Cooperative Routing in Wireless Networks

by

Amir E. Khandani

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Science Master in Electrical Engineering and Computer Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 2004

Author . . . .
Department of Electrical Engineering and Computer Science
May 7, 2004

Certified by . . . .
Eytan Modiano
Associate Professor
Thesis Supervisor

Certified by . . . .
Lizhong Zheng
Assistant Professor
Thesis Supervisor

Certified by . . . .
Jinane Abounadi
Postdoctoral Lecturer
Thesis Supervisor

Accepted by . . . .
Arthur C. Smith
Chairman, Department Committee on Graduate Students

Cooperative Routing in Wireless Networks

by

Amir E. Khandani

Submitted to the Department of Electrical Engineering and Computer Science
on May 7, 2004, in partial fulfillment of the
requirements for the degree of
Science Master in Electrical Engineering and Computer Science

Abstract

In this thesis, we study the problem of energy efficiency and reliability in wireless ad-hoc networks. First, we introduce the idea of wireless cooperation advantage. We formulate the problem of finding the minimum energy cooperative route for a wireless network under idealized channel and receiver models. Fundamental to the understanding of the routing problem is the understanding of the optimal power allocation for a single message transmission between two sets of nodes. We present the solution to this problem, and use that as the basis for solving the minimum energy cooperative routing problem. We analytically obtain the energy savings in regular line and regular grid networks. We propose heuristics for selecting the cooperative route in random networks and give simulation results confirming significant energy savings achieved through cooperation.

In the second part, we study the problem of route reliability in a multi-hop network. We look at the reliability issue at the link level and extend those result to a wireless network setting. In the network setting, we first define and analyze the reliability for a fixed route and then propose algorithms for finding the optimal route between a source-destination pair of nodes. The relationship between the route reliability and consumed power is studied. The idea of route diversity is introduced as a way to improve the reliability by taking advantage of the broadcast property, the independence of fading state between different pairs of nodes, and space diversity created by multiple intermediate relay nodes along the route. We give analytical results on improvements due to route diversity in some simple network topologies.

Thesis Supervisor: Eytan Modiano
Title: Associate Professor

Thesis Supervisor: Lizhong Zheng
Title: Assistant Professor

Thesis Supervisor: Jinane Abounadi
Title: Postdoctoral Lecturer

Acknowledgments

I would like to thank my advisor Professor Eytan Modiano for his research example, tremendous support, mentorship and positive impact on me life. He made it possible for me to come to MIT and his guidance made my research at MIT enjoyable. I greatly appreciate his openness, friendship and understanding. I would like to thank Professor Lizhong Zheng who kindly accepted to be my co-advisor. His technical insights and encouragements have been a great source of motivation for my work. I feel lucky to have the opportunity to know him and to interact with him. I would like to thank Dr. Jinane Abounadi who was always a source of inspiration and who patiently helped me with all aspects of my research. I very much appreciate all her assistance and constructive comments. I would also like to thank Professor David Forney, my academic advisor, for his encouragement to explore all opportunities available to me at MIT and for his contribution to my success.

My friends at MIT made the last two years memorable: Ali, Rouz, Andrew, Emmanuel, Murtaza, Sashi, Jun, Alex, Anand, Cemal, and all other people I have come to know in Boston.

I would also like to thank my family whose support I have enjoyed throughout the years.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In this thesis, we study the problem of reliability and energy efficiency in wireless ad-hoc networks. In an ad-hoc network, nodes often spend most of their energy on communication [1]. In most applications, such as sensor networks, nodes are usually small and have limited energy supplies. In many cases, the energy supplies are non-replenishable and energy conservation is a determining factor in extending the life time of these networks. For this reason, the problem of energy efficiency and energy efficient communication in ad-hoc networks has received a lot of attention in the past several years. This problem, however, can be approached from two different angles: energy-efficient route selection algorithms at the network layer or efficient communication schemes at the physical layer. While each of these two areas has received a lot of attention separately, not much work has been done in jointly addressing these two problems. Our analysis in this thesis tackles this less studied area.

Motivated by results from propagation of electromagnetic signals in space, the amount of energy required to establish a link between two nodes is usually assumed to be proportional to the distance between the communicating nodes raised to a constant power. This fixed exponent, referred to as the path-loss exponent, is usually assumed to be between 2 to 4. Due to this relationship between the distance between nodes and the required power, it is usually beneficial, in terms of energy savings, to relay the information through multi-hop route in an ad-hon network. Multi-hop routing extends the coverage by allowing a node to establish a multi-hop route to communicate with nodes that would have otherwise been outside of its transmission range. Finding the minimum energy route between two nodes is equivalent to finding the shortest path in a graph in which the cost associated with a link between two nodes is proportional to the distance between those nodes raised to the path-loss exponent. Figure 1-1 shows an example of a multi-hop route between two nodes.

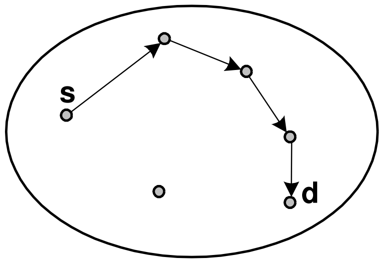

Figure 1-1: Multi-hop Relaying

The problem becomes more interesting once some special properties of the wireless medium are taken into account. In particular, there are three properties of the wireless physical layer that have motivated our work in this thesis: the wireless broadcast property, the benefits of transmission side diversity, and multi-path fading.

A wireless medium is a broadcast medium in which signal transmitted by a node is received by all nodes within the transmission radius. For example, in figure 1-2, the signal transmitted by s is received by both nodes 1 and 2. This property, usually referred to as the *Wireless Broadcast Advantage* (WBA), was first studied in a network context in [3]. Clearly, this property of the wireless physical medium significantly changes many network layer route selection algorithm. The problem of finding the minimum energy multi-cast and broadcast tree in a wireless network is studied in [3] and [4]. This problem is shown to be NP-Complete in [5] and [6]. WBA also adds substantial complexity to route selection algorithms even in non-broadcast scenarios. For example, this model is used in [7] in the context of selecting the minimum energy link and node disjoint paths in a wireless network.

Another interesting property of the wireless medium is the benefit of space diversity at the physical layer. This type of diversity is achieved by employing multiple antennas on the transmitter or the receiver side. It is well known that transmission

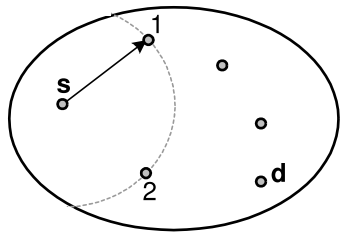

Figure 1-2: Wireless Broadcast Advantage

side diversity, i.e. using multiple antennas on the transmitter, results in significant energy savings (see [2]). In the network setting studied in this thesis, we assume that each node is only equipped with a single antenna. Hence, a straight forward extension of multiple-antenna results to a network setting is not possible. However, it might be possible that several nodes can cooperate with each other in transmitting the information to other nodes, and through this cooperation effectively achieve similar energy savings as a multiple antenna system.

In the problem studied in chapter 2, we intend to take advantage of the wireless broadcast property and the transmission side diversity created through cooperation to reduce the end-to-end energy consumption in routing the information between two nodes. To make it clear, let's look at a simple example. For the network shown in figure 1-1, assume the minimum energy route from s to d is determined to be as shown. As discussed previously, the information transmitted by node s is received by nodes 1 and 2. After the first transmission, nodes s, 1 and 2 have the information and can cooperate in getting the information to d. For instance, these 3 nodes can cooperate with each other in transmitting the information to node 3 as shown in figure 1-3.

Several questions arise in this context: how much energy savings can be realized by allowing this type of cooperation to take place? What level of coordination among the cooperating nodes is needed? And how must the route selection be done to maximize the energy savings?

These are the problems that we look at in chapter 2. We develop a formulation that captures the benefit of cooperative transmission and develop an algorithm for

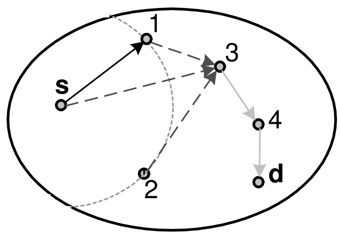

Figure 1-3: Cooperative Transmission

selecting the optimal route under this setting. We formulate the problem of finding the minimum energy cooperative route as two separate minimization problems. First, we look at the problem optimally transmit of information between two sets of nodes. We refer to the set of nodes that has correctly received the information as the reliable set. A separate problem is how to decide which nodes must be added to the reliable set in each transmission such that the information is routed to the final destination with minimum overall energy. We use dynamic programming to solve this second minimization problem. We present analytical results for the lower-bound of savings in networks with regular line or grid topology. We also propose two heuristics for finding the optimal path in arbitrary networks and present simulation results for the average energy savings of those heuristics.

Another property of a wireless medium that is often not taken into account by the network layer algorithms is the multi-path fading effect. The received signal in a wireless link is the combined sum of signals reflected by different scatterers in the propagation environment. A channel is said to be in deep fading state when the reflected signals add destructively at the receiver. Naturally, a higher transmission power is required to establish a link between two nodes when the channel between the two nodes is in deep fading. Since the fading state of the channel changes over time, the amount of energy required to transfer a unit of information between the two nodes changes overtime as well.

Depending on how fast channel changes occur, the time varying nature of the wireless channel can be addressed in two different ways. If the channel changes relatively fast, coding can be done to average the effect of fading. This type of averaging effect is the motivation behind the ergodic capacity model for fading channels (see [14]). To achieve this type of average behavior, however, very long delays might be imposed on the data.

Another possibility arises when the time scale over which the fading states vary is much longer than the time that it takes to transmits a unit of information between two nodes. Assuming that the receiver is able to measure the channel state and there is a feedback mechanism for the receiver to send this information back to the transmitter, the transmitter can leverage this information to adjusted the transmitted power based on the present channel state. This approach, however, requires the channel state information at the transmitter. If we assume that such channel knowledge is not available at the transmitter, there is no way that the transmitter can adjust its power to compensate for very bad channel states. The appropriate model for the wireless link in this scenario is the *capacity-versus-outage model*, see [15], [16], [14]. In this model, the instantaneous capacity of a wireless link is treated as a random variable. A link is said to be in outage when the instantaneous capacity supported by the link is less than the transmission rate. The reliability of a link, i.e. the probability of correct reception at the receiver, is modeled as a function of the transmission rate, the transmitted power, the distance between the communicating nodes, and the channel fading state. By adjusting the transmission rate or power, the transmitter can control the probability of successful reception at its intended receiver.

A network layer routing algorithm based on the capacity-versus-outage model at the link layer is studied in chapter 3. Our analysis starts by looking at the reliability of a point-to-point communication link. In particular, we are interested in how the reliability of a point-to-point link depends on the fading and the distance between the communicating nodes. Once the results for a point-to-point link are established, we extend the reliability results to a network setting. In a network setting, we first define and analyze the reliability for a fixed route and then propose algorithms for finding the optimal route between a source-destination pair of nodes under different constraints on the end-to-end consumed power and reliability.

An interesting aspect of the capacity-versus-outage model is the relationship between the transmitted power and link reliability. This type of analysis gives insight into how much power is required to achieve a certain level of reliability at the link level. It is known that in a Rayleigh fading link, the link outage probability decays as the inverse of the transmitted power, see [19]. If, however, either the transmitter or the receiver is equipt with multiple antenna, this trade-off become much more favorable, see [19] for details.

To extend the same analysis to the network layer, we study how the end-to-end reliability changes with the transmitted power. After defining the route reliability and proposing route selection algorithms in chapter 3, we study the relationship between the route reliability and consumed power. It is shown that the trade-off between reliability and power is similar to the trade-off in a point-to-point link, i.e. the end-to-end route outage probability decays as the inverse of the transmitted power.

The idea of *Route Diversity* is introduced as a way to improve route reliability by taking advantage of wireless broadcast property and the independence of fading states between different pairs of nodes. To clarify this idea, let's go back to our simple example. Assume that route $\{\mathsf{s} \rightarrow \mathsf{1} \rightarrow \mathsf{2} \rightarrow \mathsf{3} \rightarrow \mathsf{d}\}$ is selected. In each transmission along this route, the transmitted signal may be received by nodes other than the intended destination, shown with dotted arrows in figure 1-4. These links introduce a level of diversity in the system at the network or route level. We give analytical results on improvements due to route diversity in some simple network topologies. More precisely, we show that route diversity can improve the trade-off between the end-to-end reliability and power in networks with line topology. The effects seen are very similar to diversity benefits of multiple-antenna systems for a point-to-point link discussed in [19].

To our knowledge, this is the first attempt to introduce the concept of route reliability and the end-to-end reliability versus power trade-off in a network setting. More precisely, this is the first time that network layer routing algorithms and route properties, such as reliability and power, are studied based on the capacity-versus-outage model at the physical layer. This model has the potential to open the door

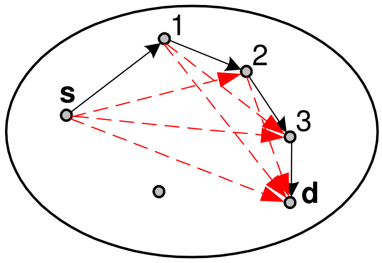

Figure 1-4: Reliability and Diversity

for a wide-range of research at the network layer.

The idea of route diversity is motivated by the work done in [10], [11], [12], and [13]. Most pervious results have been focused on two-hop networks and the analysis has been based on information theory results for relay channels. [10], [11], and [12] look at the effect of cooperation among nodes in increasing the capacity or reducing the outage probability in a fading network. In [10] and [12], the authors described several protocols for benefiting from the space diversity created by the relays in an ad-hoc network. They look at the trade-off between the capacity and the outage improvement in a two-hop ad-hoc network and several related protocols and coding schemes. This analysis ignores the deterministic part of link attenuation due to the distance between nodes and assume all link fading factors are independent and identically distributed Rayleigh random variables. [13] looks at asymptotic benefit of relay nodes in improving the capacity in an ad-hoc network. Their analysis only takes into account the deterministic part of link attenuation due to the distance between nodes. Their results mainly deal with how the capacity scales as a function of the number of nodes in the network.

Here we consider both fading and path-loss in our analysis and extend the results beyond 2-hop network topologies. The protocol discussed in our work is very similar to the decode-and-forward protocol described in [10]. The assumptions made in our treatment and the differences between our work and previous work are discussed in more details as appropriate in the context of this thesis.

Chapter 2

Energy Saving Through Cooperation

In this chapter, we look at the problem of cooperative transmission and energy savings due to the cooperation in multi-hop wireless networks. To clarify our approach, let's look at a simple example.

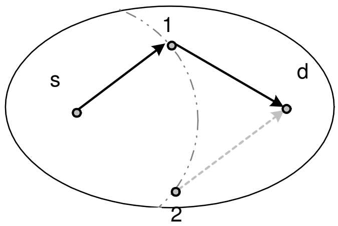

Figure 2-1: Cooperative Routing

Figure 2-1 depicts a simple 4-node wireless network, where s and d are the source and the destination nodes, respectively. We assume that there is no randomness in the fading states or distance between nodes. This implies that either there is no fading, as would be the case in free-space propagation, or the state of fading channels is completely known to all transmitters and remains constant for relatively long period of time compared to a typical transmission block length. In figure 2-1, assume that the minimum energy path from s to d is determined to be through node 1, i.e. $\{s \to 1 \to d\}$. Node 2, which is also located within the transmission radius of s to 1, receives the information transmitted from s at no additional cost. This property of a wireless medium is usually referred to as *Wireless Broadcast Advantage* (WBA) (see [3]). Cooperation between nodes 1 and 2 in the second hop creates transmission-side diversity and may result in some energy saving in the second step of relaying the information to d. We refer to potential energy saving due this type of cooperation among nodes as *Wireless Cooperation Advantage* (WCA). Under this setting, each node can participate in the cooperative transmission after it has completely received the information. The problem of finding the optimal path is a multi-stage decision making problem, where at each stage a set of nodes may cooperate to relay the information to another node or another set of nodes. The tradeoff is between spending more energy in each transmission slot to reach a larger set of nodes, and the potential savings in energy in subsequent transmission slots due to larger cooperative set of nodes. The minimum energy cooperative route may be viewed as a sequence of sets of cooperating nodes along with an appropriate allocation of transmission powers.

We first develop a formulation that captures the benefit of the wireless cooperative advantage. We then formulate the optimal route selection algorithm for a general network and look at the achievable savings in networks with very simple topologies. Two heuristic algorithms for selecting the route in a general network along with some simulation results are also presented.

The idea of wireless broadcast advantage was first introduced in [3]. The problem of finding the optimal multi-cast and broadcast tree in a wireless network and the added complexity due to WBA has been studied extensively in [3] and [4]. This problem is shown to be NP-Complete in [5] and [6]. [8] looks at the same problem under the assumption that nodes can collect power in different transmission slots. An overview of different transmission side diversity techniques is given in [2]. An architecture for achieving the required level of coordination among the cooperating nodes is discussed in [9].

2.1 Cooperative Transmission

Consider a wireless ad-hoc network consisting of arbitrarily distributed nodes where each node has a single omni-directional antenna. We assume that each node can dynamically adjust its transmitted power to control its transmission radius. It is also assumed that multiple nodes cooperating in sending the information to a single receiver node can precisely delay their transmitted signal to achieve perfect phase synchronization at the receiver. Under this setting, the information is routed from the source node to the destination node in a sequence of transmission slots, where each transmission slot corresponds to one use of the wireless medium. In each transmission slot/stage, either a node is selected to broadcast the information to a group of nodes or a subset of nodes that have already received the information cooperate to transmit that information to another group of nodes. As explained shortly, under our assumption it is the only reasonable to restrict the size of the receiving set to one node when multiple nodes are cooperating in the transmission. So, each transmission is either a broadcast, where a single node is transmitting the information and the information is received by multiple nodes, or a cooperative, where multiple node simultaneously send the information to a single receiver. We refer to the first case as the *Broadcast Mode* and the second case at the *Cooperative Mode*. In the *Broadcast Mode*, we take advantage of the known *Wireless Broadcast Advantage*. In the *Cooperative Mode*, we benefit from the newly introduced concept of *Wireless Cooperative Advantage*.

The routing problem can be viewed as a multi-stage decision problem, where at each stage the decision is to pick the transmitting and the receiving set of nodes as well as the transmission power levels among all nodes transmitting in that stage. The objective is to get the information to the destination with minimum energy. The set of nodes that have the information at the k^{th} stage is referred to as the k^{th}-stage *Reliable Set*, $\mathsf{S_k}$, and the routing solution may be expressed as a sequence of expanding reliable sets that starts with only the source node and terminates as soon as the reliable set contains the destination node. We denote the transmitting set by S and the receiving set by T. The link cost between S and T, $\mathsf{LC}(\mathsf{S}, \mathsf{T})$, is the minimum power needed for transmitting from S to T.

In this chapter, we make several idealized assumptions about the physical layer model. The wireless channel between any transmitting node, labeled s_i, and any receiving node, labeled t_j, is modeled by two parameters, its magnitude attenuation factor α_{ij} and its phase delay θ_{ij}. We assume that the channel parameters are estimated by the receiver and fed back to the transmitter. This assumption is reasonable for slowly varying channels, where the channel coherence time is much longer than the block transmission time. We also assume a free space propagation model where the power attenuation α_{ij}^2 is proportional to the inverse of the square of the distance between the communicating nodes s_i and t_j. For the receiver model, we assume that the desired minimum transmission rate at the physical layer is fixed and nodes can only decode based on the signal energy collected in a single channel use. We also assume that the received information can be decoded with no errors if the received Signal-to-Noise ration, SNR, level is above a minimum threshold $\mathsf{SNR}_{\mathsf{min}}$, and that no information is received otherwise. Without loss of generality, we assume that the information is encoded in a signal $\phi(t)$ that has unit power $P_\phi = 1$ and that we are able to control the phase and magnitude of the signal arbitrarily by multiplying it by a complex scaling factor w_i before transmission. The transmitted power by node i is $|w_i|^2$. The noise at the receiver is assumed to be additive, and the noise signal and power are denoted by $\eta(t)$ and P_η, respectively. This simple model allows us to find analytical results for achievable energy savings in some simple network topologies.

