SOUND TRANSMISSION LOSS CHARACTERISTICS OF THREE LAYER
COMPOSITE WALL CONSTRUCTIONS

by

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ABSTRACT

The sound transmission loss (TL) characteristics of three layer composite wall constructions are considered. The constructions considered have symmetric cross-sections in that the face sheets are identical and the core is homogeneous. Analytical models that predict TL are developed for constructions with homogeneous isotropic cores or homogeneous orthotropic cores.

The effects of symmetric and antisymmetric motions of the face sheets on the TL are evaluated. Symmetric out of plane displacements of the face sheets are 180° out of phase. Symmetric in-plane displacements of the face sheets are in phase. Antisymmetric displacements have the opposite phase relationships. For composite constructions with symmetric cross-sections, symmetric and antisymmetric motions are uncoupled.

The TL is reduced by coincidence effects for frequencies where the *in vacuo* free wavespeeds of vibration for either symmetric or antisymmetric motions exceed the speed of sound. Experimental results for composite panels with isotropic and orthotropic cores were compared with analytical predictions with acceptable agreement.

A design process for composite panels with high TL is developed. TL values, greater than mass law by as much as 10 dB, are predicted when the speed of sound exceeded the free wavespeeds of the symmetric and antisymmetric modes. The response of both modes to incident acoustic energy is mass controlled. A cancellation of symmetric and antisymmetric motions occurs on the opposite face sheet from which the acoustic energy is incident and the TL exceeds mass law. A composite panel was fabricated and provided a verification of the design process. Greater than mass law TL was obtained.

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LIST OF SYMBOLS

The arabic numbers at the end of each definition indicate the chapter in which the symbol first appears with the given definition.

$a_{\pm}$	amplitude of potential function describing dilatational motion in core (2)
$\bar{A}$	vector displacement field for dilatational motion (2)
A	contact area between core sample and rigid plates (2)
A_{mn}	Fourier coefficients for static load (6)
$AL0, AL0$	coefficients relating normal stress to normal displacement of face sheet (2)
$AS0, AS1$	coefficients relating $\left\{ \begin{array}{l} \text{normal} \\ \text{shear} \end{array} \right\}$ stress to $\left\{ \begin{array}{l} \text{extensional} \\ \text{normal} \end{array} \right\}$ displacement of face sheet (2)
$b_{\pm}$	amplitude of potential function describing equivoluminal motion in core (2)
$\bar{B}$	vector displacement field for equivoluminal motion (2)
c_o	speed of sound in air (2)
c_T	trace phase speed of incident acoustic plane wave (2)
c_D	phase speed for equivoluminal or shear waves in an isotropic medium (2)
d_o, d_i	amplitude of normal displacements of top and bottom face sheets (2)
$d_{\text{sym}}, d_{\text{asym}}$	amplitude of symmetric and antisymmetric normal displacements of face sheets (2)

LIST OF SYMBOLS (CONTINUED)

D_p	bending rigidity for thin plate (face sheets) (2)
D_1	displacement amplitude for rigid plates (4)
d	diameter of cylindrical rigid plate (4)
E_{11}^{fs}	elastic stiffness for face sheet (2)
E_{fs}	Young's modulus for face sheet (2)
E_c	Young's modulus for the core (2)
E_{ij}	elastic stiffness (3)
f_o	force per unit area (3)
$f_{o,sym}$	symmetric force per unit area (2)
$f_{o,asym}$	antisymmetric force per unit area (2)
f	frequency in Hz (2)
G_c	shear modulus for the core (2)
h	rigid square plate width (4)
I_{inc}	incident acoustic intensity (2)
I_{trans}	transmitted acoustic intensity (2)
I_{rad}	radiated acoustic intensity (4)
$\bar{i}, \bar{j}, \bar{k}$	unit vectors in the x_1, x_2, x_3 direction
k_1, k_2, k_3	wavenumbers in x_1, x_2, x_3 directions (2)
k_o	acoustic wavenumber (2)

LIST OF SYMBOLS (CONTINUED)

k_3^d, k_3^s	dilatational and equivoluminal wavenumbers in x_3 direction (2)
K_1, K_2, K_c	spring constants (2)
KE	kinetic energy (3)
L	thickness of the core (2)
L_o, L_1	amplitudes of symmetric and antisymmetric extensional displacements of face sheets (2)
ℓ, m	direction cosines (3)
m_1, m_2	lumped masses (2)
P_{inc}	} acoustic plane wave pressures (2)
$P_{reflected}$	
$P_{radiated, top}$	
$P_{radiated, bottom}$	
P_o	amplitude of incident acoustic plane wave (2)
$P_{r,t}$	amplitude of radiated acoustic plane wave from top face sheet (2)
$P_{r,b}$	amplitude of radiated acoustic plane wave from bottom face sheet (2)
PE	potential energy (3)
q_r	generalized displacement in Lagrange's equation (3)
Q_r	generalized force in Lagrange's equation (3)
R	room constant for reverberation room (4)
R_{sym}, R_{asym}	symmetric and antisymmetric resistive components of impedances (2)

LIST OF SYMBOLS (CONTINUED)

S_{ω}	area of test opening between reverberation rooms (4)
S_T	total surface area in reverberation room (4)
ST_0, ST_1	coefficient relating shear stress to extensional displacement of the face sheets (2)
t	time variable (2)
T_r	reverberation time for reverberation room (4)
t_p	thickness of face sheet (2)
u, v, w	particle displacements in core in x_1, x_2, x_3 directions (2)
u_0, u_1, u_2	particle displacement in the top face sheet, bottom face sheet and core in x_1 direction (3)
v_0, v_1, v_2	particle displacement in the top face sheet, bottom face sheet and core in x_2 direction (3)
w_0, w_1, w_2	particle displacement in the top face sheet, bottom face sheet and core in x_3 direction (3)
V_{sym}, V_{asym}	symmetric and antisymmetric velocities of the face sheets (2)
V_r	volume of reverberation room (4)
W	potential energy density (3)
W_{sym}, W_{asym}	symmetric and antisymmetric static deflections (6)
W_{rad}	radiated acoustic power (4)
x_1, x_2, x_3	Cartesian coordinates (2)
X_{sym}, X_{asym}	symmetric and antisymmetric reactive impedance components (2)

LIST OF SYMBOLS (CONTINUED)

$Z_{\text{sym}}, Z_{\text{asym}}$	symmetric and antisymmetric impedances (2)
Z_{comp}	impedance expression for the composite panel including radiation loading (2)
<u>Greek Symbols</u>	
$\bar{\alpha}$	average absorption coefficient for reverberation room (4)
$\alpha_{\text{sym}}, \alpha_{\text{asym}}$	symmetric and antisymmetric normal displacements of the face sheets (3,6)
$\beta_{\text{sym}}, \beta_{\text{asym}}$	symmetric and antisymmetric extensional displacements of the face sheets (3,6)
$\gamma_{\text{sym}}, \gamma_{\text{asym}}$	symmetric and antisymmetric extensional displacements of the face sheets (3,6)
ϵ_i	elastic strain $i = 1, \dots, 6$ (3)
ζ	in-plane extensional displacement of the face sheets (2)
ζ, ζ_1, ζ_2	variables describing in-plane displacements in the core (3,6)
η	normal displacement of top and bottom face sheets (2)
η_0, η_1	normal displacement of top and bottom face sheets (2)
$\eta_{\text{symmetric}} \left. \begin{array}{l} \eta_{\text{symmetric}} \\ \eta_{\text{antisymmetric}} \end{array} \right\}$	symmetric and antisymmetric normal displacements (2)
η_c	loss factor for the core (2)
η_p	loss factor for the face sheet (2)

LIST OF SYMBOLS (CONTINUED)

η_T	total loss factor for the composite (3)
η_{mech}	loss factor associated with internal damping of the composite
η_{rad}	loss factor for the composite associated with the radiation of acoustic energy from one face sheet (3)
θ, ϕ	angles in a spherical coordinate system (2)
λ	Lamé constant (2)
λ_{fs}	Lamé constant for the face sheet (2)
λ_c	Lamé constant for the core (2)
λ_D	wavelength for dilatational motion (2)
λ_1	trace wavelength in x_1 direction (3)
ν	Poisson's ratio (2)
ν_p	Poisson's ratio for the face sheet (3)
ν_c	Poisson's ratio for the core (2)
ρ_p	surface density for the face sheet (mass per unit area) (2)
ρ_c	density for the core (mass per unit volume) (2)
ρ_s	surface density for the composite (mass per unit area) (4)
ρ_0	density of air (mass per unit volume) (2)
σ_1	} normal stresses (2)
σ_2	
σ_3	

LIST OF SYMBOLS (CONTINUED)

σ_{rad}	radiation efficiency (4)
τ	transmission coefficient (2)
τ_{13}	shear stress (2)
$\tau_i \quad i = 1, \dots, 6$	notation describing both normal and shear stresses (3)
Φ	potential function for dilatational wave motions (2)
Ψ	potential function for equivoluminal wave motions (2)
ω	radian frequency (2)

Additional Notations

i	$\sqrt{-1}$
$\langle \rangle_{\mathbf{x}, t}$	connotes average in space and time

CHAPTER 1

INTRODUCTION

The sound insulating characteristics of walls, floors and ceilings in buildings and of the walls of machinery enclosures are an important aspect of their design. For floors or equipment mounting platforms the primary function is the support of static loads. An important secondary function would be the blocking or insulation of sound. For walls, doors and other partitions the blocking of sound may be the principle concern. For both situations the intent of the design is to provide the required load-bearing capability and sound insulating characteristics with a minimum of weight and cost.

At present, studded wall constructions are available that may provide a sufficient attenuation of sound across the wall. These constructions require on-site fabrication and tend to be costly with an unreliable control over the work quality. Premade partitions are available that involve double wall constructions with wood or metal face sheets separated by an absorption filled air-space. These partitions provide sufficient transmission loss (TL) but must be manufactured to exact dimensions, as subsequent on-site cutting is difficult and expensive. A composite or layered wall construction has advantages over the above constructions

in that it may be premade and cut to size at the construction site. Such a construction would involve wood, metal or plastic face sheets and a lightweight core material. The core may be a cellular honeycomb material or closed or open cell foam materials. Two layered wall designs for high TL exist already; they are the Shear Wall and the Coincidence Wall. These will be described later on.

1.1 Background

A summary of work done in understanding the transmission of sound through single layer walls is found in Beranek [1]. Mass law or limp wall effects at low frequencies and coincidence effects at higher frequencies are described for walls of infinite extent. The internal damping in the wall plays a significant role in determining the transmission loss (TL) characteristics of the wall in and above the coincidence region.

Coincidence refers to the condition where the free wave-speed of vibration in a panel of infinite extent equals the trace wavespeed of an incident acoustic wave. It is a "resonance" condition characterized by nearly perfect transmission of an incident acoustic wave through the panel. Structural damping in the panel determines the extent to which the transmission is imperfect.

Watters and Kurtze [2] studied the transmission loss characteristics of three-layer walls in which shear

deformation in the middle layer or core acted to reduce the free wavespeed of vibration in the walls. The wall is designed so that the coincidence region is shifted to higher frequencies where the reduction in transmission loss associated with coincidence effects is of less concern. The transmission loss in the principle speech bands is governed by limp wall or mass law behavior. In these studies the core is considered to be rigid in the direction perpendicular to the plane of composite; the normal displacements of the top and bottom face sheets are identical. This mode of vibration is referred to as the antisymmetric mode in this thesis. The wall is referred to as the Shear Wall. Warnaka and Holmer [3,4] developed a three-layer wall design with shear deformation in the core, where coincidence first occurs below the frequency region of interest. In the region above where coincidence first occurs TL values greater than mass law TL are achievable when the wall is heavily damped. The acoustic behavior of the wall is coincidence controlled and the wall is referred to as the Coincidence Wall. Shear deformation of the core was accounted for by Holmer in the use of an equivalent bending stiffness that incorporated the shear stiffness of the core. The equivalent stiffness was seen to depend on the wavelength of the flexural deformation in the wall. At long wavelengths the equivalent bending stiffness is principally

determined by the stiffness of the face sheets separated by a core of constant thickness. The neutral plane for bending deformations lies in the mid-plane of the core for the symmetric wall constructions considered. At intermediate wavelengths, the shear stiffness of the core material determines the equivalent bending stiffness of the composite three layer wall. At short wavelengths, the equivalent stiffness is again determined by bending deformation in the face sheets and the equivalent stiffness is the sum of the bending stiffnesses of the face sheets alone. The neutral planes for bending occur at the mid-planes of the face sheets. The technique used by Holmer to determine the equivalent bending stiffness of the layered wall was first developed by Ross, Ungar, and Kerwin [5].

Manning [6] studied the optimizing processes for the Coincidence Wall using the tools described by the above authors. Expressions for the loss factor for the composite wall in terms of the loss factors of the individual layers were given. Principles involved in optimizing the transmission loss were discussed for multilayered walls with 3, 4, and 5 layers.

Ford, Lord, and Walker; Smolenski and Krokosky; and Dym and Lang [7,8,9] have studied the effects of the compliance of the core in the thickness direction of the wall. An additional mode of vibration, the symmetric mode,

exists for which the normal displacements of the face sheets are 180° out of phase. The symmetric mode offers another way for coincidence effects to reduce the transmission loss of a three layer wall. Ford, Lord, and Walker; and Smolenski and Krokosky computed phase speeds for free travelling waves associated with this additional symmetric mode of vibration. Dym and Lang evaluated the transmission loss for three layer walls including the effects of both the symmetric and anti-symmetric modes of vibration.

1.2 Brief Outline of the Thesis

Chapter 2 describes an analytical model for a three layer wall construction made from isotropic face sheets and a homogeneous isotropic core. Motions in the core are described by the superposition of motions resulting from dilatational and equivoluminal (distortional) plane travelling waves. Motions in the face sheets are characterized by bending and extensional displacements. Expressions for the TL for a composite panel of infinite extent are obtained in terms of mechanical impedances of the composite for the symmetric and antisymmetric modes of vibration.

Chapter 3 describes an analytical model for a three layer wall with an orthotropic core and isotropic face sheets. The model extends the work of previous researchers [7,8,9] to apply for wall constructions with orthotropic

cores. As in Chapter 2 the TL is evaluated in terms of the mechanical impedances.

Chapter 5 presents experimental results for comparison with the predictions of Chapters 2 and 3.

Chapter 6 describes a new design process for three layer wall constructions with high TL that is based on insights developed from the models of Chapters 2 and 3. Predicted values for TL may, for particular frequencies, significantly exceed mass law TL. A panel was fabricated and tested, the results from which are presented in Chapter 6. Static load-bearing capabilities for a panel of finite area are evaluated.

CHAPTER 2

COMPOSITE PANEL WITH A HOMOGENEOUS ISOTROPIC CORE

2.1 Introduction

Previous researchers [7,8,9], in analytically evaluating transmission loss (TL) for a composite panel, assumed that deformations in the core of the composite were linearly dependent on the coordinate perpendicular to the face sheets. This assumption excludes the presence of and the possible effects on the TL of wave bearing motions in the core in the direction perpendicular to face sheets. The assumption is appropriate when the wavelength of wave bearing motions in the core is large compared with the thickness of the core. Wave bearing motions in the two in-plane directions are included. The analytical procedures followed in this chapter are completely general in allowing for wave-bearing motions in the core that are both dilatational (irrotational) and equivoluminal (distortional) in nature. The analysis is restricted to apply for core materials that are homogeneous and isotropic. The face sheets are modelled as homogeneous, isotropic thin plates for bending displacements normal to the face sheets and for extensional displacements in the plane of the face sheets.

2.2 Formulation for the TL of a Composite Panel of Infinite Extent

2.2.1 Acoustic pressure disturbances and the transmission coefficient

The geometry depicting the problem of determining TL for a composite panel of infinite extent is shown in Figure 2.1. The excitation is a downward travelling acoustic plane wave incident on the panel from an angle θ with respect to the normal to the plane of the panel (the x_3 axis), and the azimuthal angle ϕ . ϕ is defined as the angle that a vertical plane containing the x_3 axis makes with respect to the vertical plane defined by the x_1 and x_3 axes. The incident acoustic plane wave is of radian frequency ω , and wavenumbers k_1 , k_2 , k_3 in the x_1 , x_2 , x_3 directions, respectively. k_1 , k_2 , k_3 and ω are related to the acoustic wavespeed c_0 , as follows:

$$\begin{aligned} k_1^2 + k_2^2 + k_3^2 &= \left(\frac{\omega}{c_0} \right)^2 \quad \text{and} \quad k_1 = k_0 \sin\theta \cos\phi \\ k_2 &= k_0 \sin\theta \sin\phi \\ k_3 &= k_0 \cos\theta \\ k_0 &= \omega/c_0 \end{aligned} \quad (2.1)$$

Under steady state conditions the response to the incident plane wave is formulated in terms of a reflected plane wave from the top face sheet, and radiated plane waves from the top and bottom face sheets that result from the normal displacements of the face sheets. The

formulation is separated into two parts. In the first the face sheets are restricted from moving and the incident plane wave is reflected from a rigid boundary such that:

$$p_{\text{reflected}} = p_o e^{i(k_1 x_1 + k_2 x_2 + k_3 (x_3 - t_p) - \omega t)} \quad (2.2)$$

an acoustic plane wave travelling upward in the $+x_3$ direction. The thickness of a face sheet is t_p ; both face sheets are taken to be of the same thickness. As the reflection is from a rigid boundary the amplitude is unchanged. The sum of the incident plus reflected plane waves, evaluated at the surface of the top face sheet ($x_3 = t_p$) equals

$$2p_o e^{i(k_1 x_1 + k_2 x_2 - \omega t)}.$$

In the second part of the formulation the face sheets are assumed to move with prescribed normal displacements,

$$\eta_o = d_o e^{i(k_1 x_1 + k_2 x_2 - \omega t)} \quad \text{for the top face sheet and}$$

$$\eta_1 = d_1 e^{i(k_1 x_1 + k_2 x_2 - \omega t)} \quad \text{for the bottom face sheet. As a result of these motions acoustic plane waves are radiated from the top and bottom face sheets of the following forms:}$$

$$p_{\text{radiated, top}} = p_{r,t} e^{i(k_1 x_1 + k_2 x_2 + k_3 (x_3 - t_p) - \omega t)}$$

$$p_{\text{radiated, bottom}} = p_{r,b} e^{i(k_1 x_1 + k_2 x_2 - k_3 (x_3 + L + t_p) - \omega t)}$$

(2.3)

where L is the thickness of the core. The amplitudes $p_{r,t}$ and $p_{r,b}$ are complex constants determined from the acoustic boundary condition applied to the motions of the top and bottom face sheets:

$$\left. \frac{\partial p}{\partial x_3} \right|_{x_3=t_p} = -\rho_0 \frac{\partial^2 \eta_0}{\partial t^2}$$

and

$$\left. \frac{\partial p}{\partial x_3} \right|_{x_3=-(L+t_p)} = -\rho_0 \frac{\partial^2 \eta_1}{\partial t^2} \quad (2.4)$$

Substitution of the expressions for η_0, η_1 and $p_{\text{radiated, top}}, p_{\text{radiated, bottom}}$ into Eq. (2.4) yields

$$p_{r,t} = -\frac{i\rho_0 c_0 \omega d_0}{\cos \theta}, \quad p_{r,b} = \frac{i\rho_0 c_0 \omega d_1}{\cos \theta} \quad (2.5)$$

The quantity $\rho_0 c_0$ is the specific acoustic impedance of air where ρ_0 is the density of air. The incident plus reflected plane waves, superimposed on the radiated plane waves and evaluated at the corresponding surfaces of the composite panel, describe the pressure disturbances acting on the composite panel under steady state conditions.

The transmission coefficient, τ , is defined as the ratio of the transmitted acoustic intensity, I_{trans} , to the incident acoustic intensity, I_{inc} . For plane waves this means that:

$$I_{\text{trans}} = \frac{|p_{r,b}|^2}{2\rho_0 c_0}, \quad I_{\text{inc}} = \frac{|p_0|^2}{2\rho_0 c_0}$$

and

$$\tau = \frac{\text{Transmitted Intensity}}{\text{Incident Intensity}} = \frac{|p_{r,b}|^2}{|p_0|^2} \quad (2.6)$$

Under general circumstances the transmission coefficient will depend on the angle θ as well as the azimuthal angle, ϕ .

The average transmitted acoustic intensity is computed as

$$\langle I_{\text{trans}} \rangle_{\theta, \phi} = \frac{\int \int I_{\text{trans}}(\theta, \phi) \cos \theta \sin \theta d\theta d\phi}{\int \int \cos \theta \sin \theta d\theta d\phi}$$

where from Eq. (2.6)

$$I_{\text{trans}}(\theta, \phi) = \tau(\theta, \phi) I_{\text{inc}}(\theta, \phi) \quad (2.7)$$

For purposes of comparisons with TL results experimentally obtained from reverberation room measurements, $I_{\text{inc}}(\theta, \phi)$ is taken to be independent of θ and ϕ . Because of the in-plane isotropy of the face sheets and the isotropy of the core of the composite panels considered in this chapter, $\tau(\theta, \phi)$ is independent of the angle ϕ . The in-plane axes, x_1, x_2 , may therefore be reoriented to set $\phi=0$. This means, from Eq. (2.1), that $k_2=0$. Also, the pressures at the face sheets and displacements exhibit an $e^{i(k_1 x_1 - \omega t)}$ dependence.

