FORCED COOLING OF UNDERGROUND ELECTRIC POWER TRANSMISSION LINES

PART II OF IV

HEAT CONDUCTION IN THE CABLE INSULATION OF FORCE-COOLED UNDERGROUND ELECTRICAL POWER TRANSMISSION SYSTEMS

by

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ABSTRACT

Forced-cooled systems for oil-filled pipe-type cable circuits have recently been considered. In such systems the conduction resistance through the paper insulation of the cables is the limiting thermal resistance. Assuming bilateral symmetry, steady-state conditions, and two-dimensional heat transfer, a FORTRAN IV computer program was written to solve the heat conduction problem in the cable insulation for arbitrary configurations of a three-cable system.

For a steel pipe, a cable system is most susceptible to overheating in the equilateral configuration with the three cables touching.

Proximity effects are very significant in forced cooling, especially when cables are not provided with a copper tape under the insulation moisture seal assembly, accounting for as much as 21% of the total oil temperature rise between refrigeration stations. This figure, however, is reduced to 8% when 0.005 inch thick copper tape is present.

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LIST OF SYMBOLS

2D = height of D_3, measured along y-axis.

D_1 = solution domain consisting of the cable insulation of Cable 1.

D_2 = solution domain consisting of the cable insulation of Cable 2.

D_3 = solution domain consisting of the inter-cable conduction path.

h = local convective film coefficient.

h_r = dimensionless radial spacing between mesh points in D_1.

h_α = dimensionless azimuthal spacing between mesh points in D_2.

h_ϕ = dimensionless azimuthal spacing between mesh points in D_1.

h_ρ = dimensionless radial spacing between mesh points in D_2.

I = current.

j1 = radial index in D_1.

j2 = radial index in D_2.

j3 = normal index in D_3.

k = thermal conductivity.

k1 = azimuthal index in D_1.

k2 = azimuthal index in D_2.

k3 = tangential index in D_3.

q = heat flow per unit length.

$\dot{q}$ = heat generation per unit volume.

r = radial coordinate in D_1.

r_1 = inner radius of Cable 1 insulation.

r_2 = outer radius of Cable 1 insulation.

$\bar{r}$ = dimensionless radial coordinate in D_1.

T = temperature.

T_{oil} = mixed-mean oil temperature outside the convective boundary layer.

T_{max} = maximum allowable system temperature.

w_d = dielectric loss per unit volume.

W = arbitrary loss per unit length.

W_c = conductor loss per unit length.

W_d = dielectric loss per unit length.

W_s = sheath loss per unit length.

x = normal coordinate in D_3.

$\bar{x}$ = dimensionless normal coordinate in D_3.

y = tangential coordinate in D_3.

$\bar{y}$ = dimensionless tangential coordinate in D_3.

α = azimuthal coordinate in D_2.

ρ = radial coordinate in D_2.

ρ_1 = inner radius of Cable 2 insulation.

ρ_2 = outer radius of Cable 2 insulation.

$\bar{\rho}$ = dimensionless radial coordinate in D_2.

$\theta = \dfrac{T - T_{oil}}{W/k}$ = dimensionless temperature.

ϕ = azimuthal coordinate in D_1.

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CHAPTER 1

INTRODUCTION

High-pressure oil-filled pipe-type cable circuits have been used for underground electrical power transmission for a number of years. Such circuits employ a steel conduit inside which are several cables, each consisting of a copper conductor wrapped with porous, oil-soaked paper insulation and a protective outer covering. The space between the cables and the pipe is filled with a dielectric oil which is under high pressure. The oil, which impregnates the paper wrapping on the cables, provides electrical insulation for the cables and also transfers the heat generated by losses in the cables to the conduit and the surrounding soil. Pressurization of the oil prevents vapor formation in the paper insulation and ensures proper electrical insulation of the cables. In this non-circulating type of system, heat which is generated in the cables is transferred from the insulation to the pipe wall by natural convection through the oil, and then from the pipe to the atmosphere by conduction through the soil. The power-carrying capacity of underground cables is limited by the maximum allowable cable temperature, which depends on the rate of heat removal from the system.

Force-cooled systems for oil-filled pipe-type cable circuits, which appear to have power capacities significantly larger than those of non-circulating systems, have recently been considered. In force-cooled systems, chilled oil is circulated through the pipe, and heat is transferred from the oil to the atmosphere at refrigeration stations. Most of the heat is transferred from the cables to the flowing oil, heat transfer to the soil being of secondary importance [1]. The cable-to-oil temperature difference for a given current and voltage is determined by the overall cable-to-oil heat transfer resistance, which is due to two effects: the resistance to conduction heat transfer through the cable insulation, and the resistance to convection heat transfer from the surface of the insulation to the bulk of the oil. Based on results of the natural convection experiments performed by Orchard and Slutz [2], it is demonstrated in Appendix A that the conduction resistance for the type of system which was considered is an order of magnitude larger than the convection resistance. Therefore the rate of heat removal from the system depends primarily on conduction, and an accurate conduction model of the cable insulation is required in order to confidently predict the cable temperature.

Conduction within the insulation is complicated by the proximity of one cable to another. When two cables come into direct contact, their mutual presence causes a large increase in the resistance to heat transfer near the point of contact. Consequently, the cable insulation near the contact point experiences a sharp increase in temperature, which in turn elevates the conductor temperature, and thermal failure of the system will ensue unless the oil temperature is appropriately adjusted. Given a system with a maximum allowable cable temperature, it is therefore desirable to know the maximum oil temperature which should be allowed in order to avoid thermal failure of the system. This involves determining the two-dimensional (i.e., radial and circumferential) steady-state temperature distribution within the cable insulation for various cable configurations, especially those which produce the most severe operating conditions. This heat conduction problem is too complicated to be solved analytically. However, the solution for arbitrary cable configurations is readily obtained by means of numerical methods.

The particular system which was studied consists of three circular conductors inside a circular conduit. The dimensions of this system are shown in Figures 1.1 and 1.2. In addition to the outer moisture seal, the cables are wrapped with skid wires, which protect the cable coverings and reduce friction when the cables are pulled into the conduit. In order to simplify the geometrical problems which arise in handling configurations of three cables, it was assumed that the system possesses bilateral symmetry, as shown in Figure 1.3. This assumption reduces the system to one and one-half cables inside half a conduit, while permitting arbitrary configurations of the one and one-half

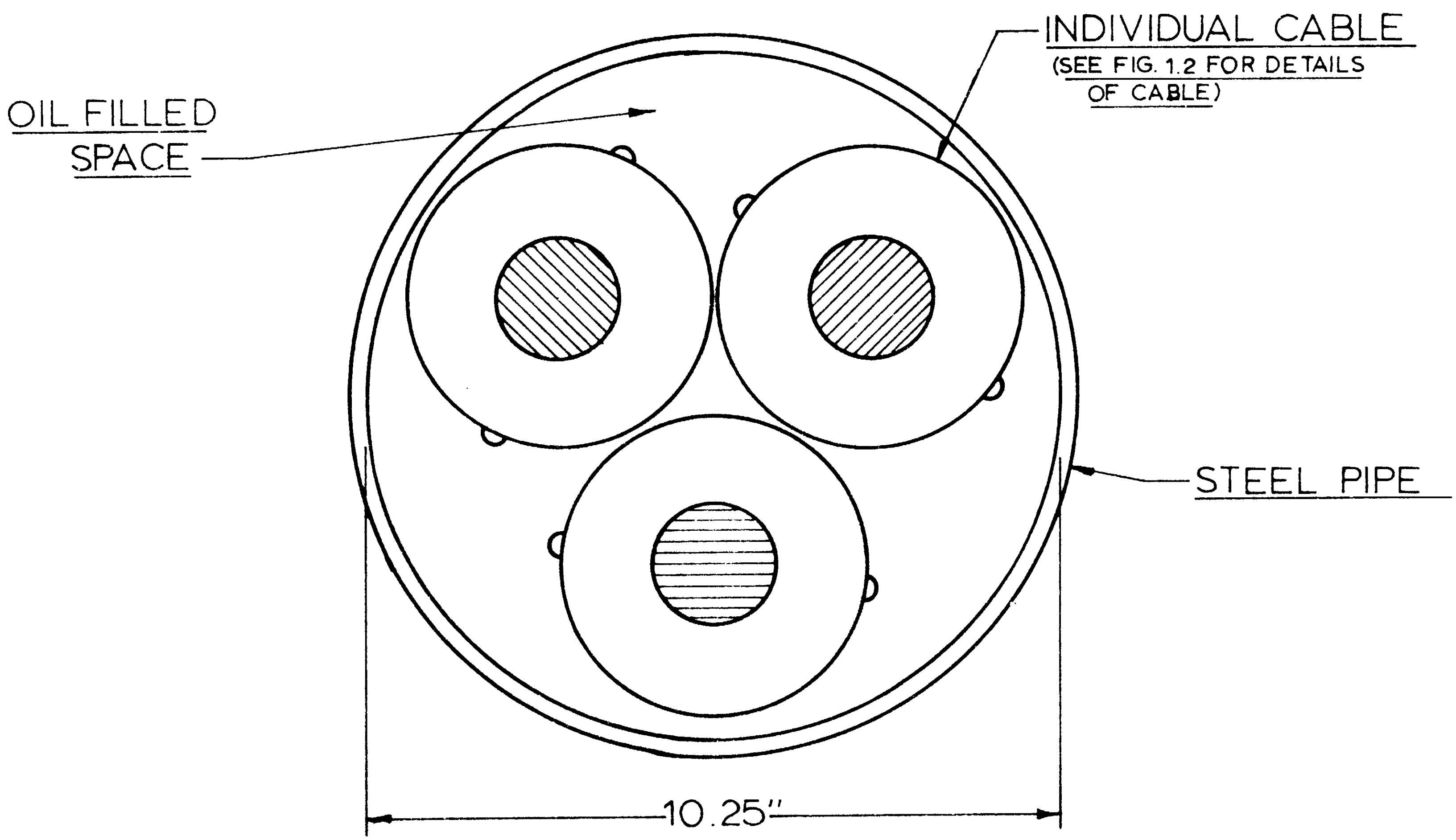


FIGURE 1.1

Cross-Section of Underground Pipe-Type Cable System

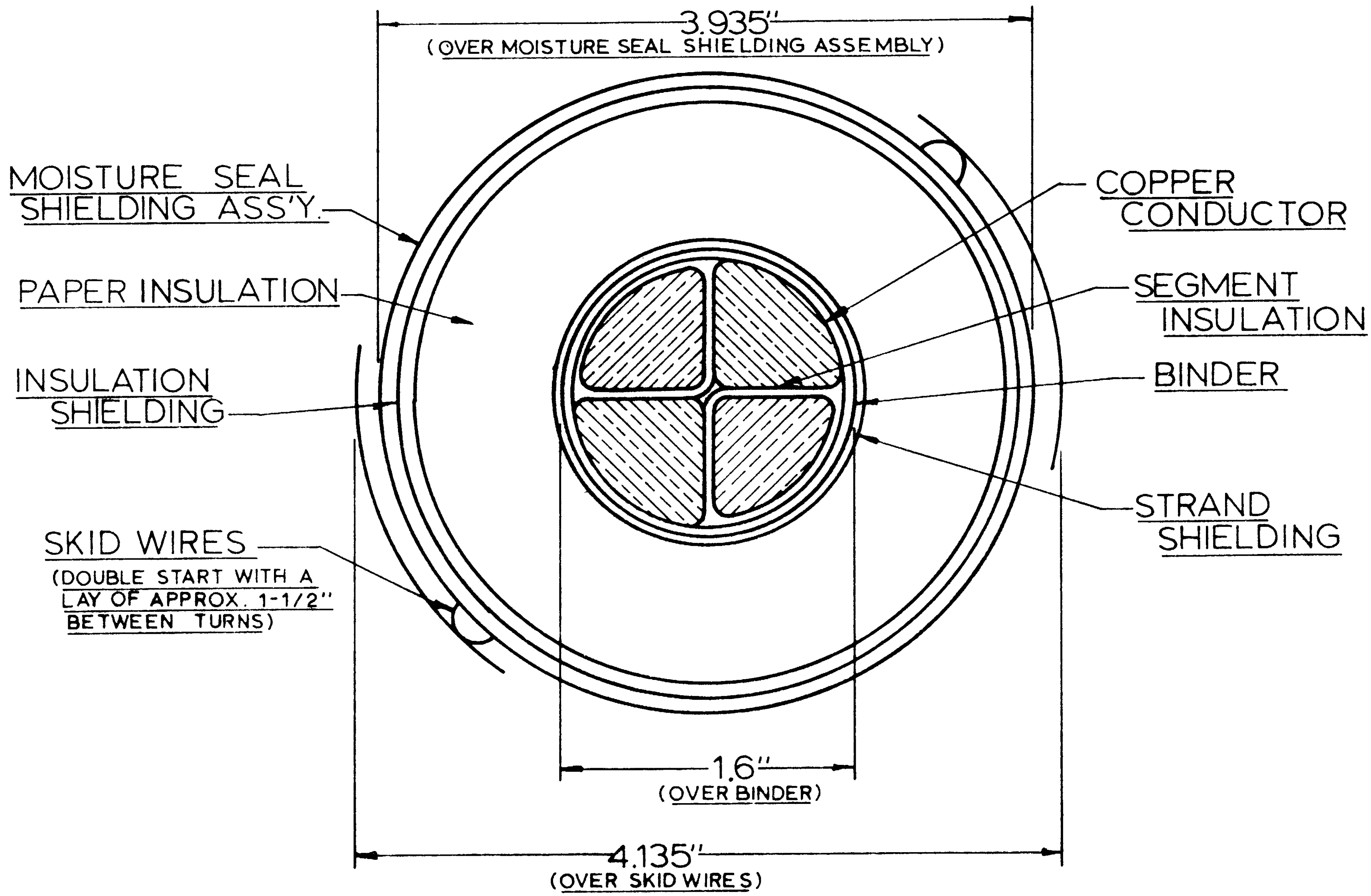


FIGURE 1.2

Cross-Section of Underground Power Cable

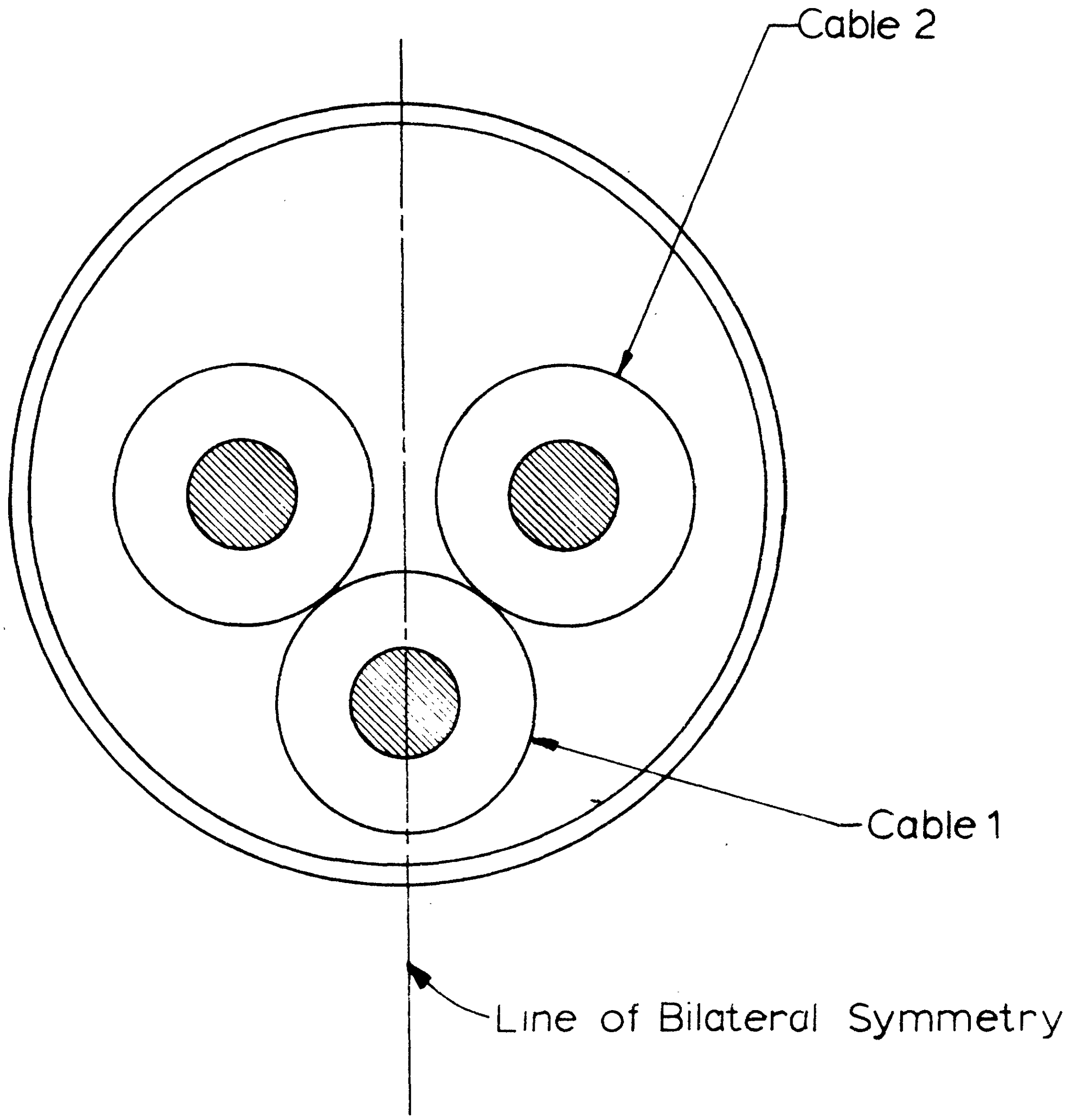


FIGURE 1.3

Bilateral Symmetry of the Underground Cable System cables. As Figure 1.3 indicates, the half- and whole cables are referred to as Cable 1 and Cable 2, respectively.

In Chapter 2 a complete formulation of the conduction problem is presented, followed in Chapter 3 by a discussion of the superposition methods which were employed in obtaining final solutions. Chapters 4 and 5 are concerned with discretizing the conduction model and with translating the discretized model into a computer program. In Chapter 6 the results of several problems are discussed, and conclusions are stated.

CHAPTER 2

FORMULATION

The Cable Insulation

In developing a conduction model for the cable insulation, the following assumptions were made: any axial conduction along the length of the cable is negligible, thus reducing the problem to two dimensions; steady-state conditions prevail in the system; the thermal conductivity throughout the insulation is taken to be uniform. Using these assumptions, an energy balance on an infinitesimal element in a cylindrical coordinate system yields the following expression, which is a special form of Poisson's equation [3]:

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} = -\frac{\dot{q}}{k} \; . \tag{2.1}$$

This equation governs the temperature distribution in the cable insulation, together with appropriate boundary conditions which operate around the various portions of the cable surface. The heat generation term $\dot{q}$ in Equation 2.1 is due to a dielectric loss which occurs throughout the insulation.

The Inter-Cable Conduction Path

In order to model the situation which exists when Cables 1 and 2 are lying together in direct contact (skid wires overlapping), a special conduction path was placed between the cable and half-cable. A conduction path was used because there is a small region between the cables in which the oil is essentially stagnant. The thermal conductivity of the path was taken to be the same as that of the insulation. The width of the path is usually taken to be the thickness of a skid wire, since this is as close as the cables come to actually touching. As an estimate of how large an angle the path should subtend along the cable surfaces, it was decided to use the angle subtended by the overlapping skid wires. For the system which was studied, this angle is approximately 25°. The inter-cable conduction path is thus an extension of the cable insulation, joining Cable 1 to Cable 2, as depicted in Figure 2.1. Since no heat sources are present within the conduction path, the governing equation for its temperature distribution is Laplace's equation [4]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 , \qquad (2.2)$$

where x and y are the normal and tangential coordinates, respectively, and where "normal" denotes an axis which joins the cable centers.