2.1.1 Link Cost Formulation

In this section, our objective is to understand the basic problem of optimal power allocation required for successful transmission of the same information from a set of source nodes $\mathsf{S} = \{s_1, s_2, \cdots, s_n\}$ to a set of target nodes $\mathsf{T} = \{t_1, t_2, \cdots, t_m\}$. In order to derive expressions for the link costs, we consider 4 distinct cases:

1. *Point-to-Point Link:* $n = 1, m = 1$: In this case, only one node is transmitting within a time slot to a single target node.

2. *Point-to-Multi-Point, Broadcast Link:* $n = 1, m > 1$: This type of link corresponds to the broadcast mode introduced in the last section. In this case, a single node is transmitting to multiple target nodes.

3. *Multi-Point-to-Point, Cooperative Link:* $n > 1, m = 1$: This type of link corresponds to the cooperative mode introduced in the last section. In this case, multiple nodes cooperate to transmit the same information to a single receiver node. We will assume that coherent reception, i.e. the transmitters are able to adjust their phases so that all signals arrive in phase at the receiver. In this case, the signals simply add up at the receiver and complete decoding is possible as long as the received SNR is above the minimum threshold SNR_{min}. Here, we do not address the feasibility of precise phase synchronization. The reader is referred to [9] for a discussion of mechanisms for achieving this level of synchronization.

4. *Multi-Point-to-Multi-Point Link:* $n > 1, m > 1$: This is not a valid option under our assumptions, as synchronizing transmissions for coherent reception at multiple receivers is not feasible. Therefore, we will not be considering this case.

Point-to-Point Link: $n = 1, m = 1$

In this case, $S = \{s_1\}$ and $T = \{t_1\}$. The channel parameters may be simply denoted by α and θ, and the transmitted signal is controlled through the scaling factor w. Although in general the scaling factor is a complex value, absorbing both power and phase adjustment by the transmitter, in this case we can ignore the phase as there is only a single receiver. The model assumptions made in Section 2.1 imply that the received signal is simply:

$$r(t) = \alpha e^{j\theta} w \phi(t) + \eta(t)\cdot$$

where $\phi(t)$ is the unit-power transmitted signal and $\eta(t)$ is the receiver noise with power P_η. The total transmitted power is $P_T = |w|^2$ and the SNR ratio at the receiver is $\frac{\alpha^2 |w|^2}{P_\eta}$. For complete decoding at the receiver, the SNR must be above the threshold value $\mathsf{SNR_{min}}$. Therefore the minimum power required, $\hat{\mathsf{P_T}}$, and hence the point-to-point link cost $\mathsf{LC(s_1, t_1)}$, is given by:

$$\mathsf{LC(s_1, t_1)} \equiv \hat{\mathsf{P_T}} = \frac{\mathsf{SNR_{min} P_\eta}}{\alpha^2}. \tag{2.1}$$

In equation 2.1, the point-to-point link cost is proportional to $\frac{1}{\alpha^2}$, which is the power attenuation in the wireless channel between $\mathsf{s_1}$ and $\mathsf{t_1}$, and therefore is proportional to the square of the distance between $\mathsf{s_1}$ and $\mathsf{t_1}$ under our propagation model.

Point-to-Multi-Point, Broadcast Link: $\mathsf{n = 1, m > 1}$

In this case, $\mathsf{S = \{s_1\}}$ and $\mathsf{T = \{t_1, t_2, \cdots, t_m\}}$, hence m simultaneous SNR constraints must be satisfied at the receivers. Assuming that omni-directional antennas are being used, the signal transmitted by node $\mathsf{s_1}$ is received by all nodes within a transmission radius proportional to the transmission power. Hence, a broadcast link can be treated as a set of point-to-point links and the cost of reaching a set of node is the maximum over the costs for reaching each of the nodes in the target set. Thus the minimum power required for the broadcast transmission, denoted by $\mathsf{LC(s_1, T)}$, is given by:

$$\mathsf{LC(s, T) = max\{LC(s_1, t_1), LC(s_1, t_2), \cdots, LC(s_1, t_n)\}}. \tag{2.2}$$

Multi-Point-to-Point, Cooperative Link: $\mathsf{n > 1, m = 1}$

In this case $\mathsf{S = \{s_1, s_2, \cdots, s_n\}}$ and $\mathsf{T = \{t_1\}}$. We assume that the n transmitters are able to adjust their phases in such a way that the signal at the receiver is:

$$\mathsf{r(t)} = \sum_{\mathsf{i}}^{\mathsf{n}} \alpha_{\mathsf{i1}} |\mathsf{w_i}| \phi(\mathsf{t}) + \eta(\mathsf{t}).$$

The total transmitted power is $\sum_{\mathsf{i=1}}^{\mathsf{n}} |\mathsf{w_i}|^2$ and the received signal power is $|\sum_{\mathsf{i=1}}^{\mathsf{n}} \mathsf{w_i} \alpha_{\mathsf{i1}}|^2$. The power allocation problem for this case is simply

$$\begin{aligned} \mathsf{min} \quad & \sum_{\mathsf{i=1}}^{\mathsf{n}} |\mathsf{w_i}|^2 \\ \mathsf{s.t.} \quad & \frac{|\sum_{\mathsf{i=1}}^{\mathsf{n}} \mathsf{w_i} \alpha_{\mathsf{i1}}|^2}{\mathsf{P_\eta}} \geq \mathsf{SNR_{min}}. \end{aligned} \tag{2.3}$$

Lagrangian multiplier techniques may be used to solve the constrained optimization problem above. The resulting optimal allocation for each node i is given by

$$|\hat{\mathsf{w}}_\mathsf{i}| = \frac{\alpha_{\mathsf{i1}}}{\sum_\mathsf{i}^\mathsf{n} \alpha_{\mathsf{i1}}^2} \sqrt{\mathsf{SNR}_{\mathsf{min}} \mathsf{P}_\eta}. \tag{2.4}$$

The resulting cooperative link cost $\mathsf{LC}(\mathsf{S}, \mathsf{t_1})$, defined as the optimal total power, is therefore given by

$$\mathsf{LC}(\mathsf{S}, \mathsf{t_1}) \equiv \hat{\mathsf{P}}_\mathsf{T} = \frac{1}{\sum_{\mathsf{i}=1}^{\mathsf{n}} \frac{\alpha_{\mathsf{i1}}^2}{\mathsf{SNR}_{\mathsf{min}} \mathsf{P}_\eta}}. \tag{2.5}$$

It is easy to see that it can be written in terms of the point-to-point link costs between all the source nodes and the target nodes (see Equation 2.1) as follows:

$$\mathsf{LC}(\mathsf{S}, \mathsf{t_1}) = \frac{1}{\frac{1}{\mathsf{LC}(\mathsf{s_1}, \mathsf{t_1})} + \frac{1}{\mathsf{LC}(\mathsf{s_2}, \mathsf{t_2})} + \cdots + \frac{1}{\mathsf{LC}(\mathsf{s_n}, \mathsf{t_1})}}. \tag{2.6}$$

A few observations are worth mentioning here. First, based on equation 2.4, the transmitted signal level is proportional to the channel attenuation. Therefore, in the cooperative mode *all* nodes in the reliable set cooperate to send the information to a single receiver. In addition, based on equation 2.6, the cooperative cost is smaller than each point-to-point cost. This conclusion is intuitively plausible and is a proof on the energy saving due to the *Wireless Cooperative Advantage*.

2.1.2 Optimal Cooperative Route Selection

The problem of finding the optimal cooperative route from the source node s to the destination node d, formulated in Section 2.1, can be mapped to a Dynamic Programming (DP) problem. The state of the system at stage k is the reliable set $\mathsf{S_k}$, i.e. the set of nodes that have completely received the information by the k^{th} transmission slot. The initial state $\mathsf{S_0}$ is simply $\{\mathsf{s}\}$, and the termination states are all sets that contain d. The decision variable at the $\mathsf{k^{th}}$ stage is $\mathsf{U_k}$, the set of nodes that will be added to the reliable set in the next transmission slot. The dynamical system evolves as follows:

$$\mathsf{S_{k+1}} = \mathsf{S_k} \cup \mathsf{U_k} \quad \mathsf{k} = 1, 2, \cdots \tag{2.7}$$

The objective is to find a sequence $\{U_k\}$ or alternatively $\{S_k\}$ so as to minimize the total transmitted power P_T, where

$$P_T = \sum_k LC(S_k, U_k) = \sum_k LC(S_k, S_{k+1} - S_k)\cdot \quad (2.8)$$

We will refer to the solution to this problem as the optimal transmission policy. The optimal transmission policy can be mapped to finding the shortest path in the state space of this dynamical system. The state space can be represented by as graph with all possible states, i.e. all possible subsets of nodes in the network, as its nodes. We refer to this graph as the *Cooperation Graph*. Figure 2-4 show the cooperation graph corresponding to the 4-node network shown in Figure 1-1.

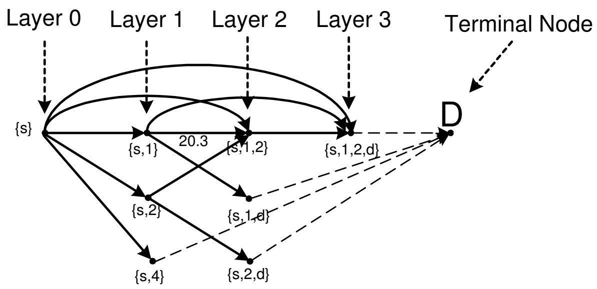

Figure 2-2: Cooperation Graph for a 4-Node Network

Nodes in the cooperation graph are connected with arcs representing the possible transitions between states. As the network nodes are allowed only to either fully cooperate or broadcast, the graph has a special layered structure as illustrated by Figure 2-4. All nodes in the k^{th} layer are of size $k + 1$, and a network with $n + 1$ nodes the cooperation graph has n layers, and the k^{th} layer has $\begin{pmatrix} k \\ n \end{pmatrix}$ nodes. Arcs between nodes in adjacent layers correspond to cooperative links, whereas broadcast links are shown by cross-layer arcs. The costs on the arcs are the link costs defined in Section 2.1.1. All terminal states are connected to a single artificial terminal state, denoted by D, by a zero-cost arc. The optimal transmission policy is simply the shortest path between nodes s and D. There are 2^n nodes in the cooperation graph for a network with $n + 1$ nodes. Therefore standard shortest path algorithms will in general have a complexity of $O(2^{2n})$. However, by taking advantage of some special properties of the cooperation graph, we are able to come up with an algorithm with complexity reduced to $\mathsf{O}(\mathsf{n}2^{\mathsf{n}})$. This algorithm is based on scanning the cooperation graph from left to right and constructing the shortest path to each nodes at the k^{th} layer based on the shortest path to nodes in the pervious layers. The *Sequential Scanning Algorithm* is outlined below.

Sequential Scanning Algorithm This is the algorithm for finding the optimal cooperative route in an arbitrary network based on finding the shortest path in the corresponding cooperation graph.

Initialize Initialize the cooperation graph data structure. Initialize the layer counter k to $\mathsf{k} = 1$.

Repeat Construct to the shortest path to all nodes at the k^{th} layer based on the shortest path to all nodes in the previous layers. Increment the counter.

Stop Stop when D is reached. i.e. when $\mathsf{k} = \mathsf{n} + 1$.

For a network with $\mathsf{n} + 1$ nodes, the main loop in this algorithm is repeated n times and at the k^{th} stage the shortest path to $\begin{pmatrix} k \\ n \end{pmatrix}$ nodes must be calculated. This operation has a complexity of order $\mathsf{O}(2^{\mathsf{n}})$, hence finding the optimal route is of complexity $\mathsf{O}(\mathsf{n}2^{\mathsf{n}})$.

Although the *Sequential Scanning Algorithm* substantially reduces the complexity for finding the optimal cooperative route in an arbitrary network, its complexity is still exponential in the number of nodes in the wireless network. For this reason, finding the optimal cooperative route in an arbitrary network becomes computationally intractable for larger networks. We will focus on developing computationally simpler and relatively efficient heuristics and on assessing their performance through simulation.

2.1.3 Example

Having developed the necessary mathematical tools, we now present a simple example that illustrates the benefit of cooperative routing. Figure 2-3 shows a simple network with 4 nodes. The arcs represent links and the arc labels are point-to-point link costs. The diagrams below show the six possible routes, P_0 through P_5. P_0 corresponds to a simple 2-hop, non-cooperative minimum energy path between s and d. P_1, P_2, and P_3 are 2-hop cooperative routes, whereas P_4 and P_5 are 3-hop cooperative routes. Figure 2-4 shows the corresponding cooperation graph for this network. Each transmission policy corresponds to a distinct path between $\{\mathsf{s}\}$ and D in this graph and the minimum energy policy of P_3 corresponds to the shortest path. Table 2.1 lists the costs of the six policies.

No.	Policy	Cost
P_0	$NonCooperative$	65
P_1	$(\{s\},\{s,2\},\{s,2,d\})$	≈ 61.5
P_2	$(\{s\},\{s,1\},\{s,1,d\})$	≈ 57.9
P_3	$(\{s\},\{s,1,2\},\{s,1,2,d\})$	≈ 55.9
P_4	$(\{s\},\{s,2\},\{s,1,2\},\{s,1,2,d\})$	≈ 73.6
P_5	$(\{s\},\{s,1\},\{s,1,2\},\{s,1,2,d\})$	≈ 65.2

Table 2.1: Transmission Policies for Figure 2-3

2.2 Analytical Results for Line and Grid Topologies

In this section, we develop analytical results for achievable energy savings in line and grid networks. In particular, we consider a *Regular Line* Topology (see Figure 2-5) and a *Regular Grid* Topology (see Figure 2-6) where nodes are equi-distant from each other. Before proceeding further, let us define precisely what we mean by energy savings for a cooperative routing strategy relative to the optimal non-cooperative

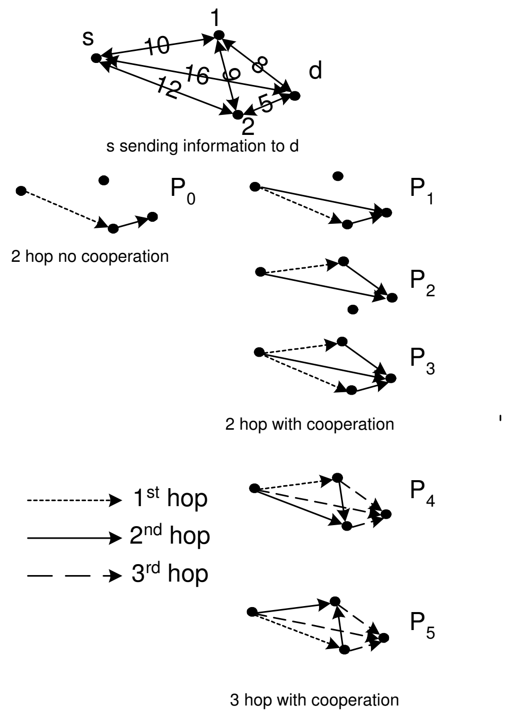

Figure 2-3: 4-Node Network Example

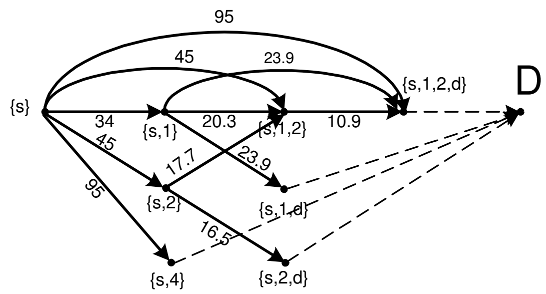

Figure 2-4: 4-Node Cooperation Graph

strategy:

$$\mathsf{Savings} = \frac{\mathsf{P_T(Non-cooperative) - P_T(Cooperative)}}{\mathsf{P_T(Non-cooperative)}}. \tag{2.9}$$

where $\mathsf{P_T(strategy)}$ denotes the total transmission power for the strategy.

2.2.1 Line Network-Analysis

Figure 2-5) shows a regular line where nodes are located at unit distance from each other on a straight line. In our proposed scheme, we restrict the cooperation to nodes along the optimal non-cooperative route. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy non-cooperative route. This cooperation strategy is referred to as the CAN (*Cooperation Along the Minimum Energy Non-Cooperative Path*) strategy.

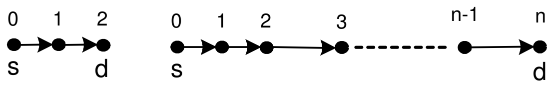

Figure 2-5: Regular Line Topology

For the 3-node line network in Figure 2-5, it is easy to show that the optimal non-cooperative routing strategy is to relay the information through the middle node. Since a longer line network can be broken down into short 2-hop components, it is clear that the optimal non-cooperative routing strategy is to always send the information to the next nearest node in the direction of the destination until the destination node is reached. From Equation 2.1, the link cost for every stage is $\frac{\mathsf{SNR_{min}P_\eta}}{\alpha^2}$, where α is the magnitude attenuation between two adjacent nodes 1-distance unit apart. Under our assumptions, α^2 is proportional to the inverse of the distance squared. Therefore,

$$\mathsf{P_T(Non-cooperative)} = \mathsf{n}\frac{\mathsf{SNR_{min}P_\eta}}{\alpha^2}. \tag{2.10}$$

With the CAN strategy, after the $\mathsf{m^{th}}$ transmission slot, the reliable set is $\mathsf{S_m} = \{\mathsf{s}, 1, \cdots, \mathsf{m}\}$, and the link cost associated with the nodes in $\mathsf{S_m}$ cooperating to send the information to the next node $(\mathsf{m}+1)$ follows from Equation 2.6 and is given by

$$\mathsf{LC}(\mathsf{S_m}, \mathsf{m}+1) = \frac{\mathsf{SNR_{min}}\mathsf{P}_\eta}{\sum_{\mathsf{i}=1}^{\mathsf{m}+1} \frac{\alpha^2}{\mathsf{i}^2}}. \tag{2.11}$$

Therefore, the total transmission power for the CAN strategy is

$$\begin{aligned} \mathsf{P_T}(\mathsf{CAN}) &= \sum_{\mathsf{m}=0}^{\mathsf{n}-1} \mathsf{LC}(\mathsf{S_m}, \mathsf{m}+1) \\ &= \frac{\mathsf{SNR_{min}}\mathsf{P}_\eta}{\alpha^2} \sum_{\mathsf{m}=0}^{\mathsf{n}-1} \frac{1}{\mathsf{C}(\mathsf{m}+1)}, \qquad (2.12) \\ & \text{where } \mathsf{C}(\mathsf{m}) = \sum_{\mathsf{i}=1}^{\mathsf{m}} \frac{1}{\mathsf{i}^2}. \qquad (2.13) \end{aligned}$$

Before moving to find the savings achieved by *CAN* in a line, we need to proves the following simple lemma regarding the existence of the average of terms for a decreasing sequence.

Lemma 1 *Let* $\mathsf{a_n}$ *be a decreasing sequence with a finite limit* c, *then* $\lim_{\mathsf{m}\to\infty} \frac{\sum_{\mathsf{n}=1}^{\mathsf{m}} \mathsf{a_n}}{\mathsf{m}} = \mathsf{c}$.