The average transmission coefficient $\bar{\tau}$ becomes:

$$\bar{\tau} = \frac{\int_0^{\theta_{lim}} \tau(\theta) \cos \theta \sin \theta d\theta}{\int_0^{\theta_{lim}} \cos \theta \sin \theta d\theta} = \frac{\langle I_{trans} \rangle_{\theta}}{I_{inc}} \quad (2.8)$$

A model of the incident acoustic intensity in a reverberant room has been assumed for which the incident acoustic intensity equals a constant value, I_{inc} , for incident angles, θ , from zero to θ_{lim} and equals zero for $\theta_{lim} < \theta < \pi/2$. θ_{lim} is typically assumed to be 78° .

The transmission loss, TL, is defined with respect to the average transmission coefficient as follows:

$$TL = 10 \log_{10} (1/\bar{\tau}) \text{ dB.} \quad (2.9)$$

2.2.2 Adaptation of impedance expressions for a composite panel to the determination of TL

The previous section describes the procedure for determining the TL in terms of the amplitudes of the incident and transmitted acoustic plane waves. The amplitude of the transmitted plane wave is determined by the amplitude of the normal displacement of the bottom face sheet (at $x_3 = -L$). The amplitudes of normal displacements of both the top and bottom face sheets are determined from a solution to the elastic deformation problem of the composite

panel where the pressure excitations on the face sheet surfaces are described in Section 2.2.1. For the purposes of this section the solution will be expressed in terms of symmetric and antisymmetric mode impedances for the composite panel.

The normal displacements of the face sheets are separated into symmetric and antisymmetric components as follows:

$$\begin{aligned} \eta_{\text{symmetric}} &= (\eta_0 - \eta_1)/2 \\ \text{and} \\ \eta_{\text{antisymmetric}} &= (\eta_0 + \eta_1)/2 \end{aligned} \quad (2.10)$$

In a similar manner we may define symmetric and antisymmetric forces per unit area acting on the top and bottom face sheets (see Figure 2.2). The symmetric and antisymmetric normal displacements and forces per unit area are in phase on the top face sheet and 180° out of phase on the bottom face sheet. The symmetric and antisymmetric impedances are defined in terms of the corresponding amplitudes of the normal displacements and the forces per unit area as follows:

$$Z_{\text{sym}} = \frac{f_{o,\text{sym}}}{v_{\text{sym}}}$$

for the symmetric mode and

$$Z_{\text{asym}} = \frac{f_{o,\text{asym}}}{v_{\text{asym}}}$$

for the antisymmetric mode; where:

$$V_{\text{sym}} = -i\omega d_{\text{sym}} \quad , \quad V_{\text{asym}} = -i\omega d_{\text{asym}} \quad (2.11)$$

V_{sym} and V_{asym} are the amplitudes of normal velocities at the face sheets for the symmetric and antisymmetric modes.

Since the composite panel is symmetric in the x_3 direction with respect to the mid-plane, the above separation into symmetric and antisymmetric components occurs naturally, as will be seen in a subsequent section. Were the core inhomogeneous or the face sheets of different materials or thicknesses the separation would not be possible. By referring to symmetric and antisymmetric motions as being separable it is meant that the motions are uncoupled and the solutions to the symmetric problem and the antisymmetric problem may be obtained independently and linearly superimposed to obtain the complete solution. The solution for the impedances depends on the material properties of the face sheets and the core as well as the frequency ω and wavenumber k_1 .

The symmetric and antisymmetric force per unit area amplitudes may be determined from the acoustic pressure disturbances described in Section 2.2.1. The total pressure disturbance acting on the top face sheet is

$$p_T e^{i(k_1 x_1 - \omega t)} = (2p_o + p_{r,t}) e^{i(k_1 x_1 - \omega t)} \quad (2.12)$$

and for the bottom face sheet:

$$p_B e^{i(k_1 x_1 - \omega t)} = p_{r,b} e^{i(k_1 x_1 - \omega t)}$$

Symmetric and antisymmetric pressure disturbance amplitudes become:

$$\begin{aligned} p_{\text{sym}} &= \frac{p_T + p_B}{2} = p_o + \frac{p_{r,t} + p_{r,b}}{2} \\ p_{\text{asym}} &= \frac{p_T - p_B}{2} = p_o + \frac{p_{r,t} - p_{r,b}}{2} \end{aligned} \quad (2.13)$$

In terms of the symmetric and antisymmetric normal velocities of the face sheets:

$$\begin{aligned} p_{\text{sym}} &= p_o + \frac{\rho_o c_o}{\cos \theta} v_{\text{sym}} \\ p_{\text{asym}} &= p_o + \frac{\rho_o c_o}{\cos \theta} v_{\text{asym}} \end{aligned} \quad (2.14)$$

Inserting the expressions for p_{sym} and p_{asym} into Eq. (2.11) and noting that at the top face sheet a positive pressure is a negative force per unit area ($f_{o,\text{sym}}$ or $f_{o,\text{asym}}$) the following results occur:

$$\left(Z_{\text{sym}} + \frac{\rho_o c_o}{\cos \theta} \right) v_{\text{sym}} = -p_o ,$$

$$\left(Z_{\text{asym}} + \frac{\rho_o c_o}{\cos \theta} \right) v_{\text{asym}} = -p_o$$

and

$$\tau(\theta) = \frac{|p_{r,b}|^2}{|p_o|^2} = \left| \frac{\frac{\rho_o c_o}{\cos \theta} (Z_{\text{sym}} - Z_{\text{asym}})}{\left(Z_{\text{asym}} + \frac{\rho_o c_o}{\cos \theta} \right) \left(Z_{\text{sym}} + \frac{\rho_o c_o}{\cos \theta} \right)} \right|^2 \quad (2.15)$$

The expression for $\tau(\theta)$ involves the impedance expression for the composite panel including radiation loading and the impedance of the acoustic half space defined by $x_3 < - (L+t_p)$. The *in vacuo* symmetric and antisymmetric mode impedances Z_{sym} , Z_{asym} , occur in series with the radiation loading impedance $\rho_o c_o / \cos \theta$. The impedance for the composite panel is determined by the in-parallel addition of the symmetric and antisymmetric mode impedances including radiation loading. The impedance of the composite is defined with respect to the motion of the bottom face sheet and incident pressure disturbance as:

$$Z_{\text{comp}} = \frac{\left(Z_{\text{asym}} + \frac{\rho_o c_o}{\cos \theta} \right) \left(Z_{\text{sym}} + \frac{\rho_o c_o}{\cos \theta} \right)}{(Z_{\text{asym}} - Z_{\text{sym}})} = \frac{p_o}{-i\omega d_1} \quad (2.16)$$

The minus sign in the denominator results from the symmetric and antisymmetric motions at the bottom face sheet being out of phase.

The expression reveals several interesting aspects of the acoustic behavior of a composite panel of infinite extent. If for a particular combination of panel properties, frequency ω , and incidence angle θ , (note: $k_1 = \omega/c_0 \sin\theta$), $Z_{\text{sym}} = Z_{\text{asym}}$ then $\tau(\theta) = 0$. No sound is transmitted. This occurs because the symmetric and antisymmetric modes respond to the incident acoustic plane wave with equal amplitude and phase of the normal displacements of the top face sheet. The impedances Z_{sym} and Z_{asym} are complex quantities. The normal displacements of the symmetric and antisymmetric modes at the bottom face sheet exactly cancel and the absence of motion means that no acoustic plane wave is radiated from the bottom face sheet.

Also, if either Z_{sym} or $Z_{\text{asym}} = 0$, $\tau(\theta)$ assumes the following form:

$$\tau(\theta) = \left| \frac{Z_{\text{sym}}}{Z_{\text{sym}} + \frac{\rho_0 c_0}{\cos\theta}} \right|^2 \quad \text{when } Z_{\text{asym}} = 0 \quad (2.17)$$

If in addition $|Z_{\text{sym}}| > \rho_0 c_0 / \cos\theta$ then $\tau(\theta) \approx 1$, a condition of high transmission of the incident acoustic plane wave through the composite. The condition

$|Z_{\text{sym}}| > \rho_0 c_0 / \cos\theta$ describes the case of light fluid

loading of the symmetric mode by the presence of the acoustic medium (namely, air). The condition $Z_{\text{asym}} = 0$ means, in the absence of internal damping in the composite and for *in vacuo* motion of the composite, that a resonance condition exists for which normal displacements, of frequency ω and wavenumber k_1 , propagate in the composite unattenuated and with phase velocity, $c_{\text{ph}} = \omega/k_1$.

2.3 Solution for the Symmetric and Antisymmetric Impedances of a Composite Panel

2.3.1 Differential equations of motion for the face sheets

As stated in the introduction to this chapter the face sheets deform in bending, producing normal displacements and in extension, producing in-plane extensional displacements. The equations of motion for the face sheets are:

$$D_p \frac{\partial^4 \eta}{\partial x_1^4} + \rho_p \frac{\partial^2 \eta}{\partial t^2} = \Sigma \begin{array}{l} \text{Normal stresses acting on} \\ \text{the face sheet} \end{array} \quad (2.18)$$

$$- E_{11}^{\text{fs}} t_p \frac{\partial^2 \zeta}{\partial x_1^2} + \rho_p \frac{\partial^2 \zeta}{\partial t^2} = \Sigma \begin{array}{l} \text{Shear stresses acting} \\ \text{on the face sheet} \end{array}$$

where ρ_p is the surface mass density of one of the face sheets (mass per unit area), D_p is the bending rigidity of the face sheet, $D_p = E_{\text{fs}} t_p^3 / 12(1-\nu_p^2)$. E_{11}^{fs} equals $\lambda_{\text{fs}} + 2\mu_{\text{fs}}$ where λ_{fs} and μ_{fs} are the Lamé constants. E_{fs} is the

Young's modulus and ν_p is Poisson's ratio for the face sheets. η is the normal displacement and ζ the in-plane extensional displacement of the face sheet. The normal stresses acting on the face sheet result from the various acoustic plane waves and from the deformations in the core, which in turn are the result of the normal and extensional motions of the face sheets. Shear stresses acting on the face sheets result only from the deformation of the core. The acoustic plane waves produce no shear stresses.

Solutions to Eq. (2.18) applied to both face sheets are assumed to be of the following form:

$$\begin{Bmatrix} \eta_0 \\ \eta_1 \end{Bmatrix} = \begin{Bmatrix} d_0 \\ d_1 \end{Bmatrix} e^{i(k_1 x_1 - \omega t)}, \quad (2.19)$$

$$\begin{Bmatrix} \zeta_0 \\ \zeta_1 \end{Bmatrix} = \begin{Bmatrix} L_0 \\ L_1 \end{Bmatrix} e^{i(k_1 x_1 - \omega t)}$$

where d_0 , d_1 , L_0 , L_1 are complex constants describing the normal and extensional motions of the top (subscript 0) and bottom (subscript 1) face sheets. The above displacements will be used as boundary conditions in solving for deformations in the core. From the core deformation, the stresses produced at the core-face sheet interfaces will be evaluated in terms of the face sheet displacements. Instead of applying the pressure disturbances that result

from the various acoustic plane waves, a normal force per unit area, $2f_0 e^{i(k_1 x_1 - \omega t)}$, will be applied only to the top face sheet. This loading has equal symmetric and anti-symmetric components, $f_0 e^{i(k_1 x_1 - \omega t)}$. The solution for η_0 and η_1 will be presented in terms of the symmetric and antisymmetric displacements from which the symmetric and antisymmetric impedances will be evaluated. These impedances will be used in Eqs. (2.15), (2.8) and finally, in Eq. (2.9) to determine TL.

2.3.2 Modelling of the isotropic core to include wave bearing motions

The solution for the normal and shear stresses produced by normal and extensional displacements of the two surfaces of an elastic isotropic layer of infinite extent will be expressed in a general form involving equivoluminal (distortional) and dilatational (irrotational) wave motions in the layer. Dilatational motions in a homogeneous isotropic elastic medium are governed by the following equation:

$$(\lambda_c + 2\mu_c) \nabla^2 \bar{A} = \rho_c \frac{\partial^2 \bar{A}}{\partial t^2} \quad (2.20)$$

where $\bar{A}$ is a vector displacement field, λ_c and μ_c are the Lamé constants.

$$\lambda_c = \frac{\nu_c E_c}{(1-\nu_c)(1+2\nu_c)} \quad \text{and} \quad \mu_c = G_c$$

where E_c is the Young's Modulus, G_c the shear modulus and ν_c Poisson's ratio for the isotropic core material. ρ_c is the density of the core material (mass per unit volume), and $\nabla = \bar{i} \frac{\partial}{\partial x_1} + \bar{j} \frac{\partial}{\partial x_2} + \bar{k} \frac{\partial}{\partial x_3}$. $\bar{A}$ may be written in terms of a potential function Φ as $\bar{A} = \nabla \Phi$.

Equivoluminal motions are described by the governing equation:

$$\mu_c \nabla^2 \bar{B} = \rho_c \frac{\partial^2 \bar{B}}{\partial t^2} \quad (2.21)$$

where $\bar{B}$ is the vector displacement field for equivoluminal displacements. $\bar{B}$ may also be written in terms of a potential function for plane waves in two dimensions:

$\bar{B} = \nabla x \bar{J} \Psi(x_1, x_3, t)$. $\bar{J}$ is a unit vector in the x_2 direction. Both equations describe non-dispersive media where the wavespeeds are $c_D^2 = \frac{\lambda_c + 2\mu_c}{\rho_c}$ for dilatational waves and $c_s^2 = \frac{\mu_c}{\rho_c}$ for equivoluminal or shear waves.

In terms of the potential functions, the particle displacements and the normal and shear stresses in the core are: for dilatational waves:

$$\begin{aligned} u &= \frac{\partial \Phi}{\partial x_1} & \frac{\sigma_1}{G_c} &= \frac{\nabla^2 \Phi}{\delta^2} - 2 \frac{\partial^2 \Phi}{\partial x_3^2} \\ v &= 0 & \frac{\sigma_3}{G_c} &= \frac{\nabla^2 \Phi}{\delta^2} - 2 \frac{\partial^2 \Phi}{\partial x_1^2} \end{aligned}$$

$$w = \frac{\partial \Phi}{\partial x_3} \quad \frac{\tau_{13}}{G_c} = 2 \frac{\partial^2 \Phi}{\partial x_1 \partial x_3} \quad (2.22)$$

$$\text{where } \delta = \frac{c_s}{c_D} = \sqrt{\frac{\mu_c}{\lambda_c + 2\mu_c}}.$$

For equivoluminal waves:

$$\begin{aligned} u &= \frac{\partial \Psi}{\partial x_3} & \frac{c_1}{G_c} &= -2 \frac{\partial^2 \Psi}{\partial x_1 \partial x_3} \\ v &= 0 & \frac{\sigma_3}{G_c} &= 2 \frac{\partial^2 \Psi}{\partial x_1 \partial x_3} \\ w &= \frac{\partial \Psi}{\partial x_1} & \frac{\tau_{13}}{G_c} &= \frac{\partial^2 \Psi}{\partial x_1^2} - \frac{\partial^2 \Psi}{\partial x_3^2}, \end{aligned} \quad (2.23)$$

and σ_1 , σ_3 are normal stresses in the x_1 , x_3 directions and τ_{13} is a shear stress.

A general solution for the core displacements, where those displacements match the displacements of the face sheets, Eq. (2.19) at the core-face sheet interfaces, is obtained by assuming that the potential functions Φ and Ψ have the following forms:

$$\Phi_{\pm} = a_{\pm} e^{i(k_1 x_1 \pm k_3^d x_3 - \omega t)}$$

and

$$\Psi_{\pm} = b_{\pm} e^{i(k_1 x_1 \pm k_3^s x_3 - \omega t)}, \quad (2.24)$$

$a_{\pm}$ and $b_{\pm}$ are four complex constants. k_3^d and k_3^s are determined from:

$$(k_3^d)^2 + k_1^2 = \left(\frac{\omega}{c_D}\right)^2, \quad (k_3^s)^2 + k_1^2 = \left(\frac{\omega}{c_S}\right)^2 \quad (2.25)$$

The x_1 dependence is the same as that of the various acoustic plane waves and the normal and extensional displacements of the face sheets. The four potentials Φ_+ , Φ_- , Ψ_+ , Ψ_- each represent plane travelling waves in the core, the particle displacements for which are given by Eq. (2.22). Two, Φ_+ and Ψ_+ , represent upward travelling waves, in the $+x_3$ direction, and the other two, Φ_- and Ψ_- , represent downward travelling waves, in the $-x_3$ direction.

The core displacements u , w are determined from the superposition of the displacements from the four travelling waves. The displacements u and w , evaluated at the core-face sheet interfaces, are set equal to the normal and extensional displacements of the face sheets, Eq. (2.19). Four equations result, which are solved to yield the amplitudes a_+ , a_- , b_+ , b_- in terms of the amplitudes of the face sheet displacements d_0 , d_1 , L_0 , L_1 . The normal and shear stresses that act on the face sheets as a result of deformations in the core are then determined from Eq. (2.22), also in terms of d_0 , d_1 , L_0 , L_1 .

The expressions relating the normal and shear stresses to the normal and extensional displacements of the face

sheets involve 6 independent complex coefficients of the forms:

$$\text{Normal Stress at interface} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = ALO \times \begin{Bmatrix} d_o \\ -d_1 \end{Bmatrix}$$

$$\text{Normal Stress at interface} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = AL1 \times \begin{Bmatrix} d_o \\ -d_1 \end{Bmatrix}$$

$$\text{Normal Stress at interface} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = -ASO \times \begin{Bmatrix} L_o \\ L_1 \end{Bmatrix}$$

$$\text{Normal Stress at interface} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = AS1 \times \begin{Bmatrix} L_o \\ L_1 \end{Bmatrix}$$

$$\text{Shear stress at interface} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = -ASO \times \begin{Bmatrix} d_o \\ d_1 \end{Bmatrix}$$

$$\text{Shear stress at interface} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = AS1 \times \begin{Bmatrix} d_o \\ d_1 \end{Bmatrix}$$

$$\text{Shear stress at interface} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = STO \times \begin{Bmatrix} L_o \\ -L_1 \end{Bmatrix}$$

$$\text{Shear stress at interface} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = ST1 \times \begin{Bmatrix} L_o \\ -L_1 \end{Bmatrix}$$

(2.26)

The normal and shear stresses assume an $e^{i(k_1 x_1 - \omega t)}$ dependence. The coefficients relating normal stresses to extensional displacements are the same as those relating shear stresses to normal displacements (AS0, AS1). The

third character in the name of the coefficient (0 or 1) refers to stresses produced at the same interface at which the displacement is applied - ending 0, or to stresses produced at the other interface from which the displacement is applied - ending 1. The coefficients describe stresses produced as a result of only the related displacement, all other displacements being set equal to zero.

The coefficients take the following forms:

$$\left(k_3^d\right)^2 = \omega^2 / \left(c_D^2(1-i\eta_c)\right) - k_1^2$$

$$\left(k_3^s\right)^2 = \omega^2 / \left(c_s^2(1-i\eta_c)\right) - k_1^2$$

where η_c is the loss factor for the core material and is assumed to be the same for shear or dilatational deformation.

$$\begin{aligned} \Delta &= 8k_1^2 k_3^d k_3^s \left(1 - \cos(k_3^d L) \cos(k_3^s L)\right) \\ &\quad + 4 \sin(k_3^d L) \sin(k_3^s L) \left(k_1^4 + (k_3^d)^2 \cdot (k_3^s)^2\right) \\ \text{ALO} &= \frac{4\rho_c \omega^2}{\Delta} \left(k_3^s k_1^2 \cos(k_3^s L) \sin(k_3^d L) \right. \\ &\quad \left. + (k_3^s)^2 k_3^d \sin(k_3^s L) \cos(k_3^d L)\right) \\ \text{ALI} &= \frac{4\rho_c \omega^2}{\Delta} \left(\left(k_3^s\right)^2 k_3^d \sin(k_3^s L) + k_1^2 k_3^s \sin(k_3^d L)\right) \end{aligned}$$

$$\begin{aligned}
 ASO &= \frac{4ik_1 k_3^s k_3^d}{\Delta} \left(4G_c (1-i\eta_c) k_1^2 - \rho_c \omega^2 \right) \left(1 - \cos(k_3^d L) \right) \\
 &\quad \times \cos(k_3^s L) \Bigg) + 4 \frac{ik_1 \sin(k_3^d L) \sin(k_3^s L)}{\Delta} \\
 &\quad \times \left(2G_c (1-i\eta_c) \left((k_3^d)^2 + (k_3^s)^2 + k_1^2 \right) - k_1^2 \rho_c \omega^2 \right) \\
 AS1 &= \frac{4ik_1 \rho_c \omega^2 k_3^d k_3^s}{\Delta} \left(\cos(k_3^d L) - \cos(k_3^s L) \right) \\
 STO &= \frac{4\rho_c \omega^2}{\Delta} \left((k_3^d)^2 k_3^s \cos(k_3^s L) \sin(k_3^d L) \right. \\
 &\quad \left. + k_1^2 k_3^d \sin(k_3^s L) \cos(k_3^d L) \right) \\
 ST1 &= \frac{4\rho_c \omega^2}{\Delta} \left(k_3^d k_1^2 \sin(k_3^s L) + (k_3^d)^2 k_3^s \sin(k_3^d L) \right)
 \end{aligned} \tag{2.27}$$

The effects of internal damping in the core have been included in the formulation by allowing the stiffness moduli, G_c and E_c to become complex. For reasons of simplification the loss factor for the core material for shear deformations was assumed to be equal to the loss factor for dilatational deformations. Thus, G_c and E_c were replaced by $G_c(1-i\eta_c)$ and $E_c(1-i\eta_c)$ in deriving the above expressions. This also means that c_s^2 and c_o^2 are replaced by $c_s^2(1-i\eta_c)$ and $c_D^2(1-i\eta_c)$. The expressions could have been determined where the loss factors in shear

and dilatation differed but owing to a lack of data dealing with such differences the above assumption was made.

