Table 6-1. The coefficients corresponding to the attributes variables in the long-run utility of HAB1 are all statistically insignificant. Furthermore, the results are as follows:

<i>model</i>	<i>Log-Lik. - # param.</i>	<i># param.</i>
HAB1	-81.53	10
HAB2	-83.75	13

TABLE 6-1
ESTIMATION RESULTS FOR THE HABIT MODELS

variables	parameters	Estimates (t-statistic)	
		HAB1	HAB2
<i>short-run utility</i>			
Bus constant	b1	6.23 (2.25)	6.02 (2.24)
out-of-vehicle travel time	b2	-0.11 (-2.06)	-0.11 (-2.07)
in-vehicle travel time	b3	-0.02 (-1.06)	-0.02 (-1.22)
cost divided by income	b4	-15.33 (-2.57)	-15.02 (-2.39)
car availability	b5	8.85 (5.37)	8.75 (4.94)
<i>long-run utility</i>			
Bus constant	b6	0.13 (0.12)	-0.25 (-0.20)
out-of-vehicle travel time	b7		-0.06 (-1.07)
in-vehicle travel time	b8		0.02 (1.11)
cost divided by income	b9		-4.58 (-0.71)
car availability	b10	2.83 (2.19)	2.63 (1.82)
<i>thresholds</i>			
<u>bus</u>			
constant	b11	-3.62 (-1.20)	-4.12 (-1.71)
car availability	b12	-1.95 (-0.29)	-1.89 (-1.28)
<u>auto</u>			
constant	b13	0.79 (0.80)	0.62 (0.49)

TABLE 6-1
continued
ESTIMATION RESULTS FOR THE HABIT MODELS

	<i>summary statistics</i>	
Number of observations	355	355
Number of parameters	10	13
Log-Likelihood at zero	-246.07	-246.07
Log-Likelihood at convergence	-71.53	-70.75
ρ^2	0.71	0.71
$\bar{\rho}^2$	0.67	0.66

Therefore, based on the comparison of the log-likelihoods adjusted for the number of parameters (Akaike information criteria), HAB1 is the habit model specification chosen and will now be compared to the logit model and the logit captivity model.

Results for the logit, the logit captivity and the habit (HAB1) model are shown in Table 6-2. In order to make it easier to compare the models, the results for the habit model are repeated in the table.

For all three models, the coefficients have the expected sign. The main explanatory variables are the cost variable (total cost divided by income) and a measure of automobile availability (number of cars in the household divided by the number of workers). Adjusting the log-likelihood at convergence for the number of parameters, the results show that the habit model performs slightly better than the captivity model and that both are clearly superior to the logit model. The results are as follows:

<i>model</i>	<i>Log-Lik. - # param.</i>	<i># param.</i>
logit	-83.07	5
captivity	-81.86	9
habit	-81.53	10

This absence of captivity to the auto mode is easily detected from the fact that the constant in the auto captivity function is 11.25 (and $1/(\exp(11.25)+1)$ is nearly zero). The constant in the transit

TABLE 6-2
ESTIMATION RESULTS FOR THE LOGIT, CAPTIVITY AND HABIT MODELS

variables	parameters	Estimates (t-statistic)		
		LOGIT	CAPTIVITY	HABIT
<i>short-run utility</i>				
Bus constant	b1	3.51 (5.30)	4.09 (4.35)	6.23 (2.25)
out-of-vehicle travel time	b2	-0.04 (-2.19)	-0.07 (-2.34)	-0.11 (-2.06)
in-vehicle travel time	b3	-0.02 (-1.13)	-0.03 (-1.61)	-0.02 (-1.06)
cost divided by income	b4	-10.22 (-2.93)	-15.40 (-3.15)	-15.33 (-2.57)
car availability	b5	6.87 (9.07)	10.24 (6.04)	8.85 (5.37)
<i>captivity functions</i>				
<u>bus</u>				
constant	b6'		1.05 (0.93)	
car availability	b7'		1.92 (1.47)	
<u>auto</u>				
constant	b8'		11.25 (2.85)	
car availability	b9'		-0.30 (-2.39)	
<i>long-run utility</i>				
Bus constant	b6			0.13 (0.12)
car availability	b7			2.83 (2.19)

TABLE 6-2
continued
ESTIMATION RESULTS FOR THE LOGIT, CAPTIVITY AND HABIT MODELS

variables	parameters	LOGIT	CAPTIVITY	HABIT
<i>thresholds</i>				
<u>bus</u>				
constant	b8			-3.62 (-1.20)
carav	b9			-1.95 (-0.29)
<u>auto</u>				
constant	b10			0.79 (0.80)
<i>summary statistics</i>				
Number of observations	355	355	355	
Number of parameters	5	9	10	
Log-Likelihood at zero	-246.07	-246.07	-246.07	
Log-Likelihood at convergence	-78.07	-72.86	-71.53	
ρ^2	0.68	0.70	0.71	
$\bar{\rho}^2$	0.66	0.66	0.67	

captivity function is a lot smaller, and the model exhibits some captivity to the bus mode. The captivity function for bus has a mean of 0.20 (with a maximum value of 0.26 and a minimum of near zero).

Therefore, the captivity model results suggest that individuals are never captive to the auto mode in the sense that if auto is available, transit is also in the choice set. The relevant choice sets are transit alone and the full choice set: transit and auto.

As car availability increases, the probability of being captive to auto is expected not to decrease (but not necessarily to increase), which is what the model predicts since the $carav$ coefficient in the captivity function is negative (although it is insignificant and very small). Similarly as car availability increases (relaxation of the car ownership constraint) it is expected that captivity to the bus mode, if any, should decrease, which is again what the model predicts since the coefficient of $carav$ for the transit captivity function is positive. This coefficient is significant and shows the importance of a measure of access to a car as an explanatory variable (in terms of being a relevant constraint) of choice set formation in a mode choice model.

The habit model results show that as car availability increases, the probability of having auto as the habitual mode increases. However, as car availability increases, it is not clear whether the thresholds for auto or for bus should be expected to increase or decrease. It can be argued, for example, that if car availability increases and if bus is the habitual mode (which becomes less likely), then the threshold should increase since an individual with better access to auto would more likely require a higher unconditional transit utility (hence a higher threshold) in order to select his habitual mode. This would imply that for an individual whose habitual mode is transit, the event of carrying out the second stage of the choice process should be more likely as access to auto improves. In any case, the car availability coefficients for the thresholds were not

significant. The carav variable in the determination of the bus threshold is kept for illustrative purposes to show that there are no requirements, as we stated earlier, that the habit thresholds be constants.

The minimum probability of having auto as the habitual mode is 0.53 (the mean value of the long-run auto choice probability is 0.66). Therefore, it always exceeds the probability of having bus as the habitual mode. The auto utility exceeds its threshold for 31% of all observations, while the bus utility is almost always (for 96% of all observations) above its threshold. This result implies that when bus is the habitual mode, the second stage of the choice process is never carried out. This is counterintuitive but can be interpreted in two ways: either bus is never the habitual and the bus threshold becomes irrelevant or since the coefficients are not statistically significant the relevant threshold to which the bus utility must be compared is actually zero, in which case the bus utility exceeds its threshold now for 71% of the population. An alternative way of interpreting the result is to view the habit as a psychological captivity. The results would then be interpreted as a sign of captivity to transit.

Therefore, the results show that if the car is available, it is not only the habitual mode but it clearly dominates all other modes. This has important consequences. It becomes essential for successfully modelling transportation mode choice (especially for the work trip) to be able to classify the population according to its access to different alternatives. This requirement is not as straightforward as it appears since the "true" access to alternatives will often be probabilistic, as in our example where the classification variable used is car availability for a given household. It is important to take into account that different household members (although with the same access to automobile according to our measure) have very different behavior according to whether or not they have a driver's license and according to whether or not they are employed (in the primary or in the secondary job within the household). These considerations, given the

large number of variables that will affect the access to the different alternatives, suggest that a better description of the choice set faced by each individual is essential in increasing our understanding of the mode choice process.