Proof: For any value of m, let $\mathsf{m_0}$ be an arbitrary integer less than m:

$$\begin{aligned} \lim_{\mathsf{m}\to\infty} \frac{\sum_{\mathsf{n}=1}^{\mathsf{m}} \mathsf{a_n}}{\mathsf{m}} &= \lim_{\mathsf{m}\to\infty} \frac{1}{\mathsf{m}} \left(\sum_{\mathsf{n}=1}^{\mathsf{m_0}} \mathsf{a_n} + \sum_{\mathsf{n}=\mathsf{m_0}+1}^{\mathsf{m}} \mathsf{a_n} \right) \\ &= \lim_{\mathsf{m}\to\infty} \frac{1}{\mathsf{m}} \sum_{\mathsf{n}=1}^{\mathsf{m_0}} \mathsf{a_n} + \lim_{\mathsf{m}\to\infty} \frac{1}{\mathsf{m}} \sum_{\mathsf{n}=\mathsf{m_0}+1}^{\mathsf{m}} \mathsf{a_n} \\ &= 0 + \lim_{\mathsf{m}\to\infty} \frac{1}{\mathsf{m}} \frac{\mathsf{m}-(\mathsf{m_0}+1)}{\mathsf{m}-(\mathsf{m_0}+1)} \sum_{\mathsf{n}=\mathsf{m_0}+1}^{\mathsf{m}} \mathsf{a_n} \\ &= \lim_{\mathsf{m}\to\infty} \frac{\mathsf{m}-(\mathsf{m_0}+1)}{\mathsf{m}} \frac{1}{\mathsf{m}-(\mathsf{m_0}+1)} \sum_{\mathsf{n}=\mathsf{m_0}+1}^{\mathsf{m}} \mathsf{a_n} \\ &= \lim_{\mathsf{m}\to\infty} \frac{\mathsf{m}-(\mathsf{m_0}+1)}{\mathsf{m}} \lim_{\mathsf{m}\to\infty} \frac{1}{\mathsf{m}-(\mathsf{m_0}+1)} \sum_{\mathsf{n}=\mathsf{m_0}+1}^{\mathsf{m}} \mathsf{a_n} \\ &= \lim_{\mathsf{m}\to\infty} \frac{1}{\mathsf{m}-(\mathsf{m_0}+1)} \sum_{\mathsf{n}=\mathsf{m_0}+1}^{\mathsf{m}} \mathsf{a_n}\,. \end{aligned} \tag{2.14}$$

Since a_n is a decreasing sequence, all terms in the final sum are less than a_{m_0}. Furthermore, $\lim_{n\to\infty} a_n = c$. So, all terms in the final sum are greater than c. Hence:

$$c \leq \lim_{m\to\infty} \frac{\sum_{n=1}^{m} a_n}{m} = \lim_{m\to\infty} \frac{1}{m-(m_0+1)} \sum_{n=m_0+1}^{m} a_n \leq a_{m_0}.$$

For increasing values of m, m_0 may be chosen such that a_{m_0} is arbitrarily close to c and the proof is established.

Theorem 1 *For a regular line network as shown in Figure 2-5, the CAN strategy results in energy savings of* $(1 - \frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)})$. *As the number of nodes in the network grows, the energy savings value approaches* $(1 - \frac{6}{\pi^2}) \approx 39\%$.

Proof: The minimum energy non-cooperative routing a regular line network with n hops has cost equal to n. The cost of the optimal cooperation scheme, i.e. the CAN strategy, is:

$$P_T(\text{Cooperative}) = \sum_{m=1}^{n} LC(\{s, \cdots, m-1\}, m) = \sum_{m=1}^{n} \frac{1}{C(m)} \tag{2.15}$$

where $C(m)$ is defined by equation 2.13. The energy savings achieved, as defined by equation 2.12, is:

$$\begin{aligned} \text{Savings}(n) &= \frac{P_T(\text{Non}-\text{Cooperative}) - P_T(\text{Cooperative})}{P_T(\text{Non}-\text{Cooperative})} && (2.16)\\ &= \frac{n - \sum_{m=1}^{n}\frac{1}{C(m)}}{n} && (2.17)\\ &= 1 - \frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)} && (2.18) \end{aligned}$$

$\frac{1}{C(m)}$ is a decreasing sequence with limit of $\frac{6}{\pi^2}$. So, based on lemma 1 we have:

$$\lim_{n\to\infty} \text{Savings}(n) = 1 - \lim_{n\to\infty} \frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)} = 1 - \frac{6}{\pi^2} \tag{2.19}$$

This establishes the claim and completes the proof.

2.2.2 Grid Network

Figure 2-6 shows a regular $n \times n$ grid topology with s and d located at opposite corners. A $n \times n$ grid can be decomposed into many 2×2 grid. Assuming that the nodes are located at a unit distance from each other, in a 2×2 grid, a diagonal transmission has a cost of 2 units, equal to the cost of one horizontal and one vertical transmission. For this reason, in an $n \times n$ grid there are many non-cooperative routes with equal cost. Figure 2-6 shows two such routes for an $n \times n$ grid.

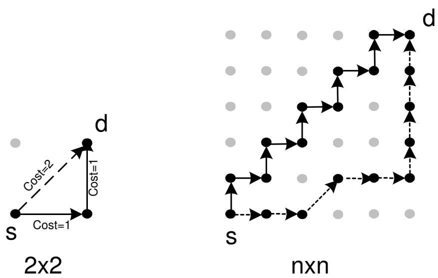

Figure 2-6: Regular Grid Topology

The minimum-energy non-cooperative route is obtained by a stair-like policy (illustrated in Figure 2-6), and its total power is $2n$. We will base our analysis for deriving the bound for saving based on this stair-like non-cooperative path. The following theorem stated the energy savings achieved by the CAN strategy applied to this non-cooperative route.

Theorem 2 *For a regular grid network as shown in Figure 2-6, the energy savings achieved by using the CAN strategy approaches* 56% *for large networks.*

Proof: Figure 2.2.2 shows an intermediate step in routing the information is a regular grid. At this stage, all the nodes with a darker shade, nodes 1 through 8, have received the information. In the next step, the information must be relayed to node 9. The cooperative cost of this stage is

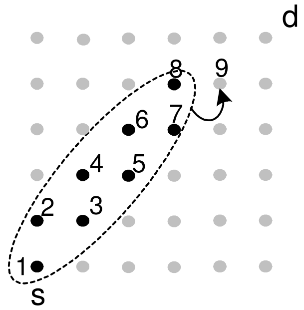

Figure 2-7: Cooperative Routing in a Grid Topology

$$
\begin{aligned}
LC(\{1,\cdots,8\},9) &= \frac{1}{\sum_{i=1}^{8}\frac{1}{LC(i,9)}} \\
&= \frac{1}{\frac{1}{1}+\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\frac{1}{13}+\frac{1}{18}+\frac{1}{25}+\frac{1}{32}} \qquad (2.20)\\
&= \frac{1}{\underbrace{\frac{1}{1}+\frac{1}{5}+\frac{1}{13}+\frac{1}{25}}+\underbrace{\frac{1}{2}+\frac{1}{8}+\frac{1}{18}+\frac{1}{32}}} \qquad (2.21)
\end{aligned}
$$

In general, the cooperative cost of the m^{th} stage of the proposed strategy is

$$
\begin{aligned}
\mathsf{C}_{\mathsf{grid}}(\mathsf{m}) &= \mathsf{LC}(\{1,\cdots,\mathsf{m}\},\mathsf{m}+1) \\
&= \frac{1}{\sum_{i=1}^{m}\frac{1}{\mathsf{LC}(i,m)}} \qquad (2.22)
\end{aligned}
$$

It is not too hard to see that the point-to-point costs have the following form

$$
\mathsf{LC}(i,m) = \left(\left\lceil\frac{m-i}{2}\right\rceil\right)^2+\left(\left\lfloor\frac{m-i}{2}\right\rfloor\right)^2 \qquad (2.23)
$$

Using Equation 2.23, Equation 2.22 can be written as

$$
\begin{aligned}
\mathsf{C}_{\mathsf{grid}}(\mathsf{m}) &= \frac{1}{\sum_{i=1}^{m}\frac{1}{\mathsf{LC}(i,m)}} \\
&= \frac{1}{\sum_{i=1}^{m}\frac{1}{\left(\left\lceil\frac{m-i}{2}\right\rceil\right)^2+\left(\left\lfloor\frac{m-i}{2}\right\rfloor\right)^2}} \\
&= \frac{1}{\sum_{k=1}^{\lceil\frac{m}{2}\rceil}\frac{1}{2k^2-2k+1}+\sum_{k=1}^{\lfloor\frac{m}{2}\rfloor}\frac{1}{2k^2}} \qquad (2.24)
\end{aligned}
$$

Comparing Equation 2.21 and Equation 2.24, it is easy to see that the first group of terms is generated by the first sum term and the second group is generated by the second sum term. $C_{grid}(m)$ is a decreasing sequence of numbers and can be shown, using Maple, to have a limit equal to 0.44.

The total cost for the cooperative route in an $n \times n$ grid is

$$P_T(\text{Cooperative}) = \sum_{m=1}^{2n} C_{grid}(m) \tag{2.25}$$

The energy saving, as defined by equation 2.9, is

$$\begin{aligned} \text{Savings}(n) &= \frac{P_T(\text{Non}-\text{Cooperative}) - P_T(\text{Cooperative})}{P_T(\text{Non}-\text{Cooperative})} \\ &= \frac{2n - \sum_{m=1}^{2n} C_{grid}(m)}{2n} \\ &= 1 - \frac{1}{2n}\sum_{m=1}^{2n} C_{grid}(m) \end{aligned} \tag{2.26}$$

Since $C_{grid}(m)$ is a decreasing sequence and $\lim_{m\to\infty} C_{grid}(m) = 0.44$, by lemma 1, the savings in the case of a regular grid, as calculated in equation 2.26, approaches $1 - 0.44 = 56\%$. This establishes the claim and completes the proof for the lower bound of achievable savings in a regular grid.

2.3 Heuristics & Simulation Results

We present two possible general heuristic schemes and related simulation results. The simulations are over a network generated by randomly placing nodes on an 100×100 grid and randomly choosing a pair of nodes to be the source and destination. For each realization, the minimum energy non-cooperative path was found. Also, the proposed heuristic were used to find co-operative paths. The performance results reported are the energy savings of the resulting strategy with respect to the optimal non-cooperative path averaged over 100,000 simulation runs.

The two heuristics analyzed are outlined below.

CAN-L Heuristic **C***ooperation* **A***long the* **N***on-Cooperative Optimal Route*:

This heuristic is based on the CAN strategy described Section 2.2. CAN-L is a variant of CAN as it limits the number of nodes allowed to participate in the cooperative transmission to L. In particular, these nodes are chosen to be the last L nodes along the minimum energy non-cooperative path. As mentioned before, in each step the last L nodes cooperate to transmit the information to the next node along the optimal non-cooperative path. The only processing needed in this class of algorithm is to find the optimal non-cooperative route. For this reason, the complexity of this class of algorithms is the same as finding the optimal non-cooperative path in a network or $\mathsf{O}(\mathsf{N}^2)$.

PC-L Heuristic **P***rogressive* **C***ooperation*:

Initialize Initialize *B*est Path to the optimal non-cooperative route. Initialize the *S*uper Node to contain only the source node.

Repeat Send the information to the first node along the current *B*est Path. Update the *S*uper Node to include all past L nodes along the current *B*est Path. Update the link costs accordingly, i.e. by considering the *S*uper Node as a single node and by using equation 2.6. Compute the optimal non-cooperative route for the new network/graph and update the *B*est Path accordingly.

Stop Stop as soon as the destination node receives the information.

For example, with $\mathsf{L} = 3$, this algorithm always combines the last 3 nodes along the current *B*est Route into a single node, finds the shortest path from that combined node to the destination and send the information to the next node along that route. This algorithm turns out to have a complexity of $\mathsf{O}(\mathsf{N}^3)$ since the main loop is repeated $\mathsf{O}(\mathsf{N})$ times and each repetition has a complexity of $\mathsf{O}(\mathsf{N}^2)$.

A variant of this algorithm keeps a window W of the most recent nodes, and in each step all subsets of size L among the last W nodes are examined and the path with the least cost is chosen. This variant has a complexity of $\mathsf{O}\left(\binom{W}{L} \times \mathsf{N}^3\right)$, where W is the window size. We refer to this variant as *Progressive Cooperation with Window*.

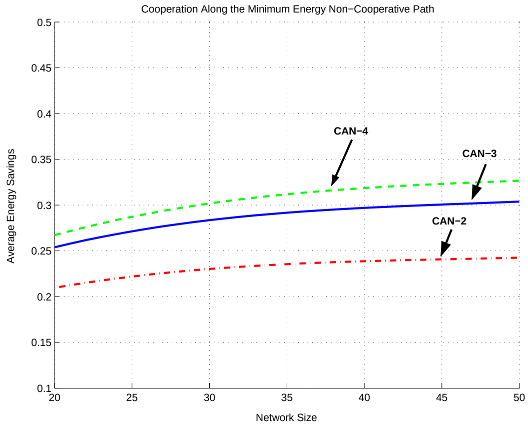

Figure 2-8: Performance of CAN

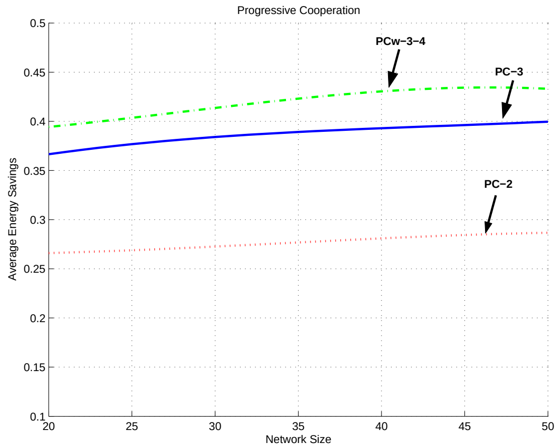

Figure 2-9: Performance of PC

Figures 2-8 and 2-9 show average energy savings ranging from 20% to 50% for CAN and PC algorithms. It can be seen that PC-2 performs almost as well as CAN-3 and PC-3 performs much between than CAN-4. This show that the method for approximating the optimal route is very important factor in increasing the savings. Figures 2-10 compares CAN, PC, and PC-W on the same chart. It is seen that PC-3-4 performs better than PC-3, which performs substantially better than CAN-4. In general, it can be seen that the energy savings increase with L, and that improvements in savings are smaller for larger values of L. As there is a trade-off between the algorithm complexity and the algorithm performance, these simulation results indicate that it would be reasonable to chose L to be around 3 or 4 for both the CAN and PC heuristics.

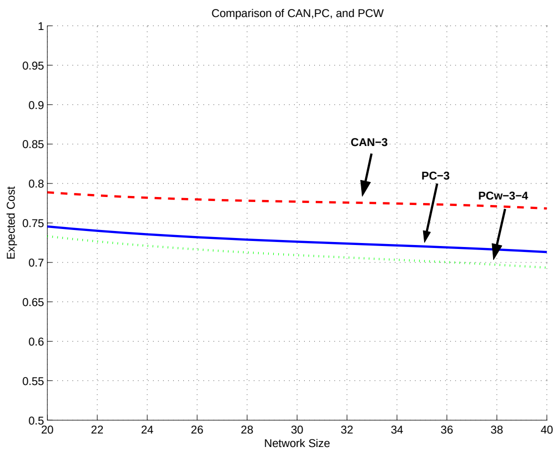

Figure 2-10: Comparison

Chapter 3

Reliability and Route Diversity in Wireless Networks

In this chapter, we look at the problem of route reliability in wireless ad-hoc networks. Our analysis starts by looking at the reliability of a point-to-point communication link. In particular, we are interested in how the reliability of a point-to-point link depends on the channel state and the distance between the two nodes. Once the result for a point-to-point link is established, we extend the reliability result to a wireless network. In a network setting, we first define and analyze the reliability for a fixed route and then propose algorithms for finding the best possible route between a source-destination pair. The relation between route reliability and consumed power is studied. The idea of *route diversity* is introduced as a way to improve route reliability by taking advantage of wireless broadcast property and the independence of fading state between different pairs of nodes. We give analytical results on improvements due to route diversity in some simple network topologies.

3.1 Point-to-Point Reliability

In this section, we look at the relationship between reliability and power in a point-to-point single-user flat fading channel. We model the communication link between two nodes as

$$\mathsf{y} = \mathsf{a}\,\mathsf{x} + \eta,$$

where x is the transmitted signal, η is additive the received noise, and a is the signal attenuation due to propagation in the wireless point-to-point link. We assume the received noise, η, is zero mean white additive Gaussian noise with average power of σ_η^2. In general, attenuation, a, depends on the distance between the communicating points and the fading state of the channel. We use d to represent the distance between the communicating nodes and f to represent the fading state of the channel. To emphasize this dependence, let's write a as a explicit function of these two parameters:

$$\mathsf{y} = \mathsf{a}(\mathsf{f}, \mathsf{d})\mathsf{x} + \eta. \tag{3.1}$$

In a real system with mobile nodes and a constantly changing propagation environment, both f and d change over time. However, we assume a system where f and d remain constant for a long period of time compared to the typical transmission block length. Furthermore, we assume that the transmission blocks are long enough that coding can be done to average over the Gaussian noise. This model is commonly referred to as the Block-Fading channel model. Given these assumptions, the link between two nodes is a simple AWGN channel and the capacity (see[21]) is given by:

$$\mathsf{C}(\mathsf{f}, \mathsf{d}, |\mathsf{x}|^2, \sigma_\eta^2) = \log(1 + \frac{|\mathsf{a}(\mathsf{f}, \mathsf{d})|^2 |\mathsf{x}|^2}{\sigma_\eta^2})$$

To simplify this notation, we decompose $\mathsf{a}(\mathsf{f}, \mathsf{d})$ into two independent components corresponding to the small and the large scale path losses (see [20].) More specifically, we assume

$$|\mathsf{a}(\mathsf{f}, \mathsf{d})|^2 = \frac{|\mathsf{f}|^2}{\mathsf{d}^\mathsf{k}},$$

where f represents the small-scale path loss, d represents the large-scale path loss, and k is the propagation power loss exponent, usually between 2 to 4. Simplifying the capacity formula using this form for a, and simplifying the notation by using $\frac{|\mathsf{x}|^2}{\sigma_\eta^2} = \mathsf{snr}$, we get:

$$C(f, d, snr) = \log(1 + \frac{|f|^2}{d^k} snr) \tag{3.2}$$

This formula gives the capacity of a point to point link for a fixed fading, f, fixed distance, d, and transmission signal-to-noise ratio of snr.

3.1.1 Outage Formulation

Equation 3.2 gives the instantaneous capacity of the point-to-point link defined by 3.1. Outage event is said to have occurred (see [15]) when the transmission rate, R bits/channel-use, is above the instantaneous capacity of the link, i.e.

$$\{Outage\} \stackrel{\text{def}}{=} \{C(f, d, snr) < R\}. \tag{3.3}$$

One parameter of interest in communication systems is the probability of error. An error occurs if the channel is in outage or if the channel is not in outage but there is a decoding error. In our analysis, we assume that the probability of error is almost 0 when channel is not in outage. Under this assumption, outage is the dominating error event. Hence:

$$P_{Error} \approx P(Outage)$$

We focus our attention on calculating the outage probability as it is a good measure, although no exact, for decoding error probability. Based on the definition of outage given in 3.3, the outage probability is given by

$$\begin{aligned} P_{Outage}(f, d, snr, R) &= P\{C(f, d, snr) < R\} \\ &= P\{\log(1 + \frac{|f|^2}{d^k} snr) < R\} \\ &= P\{\frac{|f|^2}{d^k} < \frac{2^R - 1}{snr}\}. \end{aligned} \tag{3.4}$$

Similar to the approach taken in [10] and other work in this area, we normalize the transmission rate by absorbing its effect into the snr term. This makes sense specially in our case since we assume that the transmission rate, R, is a fixed parameter of the system. So we define

$$\mathsf{snr}_{\mathsf{norm}} = \frac{\mathsf{snr}}{2^{\mathsf{R}} - 1}. \tag{3.5}$$

Without lose of generality, equation 3.4 simplifies to:

$$\mathsf{P}_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr}_{\mathsf{norm}}) = \mathsf{P}\{\frac{|\mathsf{f}|^2}{\mathsf{d}^{\mathsf{k}}} < \frac{1}{\mathsf{snr}_{\mathsf{norm}}}\}. \tag{3.6}$$

For notational convenience, we drop the norm subscript from $\mathsf{snr}_{\mathsf{norm}}$. To simplify subsequent derivation, define the indicator function for outage and success as follows:

$$1_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr}) = \begin{cases} 1 & \text{if } \frac{|\mathsf{f}|^2}{\mathsf{d}^{\mathsf{k}}}\mathsf{snr} < 1 \\ 0 & \text{else} \end{cases} \tag{3.7}$$

and

$$1_{\mathsf{Succ}}(\mathsf{f}, \mathsf{d}, \mathsf{snr}) = 1 - 1_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr}). \tag{3.8}$$

3.1.2 Randomness and Reliability

Based on our formulation from the pervious section, we now derive the outage probability for four different scenarios.