The displacements at the core-face sheet interfaces have an $e^{i(k_1 x_1 - \omega t)}$ dependence. These displacements are travelling waves moving in the $+x_1$ direction with phase velocity $c_T = \omega/k_1$. Remembering that $k_1 = (\omega/c_o) \sin\theta$, c_T becomes $c_o/\sin\theta$. The expressions for $(k_3^d)^2$ and $(k_3^s)^2$, in the absence of internal damping become:

$$(k_3^d)^2 = \omega^2 \left(\frac{1}{c_D^2} - \frac{\sin^2 \theta}{c_o^2} \right), \quad (k_3^s)^2 = \omega^2 \left(\frac{1}{c_s^2} - \frac{\sin^2 \theta}{c_o^2} \right) \quad (2.28)$$

If $c_s > c_o$ and/or $c_D > c_o$ then there will be an incidence angle for which $(k_3^s)^2$ and/or $(k_3^d)^2$ is negative. This corresponds to the situation where $c_s > c_T$ and/or $c_D > c_T$. In this circumstance, k_3^s and/or k_3^d is purely imaginary. Referring to the expressions for the potential functions this means that the shear and/or dilatational waves are travelling waves in the x_1 direction, but decay exponentially in the x_3 direction. For incidence angles such that k_3^s and/or k_3^d are real, the shear and/or dilatational waves are travelling waves in the x_1 direction and also in the x_3 direction. These results occur independent of frequency, depending only on the values of c_s, c_D, c_o and the angle of incidence.

To gain an understanding of what these coefficients mean consider the case where $k_1 = 0$. This describes the case of normal incidence ($\theta=0$), for which the displacements in the core depend only on x_3 . The expressions for the coefficients become:

$$\begin{aligned}
 AL0 &= \frac{\rho_c c_D \omega \cos\left(\frac{\omega}{c_D} L\right)}{\sin\left(\frac{\omega}{c_D} L\right)} \\
 AL1 &= \frac{\rho_c c_D \omega}{\sin\left(\frac{\omega}{c_D} L\right)} \\
 AS0 &= 0 \\
 AS1 &= 0 \\
 ST0 &= \frac{\rho_c c_s \cos\left(\frac{\omega}{c_s} L\right)}{\sin\left(\frac{\omega}{c_s} L\right)} \\
 ST1 &= \frac{\rho_c c_s \omega}{\sin\left(\frac{\omega}{c_s} L\right)} \tag{2.29}
 \end{aligned}$$

For small values of $(\omega/c_D)L$ and $(\omega/c_s)L$ these expressions become

$$\begin{aligned}
 AL0 &= \frac{\lambda_c + 2\mu_c}{L} - \frac{1}{3} \rho_c \omega^2 L \\
 AL1 &= \frac{\lambda_c + 2\mu_c}{L} + \frac{1}{6} \rho_c \omega^2 L
 \end{aligned}$$

$$\begin{aligned} ST0 &= \frac{G_c}{L} - \frac{1}{3} \rho_c \omega^2 L \\ ST1 &= \frac{G_c}{L} + \frac{\rho_c \omega^2 L}{6} \end{aligned} \quad (2.30)$$

These are the expressions that would result for elastic springs, of thickness L , in dilatation ($AL0$, $AL1$) and shear ($ST0$, $ST1$) where first order effects of the mass of the elastic material are included. The displacements are linearly distributed in the x_3 direction.

The complete expressions for $k_1 = 0$ as given in Eqs. 2.29 include wave bearing effects which may be seen if $\frac{\omega}{c_D} L = n\pi$, $n = 1, 2, \dots$ or $\frac{\omega}{c_D} L = (2m-1) \frac{\pi}{2}$, $m = 1, 2, \dots$. The case where n or $m = 0$ is considered in the low frequency case described by Eq. (2.30). For $\frac{\omega}{c_D} L = n\pi$, L equals $\frac{n\lambda_D}{2}$, where λ_D is the wavelength of dilatational motion at frequency ω and equals $\frac{2\pi c_D}{\omega}$. The normal stresses produced by a finite normal displacement of the face sheets become infinite, in the absence of internal damping. This condition describes a standing wave resonance situation in the core where the boundary conditions for dilatational motion are for zero motion at the face sheets.

The condition $\frac{\omega}{c_D} L = (2m-1) \frac{\pi}{2}$ corresponds to the situation where L equals $(2m-1) \frac{\lambda_D}{4}$. This also describes a standing wave resonance condition where the core thickness,

L , is an odd integral number of quarter wavelengths for dilatational motion at frequency ω . One face sheet is a node point and the other an antinode; which explains why no stress is produced at the antinode ($AL0 = 0$) yet stress is produced at the node ($AL1 \neq 0$).

Similar statements may be made concerning $ST0$ and $ST1$ for $k_1 = 0$ for standing wave resonances of shear motions in a layer of thickness L . The results for $k_1 \neq 0$ show a similar behavior though more complicated in nature. For $k_1 \neq 0$ the resonances involve a combination of dilatational and shear motions.

Figures 2.3 a-f show the 6 coefficients as functions of frequency and incidence angle, which with frequency determines k_1 . The results are for a closed cell styro-foam material. For this material the dilatational wave-speed c_D equals 884 m/sec and the shear wavespeed, c_s equals 440 m/sec. In the absence of internal damping in the core $AL0$, $AL1$, $ST0$, $ST1$ are real quantities while $AS0$, $AS1$ are purely imaginary. At low frequency the coefficients $AL0$, $AL1$, $ST0$, $ST1$ assume values defined by Eq. (2.30) independent of the incidence angle. When $\theta = 0$ and the frequency is increased, the values change, initially according to Eqs. (2.30); $AL0$ decreases, $AL1$ increases, $ST0$ decreases and $ST1$ increases.

At sufficiently high frequency the variations of these four coefficients with incidence angle become pronounced as the effects of standing wave resonances in the core become extant. These resonances occur at lower frequencies in the coefficients that describe shear stresses (ST_0 , ST_1) than in the coefficients describing normal stresses (AL_0 , AL_1). This occurs because $c_s < c_D$, and the frequency for which the shear wavelength is of the order of one-fourth the core thickness, is lower than for the dilatational wavelength.

The resonances occur for incidence angles where the coefficients are zero or infinity. The incidence angles where the coefficients are infinite correspond to zeros of the quantity Δ as defined in Eq. (2.27). This quantity is found in the expressions for all six coefficients, and therefore, all exhibit an infinite value for values of frequency and incidence angle where $\Delta = 0$. This condition describes standing wave resonant motions in the core where the displacements at the core-face sheet interfaces are zero. From the figures this is seen to occur at a frequency of 4000Hz and incidence angle of approximately 16.5 degrees.

The coefficients AS_0 , AS_1 are purely imaginary and from Eq. (2.27), it is seen that they depend on ik_1 times a real quantity, in the absence of internal damping. As

displacements and stresses have an $e^{i(k_1 x_1 - \omega t)}$ dependence, multiplying by ik_1 is equivalent to taking the spatial derivative $\frac{\partial}{\partial x_1}$. The stresses that AS0 and AS1 describe therefore depend on the spatial derivative of the face sheet displacements. The manner by which normal displacements produce shear stresses is related to the dependence on the spatial derivative. Figure 2.4 depicts the situation.

The material underneath the trough of the normal displacement of face sheet 0 is compressed. The core material underneath the crest is placed in a state of tension. Were the core material gaseous it would move laterally from underneath the trough to underneath the crest. As it is not gaseous, and is constrained from slipping with respect to the face sheet, it resists the tendency to move laterally. It does so by the development of shear stresses. For $k_1 = 0$ the compression of the core is uniform in x_1 and there is no tendency for lateral motion and hence, no shear stresses are developed (AS0 = AS1 = 0).

To relate the production of normal stress by extensional motions of the face sheet, again refer to Figure 2.4. At points 1, 2, 3 the face sheet extensional displacements are zero; in between the extensional displacements are in the directions indicated by the arrows. At point 1,

the core material beneath is compressed in the x_1 direction; at point 2, the core material beneath is pulled apart and a state of tension exists in the x_1 direction. The states of normal strain in the x_1 direction imply, for an isotropic material with non-zero λ and therefore, non-zero Poisson's ratio, ν , that normal stresses will exist in the x_3 direction.

2.3.3 Determination of symmetric and antisymmetric impedance expressions

With the behavior of the core to various displacements at the core-face sheet interfaces described, a solution to the equations of motion for the face sheets may proceed. The equation describing normal displacements of the top face sheet becomes, with the use of Eqs. (2.18) and (2.26):

$$\left\{ D_p k_1^4 - \rho_p \omega^2 + AL0 - \frac{ik_1 t_p}{2} AS0 \right\} d_0 - \left\{ AL1 + \frac{ik_1 t_p}{2} AS1 \right\} d_1 - AS0 L_0 + AS1 L_1 = 2f_0 \quad (2.31)$$

The equation for extensional displacements of the top face sheet is:

$$\left\{ E_{11}^{fs} t_p k_1^2 - \rho_p \omega^2 + ST0 \right\} L_0 - ST1 L_1 + \left\{ \frac{ik_1 t_p}{2} ST0 + AS0 \right\} d_0 + \left\{ \frac{ik_1 t_p}{2} ST1 + AS1 \right\} d_1 = 0 \quad (2.32)$$

For the bottom face sheet the equation for normal displacements becomes:

$$\begin{aligned}
 & - \left\{ AL1 + \frac{ik_1 t_p}{2} AS1 \right\} d_o + \left\{ D_p k_1^4 - \rho_p \omega^2 \right. \\
 & \quad \left. + AL0 - \frac{ik_1 t_p}{2} AS0 \right\} d_1 - AS1 L_o + AS0 L_1 \\
 & = 0
 \end{aligned} \tag{2.33}$$

and the equation for extensional displacements is:

$$\begin{aligned}
 & - ST1 L_o + \left\{ E_{11}^{fs} t_p k_1^2 - \rho_p \omega^2 + ST0 \right\} L_1 \\
 & - \left\{ \frac{ik_1 t_p}{2} ST1 + AS1 \right\} d_o - \left\{ \frac{ik_1 t_p}{2} ST0 \right. \\
 & \quad \left. + AS0 \right\} d_1 = 0
 \end{aligned} \tag{2.34}$$

In these equations there are the terms $\frac{ik_1 t_p}{2} ST0 \begin{Bmatrix} d_o \\ d_1 \end{Bmatrix}$,

$\frac{ik_1 t_p}{2} ST1 \begin{Bmatrix} d_o \\ d_1 \end{Bmatrix}$, $\frac{ik_1 t_p}{2} AS1 \begin{Bmatrix} d_o \\ d_1 \end{Bmatrix}$ and $\frac{ik_1 t_p}{2} AS0 \begin{Bmatrix} d_o \\ d_1 \end{Bmatrix}$.

These result from the fact that during bending of the face sheets a lateral displacement at the core-face sheet interface occurs due to the finite thickness of the face sheet. The neutral axis for bending of a face sheet lies in the mid-plane of the face sheet.

Owing to the symmetry of the composite, the solutions to Eqs. (2.31) - (2.34) may be formulated in terms of symmetric and antisymmetric displacements:

$$L_{\text{sym}} = \frac{L_o + L_1}{2}$$

and

$$d_{\text{sym}} = \frac{d_o - d_1}{2}$$

for symmetric displacements, and

(2.35)

$$L_{\text{asym}} = \frac{L_o - L_1}{2}$$

and

$$d_{\text{asym}} = \frac{d_o + d_1}{2}$$

for antisymmetric displacements.

The equations in symmetric form are obtained by subtracting Eq. (2.33) from Eq. (2.31) and dividing by 2; and adding Eq. (2.32) to Eq. (2.34) and also dividing by 2.

$$\left\{ D_p k_1^4 - \rho_p \omega^2 + AL_0 + AL_1 + \frac{ik_1 t_p}{2} (AS1-AS0) \right\} d_{\text{sym}} + \left\{ AS1-AS0 \right\} L_{\text{sym}} = f_o \quad (2.36)$$

$$\left\{ E_{11}^{fs} t_p k_1^2 - \rho_p \omega^2 + ST_0 - ST_1 \right\} L_{\text{sym}} + \left\{ AS0-AS1 + \frac{ik_1 t_p}{2} (ST_0-ST_1) \right\} d_{\text{sym}} = 0 \quad (2.37)$$

Symmetric normal displacements at the top face sheet are 180° out of phase with the normal displacements at the bottom face sheet. Symmetric extensional displacements are in phase for the top and bottom face sheets. Antisymmetric

displacements have opposite phase relationships between top and bottom face sheets as compared to symmetric displacements.

The equations in antisymmetric form are obtained by adding Eq. (2.33) to Eq. (2.31), then dividing by 2; and subtracting Eq. (2.34) from Eq. (2.32) and again dividing by 2.

$$- \left\{ AS0 + AS1 \right\} L_{\text{asym}} + \left\{ D_p k_1^4 - \rho_p \omega^2 + AS0 - AL1 - \frac{ik_1 t_p}{2} (AS0+AS1) \right\} d_{\text{asym}} = f_o \quad (2.38)$$

$$\left\{ E_{11}^{fs} t_p k_1^2 - \rho_p \omega^2 + ST0 + ST1 \right\} L_{\text{asym}} + \left\{ \frac{ik_1 t_p}{2} (ST0+ST1) + AS0 + AS1 \right\} d_{\text{asym}} = 0 \quad (2.39)$$

The two sets of Eqs. (2.36), (2.37) and (2.38), (2.39) are the uncoupled equations for symmetric and antisymmetric motions of the composite, respectively. The equations are solved to give the symmetric and antisymmetric mode impedances:

$$Z_{\text{sym}} = \frac{f_o}{-i\omega d_{\text{sym}}}$$

and

$$Z_{\text{asym}} = \frac{f_o}{-i\omega d_{\text{asym}}}$$

(2.40)

Figures 2.5 a,b show the symmetric and antisymmetric impedances where the composite is made of 1/4" thick plywood face sheets and the 3" thick closed cell styrofoam core described in Section 2.3.2. In the absence of internal damping the impedances are purely imaginary, involving stiffness reactance or mass reactance expressions. For a stiffness controlled impedance the result is a positive imaginary number, and for a mass controlled impedance the result is a negative imaginary number.

For $k_1 = 0$, normal incidence, the normal displacements become uncoupled from extensional displacements for both the symmetric and antisymmetric modes. The equations for normal displacements are:

for the symmetric mode:

$$\left\{ -\rho_p \omega^2 + AL_0 + AL_1 \right\} d_{\text{sym}} = f_o \quad (2.41)$$

for the antisymmetric mode:

$$\left\{ -\rho_p \omega^2 + AL_0 - AL_1 \right\} d_{\text{asym}} = f_o$$

At low frequency and for $k_1 = 0$, using the low frequency expressions for AL_0 and AL_1 from Eq. (2.30), Eq. (2.41) becomes

$$\left\{ \frac{2(\lambda_c + 2\mu_c)}{L} - \omega^2 \left(\rho_p + \frac{1}{6} \rho_c L \right) \right\} d_{\text{sym}} = f_o$$

$$- \omega^2 \left(\rho_p + \frac{1}{2} \rho_c L \right) d_{\text{asym}} = f_o \quad (2.42)$$

The impedance expressions become

$$Z_{\text{sym}} = \frac{1}{\omega} \frac{2(\lambda_c + 2\mu_c)}{L} - i\omega \left(\rho_p + \frac{1}{6} \rho_c L \right)$$
$$Z_{\text{asym}} = -i\omega \left(\rho_p + \frac{1}{2} \rho_c L \right) \quad (2.43)$$

For the symmetric mode impedance the stiffness reactance dominates while for the antisymmetric mode impedance the mass reactance dominates. The symmetric force per unit area loading of the composite panel compresses the face sheets against the core. The antisymmetric force per unit area loading of the composite acts to accelerate the mass of the composite.

From Figures 2.5 a,b at low frequency the behavior described by Eq. (2.43) occurs approximately independent of incidence angle. As the frequency is increased and for $k_1 = 0$, the impedances change according to Eq. (2.43); Z_{sym} decreases and Z_{asym} becomes a larger negative number.

At higher frequency the effects of coupling between normal displacements and extensional displacements become apparent. These are present in the symmetric mode impedance at 1500 Hz and for an incidence angle of $\sim 6^\circ$. The symmetric mode impedance assumes a zero value and an effectively infinite value in the region near 6° . The situation is very similar to that described in Figure 2.6

for a two degree of freedom lumped mass-spring resonance system. If mass 1 is driven at the frequency $\omega = \left\{ \frac{K_2 + K_c}{m_2} \right\}^{1/2}$ it will not move because the motion of mass two results in spring forces through the coupling stiffness k_c , acting on mass 1 which cancel the applied force. The driving point impedance at this frequency is infinite. The system as a whole is characterized by two resonances, one associated principally with the motion of mass one, the other with mass two. If mass 1 is driven at the resonance frequency associated with the motion of mass two the driving point impedance will be zero. The driving point impedance assumes a zero value and an infinite value at two different frequencies both associated principally with motions of mass two.

For the distributed composite panel involving a resonance of normal displacements and a resonance of extensional displacements, variations in the excitation near resonance may be produced by varying k_1 or ω . The behavior of the symmetric mode impedance at 1500 Hz near $\theta = 6^\circ$ is explained by analogy with the lumped spring-mass system. The zero of the impedance corresponds to a resonance of the coupled system in which the motion is principally extensional. The infinite value of impedance relates to the cancellation of the excitation force, $f_o e^{i(k_1 x_1 - \omega t)}$, by

coupling forces generated by extensional motions. The phase of the excitation forces with respect to the coupling forces is important, the two forces being out of phase when the impedance is infinite and in phase for the zero of impedance. The two effects occur close together in that both are principally characterized by extensional displacements. The resonance characterized by normal displacements occurs for significantly different excitation conditions, k_1 and ω .

At 4000 Hz, and for an incidence angle near 16° , the symmetric mode impedance exhibits this same behavior. The behavior of the core in this region was seen to be determined by standing wave resonance effects involving both shear and dilatational motions. Slightly above resonance, the effect of the standing wave resonance in the core is to add mass to the uncoupled extensional displacements of the face sheets, ST_0-ST_1 is a large negative number. Extensional displacements in the face sheets are significantly stiffness controlled in this region and the addition of mass from the core causes the uncoupled extensional displacements of the composite to become resonant. Resonance of the uncoupled extensional displacements in the composite panel means, from analogy with the lumped mass-spring system, that the impedance for normal displacements becomes infinite. The applied force per unit area is

cancelled by normal stresses generated by resonant extensional displacements of the composite. The extensional displacements are resonant as a result of the effect of the added mass of the core on the stiffness controlled displacements of the face sheets.

The effects just described occur for the symmetric mode impedance. These effects are not seen in the antisymmetric mode impedances for the values of ω and k_1 plotted. The core behaves significantly differently for symmetric and antisymmetric displacements. Effects similar to those that occurred for the symmetric mode impedance do occur for the antisymmetric mode but for different values of ω and k_1 .

2.3.4 Phase speeds for resonant motions in a composite panel

The most important behavior of the impedances relates to the zeroes of impedance that occur for both the symmetric and antisymmetric impedances near an incidence angle of 60° and for a frequency of 4000 Hz. These zeroes correspond to resonances of the composite panel that are principally characterized by normal displacements of the face sheets. They are important in determining the TL behavior of the composite panel. Their location in ω and k_1 determines the phase speed $c_{ph} = \omega/k_1$, for which resonant motion in the composite may exist without excitation, in

the absence of internal damping. The resonant motion has an $e^{i(k_1 x_1 - \omega t)}$ dependence. Figure 2.7 shows the locus of such phase speeds vs frequency for the symmetric and anti-symmetric resonant motions.

Resonant motions for the symmetric mode do not exist for frequencies less than a certain value. From Section 2.3.3, it was seen that for low frequency the symmetric mode impedance was dominated by the stiffness reactance of the core. The frequency must increase until stiffness and mass reactance terms cancel and the impedance equals zero. At ~1450 Hz, the symmetric mode phase speed becomes infinite. An infinite value of phase speed corresponds to normal incidence $k_1 = 0$, where the motions in the composite are uniform in x_1 . The frequency for which this occurs is determined from the following equation:

$$-\rho_p \omega^2 + AL0 + AL1 = 0 \quad (2.44)$$

where AL0 and AL1 are given by Eqs. (2.29). For the case where the face sheet mass per unit area is significant in comparison with the mass per unit area of the core, wave bearing effects in the core may be neglected and the low frequency expressions for AL0, AL1 as given in Eqs. (2.30) may be used in Eq. (2.44) to yield:

$$\omega = \left(\frac{2(\lambda_c + 2\mu_c)}{L} \right) / \left(\rho_p + \frac{1}{6} \rho_c L \right)^{1/2} \quad (2.45)$$

This is the expression for the resonance frequency of a mass-massy spring oscillator assuming a linear distribution of the displacements in the massy spring.

It is also noticed that for this frequency and frequencies just less than this frequency, the phase speed of the symmetric mode is double valued. This means that for a given frequency, there are two wave numbers k_1 , for which the impedance is zero.

At higher frequency, the phase speed of the symmetric mode approaches a limiting form determined by bending motion of the face sheets:

$$c_{ph} = \left(\frac{D_p}{\rho_p} \right)^{1/4} \sqrt{\omega}$$

where

(2.46)

$$D_p = \frac{E_{fs} t_p^3}{12(1-\nu_p^2)}$$

The phase speed of the antisymmetric mode at high frequency approaches the same limiting form. The potential energy is dominated by the potential energy of bending in the face sheets.