In comparing the logit captivity and the habit models, an issue is whether the results of the two models contradict or support each other. For example, is the fact that nobody appears to be captive to auto in contradiction with the fact that auto is the habitual mode for a significant portion of the data population ? The answer is no, since having auto has the one's habitual mode and choosing auto does not say anything about the availability of transit. The fact that the short-run choice coincides with the long-run choice is not a statement about choice sets.

6-7 Models Prediction Results

It is necessary to look at differences in model predictions in order to compare the various models presented in this chapter. Both the captivity and the habit model have been shown to be statistically superior to the logit model. This section investigates whether aggregate measures of behavioral responses for these two models are significantly different than the same measures for the logit model. The comparison of the models is done, similarly to what has been done in Chapter 4 and in Chapter 5, on the basis of predicted ridership changes to various policy scenarios.

The policy scenarios chosen are as follows :

- i) a doubling of out-of-vehicle time for transit;
- ii) a doubling of the cost for transit.

The reasons for selecting large changes in the magnitude of the attributes is to highlight the differences in predictions of the three models as opposed to simply testing the models responses under less extreme scenarios.

Table 6-3 shows the predicted percentage changes in transit ridership for each of the scenarios.

The coefficient of transit out-of-vehicle time is small for each of the three models (-0.044 for the logit, -0.68 for the captivity and -0.11 for the habit). Therefore, as expected, all the three models predict a moderate decrease in the transit ridership. This decrease is in turn almost identical, about -12%, for the three models.

The cost variable is divided by income for the three model specifications, and in spite of the large coefficients, the predicted decreases in transit ridership are low and again similar, about -3.3%, for the three models.

Thus the results show nearly identical predictions for the three models. More moderate predicted responses for the habit model seems logical since this is consistent with the habit hypothesis that there are inertia to changes.

TABLE 6-3
COMPARISON OF MODELS PREDICTION RESULTS

100% increase in ovt for transit

	logit	captivity	habit
change in share (transit)	-11.84%	-12.19%	-11.98%

100% increase in cost for transit

	logit	captivity	habit
change in share (transit)	-3.44%	-3.39%	-3.20%

6-8 Conclusions

Chapter 6 has developed a discrete choice model based on the notion of choice inertia or habit. The behavioral hypothesis underlying the habit model is that unless the utility of an individual's habitual mode drops below a certain threshold, in which case there will be a utility maximizing choice, an individual will automatically select his or her habitual mode. Therefore, the habit model combines a satisficing aspect at the "psychological" choice set formation level and a random utility maximization if the "psychological" choice set involves more than one (the habitual) alternative.

The empirical results of this chapter show the differences between a logit captivity and a habit model. The latter is shown to be marginally statistically superior whereas both models constitute a clear improvement over the logit specification.

Chapter 6 has explored a lot of new ground, but it did not derive a complete dynamic choice model since habit formation itself was not modelled. However it has been argued that there are important dynamic aspects in choice behavior. The idea of dynamic changes in the choice set, captured by changes in information availability, lead us to the concept of habit formation. The temporal aspects of travel choices and the concept of habit formation described in the chapter highlight limitations of most of the models (short-run) estimated in the transportation demand modelling literature. This, however, does not imply that short-run models should be rejected. In fact, the two types of models (short-run and long-run) are complementary. Short-run models are appropriate for describing decisions that are made at a specific point in time while long-run choice models are more appropriate when dynamic changes (i.e change in the availability of information about alternatives or changes in the lifestyle of individuals) are important.

CHAPTER 7

CONCLUSION

7-1 Summary

This work has placed in theoretical and empirical perspective the process of choice set generation in discrete choice situations.

Chapter 2 has reviewed the state of the art in modelling the choice set generation stage of the choice process. Recent developments in the literature have emphasized a constraint based approach to choice set formation. (See, for example, Swait, 1984; Swait and Ben-Akiva, 1987a and 1987b; and Kitamura and Lam, 1984). Chapter 2 describes the nature of constraints to urban travel, and it shows that these not only include objective limitations to alternative availabilities (e.g. possession of a driver's license) but also subjective limitations that come from the individuals' attitudes and perceptions. The literature review shows that the influence of constraints and the influence of perceptions and attitudes on the choice set generation process have mostly been addressed as separate issues. The emphasis has been on the determination of attitudes and preferences of individuals with respect to certain modes. The literature has essentially been of a descriptive nature with little attempt at using psychometric data in the estimation of choice models. Chapter 2 suggests to incorporate constraints and attitudes in a single choice set generation modelling framework which uses simultaneously psychometric data and revealed preference data.

The derivation of such a framework is the task accomplished in Chapter 3. First the chapter presents the notion of random constraints. The framework developed in Chapter 3 is based on a choice behavior paradigm that includes the influence on choice set generation of latent psychological factors representing attitudes and perceptions. The use of indicators is suggested to estimate the resulting latent variable model. Essentially, our methodology is an approach that estimates choice models using simultaneously several types of indicators (such as responses to survey questions on perceived availabilities of alternatives) as opposed to the exclusive use of the observed choice indicator. The next two chapters of the dissertation are an empirical application of the integrated framework.

The purpose of the empirical work of Chapters 4 and 5 is to assess the practicality of the integrated framework proposed in Chapter 3. Chapter 4 presents a probabilistic choice set generation model of mode to work for the city of Baltimore, Maryland. This model serves a benchmark for the empirical work of Chapter 5 where several extensions of the model, each relying on the uses of choice set formation indicators are tested. The results of Chapter 4 confirm that probabilistic choice set models constitute a significant improvement over the multinomial logit.

The model extensions presented in Chapter 5 differ in their specification of the measurement equations that relate the choice set indicators and the latent perceived availabilities. All structural equations are identical to the structural equations of the model of Chapter 4. Specifically, three models are estimated, each corresponding to a different way of modelling the influence of desirability on the responses to the alternative availabilities questions. The results show that the use of the indicators improves significantly the precision of the parameters estimates. However, estimations of these models proved to be complicated, and the increased precision is obtained at a cost. The results show that the predicted alternative availabilities

responses are most consistent with the data for a model specification for which the influence of desirability on the perceived alternative availabilities is captured by an unconstrained (calculated on the universal choice set) choice probability.

Chapter 6 argues that there are important dynamic aspects in choice behavior. The idea of dynamic changes in the choice set, captured by changes in information availability, leads us to the concept of habit formation. A habit model, based on the idea that choice situations are often of a routine nature, is derived. For this model, the habitual mode corresponds to a long-run mode choice (which can also be modelled with logistic random utilities). Short-run choices correspond to daily fluctuations around that choice. These fluctuations can occur if the utility of the habitual mode falls below a certain threshold and if the utility of at least one other alternative exceeds the utility of the habitual mode. A comparison of estimation results for a logit, a captivity and a habit model for mode choice to work in Maceio, Brazil show that the latter fits the data best (although the fit of the logit captivity is almost the same).

The temporal aspects of travel choices mentioned above are related to the work of the previous chapters in the following way. Short-run models are the models that are usually specified in most applications of transportation demand modelling. This is the case for the models developed in this thesis. Chapter 6 argues that these models are appropriate for describing decisions that are made at a specific point in time while long-run choice models are more appropriate when dynamic changes (i.e change in the availability of information about alternatives or changes in the lifestyle of individuals) are important.

7-2 Contribution of the Thesis

We first parametrized the previously proposed random constraint framework of choice set generation modelling which analyzes choice behavior at the individual level with an explicit treatment of the heterogeneous constraints that influence choice set formation.

The most important contribution of the thesis is methodological. Probabilistic choice set models have been estimated with subjective data on alternative availabilities. The information contained in survey responses to questions on perceived choice sets has been used in measurement equations which relate these responses to the true (unobservable) alternative availabilities.

The implementation of our framework involves observable and latent variables. The framework is described as an integrated framework because it has the possibility of taking fully into account objective and subjective constraints. The main idea is that the latent variables (essentially the unobservable constraints that determine the alternative availabilities) can be inferred from observed indicators. It is suggested that the estimation of demand models be carried out by using, as much as possible, the information provided by these indicators instead of relying exclusively on the information provided by the observed choice. The use of other data sources, as a complement to revealed preference data is an important aspect of our approach. The main difference with previous work in the study of attitudinal responses is that responses have been used in the past as predictors (i.e causes) as opposed to being used as indicators (i.e manifestations of latent variables). For example, our models use reported car availability as an indicator of latent availability instead of using it as an independent variable.