The Solution Domain

Two additional assumptions underlie the conduction model. The first is that the oil is assumed to be well-mixed, so that the oil temperature outside the convective

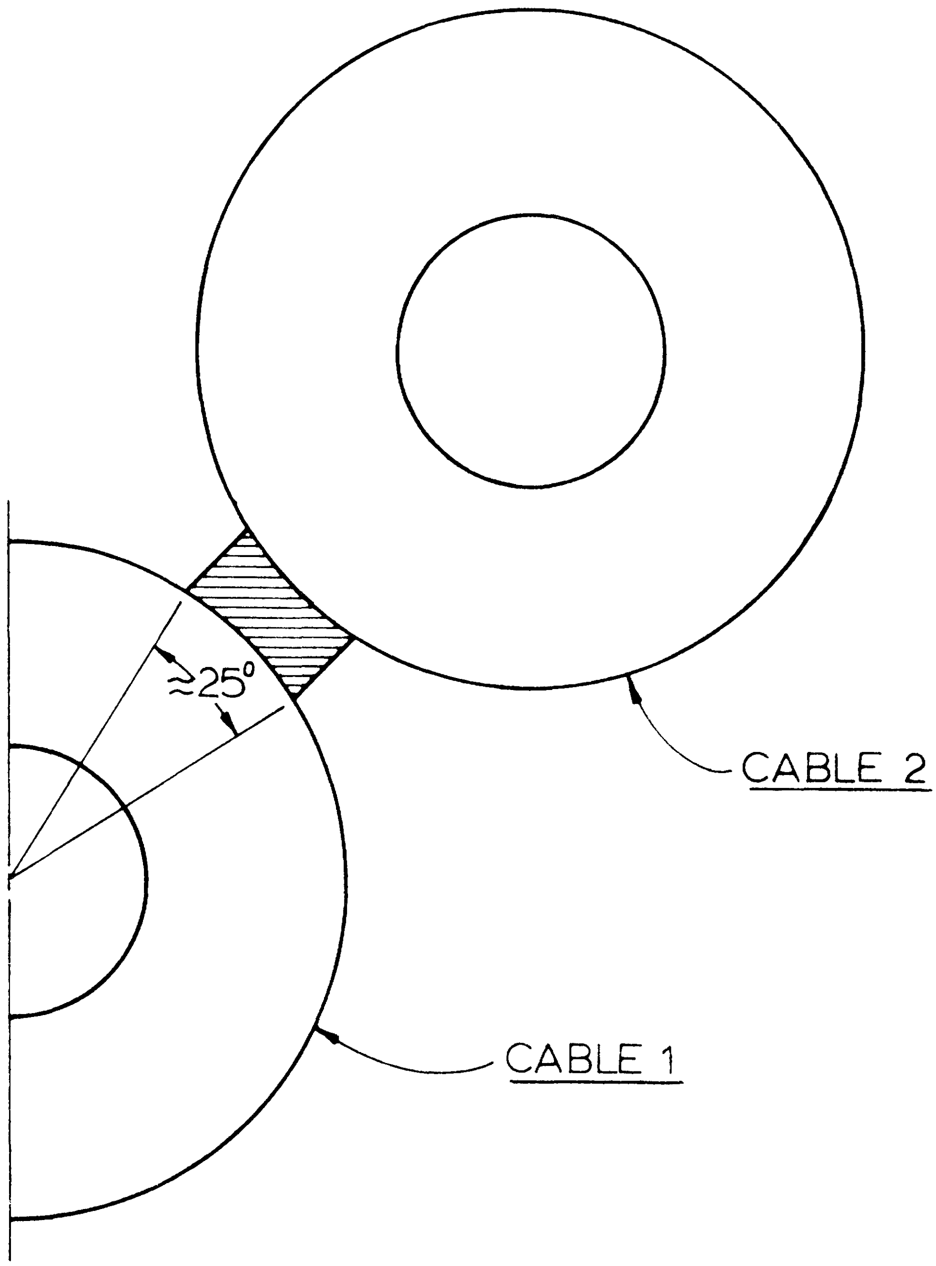


FIGURE 2.1

The Inter-Cable Conduction Path (Shaded) boundary layer is uniform at a given cross-section in the system. The second is that, because of the very high conductivity of copper, each conductor is assumed to be at a single, uniform temperature (though the two conductor temperatures are not, in general, equal). These two temperatures are obtained from a knowledge of the losses at each conductor, and this is discussed in Chapter 3 under the subject of superposition. The point to be made here is that the two conductor temperatures are not unknown quantities in the temperature field. Therefore the conduction problem has as its solution domain only the paper insulation surrounding the conductors and the inter-cable conduction path. For purposes of nomenclature, the insulation of Cable 1 is referred to as D_1 (Domain 1), that of Cable 2 is referred to as D_2 (Domain 2), and the region comprising the inter-cable conduction path is called D_3 (Domain 3). The solution domains D_1, D_2, and D_3, together with their associated coordinate systems, are shown in Figure 2.2.

Boundary Conditions

The solution domains D_1 and D_2 are divided into regions of varying size according to the type of boundary condition which is acting at the cable surface. A set of regional divisions for both cables is illustrated in Figure 2.3, and the boundary conditions associated with the various regions are listed in Table 1. Regions II of D_1 and D_2 are not included in the table, because they join the inter-cable conduction path and therefore have no surface

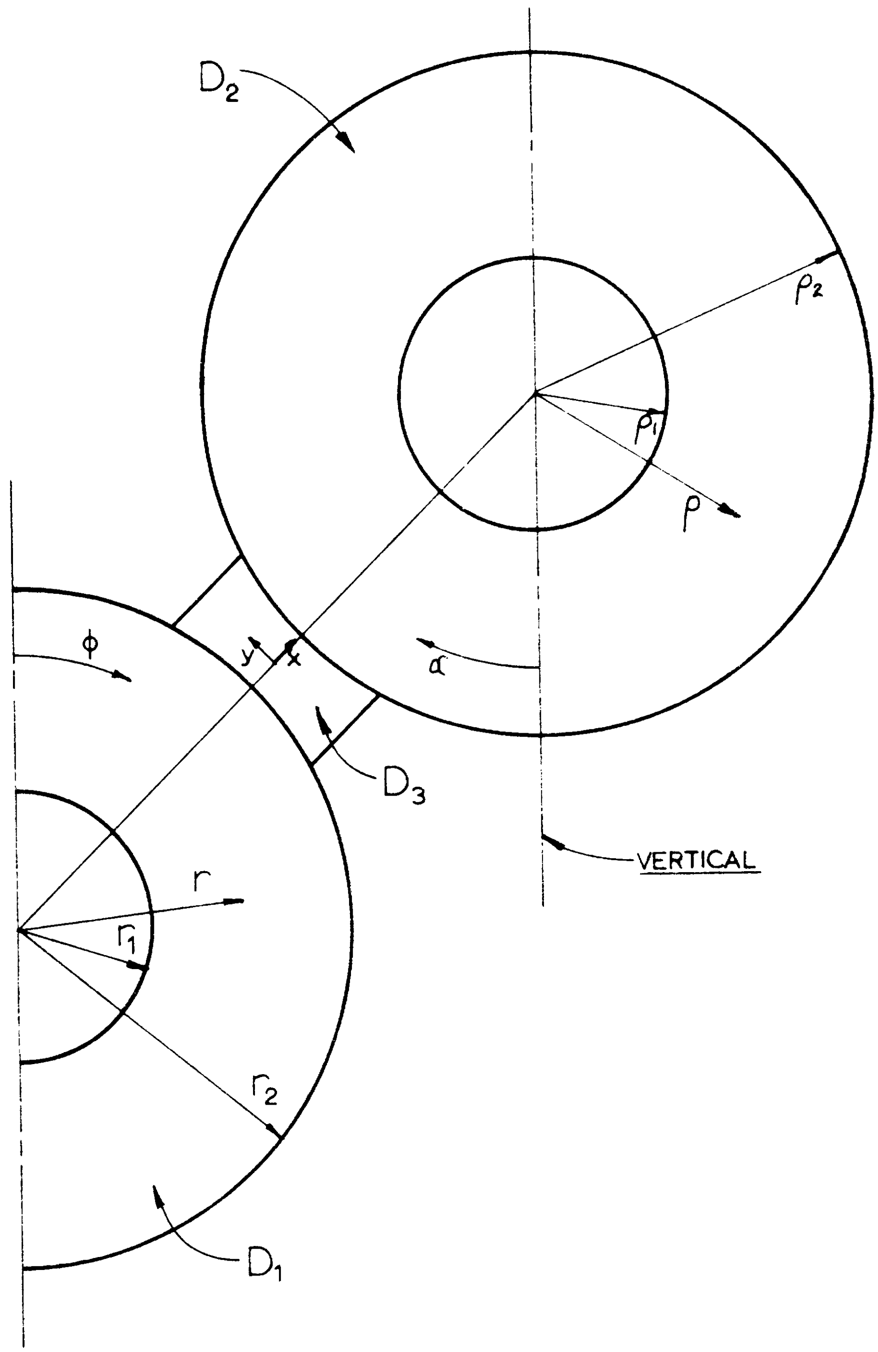


FIGURE 2.2

Coordinate Systems in D_1, D_2, and D_3

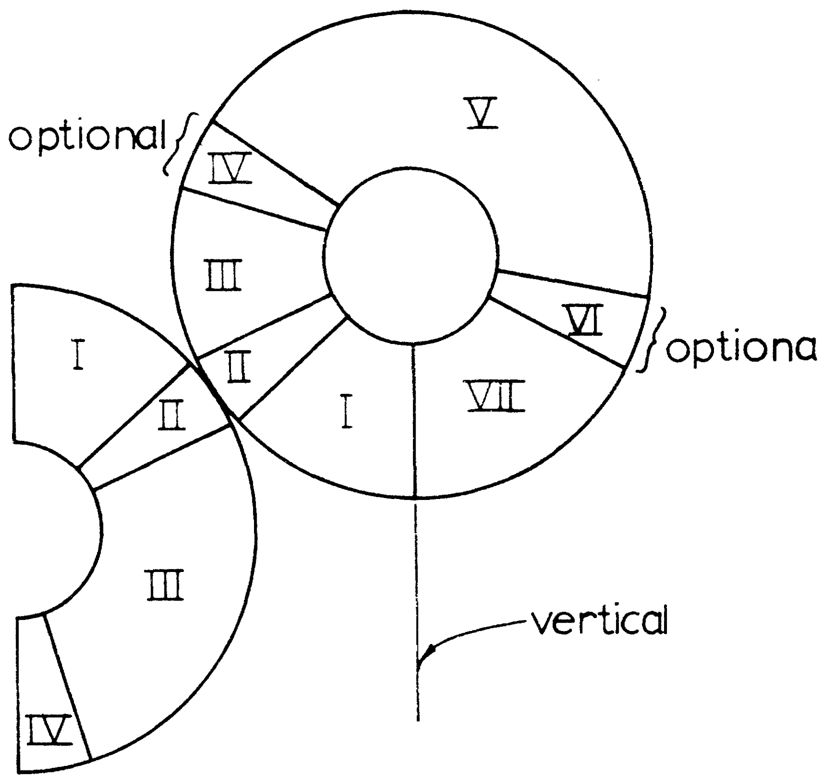


FIGURE 2.3

Regional Divisions in D_1 and D_2

TABLE 1

BOUNDARY CONDITIONS IN D_1 AND D_2

D_1	
Region I	Convection
Region III	Convection
Region IV	Cable-Conduit
D_2	
Region I	Convection
Region III	Convection
Region IV (if used)	Cable-Cable
Region V	Convection
Region VI (if used)	Cable-Conduit
Region VII	Convection

boundaries. The cable-cable and cable-conduit boundary conditions are depicted in Figure 2.4.

There are several different surface boundary conditions, but it can be shown that they are all convective in form. The convective boundary condition itself is obtained from an energy balance at the cable surface:

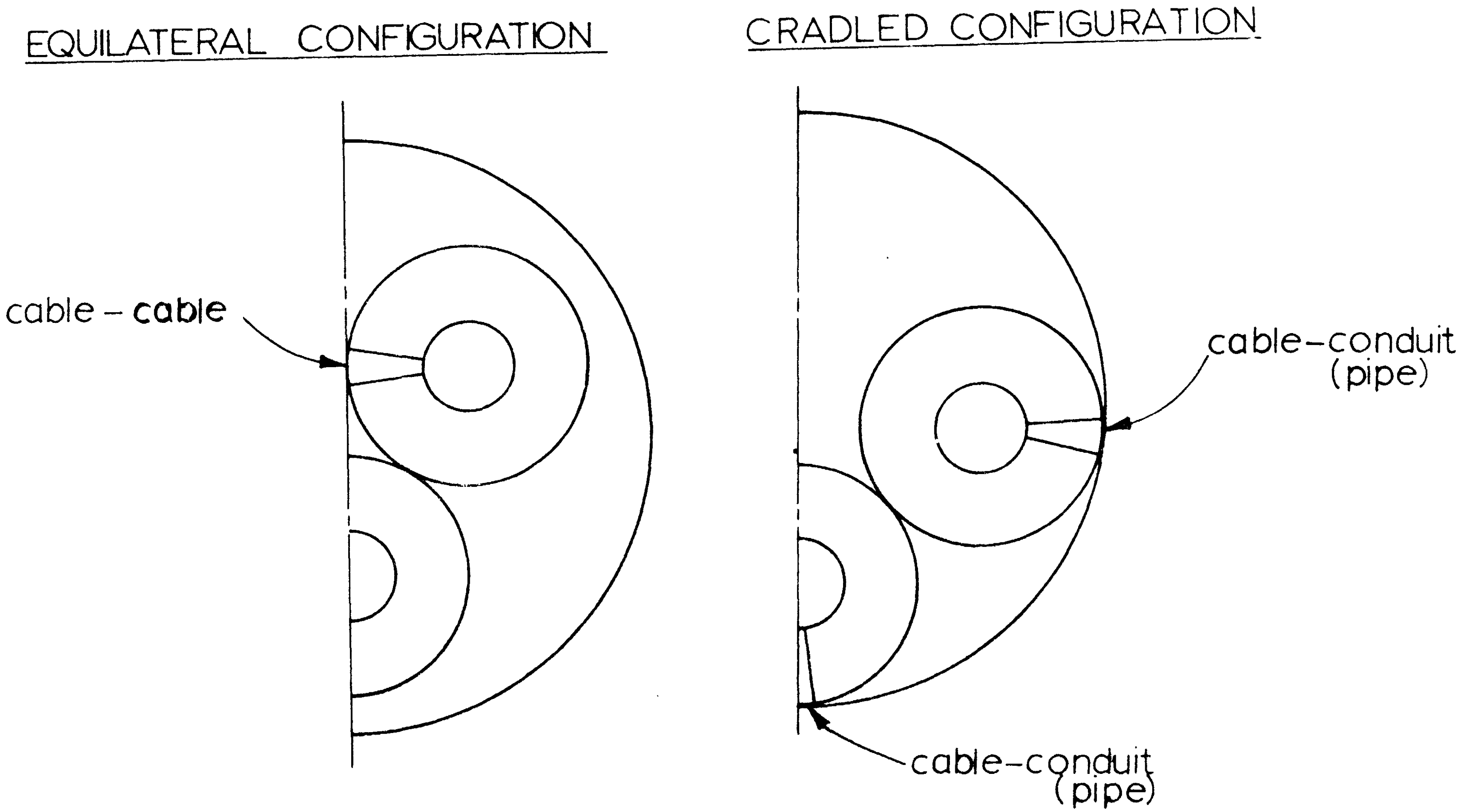
$$\frac{q}{P}\bigg|_{r_2,\phi} = -k\,\frac{\partial T}{\partial r}\bigg|_{r_2,\phi} = h\Big[T(r_2,\phi) - T_{oil}\Big], \qquad (2.3)$$

where r_2 is the outer radius of the Cable 1 insulation, P is the perimeter of the cable, and h is the local film coefficient, which may vary around the periphery of the cable.

The cable-cable boundary condition occurs when the three cables are in an equilateral configuration. A small arc along the surface of Cable 2 then lies immediately adjacent to the line of symmetry. The arc length is taken to be the same as that for the inter-cable conduction path. Since no heat flow crosses a line of symmetry, this boundary is taken to be an insulated one, which is just a convective boundary with a local film coefficient of zero. While this boundary condition differs considerably in form from the conduction mechanism operating in the inter-cable conduction path, both mechanisms have the same effect on the temperature distribution. For in the equilateral configuration, symmetrical conditions on either side of the conduction path act to prevent any flow of heat across the tangential axis

FIGURE 2.4

The Cable-Cable and Cable-Conduit Boundary Conditions



(y-axis) of D_3. The trilateral symmetry of an equilateral configuration is thus preserved by modelling the cable-cable effect as a convective boundary. An alternative method would be to employ an optional conduction path for the cable-cable effect, but this would introduce unnecessary complication.

The cable-conduit boundary condition, which exists when either cable is lying directly against the conduit, is influenced by the following factors: convection cooling near the point of contact; the thermal conductivity of the conduit, which for steel is large; potentially large AC losses in the conduit itself; and heat conduction from the conduit to the adjacent soil. This situation is examined in Appendix B, where a portion of the conduit wall is thermally modelled as a fin, and the thermal resistance through the fin is compared to the thermal resistance across the cable insulation for a given set of AC losses. In the most conservative case, the resistance from the fin base to the oil is an order of magnitude smaller than the resistance across the insulation. Thus the cable-conduit boundary condition for a steel conduit is essentially a convective one with a slightly modified film coefficient. The arc on the cable surface affected by this boundary condition is again taken to be the same as that for the inter-cable conduction path.

There are two surface boundary conditions in D_3, each a convective one. These are depicted in Figure 2.5.

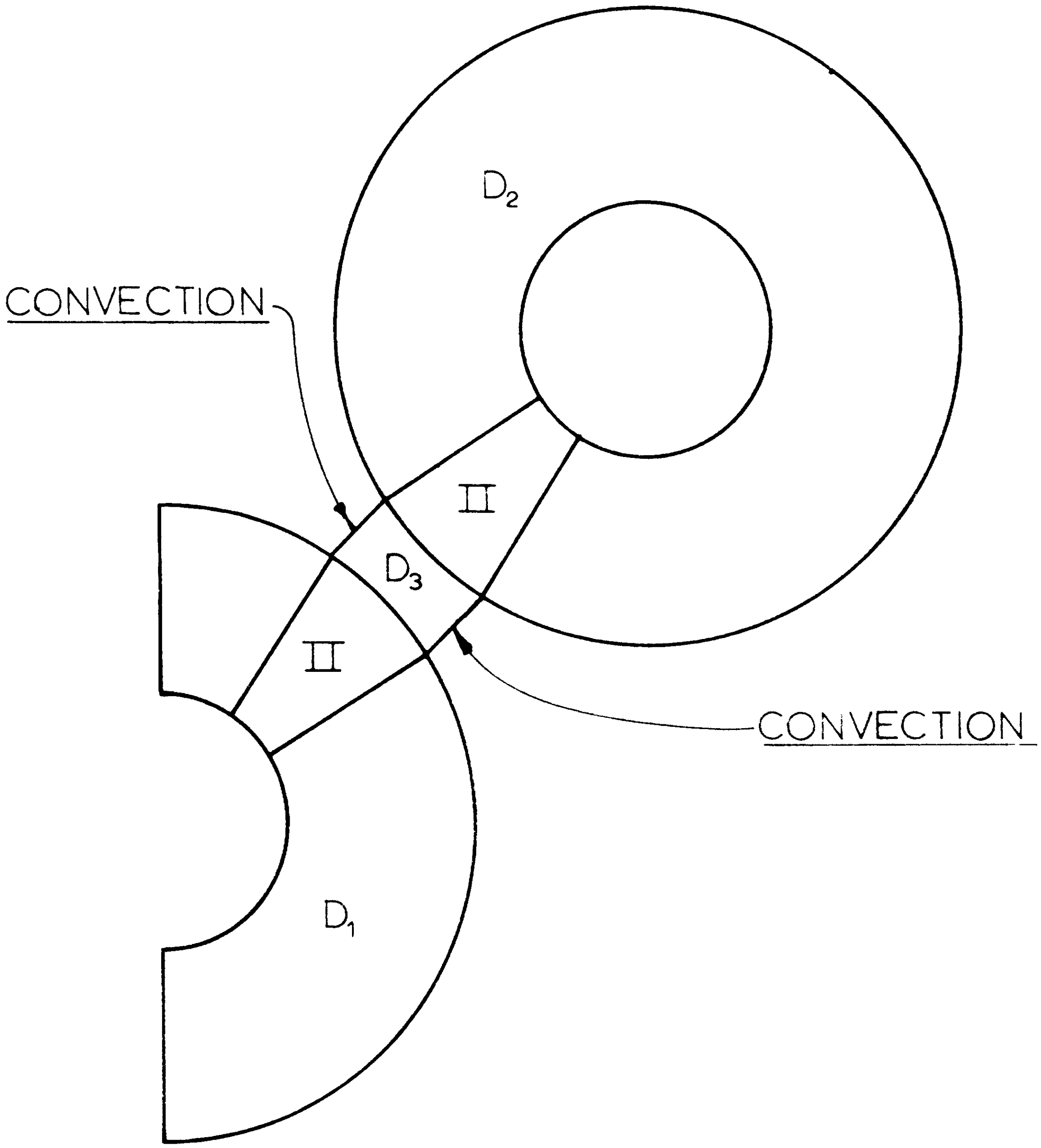


FIGURE 2.5

Convective Surfaces of D_3

An energy balance at the surface of D_3 gives

$$-k \left.\frac{\partial T}{\partial y}\right|_{\text{surface}} = \pm h\left[T(\text{surface}) - T_{\text{oil}}\right] , \qquad (2.4)$$

where (+) and (-) apply for positive and negative y, respectively, and h is a variable film coefficient.