- **Fixed and known fading, f, and distance, d:**

  If both f and d are fixed and known, there is no randomness and the outage event becomes a degenerate random event with the following probability:

$$\mathsf{P}_{\mathsf{Outage}} = 1_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr}) = \begin{cases} 1 & \frac{|\mathsf{f}|^2}{\mathsf{d}^{\mathsf{k}}}\mathsf{snr} < 1 \\ 0 & \text{else} \end{cases} \tag{3.9}$$

- **Fading, f random and distance, d, known to the transmitter:**

  The outage probability in this case is obtained by taking the expectation of the previous case over different values of f. We have:

$$\begin{aligned} \mathsf{P}_{\mathsf{Outage}} &= \mathsf{E}_{\mathsf{f}}[1_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr})] \\ &= \mathsf{P}\left(\frac{|\mathsf{f}|^2}{\mathsf{d}^{\mathsf{k}}} < \frac{1}{\mathsf{snr}}\right) \\ &= \mathsf{F}_{|\mathsf{f}|^2}(\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}). \end{aligned}$$

where $\mathsf{F}_{|\mathsf{f}|^2}$ is the CDF of $|\mathsf{f}|^2$. For Rayleigh fading with $\mathsf{E}\left[|\mathsf{f}|^2\right] = \mu$, :

$$\mathsf{F}_{|\mathsf{f}|^2}(\mathsf{x}) = 1 - \exp(\frac{\mathsf{x}}{\mu}).$$

Hence:

$$\mathsf{P}_{\mathsf{Outage}} = 1 - \exp(\frac{-\mathsf{d}^{\mathsf{k}}}{\mu\ \mathsf{snr}}).$$

- **Fading, f, known to the transmitter and distance, d, random:**

  In this case, we get:

$$\begin{aligned} \mathsf{P}_{\mathsf{Outage}} &= \mathsf{E}_{\mathsf{d}}[1_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr})] \\ &= \mathsf{P}\left(\frac{|\mathsf{f}|^2}{\mathsf{d}^{\mathsf{k}}} < \frac{1}{\mathsf{snr}}\right) \\ &= 1 - \mathsf{F}_{\mathsf{d}}(\sqrt[\mathsf{k}]{|\mathsf{f}|^2\ \mathsf{snr}}), \end{aligned} \tag{3.10}$$

  where F_{d} is the CDF of the distance between the communicating nodes.

- **Both Fading, f, and distance, d, are random:**

  Taking the expectation over both the value of f and d, we get:

$$\mathsf{P}_{\mathsf{Outage}} = \mathsf{E}_{\mathsf{d}}[\mathsf{E}_{\mathsf{f}}[1_{\mathsf{Outage}}(\mathsf{f}, \mathsf{d}, \mathsf{snr})]].$$

  For Rayleigh fading with $\mathsf{E}\left[|\mathsf{f}|^2\right] = \mu$, we get

$$\mathsf{P}_{\mathsf{Outage}} = \mathsf{E}_{\mathsf{d}}[1 - \exp(\frac{-\mathsf{d}^{\mathsf{k}}}{\mu\ \mathsf{snr}})].$$

Table 3.1 summarizes the effect of channel fading or location randomness on the outage probability.

	Fading Known, $\lvert f \rvert^2 = \mu$	Raleigh Fading, $\mathsf{E}[\lvert \mathsf{f} \rvert^2] = \mu$
Distance Known	$\begin{cases} 1 & \frac{\mu\ \mathsf{snr}}{\mathsf{d}^\mathsf{k}} < 1 \\ 0 & \text{else} \end{cases}$	$1 - \mathsf{exp}(\frac{-\mathsf{d}^\mathsf{k}}{\mu\ \mathsf{snr}})$
Random Distance	$1 - \mathsf{F_d}\left(\sqrt[k]{\mu\ \mathsf{snr}}\right)$	$\mathsf{E_d}[1 - \mathsf{exp}(\frac{-\mathsf{d}^\mathsf{k}}{\mu\ \mathsf{snr}})]$

Table 3.1: Outage Probability under Various Channel Models

Notice that snr and μ always appear as a single product form in all four scenarios. To simplify the notation, we can absorb the effect of μ into the value of snr by defining $\mathsf{snr_{norm}}$ as

$$\mathsf{snr_{norm}} = \mu\ \mathsf{snr}.$$

Using this simplified notation, table 3.2 and 3.3, summarizes the results for outage and success probability, respectively:

	Fading Known	Raleigh Fading
Distance Known	$\begin{cases} 1 & \frac{\mathsf{snr_{norm}}}{\mathsf{d}^\mathsf{k}} < 1 \\ 0 & \text{else} \end{cases}$	$1 - \mathsf{exp}(\frac{-\mathsf{d}^\mathsf{k}}{\mathsf{snr_{norm}}})$
Random Distance	$1 - \mathsf{F_d}\left(\sqrt[k]{\mathsf{snr_{norm}}}\right)$	$\mathsf{E_d}[1 - \mathsf{exp}(\frac{-\mathsf{d}^\mathsf{k}}{\mathsf{snr_{norm}}})]$

Table 3.2: Outage Probability for a Point-to-Point Link

	Fading Known	Raleigh Fading
Distance Known	$\begin{cases} 1 & \frac{\mathsf{snr}}{\mathsf{d}^\mathsf{k}} < 1 \\ 0 & \text{else} \end{cases}$	$\mathsf{exp}(\frac{-\mathsf{d}^\mathsf{k}}{\mathsf{snr}})$
Random Distance	$\mathsf{F_d}\left(\sqrt[k]{\mathsf{snr}}\right)$	$\mathsf{E_d}[\mathsf{exp}(\frac{-\mathsf{d}^\mathsf{k}}{\mathsf{snr}})]$

Table 3.3: Success Probability for a Point-to-Point Link

Note that for notational convenience, we have dropped the norm subscript from $\mathsf{snr}_{\mathsf{norm}}$ in the above tables.

It is important to note that the probability of successful reception for a Rayleigh fading link with fixed distance is given by the simple expression

$$\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}, \mathsf{snr}) = \exp(-\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}). \tag{3.11}$$

3.1.3 Fading vs. No-Fading

Table 3.2 gives the expressions for the outage probability under various channel models. The actual probabilities, however, are in terms of the distribution for inter-node distance and it is not clear how these entries compare. In this part we give a more in-depth comparison of outage probability under various scenarios. The following theorem states the effect of randomness in fading and distance on point-to-point reliability.

Theorem 3 *Effect of Randomness on point-to-point reliability:*

1. *For a point-to-point link with Rayleigh fading and fixed distance, in the high-snr regime, the outage probability is* $\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}$.

2. *With Rayleigh fading and random distance, in the high-snr regime, the outage probability is* $\frac{\mathsf{E}[\mathsf{d}^{\mathsf{k}}]}{\mathsf{snr}}$ *as long as* $\mathsf{E}[\mathsf{d}^{\mathsf{k}}]$ *is finite.*

Proof:

1. Form Equation 3.10, we have

$$\begin{aligned} \mathsf{P}_{\mathsf{Outage}} &= 1 - \exp(\frac{-\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}) \\ &= \frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}} + \mathsf{O}\left(\left(\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}\right)^2\right) \\ &\approx \frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}} \end{aligned} \tag{3.12}$$

The approximation is valid for small values of $\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}$. For any fixed and finite d, this approximation is valid in the high-snr regime. This is in fact a very well known result, [19], and we have only mentioned this here for the sake of completeness.

2. For a fixed distance, the outage probability is given as

$$P_{Outage} = 1 - \exp(\frac{-d^k}{snr}) \tag{3.13}$$

Using the Taylor expansion to bound the exponential function, we have:

$$1 - x \leq \exp(-x) \leq 1 - x + \frac{x^2}{2} \quad \forall x \geq 0$$

Hence:

$$\frac{d^k}{snr} - \frac{1}{2}\left(\frac{d^k}{snr}\right)^2 \leq P_{Outage} \leq \frac{d^k}{snr}$$

Taking the expectations of both sides, we have:

$$\frac{E[d^k]}{snr} - \frac{1}{2}\left(\frac{E[d^k]}{snr}\right)^2 \leq P_{Outage} \leq \frac{E[d^k]}{snr}$$

Assuming $E[d^k]$ is finite, for large values of snr we can drop the second order term in the lower bound. Therefore, in the high-snr regime, we have the following approximation:

$$P_{Outage} \approx \frac{E_d[d^k]}{snr}$$

In fact, it can be shown that the outage probability when only distance is random decays faster than $\frac{E_d[d^k]}{snr}$.

Theorem 4 *For any distance distribution with finite mean, the outage probability without fading is smaller than* $\frac{E_d[d^k]}{snr}$.

Proof: From equation 3.10, we have:

$$\begin{aligned} P_{Outage} &= 1 - F_d(\sqrt[k]{snr}) \\ &= P(d > \sqrt[k]{snr}) \\ &= P(d^k > snr) \end{aligned}$$

The Markov inequality states that for any non-negative random variable, Y, with finite mean, $\mathsf{E}[\mathsf{Y}]$, the following inequality holds:

$$\mathsf{P}(\mathsf{Y} \geq \mathsf{y}) \leq \frac{\mathsf{E}[\mathsf{Y}]}{\mathsf{y}}$$

Applying this to our outage probability, we have:

$$\mathsf{P}_{\mathsf{Outage}} \quad \leq \quad \frac{\mathsf{E}_{\mathsf{d}}[\mathsf{d}^{\mathsf{k}}]}{\mathsf{snr}}$$

Although the bound in derived here is generally very loose, this proof is sufficient to show that without fading, outage decays faster than snr^{-1} and with Rayleigh fading, outage only decays as snr^{-1}. Hence, there is no circumstance in which we may benefit from fading in a point-to-point link.

3.1.4 Example

To illustrate the effect of fading and random distance on link reliability, and since we use the exponential internode distance in the next few section, we re-state the outage probability results from table 3.2 for the case of exponential distance. For the fading scenarios, we have only given the outage probability approximation in the high-snr region. Note that for an exponential random variable d with parameter λ

$$\begin{aligned} \mathsf{F}_{\mathsf{d}}(\mathsf{x}) &= 1 - \exp(-\lambda\, \mathsf{x}), \\ \mathsf{E}_{\mathsf{d}}[\mathsf{d}^{\mathsf{k}}] &= \frac{\mathsf{k}!}{\lambda^{\mathsf{k}}}. \end{aligned}$$

	No Fading	Raleigh Fading
Fixed Distance, $d = \frac{1}{\lambda}$	$\begin{cases} 1 & \lambda^{\mathsf{k}}\ \mathsf{snr} < 1 \\ 0 & \text{else} \end{cases}$	$\frac{1}{\lambda^{\mathsf{k}}\ \mathsf{snr}}$
Exponential Distance, $E[d] = \frac{1}{\lambda}$	$\exp\left(-\lambda \sqrt[\mathsf{k}]{\mathsf{snr}}\right)$	$\frac{\mathsf{k}!}{\lambda^{\mathsf{k}}\ \mathsf{snr}}$

Table 3.4: Outage Probability with Exponential Distance and Rayleigh Fading

Figure 3-1 illustrate the exact outage probability for the case of $\lambda = 1$ and $\mathsf{k} = 2$. Several observations are worth mentioning here.

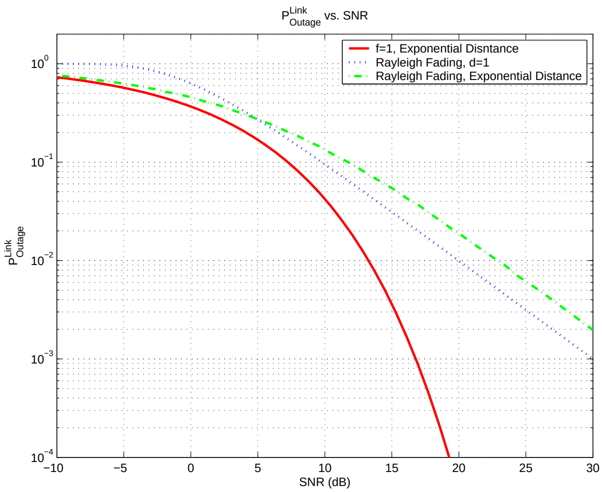

Figure 3-1: Point-to-Point Reliability

1. The effect of fading is significant. In all cases, the outage probability is much lower when there is no fading. With no fading, the outage probability decreases exponentially with snr. With Rayleigh fading, however, the outage probability only decrease linearly with snr^{-1}.

2. The outage probability for non-fading link is always less than the outage probability for fading link, i.e. there is no snr regime under which fading improves the reliability.

3. When there is fading, at low snr, the random distance case has a lower outage probability than the fixed distance. Intuitively, in this case a transmission is only successful when the communicating nodes are located very close to each other. At high snr, the effect of random distance is a factor of $\mathsf{k}!$ increase in the outage probability.

4. In all cases, the outage probability is a function of $\lambda^{\mathsf{k}}\,\mathsf{snr}$. We talk more about the significance of this factor when we look at route reliability in a line network.

3.2 Reliability at the Network Layer

In the first section, we developed the concept of reliability for a point-to-point wireless communication link. In this section, we take this concept one step further and look at reliability in a multi-hop wireless network. We assume that the fading factors for different links are independent and identically distributed Rayleigh random variables. A route is a sequence of nodes that information is relayed through from source to destination, i.e.

$$\begin{aligned} \mathsf{Route} &= (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{h-1}, \mathsf{r}_h) \\ &= (\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{h-1}, \mathsf{d}). \end{aligned}$$

where, $\mathsf{r}_0 = \mathsf{s}$, $\mathsf{r}_h = \mathsf{d}$, and h is the number of hops. We assume the network operates based on a time division protocol under which successive transmissions along a route happen in consecutive transmission slots. Route $(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{h-1}, \mathsf{d})$ is identical to a sequence of h point-to-point links, where for the i^{th} link, relay $\mathsf{i}-1$ is the transmitter and relay i is the receiver, $\mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i}$ is the transmitted signal-to-noise power, and $\mathsf{d}_{\mathsf{r}_{i-1}\mathsf{r}_i}$ is the distance between the nodes. We define the event of successful end-to-end transmission as the event that all h transmissions are successful and the *End-to-End Reliability* is defined as the probability of this event . Based on our assumption regarding the independence of the fading factor for different links, and using results from equation 3.11, the end-to-end reliability can be written as:

$$\begin{aligned} \mathsf{Reliability}^{(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{h-1}, \mathsf{r}_h)} &= \prod_{i=1}^{h} \exp\left(-\frac{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}{\mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i}}\right) \\ &= \exp\left(-\sum_{i=1}^{h} \frac{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}{\mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i}}\right). \end{aligned} \tag{3.14}$$

The corresponding total amount of power spent for successful end-to-end transmission is

$$\mathsf{SNR}_{\mathsf{Total}}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{h-1},\mathsf{r}_h)} = \sum_{i=1}^{h} \mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i}. \quad (3.15)$$

We refer to this quantity as the *end-to-end power*. In section one, we analyzed the relationship between reliability and power in a point-to-point link. In the network setting, we are interested in the relationship between the end-to-end reliability and power. There are three different problems that we look at in the next section:

1. What is the end-to-end reliability if the maximum transmitted power per link is fixed?
2. What is the maximum end-to-end reliability for a fixed end-to-end power?
3. What is the minimum end-to-end power required to achieve a guaranteed level of end-to-end reliability?

The first problem is motivated by the fact that in some cases the transmitted power by each node might be limited due to hardware constraints or to limit the interference level to other nodes. The second problem is a power allocation problem, where the objective is to maximize the end-to-end reliability of a route subject to a total power constraint. The last problem is also a power allocation problem, where the objective is to minimize the total consumed power subject to a guaranteed level of end-to-end reliability.

3.2.1 Route Reliability

In this part, we look at the end-to-end reliability and power in the three scenarios discussed at the end of the last section.

1. **End-to-End Reliability for a Fixed Maximum Transmission Power Per Link**

   Assuming the transmitted signal-to-noise ratio at each link is limited to $\mathsf{SNR}_{\mathsf{Link-Max}}$, the corresponding route reliability can be readily calculated using equation **??**.

The end-to-end reliability in this case is given by

$$\begin{aligned}\mathsf{Reliability}^{(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})} &= \mathsf{exp}\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{SNR}_{\mathsf{Link-Max}}}\right) \\ &= \mathsf{exp}\left(-\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{SNR}_{\mathsf{Link-Max}}}\right). \end{aligned} \tag{3.16}$$

Lemma 2 *For a fixed route* $(\mathsf{r_0}, \mathsf{r_1}, \cdots, \mathsf{r_{h-1}}, \mathsf{r_h})$ *and fixed maximum transmitted snr per node,* $\mathsf{SNR}_{\mathsf{Link-Max}}$*, the end-to-end reliability is* $\mathsf{exp}\left(-\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{SNR}_{\mathsf{Link-Max}}}\right)$.

2. **Maximum End-to-End Reliability for a Fixed End-to-End Power**

The problem of achieving maximum end-to-end reliability for a fixed end-to-end power is formulated by the following constrained optimization problem:

$$\begin{aligned} \mathsf{max} \quad & \mathsf{Reliability}^{(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})} \\ \mathsf{s.t} \quad & \mathsf{SNR}^{(\mathsf{r_0},\mathsf{r_1},\cdots,\mathsf{r_{h-1}},\mathsf{r_h})}_{\mathsf{Total}} \leq \mathsf{SNR}_{\mathsf{Total-Max}}, \end{aligned} \tag{3.17}$$

where the optimization is done over the values of the transmitted snr by the intermediate relay nodes along the route. Using equation 3.14 and equation 3.15, we rewrite this optimization problem in terms of the transmitted powers as:

$$\begin{aligned} \mathsf{max} \quad & \mathsf{exp}\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{snr}_{\mathsf{r_{i-1}r_i}}}\right) \\ \mathsf{s.t} \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r_{i-1}r_i}} \leq \mathsf{SNR}_{\mathsf{Total-Max}}. \end{aligned}$$

The objective is equivalent to minimizing the inner sum term. Furthermore, the inequality in the constraint can be replaced with equality. So, the problem is equivalent to:

$$\begin{aligned} \mathsf{min} \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}{\mathsf{snr}_{\mathsf{r_{i-1}r_i}}} \\ \mathsf{s.t} \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r_{i-1}r_i}} = \mathsf{SNR}_{\mathsf{Total-Max}}. \end{aligned}$$

where the optimization is done over the values of the transmitted snr by the intermediate relay nodes along the route. Setting up the Lagrangian for this problem, we have:

$$\mathsf{L}(\mathsf{snr}_{\mathsf{r}_0\mathsf{r}_1}, \cdots, \mathsf{snr}_{\mathsf{r}_{h-1}\mathsf{r}_h}, \lambda) = \sum_{i=1}^{h} \frac{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}{\mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i}} + \lambda \left(\sum_{i=1}^{h} \mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i} - \mathsf{SNR}_{\mathsf{Total-Max}} \right).$$

The first order condition is

$$\frac{\partial \mathsf{L}}{\partial \mathsf{snr}_{\mathsf{r}_{i-1}\mathsf{r}_i}} = -\frac{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}{\mathsf{snr}^2_{\mathsf{r}_{i-1}\mathsf{r}_i}} + \lambda = 0.$$

The optimal transmitted signal-to-noise power is given by

$$\widehat{\mathsf{snr}}_{\mathsf{r}_{i-1}\mathsf{r}_i} = \sqrt{\frac{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}{\lambda}}.$$

Substituting this back into the constraint equation and solving for the optimal λ, we get

$$\widehat{\lambda} = \left(\frac{\sum_{i=1}^{h} \sqrt{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}}{\mathsf{SNR}_{\mathsf{Total-Max}}} \right)^2.$$

The optimal transmitted signal-to-noise ratio is

$$\widehat{\mathsf{snr}}_{\mathsf{r}_{i-1}\mathsf{r}_i} = \mathsf{SNR}_{\mathsf{Total-Max}} \frac{\sqrt{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}}{\sum_{i=1}^{h} \sqrt{\mathsf{d}^{k}_{\mathsf{r}_{i-1}\mathsf{r}_i}}}. \tag{3.18}$$

For this set of power allocation, the end to end reliability of the route is

$$\begin{aligned}
\mathsf{Reliability}_{\mathsf{Optimal}}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} &= \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\widehat{\mathsf{snr}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right) \\
&= \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\sum_{\mathsf{j}=1}^{\mathsf{h}}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{j}-1}\mathsf{r}_\mathsf{j}}}}{\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}} \frac{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right) \\
&= \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\left(\sum_{\mathsf{j}=1}^{\mathsf{h}}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{j}-1}\mathsf{r}_\mathsf{j}}}\right)\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right) \\
&= \exp\left(-\frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right). \qquad (3.19)
\end{aligned}$$

For easier future reference, we state this result in lemma 3.