The empirical work is an important part of this research. The estimation of the integrated framework models has resulted in an increased efficiency of the parameters estimates. The

estimation results of Chapter 5, using data collected in Baltimore in 1977, have shown that respondents have a strong bias in favor of the auto mode in the sense that they are more likely to perceive it as available even if it is not, while the opposite holds for transit. The habit model has confirmed the routine nature of the mode to work choice and that if car availability increases, the auto mode will become the habitual mode.

This research introduces innovative modelling approaches to the transportation field. The two main ideas explored by the thesis are the use of psychometric data in the estimation of choice set generation models and the derivation of a habit model. The strength of the thesis is that these ideas are made operational by first providing a technical derivation of the models and then by going through empirical applications to test and implement the models.

7-3 Suggestions for Further Research

This thesis has only begun to investigate new modelling approaches to choice set formation in discrete choice situations. As we have indicated it in Chapter 3, our framework is a special case of a more general framework. Therefore, our empirical work in Chapter 5 only corresponds to one submodel of this integrated framework.

Other submodels include a model with joint use of stated and of revealed preference data and a psychometric model that explains the formation of perceptions and attitudes. The empirical implementation of these models is in many ways similar to the empirical work carried out in this thesis. Again, the main idea is the use of indicators to infer the latent variables of the problems. Therefore, a first direction for continuing research in the area is to implement along the lines of

our work these submodels. In addition, it would be fruitful if the empirical work could be carried out for discretionary choice contexts other than mode choice to work. For example, one might want to analyze residential location or shopping or recreational travel choice.

At this stage, the research has demonstrated the potential of our modelling and estimation strategy in terms of increased precision of the parameters estimates. However, the substantial difficulties of estimating these models hinder the future use of similar model since specification search is very difficult. Therefore, more experience is needed with these models. Furthermore, our choice set generation model was kept relatively simple and further exploration of choice set restrictions (again, possibly in choice situations other than mode to work choice) and further modelling of the constraints is suggested.

Chapter 6 explored a lot of new ground, but it did not derive a complete dynamic choice model since habit formation itself was not modelled. To do so is an entirely new research agenda. First, it involves the use of panel data with choice observations over time and requires that the values of the attributes be known over that same period. Second, the approach must not only incorporate attitudinal data in the analysis but also look at aspects of the choice behavior, such as lifestyle changes, which are not commonly accounted for in the present transportation demand models.

This corresponds to a dynamic version of the general paradigm mentioned in Chapter 3 (and presented in Appendix 1), which is, of course, a very ambitious undertaking.

APPENDIX 1

INTEGRATED FRAMEWORK FOR TRAVEL BEHAVIOR ANALYSIS

A1-1 Introduction

The purpose of this appendix, as indicated in Chapter 3, is to develop an integrated framework for travel behavior analysis for which a special case is the framework developed in Chapter 3. Section A1-2 motivates the derivation of an integrated framework by showing the possibility of integrating, in a single approach, market research techniques and econometric modelling techniques. Section A1-3 reviews the most widely used market research techniques. Section A1-4 develops the integrated framework itself. Section A1-5, by describing several submodels, gives examples of applications of the integrated framework. It shows how the approach adopted in Chapter 3 is a special case of the more general approach. Section A1-6 concludes the appendix.

A1-2 Motivation

In recent years, there has been an increasing interest among transportation researchers in techniques which have been extensively applied in market research. (See, for example, Bates, 1983; Benjamin and Sen, 1982; Dix, 1981; Hensher and Stopher, 1979; Kawakami and Hirobata, 1984; Koppelman and Hauser, 1979 and Michaels and Allaman, 1983). However, discrete

choice models that have been widely applied to travel demand analysis have made little use of psychometric data on perceptions and attitudes. (See, for example, Ben-Akiva and Lerman, 1985). On the other hand, psychometric data collected in market research studies have usually not been incorporated into an econometric modelling framework that permits quantitative forecasting. One exception is the use of random utility models with stated preference data to model individual preferences (e.g., Louviere and Hensher, 1983; and Kroes and Sheldon, 1986). Although econometric models and market research techniques tend to utilize different types of data, there are sufficient points of common interest to propose the integration of both methodologies in the same framework.

This appendix has two objectives:

- to develop an integrated theoretical framework with latent psychological factors for predictive travel demand analysis; and
- to show how existing and innovative data analysis methods can be used to implement this framework.

Our overall modelling approach has the following characteristics:

- Analysis of behavior is at the level of the individual decision maker with explicit treatment of heterogeneous preferences and situational constraints.
- Use of both stated and revealed preferences data to estimate and validate choice models.
- Use of both latent attitudinal and measurable socioeconomic variables to explain travel behavior.

Before developing our framework, it is necessary to briefly review, in the next section, market research methods since our approach is based on a combination of these techniques with discrete choice analysis. Obviously, the way in which the different market research techniques can be

combined with discrete choice analysis to form a coherent modelling approach will vary from case to case. The differences between the cases are the ways in which attitudes and perceptions are linked to the preferences model.

A1-3 Brief Overview of Market Research Methods

A variety of data types and data analysis techniques are being used in market research studies. The methods that are the most useful for our purpose are listed in Figure A1-1.

The technique most widely used for modelling attitudes and perceptions is factor analysis. An introduction to factor analysis may be found in Johnson and Wichern (1982). Factor analysis is very effective in identifying general attitudes that relate to choices, but has limited use in quantifying the tradeoffs that consumers are willing to make between attributes. For example, factor analysis may identify comfort as a relevant variable for transportation mode choice analysis, but it cannot quantify how many travellers are willing to pay for an incremental improvement of the level of comfort.

Multidimensional Scaling (MDS) is an indirect approach to analyse individual's perceptions. It relies on dissimilarity matrices (square table of dissimilarity, or similarity, ratings between pairs of objects). MDS is a set of procedures in which a respondent's ratings of the overall dissimilarity, or similarity, between a set of objects is used to locate specific objects in a multidimensional perceptual space. These points are determined such that the distances between objects in the perceptual space match the direct dissimilarity, or similarity, judgments as closely as possible. An overview of MDS theory can be found in Green and Rao (1982).