In addition to the surface boundary conditions, there are two internal boundary conditions. These are at the line of symmetry and at the conductors. The symmetry boundary condition occurs in Cable 1, where the line of symmetry bisects the cable and forms a portion of the insulation boundary. This boundary, of course, is perfectly insulated. The boundary condition at each conductor, as was stated previously, is one of uniform temperature.

The aforementioned governing equations and boundary conditions, along with the requirement that the temperature between D_1 and D_3 and between D_2 and D_3 be single-valued, constitute a complete formulation of the conduction problem.

Variations on the Problem Statement

Although the heat conduction problem was originally posed with the oil temperature as the unknown quantity, it is possible to specify the oil temperature and solve for other quantities. Assuming that the voltage is constant for a given system, there are three variables: maximum allowable oil temperature, maximum cable temperature, and maximum allowable current. Given any two of these, the third may be found. In subsequent discussions it will be necessary to specify which of the three variables is unknown, and so the following solutions are defined for reference: Solution 1 finds the maximum cable temperature, given the current in the circuit and the oil temperature; Solution 2 finds the maximum allowable current, given the oil temperature and the maximum allowable cable temperature; Solution 3 finds the maximum allowable oil temperature, given the current in the circuit and the maximum allowable cable temperature.

Nondimensional Formulation

In preparation for numerical solution, the heat conduction problem is cast into nondimensional form. Such a formulation can be obtained by introducing the following dimensionless variables:

$$\bar{r} = \frac{r}{r_2}, \quad \bar{\rho} = \frac{\rho}{\rho_2}, \quad \bar{x} = \frac{x}{2A}, \quad \bar{y} = \frac{y}{2D}, \quad \theta = \frac{T - T_{oil}}{W/k}, \tag{2.5}$$

where 2A denotes the minimum width of D_3 (at the x-axis), 2D is the height of D_3 (along the y-axis), and W is an arbitrary loss per unit length (Btu/hr-ft). The form of the governing equation in D_1 and D_2 is then

$$\bar{r}^2 \frac{\partial^2\theta}{\partial\bar{r}^2} + \bar{r}\frac{\partial\theta}{\partial\bar{r}} + \frac{\partial^2\theta}{\partial\phi^2} = -\frac{(r_2\bar{r})^2\dot{q}}{W}, \tag{2.6}$$

whereas the governing equation in D_3 becomes

$$\frac{\partial^2\theta}{\partial\bar{x}^2} + \frac{4A^2}{D^2}\frac{\partial^2\theta}{\partial\bar{y}^2} = 0 \ . \tag{2.7}$$

The form of the standard convective boundary condition becomes

$$-\frac{k}{r_2}\frac{\partial\theta}{\partial\bar{r}}\bigg|_{1,\phi} = h\theta(1,\phi) \ . \tag{2.8}$$

CHAPTER 3

SUPERPOSITION OF SOLUTIONS

General

In the solution of linear problems, such as this problem of conduction with uniform thermal conductivity, it is often convenient to employ the principle of superposition. This reduces the overall problem to a number of simpler problems, each having the same geometry as the overall problem, whose individual solutions may be linearly combined to form the overall solution. The required number of separate solutions is equal to the number of nonhomogeneities, or potentials, in the overall problem. In the conduction problem which has been posed, there are three potentials: the two conductor temperatures and the volumetric heating effect. The overall problem may thus be decomposed into three component problems. Solutions to these component problems need to be generated only once for a particular cable geometry and voltage (dielectric loss); the total solution for any arrangement of current-produced losses can then be achieved by suitably combining the three component solutions.

In the following sections the superposition technique for obtaining Solution 1 (which finds the cable temperature) is presented. It is then rigorously demonstrated that the overall governing equation and boundary conditions are obtained from a linear combination of the governing equations and boundary conditions of the three component problems. For brevity the following notation is introduced: $\underset{\sim}{x}$ is a generalized position vector for the overall solution domain (comprised of D_1, D_2, and D_3); $\underset{\sim}{x} \in C_i(\underset{\sim}{x})$ denotes all points in the solution domain which lie on the curve $C_i(\underset{\sim}{x})$; n_i is an outward normal to the curve $C_i(\underset{\sim}{x})$; ∇^2 is the Laplacian operator. The nine curves $C_i(\underset{\sim}{x})$ which comprise the boundaries of the solution domain are shown in Figure 3.1. The nine normals n_i are all dimensionless: normals to curves in D_1 are nondimensionalized with r_2, normals to curves in D_2 with ρ_2, and normals the two curves in D_3 with the length 2D.

The Overall Problem

The governing equation for the overall problem is the following:

$$\nabla^2\theta(\underset{\sim}{x}) = f(\underset{\sim}{x}) , \tag{3.1}$$

where $f(\underset{\sim}{x})$ describes forcing effects throughout the domain. $\theta(\underset{\sim}{x})$ also satisfies boundary conditions on the nine curves $C_i(\underset{\sim}{x})$. On the curve $C_1(\underset{\sim}{x})$ the condition is

$$\theta(x)\big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} = \theta_{01} , \tag{3.2}$$

where θ_{01} is some uniform (as yet unknown), dimensionless temperature. On the remaining curves the boundary

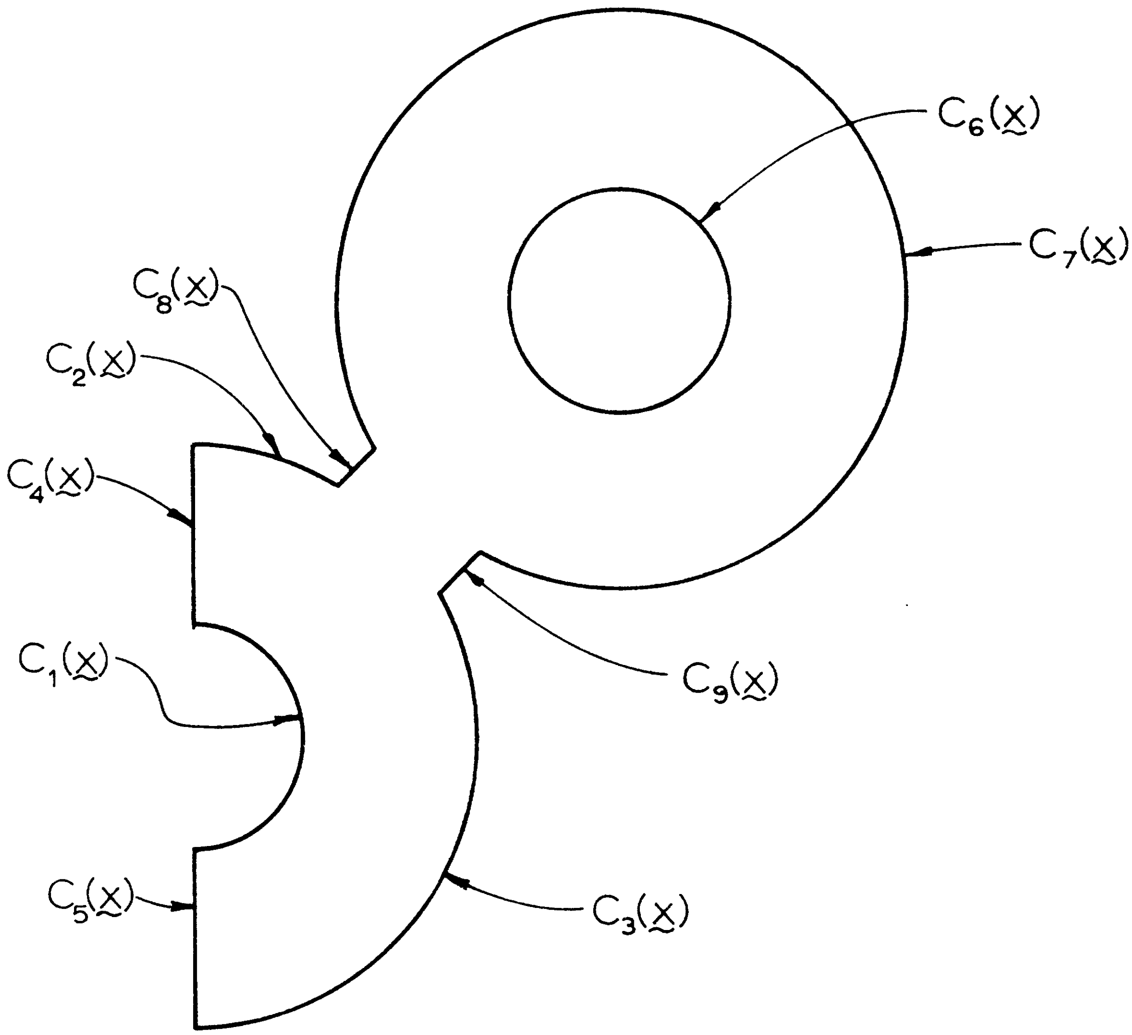


FIGURE 3.1

Curves $C_i(\underset{\sim}{x})$ Comprising the Boundaries of the Solution Domain conditions are

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_2}\right|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} \qquad (3.3)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} \qquad (3.4)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x}\in C_4(\underset{\sim}{x})} = 0 \qquad (3.5)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x}\in C_5(\underset{\sim}{x})} = 0 \qquad (3.6)$$

$$\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_6(\underset{\sim}{x})} = \theta_{02}\,, \qquad (3.7)$$

where θ_{02} is a uniform (as yet unknown), dimensionless temperature.

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})} = -\frac{h\rho_2}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})} \qquad (3.8)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} = -\frac{2Dh}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} \qquad (3.9)$$

$$\left.\frac{\partial \theta(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} = -\frac{2Dh}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} \tag{3.10}$$

The Component Problems

The overall problem is decomposed into three component problems, each of which has only one potential and is individually solvable. The component solutions are $\theta A(\underset{\sim}{x})$, $\theta B(\underset{\sim}{x})$, and $\theta C(\underset{\sim}{x})$.

$\theta A(\underset{\sim}{x})$ is the solution for the physical situation in which the Cable 1 conductor is hot, the Cable 2 conductor is cold (at the oil temperature) and in which there is no dielectric loss. $\theta A(\underset{\sim}{x})$ satisfies the homogeneous governing equation

$$\nabla^2 \theta A(\underset{\sim}{x}) = 0 \ , \tag{3.11}$$

and it satisfies the following boundary conditions:

$$\theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_1(\underset{\sim}{x})} = A_o \ , \tag{3.12}$$

where A_o is some arbitrary dimensionless temperature.

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_2}\right|_{\underset{\sim}{x} \in C_2(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_2(\underset{\sim}{x})} \tag{3.13}$$

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} = -\frac{hr_2}{k}\, \theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} \tag{3.14}$$

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x} \in C_4(\underset{\sim}{x})} = 0 \tag{3.15}$$

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x} \in C_5(\underset{\sim}{x})} = 0 \tag{3.16}$$

$$\theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} = 0 \tag{3.17}$$

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} = -\frac{h\rho_2}{k}\, \theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} \tag{3.18}$$

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} = -\frac{2Dh}{k}\, \theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} \tag{3.19}$$

$$\left.\frac{\partial \theta A(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} = -\frac{2Dh}{k}\, \theta A(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} \tag{3.20}$$

$\theta B(\underset{\sim}{x})$ is the solution for the physical situation in which the Cable 1 conductor is cold (at the oil temperature), the Cable 1 conductor is hot, and in which there is no dielectric loss. The component solution $\theta B(\underset{\sim}{x})$ satisfies the homogeneous governing equation

$$\nabla^2 \theta B(\underset{\sim}{x}) = 0 \;, \tag{3.21}$$

as well as the following boundary conditions:

$$\theta B(\underset{\sim}{x}) \Big|_{\underset{\sim}{x} \in C_1(\underset{\sim}{x})} = 0 \tag{3.22}$$

$$\left. \frac{\partial \theta B(\underset{\sim}{x})}{\partial n_2} \right|_{\underset{\sim}{x} \in C_2(\underset{\sim}{x})} = - \frac{hr_2}{k} \, \theta B(\underset{\sim}{x}) \Big|_{\underset{\sim}{x} \in C_2(\underset{\sim}{x})} \tag{3.23}$$

$$\left. \frac{\partial \theta B(\underset{\sim}{x})}{\partial n_3} \right|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} = - \frac{hr_2}{k} \, \theta B(\underset{\sim}{x}) \Big|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} \tag{3.24}$$

$$\left. \frac{\partial \theta B(\underset{\sim}{x})}{\partial n_4} \right|_{\underset{\sim}{x} \in C_4(\underset{\sim}{x})} = 0 \tag{3.25}$$

$$\left. \frac{\partial \theta B(\underset{\sim}{x})}{\partial n_5} \right|_{\underset{\sim}{x} \in C_5(\underset{\sim}{x})} = 0 \tag{3.26}$$

$$\theta B(\underset{\sim}{x}) \Big|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} = B_o \;, \tag{3.27}$$

where B_o is an arbitrary dimensionless temperature.

$$\left. \frac{\partial \theta B(\underset{\sim}{x})}{\partial n_7} \right|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} = - \frac{h\rho_2}{k} \, \theta B(\underset{\sim}{x}) \Big|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} \tag{3.28}$$

$$\left.\frac{\partial \theta B(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} = -\frac{2Dh}{k}\,\theta B(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} \tag{3.29}$$

$$\left.\frac{\partial \theta B(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})} = -\frac{2Dh}{k}\,\theta B(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})} \tag{3.30}$$

Finally, $\theta C(\underset{\sim}{x})$ is the solution for the physical situation in which both conductors are cold (at the oil temperature), but in which there is a prescribed dielectric loss. The component solution $\theta C(\underset{\sim}{x})$ satisfies the nonhomogeneous governing equation

$$\nabla^2 \theta C(\underset{\sim}{x}) = f(\underset{\sim}{x}) \tag{3.31}$$

and the following boundary conditions:

$$\theta C(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} = 0 \tag{3.32}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_2}\right|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta C(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} \tag{3.33}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta C(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} \tag{3.34}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x} \in C_4(\underset{\sim}{x})} = 0 \tag{3.35}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x} \in C_5(\underset{\sim}{x})} = 0 \tag{3.36}$$

$$\theta C(\underset{\sim}{x}) \Big|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} = 0 \tag{3.37}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} = -\frac{h\rho_2}{k} \left.\theta C(\underset{\sim}{x})\right|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} \tag{3.38}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} = -\frac{2Dh}{k} \left.\theta C(\underset{\sim}{x})\right|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} \tag{3.39}$$

$$\left.\frac{\partial \theta C(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} = -\frac{2Dh}{k} \left.\theta C(\underset{\sim}{x})\right|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} \tag{3.40}$$

Validity of the Superposition Method

Having described the overall problem and the three component problems, it remains to demonstrate the validity of the superposition method. The three component solutions are linearly combined to form the total solution according to [5]:

$$\theta(\underset{\sim}{x}) = a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x}) , \tag{3.41}$$

where a_1 and a_2 are two arbitrary constants, to be determined from two additional boundary conditions in the overall problem. That Equation 3.41 is indeed valid is proven by substituting it directly into the overall governing equation and boundary conditions. The following results are then obtained:

$$\nabla^2\theta(\underset{\sim}{x}) = \nabla^2[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})] = f(\underset{\sim}{x}) \quad \text{Check} \tag{3.42}$$

$$\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} = [a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})]\Big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})}$$

$$= a_1 A_o \quad \text{Check,} \tag{3.43}$$

provided $a_1 A_o = \theta_{01}$. This presents no problem, since θ_{01} is unknown, and a_1 and A_o are both arbitrary.

$$\frac{\partial\theta(\underset{\sim}{x})}{\partial n_2}\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} = \frac{\partial}{\partial n_2}[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})]\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})]\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}\,\theta(\underset{\sim}{x})\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} \quad \text{Check} \tag{3.44}$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} = \frac{\partial}{\partial n_3}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} \qquad \text{Check} \qquad (3.45)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x}\in C_4(\underset{\sim}{x})} = \frac{\partial}{\partial n_4}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_4(\underset{\sim}{x})}$$

$$= 0 \qquad \text{Check} \qquad (3.46)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x}\in C_5(\underset{\sim}{x})} = \frac{\partial}{\partial n_5}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_5(\underset{\sim}{x})}$$

$$= 0 \qquad \text{Check} \qquad (3.47)$$

$$\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_6(\underset{\sim}{x})} = \left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_6(\underset{\sim}{x})}$$

$$= a_2B_o \qquad \text{Check,} \qquad (3.48)$$

provided $a_2B_o = \theta_{02}$. This also causes no difficulty, since θ_{02} is unknown, and a_2 and B_o are arbitrary.

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})} = \frac{\partial}{\partial n_7}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})}$$

$$= -\frac{h\rho_2}{k}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})}$$

$$= -\frac{h\rho_2}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})} \qquad \text{Check} \qquad (3.49)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} = \frac{\partial}{\partial n_8}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} \qquad \text{Check} \qquad (3.50)$$

$$\left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})} = \frac{\partial}{\partial n_9}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\left[a_1\theta A(\underset{\sim}{x}) + a_2\theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\,\theta(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})} \qquad \text{Check.} \qquad (3.51)$$

It is thus established that the superposition technique just described is indeed valid. However, the two conductor temperatures $\theta_{01} = a_1 A_o$ and $\theta_{02} = a_2 B_o$ have yet to be found.