Lemma 3 *For a fixed route* $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$ *and for a fixed end-to-end power of* $\mathsf{SNR}_{\mathsf{Total-Max}}$*, the maximum end-to-end reliability is*

$$\mathsf{Reliability}_{\mathsf{Optimal}} = \exp\left(-\frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right),$$

and the optimal power allocation that achieves this reliability is

$$\widehat{\mathsf{snr}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} = \mathsf{SNR}_{\mathsf{Total-Max}} \frac{\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}}{\sum_{\mathsf{i}=1}^{\mathsf{h}}\sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}}.$$

3. **Minimum End-to-End Power for a Guaranteed End-to-End Reliability**

   This problem can also be formulated as a constraint optimization problem as given below:

$$\begin{aligned}
\min \quad & \mathsf{SNR}_{\mathsf{Total}}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} \\
\text{s.t.} \quad & \mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} \geq \mathsf{Reliability}_{\mathsf{Min}}, \qquad (3.20)
\end{aligned}$$

where the optimization is done over the values of the transmitted snr by the intermediate relay nodes along the route. Using equation **??** and equation 3.15 to rewrite this problem, we get:

$$\begin{aligned} \min \quad & \sum_{i=1}^{h} \mathsf{snr}_{r_{i-1}r_i} \\ \text{s.t.} \quad & \exp\left(-\sum_{i=1}^{h} \frac{d^k_{r_{i-1}r_i}}{\mathsf{snr}_{r_{i-1}r_i}}\right) \geq \mathsf{Reliability}_{\mathsf{Min}}. \end{aligned} \tag{3.21}$$

Since exponential is a monotonically increasing function, the constraint must be satisfied with equality at the optimal solution. So, the optimization problem is equivalent to:

$$\begin{aligned} \min \quad & \sum_{i=1}^{h} \mathsf{snr}_{r_{i-1}r_i} \\ \text{s.t.} \quad & \sum_{i=1}^{h} \frac{d^k_{r_{i-1}r_i}}{\mathsf{snr}_{r_{i-1}r_i}} = -\ln(\mathsf{Reliability}_{\mathsf{Min}}). \end{aligned} \tag{3.22}$$

The Lagrangian for this problem is given by:

$$L(\mathsf{snr}_{r_0r_1}, \cdots, \mathsf{snr}_{r_{h-1}r_h}, \lambda) = \sum_{i=1}^{h} \mathsf{snr}_{r_{i-1}r_i} + \lambda\left(\sum_{i=1}^{h} \frac{d^k_{r_{i-1}r_i}}{\mathsf{snr}_{r_{i-1}r_i}} + \ln(\mathsf{Reliability}_{\mathsf{Min}})\right). \tag{3.23}$$

The partial derivatives with respect to the transmitted snr at each intermediate relay is:

$$\frac{\partial L}{\partial \mathsf{snr}_{r_{i-1}r_i}} = 1 - \lambda \frac{d^k_{r_{i-1}r_i}}{\mathsf{snr}^2_{r_{i-1}r_i}}.$$

Setting these first order conditions to 0 and solving for the optimal transmitted snr, we get:

$$\widehat{\mathsf{snr}}_{r_0r_1} = \sqrt{\lambda\, d^k_{r_{i-1}r_i}}$$

Substituting these into the constraint and solving for the optimal λ, we have:

$$\begin{aligned}
\sum_{i=1}^{h} \frac{d^k_{r_{i-1}r_i}}{\widehat{snr}_i} &= -\ln(\text{Reliability}_{\text{Min}}) \\
\sum_{i=1}^{h} \frac{d^k_{r_{i-1}r_i}}{\sqrt{\widehat{\lambda}\ d^k_{r_{i-1}r_i}}} &= -\ln(\text{Reliability}_{\text{Min}}) \\
\frac{\sum_{i=1}^{h} \sqrt{d^k_{r_{i-1}r_i}}}{\sqrt{\widehat{\lambda}}} &= -\ln(\text{Reliability}_{\text{Min}}) \\
\sqrt{\widehat{\lambda}} &= \frac{\sum_{i=1}^{h} \sqrt{d^k_{r_{i-1}r_i}}}{-\ln(\text{Reliability}_{\text{Min}})}.
\end{aligned}$$

The optimal transmitted signal-to-noise ratio for each node is:

$$\widehat{snr}_{r_{i-1}r_i} = \frac{\sum_{i=1}^{h} \sqrt{d^k_{r_{i-1}r_i}}}{-\ln(\text{Reliability}_{\text{Min}})} \sqrt{d^k_{r_{i-1}r_i}}. \tag{3.24}$$

The resulting optimal end-to-end power is given by:

$$\begin{aligned}
\widehat{SNR}_{\text{Total}} &= \sum_{i=1}^{h} \widehat{snr}_{r_{i-1}r_i} \\
&= \sum_{i=1}^{h} \left(\frac{\sum_{j=1}^{h} \sqrt{d^k_{r_{j-1}r_j}}}{-\ln(\text{Reliability}_{\text{Min}})} \right) \sqrt{d^k_{r_{i-1}r_i}} \\
&= \frac{\left(\sum_{i=1}^{h} \sqrt{d^k_{r_{i-1}r_i}}\right)^2}{-\ln(\text{Reliability}_{\text{Min}})}.
\end{aligned} \tag{3.25}$$

For easier future reference, we state this result in lemma 4.

Lemma 4 *For a fixed route* $(r_0, r_1, \cdots, r_{h-1}, r_h)$*, the minimum required total power to guarantee end-to-end reliability of* $\text{Reliability}_{\text{min}}$ *is:*

$$\widehat{SNR}_{\text{Total}} = \frac{\left(\sum_{i=1}^{h} \sqrt{d^k_{r_{i-1}r_i}}\right)^2}{-\ln(\text{Reliability}_{\text{Min}})}$$

and the optimal power that achieves this total consumed power is

$$\widehat{\mathsf{snr}}_{\mathsf{r_{i-1}r_i}} = \frac{\sum_{\mathsf{i=1}}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}}{-\ln(\mathsf{Reliability_{Min}})} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}$$

3.2.2 Optimal Reliability-Power Curve

A careful reader might notice that the two optimization problems that we looked at in the last section, formulated in 3.17 and 3.20, are in fact dual problems. To clarify this point, we present a graphical illustration of the relationship between the end-to-end reliability and power.

For any fixed route, different power allocation schemes result in different end-to-end reliability and consumed power. If we were to characterize each power allocation scheme only by the total consumed power and the resulting end-to-end reliability, each allocation scheme could be represented by a point in the two dimensional plot of the end-to-end reliability vs. the total snr. Certain allocation schemes are optimal, i.e. minimize the total power consumed to achieve a guaranteed end-to-end reliability or maximize the end-to-end reliability for a fixed consumed power.

In problem 2, we found the optimal power allocation to maximize the end-to-end reliability for a given end-to-end power. This corresponds to moving along the vertical line in figure 3-2 and finding the allocation scheme that maximizes the reliability for $\mathsf{SNR_{Total-Max}}$ We found that the resulting end-to-end reliability for this optimal allocation is:

$$\mathsf{Reliability_{Optimal}} = \exp\left(-\frac{\left(\sum_{\mathsf{i=1}}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r_{i-1}r_i}}}\right)^2}{\mathsf{SNR_{Total-Max}}}\right) \tag{3.26}$$

In problem 3, we found the optimal power allocation to minimizes the total power subject to a guaranteed end-to-end reliability. Graphically, this optimization corresponds to moving along the horizontal line in figure 3-2 and finding the allocation scheme that minimized the total consumed power for the end-to-end reliability of

$\mathsf{Reliability_{min}}$ We found that the reliability and power corresponding to the optimal allocation are related by the following relationship:

$$\widehat{\mathsf{SNR}}_{\mathsf{Total}} = \frac{\left(\sum_{i=1}^{h}\sqrt{d_{r_{i-1}r_i}^{k}}\right)^2}{-\ln(\mathsf{Reliability_{Min}})}. \tag{3.27}$$

Clearly, the curve specified by equation 3.26 and equation 3.27 are identical. This set of optimal power allocation can be represented by a single curve in the two dimensional plot of the end-to-end reliability vs. total power as shown in figure 3-2. We refer to this curve as the *Optimal Reliability-Power Trade-off* curve. The relationship between the end-to-end reliability and consumed power for power allocation schemes on this curve is given by:

$$\mathsf{Reliability} = \exp\left(-\frac{\left(\sum_{i=1}^{h}\sqrt{d_{r_{i-1}r_i}^{k}}\right)^2}{\mathsf{SNR_{Total}}}\right) \tag{3.28}$$

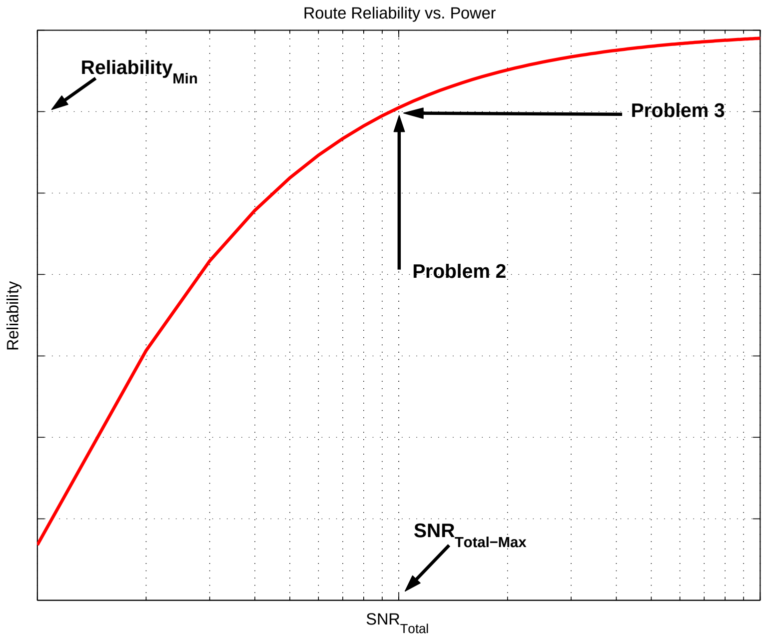

Figure 3-2: Route Reliability vs. Power

3.2.3 Route Outage-Power Trade-off

It is insightful to look at the relationship between the end-to-end reliability and power to better understand the difference between different power allocation schemes. For the case that the maximum transmitted power at each link was limited to $\mathsf{SNR}_{\mathsf{Max-Link}}$, equation 3.16 gives the end-to-end reliability. We get more insight into the relationship between power and reliability by looking at the *route outage* probability. The route outage probability, ρ, is defined as:

$$\rho^{(r_0,r_1,\cdots,r_{h-1},r_h)} = 1 - \mathsf{Reliability}^{(r_0,r_1,\cdots,r_{h-1},r_h)}.$$

Writing 3.16 in terms of the outage probability, we have:

$$\begin{aligned} 1-\rho &= \exp\left(-\frac{\sum_{i=1}^{h} d_{r_{i-1}r_i}^{k}}{\mathsf{SNR}_{\mathsf{Max-Link}}}\right), \\ \ln(1-\rho) &= -\frac{\sum_{i=1}^{h} d_{r_{i-1}r_i}^{k}}{\mathsf{SNR}_{\mathsf{Max-Link}}}. \end{aligned}$$

For small values of ρ, we can use the approximation of $\ln(1-\rho) \approx \rho$ to simplify this relation to:

$$\rho \approx \frac{\sum_{i=1}^{h} d_{r_{i-1}r_i}^{k}}{\mathsf{SNR}_{\mathsf{Max-Link}}}. \tag{3.29}$$

The relationship between reliability and power with optimal power allocation is given by equation 3.28. Writing 3.28 in terms of the route outage probability, we have

$$\begin{aligned} 1-\rho &= \exp\left(-\frac{\left(\sum_{i=1}^{h}\sqrt{d_{r_{i-1}r_i}^{k}}\right)^2}{\mathsf{SNR}_{\mathsf{Total}}}\right), \\ \ln(1-\rho) &= -\frac{\left(\sum_{i=1}^{h}\sqrt{d_{r_{i-1}r_i}^{k}}\right)^2}{\mathsf{SNR}_{\mathsf{Total}}}. \end{aligned}$$

Using the same approximation for small ρ, this relation can be simplified to:

$$\rho \approx \frac{\left(\sum_{i=1}^{h} \sqrt{d_{r_{i-1}r_i}^{k}}\right)^2}{\mathsf{SNR}_{\mathsf{Total}}}. \tag{3.30}$$

From 3.29 and 3.30, we observe that route outage decays as $\mathsf{SNR}_{\mathsf{Max-Link}}^{-1}$ and $\mathsf{SNR}_{\mathsf{Total}}^{-1}$, respectively, in the regimes where the route is highly reliable. It is not surprising that we observe this type of relation as these relationships are very similar to what we observed in the first section for a point-to-point link. In the last section, it is shown how diversity at the route level can fundamentally change the form of this trade-off.

3.2.4 Optimal Route Selection

In this section, we look at the problem of finding the optimal route between a source and destination pair of nodes in a multi-hop wireless network. We have looked at this problem under the following 3 scenarios.

1. Finding the most reliable route when the maximum transmitted power in each link is fixed.

2. Finding the most reliable route for a fixed end-to-end power.

3. Finding the minimum power route for a guaranteed end-to-end reliability.

1. **Maximum End-to-End Reliability for a Fixed Maximum Transmission Power Per Link**

   In 3.16, we showed that for any route $(r_0, r_1, \cdots, r_{h-1}, r_h)$, the end-to-end reliability is a monotonically decreasing function of $\sum_{i=1}^{h} d_{r_{i-1}r_i}^{k}$. This quantity can be treated as the cost metric for the route and the most reliable route between two nodes is the route that minimizes this cost metric. We refer to route selection algorithm based on this cost metric as the **M***ximum* **R***eliability for Fixed* **L***ink SNR* **R***oute*, *MRLR*, algorithm.

Theorem 5 *The most reliable route between nodes* s *and* d *in a fixed multi-hop wireless network where the fading parameters of different links are independent Rayleigh random variables and the maximum transmitted* snr *at each node is limited to* $\mathsf{SNR}_{\mathsf{Max-Link}}$ *is the route*

$$(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d}) = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_{\mathsf{h}})$$

that minimizes

$$\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_{\mathsf{i}}},$$

and the reliability of this route is given by

$$\mathsf{Reliability}^{(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_{\mathsf{h}})} = \exp\left(-\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_{\mathsf{i}}}}{\mathsf{SNR}_{\mathsf{Max-Link}}}\right).$$

Proof:

From 3.16, we know that route reliability is a decreasing function of $\sum_{i=1}^{h} d^{k}_{r_{i-1}r_i}$. The most reliable route, i.e. the minimum outage probability route, is the route that minimizes this sum. Furthermore, based on 3.16, the reliability of this route is as claimed.

2. **Maximum End-to-End Reliability for a Fixed End-to-End Power**

We now turn our attention to a slightly different problem. Assuming that the maximum amount of power that can be spent in relaying the information from the source node to the destination node is limited to $\mathsf{SNR}_{\mathsf{Total}}$, our goal is to find the most reliable route between a source-destination pair of nodes. From lemma 3, we know for any route $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_{\mathsf{h}})$, the end-to-end reliability is a monotonically decreasing function of $\sum_{i=1}^{h} \sqrt{d^{k}_{r_{i-1}r_i}}$. The maximum reliability route is the route that minimizes this sum. We refer to this route selection algorithm as the **M***aximum* **R***eliability for Fixed* **E***nd-to-End SNR* **R***oute*, *MRER*, algorithm.

Theorem 6 *The most reliable route between nodes* s *and* d *in a fixed multi-hop wireless network where the fading parameters of different links are independent Rayleigh random variables and the maximum end-to-end power is limited to* $\mathsf{SNR}_{\mathsf{Total-Max}}$ *is the route*

$$(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d}) = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_{\mathsf{h}})$$

that minimizes

$$\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_{\mathsf{i}}}},$$

and the corresponding end-to-end reliability is given by

$$\mathsf{Reliability} = \exp\left(-\frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_{\mathsf{i}}}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right).$$

Proof:

From lemma 3, we know that route reliability is a decreasing function of $\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_{\mathsf{i}}}}$. The most reliable route, i.e. the minimum outage probability route, is the route that minimizes this sum. Furthermore, based on lemma 3, the reliability of this route is as claimed.

3. **Minimum End-to-End Power for a Guaranteed End-to-End Reliability**

Lastly, we look at the problem of selecting the route between two nodes that requires the least amount of end-to-end power to achieve a desired level of end-to-end reliability. From lemma 4, we know that for any route $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_{\mathsf{h}})$, the end-to-end route power is a monotonically increasing function of $\sum_{i=1}^{h} \sqrt{d^k_{r_{i-1}r_i}}$. Hence, the minimum power route is the route among all possible routes between two nodes that minimizes this sum. We refer to this route selection scheme as **M***inimum* **P***ower for Fixed End-to-End Reliability* **R***oute*, *MPR*, algorithm.

Theorem 7 *The minimum power route between nodes* s *and* d *in a fixed multi-hop wireless network where the fading parameters for different links are independent Rayleigh random variables to achieve guaranteed end-to-end reliability of* $\mathsf{Reliability_{Min}}$ *is the route*

$$(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d}) = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$$

that minimizes

$$\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}},$$

and the corresponding end-to-end power is given by

$$\mathsf{SNR_{Total}} = \frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{-\ln(\mathsf{Reliability_{Min}})}.$$

Proof:

From theorem 4, we know that route power is an increasing function of $\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^{\mathsf{k}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}$. So, the minimum power route is the route that minimizes this sum. Furthermore, based on theorem 4, the total power for this route is as claimed.

3.2.5 Routing Algorithms

In section 3.2.3, we looked at three routing formulations: MRLR (Theorem 5), MRER (Theorem 6), and MPR (Theorem 7). The route selection metric was shown to only be a function of the distance between nodes in all three algorithms. A shortest path algorithm can be used in a straightforward way to find the optimal route in all three cases. See [18] for several variations of the shortest path algorithm. The complexity of finding the optimal route is $\mathsf{O}(\mathsf{N}^2)$, where N is the number of nodes in the network.