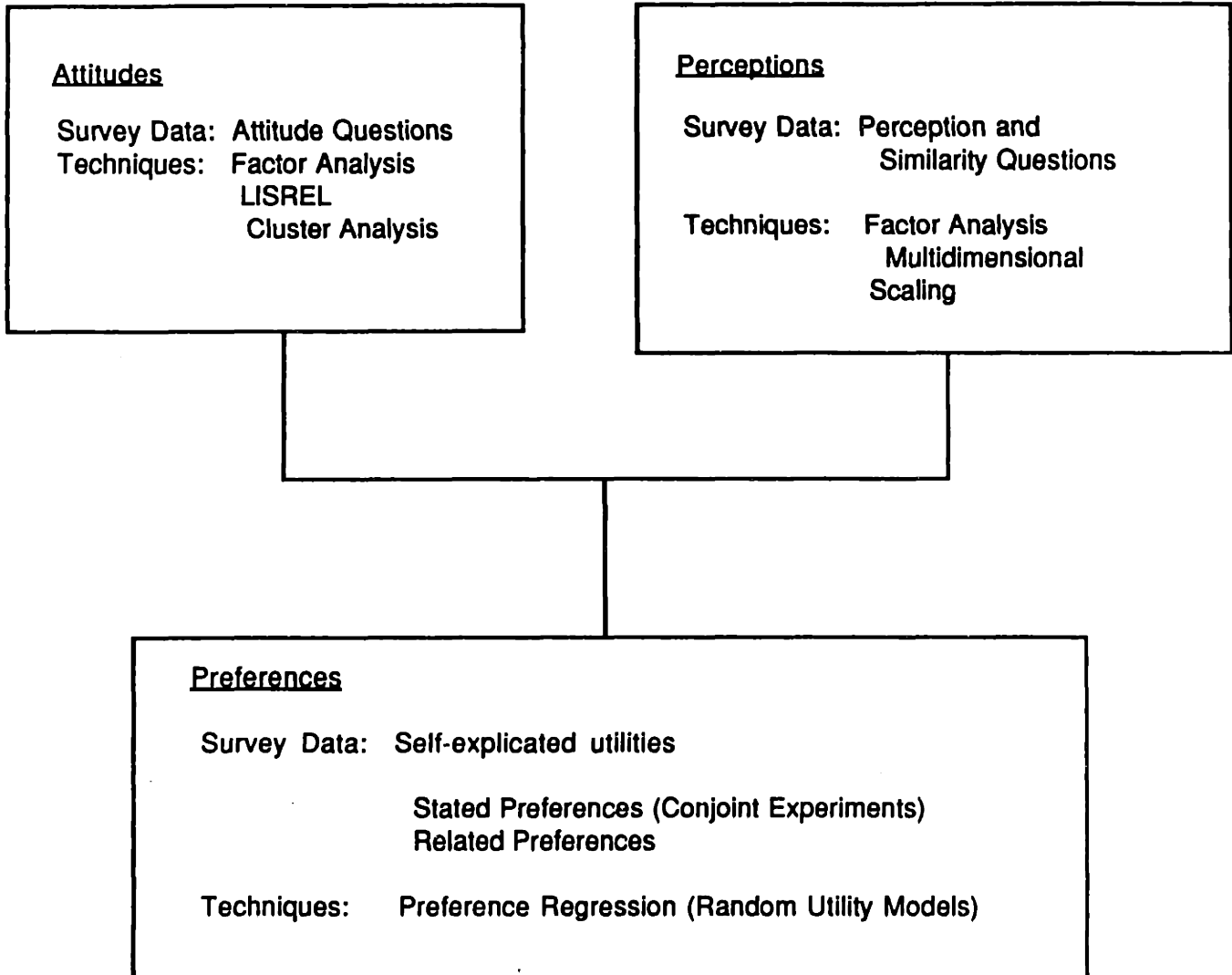


FIGURE A1-1

SURVEY DATA AND ANALYSIS METHODS

A generalization of factor analysis is a system of simultaneous linear equations with latent variables. Such a system may include both structural equations, relating latent and observable variables, and measurement equations which express indicator variables as function of latent and observable structural variables. One of the most widely used estimation techniques for linear systems of interdependent equations with latent variables is LISREL (Joreskog and Sorbom, 1982). Generalizations of LISREL that might be needed when combining discrete choice analysis and linear latent variable models are presented in Muthen (1983) (i.e., a LISREL model with categorical indicators) and in Bartholomew (1983) (i.e., an approach called latent class analysis that includes both categorical indicators and categorical latent variables).

The identification of individuals with different attitudes, perceptions, and preferences is often achieved by the use of a market segmentation approach. The objective is to classify individuals into a mutually exclusive and exhaustive set of market segments that are (relatively) homogenous in their behavioral response to changes in the transportation system. The techniques being employed range from cross-tabulation of attitudinal responses and socioeconomic factors to a variety of cluster analysis techniques and classification procedures. Classification is usually performed using discriminant analysis but can also be done more efficiently with fewer assumptions using logit analysis.

One particularly popular method for measuring preferences is known as conjoint analysis. An introduction to conjoint analysis may be found in Green and Wind (1975). In conjoint analysis, respondents are presented with descriptions of several hypothetical alternatives, each of which has different attributes. For example, various airport public access means might be described, each with a specific fare, travel time, frequency, comfort, etc. The respondent would be asked to indicate his relative preference for each of the alternatives. The responses are used to infer the

implicit weights which respondents place on each of the attributes in expressing their preferences. Conjoint analysis requires that at an earlier stage the relevant attributes be identified, using for example, one of the methods mentioned above.

A1-4 Development of an Integrated Framework

The development in this paper is based on the basic paradigm of individual behavior depicted in Figure 3-1 of Chapter 3. Similar paradigms have been used in many previous studies (see, for example, the review in Cambridge Systematics, 1986). It identifies four types of latent psychological factors:

- (i) Attitudes, Needs and Beliefs
- (ii) Perceptions
- (iii) References
- (iv) Behavioral intentions

The core of the integrated approach is the extension of the basic paradigm that is presented in Figure A1-2. The principal group of factors that directly or indirectly affect individual behavior and the postulated relationships are shown in this paradigm. This path diagram consists of several submodels. The first model is a psychometric model that explains the formation of perception and attitudes. The second model defines the utility and the choice. The third model incorporates the constraints into the choice process.

The empirical implementation of this behavioral paradigm involves both the observable and the latent variables shown in Figure A1-2. The latent variables can be inferred from observed data

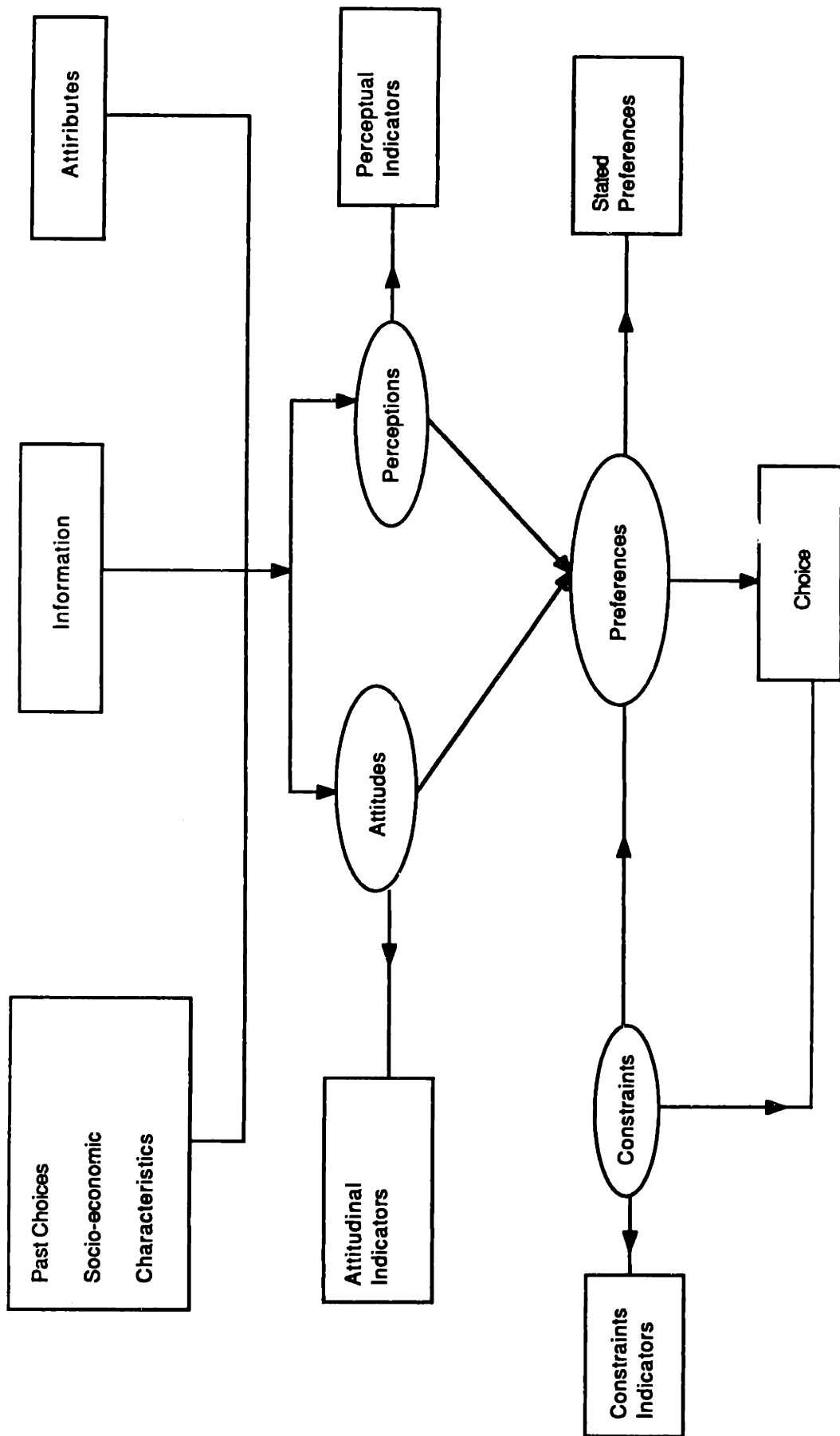


FIGURE A1-2.
INTEGRATED FRAMEWORK FOR BEHAVIORAL ANALYSIS

which include indicator variables by using one of the estimation methods that were described in the previous section. The way in which different techniques are combined to form a coherent analysis approach will naturally vary from study to study.