At lower frequency, the antisymmetric mode phase speed varies less with frequency. In this region, the storage of potential energy is principally in shear deformation in the core. Shear waves in a medium, in this case a composite panel, are non-dispersive and this explains the

near constancy of the phase speed with frequency.

At low frequency, the antisymmetric mode phase speed again assumes a dispersive form associated with bending motions. In this situation, the neutral axis for bending lies in the mid-plane of the composite. The expression for the phase speed becomes:

$$c_{ph} = \left(\frac{D'_p}{\rho_p} \right)^{1/4} \sqrt{\omega}$$

where

(2.47)

$$D'_p = \frac{E_{fs} t_p (L + t_p)^2}{4(1 - \nu_p^2)}$$

The storage of potential energy in bending in the core is neglected in this expression since Young's modulus for the core is typically significantly less than the Young's modulus for the face sheets.

2.4 Transmission Coefficients Determined From the Impedance Expressions

The combination of ω and k_1 that result from an acoustic plane wave incidence are limited to values for which $\omega/k_1 = c_T \geq c_o$. If at a particular frequency the phase speed for the symmetric and/or antisymmetric modes, c_{ph} , is greater than the acoustic wave speed c_o , then an incidence angle exists for which $c_T (= c_o / \sin \theta)$ equals c_{ph} . A coincidence condition exists in that the pressure disturbances on the face sheets resulting from the acoustic plane

waves resonantly excite symmetric and/or antisymmetric motions. In the absence of internal damping in the composite panel the particular mode, for which $c_{ph} = c_T$, of the composite panel appears acoustically transparent to the incident wave. The transmission through the composite is not perfect depending on the impedance of the other mode according to Eqs. (2.15) and (2.17).

For the composite panel made with a styrofoam core and plywood face sheets, the transmission coefficient obtained from Eq. (2.15) is shown in Figures 2.8 a,b. The transmission coefficient is shown for the case of zero internal damping. At low frequency, the expressions for the impedances given by Eqs. (2.43), when substituted into Eq. (2.15) yield:

$$\tau(\theta) = \frac{1}{1 + \left(\frac{\left| \rho_p + \frac{1}{2} \rho_c L \right| \omega \cos \theta}{\rho_o c_o} \right)^2} \quad (2.48)$$

In deriving this result, it was assumed that the frequency was sufficiently small so that $|Z_{sym}| \gg \frac{\rho c}{\cos \theta}$. This expression is the limp wall mass law transmission coefficient [1].

The coupling effects, seen in the result for the symmetric mode impedance at 1500 Hz and 6° incidence angle, are also evident in the transmission coefficient. The zero

of impedance corresponds to the peak in the transmission coefficient. The symmetric and antisymmetric mode impedances have equal values for a particular incidence angle near 6° . According to Eq. (2.15), when the impedances are of equal amplitude and phase, the transmission coefficient is zero. For incidence angles greater than 6° the impedances are of the same sign and the dip in the transmission coefficient is the result of the cancellation of symmetric and antisymmetric displacements at the bottom face sheet. The rise in the transmission coefficient for larger incidence angles is the result of the increasing value of $\rho_0 c_0 / \cos \theta$ in the numerator of Eq. (2.15).

$\rho_0 c_0 / \cos \theta$ is the impedance expression for the acoustic half space $x_3 < - (L+t_p)$. It also describes the radiation loading for both symmetric and antisymmetric motions of the composite. The increase in transmission coefficient is the result of a matching of the impedance of the acoustic half space with the impedance of the composite, including radiation loading effects. The impedance of the composite in this region is controlled by the *in vacuo* impedances of the composite. As θ further increases an impedance mismatch develops as the radiation loading terms dominate in the expression for the impedance of the composite and the transmission coefficient decreases. The denominator of Eq. (2.15) is increasing faster than the numerator.

At high frequencies the distinguishing features are determined by the locations of the zeroes of the impedances. These determine the locations of coincidence peaks in the transmission coefficient. At 4000 Hz the transmission coefficient peaks correspond to the zeroes of the symmetric and antisymmetric mode impedances near 60° and to the zero in the symmetric mode impedance near 16° . As frequency is changed the location of the transmission coefficient peaks shift in incidence angle according to:

$$\sin\theta = \frac{c_o}{c_{ph}} \quad \text{where } c_{ph} \text{ varies with frequency according to}$$

Figure 2.7. This applies for the transmission coefficient peaks corresponding to zeroes of the impedances associated with normal displacement resonances of the composite. The behavior of the peaks at 4000 Hz and 16° incidence angle and 1500 Hz and 6° with frequency is defined differently, not according to Figure 2.7.

The results, shown in Figure 2.8 b for frequencies in the region near 1300 Hz where the symmetric mode phase speed is greater than c_o display an interesting behavior. As frequency is increased to a value where the symmetric mode is resonant, the transmission coefficient increases in value at the incidence angle defined by:

$$\sin\theta = \frac{c_o}{c'_{ph}} \quad . \quad c'_{ph} \text{ is the symmetric mode phase speed at}$$

which the symmetric mode first becomes resonant (providing

this phase speed is greater than c_0). This is the case for the phase speed results of Figure 2.7 where c'_{ph} is approximately 437 m/sec, corresponding to an incidence angle of $\sim 52^\circ$.

For frequencies greater than the frequency for which the symmetric mode is first resonant, the symmetric mode phase speed is multi-valued. To the extent that both phase speeds are greater than c_0 two transmission coefficient peaks will exist. The greater of the two phase speeds will shift to smaller incidence angles approaching normal incidence as the phase speed becomes infinite. For frequencies greater than the frequency for which this phase speed becomes infinite, the transmission coefficient peak disappears. The transmission coefficient peak associated with the smaller phase speed shifts to higher incidence angles as the phase speed decreases. If the phase speed decreases to where it is less than c_0 the transmission coefficient peak shifts to an incidence angle of 90° and disappears until the phase speed again equals c_0 .

The effect of including internal damping on the motions of the core and face sheets is to reduce the transmission coefficient peaks. The mode impedances do not become zero at resonance as a result of damping. The effect of damping may be seen from Eq. (2.15) by describing Z_{asym} and Z_{sym} in terms of real parts (R_{sym} , R_{asym}), where the real parts are associated with damping, and imaginary parts

(X_{sym} , X_{asym}). For resonance conditions for the symmetric mode where the imaginary part of Z_{sym} , X_{sym} is zero the expression for $\tau(\theta)$ becomes:

$$\tau(\theta) = \left| \frac{\frac{\rho_o c_o}{\cos \theta} (R_{\text{sym}} - Z_{\text{asym}})}{\left(R_{\text{sym}} + \frac{\rho_o c_o}{\cos \theta}\right) \left(Z_{\text{asym}} + \frac{\rho_o c_o}{\cos \theta}\right)} \right|^2 \quad (2.49)$$

If $|Z_{\text{asym}}| \gg \frac{\rho_o c_o}{\cos \theta}$ and $|Z_{\text{asym}}| \gg R_{\text{sym}}$ the following simplified expression results:

$$\tau(\theta) = \frac{1}{\left(1 + \frac{R_{\text{sym}}}{\frac{\rho_o c_o}{\cos \theta}}\right)^2} \quad (2.50)$$

The effect of internal damping is insignificant except for the above conditions of resonant motion in either the symmetric or antisymmetric modes.

For the transmission coefficient peak at 4000 Hz and 16.5° incidence angle, which results from standing wave resonant motions in the core, the inclusion of internal damping in the core significantly reduces the transmission coefficient. The transmission coefficient peaks associated with resonant motions of the composite, at a frequency of 4000 Hz near 60° and 1500 Hz near 6° , are reduced by internal damping, but to a lesser extent.

2.5 TL Values Determined from Transmission Coefficient

A computer program was written to aid in evaluating the integral for $\bar{T}$ described by Eq. (2.8). The TL was evaluated according to Eq. (2.9). The resulting values for TL for the styrofoam core-plywood face sheet composite, as a function of frequency, are shown in Figure 2.9. At low frequency the transmission coefficient approaches the limp wall mass law form given by Eq. (2.48). The TL at low frequency accordingly approaches field incidence mass law transmission loss [1]. From Eq. (2.48) for values of ρ_p , ρ_c , ω and θ where $\tau(\theta) \ll 1$, $\tau(\theta)$ increases by a factor of 4 for a factor of 2 increase in either $(\rho_p + \frac{1}{2} \rho_c L)$ or frequency ω for fixed incidence angle. The TL increases by 6 dB for a factor of 2 increase in either total mass per unit area of the composite, $2(\rho_p + \frac{1}{2} \rho_c L)$ or frequency ω .

As frequency is increased the transmission peak associated with resonant excitation of the symmetric mode limits the increase in TL. A coincidence dip in the TL occurs where the phase speed for the symmetric mode is greater than c_o , that is, where the symmetric mode is coincidently excited. The extent to which the TL is reduced in this coincidence region depends on the internal damping.

For frequencies where the symmetric mode phase speed dips below c_o , according to Figure 2.7, there are no transmission coefficient peaks and the TL achieves large values.

The values for TL are seen to exceed levels predicted by the field incidence mass law TL. This is due to the cancellation of symmetric and antisymmetric mode normal displacements on the bottom face sheet. In this frequency region the excitation, as defined by $c_T = \frac{c_o}{\sin\theta} > c_{ph_{sym}}$, $c_{ph_{asym}}$, is such that the reactive components of the impedances will cancel in Eq. (2.15). The real components of impedances, associated with damping, will have the same sign for both modes and will also cancel in the expression for $\tau(\theta)$, Eq. (2.15).

As frequency is increased to where the phase speeds for both modes increase and become greater than c_o , transmission coefficient peaks again appear in $\tau(\theta)$. These again reduce the resulting value of TL. In that the phase speeds for both modes at these higher frequencies are determined principally by bending motions in the face sheets, the coincidence effect is referred to as the face sheet coincidence effect. Further increase in frequency results in an increase in TL for the composite panel that is very similar to the result that occurs for a single layer panel above coincidence [1]. The TL in this region is strongly dependent upon internal damping in that the transmission coefficient peak is controlled by damping.

The effects of internal damping on TL are principally restricted to the coincidence regions where transmission coefficient peaks occur. The effect on TL of standing wave resonant motions in the core and of extensional resonances in the face sheets are largely nonexistent. This is the result of the inclusion of internal damping and the fact that these phenomena produce very narrow transmission coefficient peaks and thus do not significantly contribute to the integral of Eq. (2.8).

2.6 Summary of Results

An analytical model of a composite panel of infinite extent with an isotropic core has been developed. The face sheets are considered to be deforming in bending and in extension. Impedance expressions for symmetric and antisymmetric motions in the composite were derived and phase speeds for symmetric and antisymmetric resonant motions, determined from the zeroes of the impedances, were obtained. The solution to the acoustic transmission problem was formulated in terms of the impedance expressions, and the effects of various resonant motions in the composite on the transmission of sound were qualitatively and quantitatively described. Coincidence dips in the TL were shown to exist in frequency regions where the phase speeds of the symmetric and antisymmetric modes were such that (if $c_{ph} > c_o$) these modes would be resonantly excited by an

incident acoustic plane wave. At low frequency the TL approached the usual limp wall mass law TL expression. It was seen that TL values greater than mass law were achievable where cancellations of symmetric and antisymmetric mode displacements on the bottom face sheet occurred.

As an additional note, the analysis presented in this chapter may be extended to apply to composite panels with additional layers in the core. The ability to do so depends only on the assumption that the additional layers are homogeneous isotropic elastic materials.

CHAPTER 3

COMPOSITE PANEL WITH A HOMOGENEOUS ORTHOTROPIC CORE

3.1 Introduction

In Chapter 2 an analysis was developed that described the acoustic behavior of a composite panel with a homogeneous isotropic core. In practice the kinds of materials used as cores in composite panels are not restricted to being isotropic. An example of a nonisotropic core material would be a cellular honeycomb material. In this chapter an analytical model is developed that predicts the acoustic behavior of a composite panel with a homogeneous orthotropic core. An orthotropic material is elastically symmetric with respect to three mutually perpendicular axes. A restriction on the development is that one of the principle axes of the orthotropic core correspond to the axis perpendicular to the plane of the composite.

The procedure followed in determining TL matches closely the procedures established by previous researchers. The analytical form for the particle displacements assumed for the core and face sheets are identical to those of other researchers, [7,8,9] and are linearly dependent on x_3 , the coordinate perpendicular to the plane of the composite. A general formulation for wave bearing motions in a layer of orthotropic material, similar to the

formulation of Chapter 2 for an isotropic material, was not known by the author. Particle displacements in the core, for the approach of Chapter 2, involve $\sin$, $\cos$, $\sinh$ and $\cosh$ terms that describe the dependence on the coordinate x_3 . These expressions, for low frequencies where the wavelength of both dilatational and shear waves is large compared to the thickness of the core, approach the linear distributions assumed in this chapter. The dependencies on the in-plane coordinates, x_1 , x_2 , are equivalent for the approaches of Chapter 2 and this chapter.

The work of the previous researchers will be extended to apply to orthotropic cores. For a composite panel with an orthotropic core the response to an incident acoustic plane wave will depend on the azimuthal angle ϕ , as well as the angle θ . The expressions for the symmetric and antisymmetric mode impedances and the transmission coefficient will include the dependence on ϕ and the transmission loss will be evaluated to include the integration in ϕ as described by Eq. (2.7).

3.2 The Elastic Stress-Strain Equations for a Homogeneous Orthotropic Material

The matrix of elastic stiffnesses that relates normal and shear strains to normal and shear stresses contains 21 independent stiffnesses for a general linear anisotropic

material. The additional symmetry of an orthotropic material reduces the number of independent stiffnesses to nine. Elastic symmetry with respect to three mutually perpendicular axes means that the elastic stiffnesses are invariant under the following transformation of axes:

$$\bar{x}_1 = -x_1, \quad \bar{x}_2 = -x_2, \quad \bar{x}_3 = -x_3 \quad (3.1)$$

where $\bar{x}_1, \bar{x}_2, \bar{x}_3$ are the transformed axes. The x_1, x_2, x_3 axes are the principle axes for the orthotropic material. The procedure for obtaining the transformed elastic stiffnesses is described by Stavsky and Hoff [10]. The stiffnesses that relate shear strain to normal stress or normal strain to shear stress are zero for an orthotropic material. The off diagonal stiffnesses relating shear strain to the shear stress in the two planes orthogonal to the plane of the shear strain are also zero. The fact that a stiffness is zero means that the strain produces a zero related stress.

The matrix of elastic stiffnesses for an orthotropic material has the following form [10]:

$$\{E\} = \begin{pmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 \\ E_{12} & E_{22} & E_{23} & 0 & 0 & 0 \\ E_{13} & E_{23} & E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{pmatrix} \quad (3.2)$$

The stress-strain relations are written as follows:

$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{pmatrix} = \{E\} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{pmatrix} \quad (3.3)$$

τ_1 , τ_2 , and τ_3 are the normal stresses in the x_1 , x_2 , and x_3 directions, respectively. The shear stresses τ_4 , τ_5 , τ_6 are defined as follows: τ_4 is the shear stress in the $\begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$ direction on the face perpendicular to direction $\begin{pmatrix} x_3 \\ x_2 \end{pmatrix}$; τ_5 is the shear stress in the $\begin{pmatrix} x_1 \\ x_3 \end{pmatrix}$ direction on the face perpendicular to direction $\begin{pmatrix} x_3 \\ x_1 \end{pmatrix}$, and τ_6 , the shear stress in $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ direction on the face perpendicular to direction $\begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$. The strains are defined with respect to the particle displacements u , v , w in the x_1 , x_2 , x_3 directions, respectively, as follows:

$$\begin{aligned} \epsilon_1 &= \frac{\partial u}{\partial x_1} \quad , \quad \epsilon_2 = \frac{\partial v}{\partial x_2} \quad , \quad \epsilon_3 = \frac{\partial w}{\partial x_3} \\ \epsilon_4 &= \frac{\partial v}{\partial x_3} + \frac{\partial w}{\partial x_2} \quad , \quad \epsilon_5 = \frac{\partial u}{\partial x_3} + \frac{\partial w}{\partial x_1} \quad , \quad (3.4) \\ \epsilon_6 &= \frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1} \end{aligned}$$

The meaning of the elastic stiffness may be explained by referring to the matrix $\{E\}$ for an isotropic material. For that case

$$\begin{aligned} E_{11} &= E_{22} = E_{33} = \lambda + 2\mu, \\ E_{12} &= E_{13} = E_{23} = \lambda, \\ E_{44} &= E_{55} = E_{66} = \mu, \end{aligned} \quad (3.5)$$

where λ , μ are the Lamé constants for the isotropic material. For an isotropic elastic material there are two independent stiffnesses λ , μ , whereas for an orthotropic material there are nine independent stiffnesses.

The stored elastic potential energy density, W , for a given strain field is:

$$\begin{aligned} 2W &= E_{11} \epsilon_1^2 + 2E_{12} \epsilon_1 \epsilon_2 + 2E_{13} \epsilon_1 \epsilon_3 + E_{22} \epsilon_2^2 \\ &\quad + 2E_{23} \epsilon_2 \epsilon_3 + E_{33} \epsilon_3^2 + E_{44} \epsilon_4^2 + E_{55} \epsilon_5^2 \\ &\quad + E_{66} \epsilon_6^2 \end{aligned} \quad (3.6)$$

The total elastic potential energy stored in a body of finite volume is computed as the integral of the potential energy density over the volume of the body.

$$\text{P.E.} = \int_{\text{Vol}} W \, d \, \text{Vol} \quad (3.7)$$

3.3 The Assumed Displacement Field and the Stored Potential Energy Density

Consistent with the assumptions of previous researchers, the displacements are written in symmetric and anti-symmetric forms. For the symmetric composite panels considered in this chapter symmetric and antisymmetric motions are uncoupled. The orientation of the composite with respect to the x_1, x_2, x_3 axes is described by Figure 2.1. The principle axes of the orthotropic core material are alligned such that x_1, x_2 lie in the plane of the composite and x_3 is perpendicular to the plane of the composite.

The displacements in two dimensions assume the following form for symmetric motions:

for the top face sheet (subscript "0")

$$\begin{aligned} u_0 &= \left(\beta_{\text{sym}} \cos k_1 x_1 - \left(x_3 - \frac{L}{2} \right) \frac{\partial w_0}{\partial x_1} \right) \\ v_0 &= 0 \end{aligned} \quad (3.8)$$

$$w_0 = \alpha_{\text{sym}} \sin k_1 x_1 .$$

For the core (subscript 2):

$$\begin{aligned} u_2 &= \left(\beta_{\text{sym}} + \zeta \cos \frac{\pi x_3}{L} \right) \cos k_1 x_1 \\ v_2 &= 0 \\ w_2 &= \frac{2\alpha_{\text{sym}}}{L} x_3 \sin k_1 x_1 . \end{aligned} \quad (3.9)$$

For the bottom face sheet (subscript 1)

$$u_1 = \left(\beta_{\text{sym}} \cos k_1 x_1 - \left(x_3 + \frac{L}{2} \right) \frac{\partial w_1}{\partial x_1} \right)$$

$$v_1 = 0 \quad (3.10)$$

$$w_1 = -\alpha_{\text{sym}} \sin k_1 x_1 .$$

The displacements in two dimensions for antisymmetric motions assume the following form:

For the top face sheet

$$u_o = \beta_{\text{asym}} \cos k_1 x_1 - \left(x_3 - \frac{L}{2} \right) \frac{\partial w_1}{\partial x_1}$$

$$v_o = 0 \quad (3.11)$$

$$w_o = \alpha_{\text{asym}} \sin k_1 x_1$$

For the core

$$u_2 = \frac{2\beta_{\text{asym}}}{L} x_3 \cos k_1 x_1$$

$$v_2 = 0 \quad (3.12)$$

$$w_2 = \alpha_{\text{asym}} \sin k_1 x_1 .$$

For the bottom face sheet

$$\begin{aligned}
 u_1 &= -\beta_{\text{asym}} \cos k_1 x_1 - \left(x_3 + \frac{L}{2} \right) \frac{\partial w_1}{\partial x_1} \\
 v_1 &= 0 \\
 w_1 &= \alpha_{\text{asym}} \sin k_1 x_1 .
 \end{aligned} \tag{3.13}$$

The assumed displacements allow for normal and extensional motions of the face sheets and the core. The displacements in the core are linearly dependent on x_3 except for symmetric displacements relating to ζ . These describe, in a limited way, wave-bearing motions in the plane of the core. The displacements of the core and face sheets are equal at the core-face sheet interfaces, $x_3 = \pm \frac{L}{2}$.