Determination of Conductor Temperatures

All that has been said of the conductor temperatures up to this point is that each one is uniform. These two temperatures, though, are uniquely determined by two additional conditions in Solution 1: the specified loss per unit axial length at each conductor. Returning momentarily to dimensional variables, let W_{C1} and W_{C2} be the specified conductor losses per unit axial length in Cables 1 and 2, respectively. Equating W_{C1} to the total heat flow per unit length transferred from the conductor of Cable 1, the following result is obtained:

$$q_1 = -k \int_0^{\pi} \left.\frac{\partial T}{\partial r}\right|_{r_1,\phi} r_1 d\phi = W_{C1} \ . \tag{3.52}$$

Likewise, for Cable 2

$$q_2 = -k \int_0^{2\pi} \left.\frac{\partial T}{\partial \rho}\right|_{\rho_1,\alpha} \rho_1 d\alpha = W_{C2} \ . \tag{3.53}$$

These two expressions are rendered dimensionless and rearranged to give

$$\int_0^{\pi} \left.\frac{\partial\theta}{\partial\bar{r}}\right|_{\bar{r}_1,\phi} d\phi = -\frac{r_2 W_{C1}}{r_1 W} ; \quad (3.54)$$

$$\int_0^{2\pi} \left.\frac{\partial\theta}{\partial\bar{\rho}}\right|_{\bar{\rho}_1,\alpha} d\alpha = -\frac{\rho_2 W_{C2}}{\rho_1 W} . \quad (3.55)$$

Finally, in terms of the present vector notation, Equations 3.54 and 3.55 become the following:

$$\int_{C_1(\underset{\sim}{x})} \left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_1}\right|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} dC_1(\underset{\sim}{x}) = +\frac{W_{C1} r_2}{W r_1} , \quad (3.56)$$

$$\int_{C_6(\underset{\sim}{x})} \left.\frac{\partial\theta(\underset{\sim}{x})}{\partial n_6}\right|_{\underset{\sim}{x}\in C_6(\underset{\sim}{x})} dC_6(\underset{\sim}{x}) = +\frac{W_{C2}\, \rho_2}{W\, \rho_1} . \quad (3.57)$$

$\theta(\underset{\sim}{x})$ may now be eliminated from these two equations in favor of $\theta A(\underset{\sim}{x})$, $\theta B(\underset{\sim}{x})$, and $\theta C(\underset{\sim}{x})$ by substituting Equation 3.41 into Equations 3.56 and 3.57:

$$\int_{C_1(\underset{\sim}{x})} \frac{\partial}{\partial n_1} \left[a_1 \theta A(\underset{\sim}{x}) + a_2 \theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x}) \right] \Bigg|_{\underset{\sim}{x} \in C_1(\underset{\sim}{x})} dC_1(\underset{\sim}{x})$$

$$= + \frac{W_{C1} r_2}{W r_1} \; ; \qquad (3.58)$$

$$\int_{C_6(\underset{\sim}{x})} \frac{\partial}{\partial n_6} \left[a_1 \theta A(\underset{\sim}{x}) + a_2 \theta B(\underset{\sim}{x}) + \theta C(\underset{\sim}{x}) \right] \Bigg|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} dC_6(\underset{\sim}{x})$$

$$= + \frac{W_{C2} \rho_2}{W \rho_1} \; . \qquad (3.59)$$

Since the component solutions $\theta A(\underset{\sim}{x})$, $\theta B(\underset{\sim}{x})$, and $\theta C(\underset{\sim}{x})$ are each known, Equations 3.58 and 3.59 are two simultaneous equations from which the arbitrary constants a_1 and a_2 are determined. Furthermore, since A_o and B_o are known quantities (the arbitrary dimensionless temperatures which were used in solutions $\theta A(\underset{\sim}{x})$ and $\theta B(\underset{\sim}{x})$, respectively), the dimensionless conductor temperatures follow directly from Equations 3.43 and 3.48: $\theta_{01} = a_1 A_o$, and $\theta_{02} = a_2 B_o$. This then completes a description of the superposition technique for Solution 1.

The technique for obtaining Solution 2 (which finds the maximum current) is somewhat more complicated, owing to the fact that current is then a variable. This makes necessary a separation of current-produced losses from voltage-produced losses, as well as a subsequent procedure for maximizing current with respect to the allowable cable temperature and the oil temperature. A full description of this solution is presented in Appendix C. Solution 3 (which finds the oil temperature) is nearly identical to Solution 1, the former requiring only a minor extension of the latter. In particular, Solution 3 is obtained by using an arbitrary oil temperature in Solution 1 and then by equally incrementing all temperatures (including the arbitrary oil temperature) until the maximum temperature in the field has reached the prescribed allowable value. The two temperature distributions therefore have the same shape, differing only by a constant.

CHAPTER 4

THE FINITE-DIFFERENCE METHOD

Discretization of Domains

The numerical method used to generate solutions for the various component problems described in Chapter 3 is the finite-difference method. It has as its first basic step discretizing the solution domain. Discretization is the reduction of a continuous system into a system which has a finite number of degrees of freedom. The basic approximation involves the replacement of a continuous domain by a network of discrete points within the domain. A one-dimensional example of this is shown in Figure 4.1. Instead of obtaining a continuous solution defined throughout the domain, approximations to the true solution are found only at these isolated points.

Discretization of D_1 and D_2 is accomplished by defining a network of radial and circumferential mesh points. Since it is desirable in terms of computational labor for the mesh to be as regular as possible, the following conventions were adopted: points along a radius are uniformly spaced, though the spacing in D_1 may be different from that in D_2; circumferential spacing of points within a particular region is uniform; the number of circumferential subdivisions in both Regions II of D_1 and D_2 and the number of tangential subdivisions in D_3 are constrained to be equal, thereby avoiding mismatches at the D_1-D_3 and D_2-D_3

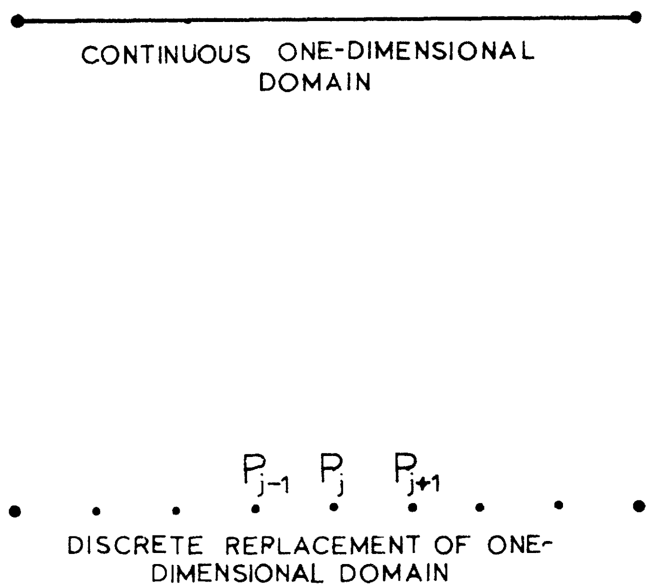


FIGURE 4.1

A Regular, One-Dimensional Finite-Difference Mesh interfaces. It should be noted that, just as the conductors were not included in the continuous domains D_1 and D_2, so are they not included in the corresponding discrete domains.

The dimensionless radial spacings h_r and h_ρ are obtained from

$$h_r = \frac{1 - \bar{r}_1}{N_1} , \qquad h = \frac{1 - \bar{\rho}_1}{N_2} , \tag{4.1}$$

where N_1 and N_2 are the number of radial subdivisions in D_1 and D_2, respectively. The dimensionless circumferential spacings h_ϕ and h_α vary in magnitude depending on which region is involved, since the regions generally are of varying size. Thus there are four values of h_ϕ in D_1 and seven values of h_α in D_2, or one for each region. A typical spacing is calculated according to

$$(h_\phi)_n = \frac{\phi_n - \phi_{n-1}}{M_n} , \tag{4.2}$$

where ϕ_n and ϕ_{n-1} are the values of ϕ at the bounding radial lines of Region n, and M_n is the number of circumferential subdivisions in Region n.

Discretization of the domain D_3 is difficult because of its irregular shape. For this reason it was approximated by the more regular shape shown in Figure 4.2. The nondimensional normal and tangential spacings h_x and h_y

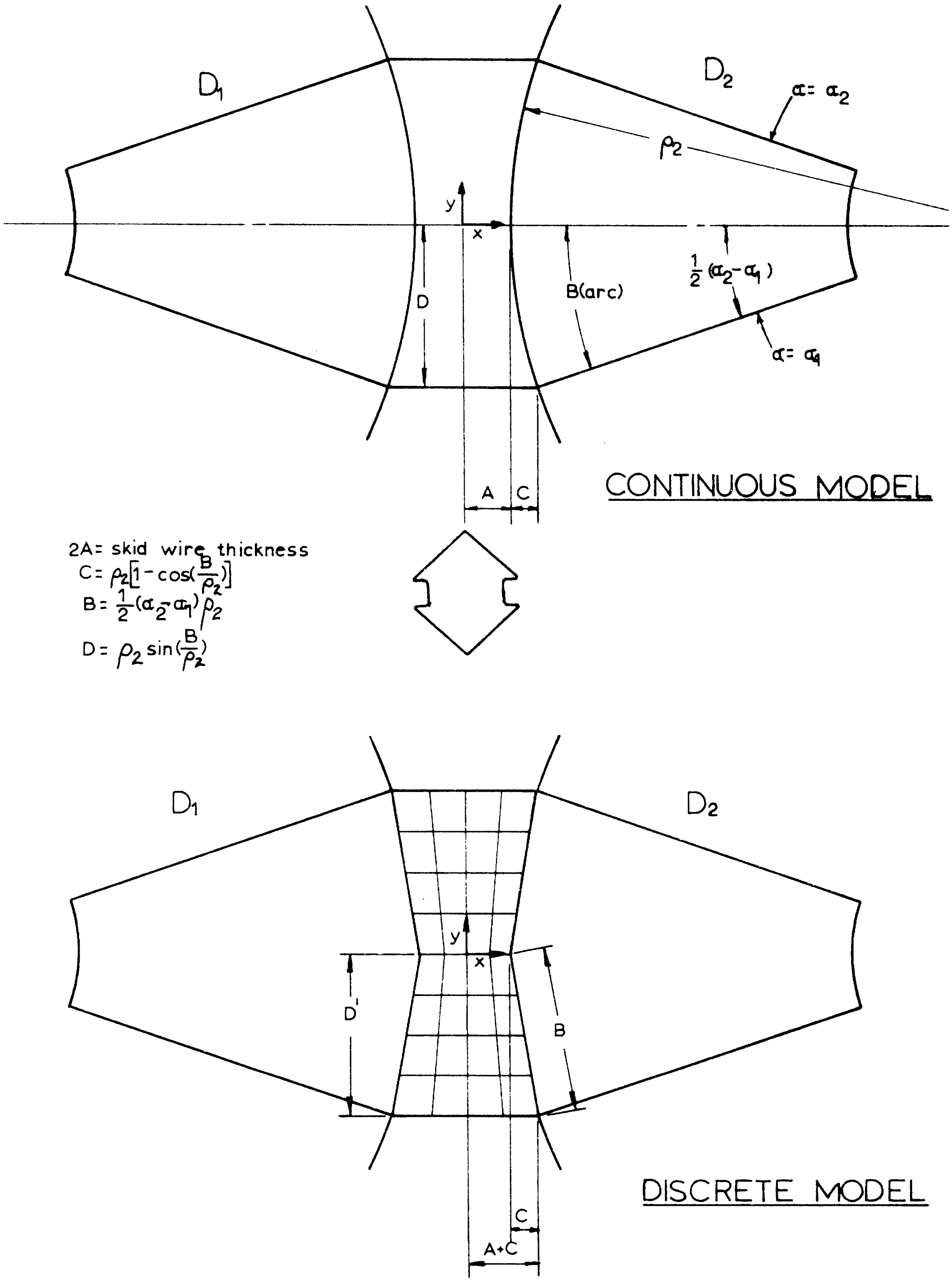


FIGURE 4.2

Discretization of the Domain D_3

are obtained by dividing the nondimensional width and height, respectively, by the appropriate number of subdivisions. It is seen in Figure 4.2 that these spacings are not uniform: h_x depends on y and h_y is a function of x. Since changes in h_y are small compared to changes in h_x, a uniform, mean value for h_y was assumed. The linear dependence of h_x on y was retained in the model.

A subtlety regarding the two convective surfaces of domain D_3 is also mentioned here briefly. At each corner of D_3 there exists a discontinuity in the area available for conduction heat transfer. This discontinuity is most conveniently accounted for in the following manner: the regular form of the governing equation, which itself assumes no discontinuity in area, is applied at each corner point. Four effective corner locations, which lie outside the corner mesh points, are thereby established, and these corner locations define the two effective surfaces for convection in D_3. The mesh points along either convective surface thus lie inside the conduction path, rather than on the boundary itself. Details of this modelling procedure are discussed in the user's instructions in Appendix E.

The discretized domains are shown with a typical mesh in Figure 4.3. The number of points to be used in a given problem is dictated by the level of accuracy required in the solution; as the mesh becomes finer, it more nearly approaches the original continuous domain. Also it is economical to use a coarse network in regions where the

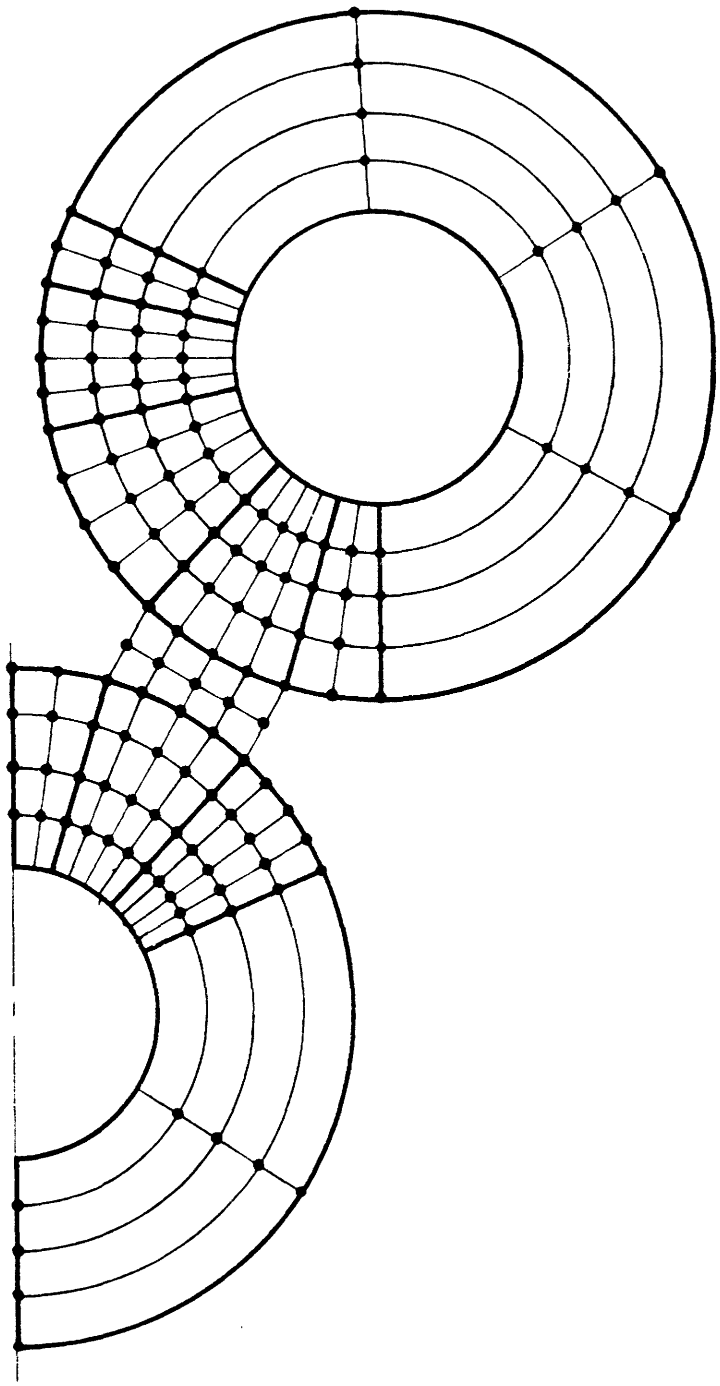


FIGURE 4.3

A Typical Mesh for the Equilateral Configuration temperature gradient is small, switching to a finer mesh where there are rapid variations in temperature. Note, for example, how the mesh points in Figure 4.3 are arranged: since the temperature distribution away from points of contact should be nearly one-dimensional, most of the points are concentrated between the cables, where large gradients are expected.

Difference Form of Governing Equations and Boundary Conditions

The reduction of a governing equation and boundary conditions for a continuous domain to those of its discrete replacement may be accomplished physically or mathematically. In the mathematical approach, which was used by the author, the continuous formulation is reduced to a discrete formulation by simply replacing derivatives with finite-difference approximations. When this is done, the original system of governing partial differential equations is reduced to a set of n simultaneous algebraic equations, where n is the number of discrete points in the mesh. Since the original continuous system is linear, the algebraic system will also be linear.

In preparation for replacing differential equations with finite difference relations, two basic one-dimensional finite-difference expressions are listed. The extension to two dimensions is straightforward. In Figure 4.1 let the points ..., P_{j-1}, P_j, P_{j+1},... be separated by a dimensionless spacing h, and let the value of

$\psi(z)$ at P_j be denoted by ψ_j. Then the first and second derivatives at P_j may be approximated by the following finite-difference expressions [6]:

$$\left(\frac{d\psi}{dz}\right)_j = \frac{\psi_{j+1} - \psi_{j-1}}{2h} + O(h^2) \; ; \tag{4.3}$$

$$\left(\frac{d^2\psi}{dz^2}\right)_j = \frac{\psi_{j+1} - 2\psi_j + \psi_{j-1}}{h^2} + O(h^2) \; ; \tag{4.4}$$

where O( ) denotes the order of the error. With the availability of these computational formulas, the process of replacing the governing equations and boundary conditions of the heat conduction problem with approximate algebraic equations is simple and direct: at each internal point the finite-difference approximation to the governing differential equation provides an algebraic equation connecting the values of ψ at the several neighboring points. For example, a typical equation at the point (j2,k2) in D_2, Region IV is:

$$\left[\frac{\bar{\rho}^2_{j2}}{h^2_\rho} - \frac{\bar{\rho}_{j2}}{2h_\rho}\right]\theta_{j2-1,k2} - 2\left[\frac{\bar{\rho}^2_{j2}}{h^2_\rho} + \frac{1}{h^2_{\alpha_4}}\right]\theta_{j2,k2} + \left[\frac{\bar{\rho}^2_{j2}}{h^2_\rho} + \frac{\bar{\rho}_{j2}}{2h_\rho}\right]\theta_{j2+1,k2}$$

$$+ \left[\frac{1}{h^2_{\alpha_4}}\right]\theta_{j2,k2-1} + \left[\frac{1}{h^2_{\alpha_4}}\right]\theta_{j2,k2+1} = - \frac{\bar{\rho}^2_{j2}\rho^2_2(\dot{q})_{j2,k2}}{W} \; , \tag{4.5}$$

where ρ_2 is the outer radius of Cable 2, $(\dot{q})_{j2,k2}$ is the local volumetric loss, W is an arbitrary loss per unit length, and the remaining symbols are explained in Figure 4.4. This result was obtained by substituting the two-dimensional forms of Equations 4.3 and 4.4 into a typical governing equation, such as Equation 2.6.

Two types of exceptional situations can arise in applying this equation. The first is that on the boundaries, not all the neighboring points of a governing equation will lie within the domain. It is then necessary to introduce finite-difference approximations to the given boundary conditions and thereby to eliminate the need for any point that lies outside the domain. For example, the standard convective boundary condition in D_1 reduces to the following equation after discretization:

$$-\theta_{j1-1,k1} + \left[\frac{2h_r hr_2}{k}\right]\theta_{j1,k1} + \theta_{j1+1,k1} = 0 \,, \qquad (4.6)$$

where $\theta_{j1,k1}$ is a temperature on the surface of D_1. $\theta_{j1+1,k1}$ is then a fictitious temperature outside D_1. However, when the governing equation is applied at the point $P_{j1,k1}$ (whose temperature is $\theta_{j1,k1}$), there will then be two simultaneous equations in the unknown $\theta_{j1+1,k1}$, and this fictitious temperature may be eliminated in favor of real temperatures within D_1. The same problem occurs at the conductors, where a finite-difference expression for the

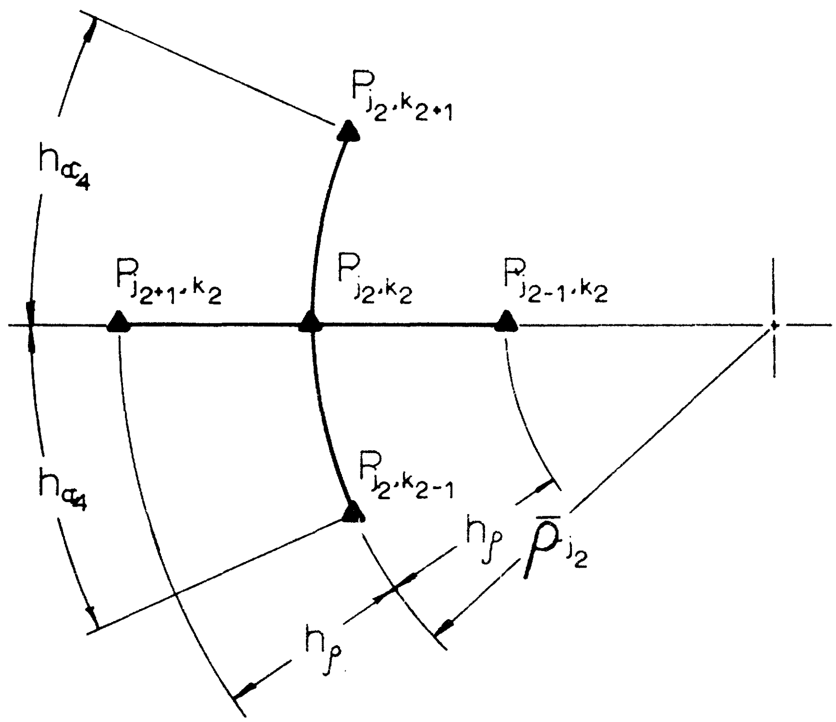


FIGURE 4.4

Nomenclature in the Neighborhood of $P_{j2,k2}$ in D_2

first derivative is required. In this case, however, the situation is less easily resolved. Since no governing equation is applied at the conductor, there is no way of eliminating a fictitious point within the conductor. This problem is addressed in Appendix D, where a suitable approximation to the conductor boundary condition is derived. The basic method involves satisfying the boundary condition at a slight distance from the conductor, and then relying on the fact that the temperature distribution is nearly one-dimensional in the immediate vicinity of the conductor.