Consider, for example, a survey that includes attitudinal questions and a conjoint experiment. The first step of the analysis could be to apply a factor analysis or LISREL technique to estimate attitudinal variables. In the second step, the conjoint data may be used to estimate the weights of the attributes. One method that is particularly well suited for this is a discrete choice model. The behavioral paradigm suggests that it would be valuable to incorporate the results of the factor analysis into the discrete choice model. For example, one would expect that respondents with high factor scores for cost consciousness would tend to favor lower cost alternatives more than would respondents with low factor scores. Thus the factor analysis results should add predictive power to the discrete choice models.

The modelling concepts that were outlined so far are considered to represent an integrated framework because attitudes, perceptions and constraints, at the individual level are fully accounted for. Such an approach explicitly allows for some of the population heterogeneities to appear in the choice model itself (as variations in model parameters or variable coefficients).

Essentially, our approach can be viewed as a methodology that formulates choice behavior models simultaneously using several types of indicators, as opposed to econometric discrete choice models for which estimation is based only on observed choice (i.e. choice indicator only).

A1-5 Applications of the Integrated Framework

The purpose of this section is to illustrate our approach by considering several simplified versions of the modelling structure, each one corresponding to one of the submodels mentioned.

The approach in deriving each of the submodel consists of isolating the latent variables of interest and specifying a model that explicitly analyzes these latent variables by using indicators for these latent variables. As can be seen from figure A1-2, there are three groups of latent variables. The first group corresponds to the attitudes and perceptions. The second group corresponds to the preferences. The third group corresponds to the latent constraints. Accordingly there are three submodels that can be formulated once indicators used to obtain additional information on each of the group of latent variables have been identified.

The first nested version of the integrated framework aims at developing models which integrate perceptions, attitudes and choice behavior in order to improve the definition of attributes and tastes in the construct of the utility function. An example of such a model can be found in MacFadden (1986). A schematic representation of this model can be obtained by eliminating from Figure A1-2 the use of stated preference data and the use of constraints indicators. It is shown in Figure A1-3. The appropriate data for perceptual and attitudinal indicators could include attribute and feelings statements such as the ones mentioned in Section 3-3 of Chapter 3.

The second nested version of the integrated framework aims at developing models which simultaneously use stated and revealed preference data. These two sources of data are complementary sources of information. Stated preference data, if available, can be viewed as indicators of the latent utility. An example of such a model can be found in Ben-Akiva and Boccara (1987). A schematic representation of this model can be obtained by eliminating from

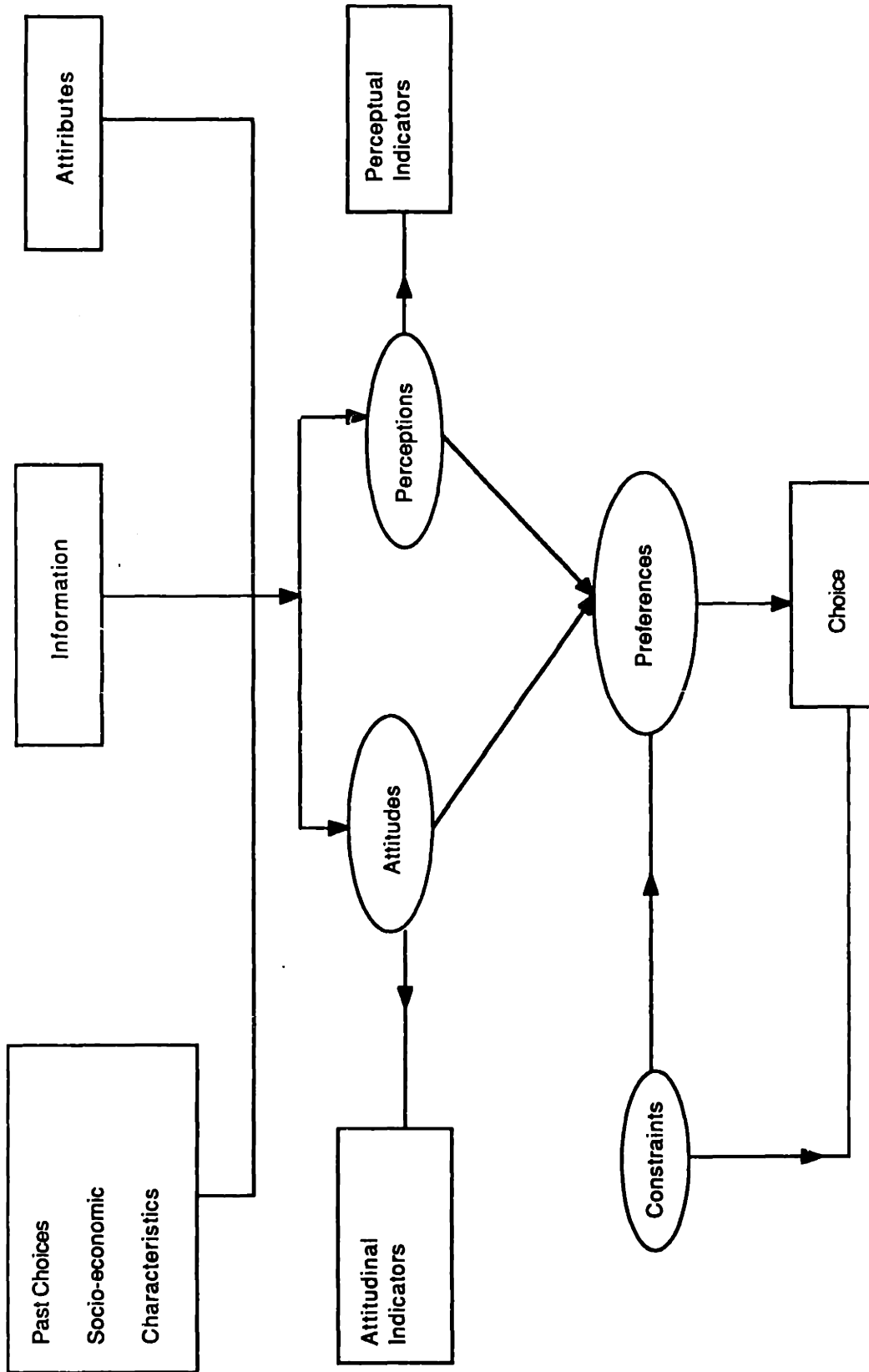


FIGURE A1-2

FRAMEWORK FOR SUBMODEL 1:
Use of Psychometric Data in Choice Models

Figure A1-2 the use of attitudinal indicators and the use of constraints indicators. It is shown in Figure A1-4. The appropriate stated preference data could include stated preference questions from a conjoint experiment.

The third nested version of the integrated framework aims at developing models that use choice set formation indicators to explain the influence of constraints on alternative availabilities. Naturally, this model corresponds to the framework described in Section 3-3 of Chapter 3. A schematic representation of this model could be obtained by eliminating the use of attitudinal indicators and the use of stated preference data from Figure A1-2. The resulting figure would be identical to Figure 3-1 of Chapter 3.

A1-6 Conclusions

This appendix has presented a paradigm that attempts to place an individual's choice process in a theoretical and behaviorally realistic perspective. The approach involves both latent variables and indicators. These latent variables can be inferred from indicators, and it is suggested that the estimation of models such as the ones given in the examples be carried out by using, as much as possible, the information provided by these indicators instead of relying exclusively on the information provided by the observed choice. This is exactly what the framework developed in Section 3-3 of Chapter 3 proposes. The use of other data sources, as a complement to revealed choice data, is the essence of our approach.

