Online Matching with Stochastic Rewards: Optimal Competitive Ratio via Path Based Formulation

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VINEET GOYAL, Columbia University
RAJAN UDWANI, Columbia University

The problem of online matching with stochastic rewards is a generalization of the online bipartite matching problem where each edge has a probability of success. When a match is made it succeeds with the probability of the corresponding edge. Introducing this model, Mehta and Panigrahi (FOCS 2012) focused on the special case of identical edge probabilities. Comparing against a deterministic offline LP, they showed that the Ranking algorithm of Karp et al. (STOC 1990) is 0.534 competitive and proposed a new online algorithm with an improved guarantee of 0.567 for vanishingly small probabilities. For the case of vanishingly small but heterogeneous probabilities Mehta et al. (SODA 2015), gave a 0.534 competitive algorithm against the same LP benchmark. For the more general vertex-weighted version of the problem, to the best of our knowledge, no results being 1/2 were previously known even for identical probabilities.

We focus on the vertex-weighted version and give two improvements. First, we show that a natural generalization of the Perturbed-Greedy algorithm of Aggarwal et al. (SODA 2011), is $(1 - 1/e)$ competitive when probabilities decompose as a product of two factors, one corresponding to each vertex of the edge. This is the best achievable guarantee as it includes the case of identical probabilities and in particular, the classical online bipartite matching problem. Second, we give a deterministic 0.596 competitive algorithm for the previously well studied case of case of fully heterogeneous but vanishingly small edge probabilities. A key contribution of our approach is the use of novel path-based formulations and a generalization of the primal-dual scheme of Devanur et al. (SODA 2013). These allow us to compare against the natural benchmark of clairvoyant (offline) algorithms that know the sequence of arrivals and the edge probabilities in advance, but not the outcomes of potential matches. These ideas may be of independent interest in other online settings with post-allocation stochasticity.

CCS Concepts: • **Theory of computation → Design and analysis of algorithms**; **Online algorithms**; *Adversary models*;

Additional Key Words and Phrases: Online Matching; Stochastic Rewards; Path based Certificate

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