The remaining exceptional case occurs at mesh points whose neighboring points on either side have different dimensionless spacings. This happens for radial spacings at the D_1-D_3 and D_2-D_3 interfaces, and it happens for circumferential spacings at the interfaces of all adjacent regions in D_1 and D_2. In such situations it is necessary to have finite-difference approximations which have been modified to fit an irregular mesh. The expressions for the first and second derivatives in a non-uniform mesh which are used in this study are readily derived, either from a Taylor's series expansion about the central point or by deduction from the mean value theorem of differential calculus. They are the following [7]:

$$\left(\frac{d\psi}{dz}\right)_j = \left[\frac{h_1}{h_2(h_1+h_2)}\right]\psi_{j+1} + \left[\frac{h_2-h_1}{h_1 h_2}\right]\psi_j - \left[\frac{h_2}{h_1(h_1+h_2)}\right]\psi_{j-1} + O(h^2) \; ; \tag{4.7}$$

$$\left(\frac{d^2\psi}{dz^2}\right)_j = \left[\frac{2}{h_2(h_1+h_2)}\right]\psi_{j+1} - \left[\frac{2}{h_1h_2}\right]\psi_j + \left[\frac{2}{h_1(h_1+h_2)}\right]\psi_{j-1} + O(h) \; ; \tag{4.8}$$

where h_1 is the dimensionless spacing between ψ_{j-1} and ψ_j, and h_2 is the spacing dimension between ψ_j and ψ_{j+1}. Also it is noted that these expressions reduce to the standard form of Equations 4.3 and 4.4 when the dimensionless spacings are uniform ($h_1 = h_2$).

CHAPTER 5

THE COMPUTER PROGRAM

General

The result of transforming the continuous formulation of the conduction problem into the corresponding finite-difference formulation is a linear set of simultaneous algebraic equations. A FORTRAN IV computer program written by the author generates this system if equations, performs the matrix inversion and multiplication to obtain various component solutions, and then combines component solutions to produce a final solution according to the superposition principle. User instructions for the program are discussed in Appendix E, and a complete listing of the source program is given in Appendix F.

The Coefficient Matrix

The set of simultaneous equations for the conduction problem may be written in the form

$$[A]\{\theta\} = \{B\} , \tag{5.1}$$

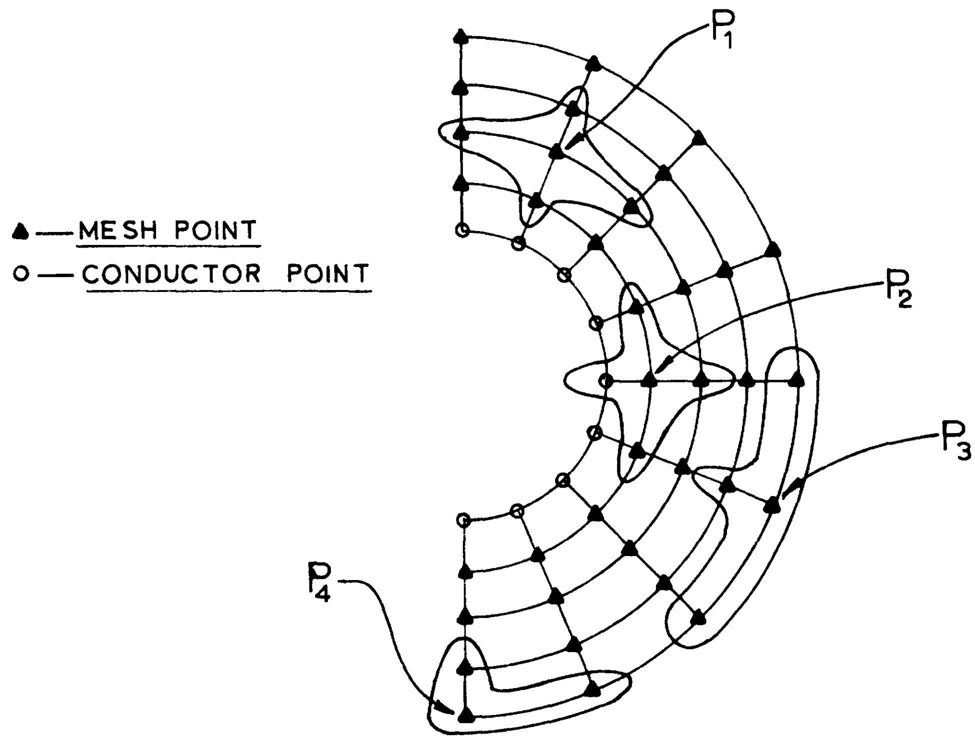
where [A] is an $n \times n$ matrix of coefficients, $\{\theta\}$ is a vector of n unknown dimensionless temperatures, and the right-hand-side vector $\{B\}$ is a vector of n forcing elements. Referring back to Figure 4.4, a typical algebraic equation was shown to be of the form

$$(a_{j-1,k})\theta_{j-1,k} + (a_{j,k})\theta_{j,k} + (a_{j+1,k})\theta_{j+1,k} + (a_{j,k-1})\theta_{j,k-1}$$

$$+ (a_{j,k+1})\theta_{j,k+1} = b_{j,k} , \qquad (5.2)$$

where $\theta_{j,k}$ is the central point at which the governing equation was applied, and the $a_{m,n}$ are the coefficients which were given in Equation 4.5. A typical equation thus involves five points – a central point and its four neighbors – and a typical row of the coefficient matrix accordingly has five non-zero elements. However, the application of a governing equation at certain mesh points produces rows with fewer than five non-zero elements. These situations are depicted in Figure 5.1, together with a typical mesh point. In this figure, points P_3 and P_4 initially had the full complement of four neighboring points, but fictitious points outside the domain were eliminated by incorporating the boundary conditions at those locations into the governing equations.

It is a simple matter to assemble the various coefficients $a_{m,n}$ into a matrix. The only requirement is that the rows be arranged so as to place the coefficients of central points (P_1, P_2, P_3, or P_4 in Figure 5.1) on the main diagonal of the matrix. Since a governing equation will necessarily involve the central point at which it is applied, it is therefore ensured that only non-zero elements will appear on the main diagonal, a necessary condition prior to matrix inversion.



P_1—TYPICAL POINT WITH FOUR NEIGHBORING MESH POINTS

P_2—POINT WITH THREE NEIGHBORING MESH POINTS AND ONE NEIGHBORING CONDUCTOR POINT

P_3—POINT WITH THREE NEIGHBORING MESH POINTS

P_4—POINT WITH TWO NEIGHBORING MESH POINTS

FIGURE 5.1

Points Affected by the Application of a Governing Equation at Various Locations in a Discrete Network

The coefficient matrix is inverted by the packaged subroutine RMINV, which uses the standard Gauss-Jordan algorithm. An automatic feature of this subroutine is the calculation of the determinant of the matrix. In order to keep the order of magnitude of this determinant within a range acceptable to FORTRAN IV, each row of the matrix and the corresponding elements of the forcing vectors are scaled so that the largest element of every row is unity. Once the matrix has been inverted, the temperatures $\{\theta\}$ are available from

$$\{\theta\} = [A]^{-1}\{B\} , \qquad (5.3)$$

where $[A]^{-1}$ denotes the inverse of $[A]$.

Forcing Vectors

In the conduction problem the total heat flow is comprised of the conductor losses, the dielectric loss, and the sheath loss. This heat flow is driven by two types of potentials: the conductor temperatures and the dielectric heating. These potentials are accounted for in the right-hand-side vectors $\{B\}$ of Equation 5.1. For purposes of reference the following vectors are defined: $\{B\}_1$ is the forcing vector for the component problem in which a conductor temperature is driving the heat flow; $\{B\}_2$ is the forcing vector for the component problem in which the heat flow is driven by dielectric heating.

The procedure for generating $\{B\}_1$ is suggested by point P_2 of Figure 5.1. The elements of $\{B\}_1$ are initially all zero. However, when a governing equation is applied at points adjacent to the conductor, such as point P_2, one of the neighboring points is the conductor itself. The conductor temperature is not an unknown, though, and when the governing equation is written in the form of Equation 5.1, the term involving the conductor temperature is carried over to the right-hand side and becomes a forcing term. The non-zero elements of $\{B\}_1$ are therefore comprised of conductor temperature-terms which have been referred to the forcing vector.

The elements of $\{B\}_2$ appear as volumetric heating terms in the difference form of Poisson's equation, Equation 4.5. It can be shown [8,9] that the dielectric loss per unit volume is of the form:

$$w_d = \frac{C}{r^2} , \tag{5.4}$$

where C is a constant for a given system, and r is the radius at a point in the insulation where the local dielectric loss per unit volume is w_d. Since the distribution of the dielectric loss is known, it is possible to integrate Equation 5.4 over any particular area to obtain the total loss per unit axial length within that area. A typical radial mesh and the areas associated with each mesh point are shown in Figure 5.2. The dielectric loss per unit length in a typical area, say A_2, is given by

$$(W_d)_2 = \int_{r_2}^{r_3} w_d dA = \int_{r_2}^{r_3} \left(\frac{C}{r^2}\right)(2\pi r)\,dr = 2\pi C \ln\left(\frac{r_3}{r_2}\right) . \quad (5.5)$$

The total dielectric loss per unit length is:

$$W_d = \int_{r_1}^{r_6} w_d dA = 2\pi C \ln\left(\frac{r_6}{r_1}\right) . \quad (5.6)$$

The loss per unit length in any particular area may be expressed as a fraction of the total loss per unit length just from information about the radial mesh. For example,

$$(W_d)_2 = \frac{(W_d)_2}{W_d} (W_d) = \frac{\ln\left(\frac{r_3}{r_2}\right)}{\ln\left(\frac{r_6}{r_1}\right)} (W_d) . \quad (5.7)$$

The computer program distributes the dielectric loss in this manner. The fraction of the total loss per unit length which occurs in each discrete area is calculated from the shape of the radial mesh. The dielectric loss per unit length for each area is then obtained by multiplying the

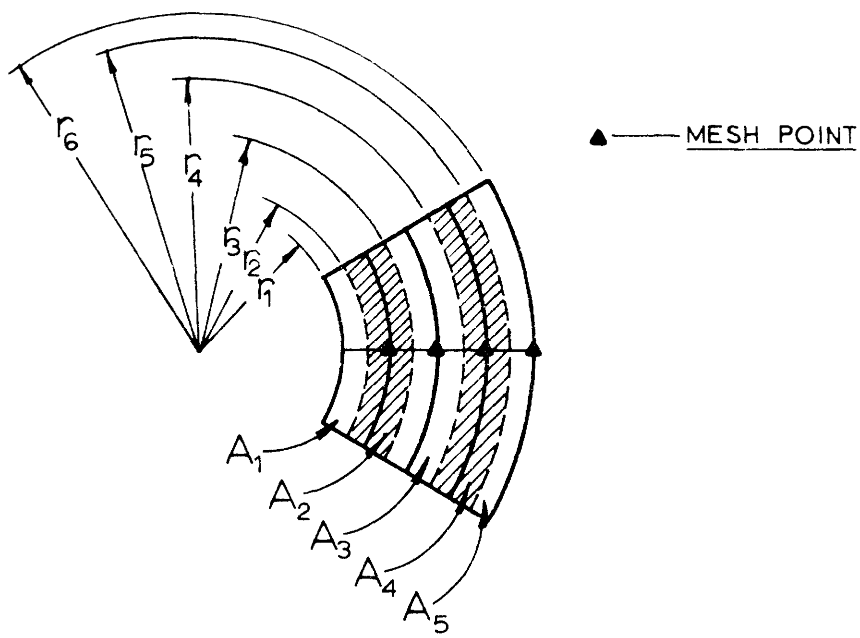


FIGURE 5.2

A Radial Mesh Illustrating the Area Associated With Each Mesh Point various fractions by the total loss per unit length, W_d, a number which is supplied as input data. Each mesh point therefore has an associated dielectric loss and the volumetric loss terms in Equation 4.5 can be generated accordingly. A question then arises, however, regarding the disposition of the loss in the innermost discrete area, A_1 in Figure 5.2. It is noticed in this figure that there is no mesh point associated with A_1. For this reason the loss which occurs in A_1 is added to the current-produced heating of the conductor. This suitably accounts for the loss, providing a conservative approximation to the true dielectric distribution.

The sheath loss is a current-produced loss which occurs on the surface of the cable. In the finite-difference model, this loss is placed in the outermost discrete area of insulation, which is associated with the mesh point on the cable surface. In Figure 5.2, for example, the sheath loss would be placed in A_5. It is then treated as a volumetric type of heating in addition to the dielectric loss for that area. A third potential is thereby introduced, which is accounted for in a third vector: $\{B\}_3$ is the forcing vector for the component problem in which sheath heating drives the heat flow.

Verification

During the course of developing the computer program, periodic tests were performed to ensure its correctness. One general technique used to verify a computer program is to solve a problem with it whose solution is already known and then to compare the two results. Several such problems, as well as a simpler checking procedure, were employed in this study.

The first major verification was a check on the coefficient matrix. This test was accomplished by printing out the matrix for a 111-point mesh and then by verifying each element by hand computation. The 111-point mesh was of sufficient size for the matrix-generation portion of the program to pass through all its decision branches. This test was repeated for meshes of decreasing size, until the matrix for the smallest possible mesh had been verified. In particular, the coefficient matrices for the following mesh sizes were verified: 111-point, 57-point, 24-point, 17-point, and 16-point. Forcing vectors were checked in a similar manner.

The second major verification of the computer program was accomplished by solving the following problem: the conductor of Cable 2 was maintained at a specified hot temperature, while that of Cable 1 was maintained at the oil temperature. Sheath and dielectric losses were not included. The height of the inter-cable conduction path was chosen so as to include an angle of 6° in either cable. This angle was judged to be sufficiently small so as to minimize the thermal effect of Cable 1 on Cable 2. Temperatures in Cable 2 diametrically opposite the intercable conduction path (and hence far-removed from the limited two-dimensional effect) were then compared to corresponding analytical temperatures from the one-dimensional solution. This comparison was repeated for successively finer meshes in order to examine the convergence of the solution. In performing the test, temperatures at similar radial points were compared, and a root-mean-square error was defined:

$$\text{RMS-error } (°\text{F}) = \left\{ \frac{\sum_{j=1}^{N} \left[(T_j)_{\text{computer}} - (T_j)_{\text{analytical}} \right]^2}{N} \right\}^{1/2}, \tag{5.8}$$

where N is the number of mesh points along a radius in Cable 2. This error was then expressed as a percentage of the total temperature drop through the insulation. The result for five different mesh sizes is shown in Figure 5.3. The errors demonstrate the typical $(1/h^2)$-dependence on mesh size, which is expected, since nearly all the finite difference expressions used in this problem are $0(h^2)$ approximations. It is further seen that the computer solution clearly converges to the analytical solution, having less than one percent error in temperature with as few as three radial subdivisions. A final run was then made with this problem to check whether a symmetrical cable configuration would produce a symmetrical temperature distribution. A coarse mesh of 18 points was used with a cable geometry such that the line joining the cable centers was a line of symmetry. In the resulting temperature distribution, the seven temperatures

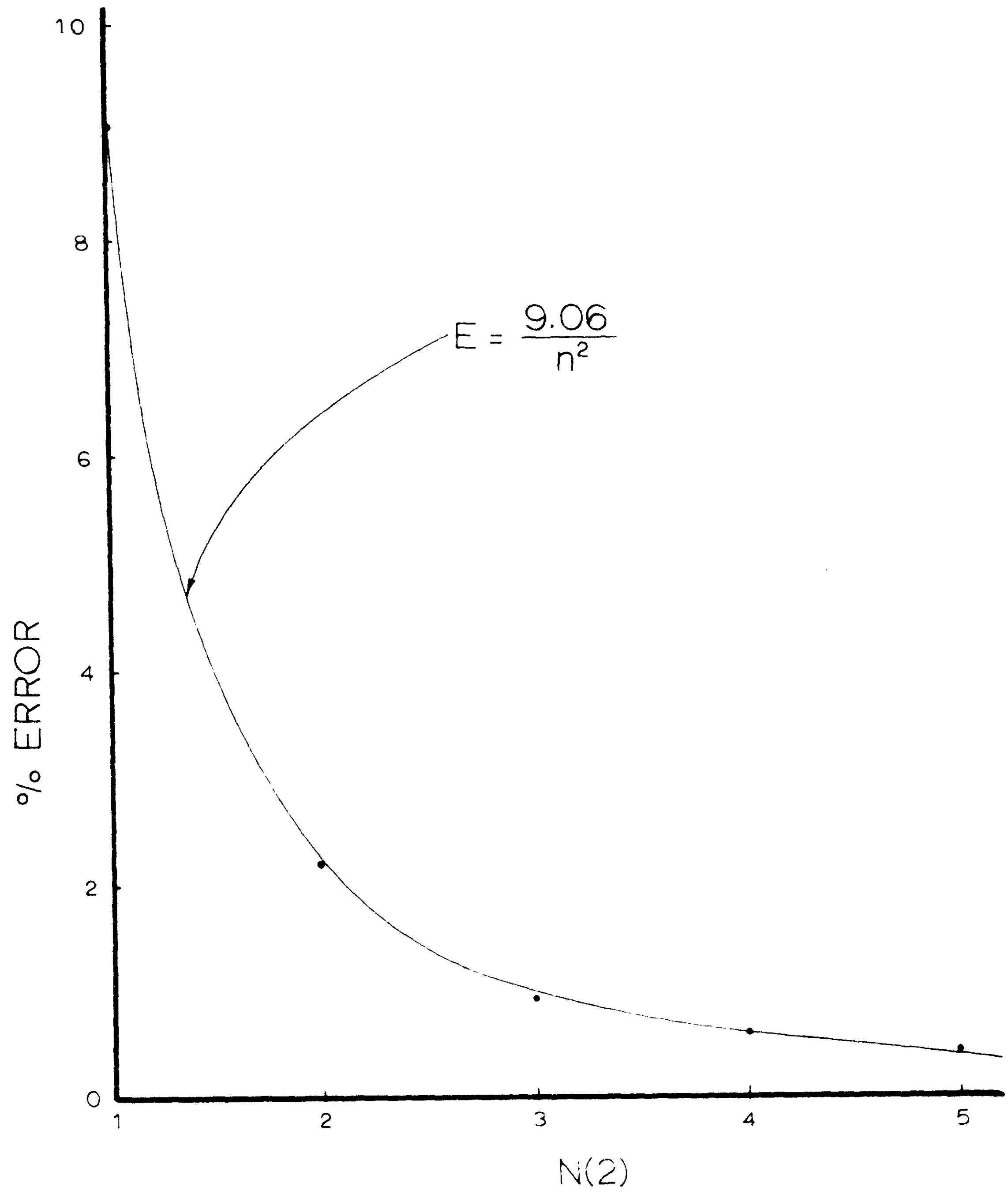


FIGURE 5.3

Percent-Error in Temperature vs. N_2, the

Number of Radial Subdivisions in D_2

above the line of symmetry agreed with their mirror images to six significant figures.