The strains corresponding to these displacements are computed according to Eq. (3.4). Consider symmetric motions:

for the top face sheet

$$\begin{aligned}
 \epsilon_1 &= \frac{\partial u_0}{\partial x_1} = -k_1 \left(\beta_{\text{sym}} - \left(x_3 - \frac{L}{2} \right) k_1 \alpha_{\text{sym}} \right) \sin k_1 x_1 \\
 \epsilon_2 &= \epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 0 .
 \end{aligned} \tag{3.14}$$

For the core

$$\begin{aligned}
 \epsilon_1 &= -k_1 \left(\beta_{\text{sym}} + \zeta \cos \frac{\pi x_3}{L} \right) \text{sink}_1 x_1 \\
 \epsilon_2 &= \epsilon_4 = \epsilon_6 = 0 \\
 \epsilon_3 &= \frac{2\alpha_{\text{sym}}}{L} \text{sink}_1 x_1 \\
 \epsilon_5 &= \left(\frac{2\alpha_{\text{sym}} x_3 k_1}{L} - \zeta \frac{\pi}{L} \sin \frac{\pi x_3}{L} \right) \text{cosk}_1 x_1 .
 \end{aligned} \tag{3.15}$$

for the bottom face sheet

$$\begin{aligned}
 \epsilon_1 &= -k_1 \left(\beta_{\text{sym}} + \left(x_3 + \frac{L}{2} \right) k_1 \alpha_{\text{sym}} \right) \text{sink}_1 x_1 \\
 \epsilon_2 &= \epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 0 .
 \end{aligned} \tag{3.16}$$

For antisymmetric motions the strains become:

for the top face sheet

$$\begin{aligned}
 \epsilon_1 &= -k_1 \left(\beta_{\text{asym}} - \left(x_3 - \frac{L}{2} \right) k_1 \alpha_{\text{asym}} \right) \text{sink}_1 x_1 \\
 \epsilon_2 &= \epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 0 .
 \end{aligned} \tag{3.17}$$

for the core

$$\begin{aligned}
 \epsilon_1 &= \frac{-2\beta_{\text{asym}} x_3 k_1}{L} \text{sink}_1 x_1 \\
 \epsilon_2 &= \epsilon_3 = \epsilon_4 = \epsilon_6 = 0 \\
 \epsilon_5 &= \left(\frac{2\beta_{\text{asym}}}{L} + k_1 \alpha_{\text{asym}} \right) \text{cosk}_1 x_1 .
 \end{aligned} \tag{3.18}$$

For the bottom face sheet

$$\begin{aligned}\epsilon_1 &= k_1 \left(\beta_{\text{asym}} + \left(x_3 + \frac{L}{2} \right) k_1 \alpha_{\text{asym}} \right) \sin k_1 x_1 \\ \epsilon_2 &= \epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 0\end{aligned}\quad (3.19)$$

The stored elastic potential energies for the core and face sheets for symmetric and antisymmetric displacements are evaluated from Eqs. (3.6) and (3.7). The potential energies for the face sheets are computed where the elastic stiffness matrix $\{E\}$, is written for an isotropic elastic material. The integral in Eq. (3.7) is over a volume defined, for the core, by a unit depth in the x_2 direction, the core thickness L in the x_3 direction and an integral number of wavelengths, λ_1 , in the x_1 direction. λ_1 is defined by $\lambda_1 = \frac{2\pi}{k_1}$. For the face sheets the integral is over the face sheet thickness, t_p , in the x_3 direction, the other limits being the same as for the core.

For the symmetric motions the stored potential energies become:

for the top face sheet:

$$\begin{aligned}PE_o &= \frac{1}{2} E_{11}^{fs} k_1^2 \left(\beta_{\text{sym}}^2 t_p - k_1 \beta_{\text{sym}} \alpha_{\text{sym}} t_p^2 \right. \\ &\quad \left. + \frac{k_1^2 \alpha_{\text{sym}}^2 t_p^3}{3} \right) \frac{\lambda_1}{2}\end{aligned}\quad (3.20)$$

For the core:

$$\begin{aligned}
 PE_2 = & \frac{E_{11} k_1^2 L}{2} \left\{ \beta_{sym}^2 + \frac{4\beta_{sym}}{\pi} + \frac{\zeta^2}{2} \right\} \frac{\lambda_1}{2} \\
 & - 2E_{13} \alpha_{sym} k_1 \left\{ \beta_{sym} + \frac{2\zeta}{\pi} \right\} \frac{\lambda_1}{2} \\
 & + \frac{2E_{33} \alpha_{sym}^2}{L} \frac{\lambda_1}{2} + \frac{E_{55}}{2} \left\{ \frac{\alpha_{sym}^2 k_1^2 L}{3} \right. \\
 & \left. - \frac{8\alpha_{sym} \zeta k_1}{\pi} + \frac{\zeta^2 \pi^2}{2L} \right\} \frac{\lambda_1}{2} .
 \end{aligned}$$

The expression for the bottom face sheet is identical to the expression obtained for the top face sheet. E_{11}^{fs} refers to the elastic stiffness for the face sheet which equals $\lambda_{fs} + 2\mu_{fs}$ for the isotropic face sheet material. The presence of λ_1 , means that the integration in x_1 is over one wavelength.

For antisymmetric motions the expressions for the stored potential energies are:

for the top face sheet:

$$\begin{aligned}
 PE_o = & \frac{E_{11}^{fs} k_1^2}{2} \left\{ \beta_{asym}^2 t_p - \beta_{asym} \alpha_{asym} k_1 t_p^2 \right. \\
 & \left. + k_1^2 \frac{\alpha_{asym}^2 t_p^3}{3} \right\} \frac{\lambda_1}{2} .
 \end{aligned} \tag{3.21}$$

For the core:

$$PE_2 = \frac{E_{11} k_1^2 \beta_{\text{asym}}^2 L}{6} \frac{\lambda_1}{2} + \frac{E_{55}}{2} \left\{ \frac{4 \beta_{\text{asym}}^2}{L} + 4 \beta_{\text{asym}} \alpha_{\text{asym}} k_1 + k_1^2 \alpha_{\text{asym}}^2 L \right\} \frac{\lambda_1}{2} .$$

The expression for the bottom face sheet is identical to that obtained for the top face sheet.

The expression for the potential energy of a face sheet for symmetric motions is of the same form as the expression for antisymmetric motions and involves, principally, potential energy in bending and extension. The expressions for the core differ substantially. The core deformations for symmetric motions principally involve a compression of the core in the x_3 direction, hence the presence of the term proportional to E_{33} in the potential energy. The core deformations for antisymmetric motions do not involve compressing the core. Deformations for the antisymmetric displacements involve shearing displacements leading to the presence of terms proportional to E_{55} . The potential energy in shear due to the antisymmetric normal displacement, α_{asym} , is three times larger than the similar term for symmetric normal displacements, α_{sym} . In addition, the potential energy in shear for antisymmetric displacements contains a term dependant upon extensional

displacement of the face sheets, β_{asym} . This term is significant in that antisymmetric extensional displacements are of opposite sign at the top and bottom face sheets leading directly to a shearing of the core.

3.4 Displacements as Functions of Time, Kinetic Energies, and Lagrange's Equation

The amplitudes of the assumed displacements α_{sym} , β_{sym} , ζ and α_{asym} , β_{asym} are assumed to be functions of time. Kinetic energy expressions for the face sheets and the core, for symmetric and antisymmetric motions are obtained from time derivatives of the above amplitudes:

$$\text{Kinetic Energy} = \frac{1}{2} \int_{\text{Vol}} \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) d\text{Vol} \quad (3.22)$$

where $\dot{u}$, $\dot{v}$, $\dot{w}$ are time derivatives of the particle displacements u , v , w .

For the symmetric displacements:

for the top face sheet:

$$\begin{aligned} \text{KE}_0 = \frac{\rho_p t_p}{2} \left\{ \dot{\beta}_{\text{sym}}^2 + \dot{\alpha}_{\text{sym}}^2 \left(1 + \frac{k_1^2 t_p^2}{3} \right) \right. \\ \left. - \dot{\beta}_{\text{sym}} \dot{\alpha}_{\text{sym}} k_1 t_p \right\} \frac{\lambda_1}{2} \quad (3.23) \end{aligned}$$

For the core:

$$\text{KE}_2 = \frac{\rho_c L}{2} \left\{ \dot{\beta}_{\text{sym}}^2 + \frac{4}{\pi} \dot{\beta}_{\text{sym}} \dot{\zeta} + \frac{\dot{\zeta}^2}{2} + \frac{\dot{\alpha}_{\text{sym}}^2}{3} \right\} \frac{\lambda_1}{2}$$

The expression for the kinetic energy of the bottom face sheet is identical to the expression for the top face sheet.

For the antisymmetric displacements:

for the top face sheet:

$$KE_o = \frac{\rho_p t_p}{2} \left\{ \dot{\beta}_{asym}^2 + \dot{\alpha}_{asym}^2 \left(1 + \frac{k_1^2 t_p^2}{3} \right) - \dot{\beta}_{asym} \dot{\alpha}_{asym} k_1 t_p \right\} \frac{\lambda_1}{2} .$$

For the core:

(3.24)

$$KE_2 = \frac{\rho_c L}{2} \left\{ \frac{\dot{\beta}_{asym}^2}{3} + \dot{\alpha}_{asym}^2 \right\} \frac{\lambda_1}{2} .$$

The Lagrange equation is:

$$\frac{d}{dt} \left(\frac{\partial KE_{TOT}}{\partial \dot{q}_r} \right) - \frac{\partial KE_{TOT}}{\partial q_r} + \frac{\partial PE_{TOT}}{\partial q} = Q_r$$

$$r = 1, 2, \dots$$

(3.25)

where KE_{TOT} , PE_{TOT} are the total kinetic and potential energies for the composite including both face sheets and the core. The q_r are the generalized displacements and correspond to α_{sym} , β_{sym} , ζ and α_{asym} , β_{asym} . $\dot{q}_r$ are the time derivatives of the generalized displacements. Q_r

describes the generalized forces per unit area acting on the system. If a normal force per unit area, $f_o \sin k_1 x_1$, is applied to the top face sheet, the virtual work is given as:

$$\int_{\text{Area}} f_o \sin k_1 x_1 \delta \alpha_{\text{sym}} \sin k_1 x_1 d\text{Area}$$

where $\delta \alpha_{\text{sym}} \sin k_1 x_1$ is the virtual displacement associated with the generalized displacement α_{sym} . The area over which the integration is performed is defined by a unit distance in the x_2 direction and an integral number of wavelengths λ_1 in the x_1 direction. The integral is evaluated to give $f_o \frac{\lambda_1}{2} \delta \alpha_{\text{sym}}$. A symmetric loading requires that a force per unit area of the same amplitude be applied to the bottom face sheet and the total virtual work for the symmetric displacements becomes $f_o \lambda_1 \delta \alpha_{\text{sym}}$. According to Lagrange's equation the generalized forces per unit area for symmetric and antisymmetric normal displacements, α_{sym} , α_{asym} , are both $f_o \lambda_1$.

Applying Lagrange's equation to the expressions for the kinetic and potential energies for symmetric motions, and letting the displacements have an $e^{-i\omega t}$ time dependence, results in the following equations:

$$\left\{ \begin{array}{l} \frac{2E_{33}}{L} + \frac{E_{55}k_1^2L}{6} + \frac{E_{11}^{fs}k_1^4t^3}{3} \\ - \frac{\rho_c \omega^2 L}{6} - \rho_p \omega^2 \left(1 + \frac{k_1^2 t^2}{3} \right) \\ -E_{13}k_1 - \frac{E_{11}^{fs}k_1^3 t^2}{2} \\ + \frac{\rho_p \omega^2 k_1 t}{2} \\ - \frac{2E_{13}k_1}{\pi} - \frac{2E_{55}k_1}{\pi} \end{array} \right\} \begin{array}{l} -E_{13}k_1 - \frac{E_{11}^{fs}k_1^3 t^2}{2} \\ + \frac{\rho_p \omega^2 k_1 t}{2} \\ E_{11}^{fs}k_1^2 t_p + \frac{E_{11}k_1^2 L}{2} \\ - \frac{\rho_c \omega^2 L}{2} - \rho_p \omega^2 \\ \frac{E_{11}k_1^2 L}{\pi} - \frac{\rho_c \omega^2 L}{\pi} \end{array} \begin{array}{l} - \frac{2E_{13}k_1}{\pi} - \frac{2E_{55}k_1}{\pi} \\ \frac{E_{11}k_1^2 L}{\pi} - \frac{\rho_c L \omega^2}{\pi} \\ \frac{E_{11}k_1^2 L}{4} + \frac{E_{55}\pi^2}{4L} - \frac{\rho_c L \omega^2}{4} \end{array} \left\{ \begin{array}{l} \alpha_{\text{sym}} \\ \beta_{\text{sym}} \\ \zeta \end{array} \right\} = \left\{ \begin{array}{l} f_o \\ 0 \\ 0 \end{array} \right\}$$

(3.27)

For antisymmetric motions:

$$\left\{ \begin{array}{l} \frac{E_{11}^{fs} k_1^4 t^3}{3} + \frac{E_{55} k_1^2 L}{2} \\ - \frac{\rho_c L \omega^2}{2} - \rho_p \omega^2 \left(1 + \frac{k_1^2 t^2}{3} \right) \\ - \frac{E_{11}^{fs} k_1^3 t^2}{2} + E_{55} k_1 \\ + \frac{\rho_p \omega^2 t}{2} k_1 \end{array} \right\} = \left\{ \begin{array}{l} - \frac{E_{11}^{fs} k_1^3 t^2}{2} + E_{55} k_1 \\ + \frac{\rho_p \omega^2 t}{2} k_1 \\ \cdot \\ E_{11}^{fs} k_1^2 t + \frac{E_{11} k_1^2 L}{6} + \frac{2E_{55}}{L} \\ - \rho_p \omega^2 - \frac{\rho_c L \omega^2}{6} \end{array} \right\} \left\{ \begin{array}{l} \alpha_{\text{asym}} \\ \beta_{\text{asym}} \end{array} \right\} \left\{ \begin{array}{l} f_o \\ 0 \end{array} \right\}$$

(3.28)

3.5 Dependence on the Angle ϕ

The above results were derived for the case where the principle axes of the orthotropic core corresponded to the coordinate system axes x_1, x_2, x_3 . The dependence of the assumed displacements on functions only of x_1 and x_3 implied that the case being considered was for an incident acoustic plane wave in the x_1, x_3 plane ($\phi = 0$). For other values of ϕ the incidence is in a vertical plane rotated about x_3 by an angle ϕ . The previously assumed displacements are now assumed to apply to particle displacements in the rotated plane. In the rotated plane, the elastic stiffnesses of the orthotropic core are different and the potential energies will also differ. The potential energy expressions for the isotropic face sheets are invariant to such a rotation of axes. The kinetic energy expressions for both the core and face sheets will remain the same.

The transformed elastic stiffnesses for an orthotropic material for a rotation about one of the principle axes may be expressed in terms of the stiffnesses for the principle axes and the direction cosines associated with the rotation [10].

$$\begin{aligned}\overline{E}_{11} &= \ell^4 E_{11} + 2\ell^2 m^2 E_{12} + m^4 E_{22} \\ \overline{E}_{13} &= \ell^2 E_{13} + m^2 E_{23} \\ \overline{E}_{33} &= E_{33}\end{aligned}\tag{3.29}$$

$$\bar{E}_{55} = m^2 E_{44} + \ell^2 E_{55}$$

where $\ell = \cos\phi$, $m = \sin\phi$ and the notation $\bar{E}_{ij}$ refers to the elastic stiffnesses in the rotated coordinate system. The potential energies for the core and the resulting equations have the same form in the rotated system where the values for the elastic stiffnesses for the core are replaced by the transformed expressions, Eqs. (3.29). In this way the dependence on ϕ is determined. It should be noted that the transformation need only be defined in the region $0 \leq \phi \leq \pi/2$. In the other quadrants the transformation is symmetric with respect to the transformation in the first quadrant. This occurs because of the symmetry an orthotropic material displays with respect to the three principle axes.

3.6 Symmetric and Antisymmetric Mode Impedances, Phase Speeds; the Transmission Coefficient and the Transmission Loss

Equations (3.27) and (3.28) may be solved to yield:

$$\begin{aligned} Z_{\text{sym}} &= \frac{f_o}{-i\omega\alpha_{\text{sym}}} , \\ Z_{\text{asym}} &= \frac{f_o}{-i\omega\alpha_{\text{asym}}} \end{aligned} \quad (3.30)$$

The impedances will depend on k_1 (and therefore, θ), and ϕ . From the zeroes of the impedances the phase speeds as functions of frequency are determined. The transmission coefficient is evaluated from the impedances according to Eq. (2.15). The transmission loss is evaluated from Eqs. (2.7), (2.8) and (2.9). These equations involve an average over the angle ϕ since the transmission coefficient depends on ϕ . The average need only be evaluated for values of ϕ from 0 to $\pi/2$ because of the symmetry of the orthotropic core. The $x_1 - x_3$ plane corresponds to $\phi = 0$ and the $x_2 - x_3$ plane corresponds to $\phi = \pi/2$.

The above procedure for computing TL may be applied to a composite panel with an isotropic core if the appropriate elastic stiffnesses are set equal to each other:

$$\begin{aligned} E_{11} &= E_{22} = E_{33} = \lambda_c + 2\mu_c; \\ E_{44} &= E_{55} = E_{66} = G_c, \\ E_{12} &= E_{23} = E_{13} = \lambda_c, \\ G &= \mu_c \end{aligned} \tag{3.31}$$

λ_c and μ_c are the Lamé constants for the isotropic core material. Figure 3.1 shows the symmetric and antisymmetric mode phase speeds for the styrofoam core - plywood face sheet composite panel. Values for the transmission loss are shown in Figure 3.2. These results closely agree

with the results of Chapter 2, Figure 2.9, for the styro-foam core - plywood face sheet composite panel. The differences between the model for the composite panel which allows for wave-bearing motions in the core and the model which assumes a linear dependence of displacements on the coordinate in the thickness direction of the core, are relatively insignificant with respect to the transmission loss of a styrofoam core - plywood face sheet composite panel.

3.6.1 The cellular honeycomb core - plywood face sheet composite panel

The expressions derived in this chapter are evaluated to predict the acoustic behavior of a composite panel with a cellular honeycomb core and plywood face sheets. The honeycomb material is fabricated from resin-impregnated and cured paper board (see Figure 3.3). The honeycomb material is assumed to be orthotropic; stiff in compression in one principle axis direction (x_3) and significantly softer in the other two principle axes directions (x_1 and x_2). The honeycomb material is soft in shear in the $x_1 - x_2$ plane and stiff in shear in the $x_1 - x_3$ and $x_2 - x_3$ planes.

Phase speeds for the symmetric and antisymmetric modes depend on the angle ϕ and are shown in Figure 3.4. The antisymmetric mode phase speed approaches the same low-

frequency and high-frequency bending phase speed limits as for the styrofoam core - plywood face sheet composite. The limits apply independent of the angle ϕ . The limits are determined by the properties of the face sheets and the thickness of the core, which are the same for the two composite panels. The mid-frequency phase speed depends on ϕ in that the shear stiffness in the $x_2 - x_3$ plane ($\phi = \pi/2$) differs from the shear stiffness in the $x_1 - x_3$ plane ($\phi = 0$). The phase speeds for the antisymmetric mode become greater than c_0 at relatively low frequency (~ 200 Hz) and exceed c_0 over the frequency range of interest.

The TL is limited as a result of coincidence effects for the antisymmetric mode. Figure 3.5 shows the predicted TL versus frequency. Values in the frequency region from 300 to 8000 Hz are less than field incidence mass law by ~ 14 dB as a result of the coincidence of the antisymmetric mode. The TL increases with frequency in this region at a rate of 6-7 dB per doubling of frequency.

For a single layer panel above coincidence, it may be shown that the TL increases at a rate of ~ 9 dB per doubling of frequency for a panel with a constant internal damping loss factor, η [6, 11]. The transmission loss assumes the form:

$$TL = TL_{fi} + 10 \log_{10} \left[\eta \left(\frac{c_{ph}^2}{c_0^2} \right) - 1 \right] + 3 \text{ dB} \quad (3.32)$$

where TL_{fi} is the field incidence mass law TL which increases at a rate of 6 dB per doubling of frequency. c_{ph} is the phase speed of a single layer panel in bending:

$$c_{ph} = \left(\frac{B}{\rho} \right)^{\frac{1}{4}} \sqrt{\omega} .$$
 For $c_{ph}^2/c_o^2 \gg 1$ the argument of the $\log_1$ in Eq. (3.32) is proportional to the frequency ω , which with the 6 dB increase per doubling of frequency associated with TL_{fi} , means that the TL increases at a rate of 9 dB per doubling of frequency. The phase speed for the anti-symmetric model of the honeycomb core composite panel in the frequency region above coincidence is dominated by shear deformation in the core and increases at a rate less than proportional to frequency. The TL would not, therefore, be expected to increase at 9 dB per doubling of frequency, but at a lesser rate.

The symmetric mode becomes resonant at high frequency (~ 7750 Hz) since the principle axis of honeycomb that is stiff in compression is oriented perpendicular to the plane of the composite. The effect of the symmetric mode coincidence effects is small as is seen from Figure 3.5. The coincidence peak in the transmission coefficient for the symmetric mode occurs for small angles θ , as $c_o/c_{ph} < 1$. The transmission coefficient peak is narrow and strongly affected by the inclusion of internal damping and therefore is not a dominant contribution to the integral for $\bar{\tau}$.

3.7 Summary of Results

An analytical model predicting TL for orthotropic core composite panels has been developed. The properties of the composite and therefore its acoustic behavior depend on the azimuthal angle ϕ . The model computes stored potential and kinetic energies for an assumed displacement field that is linearly dependent on the coordinate perpendicular to the plane of the composite. Wave-bearing effects in the core in this coordinate direction are excluded. The model was used to predict the acoustic behavior of an isotropic styrofoam core composite panel and the results were compared with the predictions of the wave-bearing model of Chapter 2. The differences in the results were relatively insignificant.

The properties of a composite panel with an orthotropic core were shown to depend on the azimuthal angle ϕ . The transmission loss was evaluated including the averaging over ϕ for a composite panel with a cellular honeycomb core and plywood face sheets. For the materials considered the symmetric mode resonance occurs at high frequency. The antisymmetric mode displayed coincidence effects for frequencies greater than 250 Hz. The coincidence effects occurred as a result of the relatively large values of the shear stiffness of the honeycomb material in the $x_1 - x_3$ and $x_2 - x_3$ planes. The predicted TL was limited by the coincidence effects of the antisymmetric mode. The TL

was less than field incidence mass law by ~14 dB for frequencies from 200 Hz to 8000 Hz. The behavior above coincidence was shown to be different from that of a single layer panel in that the TL increased with frequency at a rate less than for the single layer panel.