To get more insight into the difference between these formulations, we can compare their cost metrics. MPR and MRER have identical cost metrics, see theorem 6 and theorem 7. This similarity is intuitively plausible: the route that has maximum reliability for a given power (MRER) is also the route that minimizes power to achieve that level of reliability (MPR). There is however an interesting difference between the cost metric used by MPR/MRER and the cost metric for MRLR. The solution for the first two problems minimizes

$$\sum_{i=1}^{h} \sqrt{d^{k}_{r_{i-1}r_i}}, \tag{3.31}$$

while the cost metric for MRLR is

$$\sum_{i=1}^{h} d^{k}_{r_{i-1}r_i}. \tag{3.32}$$

Comparing 3.31 and 3.32, it is clear that in 3.31 the distance between nodes is raised to a smaller power. Assuming all distance are normalized such that all distance are greater than one, MRLR associates a larger penalty for selecting long hops. A route selected based on this criteria would tend to use more but shorter hops in comparison with a MPR/MRER route. This difference is due to the fact that the transmitted snr can be adjusted to compensate for the higher attenuation in MRER/MPR whereas the transmitted snr per link is fixed in MRLR.

As the last point, we would like to mention some implementation issues regarding these algorithms. The process of routing a packet between two nodes consists of two steps: route selection and relaying. The route selection processes can be done in a distributed way using any distributed shortest path algorithm. Hence, selecting the optimal route does not require knowledge of the entire network topology. Once the optimal route is selected, the information must be relayed by the nodes along the selected route. For MRER and MPR routes, optimization was done to find the power allocation for each link along the optimal route. The optimal transmitted snr at each hop is given in Theorem 6 and Theorem 7. The difficulty lies in the fact that these optimal transmitted snr values depend on some non-local information, namely, the distance between all relay pairs and the desired end-to-end reliability or power. This information must be made available to each hop along the selected route. We will not get into the details of this issue here.

3.3 Route Diversity

Before introducing the idea of route diversity, let's review our findings so far. In section 3.1, we looked at the reliability of a point-to-point link and analyzed the effect of randomness in the channel state of distance on this reliability. We observed that Rayleigh fading has a significant impact on the relationship between reliability and power.

In section 3.2, we extended the idea of reliability from the physical layer, i.e. point-to-point link, to the network layer. We introduced the concept of end-to-end route reliability and looked at several route selection algorithms. We also looked at the relationship between the route outage probability and power. For the case of fixed transmitted snr at each link, we observed that the outage probability decreases as the inverse of the maximum transmitted snr per link (see equation 3.30.) For the case of a fixed end-to-end power, we also observed a similar relationship in 3.29.

In a multi-hop wireless network it may be possible to improve the end-to-end reliability by benefiting from the wireless broadcast advantage and independence between Rayleigh fading channels. This is the motivation behind the *Route Diversity* idea that we introduce in this section. For our analysis in this section, we assume that there is a fixed maximum transmitted snr for each node.

To understand the shortcoming of the routing process that we focused on in Section 3.2, we should look at the details of the routing process in a multi-hop network. First, the most reliable route is selected based on MRLR algorithm. Suppose the selected route is:

$$\mathsf{Route}_{\mathsf{optimal}} = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_\mathsf{h}),$$

where $\mathsf{r}_0 = \mathsf{s}$, $\mathsf{r}_\mathsf{h} = \mathsf{d}$, and there are h hops in this route.

As discussed in Section 2, subsequent transmissions take place in different time-slot. For instance, in the i^{th} time-slot, the information signal is transmitted by node $\mathsf{r}_{\mathsf{i}-1}$ and is received by r_i. The information is successfully routed to the destination if it is received by the destination in time-slot h. This event requires h successful point-to-point transmissions. The probability of this event was calculated in the previous section (see equation 3.16.) We refer to this routing scheme as the *Simple* or *Non-Diversified Routing Scheme.*

To understand the shortcoming of the process explained above, consider a relay node such as r_i. In the simple routing scheme, this node can only receive the information via relay r_{i-1}. Motivated by the broadcast property of the wireless medium, we propose to modify the non-diversified routing scheme in the following way: instead of limiting relay r_i to only listen for the transmitted signal in the i^{th} transmission slot, we also consider the possibility that this node can receive the information during earlier transmission slots by other relays along the selected route. This gives each node along the route several chances to receive the information and hence introduces a level of diversity into the routing process. To simplify our analysis, we ignore the possibility of energy accumulation over several transmission slot, and partial decoding. In particular, we assume if the received signal power in a single transmission is high enough to allow full decoding of the information, then the information is received successfully; otherwise, the received signal is discarded.

To clarify this idea, let's look at a simple example. Assume that in the network shown in figure 3-3 the most reliable route is selected by the source as shown in the left-hand-side diagram. Without diversity, a successful end-to-end route would require 3 successful point-to-point transmissions. By adding the possibility of diversity as shown in the right-hand-side diagram in figure 3-3, we increase the probability of successful reception by d by including several new paths. For instance, d can now receive the information directly from s in the first transmission slot, from r_0 in the second transmission slot or from r_1 in the third transmission slot. We refer to this routing scheme as the *Diversified Routing Scheme.*

3.3.1 Reliability Formulation with Diversity

Before looking at the end-to-end reliability in a diversified route, we need to give a more precise description of the relaying process in the diversified routing scheme. Assume $\mathsf{Route}_{\mathsf{optimal}} = (r_0, r_1, \cdots, r_h)$ is selected as the most reliable route to the destination. In the diversified routing scheme that we analyze in this and the next section,

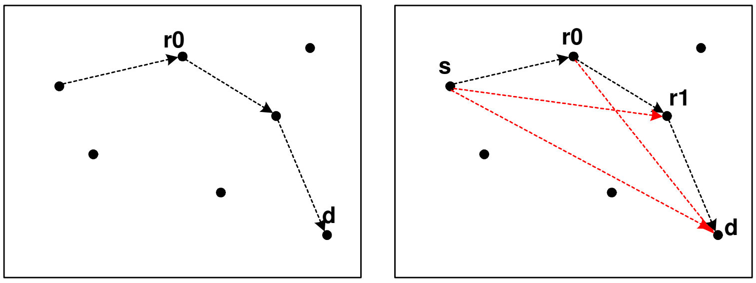

Figure 3-3: Route Diversity

nodes operate according to the following rules: node i, $i \geq 1$, transmits in slot $i+1$ if it has received the information in time slots $\{1, \cdots, i\}$. Otherwise, no transmission takes place in time slot $i+1$. Given this protocol, successful relaying of the information from r_0 to r_h takes no more than h time slots and no more than h units of transmitted power. This route is defined to be in outage if the information is not received by r_h by the end of time slot h. The end-to-end outage probability is defined as the probability of this event.

Defining the diversified route according to these rules is the simplest way that would allow us to compare the end-to-end outage in simple and diversified route on fair basis, We can compare the end-to-end outage probability since both routing schemes use the same number of transmission slots and the same amount of total power. There are other ways to take advantage of diversity, for instance to reduce the end-to-end delay or reduce the end-to-end power consumption. However, since our main focus in this part is to study the end-to-end reliability, it will keep all other parameters constant and solely focus on the reliability improvement.

For route $(r_0, r_1, \cdots, r_h)$, let $R[i,t]$ be the probability that node i has received the information by time-slot t. This probability can be calculate by the following recursive expression:

$$R[i,t] = \begin{cases} R[i,t-1] + (1 - R[i,t-1])R[t-1,t-1]P_{Succ}(d_{r_i r_{t-i}}, snr) & t \leq i \\ R[i,i] & \text{else} \end{cases} \tag{3.33}$$

where $P_{Succ}(d, snr)$ is given by equation 3.11. This expression requires some clarifications. Note that node slotted to transmit in time t is node $\mathsf{t}-1$. If node i has not received the information by time $\mathsf{t}-1$, then the only way it can receive the information by time t is node $\mathsf{t}-1$ has received the information by time $\mathsf{t}-1$, and the transmission from node $\mathsf{t}-1$ to node i is successful. Different components in the first part of the expression correspond to the probability of these event. $\mathsf{R}[\mathsf{i},\mathsf{t}-1]$ is the probability that node i has received the information prior to time-slot t, $\mathsf{R}[\mathsf{t}-1,\mathsf{t}-1]$ is the probability that node $\mathsf{t}-1$ has received the information by time-slot $\mathsf{t}-1$ and would transmit in time-slot t, and $\mathsf{d}_{\mathsf{r}_\mathsf{i}\mathsf{r}_{\mathsf{t}-\mathsf{i}}}$ is distance between r_i and $\mathsf{r}_{\mathsf{t}-\mathsf{i}}$. The route reliability and outage probability are defined as:

$$
\begin{aligned}
\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_\mathsf{h})} &= \mathsf{R}[\mathsf{h},\mathsf{h}], &(3.34)\\
\rho^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_\mathsf{h})} &= 1-\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_\mathsf{h})}\\
&= 1-\mathsf{R}[\mathsf{h},\mathsf{h}] &(3.35)
\end{aligned}
$$

These results can only be applied to a fixed network. In our analysis, we will look at the end-to-end reliability in networks with randomly distributed nodes. In this case, we can use the law of iterative expectations to find the end-to-end reliability by first conditioning on the topology, i.e. the node locations, and then taking the expectation over the location of nodes. The more appropriate definition for end-to-end reliability or the end-to-end outage in that setting are:

$$
\begin{aligned}
\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_\mathsf{h})} &= \mathsf{E}_{\text{Node Locations}}\left[\mathsf{R}[\mathsf{h},\mathsf{h}]\right], &(3.36)\\
\rho^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_\mathsf{h})} &= 1-\mathsf{E}_{\text{Node Locations}}\left[\mathsf{R}[\mathsf{h},\mathsf{h}]\right]. &(3.37)
\end{aligned}
$$

3.3.2 Example

In this part we give two examples to illustrate how the proposed route diversity can significantly improve the reliability. Our examples focus on two very simple 2-hop random networks. In both networks, figures 3-4 and 3-5, s and d are located two unit distance apart. In the first network, figure 3-4, the relay node is placed uniformly on the line connecting s to d. In the second example, figure 3-5, the relay node is placed uniformly inside the circle centered at the mid-point between s and d. We assume path-loss exponent $\mathsf{k} = 2$. The most reliable route in both networks is $(\mathsf{s}, \mathsf{r}, \mathsf{d})$.

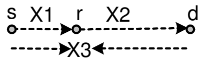

Figure 3-4: 2-Hop Line Network

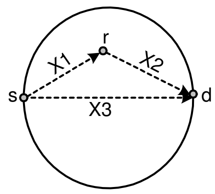

Figure 3-5: 2-Hop Disk Network

The outage probability for the non-diversified route is found by first conditioning on the location of the nodes and applying the result from equation 3.16. We have:

$$\rho^{(\mathsf{s},\mathsf{r},\mathsf{d})}_{\mathsf{Non-Diversified}} = 1 - \mathsf{E}_{\mathsf{x}_1,\mathsf{x}_2}[\exp(-\frac{\mathsf{x}_1^2 + \mathsf{x}_2^2}{\mathsf{snr}})] \tag{3.38}$$

$$\approx \mathsf{E}_{\mathsf{x}_1,\mathsf{x}_2}[\frac{\mathsf{x}_1^2 + \mathsf{x}_2^2}{\mathsf{snr}}], \tag{3.39}$$

where the approximation is valid in the high-snr regime.

In the diversified scheme, successful relaying requires either a successful direct transmission $\{\mathsf{s} \to \mathsf{d}\}$, or successful multi-hoping, $\{\mathsf{s} \to \mathsf{r}\}$ followed by $\{\mathsf{r} \to \mathsf{d}\}$. This probability can be calculated by using the recursive expression 3.33. Thus:

$$\begin{aligned}
\mathsf{R}[1,0] &= 0 \\
\mathsf{R}[2,0] &= 0 \\
\mathsf{R}[1,1] &= \mathsf{R}[1,0] + (1 - \mathsf{R}[1,0])\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_{\mathsf{sr}}, \mathsf{snr}) \\
&= \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_1, \mathsf{snr}) \\
\mathsf{R}[2,1] &= \mathsf{R}[2,0] + (1 - \mathsf{R}[2,0])\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_{\mathsf{sd}}, \mathsf{snr}) \\
&= \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_3, \mathsf{snr}) \\
\mathsf{R}[2,2] &= \mathsf{R}[2,1] + (1 - \mathsf{R}[2,1])\mathsf{R}[1,1]\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_{\mathsf{rd}}, \mathsf{snr}) \\
&= \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_3, \mathsf{snr}) + (1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_3, \mathsf{snr}))\mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_1, \mathsf{snr})\mathsf{P}_{\mathsf{Succ}}(\mathsf{x}_2, \mathsf{snr}),
\end{aligned} \tag{3.40}$$

where $\mathsf{P_{Succ}}(\mathsf{d}, \mathsf{snr})$ is defined by equation 3.11. The end-to-end reliability, defined in equation 3.36, is:

$$\begin{aligned}
\mathsf{Reliability}^{(s,r,d)}_{\mathsf{Diversified}} &= \mathsf{E}_{x_1,x_2,x_3}[\mathsf{R}[2,2]], \\
&= \mathsf{E}_{x_1,x_2,x_3}[\exp(-\frac{x_3^2}{\mathsf{snr}}) + (1-\exp(-\frac{x_3^2}{\mathsf{snr}}))\exp(-\frac{x_1^2+x_2^2}{\mathsf{snr}})].
\end{aligned} \tag{3.41}$$

The end-to-end outage probability is:

$$\begin{aligned}
\rho^{(s,r,d)}_{\mathsf{Diversified}} &= 1 - \mathsf{Reliability}^{(s,r,d)}_{\mathsf{Diversified}}(\mathsf{snr}) \\
&= 1 - \mathsf{E}_{x_1,x_2,x_3}[\exp(-\frac{x_3^2}{\mathsf{snr}}) + (1-\exp(-\frac{x_3^2}{\mathsf{snr}}))\exp(-\frac{x_1^2+x_2^2}{\mathsf{snr}})]
\end{aligned} \tag{3.42}$$

$$\begin{aligned}
&= \mathsf{E}_{x_1,x_2,x_3}[(1-\exp(-\frac{x_3^2}{\mathsf{snr}}))(1-\exp(-\frac{x_1^2+x_2^2}{\mathsf{snr}}))] \\
&\approx \mathsf{E}_{x_1,x_2,x_3}[\frac{x_3^2(x_1^2+x_2^2)}{\mathsf{snr}^2}],
\end{aligned} \tag{3.43}$$

where the approximation is valid in high-snr regime. Note we used the first two terms of Taylor expansion for $\exp(x) = 1 + x + \frac{1}{2}x^2 + O(x^3)$ to arrive at the final result.

The results of 3.38, 3.39, 3.42, 3.43 are valid for any two-hop network. For the two networks shown in figure 3-4 and 3-5, these expectations can be easily calculated. The resulting end-to-end outage probabilities are:

$$\begin{aligned}
\rho^{\mathsf{Line}}_{\mathsf{Non-Diversified}} &\approx \frac{2.7}{\mathsf{snr}}, \\
\rho^{\mathsf{Circle}}_{\mathsf{Non-Diversified}} &\approx \frac{1.2}{\mathsf{snr}}, \\
\rho^{\mathsf{Line}}_{\mathsf{Diversified}} &\approx \frac{10.6}{\mathsf{snr}^2}, \\
\rho^{\mathsf{Circle}}_{\mathsf{Diversified}} &\approx \frac{4.7}{\mathsf{snr}^2}.
\end{aligned} \tag{3.44}$$

Figure 3-6 and 3-7 show the exact values for the end-to-end outage probability obtained through equations 3.38 and 3.42, as well as the approximations for the end-to-end outage probability obtained through equations 3.39 and 3.43. It is clear that our approximation are quite good for high values of snr. A few observations are worth mentioning at this point.

1. Route diversity significantly improves the end-to-end reliability in both networks. In the high-snr regime, the end-to-end outage probability decays as snr^{-2} for the diversified route. This is a significant improvement over the snr^{-1} decay observed in the absence of diversity.

2. A question that might come to mind is: How high is high snr? There is no exact answer to this question for a general route. Based on figure 3-6 and 3-7, it is clear that in this case the high-snr approximations are quite good for the regions of snr corresponding to 1% end-to-end outage. The main significance of finding this type of result is to find the asymptotic trade-off between the end-to-end reliability and consumed power. We do a similar asymptotic analysis in the next section when comparing the simple and diversified route reliability in a line network. These approximations are not necessarily good for any finite values of snr and may only become accurate at very large levels of transmitted snr.

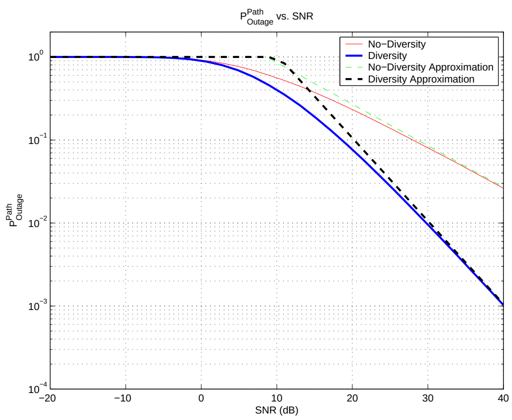

Figure 3-6: Line Network Reliability

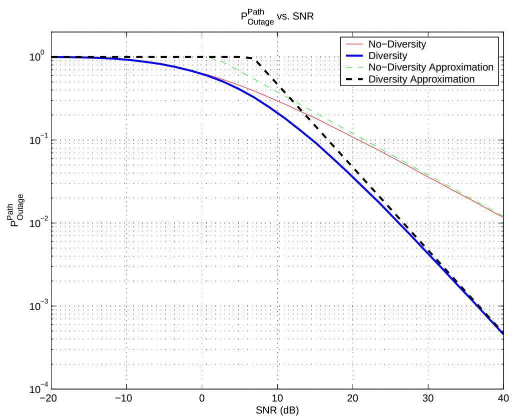

Figure 3-7: Disk Network Reliability

3.4 Multi-Hop Line Network

We looked at the reliability and reliability-power relationship for a point-to-point link in section 1. We observed that the outage probability decays as snr^{-1} in a Rayleigh fading channel. We then turned our attention to a multi-hop network scenarios, where we looked at the problem of routing and optimal route selection. It was shown that a similar relationship governs the trade-off between the end-to-end reliability and power in a multi-hop route (see section 1.2.2.) Route diversity was introduced in section 3 as one possible venue to improve the trade-off between the end-to-end reliability and power. We looked at how diversity improves the reliability in some very simple 2-hop networks.

In this section, we look at the relationship between the end-to-end reliability and power in another simple topology: multi-hop line network. In particular we compare two specific topologies: deterministic and Poisson line network. A deterministic line network with density λ is a line network in which nodes are located at fixed distance of $\frac{1}{\lambda}$ from each other. In a Poisson line network with density λ, the distance between neighbors are independent exponential random variables with parameter λ.

For the analysis in this section, we assume that there is a limit on the maximum power transmitted by each node, and denote the transmitted signal-to-noise ratio corresponding to this maximum transmission power level by snr. We start by looking at the reliability of a non-diversified route as a function of the maximum transmitted power, the number of hops, and network topology under the assumption of Rayleigh fading or no fading. We then find the end-to-end reliability for diversified route and compare that result with the simple routing scheme, i.e. multi-hoping with out diversity, to get a better understanding of route diversity benefits.