The third major verification of the program was a cross-check, using the output from Solution 1 (which finds temperature) as the input for Solution 2 (which finds current). For this test a 79-point mesh was employed, and all losses in both cables were included. Using an oil temperature of 140°F and a current of 942 amperes in each cable, Solution 1 predicted a maximum temperature of 189.451°F for the system. This maximum temperature, together with the oil temperature of 140°F, was then used as input for Solution 2, which predicted a maximum allowable current of 942 amperes in each cable. The solutions mutually agreed to six significant figures, thereby demonstrating the reciprocal validity of the program.

The final major verification procedure was to approximate the one-dimensional solution for a single cable with all losses, by shrinking to a minimum the height of the inter-cable conduction path. In the computer program the conduction path is presumed to have some non-zero height, so the included angle in either cable was taken to be 2°. Again temperatures in Cable 2 opposite the inter-cable conduction path were compared to corresponding analytical temperatures, and the same error criteria (RMS-error as a percentage of the total drop through the insulation) were employed. The test was made for three cases: dielectric loss only, conductor losses only, and then all losses.

Using a mesh comprised of four radial subdivisions in the insulation, the following results were obtained: dielectric loss only — 2.6% error in temperature solution; conductor losses only — 0.4% error in temperature solution, 1.0% error in current solution; all losses — 0.8% error in temperature solution, 2.0% error in current solution. Temperatures in the dielectric solution are elevated above their analytical counterparts because of the referral of loss near the conductor into the conductor itself. However, when the dielectric loss is considered proportionately with all other losses, the temperature error is observed to be reasonably small (less than one percent for this mesh). The currents predicted in this test are seen to be less accurate than the corresponding predicted temperatures. This circumstance, though, reflects a limitation of the test itself rather than one of the model. That is, the temperature distribution is expected to smooth out into a one-dimensional form in a region far-removed from disturbing effects. However, the current solution depends on the entire temperature distribution. Since an inter-cable conduction path of any non-zero size will inevitability produce local distortions in the temperature distribution, it is ultimately futile to expect very close agreement with a one-dimensional current solution. Agreement could be demanded if the size of the conduction path could be made identically zero, but the model does not possess this capability, having been designed for the solution of real two-dimensional problems. Given this limitation, the current solutions demonstrate excellent agreement with the one-dimensional results. The credibility generated by this test, together with all the evidence previously stated, suffices to establish the validity of the computer program.

CHAPTER 5A

COPPER TAPE EFFECTS

Effective Conductivity

Many cable systems used in the underground power transmission have a thin copper tape wrapped around the cable insulation directly under the moisture seal assembly. The tape is included to circumferentially smooth out the electric potential and to provide electric ground. Having a very high thermal conductivity the copper tape provides a mechanism for transferring heat away from high temperature regions. It therefore causes a redistribution of the temperature, tending toward the one-dimensional form. Order of magnitude calculations indicate that the presence of a five mil (t = 0.005") copper tape (thermal conductivity k_{cu} = 220 BTU/hr ft°F) under the moisture seal assembly have a significant effect on the temperature distribution of the cable:

Consider a typical system of Fig. 5A.1,

r_1 = 0.9125 in

r_2 = 2.0675 in

k = 0.1153 BTU/hr ft°F

The thermal resistance per unit length in the circumferential direction without the tape R_c is approximately

$$R_c = \frac{1}{k(r_2-r_1)} = 90 \text{ hr°F/BTU}$$

The circumferential thermal resistance including the copper tape R_c' is

$$\frac{1}{R'_c} \simeq k(r_2 - r_1 - t) + k_{cu} t$$

$$R'_c \simeq \frac{1}{k(r_2 - r_1 - t) + k_{cu} t} \simeq 9.7 \text{ hr°F/BTU}$$

Since R_c and R'_c are significantly different, the presence of a highly conductive medium under the cable moisture seal assembly must be taken into account.

An order of magnitude calculation also indicates that the copper tape effect in the radial direction is negligible: whereas in the circumferential direction the resistance of copper and insulation are parallel, in the radial direction the two resistances are connected in series. The radial thermal resistance per unit length without the tape R_r is

$$R_r = \frac{\ln r_2/r_1}{2\pi k} = 1.129002568 \text{ hr °F/BTU}$$

The radial thermal resistance per unit length including the tape, R'_r, is

$$R'_r = \frac{\ln \frac{r_2 - t}{r_1}}{2\pi k} + \frac{\ln \frac{r_2}{r_2 - t}}{2\pi k_{cu}} = 1.12900432 \text{ hr°F/BTU}$$

Since the tape is very thin only the mesh points located around the outside circumference of the cable insulation are affected. In Fig. 5A.1 the thermal conduction path between point 0 and 1 consists of the copper tape (thickness t) and a layer of insulation (thickness L_r-t). The path length is $L_{\alpha l}$. The thermal resistance of the two conducting media R' is

$$\frac{L_{\alpha l}}{R} = k(L_r - t) + k_{cu} t = k_{eff} L_r$$

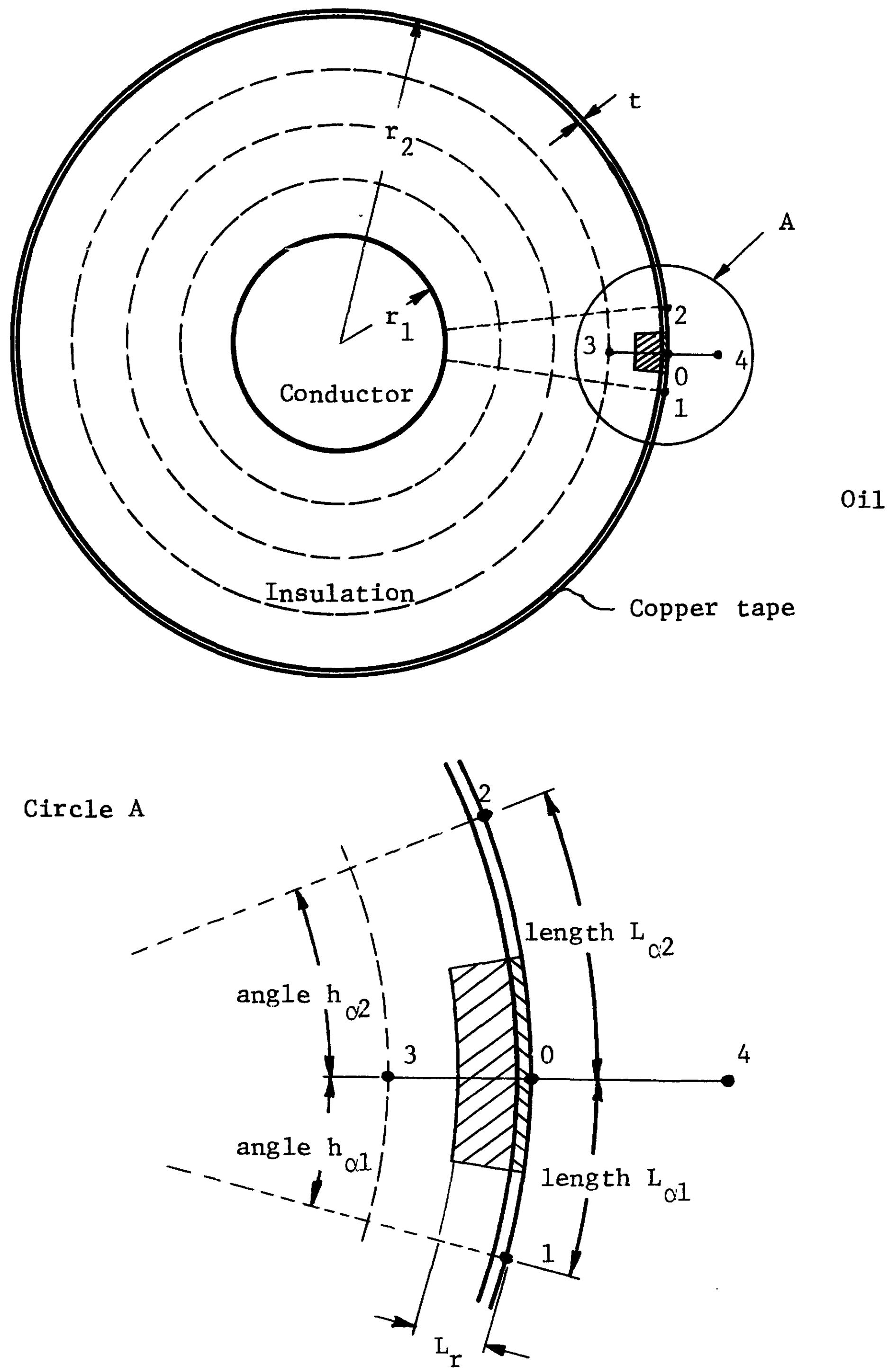


Fig. 5A.1. Development of the expression for the combined effective conductivity of a layer of the cable insulation and the thin copper tape.

where k_{eff} is the effective conductivity of the entire thermal conduction path betwee 0 and 1.

In the computer program $L_r = (r_2 - r_1)/N$; where N is the number of radial divisions.

Hence,

$$k_{eff} = \frac{k_{cu}t + k(\frac{r_2 - r_1}{N} - t)}{\frac{r_2 - r_1}{N}} \qquad (5A.1)$$

Equation (5A.1) applies only for points on the insulation outer circumference and in the circumference direction only, al other mesh poinst are unaffected.

<u>Finite Difference Equations</u>

For the mesh points located on the outside circumference some of the finite difference expressions (4.3) (4.4) or (4.8) used in discretizing of the governing equation (2.6) and the boundary condition must be adjusted to account for the higher conductivity in the circumferential direction. Thus for the point 0 in Figure 5A.1 the finite difference approximations in non-dimensional form ($L_r = h_r r_2$, $L_\alpha = h_\alpha r_2$)

$$\frac{\partial^2 \Theta}{\partial r^2} = \frac{\Theta_4 - 2\Theta_0 + \Theta_3}{h_r^2}$$

$$\frac{\partial \Theta}{\partial r} = \frac{\Theta_4 - \Theta_3}{2h_r}$$

$$\frac{\partial^2\Theta'}{\partial\alpha^2} = \frac{\Theta'_2 - 2\Theta'_0 + \Theta_1}{h^2_\alpha}$$

$$\frac{\partial^2\Theta}{\partial\alpha^2} = \frac{2}{h_{\alpha 2}(h_{\alpha 1}+h_{\alpha 2})}\ \Theta_2 - \frac{2}{h_{\alpha 1}h_{\alpha 2}}\ \Theta_0 + \frac{2}{h_{\alpha 1}(h_{\alpha 1}+h_{\alpha 2})}\ \Theta_1 \tag{5A.2}$$

where Θ is given by (2.5) and $\Theta' = \dfrac{T - T_{oil}}{W/k_{eff}}$.

Then using (5A.2), the equation (2.6) about a regular boundary point 0 ($h_{\alpha 1} = h_{\alpha 2}$) becomes,

$$\bar{r}^2 \left[\frac{\Theta_4 - 2\Theta_0 + \Theta_3}{h^2_r}\right] + \bar{r}\left[\frac{\Theta_4 - \Theta_3}{2h_r}\right] + \left[\frac{\Theta'_2 - 2\Theta_0 + \Theta'_1}{h^2_\alpha}\right] = \frac{(r_2\bar{r})^2\dot{q}}{W} \tag{5A.3}$$

For a boundary point at a regional interface ($h_{\alpha 1} \neq h_{\alpha 2}$):

$$\bar{r}^2 \left[\frac{\Theta_4 - 2\Theta_0 + \Theta_3}{h^2_r}\right] + \bar{r}\left[\frac{\Theta_4 - \Theta_3}{2h_r}\right] + \left[\frac{2}{h_{\alpha 2}(h_{\alpha 1}+h_{\alpha 2})}\right]\Theta'_2 - \frac{2}{h_{\alpha 1}h_{\alpha 2}}\Theta'_0$$

$$+ \frac{2}{h_{\alpha 1}(h_{\alpha 1}+h_{\alpha 2})}\ \Theta'_1] = -\frac{(r_2\bar{r})^2\dot{q}}{W} \tag{5A.4}$$

The boundary condition (2.8) is unaffected for all boundary points:

$$-\frac{k}{r^2}\ \frac{\Theta_4 - \Theta_3}{2h_r} = h\Theta_0 \tag{5A.5}$$

Eliminating Θ_4 from (5A.3) and (5A.4) using (5A.5) and noticing that from

$$\Theta = k\frac{\Delta T}{W} \qquad \Theta' = k_{eff}\frac{\Delta T}{W}$$

obtain

$$\Theta' = \frac{k_{eff}}{k}\Theta \tag{5A.6}$$

Hence for $h_{\alpha 1} = h_{\alpha 2} = h_\alpha$

$$-[\frac{2h_r h r_2}{k}(\frac{\bar{r}^2}{h_r^2} + \frac{\bar{r}}{2h_r}) + \frac{k_{eff}}{k}(\frac{2}{h_\alpha^2}) + \frac{2\bar{r}^2}{h_r^2}]\Theta_0 +$$

$$\frac{k_{eff}}{k}[\frac{1}{h_\alpha^2}]\,\Theta_1 + \frac{k_{eff}}{k}[\frac{1}{h_\alpha^2}]\,\Theta_2 + [\frac{2\bar{r}^2}{h_r^2}]\,\Theta_3 = -\frac{(r_2\bar{r})\,\dot{q}}{W} \qquad (5A.7)$$

and for $h_{\alpha 1} \neq h_{\alpha 2}$

$$-[\frac{2h_r h r_2}{k}(\frac{\bar{r}^2}{h_r^2} + \frac{\bar{r}}{2h_r}) + \frac{2\bar{r}^2}{h_r^2} + \frac{k_{eff}}{k}(\frac{2}{h_{\alpha 1}h_{\alpha 2}})]\Theta_0 +$$

$$+ \frac{k_{eff}}{k}[\frac{2}{h_{\alpha 1}(h_{\alpha 1}+h_{\alpha 2})}]\Theta_1 + \frac{k_{eff}}{k}[\frac{2}{h_{\alpha 2}(h_{\alpha 1}+h_{\alpha 2})}]\Theta_2 + [\frac{2\bar{r}^2}{h_r^2}]\Theta_3 =$$

$$-\frac{(r_2\bar{r})^2\,\dot{q}}{W} \qquad (5A.8)$$

<u>Computer Program Modification</u>

From (5A.7) and (5A.8) it is apparent that to account for the presence of a thin high-conductivity tape under moisture seal of the cable insulation it is only necessary to multiply the original coefficients (corresponding to the homogeneous insulation material) of Θ_1's and Θ_2's by $\frac{k_{eff}}{k}$ and to add $\frac{2}{h_{\alpha 1}h_{\alpha 2}}(1 - \frac{k_{eff}}{k})$ to both (5A.7) and (5A.8) for if $h_{\alpha 1} = h_{\alpha 2} = h_\alpha$, it follows that $\frac{2}{h_{\alpha 1}h_{\alpha 2}} = \frac{2}{h_\alpha^2}$. The simple complementation of this procedure can be eased by the following manipulation.

From (5A.8) the coefficients of Θ_1's and Θ_2's for $\frac{k_{eff}}{k} = 1$ (original coefficient for $h_{\alpha 1} = h_{\alpha 2} = h_\alpha$, reduces to $\frac{1}{h_\alpha^2}$ which are the original coefficients of Θ_1's and Θ_2's in (5A.7). It is possible therefore to use (5A.8) for all circumferential boundary points.

Further, let the original coefficients of Θ_1's and Θ_2's be

$$\frac{2}{h_{\alpha 1}(h_{\alpha 1}+h_{\alpha 2})} = 0_1 \qquad (5A.9)$$

$$\frac{2}{h_{\alpha 2}(h_{\alpha 1}+h_{\alpha 2})} = 0_2$$

Also, let the new coefficients of Θ_1's and Θ_2's be

$$\frac{2\ f}{h_{\alpha 1}(h_{\alpha 1}+h_{\alpha 2})} = P_1 = 0_1 f$$

$$(5A.10)$$

$$\frac{2\ f}{h_{\alpha 2}(h_{\alpha 1}+h_{\alpha 2})} = P_2 = 0_2 f$$

where $f = \frac{k_{eff}}{k}$

Let $Q = \frac{2}{h_{\alpha 1} h_{\alpha 2}} (1 - f)$ = factor to be added to the original coefficient of Θ_2's.

Then from (5A.9) and (5A.10) $0_1 + \frac{2}{h_{\alpha 1}(h_{\alpha 2}+h_{\alpha 1})} = P_1$

or

$$(0_1 - P_1)\left(\frac{h_{\alpha 1}}{h_{\alpha 2}} + 1\right) = \frac{2(1-t)}{h_{\alpha 1} h_{\alpha 2}} = Q \qquad (5A.11)$$

and similarly $(0_2 - P_2)\left(\frac{h_{\alpha 2}}{h_{\alpha 1}}\right) = Q$ (5A.12)

Eliminating $h_{\alpha 1}$ and $h_{\alpha 2}$ between (5A.11) and (5A.12) obtain,

$$Q = 0_1 + 0_2 - P_1 - P_2 \qquad (5A.13)$$

Thus to modify the original matrix of coefficients of Θ's it is necessary to multiply the original coefficients of Θ_1's and Θ_2's by $f = \frac{k_{eff}}{k}$ and to add the difference $(0_1 - P_1)$ and $(0_2 - P_2)$ between the original coefficient 0_1 and 0_2 of Θ_1's and Θ_2's and the new coefficients $P_1 = 0_1 f$ and $P_2 = 0_2 f$ to the original coefficients of Θ_0's. An important property of the matrix A was used to find the locations of Θ_0's, Θ_1's and Θ_2's: the numbering system is such that all coefficients of Θ_0's lie on the main diagonal and the coefficients of Θ_1's immediately precede Θ_0's in each row and the coefficients of Θ_2's immediately follow Θ_0's in each row with the exception of the row corresponding to the governing equation written about the point in cable 2 at an angle $\alpha = 0$. Also all Θ_0's are circumferential boundary points.

Verifications

The first major verification was a check on the coefficient matrix. This test was, as in Chapter 5, accomplished by printing out the matrix for a 42 and 17-point mesh and then by verifying each element by hand computations.

The second major verification was an energy balance performed on selected mesh elements, again by hand computation. The final major verification was accomplished by solving the following problem: the intercable conduction path was reduced to a very small size so that approximately uniform convective boundary conditions existed at every circumferential point. The problem was run with and without the tape and both solutions were converging to the 1-D solution as the intercable conduction path was being reduced.

CHAPTER 6

RESULTS AND CONCLUSIONS

Evaluation Criteria

As a first criterion for evaluating the severity of a given set of system operating conditions, the numerical solutions produced by the computer program are compared to the corresponding analytical solutions for a single, undisturbed cable. The one-dimensional temperature and current solutions for a single cable are presented in Appendix G. Since the undisturbed cable represents an optimum operating condition, the one-dimensional solution provides an upper bound for system performance.

A second criterion for evaluation may be obtained from a modification of the one-dimensional solutions, in which a conservative allowance for two-dimensional effects is made. This modification is effected in the following manner: it is reasoned that an effect on any portion of the cable surface which disturbs the one-dimensional temperature distribution is less severe than the effect of insulating that portion of the surface. The conservative approximation is then made that such disturbances do indeed effectively insulate appropriate portions of the surface, and that the entire sector defined by such an insulated arc is likewise insulated. All losses which would have occurred in the insulated sector are then placed into the undisturbed fraction of the cable, and the one-dimensional solutions are employed with appropriately scaled-up losses. This procedure is explained fully in Appendix H, where the conservative approximations for maximum temperature and current are derived. Since by physical argument the disturbed portions of the cable surface cannot have a more stringent condition imposed than that of being insulated, the approximate formulas of Appendix H provide a lower bound for system performance.