CHAPTER 4

EXPERIMENTAL PROCEDURES

4.1 Introduction and Description of the TL Facility

Transmission loss measurements were performed in the sound transmission facility at M.I.T. The facility consists of two adjacent reverberation rooms each with a volume of approximately 100m^3 and a surface area of 140m^2 . The test opening in the wall between the rooms, as originally constructed, was 9 ft. by 14 ft. The remaining wall area between the rooms was a double walled 10" thick brick construction, isolated from the floor and ceiling by cork. The common floor and ceiling between the two rooms was of 8 inch thick concrete. The walls were made of 8 inch thick hollow concrete blocks. The double doors to the rooms were made of 1-1/2 inch thick pine covered with sheet steel. The door openings measured 7'9" wide by 8 ft. high. In the back of each room there was an additional set of sheet metal covered pine doors measuring 2.5 ft. by 7 ft. The doors for both rooms opened onto common hallways, both front and back.

The test opening was modified to accommodate 4 ft. by 8 ft. test panels. The original opening was reduced with a four layer 1/2" thick gypsum board construction. The inner two layers were separated with 2" by 4" studs and the outer layers separated from the inner layers with 1" thick spacers.

The spaces in between the layers were partially filled with fiberglass. The frame around the test opening was 10" deep.

4.2 Transmission Loss Measurements

One of the reverberation rooms was used as the sending room, the other as the receiving room for acoustic energy. A speaker located in a corner of the sending room was used to excite the acoustic pressure field in that room. Sound was transmitted through the test panel to the receiving room. Spatial averages of mean square pressures in the sending and receiving rooms were used to compute the transmission loss according to:

$$TL = NR + 10 \log_{10} \left\{ \frac{S_w}{R} \right\} \quad (4.1)$$

The noise reduction NR is defined as:

$$NR = 10 \log_{10} \left(\langle P_{sr}^2 \rangle_{x,t} / \langle P_{rr}^2 \rangle_{x,t} \right)$$

where $\langle P_{sr}^2 \rangle_{x,t}$ and $\langle P_{rr}^2 \rangle_{x,t}$ are the space average mean square pressures in the sending and receiving rooms, respectively. R is the room constant for the receiving room and S_w the area of the test opening between the rooms. R is determined from the room volume V_r , the total wall surface area S_T , and the reverberation times T_r , all for the receiving room:

$$R = \frac{S_T \bar{\alpha}}{1 - \bar{\alpha}}$$

where

$$\bar{\alpha} = \frac{60V_r}{1.086 C_o S_T T_r} \quad (4.3)$$

is the average absorption coefficient for the absorbing surfaces of the receiving room [1]. The transmission loss computed in this way is equivalent to the TL defined by:

$$TL = 10 \log_{10} \left(\frac{1}{\tau} \right)$$

where (4.4)

$$\tau = \frac{\text{Transmitted Acoustic Power}}{\text{Incident Acoustic Power}}$$

This is true when the pressure fields in the reverberation rooms are diffuse.

The diffuseness of a sound field in a reverberation room is dependent upon the number of acoustic modes resonantly excited. For the 100m³ reverberation rooms with 1/3 octave bands of excitation there are approximately seven resonant modes in the 1/3 octave band with a center frequency of 100 Hz. As the 1/3 octave band center frequency is increased the number of modes increases proportional to f_c^3 so that there are many thousands of modes when $f_c = 1000$ Hz. In practice the number of modes generally required for the sound field to be sufficiently diffuse is 20. The 100 Hz and 125 Hz 1/3 octave bands do not satisfy this criterion and measurements in these bands should be treated with suspicion.

The procedures for measuring transmission loss using the sound transmission facility at M.I.T. were established by A. Rinsky in his doctoral thesis [12]. Figure 4.1 shows a schematic of the instrumentation chain used in measuring pressure levels in the sending and receiving rooms, for 1/3 octave bands of excitation. Five microphone locations in each room were used for measuring the space average mean square pressures yielding 95% confidence limits for TL measurements that were ± 2.5 dB for the lowest 1/3 octave bands, and less than ± 1 dB above 500 Hz [12]. The confidence limits depend on the absorption in the rooms and the rooms for these experiments were in approximately the same configurations as when they were tested by Rinsky. In order to insure the statistical independence of the measurements at the five microphone locations, the positions were spatially separated from each other and the nearest wall surfaces by $\sim 1/2$ the wave length of sound at the lowest frequency of interest, (a frequency of 100 Hz, a wavelength of 5 ft. or 1.7m). To avoid measurements in the direct field of the speaker the microphone locations were restricted to being ~ 5 ft. (1.7m) or more from the speaker. The relative humidity in the receiving room was controlled to be approximately 65% during the measurement of pressure levels.

4.2.1 Flanking transmission

Ideally, sound energy ought to transmit to the receiving room only through the test panel. In practice, energy is also transmitted to the receiving room via other flanking transmission paths, the floor, ceiling, the wall structure of the adjoining wall, doors and common hallways, etc. For accurate transmission loss measurements the energy in the receiving room that is transmitted through the test panel should be a factor of 10 greater than the energy from the flanking transmission paths. Flanking transmission limits use of the facility to panels which transmit sufficient energy in comparison with the energy from flanking paths.

To determine the flanking transmission a very high TL panel was placed in the test opening and the TL evaluated. This provides a lower bound on the flanking transmission TL, in that the acoustic energy in the receiving room may actually have come through the high TL panel and not via flanking transmission paths. The high TL panel for this experiment consisted of a five layer 3/4" thick gypsum board composite with fiberglass in the 1" thick spaces between adjacent gypsum board sheets. Each sheet of gypsum board was sealed around its edges with plasticene modelling clay to prevent the leakage of sound around the edges. This procedure was followed in sealing all the panels subsequently

tested. The resulting lower bound of flanking TL is shown in Figure 4.2. TL values for test panels which exceed or are within 10 dB of the results of Figure 4.2, are possibly contaminated by sound energy from flanking transmission paths. The flanking transmission experiments were performed by G. Chisholm as part of an undergraduate thesis project at M.I.T.

4.2.2 TL for 1/4" and 1/2" thick plywood sheets

To determine if the 10" depth of the frame of the test opening would affect the measurements of TL, single leaf sheets of plywood (1/4" and 1/2" thick) were tested for locations 1" and 4" from the receiving room edge of the test opening. The results are shown in Figures 4.3 and agree with predictions by Beranek [1]. At very low frequencies finite panel size effects result in deviations from the low frequency mass law transmission loss behavior. In the coincidence region the TL exhibits a plateau behavior that extends over a frequency range consistent with the stated range for plywood, from Beranek [1]. The plateau levels are approximately 3-5 dB greater than stated levels for plywood, perhaps as a result of additional damping provided by the modelling clay seals at the edges of the panels. At high frequencies, above the coincidence plateau region, the increase in TL is consistent with expectations. The results for the two locations in the frame agree

indicating that the 10" depth of the frame did not affect the measurements on the ~3-1/2" thick composite panels which are part of the experimental program of this thesis.

4.3 Radiation Efficiency Measurements

Sound is transmitted to the receiving room, neglecting flanking transmission, as a result of the induced motion of the test panel. The radiation efficiency σ_{rad} , by which the motion of the test panel radiates sound is defined as follows [1]:

$$\sigma_{\text{rad}} = \frac{I_{\text{rad}}}{\rho_0 c_0 \langle v^2 \rangle_{\underline{x}, t}} \quad . \quad (4.5)$$

I_{rad} is the radiated acoustic intensity, and $\langle v^2 \rangle_{\underline{x}, t}$ is the space average of mean square velocities on the panel. The intensity I_{rad} is written in terms of the total radiated power W_{rad} , radiated from a finite panel of area S_w as:

$$I_{\text{rad}} = \frac{W_{\text{rad}}}{S_w} \quad . \quad (4.6)$$

The radiation efficiency compares the average intensity radiated from a vibrating test panel to the intensity radiated by an infinite rigid piston moving with the same mean square velocity as the test panel.

For an infinite panel with a plane travelling wave velocity $v_0 e^{i(k_1 x_1 - \omega t)}$, the radiated intensity is zero when

the phase speed of the wave motions $c_{ph} = \omega/k_1$, is less than c_0 . The radiation efficiency is also zero. The pressure disturbances in the acoustic medium are confined to the region near the panel; there is no radiation of acoustic energy away from the panel. However, for phase speeds greater than c_0 a pressure disturbance, in the form of an acoustic plane wave, is radiated away from the panel. The intensity of the wave is:

$$I_{rad} = \frac{\rho_0 c_0 v_0^2}{2 \cos \theta} \quad (4.7)$$

θ is the angle with respect to direction perpendicular to the panel at which the acoustic plane wave is radiated.

θ is defined by $\sin \theta = \frac{k_1}{k_0} = \frac{c_0}{c_{ph}}$ where $k_0 = \frac{\omega}{c_0}$. The space average of mean square velocities is $v_0^2/2$ and the radiation efficiency becomes:

$$\sigma_{rad} = \frac{1}{\cos \theta} \quad (4.8)$$

When c_{ph} corresponds to the phase speed of bending waves, as frequency increases, c_{ph} increases proportional to $\sqrt{\omega}$, θ approaches zero, and the radiation efficiency approaches 1.

For a finite panel and for resonant modes below coincidence the radiation efficiency is non-zero as radiation of sound occurs from regions near the edges of the panel. The

theoretical radiation efficiency varies with frequency as described in Figure 4.4 [1], for a typical panel in bending. Above coincidence the result is consistent with the infinite panel result, Eq. (4.8).

Experimentally determined radiation efficiencies involve measurements of $\langle v^2 \rangle_{x,t}$ and W_{rad} . $\langle v^2 \rangle_{x,t}$ is determined from a space average of mean square accelerations on the panel. For purposes of statistical accuracy accelerations were averaged for, at least, five different accelerometer locations on the panel. Velocity averages in 1/3 octave bands are determined from the acceleration averages by dividing by ω_c^2 where ω_c is the center radian frequency of the 1/3 octave band.

$$\langle v^2 \rangle_{x,t} = \langle a^2 \rangle_{x,t} / \omega_c^2 \quad (4.9)$$

where $\langle a^2 \rangle_{x,t}$ is the space average of mean square accelerations. For composite panels with two face sheets, accelerations for the purpose of computing σ_{rad} were measured on the face sheet facing the receiving room. The instrumentation chain used in the measurement of accelerations is shown in Figure 4.1

The radiated power W_{rad} , in 1/3' octave bands is determined from the space average of mean square pressures in the receiving room as [1]:

$$W_{\text{rad}} = \frac{\langle p^2 \rangle_{x,t}}{\rho_o c_o} \left\{ \frac{4}{R} \right\} \quad (4.10)$$

In the course of measuring acceleration levels on the composite panels with 1/4" thick fir plywood face sheets, a problem developed involving a local resonance of the mass of the two gm accelerometer against a local stiffness of the plywood. The local stiffness of the plywood was apparently related to the poor quality of the outer ply of the plywood face sheet to which the 2 gm accelerometer was epoxied. The plywood was an interior-exterior variety where the quality of the outer ply differed substantially. In an attempt to increase the local stiffness by increasing the contact area, an 18 gm, 1" square by 3/8" thick steel block was epoxied to the plywood. The accelerometer was then epoxied to the steel block and the acceleration levels measured. Unfortunately, for the accelerometer on steel block configuration, the total mass was sufficient to affect the acceleration measurements at high frequencies. This problem also existed for the 2 gm accelerometer mounted directly on the plywood. Late in the thesis effort an accelerometer weighing only .1 gm (Wilcoxon model 91) was obtained. Comparative measurements of acceleration levels on the previously tested composite panels were made, it being assumed that the small size of the .1 gm accelerometer would not cause resonance or mass loading problems.

The relative acceleration levels from these experiments are shown in Figures 4.5. The difference in the sensitivities of the .1 gm and 2 gm accelerometers has been taken into account. For low frequencies the two accelerometers measured nominally the same levels. The 2 gm accelerometer on steel block configuration also yielded the same results as the .1 gm and the 2 gm accelerometers at low frequencies. At higher frequencies both mounting configurations on the poor quality outer ply exhibited resonance effects. The measured acceleration levels exceeded the actual levels as a result of the resonance of the mass of the accelerometer mounting system against the local stiffness of the poor quality ply of the plywood. The actual levels were assumed to be measured by the .1 gm accelerometer. Above the resonance the measured levels were less than the actual levels due to the mass of the accelerometer mounting systems. For the plywood with the higher quality ply facing outward, resonance effects were not seen. Instead, at high frequencies, the measured acceleration levels were less than the actual levels as a result of the added mass of the accelerometer mounting system. Acceleration data taken in computing radiation efficiencies was corrected according to Figures 4.5 to account for resonance and added mass effects.

4.4 Internal Damping and Reverberation Time Measurements

The procedure followed in measuring reverberation times, T_r , for the receiving room was a standard one. The instrumentation chain for the measurements is shown in Figure 4.6. Measurements were taken five different times for a 1/3 octave band and at a given microphone location, and averaged to determine T_r . The relative humidity was controlled to be 65% during the measurements.

Measurements of the internal damping in the composite panels involved measuring significantly shorter reverberation times than for the decay of acoustic energy in the receiving room. The instrumentation chain differed as is shown in Figure 4.6. The panel, in place in the test opening with its edges sealed with modelling clay, was excited by a shaker with one third octave bands of noise. The reverberation time was measured based on the initial rate of energy decay. The initial decay was limited as a result of the reverberant acoustic fields in the sending and receiving rooms that were excited by the shaker induced motion of the panel. Energy in the rooms decayed at a much slower rate than energy in the panel and the panel energy would decay rapidly to a level determined by the pressure levels in the reverberant rooms, and then decay at a rate determined by

the decay of the pressure levels in the reverberation rooms. In an attempt to reduce the reverberant pressure levels the doors were left open for these measurements and fiber-glass was added to the rooms.

Care was taken to insure that the response times of the equipment used in the measurements did not affect the results. The loss factor, η , for the composite panels were determined from the reverberation times according to:

$$\eta = \frac{2.2}{f_c T_r} \quad (4.11)$$

where f_c is the center frequency of the 1/3 octave band.

For the composite panels the measured loss factor η involves energy dissipated internally as well as energy dissipated in the radiation of acoustic energy away from both face sheets:

$$\eta = \eta_{\text{mech}} + 2\eta_{\text{rad}} \quad (4.12)$$

η_{mech} is the loss factor for energy dissipated internally and η_{rad} the loss factor for energy dissipated through acoustic radiation to one side of the panel. η_{rad} may be determined from the radiation efficiency of the composite panel [1]:

$$\eta_{\text{rad}} = \frac{\rho_o c_o \sigma_{\text{rad}}}{\rho_s \omega_c} \quad (4.13)$$

ρ_s is the total surface mass density for the composite panel.

4.5 Phase Speed Measurements

Beam sections were fabricated for the composite panel constructions tested in this thesis. The sections measured 8 ft long by 4" wide by 3-1/2" thick. The beam sections were excited into resonances with free-free boundary conditions. The mode shapes at resonance and the resonance frequencies were determined where the resonances were distinguishable. A schematic of the experimental set-up is shown in Figure 4.7.

For a free-free beam section deforming in bending, the resonant mode shapes are complicated expressions involving $\sin$, $\cos$, $\sinh$, and $\cosh$ terms. The distances between node points varies across the length of the beam. From the mode number n , which specifies the number of node points, the wavenumber k_n is determined [13]. From the resonance frequency ω_n and wavenumber k_n the phase speed $c_{ph} = \omega_n / k_n$, is determined. By repeating the process for various resonances a plot of c_{ph} versus frequency is obtained.

The analytically predicted phase speeds for the antisymmetric mode (see Figures 2.7 and 3.4) indicate that the composite panel deforms in bending in the limit of low frequencies and high frequencies. For frequencies in between, shear deformations in the core are important and the phase speed becomes nearly constant with frequency. A free-

free beam section deforming in shear would display different resonant mode shape characteristics from those for a beam section deforming in bending. The mode shapes will involve only sine or cosine terms. For a given mode number n the wave-number k_n will differ from the value determined for a beam section in bending. Hence, the phase speeds will differ for a resonance at frequency ω_n , depending on whether it is characterized by bending deformations or shear deformations. Motion in the composite panel involves both kinds of deformations and the actual phase speed lies between the results based on bending deformations and those based on shear deformations alone (see Figures 5.3 and 5.9). A decision as to the nature of the deformation was not possible based on a detailed description of the location of the node points. The symmetric mode phase speeds were not measurable as symmetric mode resonances first occurred for frequencies well above the point where the resonances became indistinguishable.

4.6 Elastic Stiffness Measurements and Results

The elastic stiffnesses of the various core materials were determined experimentally. The procedure used was to measure the resonance frequencies of a thin layer of the core material sandwiched between two rigid massive plates (see Figure 4.8). The resonance frequencies were expressible in terms of the masses of the identical plates, the core

thickness and the elastic stiffness of the core. The shear stiffness was determined from the resonance of the sandwich involving lateral, in-plane motions of the rigid plates. The stored potential energy in the layer is the result of shear deformation. The stiffness in compression was determined from the resonance of the sandwich involving motions of the plates normal to the plane of the plates. The thickness of the layer of core material was kept small with respect to a typical dimension of the contact area between the plates and the core material layer. The plates were either 6" square or 6" in diameter and the layer typically 1" thick.

For normal motion resonances the expression for the resonance frequency was:

$$\omega^2 = \frac{E_{11} A^2}{m_1 L} \quad (4.14)$$

where m_1 is the mass of one of the plates. A is the contact area between the plates and the core layer, and L is the thickness of the core layer. E_{11} is the stiffness in compression, which for an isotropic material equals $\lambda + 2\mu$. In deriving Eq. (4.14) the displacements in the core layer were assumed to be linearly dependent on x_3 and independent of x_1 and x_2 ; thus neglecting edge effects.

For shear resonances the expression for the resonance frequency, where the plates were cylindrical discs, was:

$$\omega^2 = \frac{G_c \left(1 + \frac{L}{d}\right)^2 A}{m_1 L \left(\frac{5}{8} + \frac{t_p^2}{3d^2}\right)}, \quad A = \frac{\pi d^2}{4} \quad (4.15)$$

where d is the diameter of the cylindrical discs and t_p is the thickness of the plates. The core layer was considered to be in a state of pure shear. The core displacements u , v , w in the x_1 , x_2 , x_3 directions, respectively, were assumed to be

$$\begin{aligned} u &= D_1 x_3 \\ v &= 0 \\ w &= \frac{L}{d} D_1 x_1 \end{aligned} \quad (4.16)$$

This implies that the displacements u , w of the plates at the edges, $x_1 = \pm \frac{d}{2}$, $x_3 = \pm \frac{L}{2}$, were equal. This was determined to be the case from experimental measurements. The potential energy for the core layer was evaluated in terms of D_1 and the shear modulus for the core, G_c . The kinetic energy for the plates was determined from the displacements in terms of D_1 . The expressions for the potential and kinetic energies were inserted into Lagrange's equation to yield the equation of motion for a single degree

of freedom oscillator with the resonance frequency given by Eq. (4.15).

For square plates the following expression for the shear resonance frequency was derived:

$$\omega^2 = \frac{3G_c \left(1 + \frac{L}{h}\right)^2 h^2}{2m_1 L \left(1 + \frac{t_p^2}{h^2}\right)} \quad (4.17)$$

where the plate dimensions are h by h by t_p thick.

The results of the above experiments for the isotropic styrofoam core material were:

$$\begin{aligned} \lambda_c + 2\mu_c &= 1.26 \times 10^7 \text{ N/m}^2 \\ G_c &= 4.0 \times 10^6 \text{ N/m}^2 \end{aligned} \quad (4.18)$$

from which Poisson's ratio is determined; $\nu_c = .27$. The styrofoam material was a common packing material with a density of 16 kg/m^3 . For the honeycomb core material where the principle axes are defined according to Figure 3.3; values for the stiffnesses are:

$$\begin{aligned} E_{33} &= 3.7 \times 10^7 \text{ N/m}^2 & E_{44} &= 7.8 \times 10^7 \text{ N/m}^2 \\ E_{22} &= 4.0 \times 10^6 \text{ N/m}^2 & E_{55} &= 3.6 \times 10^7 \text{ N/m}^2 \\ E_{11} &= \text{Assumed to be} & E_{66} &= 2.0 \times 10^5 \text{ N/m}^2 \\ &\text{equal to } E_{22} & & \end{aligned} \quad (4.19)$$

It was not possible to experimentally determine the stiffnesses E_{12} , E_{23} , and E_{13} . Values for these were assumed to be $.1 \times E_{22}$. Their effect on analytically predicted results was found to be small and so this assumption was not critical. The honeycomb material is a resin impregnated and cured paper board material. Its stiffnesses depend upon the base paper board, the percent of resin content, by weight, and the size of the hexagonal cells to which the honeycomb is expanded before curing. The material used was manufactured by Union Camp Corp. and was designated as a 99-11-3/4 honeycomb. The 99 refers to the base paper board weight, 11 is the percent of total weight that is resin and 3/4 is the cell size in inches. The density of the material was approximately 28 kg/m^3 .