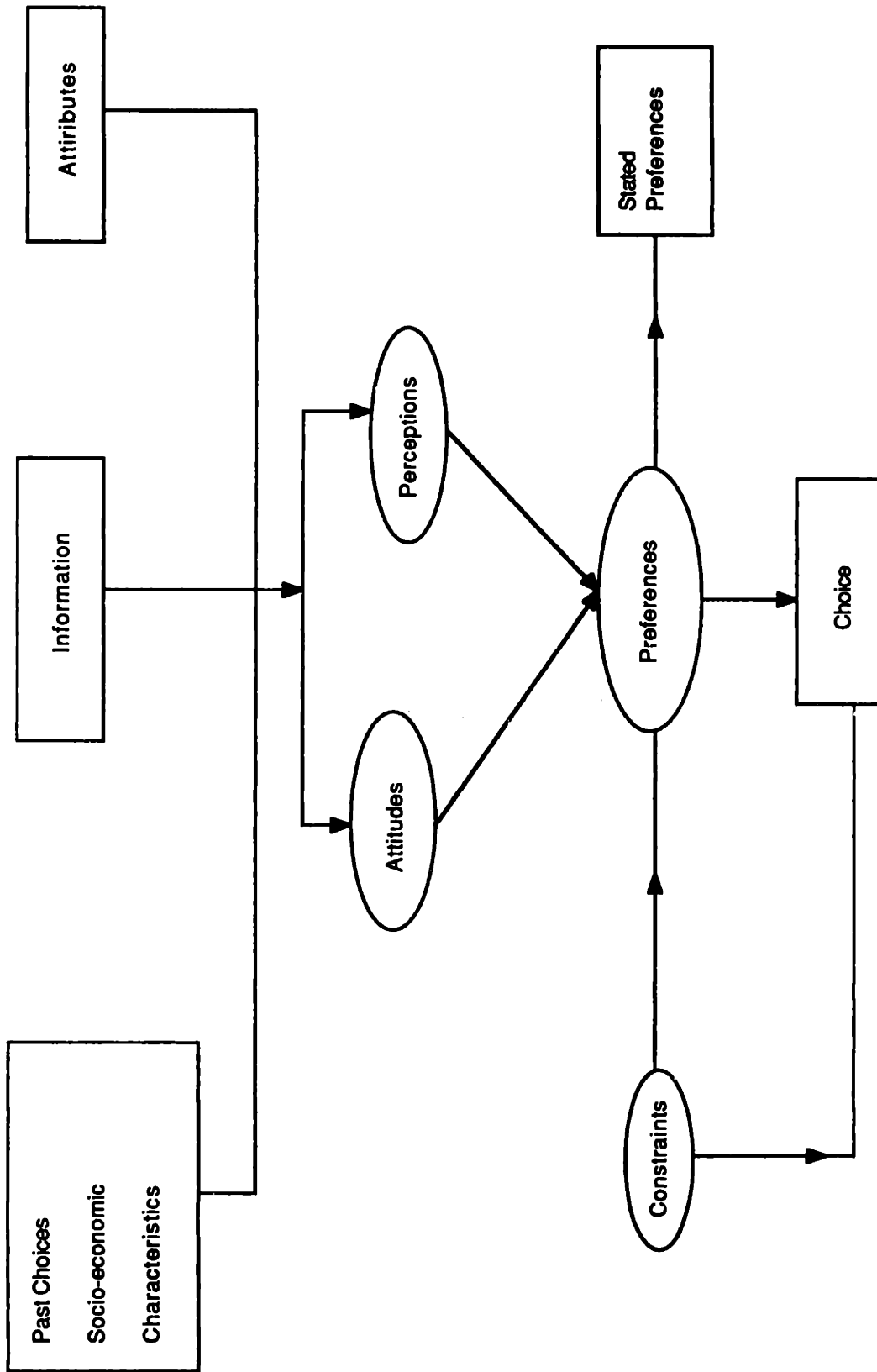


FIGURE A1-4
 FRAMEWORK FOR SUBMODEL 2:
 Simultaneous Use of Revealed and Stated Preference Data

APPENDIX 2

DERIVATION OF LOG-LIKELIHOODS FOR THE INTEGRATED FRAMEWORK MODELS

A2-1 Introduction

The purpose of this appendix is to derive the log-likelihood for the three model specifications, IFM1, IFM2, and IFM3 estimated in Chapter 5.

Section A2-2 derives the general formula for the log-likelihood and shows that the conditional probabilities

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0)$$

and

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1)$$

must be evaluated.

Section A2-3 evaluates these conditional probabilities for IFM1, IFM2 and for IFM3 and derives the log-likelihood for each of these models.

A2-2 Derivation of the Log-likelihood (general formula)

We have 3 indicators

$$Y_n, r_{in}^-$$

for the 2 non-chosen alternatives which became, with notation introduced in Chapter 3, 3

independent pairs:

$$(Y_{in}, r_{in})$$

For each pair, there are 3 possible events :

$$r_{in} = 1, Y_{in} = 1$$

$$r_{in} = 1, Y_{in} = 0$$

$$r_{in} = 0, Y_{in} = 0$$

For the first event, the probability is simply

$$P_{in} = Prob(Y_{in} = 1).$$

For the other two events, the probabilities are computed using Bayes's formula and we have:

$$Prob(r_{in} = 1, Y_{in} = 0) =$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1) \sum_{C \in C_i} Prob(Y_{in} = 0 | C) Prob(C) +$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0) \sum_{C \in \bar{C}_i} Prob(Y_{in} = 0 | C) Prob(C)$$

where

C_i is the set of choice sets which include alternative i ; and

\bar{C}_i is the set of choice sets which do not include alternative i .

Similarly,

$$Prob(r_{in} = 0, Y_{in} = 0) =$$

$$Prob(r_{in} = 0 | Y_{in} = 0, A_{in} = 1) \sum_{C \in C_i} Prob(Y_{in} = 0 | C) Prob(C) +$$

$$Prob(r_{in} = 0 | Y_{in} = 0, A_{in} = 0) \sum_{C \in \bar{C}_i} Prob(Y_{in} = 0 | C) Prob(C)$$

$$\forall C \in \bar{C}_i, Prob(Y_{in} = 0 | C) = 1$$

This simplifies the second term of each of these two expressions since

$$\sum_{C \in \bar{C}_i} Prob(C) = Prob(A_{in} = 0)$$

For the first term, however, each of the conditional probabilities $Prob(Y_{in} = 0 | C)$ for $C \in C_i$ must be computed separately.

For example, for the DA (drive alone) alternative, the conditioning choice sets are C1:

{DA,SR,T} and C2: {DA,SR} and

$$Prob(Y_{DA} = 0 | C2) = 1 - \frac{e^{U_{DA}}}{e^{U_{DA}} + e^{U_{SR}}}$$

The generic term for the log-likelihood is

$$r_{in} Y_{in} \log P_{in} + r_{in} (1 - Y_{in}) \log(Prob(r_{in} = 1, Y_{in} = 0)) + (1 - r_{in}) (1 - Y_{in}) \log(Prob(r_{in} = 0, Y_{in} = 0))$$

The log-likelihood is equal to:

$$\sum_{n=1}^3 \sum Y_{in} \cdot \log P_{in} +$$

$$\sum_{n=1}^3 \sum r_{in} Y_{in} \log P_{in} + r_{in} (1 - Y_{in}) \log(Prob(r_{in} = 1, Y_{in} = 0)) + (1 - r_{in}) (1 - Y_{in}) \log(Prob(r_{in} = 0, Y_{in} = 0))$$

where the first summation is for individuals for which indicators are not available and the second summation is for individuals for which indicators are available.

A2-3 Derivation of the Log-likelihood for IFM1, IFM2 and IFM3

As it is shown in the previous section, the log-likelihood involves the computation of

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0)$$

and of

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1)$$

This section derives these expressions for IFM1, IFM2 and for IFM3.