Results

While the primary product of this study is the computer program itself, a total of 16 cable problems were solved by the author in order to provide preliminary information about some typical operating conditions. These 16 problems break down into the following: Solution 2 (for maximum current) and Solution 3 (for maximum oil temperature) were employed for four configurations of cables, and two cable systems were considered. The four cable configurations were equilateral, cradled, open, and equilateral-pipe. These are depicted in Figure 6.1. The systems considered were a 2500 MCM system (System 1) and a 2000 MCM system (System 2). Values for the physical parameters associated with these two systems are listed in Table 2. The thermal parameters were taken to have the following values for all 16 problems: 0.1153 Btu/hr-ft-°F for the thermal conductivity of the insulation, and 5.0 Btu/hr-ft^2-°F for the thermal film coefficient in convective regions. Also the conservative assumption of a thermally nonconducting conduit

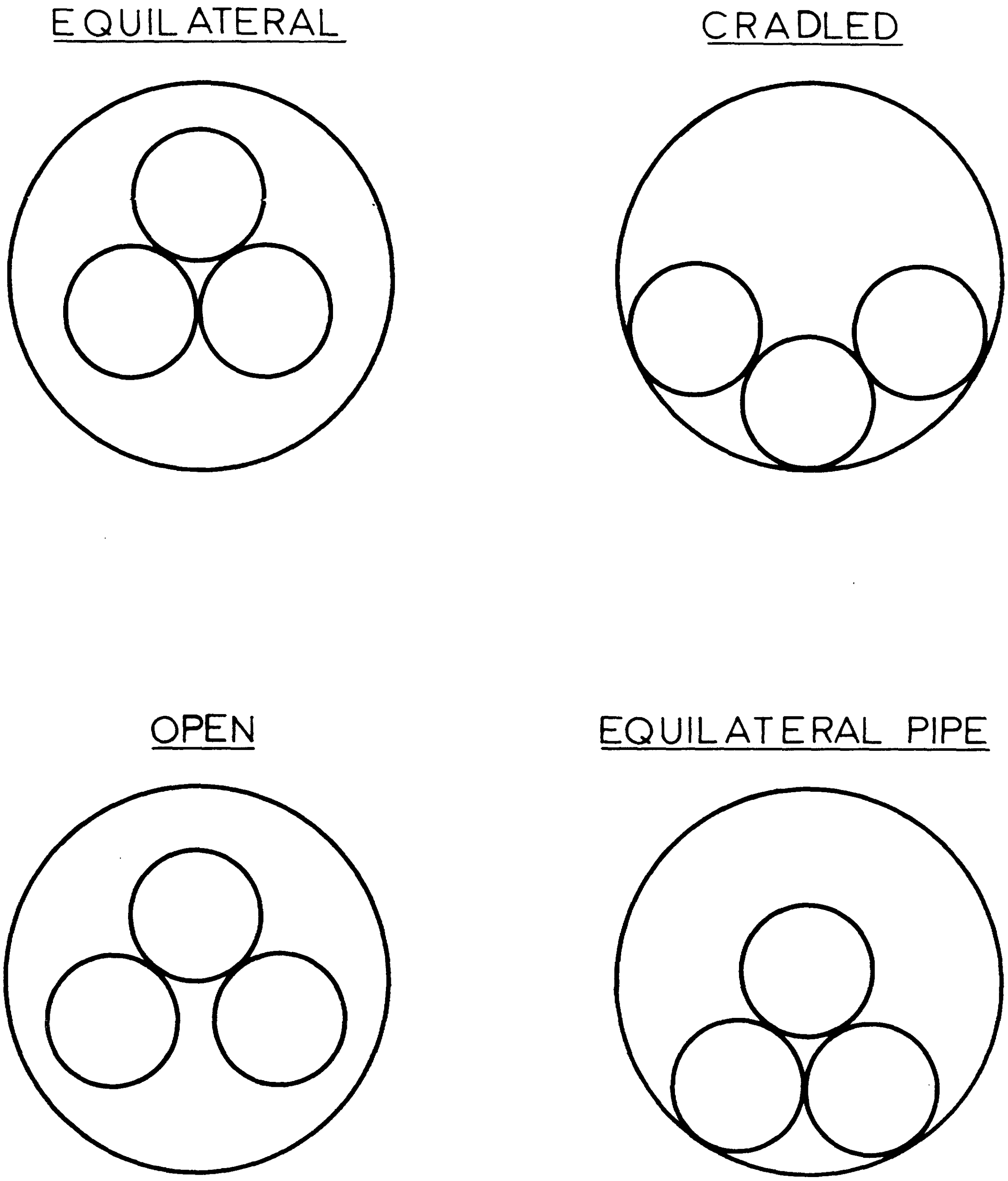


FIGURE 6.1

Nomenclature for Cable Configurations

TABLE 2

Values for The Physical Parameters

of Systems 1 and 2

	System 1	System 2
Inner Radius of Cable Insulation (in)	0.9125	0.8155
Outer Radius of Cable Insulation (in)	2.0675	1.9675
Skid Wire Thickness (in)	0.10	0.10
DC Resistance of Conductor (μΩ/ft)	5.36	6.63
AC/DC Ratio at Conductor	1.19	1.13
AC/DC Ratio at Sheath	1.24	1.18
Typical Dielectric Loss (watts/conductor-ft)	3.18	3.18
Typical Current (amps)	942	888
Thermal Conductivity of Insulation (BTU/hr-ft°F)	0.1153	0.1153
Thermal Conductivity of Copper Tape (BTU/hr-ft°F)	220.0	220.0
Copper Tape Thickness (in)	0.005	0.005

was made. This latter assumption is significant only for the cradled and equilateral-pipe configurations, where the cables actually touch the conduit wall. A nonconducting wall substantially increases the thermal resistance in the vicinity of the contact point, thereby producing a local region of high temperature and hindering the removal of heat from the cable.

Results for the 16 problems are given in Tables 3 - 6. In these tables the first column of percentages is a comparison of computer solutions to corresponding one-dimensional solutions. These negative percentages are a measure of how much worse the given operating condition is than the best possible condition. The second column of percentages is a comparison of computer solutions to the conservative approximate solutions from Appendix H. These percentages, which are positive, provide a measure of how much better the given operating condition is than the estimated worst condition.

Upon examining the four tables, the equilateral-pipe configuration is immediately identified as the most severe operating configuration. This is expected, since the greatest obstruction of cable surface area occurs in this configuration. The other configurations follow in logical sequence: equilateral, cradled, and open. It is shown in Appendix B and also in the heat transfer report [2] that a steel pipe is very effective in conducting heat away from the cables to the bulk of the oil. So had the problems

TABLE 3

Solution 2 For Four Cable Configurations - System 1

	I (amps)	$\frac{I-I_{1-D}}{I_{1-D}}$	$\frac{I-I_*}{I_*}$
Open -- no tape	1075.4	-5.0%	+0.2%
Open -- tape	1105.0	-2.4%	+0.3%
Cradled (Nonconducting Pipe) -- no tape	1030.2	-9.0%	+2.0%
Cradled (Nonconducting Pipe) -- tape	1086.0	-4.1%	+7.5%
Equilateral -- no tape	978.2	-13.6%	+3.7%
Equilateral -- tape	1071.3	-5.4%	+13.5%
Equilateral-Pipe (Nonconducting Pipe) -- no tape	946.4	-16.4%	+8.6%
Equilateral-Pipe (Nonconducting-Pipe) -- tape	1057.0	-6.6%	+21.3%
One-Dimensional	(1132.1)	-	-

T_{oil} = 140°F

T_{max} = 185°F

I - current from computer solution

I_{1-D} - current from one-dimensional solution

I_* - current from conservative approximate solution

TABLE 4

Solution 3 For Four Cable Configurations - System 1

	T_{oil} (°F)	$\frac{T_{oil}-(T_{oil})_{1-D}}{T_o-(T_{oil})_{1-D}}$	$\frac{T_{oil}-(T_{oil})_*}{T_o-(T_{oil})_*}$
Open -- no tape	148.3	-8.9%	0.0%
Open -- tape	149.9	-4.2%	+4.4%
Cradled (Nonconducting Pipe) -- no tape	145.7	-16.6%	+2.8%
Cradled (Nonconducting Pipe) -- tape	148.9	-7.1%	+10.7%
Equilateral -- no tape	142.4	-26.4%	+4.8%
Equilateral -- tape	148.1	-9.5%	+17.8%
Equilateral-Pipe (Nonconducting Pipe) -- no tape	140.3	-32.6%	+11.5%
Equilateral-Pipe (Nonconducting Pipe) -- tape	147.3	-11.9%	+25.4%
One-Dimensional	(151.3)	-	-

I = 942 amps

T_{max} = 185°F

T_{oil} - oil temperature from computer solutions

$(T_{oil})_{1-D}$ - oil temperature from one-dimensional solution

$(T_{oil})_*$ - oil temperature from conservative approximate solution

T_o - conductor temperature

TABLE 5

Solution 2 For Four Cable Configurations - System 2

	I (amps)	$\frac{I-I_{1-D}}{I_{1-D}}$	$\frac{I-I_*}{I_*}$
Open -- no tape	950.2	-5.0%	+0.3%
Open -- tape	975.3	-2.5%	+3.0%
Cradled (Nonconducting Pipe) -- no tape	911.9	-8.8%	+2.4%
Cradled (Nonconducting Pipe) -- tape	959.2	-4.1%	+7.7%
Equilateral -- no tape	864.6	-13.6%	+4.0%
Equilateral -- tape	946.6	-5.4%	+13.9%
Equilateral-Pipe (Nonconducting Pipe) -- no tape	837.2	-16.3%	+9.3%
Equilateral-Pipe (Nonconducting Pipe) -- tape	938.8	-6.7%	+21.9%
One-Dimensional	(1000.4)	(0.0%)	-

T_{oil} = 140°F

T_{max} = 185°F

I - current from computer solution

I_{1-D} - current from one-dimensional solution

I_* - current from conservative approximate solution

TABLE 6

Solution 3 For Four Cable Configurations - System 2

	T_{oil} (°F)	$\frac{T_{oil}-(T_{oil})_{1-D}}{T_o-(T_{oil})_{1-D}}$	$\frac{T_{oil}-(T_{oil})_*}{T_o-(T_{oil})_*}$
Open -- no tape	144.4	-8.8%	+0.3%
Open -- tape	146.1	-4.3%	+4.5%
Cradled (Nonconducting Pipe) -- no tape	141.8	-15.8%	+3.4%
Cradled (Nonconducting Pipe) -- tape	145.1	-7.0%	+10.8%
Equilateral -- no tape	138.2	-25.5%	+5.8%
Equilateral -- tape	144.2	-9.4%	+17.9%
Equilateral-Pipe (Nonconducting Pipe) -- no tape	136.0	-31.4%	+12.4%
Equilateral-Pipe (Nonconducting Pipe) -- tape	143.3	-11.8%	+25.4%
One-Dimensional	(147.7)	-	-

I = 888 amps

T_{max} = 185°F

T_{oil} = oil temperature from computer solution

$(T_{oil})_{1-D}$ - oil temperature from one-dimensional solution

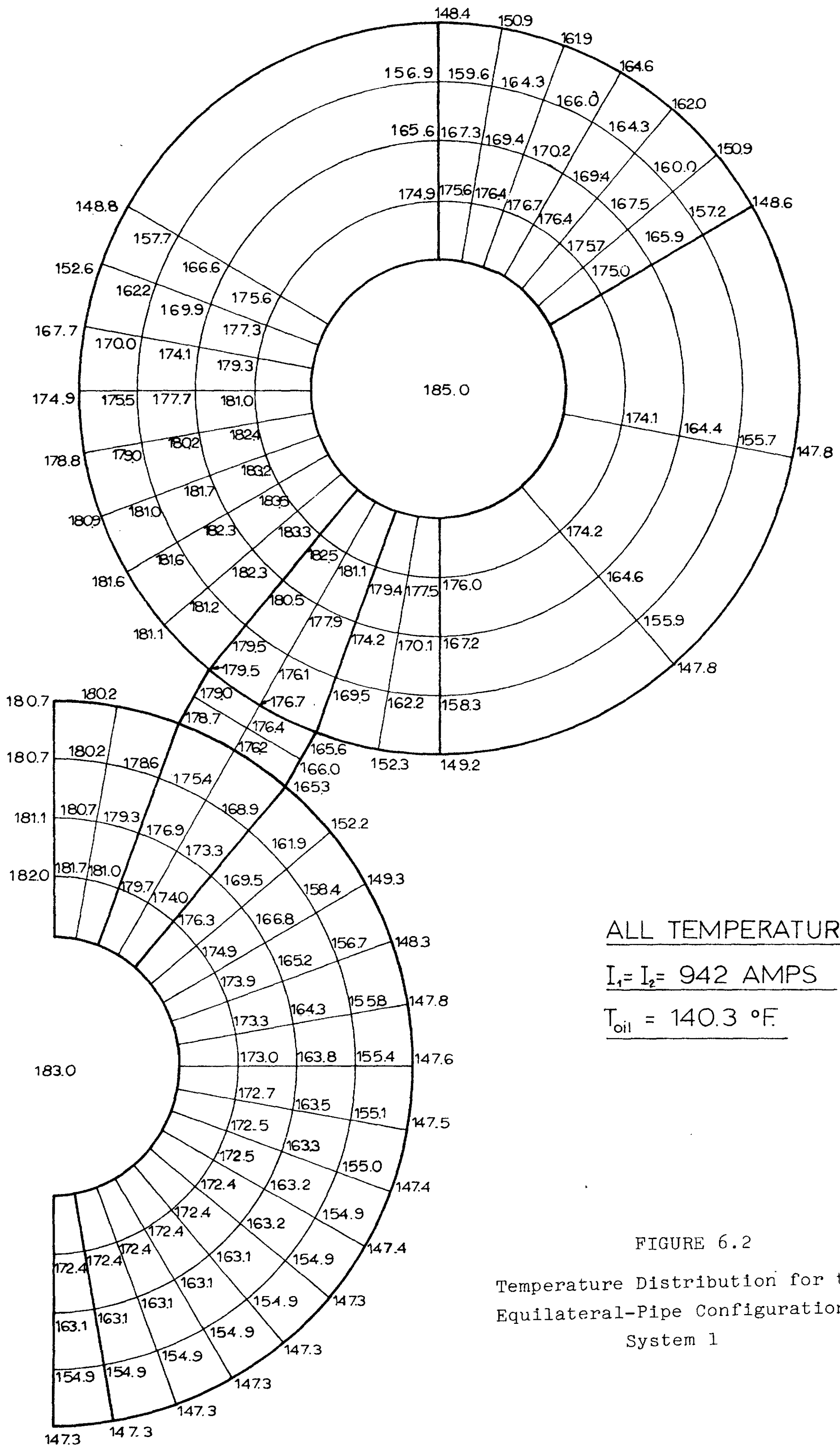
$(T_{oil})_*$ - oil temperature from conservative approximate solution

T_o - conductor temperature

been solved using a thermally conducting conduit material such as steel, then the results for the equilateral-pipe configuration would have been essentially the same as those for the equilateral case, and the latter would have been the most severe configuration. A complete temperature distribution for the equilateral-pipe configuration without the high conductivity tape under the cable moisture seal assembly of System 1 is displayed in Figure 6.2.

Two observations are made regarding the conservative approximate solutions. The first is that in cases without tape they are reasonably accurate, being conservative by 9.3% in the least accurate case (Solution 2, System 2, Equilateral-Pipe). They are therefore useful whenever a highly refined solution is not required. The second observation is that the conservative solutions become less accurate as the configurations become more severe. This tendency is readily explained, for as the surface area of a cable is increasingly obstructed, two-dimensional effects grow stronger. Since the conservative approximations are based on the one-dimensional solutions, they become increasingly deviant with the severity of the configuration. So despite the conservative nature of the approximate solutions, they are not recommended for design purposes.

Also it is noticed that there is a large discrepancy between corresponding percentages in Solution 2 and Solution 3: temperature deviations are somewhat larger than current deviations. Such a discrepancy, however, should not be a surprising one. Consider that both solutions are applied to a given configuration. In Solution 3 the heat flow is constant, and the temperature distribution must be linearly adjusted so as to account for the two-dimensional constraints imposed



ALL TEMPERATURES IN °F.

$I_1 = I_2 =$ 942 AMPS

T_{oil} = 140.3 °F.

FIGURE 6.2

Temperature Distribution for the Equilateral-Pipe Configuration - System 1

by the configuration. In Solution 2, though, the heat flow is the adjustable quantity of the total heat flow, only the current-produced fraction (usually about 2/3) is variable, and thus the current-produced losses must be disproportionately adjusted so as to align the overall heat flow according to the two-dimensional constraints of the configuration. Furthermore, the current itself varies as the second root of the variable heat flow. Therefore the relationship between the two solutions is a complicated one, and there is no reason to expect any similarity between their respective deviations.

Finally, it is evident from the tables that cable proximity effects are very significant in forced cooling especially for cables without the high thermal conductivity tapes. In System 2 with no tape, for example, the maximum allowable oil temperature is 11.7°F lower for the equilateral-pipe configuration than for the one-dimensional case.

Since force-cooled systems are typically designed for an axial oil temperature rise of about 45°F between refrigeration stations, the 11.7°F difference itself would accout for 26% of the axial oil temperature rise. On the other hand, in the same system with the tape present the maximum allowable oil temperature is only 4.4°F lower than in ID case, or about 10% of the axial oil temperature rise. For the more realistic equilateral configuration the same figures are 9.5°F or 21.6% without the tape and 3.5°F or 8% with the tape. This means that depending on whether the tape is or is not present under the cable moisture seal assembly System 2 in the equilateral configuration would require either an 8% or 21% higher flow rate, or either an 8% or 21% shorter axial distance between refrigeration stations then the same system in a completely free (one-dimensional) configuration.

Approximate allowance for cable proximity effects must therefore be made in the overall design of force-cooled systems. Since there is a significant improvement in the results when the copper tape is wrapped around the cable insulation, such cables are from thermal considerations, the more suitable for force-cooled power transmission work.

Isothermal lines for the four oil temperature solutions of System 1 are shown in Figures 6.3 - 6.7. One-dimensional portions of the various solutions may readily be identified in these figures by isotherms which are circular arcs. As expected, all the distributions smooth out into one-dimensional form away from points of cable contact and conduit contact. Regions of high temperature within the insulation are identified by isotherms which depart significantly from the circular shape, protruding outward from the cable centers. This effect is observed to be most prevalent in the equilateral-pipe configuration, decreasing in strength in the equilateral, cradled, and open configurations, respectively. Thus the isothermal lines in themselves provide a vivid illustration of the severity of the various configurations. Shown with the isotherms are adiabatic lines. These lines are everywhere normal to the isotherms, and they represent curves along which the heat flow travels. It is noted that not all adiabatic lines originate at the conductors. This circumstance is attributable to the dielectric heating, which occurs within the insulation itself.