CHAPTER 5

EXPERIMENTAL RESULTS

5.1 The Styrofoam Core-Plywood Face Sheet Composite Panel

A composite panel with 1/4" fir plywood face sheets and a 3" thick styrofoam core was fabricated and tested to provide experimental results for comparison with the predictions of Chapter 2. The styrofoam material was considered to be homogeneous and isotropic. It had a cellular structure where the cells were nearly spherical with a nominal diameter of 3/32". The panel measured 4' by 8' in area. The face sheets were glued to the styrofoam core using Elmers glue, a common organic glue. During the gluing process, care was taken to insure a uniform bonding over the area of the panel. The panel was placed on a flat machine bed and approximately uniformly loaded on top during the curing period.

5.1.1 TL results for the styrofoam core composite panel

TL measurements for the panel were performed according to the procedures described in Chapter 4. The resulting TL values in 1/3 octave bands are shown in Figure 5.1. Also plotted is the field incidence mass law TL for the panel. A comparison with Figure 4.2 for the flanking TL reveals that flanking transmission is possibly a contaminating factor for low frequencies, i.e., for 1/3 octave band center frequencies less than 250 Hz and for high frequencies

greater than 2000 Hz. Since the flanking transmission loss curve is a lower bound it is not meaningful to correct the TL results for flanking transmission. The analytical prediction from Chapter 2 is also plotted on Figure 5.1. The analytical result has not been averaged in frequency. The effect of averaging in frequency, in order to compare with 1/3 octave band data, is to round off the sharp coincidence dips found in the analytical results.

At low frequencies both the experimental and analytical results approximate field incidence mass law TL. For the lowest 1/3 octave band the experimental result deviates from mass law as a result of an insufficient number of room and panel modes. The insufficient number of room and panel modes is the result of the finite sizes of the room and the panel.

In the region from 800 - 1500 Hz, a coincidence dip associated with the symmetric mode resonance is present in the experimental and analytical results. The experimental result is broad with frequency while the predicted coincidence dip is much narrower. This is consistent with results obtained for finite size single layer panels near coincidence [1].

The symmetric mode phase speed, Figure 2.3.5, decreased to values less than c_0 for frequencies just above the coincidence dip. The absence of coincidence effects in the

analytical prediction causes the TL to increase rapidly as the phase speed becomes less than c_o . The experimental results exhibit this same behavior and the agreement between theory and experiment is good.

At frequencies near and above 2500 Hz a major difference between theory and experiment develops. The experimental results exceed the prediction by at least 10 - 15 dB over most of the range from 3150 Hz to 8000 Hz, taking into account that the predicted results are not averaged in frequency. The analytically predicted coincidence dip is associated with the increase in the phase speeds for both the symmetric and antisymmetric modes to values greater than c_o . The presence of transmission coefficient peaks associated with resonant excitation of both modes limits the TL and causes the coincidence dip. Experimentally a coincidence dip was not observed. The data at these frequencies was possibly contaminated by flanking transmission but this would imply that the actual TL was greater than the measured values, not less.

An explanation for the difference does not exist at present. A possible explanation may develop from the following considerations. The symmetric and antisymmetric modes in this region are principally controlled by the behavior of the face sheets. This is seen from the plots of phase speeds, which show that the results for both modes

approach a high frequency limit determined by the properties of the face sheets (see Figure 2.7). The phase speeds for both modes are nearly equal. For an ideal finite panel a symmetric mode resonance with the same mode shape as for an antisymmetric mode resonance is resonant at a frequency that is nearly equal to the resonance frequency of the antisymmetric mode. For each resonant symmetric mode in this frequency region there is a resonant antisymmetric mode with the same mode shape and approximately the same resonant frequency. An ideal panel means that the boundary conditions are ideal and that the material properties of the panel are homogeneous. A coupling of the motions of the two resonant modes will occur as a result of the internal as well as radiation damping. The symmetric and antisymmetric motions have opposite phase characteristics for motion of the top face sheet with respect to motion of the bottom face sheet. To the extent that the phase of the responses of the symmetric and antisymmetric modes to a common excitation is the same, motions at the bottom face sheet will cancel. The TL associated with the common excitation will increase as the degree of coupling increases. This behavior occurs for all resonant modes in this frequency region. The modes occur in pairs of symmetric and antisymmetric modes and are coincidentally excited by the incident sound field. For the infinite panel prediction the same cancellation occurs, as is seen

from the numerator of Eq. (2.15). To the extent that there are a sufficient number of resonant modes of vibration at these frequencies the analytical result for a composite panel of infinite extent should agree with the experimental result for a panel of finite size. The number of modes resonant in a 1/3 octave band centered at 3150 Hz (~200), based on the high frequency face sheet controlled phase speed limit, exceeds the generally required number (~20) by a substantial amount. The effects of any inhomogeneities in the materials used or the gluing have not been accounted for in the predictions.

5.1.2 Radiation efficiency results

Figure 5.2 shows experimentally obtained radiation efficiencies for the 3" thick styrofoam core panel. The results have been corrected to account for accelerometer mounting problems, (Section 4.3). The radiation efficiency increases with increasing frequency to a value of 0 dB near 1250 Hz. Resonant symmetric motions first occur in this frequency region and with phase speeds that exceed c_0 . Above this frequency the radiation efficiency decreases to values less than 0 dB which is associated with the decrease in the symmetric mode phase speed to values less than c_0 . The anti-symmetric mode phase speed is less than c_0 until near 3150 Hz when the phase speeds for both modes exceed c_0 . The radiation efficiency again increases to values slightly greater than 0 dB.

5.1.3 Phase speed results

Experimentally obtained phase speed results for the antisymmetric mode are shown in Figure 5.3. The value for the shear stiffness G_c used to analytically predict the phase speed was chosen so that the analytical results agreed with the experimentally obtained phase speeds. The value for G_c was 3.1×10^6 N/m². This value differs from the value obtained from the rigid plate styrofoam core sandwich experiments of Section 4.6.1. The above value was used in computations for TL because it was felt that it was a more accurate measure of the behavior of the styrofoam when fabricated in a composite panel with plywood face sheets.

Internal damping values for the 3" styrofoam core-plywood face sheet composite panel were determined from the half power or 3 dB down points of the resonances of the beam section. The internal damping η_c is approximately .03 in the frequency range from 100 Hz to 800 Hz. As the panel motion was principally controlled by shear deformation in the core, this value was assigned to the internal damping for the styrofoam in the analytical computations for TL. The value for internal damping for the plywood was taken to be $\eta_p = .02$ [1].

5.1.4 1-1/2" thick styrofoam core composite panel

A second styrofoam core panel was fabricated and tested. The thickness of the styrofoam core was reduced

to 1-1/2". TL values for this panel are shown in Figure 5.4. The analytical prediction for TL is also shown in Figure 5.4. At low frequency both the analytical and experimental results approach the field incidence mass law result. For the lowest frequency bands the effects of an insufficient number of panel modes or room modes are seen. The coincidence dip associated with the symmetric mode has been shifted to the 1/3 octave band centered at 2000 Hz. Decreasing the thickness of the core by a factor of 2 increases the resonance frequency for symmetric motion when $k_1 = 0$, $c_{ph} = \infty$ by a factor of $\sim\sqrt{2}$ according to Eq. (2.45). For the 3" thick styrofoam core panel, this frequency was approximately 1450 Hz. The frequency for which $c_p = \infty$ for the 1-1/2" for styrofoam core panel would therefore be ~ 2050 Hz. This is seen from Figure 5.5 which shows the analytically predicted phase speeds for the 1-1/2" styrofoam core panel. The symmetric mode first becomes resonant at 2050 Hz. This behavior differs from the 3" styrofoam core panel in that the result is not double valued for frequencies less than this resonance frequency. At high frequencies above the symmetric mode resonance the experimental TL does not appreciably show the effects of the face sheet coincidence. Experimental TL values are greater than predicted values by 6-8 dB.

Experimentally obtained radiation efficiencies are shown in Figure 5.6. The results are consistent with the analytically obtained phase speed results. The radiation

efficiency increases to a peak value of 4 dB where the symmetric mode becomes resonant, above which the radiation efficiency equals approximately 1 dB. The symmetric mode phase speed does not dip below c_0 as frequency is increased and the radiation efficiency therefore does not dip below 0 dB.

5.2 Honeycomb Core-Plywood Face Sheet Composite Panel

A composite panel with 1/4" fir plywood face sheets and a 3" thick honeycomb core was fabricated and tested. The results are compared to the results of Chapter 3. The honeycomb material used was a Union Camp Corp. honeycomb of grade 99-11-3/4 (see Section 4.6). The fabrication process was the same as that for the styrofoam core panels.

5.2.1 TL results for the honeycomb core composite panel

The results of the TL measurements are shown in Figure 5.7. A comparison with the flanking TL, Figure 4.2, shows that flanking transmission may be contaminating the data for the low frequency 1/3 octave bands below 250 Hz. The analytical prediction for TL is also shown on Figure 5.7 and agrees quite well with the experimental result. The TL is less than field incidence mass law TL as a result of the coincidence of the antisymmetric mode, as explained in Section 3.6.1. At low frequency an insufficient number of panel and room modes affect the data. The low frequency experimental results exceed the analytical prediction by approximately 7 dB.

5.2.2 Radiation efficiency results

The experimentally obtained radiation efficiency is plotted in Figure 5.8. The result increases to a 0 dB level near 250 Hz. This corresponds to the region where the analytically predicted antisymmetric mode phase speeds, for $\phi = 90^\circ$, exceed c_0 , Figure 3.5. For frequencies greater than 250 Hz values for the radiation efficiency are in the range from 1 to 3 dB except for the 6.3 kHz and 8 kHz 1/3 octave bands where the radiation efficiency decreases to below 0 dB. This behavior is considered to be anomalous probably resulting from accelerometer mounting problems.

5.2.3 Phase speed results

A beam section made with a honeycomb core and plywood face sheets was tested. The beam was 3-1/2" thick by 4" wide by 8 ft. long. The phase speeds were measured according to the procedures of Section 4.5. The orientation of the honeycomb with respect to the length of the beam was such that the results correspond to the $\phi = 0$ case. In analytically determining the phase speeds the value for shear stiffness E_{55} was chosen so that the predictions would agree with the experimentally obtained results, Figure 5.9. The value for E_{55} obtained in this way was less than the value obtained from the rigid plate-honeycomb core sandwich experiments (Section 4.6). The value for E_{44} used in the TL predictions was assumed to be in the same proportion to the rigid plate sandwich experimental result as for E_{55} . This

assumption was not made for the softer shear stiffness E_{66} in that the rigid plate sandwich experimental result agreed with a published value from the manufacturer. Values for the other stiffnesses were not available from the manufacturer.

5.2.4 Internal damping results

Loss factors for the honeycomb core-plywood face sheet composite panel are plotted in Figure 5.10. The panel was sealed around its edges with modelling clay for these experiments. In computing the TL the internal mechanical damping η_{mech} , for the core was assumed to lie between .05 and .03. The two analytical TL results plotted on Figure 5.7 are for $\eta_c = 0.5$ and .03. The result for high frequencies would correspond to the " $\eta_c = .03$ " results, while for lower frequencies the " $\eta_c = .05$ " result would apply. The internal damping in the core was set equal to the mechanical damping for the composite because the motion of the composite was principally controlled by shear deformation in the core. The internal damping for the plywood face sheets was taken to be $\eta_p = .02$ [1].

5.3 Summary

Four feet by eight feet composite panels with styrofoam and honeycomb cores and plywood face sheets were fabricated and tested. The TL results provided acceptable agreement with theoretical results with a notable exception. For the styrofoam panel for high frequencies where both symmetric

and antisymmetric modes become coincident, the experimental results significantly exceed the prediction. The predicted coincidence effects, which are principally determined by deformation in the face sheets for both modes, are not seen experimentally. The experimental results exceed the predicted levels by at least 10-15 dB for the 3" thick styrofoam core panel and 6-8 dB for the 1-1/2" thick styrofoam core panel. An explanation for this difference is not available and this represents an area where further work should be performed. The coincidence effects for the styrofoam core panels associated with the symmetric mode at low frequencies are found in both theory and experiment. The predicted result is more sharply defined with frequency than the experimental result.

Predictions for the honeycomb core composite panel agree well with the experimental results. The TL is limited by the coincidence of the antisymmetric mode over substantially the entire frequency range of interest. For this composite panel, the symmetric mode is resonantly excited at high frequency. The coincidence effects associated with the symmetric mode are relatively negligible.

For the panels with both types of core constructions finite panel size effects (i.e., an insufficient number of panel modes) and an insufficient number of room modes affected the experimental TL results at low frequencies.

The procedure for setting the values of internal damping in computing TL are, at best, approximate. The effect of damping is greatest in regions where coincidence exists. Damping acts to reduce coincidence peaks in the transmission coefficient, thereby increasing TL. The major discrepancy that developed between theory and experiment at high frequency for the styrofoam panels is not explained by inaccuracies in the value of internal damping used in the predictions. The values that would be required for the results to agree would be of the same order of magnitude as the discrepancy itself. It is not likely that the damping was underestimated by ~15 dB or even 6-8 dB.

CHAPTER 6

NEW COMPOSITE PANEL DESIGN FOR HIGH TL

6.1 Introduction

The analytical and experimental results from the previous chapters describe the acoustic behavior of two common composite panel configurations. Both panels tested have serious deficiencies in the principle speech bands from 500 to 2000 Hz. The deficiencies, in the form of low TL, were associated with two distinct coincidence phenomena. For the styrofoam core composite panel, it was the coincidence of the symmetric mode that was responsible for limiting the TL. For the honeycomb core composite panel coincidence effects associated with the antisymmetric mode limited the TL over a broad range of frequencies.

Two approaches aimed at improving the TL for a composite panel dealing with the antisymmetric mode have been developed. The "Shear Wall" as developed by B.G. Watters [2], is a composite panel in which the antisymmetric mode phase speed in the mid-frequency region is kept below c_0 by controlling the shear stiffness of the core material. The coincidence effects are shifted to higher frequencies and the TL in the mid-frequency region is mass law controlled. The symmetric mode resonance occurs at high frequency as the core material is stiff in

compression in the axis perpendicular to the plane of the panel. Applying this approach to improve the TL of the honeycomb core panel would mean reducing the shear stiffness of the honeycomb core material. Coincidence effects would be shifted to higher frequencies where the high frequency limit for the antisymmetric mode phase speed would predict the onset of the coincidence effects. The ability to shift the coincidence effects to higher frequency and thus extend the mass law TL behavior of the panel is restricted by the properties of the face sheets.

The other approach was the development of the "Coincidence Wall" by Warnaka, Holmer and Manning [3,4,6]. The behavior of a composite panel when the antisymmetric mode is coincident is strongly dependent upon the amount of internal damping. The greater the internal damping, the greater the TL. This approach utilizes this fact plus the fact that the TL above coincidence depends on the phase speed of the antisymmetric mode, see Eq. (3.32). The design procedure includes adding additional layers to the core that are highly viscous and significantly increase the internal damping of the composite, thus increasing the TL. An important part of the design is the optimization of the parameters of the various layers so that the damping layer has a maximal effect. The resulting designs tend to produce constructions that are

expensive to fabricate, in that they involve additional layers.

The approach to improving the TL of a composite panel developed here will involve considerations of the effects of both the antisymmetric and symmetric modes. The intent is to reduce the stiffness of the core in compression so that the symmetric mode resonance and associated coincidence effects are moved to frequencies below the principle speech bands. At the same time the shear stiffness of the core will be kept large enough to maintain the static load-bearing capacity of the composite. The shear stiffness will be kept small enough though so that the antisymmetric mode coincidence is shifted to high frequency, as with the shear wall. For the principle speech bands the antisymmetric mode and symmetric mode phase speeds are less than c_0 and the TL, in fact, exceeds mass law for reasons having to do with interactions between the modes.

6.2 A Suggested Design

The suggested design involves a modified honeycomb core material and plywood face sheets. The honeycomb is reoriented so that one of the two principle axes which is soft in compression is aligned perpendicular to the plane of the composite. This reduces the resonance frequency of the symmetric mode. The antisymmetric mode phase speed in the mid-frequency region strongly depends on the angle ϕ ,

as shear deformations for two perpendicular in-plane directions ($\phi = 0^\circ, 90^\circ$) involve one of the larger shear stiffnesses and the smallest shear stiffness. The orientation of the honeycomb with respect to the face sheets is shown in Figure 6.1. The grade of the honeycomb material used for the experiments in Chapter 5, was found to be too stiff. Phase speeds for the reoriented honeycomb core-plywood face sheet panel are shown in Figure 6.2. The phase speeds were predicted according to the model described in Chapter 3. For $\phi = 0$ the mid-frequency value for the antisymmetric mode phase speed exceeds c_0 near 250 Hz. The symmetric mode resonance occurs near 810 Hz. The coincidence effects associated with both modes would seriously limit the TL in the principle speech bands. The predicted TL is shown in Figure 6.3 and is seen to be dominated by coincidence effects over a broad range of frequencies.

To improve the behavior of a composite panel with this configuration, the elastic stiffnesses of the honeycomb are reduced. If the stiffnesses are reduced by a factor of three, the phase speeds for the composite panel are given in Figure 6.4. The density of the honeycomb material has been reduced to 20 Kg/m^3 from 27.7 Kg/m^3 . The antisymmetric mode phase speed for $\phi = 0$, the stiffer direction for shear deformation, now first exceeds c_0 near 2000 Hz and the symmetric mode resonance is lowered to near 470 Hz. In the

frequency range from 500 to 2000 Hz coincidence effects for either mode are not present. The TL for this hypothetical honeycomb core composite panel is shown in Figure 6.5.

Near 470 Hz, a coincidence dip associated with symmetric mode occurs. Near 3000 Hz a coincidence dip also exists that is broad in frequency and is the result of the coincidence of both the symmetric and antisymmetric modes. In the principle speech bands, the predicted TL is large and exceeds mass law TL by as much as 8 to 9 dB.

The fact that the predicted TL exceeds mass law TL is explained with reference to Eq. (2.15). In this region, the phase speeds for both modes for all angles ϕ are less than c_0 . For all coincidence angles ϕ , θ , the responses of both modes are mass controlled. The imaginary or reactive components of the impedances for both modes will have the same negative sign. The real components, associated with internal damping, have the same sign for all circumstances. In Eq. (2.15), a cancellation of the impedance expressions in the numerator occurs and τ is reduced in comparison with the mass law result. Physically, this means that both modes respond with similar phase to the incident and reflected acoustic plane waves and the motions of the symmetric and antisymmetric modes add out of phase on the bottom face sheet. The transmitted sound is reduced as a result of this cancellation of motions at the bottom face sheet.

Figure 6.6 shows computed values for $\tau(\theta)$. Also shown is the mass law transmission coefficient. The result for $\phi = 90^\circ$ shows that the transmission is less than mass law. The results for $\phi = 0^\circ$ shows the presence of zeroes in τ . The zeroes occur when the impedances for both modes are equal. These values for τ were obtained for the case of zero internal damping. The effect of damping is small in this region as neither of the modes is being resonantly or coincidently excited.

6.3 Static Load Bearing Capability of a Composite Panel of Finite Area

The suggested design involves softening the core material to achieve an improved acoustic behavior. The strength of the composite panel for a static loading becomes of serious concern. The response of the panel to a static load applied to the surface of a face sheet is evaluated from the model developed in Chapter 3. The terms corresponding to kinetic energies are set equal to zero. The generalized displacements and the applied force are no longer functions of time.