3.4.1 Simple Route Reliability

In this section we look at the end-to-end reliability of a simple route in a line network with N hops as shown in figure 3-8. We look at the following 4 different scenarios:

1. Deterministic line with density λ and no fading.

2. Deterministic line with density λ and Rayleigh fading.

3. Poisson line with density λ and no fading.

4. Poisson line with density λ and Rayleigh fading.

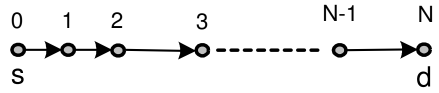

Figure 3-8: N-Hop Route in Line Network

The simple route connecting node $\mathsf{0}$ to node N consists of N point-to-point links. The event of end-to-end outage is equivalent to outage at any of these intermediate point-to-point links. Define

$$\mathsf{O_i} \equiv \mathsf{Outage}(\mathsf{i}-1 \rightarrow \mathsf{i})$$

as the event that the link between relays $\mathsf{i}-1$ and i is in outage. The end-to-end route is in outage if any intermediate link is in outage. Hence

$$\begin{aligned} \rho^{(0,1,\cdots,\mathsf{N}-1,\mathsf{N})} &= \mathsf{P}\left(\underbrace{\mathsf{Outage}(0 \rightarrow 1)}_{\mathsf{O_1}} \cup \cdots \cup \underbrace{\mathsf{Outage}(\mathsf{N}-1 \rightarrow \mathsf{N})}_{\mathsf{O_N}}\right) \\ &= \mathsf{P}\left(\mathsf{O_1} \cup \cdots \cup \mathsf{O_N}\right). \end{aligned} \tag{3.45}$$

We have already looked at the outage probability for each link in section 3.1.4. In all 4 scenarios described above, individual links have the same outage probability. So

$$\mathsf{P}(\mathsf{O_i}) = \mathsf{P_{Outage}}(\mathsf{Link}), \quad \forall 1 \leq \mathsf{i} \leq \mathsf{N},$$

where $\mathsf{P_{Outage}}(\mathsf{Link})$ is calculated in table 3.4.

Let's first analyze the end-to-end outage probability in a deterministic network without fading. This is a very degenerate problem as all the intermediate links, and hence the entire route, are either up or down, i.e. $\mathsf{P}(\mathsf{O_i}|\mathsf{O_1}) = 1 \quad \forall \mathsf{i} \geq 1$. Hence

$$\rho^{(0,1,\cdots,N-1,N)} = P(O_1) = P_{Outage}(Link) \quad (3.46)$$

For the other 3 cases, deterministic or Poisson line with fading or Poisson line without fading, we use the union bound approxmation to get the end-to-end outage probability. We have

$$\rho^{(0,1,\cdots,N-1,N)} \approx N P_{Outage}(Link), \quad (3.47)$$

where the approximation is valid when $P_{Outage}(Link)$ is small, i.e. in the high-snr regime.

Using relation 3.46 and 3.47 and table 3.4, the end-to-end outage can be found for all 4 scenarios of interest. Table 3.5 summarizes these results.

	No Fading	Raleigh Fading
Deterministic Network, Density λ	$\begin{cases} 1 & \lambda^k \, snr < 1 \\ 0 & \text{else} \end{cases}$	$N \, \frac{1}{\lambda^k \, snr}$
Poisson Network, Density λ	$N \exp\left(-\sqrt[k]{\lambda^k \, snr}\right)$	$N \, \frac{k!}{\lambda^k \, snr}$

Table 3.5: Route Outage Probability in Line Network

Table 3.5 summarizes the relationship between the number of hops. i.e. the size of the network, the transmitted snr, the network density, and end-to-end outage probability. Several observations are worth mentioning here:

1. For a fixed N, the end-to-end outage probability decreases exponentially with snr in non-fading scenarios compared to linear decrease with snr^{-1} in a fading network. Hence, to achieve a guaranteed end-to-end outage for a fixed number of hops, N, a much lower power is required in the non-fading case. For a fixed snr and desired end-to-end outage probability, information can be relayed over many more hops in the non-fading scenario than the fading case.

2. In all different scenarios, the outage probability depends on $\lambda^{\mathsf{k}}\mathsf{snr}$. This is the same as our earlier observation in section 3.1.4. This is plausible since the reliability should depend both on the transmitted power and the distance between nodes. $\lambda^{\mathsf{k}}\mathsf{snr}$ shows the dependence on the first factor through snr and on the second factor through λ. We call $\lambda^{\mathsf{k}}\mathsf{snr}$ the *Power-Density Profile* of a line network. For notational convenience, we assign a symbol to this network parameter,

$$\Psi_{\mathsf{k}}(\lambda, \mathsf{snr}) \stackrel{\mathsf{def}}{=} \lambda^{\mathsf{k}}\ \mathsf{snr}. \tag{3.48}$$

In the next two sections, we show that route diversity fundamentally changes the relationship between the end-to-end reliability and the power-density profile.

3.4.2 Full Diversity

The end to end reliability for the diversified routing scheme can be calculated based on equations 3.36, 3.34, and 3.33. The main difficulty is that solving the recursion for a large network is tedious. We first find the exact value for the end-to-end outage probability for small networks. Based on those results, we conjecture the relationship between the end-to-end outage probability and power in larger networks. A deterministic network can be treated as a degenerate random network. Hence, we only carry out the analysis for random network knowing that all the results can be extended to deterministic networks in a straightforward way.

We start by looking at 2-hop line network as shown in figure 3-9.

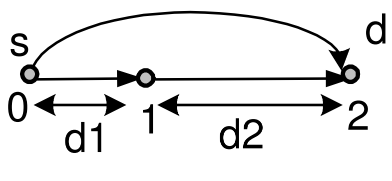

Figure 3-9: 2-Hop Poisson Line Network

From equation 3.36, the end-to-end route outage probability is:

$$\begin{aligned}\rho^{(0,1,2)} &= 1 - \mathsf{E}_{\mathsf{NodeLocations}}\left[\mathsf{R}[2,2]\right] \\ &= 1 - \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\mathsf{R}[2,2]\right]\cdot \end{aligned} \tag{3.49}$$

Using relation 3.33, $\mathsf{R}[2,2]$ can be calculated as a function of the distances between nodes:

$$\begin{aligned}\mathsf{R}[1,0] &= 0, \\ \mathsf{R}[2,0] &= 0, \\ \mathsf{R}[1,1] &= \mathsf{R}[1,0] + (1 - \mathsf{R}[1,0])\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_1, \mathsf{snr}), \\ &= \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_1, \mathsf{snr}), \\ \mathsf{R}[2,1] &= \mathsf{R}[2,0] + (1 - \mathsf{R}[2,0])\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}), \\ &= \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}), \\ \mathsf{R}[2,2] &= \mathsf{R}[2,1] + (1 - \mathsf{R}[2,1])\mathsf{R}[1,1]\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2, \mathsf{snr}), \\ &= \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}) + (1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}))\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_1, \mathsf{snr})\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2, \mathsf{snr}) \cdot \end{aligned} \tag{3.50}$$

Substituting this back into 3.49, we get:

$$\begin{aligned}\rho^{(0,1,2)} &= 1 - \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\mathsf{R}[2,2]\right] \\ &= 1 - \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}) + (1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}))\,\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_1, \mathsf{snr})\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2, \mathsf{snr})\right] \\ &= \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}) - ((1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}))\,\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_1, \mathsf{snr})\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2, \mathsf{snr})\right] \\ &= \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[(1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2 + \mathsf{d}_1, \mathsf{snr}))\,(1 - \mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_1, \mathsf{snr})\mathsf{P}_{\mathsf{Succ}}(\mathsf{d}_2, \mathsf{snr}))\right] \\ &= \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\left(1 - \exp(-\frac{(\mathsf{d}_2 + \mathsf{d}_1)^{\mathsf{k}}}{\mathsf{snr}})\right)\left(1 - \exp(-\frac{\mathsf{d}_1^{\mathsf{k}}}{\mathsf{snr}})\exp(-\frac{\mathsf{d}_2^{\mathsf{k}}}{\mathsf{snr}})\right)\right] \qquad (3.51) \\ &\approx \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\frac{(\mathsf{d}_2 + \mathsf{d}_1)^{\mathsf{k}}(\mathsf{d}_1^{\mathsf{k}} + \mathsf{d}_2^{\mathsf{k}})}{\mathsf{snr}^2}\right], \qquad (3.52)\end{aligned}$$

where we used the first two terms in the Taylor expansion of exponential function to find this approximation for the high-snr regime. For a deterministic line with density λ, assuming $\mathsf{k} = 2$, the end-to-end outage probability is

$$\begin{aligned}
\rho^{(0,1,2)} &\approx \frac{(\frac{1}{\lambda} + \frac{1}{\lambda})^2(\frac{1}{\lambda}^2 + \frac{1}{\lambda}^2)}{\mathsf{snr}^2} \\
&\approx \frac{8}{\lambda^4\ \mathsf{snr}^2} \\
&\approx \frac{8}{\Psi_2(\lambda, \mathsf{snr})^2}.
\end{aligned} \tag{3.53}$$

In a Poisson line with density λ, d_1 and d_2 are independent exponential random variables with parameter λ. In this case, again assuming $\mathsf{k} = 2$, the outage probability simplifies to:

$$\begin{aligned}
\rho^{(0,1,2)} &\approx \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\frac{(\mathsf{d}_2 + \mathsf{d}_1)^2(\mathsf{d}_1^2 + \mathsf{d}_2^2)}{\mathsf{snr}^2}\right] \\
&\approx \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\frac{(\mathsf{d}_2^2 + \mathsf{d}_1^2 + 2\mathsf{d}_2\mathsf{d}_1)(\mathsf{d}_1^2 + \mathsf{d}_2^2)}{\mathsf{snr}^2}\right] \\
&\approx \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2}\left[\frac{\mathsf{d}_2^2\mathsf{d}_1^2 + \mathsf{d}_1^4 + 2\mathsf{d}_2\mathsf{d}_1^3 + \mathsf{d}_2^4 + \mathsf{d}_1^2\mathsf{d}_2^2 + 2\mathsf{d}_2^3\mathsf{d}_1}{\mathsf{snr}^2}\right] \\
&\approx \frac{80}{\lambda^4\ \mathsf{snr}^2} \\
&\approx \frac{80}{\Psi_2(\lambda, \mathsf{snr})^2}.
\end{aligned}$$

Table 3.6 gives a side-by-side comparison between the end-to-end outage for the simple route, based on table 3.5, and the diversified route, based on the analysis in this section. Note that for the simple routing scheme outage probability decays as $\Psi_2(\lambda, \mathsf{snr})^{-1}$. In the diversified routing scheme, however, outage probability decays as $\Psi_2(\lambda, \mathsf{snr})^{-2}$.

Figure 3-10 gives both the exact and the approximation for outage in a 2-hop Poisson line with $\lambda = 1$. Note that the diversified route has a much lower outage probability than the simple route for any snr level. Furthermore, our approximations are quite good even for the snr values corresponding to 1% end-to-end outage.

	Simple Route	Diversified Route
Deterministic Network, Density λ	$\frac{2}{\Psi_2(\lambda,\mathrm{snr})}$	$\frac{8}{\Psi_2(\lambda,\mathrm{snr})^2}$
Poisson Network, Density λ	$\frac{4}{\Psi_2(\lambda,\mathrm{snr})}$	$\frac{80}{\Psi_2(\lambda,\mathrm{snr})^2}$

Table 3.6: Outage Probability with or without Diversity in 2-Hop Line

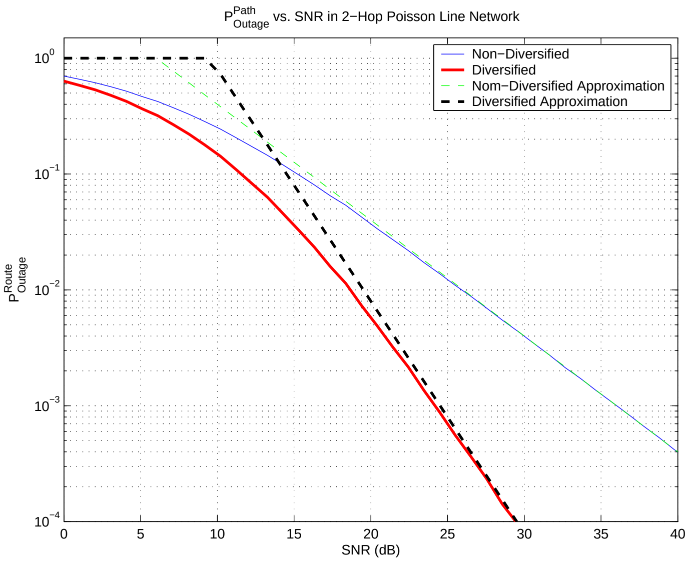

Figure 3-10: Reliability Improvement in 2-Hop Line Network

The same technique can be used for finding the end-to-end outage probability for routes with more hops. For a 3-Hop line network, following the same steps, we get the following approximation:

$$\rho^{(0,1,2,3)} \approx \mathsf{E}_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{d}_3}\left[\frac{(\mathsf{d}_1+\mathsf{d}_2+\mathsf{d}_3)^{\mathsf{k}}((\mathsf{d}_2+\mathsf{d}_3)^{\mathsf{k}}+\mathsf{d}_1^{\mathsf{k}})\mathsf{d}_3^{\mathsf{k}}}{\mathsf{snr}^3}\right], \quad (3.54)$$

where d_1,d_2, and d_3 are the internode distances.

For a deterministic line with density λ, this approximation simplifies to:

$$\begin{aligned}\rho^{(0,1,2,3)} &\approx \frac{45}{\lambda^6\,\mathsf{snr}^3}\\ &\approx \frac{45}{\Psi_2(\lambda,\mathsf{snr})^3}. \quad (3.55)\end{aligned}$$

and for a Poisson line with density λ, it simplifies to:

$$\rho^{(0,1,2,3)} \approx \frac{2464}{\lambda^6 \; \mathsf{snr}^3} \approx \frac{2464}{\Psi_2(\lambda, \mathsf{snr})^3}. \tag{3.56}$$

Figure 3-11 gives both the exact and high-snr approximation for a 3-hop Poisson line network with $\lambda = 1$. Again, we see that the approximation is quite good even for modest levels of end-to-end reliability.

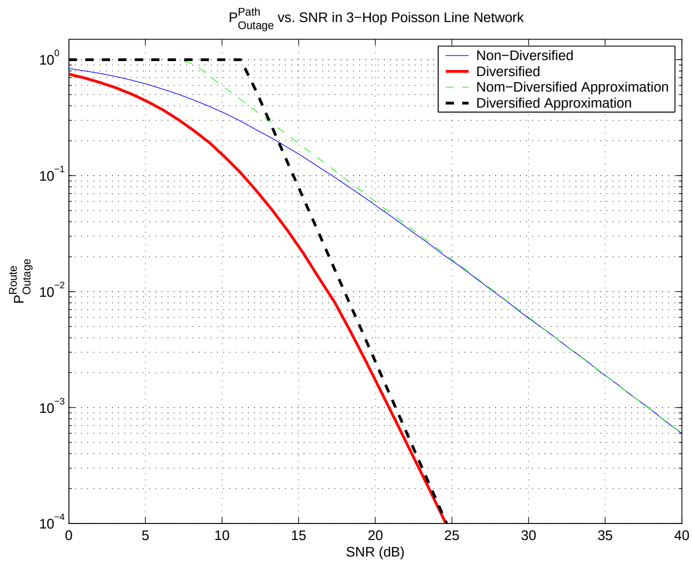

Figure 3-11: Reliability Improvement in 3-Hop Line Network

Based on our analysis of 2 and 3-hop networks, we would like to make the following observations:

1. We conjecture that for a route with N hops, the outage probability decays as $\Psi_k(\lambda, \mathsf{snr})^{-\mathsf{N}}$ in the high-snr regime.

2. In order to achieve this type of behavior, the high-snr approximation for individual link outage probability must be valid for the worst link, i.e. for the link between the source, node $\mathsf{0}$, and the destination, node N. This requires a very high level of transmitted snr. In fact, a higher value of snr is needed as N increases. Therefore, this behavior is not scalable to large networks. In

section 3.4.3, we look at what type of gain can be achieved if not all, but only a limited number of links in each transmission are assumed to be operating in the high-snr regime. In the last section of this chapter, we look at some preliminary analysis outside of the high-snr regime.

3.4.3 Limited Diversity

In the previous section we worked out the exact and high-snr approximation for the outage probability for diversified routes in deterministic and Poisson line networks. We conjectured that for a route with N hops, the end-to-end outage probability decreases as $\Psi_{\mathsf{k}}(\lambda, \mathsf{snr})^{-\mathsf{N}}$ in the high-snr regimes. However, to achieve this type of behavior, the high-snr approximation for link outage probability must be valid for all single point-to-point transmission. For large values of N, a very high transmitted power is required to achieve this behavior.

In this section, we look at the improvement possible due to limited diversity. In limited diversity with degree L, a node i can only receive the information from nodes $\{\mathsf{i}-\mathsf{L}+1, \cdots, \mathsf{i}-1\}$. For example, figure 3-12 shows the paths in a 6-Hop line network with a cooperation limit of 2. The motivation behind this is two-fold: first, this type of diversity requires coordination only among nodes located close to each other, which might be more reasonable than coordination among all nodes along the selected route as needed in unlimited diversity. Furthermore, this approach allows us to show some interesting analytical results for the benefits of route diversity in a multi-hop line network.

The exact analysis of the end-to-end reliability is complicated due to strong correlation between the different events that contribute to successful end-to-end relaying. For example in figure 3-12, we are interested in calculating the probability that a route exists between node $\mathsf{0}$ and node $\mathsf{6}$ using any of the point-to-point links shown in the figure. The probability of success for these links are strongly correlated. This correlation arises from the dependence of the point-to-point link success probability on the same underlying set of random variables, i.e. the node locations. For example, in figure 3-12, distance between nodes $\mathsf{1}$ and $\mathsf{2}$, d_{12}, affects the probability of success for links $\{1 \rightarrow 2\}$, $\{0 \rightarrow 2\}$, and $\{1 \rightarrow 3\}$.

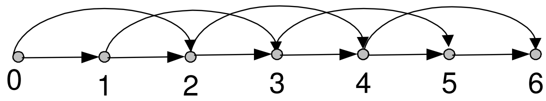

Figure 3-12: Limited Cooperation in a Line Network

To eliminate this correlation, we consider the diversity scheme shown in figure 3-13, where we have eliminated the links between $\{1 \rightarrow 3\}$ and $\{3 \rightarrow 5\}$. Hence, the total reliability of all the routes shown in figure 3-13 is less than the reliability of routes shown in figure 3-12. In a way, by finding the probability of outage for figure 3-13, we find an upper bound for the probability of outage for figure 3-12.

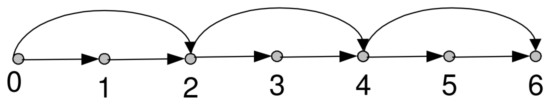

Figure 3-13: Upper Bounding the Outage Probability in a Line Network

It is not hard to observe that

$$\begin{aligned} \mathsf{Reliability}^{(0,1,\cdots,\mathsf{N})} &= \mathsf{E}_{\mathsf{NodeLocations}}\left[\mathsf{R}[\mathsf{N},\mathsf{N}]\right] \\ &\geq \mathsf{Reliability}^{(0,\cdots,\mathsf{L})}\mathsf{Reliability}^{(\mathsf{L},\cdots,2\mathsf{L})}\cdots\mathsf{Reliability}^{(\mathsf{N}-\mathsf{L},\cdots,\mathsf{N})}\ . \end{aligned} \tag{3.57}$$

Of course, for this to make sense N must to divisible by L. Essentially, we have divided the line into segments of size L and used the result from unlimited diversity in each segment. For the case of a Poisson line, the internode distances are independent from each other. We have:

$$\mathsf{Reliability}^{(0,\cdots,\mathsf{L})} = \mathsf{Reliability}^{(\mathsf{L},\cdots,2\mathsf{L})} = \cdots = \mathsf{Reliability}^{(\mathsf{N}-\mathsf{L},\cdots,\mathsf{N})} = \mathsf{E}_{\mathsf{NodeLocations}}\left[\mathsf{R}[\mathsf{L},\mathsf{L}]\right],$$

hence,

$$\mathsf{Reliability}^{(0,1,\cdots,\mathsf{N})} \geq \left(\mathsf{E}_{\mathsf{NodeLocations}}\left[\mathsf{R}[\mathsf{L},\mathsf{L}]\right]\right)^{\frac{\mathsf{N}}{\mathsf{L}}}. \tag{3.58}$$

Using these bounds and the result from the section 3.4.2, we can find the bound for end-to-end outage probability. For $\mathsf{L} = 2$, we have:

$$\begin{aligned}
\mathsf{Reliability}^{(0,1,\cdots,N)} &\geq \left(\mathsf{E}_{\mathsf{NodeLocations}}\mathsf{R}[2,2]\right)^{\frac{N}{2}} \\
&\geq \left(1 - \mathsf{E}_{x_1,x_2}\left[\frac{(x_1+x_2)^k(x_2^k+x_1^k)}{\mathsf{snr}^2}\right]\right)^{\frac{N}{2}} \\
&\geq 1 - \frac{N}{2}\mathsf{E}_{x_1,x_2}\left[\frac{(x_1+x_2)^k(x_2^k+x_1^k)}{\mathsf{snr}^2}\right] \\
\rho^{(0,1,\cdots,N)} &\leq \frac{N}{2}\mathsf{E}_{x_1,x_2}\left[\frac{(x_1+x_2)^k(x_2^k+x_1^k)}{\mathsf{snr}^2}\right]
\end{aligned} \tag{3.59}$$

We used the fact that $(1-p)^N \geq 1 - Np$ for all $0 \leq p \leq 1$. This fact can be shown by collecting all the terms on the left-hand-side and taking the first derivative. For $k = 2$, we have:

$$\rho^{(0,1,\cdots,N)} \leq \frac{N}{2}\frac{8}{\lambda^2\,\mathsf{snr}^2} \tag{3.60}$$

in a deterministic line with density λ and

$$\rho^{(0,1,\cdots,N)} \leq \frac{N}{2}\frac{80}{\lambda^2\,\mathsf{snr}^2} \tag{3.61}$$

in a Poisson line with density λ.