IFM1

$$r_{in}^* = \alpha_i + \beta_i A_{in} - \varepsilon_{in}$$

$$\Phi(\alpha_i) = Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0)$$

$$\Phi(\alpha_i + \beta_i) = Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1)$$

IFM2

$$\begin{aligned}r_{in}^* &= A_{in}(\alpha_1 + \beta_1 P_{in}^-) + (1 - A_{in})(\alpha_2 + \beta_2 P_{in}^-) - \varepsilon_{in} \\ &= A_{in}(\alpha_{1i} + \beta_1 P_{in}^- - \varepsilon_{in}) + (1 - A_{in})(\alpha_2 + \beta_2 P_{in}^- - \varepsilon_{in})\end{aligned}$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 0)$$

$$= \Phi(\alpha_2 + \beta_2 P_{in}^-)$$

$$Prob(r_{in} = 1 | Y_{in} = 0, A_{in} = 1)$$

$$= \Phi(\alpha_1 + \beta_1 P_{in}^-)$$

IFM3

Specification is identical to IFM2 except that the long run choice probabilities are replaced by the actual choice probabilities and we have:

$$\begin{aligned}r_{in}^* &= A_{in}(\alpha_{1i} + \beta_{1i} P_{in}) + (1 - A_{in})(\alpha_{2i} + \beta_{2i} P_{in}) - \varepsilon_{in} \\ &= A_{in}(\alpha_{1i} + \beta_{1i} P_{in} - \varepsilon_{in}) + (1 - A_{in})(\alpha_{2i} + \beta_{2i} P_{in} - \varepsilon_{in})\end{aligned}$$

Note that when conditioning on $A_{in} = 0$, the coefficient of P_{in} is not identified (since $P_{in} = 0$ when $A_{in} = 0$). Therefore β_{2i} is not identified and we have:

$$\text{Prob}(r_{in} = 1 | Y_{in} = 0, A_{in} = 0)$$

$$= \Phi(\alpha_{2i})$$

$$\text{Prob}(r_{in} = 1 | Y_{in} = 0, A_{in} = 1)$$

$$= \Phi(\alpha_{1i} + \beta_{1i} P_{in})$$

APPENDIX 3

CONDITIONAL PROBABILITY OF CHOICE OF HABITUAL MODE

A3-1 Introduction

The purpose of this appendix is to derive the expression for

$$P_n(U_i \leq U_j \leq a_j)$$

This expression is the probability that the best mode is chosen given that its utility was below the "no comparison of alternatives" threshold.

This expression is needed to calculate $P_n(j | h = j)$ which is the probability that an individual n chooses alternative j given that his or her habitual mode is j .

Once the probabilities ($Q_n(i), Q_n(j) = 1 - Q_n(i)$) of each alternative being the habitual one are specified, the probability of alternative j being chosen is simply

$$P_n(j) = Q_n(i)P_n(j | h = i) + Q_n(j)P_n(j | h = j)$$

A3-2 Derivation of the Conditional Probability of Choice of Habitual Mode

Assuming independence of ϵ_i, ϵ_j (iid Gumbel) we have

$$P_n(U_i \leq U_j \leq a_j) = P_n(\epsilon_i \leq V_j - V_i + \epsilon_j, \epsilon_j \leq a_j - V_j)$$

$$P_n(U_i \leq U_j \leq a_j) = \int_{-\infty}^{a_j - V_j} \int_{-\infty}^{V_j - V_i + \epsilon_j} f(\epsilon_i, \epsilon_j) d\epsilon_i d\epsilon_j$$

with

$$f(\epsilon_i, \epsilon_j) = \mu e^{-\mu \epsilon_i} e^{-e^{-\mu \epsilon_i}} \mu e^{-\mu \epsilon_j} e^{-e^{-\mu \epsilon_j}}$$

with

$$\mu e^{-\mu \epsilon_i} e^{-e^{-\mu \epsilon_i}} = \frac{d}{d\epsilon_i} \left(e^{-e^{-\mu \epsilon_i}} \right)$$

therefore

$$P_n(U_i \leq U_j \leq a_j) = \int_{-\infty}^{a_j - V_j} \int_{-\infty}^{V_j - V_i + \epsilon_j} \mu e^{-\mu \epsilon_j} e^{-e^{-\mu \epsilon_j}} \left(e^{-e^{-\mu \epsilon_i}} \right) d\epsilon_i d\epsilon_j$$

$$P_n(U_i \leq U_j \leq a_j) = \int_{-\infty}^{a_j - V_j} \mu e^{-\mu \epsilon_j} e^{-e^{-\mu \epsilon_j}} \left[e^{-e^{-\mu \epsilon_i}} \right]_{-\infty}^{V_j - V_i + \epsilon_j} d\epsilon_j$$

$$P_n(U_i \leq U_j \leq a_j) = \int_{-\infty}^{a_j - V_j} \mu e^{-\mu \epsilon_j} e^{-e^{-\mu \epsilon_j}} e^{-e^{-\mu(V_j - V_i + \epsilon_j)}} d\epsilon_j$$

$$\begin{aligned} e^{-e^{-\mu \epsilon_j}} e^{-e^{-\mu(V_j - V_i + \epsilon_j)}} &= e^{-\left(e^{-\mu \epsilon_j} + e^{-\mu(V_j - V_i + \epsilon_j)} \right)} \\ &= e^{-\left(e^{-\mu \epsilon_j} \left(1 + e^{-\mu(V_j - V_i)} \right) \right)} \end{aligned}$$

and

$$P_n(U_i \leq U_j \leq a_j) = \int_{-\infty}^{a_j - V_j} \mu e^{-\mu \epsilon_j} e^{-e^{-\mu \epsilon_j} \left(1 + e^{-\mu(V_j - V_i)} \right)} d\epsilon_j$$

Let $X_j = e^{-\mu \epsilon_j}$ then $-\mu e^{-\mu \epsilon_j} d\epsilon_j = dX_j$ and

$$P_n(U_i \leq U_j \leq a_j) = \int_{e^{-\mu(a_j - v_j)}}^{+\infty} e^{-x_j \left[1 + e^{-\mu(v_j - v_i)} \right]} dx_j \quad \text{and}$$

$$P_n(U_i \leq U_j \leq a_j) = \frac{1}{1 + e^{-\mu(v_j - v_i)}} e^{-\mu(a_j - v_j) \left[1 + e^{-\mu(v_j - v_i)} \right]}$$

The first term in the product is simply $P_n(U_j \geq U_i)$ i.e.

$$\frac{e^{\mu v_j}}{e^{\mu v_i} + e^{\mu v_j}}$$

for the second term we have

$$\begin{aligned} & e^{-\mu(a_j - v_j) \left[1 + e^{-\mu(v_j - v_i)} \right]} \\ &= e^{-\mu(a_j - v_j)} \cdot e^{-\mu(a_j - v_i)} \end{aligned}$$

but

$$\begin{aligned} P_n(U_j \leq a_j) &= \int_{-\infty}^{a_j - v_j} \mu e^{-\mu \epsilon_j} e^{-\epsilon_j} d\epsilon_j \\ &= e^{-\mu(a_j - v_j)} \end{aligned}$$

so that

$$P_n(U_i \leq U_j \leq a_j) = P_n(U_j \geq U_i) \cdot P_n(U_j \leq a_j) \cdot P_n(U_i \leq a_j)$$

and

$$P_r(j | h = j) = P_n(U_j \geq a_j) + P_n(U_j \geq U_i) \cdot (1 - P_n(U_j \geq a_j)) \cdot (1 - P_n(U_i \geq a_j))$$

$P_n(i | h = i)$ is defined in a similar way.

REFERENCES

Bates, J., "Stated Preference Technique for the Analysis of Transport Behaviour", prepared by J. Bates Services, Gillingham U.K., 1983.

Ben-Akiva, M., "Structure of Passenger Travel Demand Models", Unpublished Ph.D. dissertation, Department of Civil Engineering, MIT, 1973.

Ben-Akiva, M., "Choice Models with Simple Choice Set Generating Processes", working paper, Dept. of Civil Engineering, MIT, 1977.

Ben-Akiva, M. and B. Boccara, "Integrated Framework for Travel Behavior Analysis", paper presented at the W.C.T.R. conference, Aix-en-Provence, France, October 1987.

Ben-Akiva, M. and S. Lerman, "Some Estimation Results of a Simultaneous Model of Auto Ownership and Mode Choice to Work", *Transportation*, Vol. 4, No 4, 1974, pp. 357-376.

Ben-Akiva, M. and S. Lerman, *Discrete Choice Analysis*, MIT Press, 1985.

Benjamin, J. and L. Sen, "A Manual of Procedures to analyze Attitudes Toward Transportation", prepared for U.S.D.O.T., 1982.