For a composite panel with finite size in two dimensions, as shown in Figure 6.1, it is necessary to extend the dependence of assumed displacements of Chapter 3 to include a dependence on the added dimension. The displacements are identical to those adopted by previous

researchers [7,8,9]. The same separation into symmetric and antisymmetric forms is performed. The displacements become:

For the symmetric mode:

top face sheet:

$$\begin{aligned} w_o &= \alpha_{\text{sym}} \sin k_1 x_1 \sin k_2 x_2 \\ u_o &= \beta_{\text{sym}} \cos k_1 x_1 \sin k_2 x_2 - \left(x_3 - \frac{L}{2} \right) \frac{\partial w_o}{\partial x_1} \\ v_o &= \gamma_{\text{sym}} \sin k_1 x_1 \cos k_2 x_2 - \left(x_3 - \frac{L}{2} \right) \frac{\partial w_o}{\partial x_2} \end{aligned}$$

bottom face sheet:

$$\begin{aligned} w_1 &= -\alpha_{\text{sym}} \sin k_1 x_1 \sin k_2 x_2 \\ u_1 &= \beta_{\text{sym}} \cos k_1 x_1 \sin k_2 x_2 - \left(x_3 + \frac{L}{2} \right) \frac{\partial w_1}{\partial x_1} \\ v_1 &= \gamma_{\text{sym}} \sin k_1 x_1 \cos k_2 x_2 - \left(x_3 + \frac{L}{2} \right) \frac{\partial w_1}{\partial x_2} \end{aligned} \quad (6.1)$$

core:

$$\begin{aligned} w_2 &= \frac{2\alpha_{\text{sym}}}{L} x_3 \sin k_1 x_1 \sin k_2 x_2 \\ u_2 &= \left(\beta_{\text{sym}} + \zeta_1 \cos \frac{\pi x_3}{L} \right) \cos k_1 x_1 \sin k_2 x_2 \\ v_2 &= \left(\gamma_{\text{sym}} + \zeta_2 \cos \frac{\pi x_3}{L} \right) \sin k_1 x_1 \cos k_2 x_2 \end{aligned}$$

For the antisymmetric mode:

top face sheet:

$$\begin{aligned} w_0 &= \alpha_{\text{asym}} \sin k_1 x_1 \sin k_2 x_2 \\ u_0 &= \beta_{\text{asym}} \cos k_1 x_1 \sin k_2 x_2 - \left(x_3 - \frac{L}{2} \right) \frac{\partial w_0}{\partial x_1} \\ v_0 &= \gamma_{\text{asym}} \sin k_1 x_1 \cos k_2 x_2 - \left(x_3 - \frac{L}{2} \right) \frac{\partial w_0}{\partial x_2} \end{aligned}$$

bottom face sheet:

(6.2)

$$\begin{aligned} w_1 &= \frac{2\alpha_{\text{asym}}}{L} x_3 \sin k_1 x_1 \sin k_2 x_2 \\ u_1 &= -\beta_{\text{asym}} \cos k_1 x_1 \sin k_2 x_2 - \left(x_3 + \frac{L}{2} \right) \frac{\partial w_1}{\partial x_1} \\ v_1 &= -\gamma_{\text{asym}} \sin k_1 x_1 \cos k_2 x_2 - \left(x_3 + \frac{L}{2} \right) \frac{\partial w_1}{\partial x_2} \end{aligned}$$

core:

$$\begin{aligned} w_2 &= \alpha_{\text{asym}} \sin k_1 x_1 \sin k_2 x_2 \\ u_2 &= \frac{2\beta_{\text{asym}}}{L} x_3 \cos k_1 x_1 \sin k_2 x_2 \\ v_2 &= \frac{2\gamma_{\text{asym}}}{L} x_3 \sin k_1 x_1 \cos k_2 x_2 \end{aligned}$$

The wave numbers k_1 and k_2 are now restricted to having specified values corresponding to the mode shapes of the finite panel.

$$k_1 = \frac{n\pi}{L_1}, \quad k_2 = \frac{m\pi}{L_2} \quad n = 1, 2, \dots, \quad m = 1, 2, \dots \quad (6.3)$$

The boundary conditions at the edges of the panel correspond to simple supports for motions normal to the plane of the composite and to a free boundary condition for extensional motions. The applied force per unit area on the top face sheet of the panel is expanded in a two dimensional Fourier series of the form:

$$f(x_1, x_2) = \sum A_{mn} \sin \frac{n\pi x_1}{L_1} \sin \frac{m\pi x_2}{L_2} \quad (6.4)$$

The generalized force for each m,n combination for both the symmetric and antisymmetric modes becomes:

$$Q_r = A_{mn} \times L_1 \times L_2 / 4 \quad (6.5)$$

The stored potential energy may be evaluated from the assumed displacements according to the procedures of Chapter 3. The integrals of the potential energy density are now for a volume defined by the panel dimensions L_1 , L_2 and the appropriate thicknesses. The expressions for the potential energy and the generalized force, with Lagrange's equation yields two matrix sets of equations for the displacements α_{sym} , β_{sym} , γ_{sym} , ζ_1 , ζ_2 , and α_{asym} , β_{asym} ,

γ_{asym} in terms of the amplitudes A_{mn} of the applied static loading. The 5 by 5 and 3 by 3 matrix equations are solved for α_{sym} and α_{asym} for each amplitude A_{mn} . The symmetric and antisymmetric mode normal displacements of the face sheets are written as sums of the form:

$$W_{\text{sym}} = \sum_n^{\infty} \sum_m^{\infty} \alpha_{\text{sym}}(n,m) \sin \frac{n\pi x_1}{L_1} \sin \frac{m\pi x_2}{L_2}$$

and (6.6)

$$W_{\text{asym}} = \sum_n^{\infty} \sum_m^{\infty} \alpha_{\text{asym}}(n,m) \sin \frac{n\pi x_1}{L_1} \sin \frac{m\pi x_2}{L_2}$$

The displacement for each face sheet is determined from the symmetric and antisymmetric displacements as:

$$\begin{aligned} \text{displacement of top face sheet} &= \\ &= (W_{\text{sym}} + W_{\text{asym}})/2 \\ \text{displacement of bottom face sheet} &= \\ &= (W_{\text{asym}} - W_{\text{sym}})/2 \end{aligned}$$

(6.7)

The displacements for the top and bottom face sheets were evaluated for a uniform static load of 40 lbs/sq ft on the top face sheet. The deflections at the center of a 4' by 8' panel with the softened and reoriented 99, 18, 3/4 honeycomb core and plywood face sheets are:

Deflection of the center of the:

	Top face sheet	Bottom face sheet
$\phi = 0^\circ$	.2 in	.197 in
$\phi = 45^\circ$	.05 in	.048 in
$\phi = 90^\circ$	.037 in	.035 in

(6.8)

The orientation given by the angle ϕ indicates the orientation of the in-plane principle axes of the honeycomb core with respect to the axes of the panel. The maximum deflection of the panel, the deflection at the center, is strongly dependent on this orientation. The deflections of the top and bottom face sheets are essentially equal indicating that the deflections of the symmetric mode are significantly less than those of the antisymmetric mode. From an evaluation of the terms in the potential energy expression for the antisymmetric mode, its stiffness was principally dependent upon the shear stiffness in the core, in particular the larger of two shear stiffnesses.

The stiffness of a rectangular panel is principally determined by the stiffer of the two in-plane directions. By orienting the honeycomb so that the vertical plane with the greater core shear stiffness is in the direction of the shorter panel dimension the stiffest configuration is achieved. This occurs for $\phi = 90^\circ$ as seen from Eqs. (6.8).

The deflection obtained is $\sim .04$ in for a total load on the top face sheet of the 4' by 8' panel of 1280 lbs.

6.4 Experimental Results for a Suggested Design

In an attempt to achieve the predicted results of Section 6.2 a composite panel was fabricated with $3/8$ " plywood face sheets and a honeycomb core. The honeycomb was a Union Camp Corporation material with a 99-11-1 grade specification. The resin content (11%) and the cell size (1") constituted the softest grade of honeycomb available from Union Camp Corporation. The 4' by 8' composite was fabricated using twelve honeycomb strips each measuring 8' by 4" with a nominal thickness of $3-5/16$ ". The strips were glued to the plywood face sheets using Elmer's glue with the orientation shown in Figure 6.7. Due to a non-uniformity in thickness over the length of the honeycomb strips as well as between strips, a substantial amount (12 lbs.) of Elmer's glue was used to insure an adequate bond between the strips and the face sheets.

The greater mass of the $3/8$ " thick plywood face sheets in comparison with the $1/4$ " thick plywood sheets was needed to insure that the symmetric mode coincidence dip occurred at sufficiently low frequency and that the antisymmetric mode phase speed for the stiff in shear, in-plane direction did not exceed c_0 until high frequency.

6.4.1 TL and radiation efficiency results

TL values for the panel are shown in Figure 6.8. Also plotted is the field incidence mass law TL for the panel. A comparison with the flanking TL reveals that flanking transmission probably affects the data over the entire frequency range. Radiation efficiencies were computed from measurements of vibration levels on the face sheet facing the receiving room, during the TL measurements. The results are shown in Figure 6.9. The computations for the radiation efficiency are affected by flanking transmission in the same way as the TL results. The vibration measurements were made using a 2 gm B&K accelerometer, attached directly to the plywood. An experiment was performed using the .1 gm accelerometer to evaluate possible effects of the mass of the 2 gm accelerometer. The 2 gm accelerometer correctly measured the acceleration levels over the frequency range of interest.

The original flanking transmission loss curve was measured just after the time when the modifications to the adjoining wall between the two reverberation rooms were made. They preceded the measurements of this section by approximately two years. The probable degradation with time, of the caulked seals in the modified section of the adjoining wall leads to an uncertainty as to the actual flanking TL.

To evaluate the effect of flanking transmission on the TL results, an additional experiment was performed in which the composite panel, located in the adjoining wall, was excited, not by the pressure field in the sending reverberation room, but by a shaker attached to the face sheet facing the sending room. Vibration levels on the receiving room face sheet and the pressure levels in the receiving room were measured and the radiation efficiencies computed according to the procedures of Section 4.3. The radiation efficiencies evaluated in this way are also shown on Figure 6.9. Flanking transmission does not affect the measurement of pressure levels in the receiving room in this experiment.

The variation of vibration levels over the surface of the face sheet was substantial, with the highest levels occurring near the point across from the drive point. Vibration levels varied over a range up to 20 dB with no appreciable dependence on frequency. To improve the statistical accuracy of the vibration levels, 10 measurement locations were used.

Assuming that the distribution of vibrational energy in the modes of the panel resulting from the shaker excitation is insignificantly different from the distribution resulting from the acoustic excitation during the TL measurements, the difference between the two sets of

radiation efficiency results is due to flanking transmission during the TL measurements. The TL results may then be adjusted to account for flanking transmission according to the differences in the radiation efficiencies. The resulting TL values are shown in Figure 6.10 and represent an optimistic estimate of the TL. The above adjustments would indicate that the flanking transmission was greater (flanking TL smaller) than that represented by Figure 4.2.

In the frequency range from 400 to 2000 Hz the TL values from Figure 6.10 exceed the mass law prediction by as much as 11 dB and by 7 to 8 dB on the average. At low frequencies, insufficient panel and room modes affect the data. The symmetric mode coincidence dip appears to occur near 160 Hz from the uncorrected TL results. At high frequencies the coincidence dip associated with the face sheets occurs near 3150 Hz. The coincidence frequency for 1/4" plywood sheets occurs near 3120 Hz. As the coincidence frequency is proportional to the reciprocal of the thickness of the sheets, the coincidence frequency for 3/8" thick plywood sheet should occur near 2080 Hz. The layer of Elmer's glue would affect the behavior of the face sheets in two ways. The additional weight (approximately 6 lbs. per face sheet) would tend to increase the coincidence frequency, while any additional stiffness that resulted from the layer of glue would tend to decrease the coincidence

frequency. Also, the properties of plywood vary from grade to grade.

The radiation efficiency determined from the shaker excitation experiment increases sharply near 200 Hz. The rise is associated with the coincidence of the symmetric mode in this region. The levels are roughly constant averaging from -3 to -6 dB until 1600 Hz when the radiation efficiency again increases. The increase continues, peaking near 4000 Hz. Above 4000 Hz the radiation efficiency decreases. In the broad range of frequency from 1600 Hz to 8000 Hz the radiation efficiency exceeds 0 dB, achieving a peak level of ~11 dB at 4000 Hz. This behavior is associated with the coincidence of both the symmetric and antisymmetric modes in this region.

6.4.2 Predicted TL for the panel

The stiffnesses of the 99-11-1 honeycomb material were experimentally determined according to the experiments of Section 4.6. The values for the stiffnesses where the honeycomb has the orientation shown in Figure 6.7 are:

$$E_{11} = \text{Assumed to be the same as } E_{33}$$

$$E_{22} = 2.37 \times 10^8 \text{ N/m}^2$$

$$E_{33} = 1. \times 10^6 \text{ N/m}^2$$

$$E_{44} = 4.6 \times 10^7 \text{ N/m}^2$$

$$E_{55} = 1.65 \times 10^5 \text{ N/m}^2$$

$$E_{66} = 2.4 \times 10^7 \text{ N/m}^2$$

$$E_{12} = E_{13} = E_{23} = .1 \times E_{33}$$

$$\rho_c = 20.8 \text{ Kg/m}^2$$

TL values were predicted from the procedures of Chapter 3 using the above values. The results did not agree with experimental results because the stiffness values used were too large. Coincidence effects occurred for frequencies not indicated by the experimental data. This discrepancy in the values for the stiffnesses was seen in comparing predicted and experimental results for the previous styro-foam and honeycomb core panel configurations. Stiffness values smaller than those predicted by the rigid plate sandwich experiments of Chapter 4 provided a better agreement between theory and experiment. The TL predicted from the smaller stiffnesses is also shown in Figure 6.10. The agreement in the region from 315-1600 Hz is acceptable. Below 315 Hz finite panel size effects are presumed to affect the data. Above 1600 Hz the discrepancy that occurred for the styrofoam core panels again occurs. The predicted levels are substantially less than the experimental levels. In addition, there is a discrepancy in the onset of the coincidence associated with the face sheets. The added mass of the glue was added to the mass of the face sheets in the predictions. Possible stiffening effects associated

with the glue and effects relating to the varying quality of the plywood were not accounted for.

6.4.3 Evaluation of TL results

The experimental TL results for the styrofoam, 99-18-3/4 honeycomb and the 99-11-1 honeycomb panels may be compared in terms of their STC ratings. The Sound Transmission Class (STC) rating is based on the standard frequency curve, Figure 6.11, and involves the TL values in the range from 125 Hz to 4000 Hz. The standard frequency curve is overlaid on the experimental TL curve and moved upward until the standard curve exceeds the experimental TL in any one third octave band by not more than 8 dB. The sum of the differences between standard and experimental curves may not exceed 32 dB, including only differences where the standard curve exceeds the experimental result. The standard curve is repositioned downward until this restriction on the sum is satisfied. The STC rating is read off the ordinate according to the pointer, with the standard frequency curve appropriately positioned.

The following ratings are obtained for the three panels tested:

	STC	Mass Law TL @ 100 Hz	Mass per Unit Area of Panel lbm/ft ²
3" styrofoam core - 1/4" plywood face sheets	20	11.5 dB	1.8
1-1/2" styrofoam core - 1/4" plywood face sheets	19	11.5 dB	1.8
3" 99-18-3/4 honeycomb core - 1/4" plywood face sheets	22.5	13.0 dB	2.1
3-5/16" 99-11-1 honeycomb core - 3/8" plywood face sheets (corrected)	41	15.0 dB	2.7
3-5/16" 99-11-1 honeycomb core - 3/8" plywood face sheets (uncorrected)	35	15.0 dB	2.7

The STC ratings for the styrofoam core panels are limited by the coincidence dips and the 8 dB requirement. The STC rating for the 99 18 3/4 honeycomb core panel is limited by coincidence effects occurring over a broad frequency range and the 32 dB requirement. The rating for the 99-11-1 honeycomb core composite panel exceeds the ratings for the other panels by about 18 to 22 dB, which represents a substantial improvement over the other panels. The difference in the weights of the panels represents at most a 3 to 5 dB shift in the comparison.

6.5 Summary

A new design procedure for three layer composite panels was suggested and an actual panel tested. The design

procedure was based of shifting the symmetric mode coincidence dip to low frequencies where it is of less concern. Coincidence effects associated with the anti-symmetric mode were delayed until high frequencies by controlling the shear stiffness of the core to keep the antisymmetric mode phase speed less than c_0 . At high frequencies the properties of the face sheets determine the onset of coincidence. In the frequency region between the symmetric mode coincidence and the face sheet coincidence both the symmetric and antisymmetric modes are mass controlled. Neither is resonantly excited by an incident acoustic plane wave as the phase speeds for both are less than c_0 . Because both modes respond with similar phase to the acoustic excitation, a cancellation of motions at the bottom face sheet occurs. As a result, the predicted TL exceeds mass law. The TL for either mode alone would be mass law controlled.

The load bearing capability of a panel of finite area was evaluated and seen to depend on the shear stiffness of the core material. Symmetric deflections resulting from static loads on a panel simply supported on all edges, were significantly less than antisymmetric deflections. Computed deflections indicated the potential usefulness of such a panel in situations where load bearing capability is not of extreme concern.

Experimental results for a modified honeycomb core composite panel with 3/8" thick plywood face sheets indicated that the computed behavior was achievable from a relatively simple design configuration. The STC rating (STC 41) exceeded the ratings of the other panels of similar construction tested in the thesis by about 20. It is recommended that as a result of flanking transmission problems the TL for the 99-11 -1 honeycomb core panel be redetermined under more favorable circumstances as regards flanking transmission.

Figure 2.1. Geometry for the incident and radiated acoustic plane waves and a composite panel of infinite extent.

Symmetric Mode

Antisymmetric Mode

Figure 2.2. Symmetric and antisymmetric motions and forces per unit area of a composite panel.

Figure 2.3a. Coefficient relating normal stress to normal displacement at the same interface.

Figure 2.3b. Coefficient relating normal stress to normal displacement at the opposite interface.

Figure 2.3c. Coefficient relating shear stress to extensional displacement at the same interface.

Figure 2.3d. Coefficient relating shear stress to extensional displacement at the opposite interface.

Figure 2.3e. Coefficient relating $\left\{ \begin{smallmatrix} \text{normal} \\ \text{shear} \end{smallmatrix} \right\}$ stress to $\left\{ \begin{smallmatrix} \text{extensional} \\ \text{normal} \end{smallmatrix} \right\}$ displacement at the same interface.

Figure 2.3f. Coefficient relating $\begin{Bmatrix} \text{normal} \\ \text{shear} \end{Bmatrix}$ stress to $\begin{Bmatrix} \text{extensional} \\ \text{normal} \end{Bmatrix}$ displacement at the opposite interface.

Figure 2.4. Core displacements for normal and extensional displacements of the face sheets.

Figure 2.5a. Symmetric mode impedance for the isotropic styrofoam core plywood face sheet composite panel. For core parameters see Figure 2.3a.

Figure 2.5b. Antisymmetric mode impedance.

Figure 2.6. Two degree of freedom lumped spring-mass oscillator.

Figure 2.7. Phase speeds for the isotropic styrofoam core plywood face sheet composite panel.

Figure 2.8a. Transmission coefficient for the styrofoam core plywood face sheet composite panel. For parameters see Figures 2.3a, 2.5a.

Figure 2.8b. Transmission coefficient. For parameters see Figures 2.3a, 2.5a.

Figure 2.9. Predicted TL for the styrofoam core plywood face sheet composite panel.

Figure 3.1. Phase speeds for the isotropic styrofoam core composite panel.

Figure 3.2. Predicted TL for the isotropic styrofoam core composite panel.

Figure 3.3. Photograph of the 99-18-3/4 honeycomb material.

Figure 3.3. Photograph of the 99-18-3/4 honeycomb material.

Figure 3.4. Phase speeds for the orthotropic honeycomb core composite panel.

Figure 3.5. Predicted TL for the orthotropic honeycomb core composite panel.

Figure 4.1. Instrumentation schematic for the measurement of TL.

Figure 4.2. Flanking transmission loss for the sound transmission facility.

Figure 4.3. TL results for 1/4" and 1/2" thick plywood sheets.

Figure 4.4. Theoretical radiation efficiency for a 4' by 8' by 1/4" thick plywood sheet.

Figure 4.5a. Comparative acceleration levels from 2 gm and .1 gm accelerometers mounted on plywood, for the 3" thick styrofoam core panel.

Figure 4.5b. Comparative acceleration levels on plywood for the 1-1/2" thick styrofoam core panel.

Figure 4.5c. Comparative acceleration levels on plywood for the 3" thick honeycomb core panel.

Figure 4.6. Instrumentation schematic for the measurement of reverberation times and internal damping.

Figure 4.7. Instrumentation schematic for the measurement of phase speeds from the resonances of a beam section.

Figure 4.8. Sketch of rigid massive plates-core material sandwich for measurement of elastic stiffnesses.

Figure 5.1. Experimental TL results for the 3" thick styrofoam core composite panel.

Figure 5.2. Radiation efficiency for the 3" thick styrofoam core composite panel.

Figure 5.3. Phase speeds for the styrofoam core beam section.

Figure 5.4. TL results for the 1-1/2" thick styrofoam core composite panel.

Figure 5.5. Predicted phase speeds for the 1-1/2" thick styrofoam core composite panel.

Figure 5.6. Radiation efficiency for the 1-1/2" thick styrofoam core composite panel.

Figure 5.7. TL results for the 3" thick 99-18-3/4 honeycomb core composite panel.

Figure 5.8. Radiation efficiency for the 99-18-3/4 honeycomb core panel.

Figure 5.9. Phase speeds for the 99-18-3/4 honeycomb core beam section.

Figure 5.10. Loss factors for the 99-18-3/4 honeycomb core panel.

Figure 6.1. Sketch of a reoriented 99-18-3/4 honeycomb core composite panel.

Figure 6.2. Phase speeds for a reoriented 99-18-3/4 honeycomb core composite panel.

Figure 6.3. TL for a reoriented 99-18-3/4 honeycomb core composite panel.

Figure 6.4. Phase speeds for a softened and reoriented 99-18-3/4 honeycomb core composite panel.

Figure 6.5. TL for a softened and reoriented 99-18-3/4 honeycomb core composite panel.

Figure 6.6a. Transmission coefficient for the softened and reoriented 99-18-3/4 honeycomb core composite panel, $\phi=0^\circ$.

Figure 6.6b. Transmission coefficient for a softened and reoriented 99-18-3/4 honeycomb core composite panel, $\phi=90^\circ$.

Figure 6.7. Photograph of the soft 99-11-1 honeycomb core - 3/8" thick plywood face sheets composite panel.

Figure 6.7. Photograph of the soft 99-11-1 honeycomb core, $\frac{3}{8}$ " thick plywood face sheets composite panel.

Figure 6.8. TL results for the soft 99-11-1 honeycomb core composite panel.

Figure 6.9. Radiation efficiency from TL experiments and shaker excitation experiments.

Figure 6.10. TL corrected for flanking transmission based on radiation efficiency results.

Figure 6.11. Standard frequency curve for STC rating of TL results.

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BIOGRAPHICAL NOTES

The author was born on October 27, 1945 and was raised on Long Island, New York. He attended undergraduate school at Rensselaer Polytechnic Institute in Troy, New York, receiving a Bachelors of Science in Mechanical Engineering in June 1967. From 1967 to 1969 he attended M.I.T. receiving the degree of Master of Science in Mechanical Engineering in June 1969. His thesis was in the area of the response of panels to turbulent boundary layer pressure fluctuations and under Prof. Patrick Leehey. From 1969 to 1971 he worked at Bolt Beranek and Newman on a wide variety of problems relating to noise control techniques on submerged vessels, and also problems relating to the more traditional acoustics in air. In September 1971 he reentered M.I.T. working on his PhD with Prof. Richard Lyon.