Similar bounds can be found for the case when diversity is limited to 3 nodes, i.e. $\mathsf{L} = 3$. Table 3.7 summarizes these results. We have also repeated some results from table 3.5 for easier comparison between limited diversity and simple routing. In figure 3-14, the exact end-to-end outage probability and bound for a 6-Hop Poisson line network under diversity limit of $\mathsf{L} = 2$ and $\mathsf{L} = 3$ are shown. These bounds are clearly not very tight. However, finding these bounds allow us to get an idea of how the relation between reliability and power changes under limited diversity scenarios. Comparing entries in table 3.7, it is clear that without diversity, the end-to-end outage probability decays as $\Psi_2(\lambda, \mathsf{snr})^{-1}$. When diversity is limited to $\mathsf{L} = 2$ nodes, the end-to-end outage probability $\Psi_2(\lambda, \mathsf{snr})^{-2}$ and for $\mathsf{L} = 3$, this tradeoff become $\Psi_2(\lambda, \mathsf{snr})^{-3}$. We conjecture that for diversity limit of L, the end-to-end outage decays as $\Psi_k(\lambda, \mathsf{snr})^{-\mathsf{L}}$.

	Simple Route	Diversified, $\mathsf{L} = 2$	Diversified, $\mathsf{L} = 3$
Deterministic Network, Density λ	$\mathsf{N}\frac{1}{\lambda^2\ \mathsf{snr}}$	$\frac{\mathsf{N}}{2}\frac{8}{\lambda^2\ \mathsf{snr}^2}$	$\frac{\mathsf{N}}{3}\frac{45}{\lambda^6\ \mathsf{snr}^3}$
Poisson Network, Density λ	$\mathsf{N}\frac{2}{\lambda^2\ \mathsf{snr}}$	$\frac{\mathsf{N}}{2}\frac{80}{\lambda^2\ \mathsf{snr}^2}$	$\frac{\mathsf{N}}{3}\frac{2464}{\lambda^6\ \mathsf{snr}^3}$

Table 3.7: Limited Diversity Route Outage Probability in Line Network

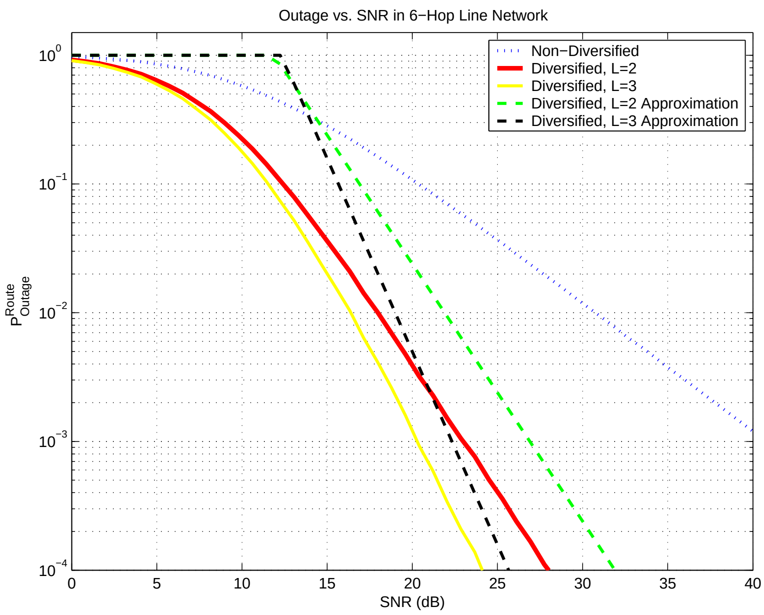

Figure 3-14: Outage for 6-Hop Poisson Line Network

3.4.4 Non High-SNR Diversity Analysis

All our analysis up to this point has been based on the high-snr assumption. In section, 3.4.2, we looked at the case of full diversity in a line network. We conjectured that the end-to-end outage probability decays as $\Psi_k(\lambda, \mathsf{snr})^{-\mathsf{N}}$. However, this behavior is only seen at very high transmission power levels and not practical to achieve for large networks, i.e. for large N. Trying to move away from this shortcoming, in section 3.4.3 we looked at the case of limited diversity. It was observed that even limited diversity has some significant benefits. It was shown that the end-to-end outage decays as $\Psi_{\mathsf{k}}(\lambda, \mathsf{snr})^{-\mathsf{L}}$ where L is the diversity level.

Clearly, assuming infinite snr or limiting diversity to only a few nodes are not very precise in any realistic system, i.e. we either get asymptotic results that only apply to high-snr regimes or we don't get the full benefit of diversity in the relay nodes as we limit the number of receivers. In this section, we try move away from these assumption and develop a more precise analysis of the benefits of diversity in a line network. We calculate the probability that a transmission by a node is not received by any node in the direction of the destination, i.e. by any node closer to the destination. In fact, we are trying to find the exact probability that all links shown in 3-15 fail. This can be related to our earlier analysis in a very natural way. In section 3.4.2 and 3.4.3, we found the probability that the first N of these links fail. We found that in the high-snr regime, this probability decays as $\Psi_{\mathsf{k}}(\lambda, \mathsf{snr})^{-\mathsf{N}}$. In this section, we move away from the high-snr assumption. We solely focus on the case of Poisson line as Poisson line is more representative of a random network. We also assume that the final destination is located very far away from the source node. We observe this probability decays exponentially in the network power-density-profile as defined in equation 3.48.

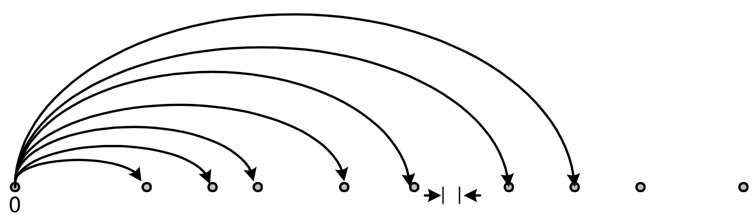

Figure 3-15: Poisson Line Network

Let $\mathsf{N}(\mathsf{a}, \mathsf{b})$ be the random variable for the number of nodes in the interval of $(\mathsf{a}, \mathsf{b}]$. It is well known, see [17], that for exponential internode distances, the number of nodes in any interval is a Poisson random variable. Furthermore, the number of nodes in non-overlapping intervals are independent of each other, To develop our result, we start by dividing the line into segments of length δ. For small values of δ, the number of nodes in a line segment of length δ is approximately a Bernoulli random variable. i.e. there is a node in the segment with probability $\lambda\delta$ or there is no node with probability $1 - \lambda\delta$. Furthermore, the number of nodes in non-overlapping line segments are independent random variables, see [17] for details. This approximation is prefect in our case as we will take the limit of $\delta \to 0$ to get the desired result. For small values of δ, let's define the disconnect event for segment located at distance $m\delta$ away from the transmitter as the event that the information is not received by any node in the line segment $(m\delta, (m+1)\delta]$. This event includes both the case that there is no node in this line segment or there is a node and transmission fails due to bad fading. This probability can be calculated as:

$$
\begin{aligned}
P_{Disconnect}(m\delta, snr) &= P\left(Disconnect | N(m\delta, (m+1)\delta] = 0\right) P\left(N(m\delta, (m+1)\delta] = 0\right) + \\
&\quad P\left(Disconnect | N(m\delta, (m+1)\delta] = 1\right) P\left(N(m\delta, (m+1)\delta] = 1\right) \\
&= (1)(1 - \lambda\delta) + (1 - P_{Succ}(m\delta, snr)(\lambda\delta) \\
&= 1 - P_{Succ}(m\delta, snr)\lambda\delta,
\end{aligned}
$$

where $P_{Succ}(d, snr)$ is defined in 3.11. Let $P_{Disconnect}(x, y, \delta)$ be the probability that the information transmitted by a node located at location 0 is not received by any node between $(x, y]$ where this segment is broken down into segments of length δ. This probability can written in terms of $P_{Disconnect}(m\delta)$ calculated above as:

$$
\begin{aligned}
P_{Disconnect}(x, y, \delta, snr) &= \prod_{i=\frac{x}{\delta}}^{\frac{y}{\delta}} P_{Disconnect}(i\delta, snr) \\
\ln(P_{Disconnect}(x, y, \delta, snr)) &= \sum_{i=\frac{x}{\delta}}^{\frac{y}{\delta}} \ln(P_{Disconnect}(i\delta, snr)) \\
&= \sum_{i=\frac{x}{\delta}}^{\frac{y}{\delta}} \ln(1 - P_{Succ}(m\delta)\lambda\delta). \qquad (3.62)
\end{aligned}
$$

Taking the limit $\delta \to 0$, we get:

$$\begin{aligned}
\ln(\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{x},\mathsf{y},\mathsf{snr})) &= \lim_{\delta\to 0} \ln(\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{x},\mathsf{y},\delta,\mathsf{snr})) \\
&= \lim_{\delta\to 0} \sum_{\mathsf{i}=\frac{\mathsf{x}}{\delta}}^{\frac{\mathsf{y}}{\delta}} \ln(1-\mathsf{P}_{\mathsf{Succ}}(\mathsf{m}\delta,\mathsf{snr})\lambda\delta) \\
&= -\int_{\mathsf{x}}^{\mathsf{y}} \lambda\ \mathsf{P}_{\mathsf{Succ}}(\mathsf{l},\mathsf{snr})\mathsf{dl} \qquad (3.63)
\end{aligned}$$

where we used the approximation of $\ln(1-\mathsf{x}) \approx -\mathsf{x}$ for small values of x in the last step.

For the case when the path-loss exponent $\mathsf{k}=2$, we have

$$\mathsf{P}_{\mathsf{Succ}}(\mathsf{d},\mathsf{snr}) = \exp(-\frac{\mathsf{d}^2}{\mathsf{snr}}),$$

and the above integral can be calculated easily based on error function:

$$\begin{aligned}
\ln(\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{x},\mathsf{y})) &= -\lambda(\frac{\sqrt{\pi\mathsf{snr}}}{2}\left(\mathsf{erfc}\left(\frac{\mathsf{y}}{\sqrt{\mathsf{snr}}}\right)-\mathsf{erfc}\left(\frac{\mathsf{x}}{\sqrt{\mathsf{snr}}}\right)\right) \\
&= -\lambda\left(\sqrt{\mathsf{snr}}\int_{\frac{\mathsf{x}}{\sqrt{\mathsf{snr}}}}^{\frac{\mathsf{y}}{\sqrt{\mathsf{snr}}}} \mathsf{e}^{-\mathsf{t}^2}\mathsf{dt}\right). \qquad (3.64)
\end{aligned}$$

It is interesting to look at this result for $\mathsf{x}=0$ and $\mathsf{y}=\infty$. For these values we have:

$$\begin{aligned}
\ln(\mathsf{P}_{\mathsf{Disconnect}}(0,\infty)) &= -\lambda\frac{\sqrt{\pi\mathsf{snr}}}{2} \\
&= -\frac{\sqrt{\pi\ \lambda^2\mathsf{snr}}}{2} \\
\mathsf{P}_{\mathsf{Disconnect}}(0,\infty) &= \exp\left(-\frac{\sqrt{\pi\ \lambda^2\mathsf{snr}}}{2}\right) \\
&= \exp\left(-\frac{\sqrt{\pi\Psi_2(\lambda,\mathsf{snr})}}{2}\right).
\end{aligned}$$

It is seen that the probability that the information transmitted by a node is not received by any node closer to the destination decays exponentially with $\Psi_2(\lambda,\mathsf{snr})$.

This is very similar to the case of non-fading point-to-point link with exponential distance (see table 3.4.) The reason behind this type of result is the infinite number of intermediate relay nodes that can receive each transmission. Of course, this is not the case in any real network as there are only a finite number of relays between any two node pairs. This analysis proves that route diversity has the potential to significantly change the relationship between the transmitter power, the network density, and the end-to-end outage, and potentially even make a fading network look a lot more like a non-fading network.

Chapter 4

Conclusion

In this thesis, we studied the problem of reliability and energy efficiency in wireless ad-hoc networks. In the first part, we introduced the idea of wireless cooperation advantage. We formulated the problem of finding the minimum energy cooperative route for a wireless network under idealized channel and receiver models. Our main assumption were that the channel states are known at the transmitter and precise power and phase control, to achieve coherent reception is possible. We focused on the optimal transmission of a single message from a source to a destination node through sets of nodes, that may act as cooperating relays. Fundamental to the understanding of the routing problem was the understanding of the optimal power allocation for a single message transmission from a set of source nodes to a set of destination nodes. We presented solutions to this problem, and used these as the basis for solving the minimum energy cooperative routing problem. We used Dynamic Programming (DP) to formulate the cooperative routing problem as a multi-stage decision problem. For a regular line and a regular grid topologies, we analytically obtained the energy savings due to cooperative transmission, demonstrating the benefits of the proposed cooperative routing scheme. In particular, we showed that in a regular line network energy savings of $1 - \frac{6}{\pi^2} \approx 39\%$ is possible. For the case of a regular grid network, we found that energy savings of $\approx 56\%$ is achievable. We proposed two heuristics for general topologies. Simulations confirmed that energy savings of close to 50% is achievable in a random network.

In the second part, we studied the problem of route reliability in a multi-hop network. Our analysis started by looking at the reliability of a point-to-point communication link. In particular, we looked at how the reliability of a point-to-point link depends on the fading parameter and the distance between the communicating nodes. The analysis of a point-to-point link was based on the widely used capacity-versus-outage model for a wireless link. Once the result for a point-to-point link was established, we extend the reliability result to a wireless network setting. In a network setting, we first defined and analyzed the reliability for a fixed route and then proposed algorithms for finding the optimal route between a source-destination pairs of nodes. We looked at three different formulation for the routing problem: finding the most reliable route for a fixed maximum transmitted snr per link, finding the most reliable route for a fixed end-to-end power, and finding the minimum power route for a guaranteed end-to-end reliability. We showed that the last two problem are dual of each other. Based on this duality, we found the optimal trade-off curve between the end-to-end reliability and the end-to-end power consumption. The relation between route reliability and consumed power was studied. It was shown that the trade-off between the end-to-end reliability and consumed power in a route is very similar to the trade-off between the transmission power and reliability in a link.

The idea of route diversity was introduced as a way to improve the end-to-end reliability by taking advantage of wireless broadcast property and the independence of the fading parameters between different pairs of nodes. We gave analytical results for improvements due to route diversity in some simple network topologies. For a line topology, it was shown that even limited diversity can significantly improve the trade-off between the end-to-end reliability and the consumed power. The results observed in this section closely resembled the reliability improvements due to space diversity in multiple-antenna system. The model proposed in this chapter can open the door to a new area of research on various network layer protocols and trade-off among different route properties, such as the end-to-end reliability, the expected delay, or the total consumed power. In this context, route diversity, i.e. the diversity created through using multiple relay nodes at the transmitter or the receiver side, appears to have the potential to fundamentally change these trade-offs.

Bibliography

[1] L.M. Feeney, M. Nilsson, "Investigating the energy consumption of a wireless network interface in an ad hoc networking environment," INFOCOM 2001, pp. 1548 -1557

[2] R.T. Derryberry, S.D. Gray, D.M. Ionescu, G. Mandyam, B. Raghothaman, "Transmit diversity in 3G CDMA systems," IEEE Communications Magazine, April 2002

[3] J.E. Wieselthier, G.D. Nguyen, A. Ephremides, "Algorithms for energy-efficient multicasting in ad hoc wireless networks," Proc. IEEE Military Communications Conference , 1999, pp. 1414-1418

[4] J.E. Wieselthier, G.D. Nguyen, A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks ," Proc. IEEE INFOCOM, 2000, pp. 585 -594

[5] Ahluwalia Ashwinder, Eytan Modiano and Li Shu, "On the Complexity and Distributed Construction of Energy Efficient Broadcast Trees in Static Ad Hoc Wireless Networks," Conference on Information Science and System, Princeton, NJ, March, 2002.

[6] Mario Cagalj, Jean-Pierre Hubaux and Christian Enz,"Minimum-energy broadcast in all-wireless networks: NP-completeness and distribution issues," in Proceedings of ACM MobiCom 2002, Atlanta, USA, September 2002

[7] Anand Srinivas and Eytan Modiano, "Minimum Energy Disjoint Path Routing in Wireless Ad Hoc Networks," ACM Mobicom, San Diego, Ca, September, 2003.

[8] Ivana Maric, Roy Yates, "Efficient Multihop Broadcast for Wideband Systems", DIMACS Workshop on Signal Processing for Wireless Transmission Rutgers University Oct. 2002

[9] T. Yung-Szu , G.J. Pottie, "Coherent cooperative transmission from multiple adjacent antennas to a distant stationary antenna through AWGN channels," Vehicular Technology Conference, 2002, pp. 130-134 vol.1

[10] J.N. Laneman, "Cooperative Diversity in Wireless Networks: Algorithms and Architectures," Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA, August 2002.

[11] J.N. Laneman, G.W. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," Wireless Communications and Networking Conference, Sept. 2000, Vol.1, pp 7-12

[12] J.N. Laneman, G.W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," IEEE Transactions on Information Theory, Oct. 2003

[13] M. Gastpar, M. Vetterli, "On the capacity of wireless networks: the relay case," Proc. INFOCOM 2002, Vol. 3, pp 1577-1586

[14] D. Tse, P. Viswanath, "Fundamental of Wireless Communications", working draft.

[15] E. Biglieri, J. Proakis, S. Shamai, "Fading channels: information-theoretic and communications aspects," Information Theory, IEEE Transactions on , Volume: 44 , Issue: 6 , Oct. 1998

[16] L.H. Ozarow, S. Shamai, A.D. Wyner, "Information theoretic considerations for cellular mobile radio," IEEE Transactions on Vehicular Technology, May 1994

[17] R. Gallagar, "Discrete Stochastoc Processes," Kluwer Academic Publisher, 1996

[18] D. Bertsekas, R. Gallagar, "Data Networks," Prentice Hall, 1992

[19] Lizhong Zheng, D.N.C Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," IEEE Transactions on Information Theory, May 2003

[20] T. S. Rappaport, "Wireless Communications: Principles and Practice", Prentice Hall, 2002.

[21] John G. Proakis, "Digital Communiation," Forth Edition, McGrawHill, 2001