Blase, J., "Hysteresis and Catastrophe Theory: A Demonstration of Habit and Thresholds Effects in Travel Behavior", paper presented at PTRC annual meeting, 1979.

Bourgin C. and X. Godard, "Structural and Thresholds Effects in the Use of Transportation Mode" published in *New Horizons in Travel research Behavior* Chapter 19, 1981, pp.353-368.

Brog, W. and E. Erl, "Applications of a Model of Individual Behavior to Explain Household Activity Patterns in an Urban Area and to Forecast Behavioral changes", presented at the International Conference on Travel Demand Analysis, St Catherine's College, Oxford, England, 1979.

Charles River Associates, "Implicit Discount Rates for Refrigerator Purchases", report prepared for the Electric Power Research Institute, August, 1986.

Domencich, T. and D. Mc Fadden, "Urban Travel Demand: A Behavioral Analysis", North Holland, Amsterdam, 1975.

Federal Highway Administration, U.S.D.O.T., Baltimore Travel Demand Data Set, 1980.

Foerster J., "Nonlinear and Noncompensatory Perceptual Functions of Evaluation and Choice" published in New Horizons in Travel research Behavior Chapter 19, 1981, pp.335-352.

Gaudry, M. and M. Dagenais, "The Dogit Model", Transportation Research B, Vol. 13B, 1979, pp.105-111.

GAUSS by L. Edlefsen and S. Jones, Applied Technical Systems, Kent, WA, 1986.

Golob, T, "The Survey of Users Choice of Alternate Transportation Modes", General Motors Research Report 950, Warren, Michigan, 1970.

Golob T. and A. Richardson, "Noncompensatory and Discontinuous Constructs in Travel Behavior Models" published in New Horizons in Travel research Behavior Chapter 19, 1981, pp.369-387.

Goodwin P., "Habit and Hysteresis in Mode Choice", Urban Studies 14, no1, 1977, pp. 95-98.

Hauser J. and F. Koppelman, "Alternative Perceptual Mapping Techniques: Relative Accuracy and Usefulness", Journal of Marketing Research, Vol 16, November 1979, pp. 495-506.

Hausmann, J., "Specification Tests in Econometrics", Econometrica 46, 1978, pp. 1251-1272.

Heckman J. , "Statistical Analysis of Discrete Panel Data" published in Structural Analysis of Discrete Data, ed Manski and Mac Fadden, 1981.

Hensher D., and P. Stopher (eds), 1979, "Behavioural Travel Modelling", Proceedings of the Third International Conference on Behavioural Travel Modelling, Tanunda, Australia, April 1977.

Hills, P., and C. Mitchell, "New Approaches to Understanding Travel Behavior", paper presented at International Conference on Behavioral Travel Modelling, Grainau, Germany, 1979.

Joreskog, K.G., "A General Method for Estimating a Linear Structural Equation System", in Structural Equation Models in the Social Sciences", A.S. Goldberger and O.D. Duncan (eds), Seminar Press, 1975, chapter 5.

Joreskog, K.G. and D. Sorbom, "Recent Developments in Structural Equation Modeling", Journal of Marketing Research, Vol 19, November 1982, pp. 404-416.

Kitamura, R. and T. Lam, "A Model of Constrained Binary Choice", Ninth International Symposium on Transportation and Traffic Theory, VNU Science Press, 1984, pp. 493-512.

Koppelman, F. and J. Hauser, "Destination Choice Behavior for Non-Grocery Shopping Trips", Transportation Research Record, Issue 673, 1979, pp. 157-165.

Koppelman, F., Hirsh, M. and J. Schofer, "Market Analysis for Chicago Suburb to Central Area Commuters", paper published at the Metropolitan Conference on Public Transportation Research, University of Chicago, 1986.

Koppelman, F. and P. Lyon, "Attitudinal Analysis of Work/School Travel", Transportation Science, Vol. 15, No 3, August 1981.

Koppelman, F. and E. Pas, "Travel-choice Behavior: Models of Perceptions, Feelings, Preference, and Choice", Transportation Research Record 765, Transportation Research Board, 1980.

Kozel, V., "The impact of constraints to travel in Maceio", Unpublished Working Paper, Center for Transportation Studies, MIT, Cambridge, MA, 1981.

Louviere, J. and D. Hensher, "Using Discrete Choice Models with Experimental Design Data to Forecast Consumer Demand for a Unique Cultural Event", *Journal of Consumer Research*, Vol 10, December 1983, pp.348-361.

Manski, C., "The Structure of Random Utility Models", *Theory and Decision*, Vol.8, 1977, pp. 229-254.

MacFadden, D.L., "The Multinomial Logit Model When the Population Contains Captive Subpopulations", unpublished memorandum, 1976.

MacFadden, D.L., "The Choice Theory Approach to Market Research", *Marketing Science*, Vol. 5, No. 4, 1986.

Meyer, R., "Theory of destination choice-set formation under informational constraints", *Transportation Research Record*, Issue 750, 1979, pp. 6-12.

Nelson, D. and K. O'Neill, "An Assessment of Empirical Choice Set Formation", *Transportation Research Forum*, 1977.

Pitschke, S. "Choice Set Formulation for Discrete Choice Models", Unpublished MSc Thesis, Department of Civil Engineering, MIT, 1980.

Richardson, A., "Search Models and Choice Set Generation", *Transportation Research A*, Vol. 16, 1982, pp.403-419.

Salomon, I., "Lifestyle as a Factor in Explaining Travel Behavior", Unpublished Ph.D. Dissertation, Department of Civil Engineering, MIT, 1981.

Sen, L. and J. Benjamin, "Travel Behavior and Market Segmentation of Low and Middle Income Residents of Richmond, Va", U.S.D.O.T., 1979.

Simon, H., "A Behavioral Model of Rational Choice", *Quarterly Journal of Economics*, Issue 69, 1955, pp.99-118.

SST statistical software tool by Dubin J. and R. Rivers, Dubins Rivers Research, Pasadena California, March 1986.

Stopher, P., "Captivity and choice in Travel-Behavior Models", Transportation Journal of ASCE, Vol.106, 1980, pp.427-435.

Stopher, P., Meyburg, A. and W. Brog, "New-Horizons in Travel Behavior Research", Lexington Books, 1979.

Supernak, J., "Rationality of Travel Choices: a Discussion Paper", Unpublished Working Paper, Department of Civil Engineering, San Diego State University, 1986.

Supernak, J. and D. Schoendorfer, "Automobile Availability and its Application in Transportation Studies", Transportation Research Record 1037, 1986, pp. 73-81.

Swait, J., "Probabilistic Choice Set Formation in Transportation Demand Models", Unpublished Ph.D. Dissertation, Department of Civil Engineering, MIT, 1984.

Swait, J. and M. Ben-Akiva, "Constraints to individual Travel Behavior in a Brazilian city", Transportation Research Record, 1986.

Swait, J. and M. Ben-Akiva, "Incorporating Random Constraints in Discrete Models of Choice Set Generation", Transportation Research B, Vol.21, 1987a, pp. 92-102.

Swait, J. and M. Ben-Akiva, "Empirical Test of a Constrained Choice Discrete Model: Mode choice in Sao-Paulo, Brazil", Transportation Research B, Vol.21, 1987b, pp. 103-115.

Ting N., "Choice Set Generation in a Random Utility Model of Mode Choice", unpublished paper, Department of Geography, University of Iowa, 1987.

Torgenson W., "Theory and Methods of Scaling", John Wiley and Sons, Inc., New York, 1958.

Train, K. "A Structural Logit Model of Auto Ownership and Mode Choice", Review of Economic Studies, Vol. 47, 1974, pp. 357-370.

Tverski, A., "Elimination by Aspects : A Theory of Choice", Psychological Review, Vol. 79, 1972, pp.281-299.

Wermuth, M. "Structure and Calibration of a Behavioral and Attitudinal Binary Mode Choice Model Between Public Transportation and Private Car", presented at PTRC Annual Meeting, warwick, England, 1978.

Williams, H. and J. Ortuzar, "Behavioral Theories of Dispersion and Misspecification of Travel Demand Models", Transportation Research B, Vol. 16B, No. 3, 1982, pp. 167-219.

Wilmot C., "Baltimore Travel Demand Data Set" Northwestern University, 1982.