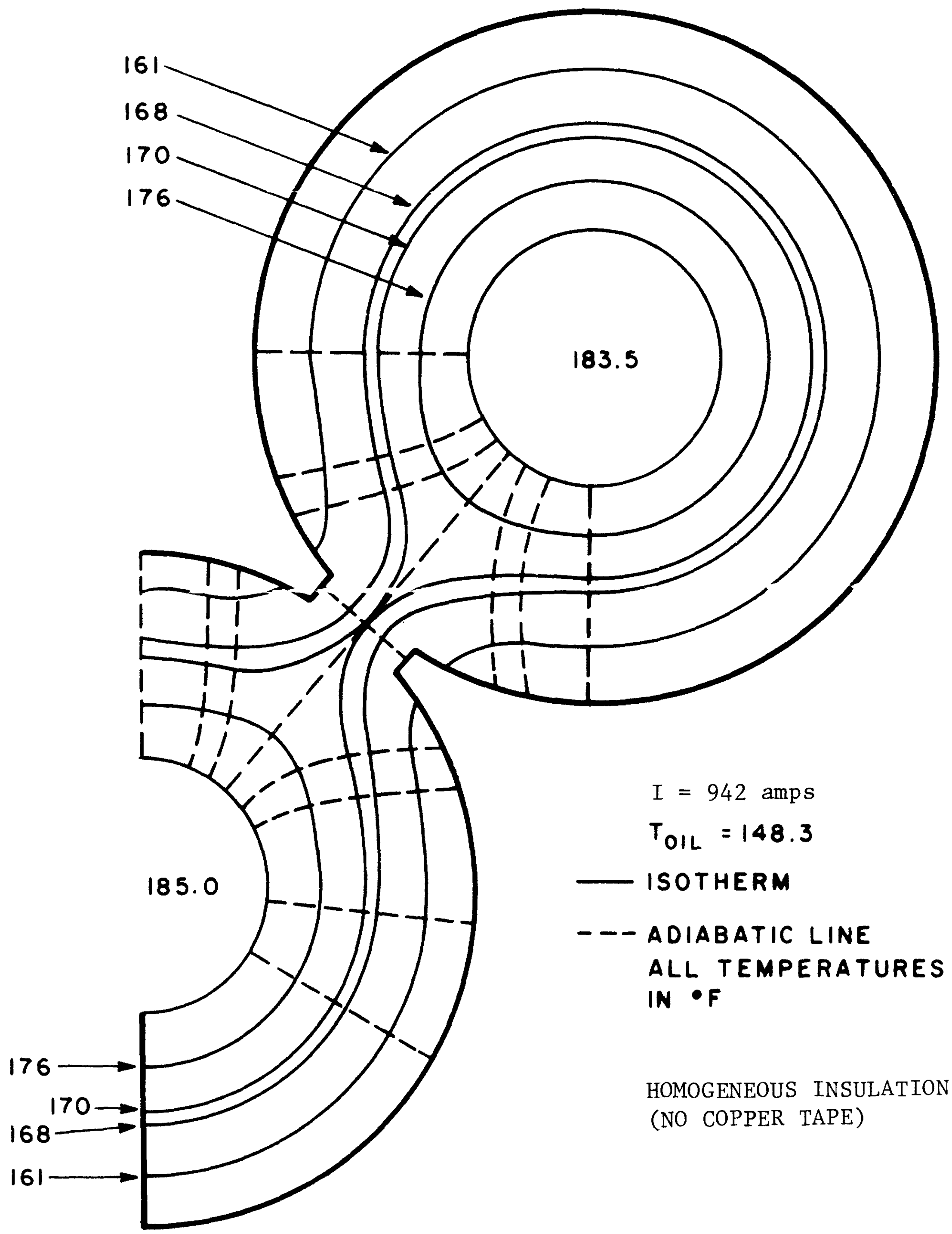


FIGURE 6.3

Isothermal and Adiabatic Lines for the
Open Configuration - System 1

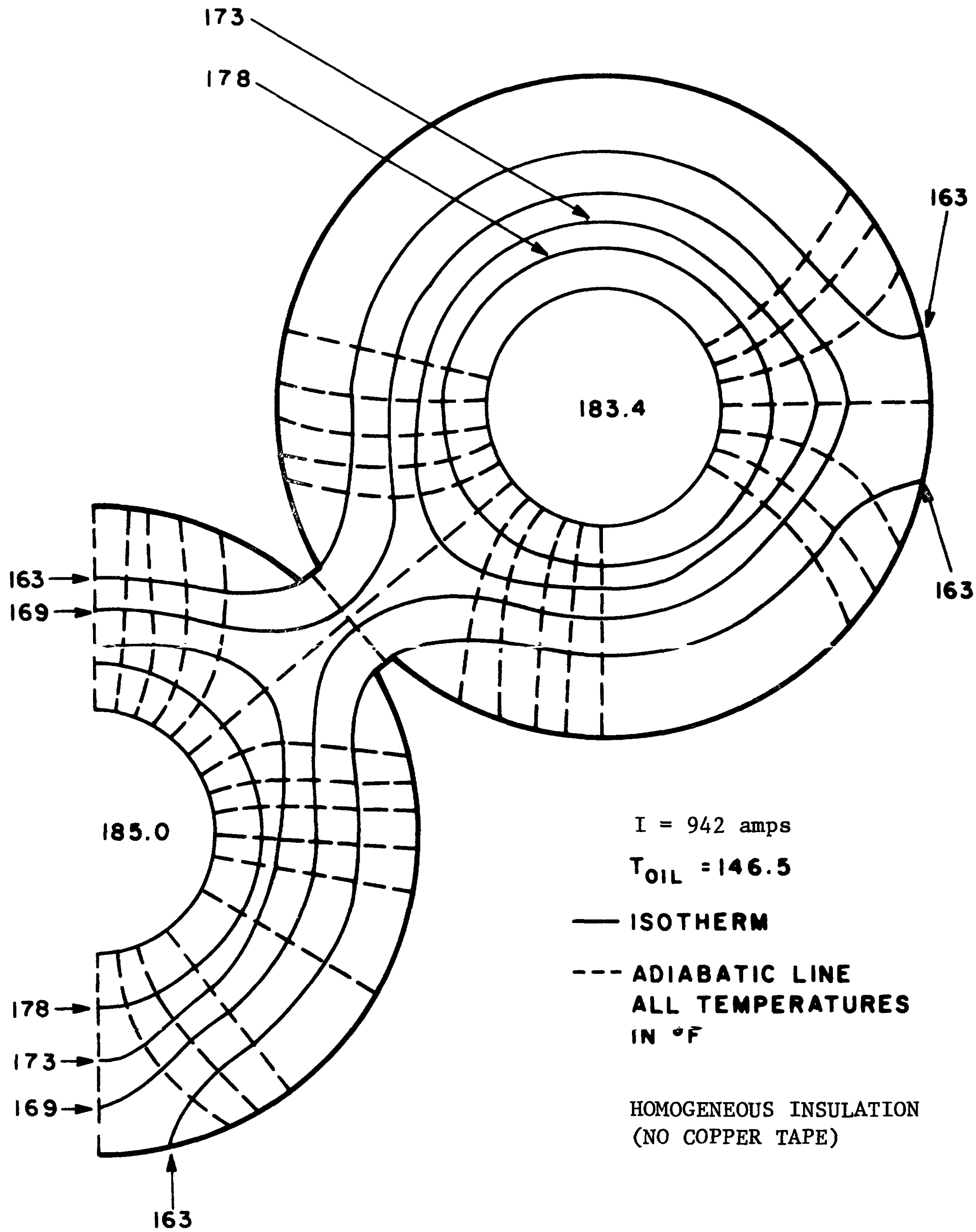


FIGURE 6.4

Isothermal and Adiabatic Lines for the Cradled Configuration - System 1

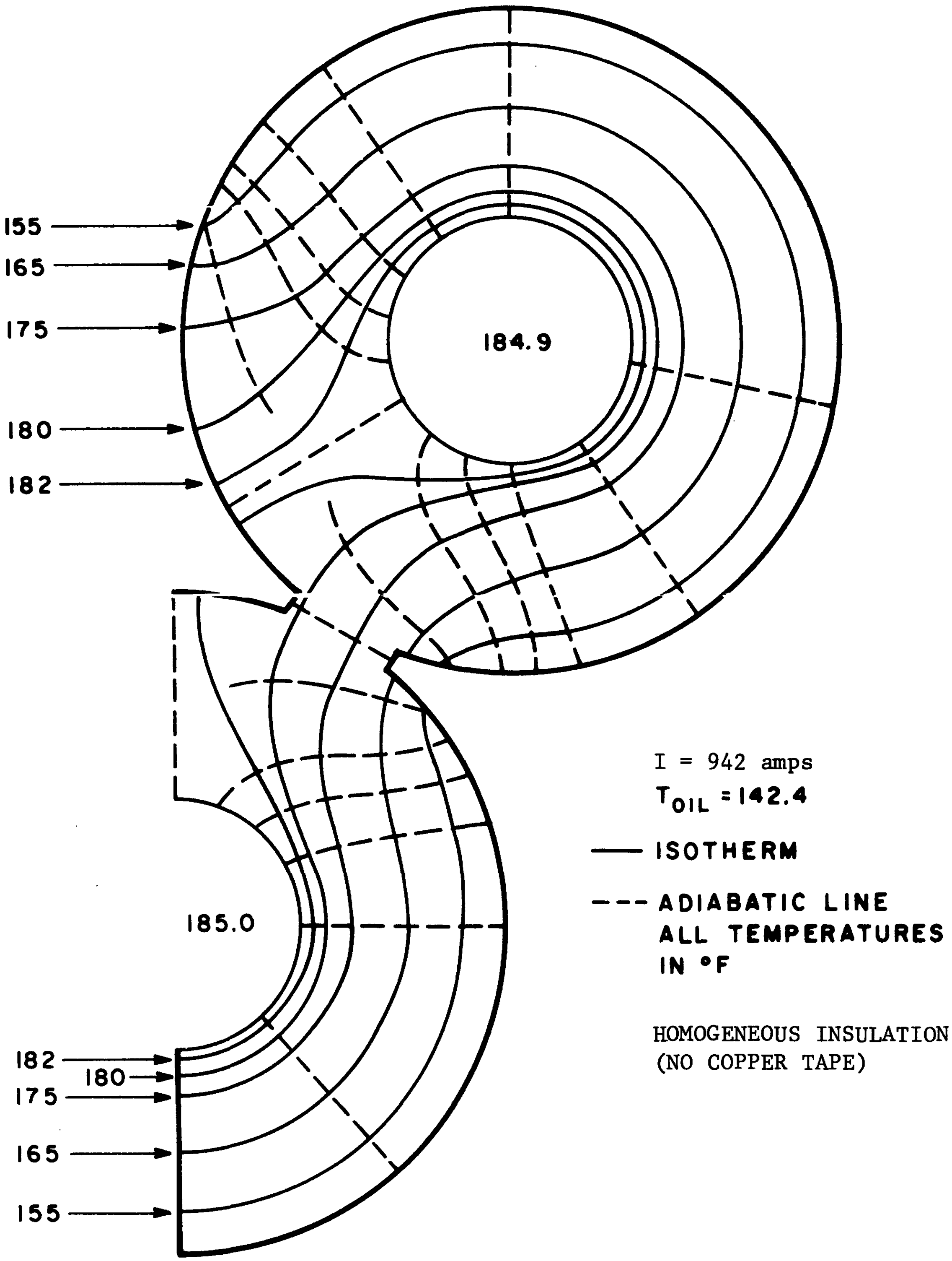


FIGURE 6.5

Isothermal and Adiabatic Lines for the Equilateral Configuration - System 1

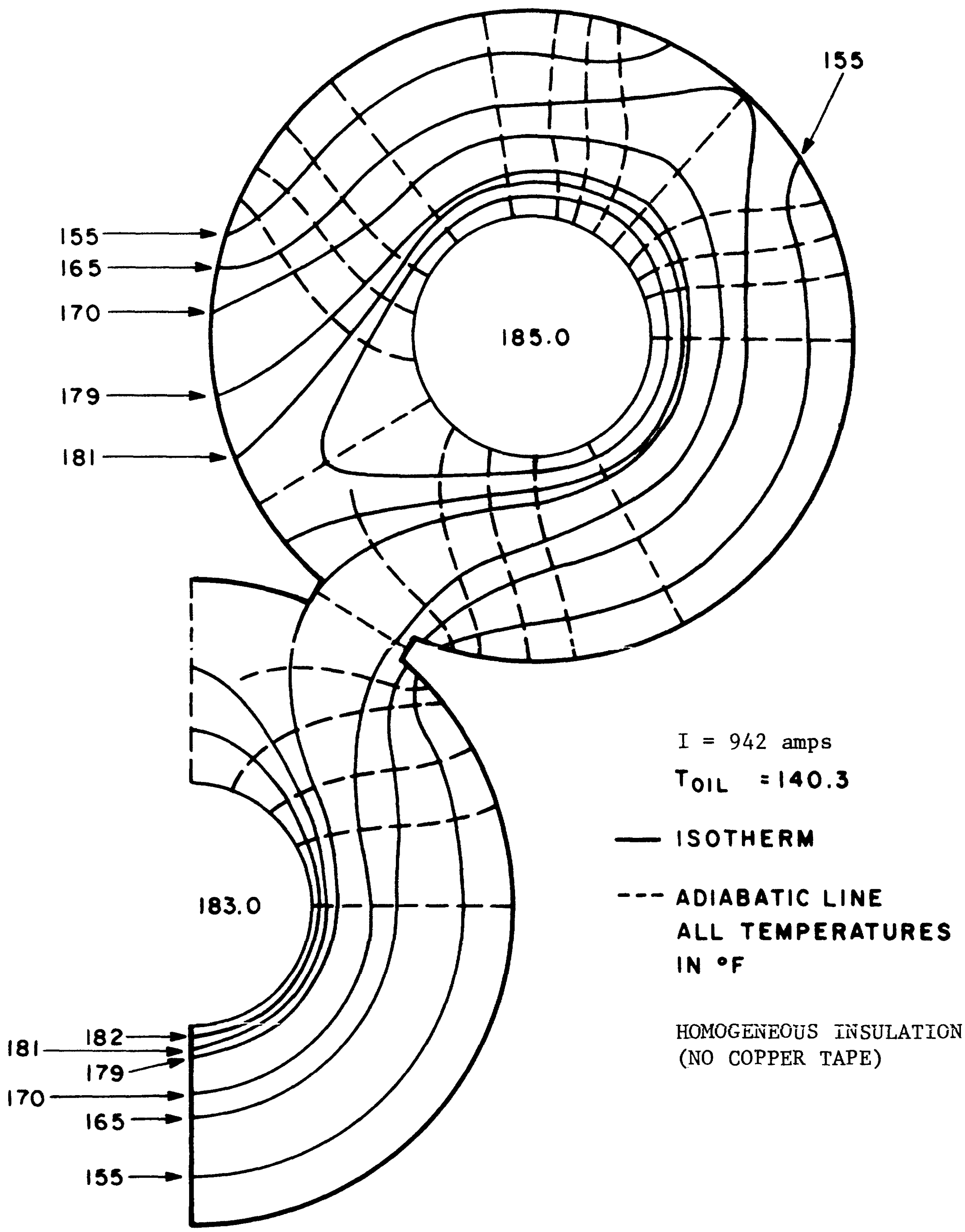


FIGURE 6.6

Isothermal and Adiabatic Lines for the Equilateral-Pipe Configuration - System 1

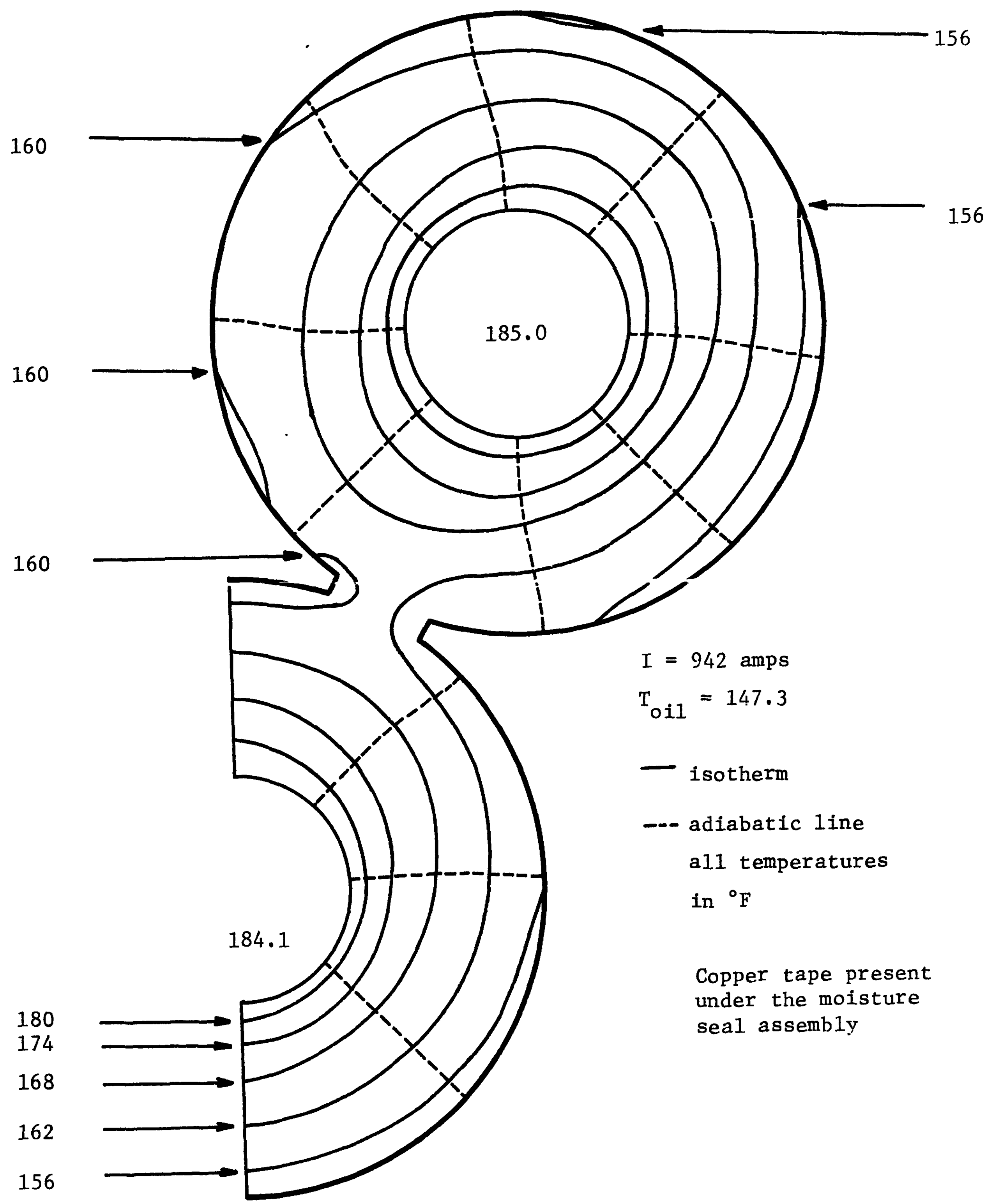


Figure 6.7

Isothermal and Adiabiatic Lines for the Equilateral-Pipe Configuration with copper tape in the moisture seal assembly - System 1.

Conclusions

On the basis of the results discussed, the following conclusions are drawn:

1. For a thermally nonconductiong conduit, a cable system is most susceptible to thermal failure in the equilateral-pipe configuration. For a thermally conducting conduit, the equilateral-pipe and equilateral configurations are equally severe, and the latter represents the worst operating configuration.
2. The conservative approximate solutions developed in Appendix H are useful for obtaining good estimates of maximum temperature and current. However, recourse should be made to the computer solutions whenever design information is required.
3. Cable proximity effects are important in forced cooling. The cable configuration can account for 21% (equilateral configuration with no copper tape) to 26% (equilateral-pipe configuration with no tape) of the total oil temperature rise between refrigeration stations.
4. The presence of a thin copper tape in the cable insulation moisture seal assembly significantly smooths out the temperature distribution in the cable insulation and thus higher maximum allowable oil temperature and higher currents are permitted than if a homogeneous cable insulation is used. Numerically the improvement is from about 4.3% for the oil temperature and approximately 9.5% for the current. If this figure is unacceptable, thicker copper tapes would smooth out temperatures even more.

Recommendations for Further Work

In order to fully exploit the convenience of having a separate region associated with each boundary condition, a provision should be written which would allow Regions IV and VI of Domain 2 to be used simultaneously. Such a provision does not presently exist, because there is only one configuration for which it would be desirable. However, the exceptional configuration is the equilateral-pipe case, in which both cable-cable and cable-conduit boundary conditions act simultaneously. Since this is the most severe condition for a thermally nonconducting conduit, it is expected that this configuration will be frequently used, and the change is probably warranted. It is noted, however, that in the present program all boundary conditions are independently specified by means of a variable film coefficient. The equilateral-pipe configuration can therefore be modelled as accurately as the user desires, and the proposed modification represents only a convenience in laying out the system geometry and mesh size. Instructions for implementing the change are given in Appendix E.

Finally, some improvement could be effected in the input-output formats of the computer program. Throughout the development of the program, attention was continually given to simplicity of I/O procedures and to ease of user operation. Yet certain aspects of the final I/O formats are less convenient than is desirable. Suggestions for their improvement are offered, again in Appendix E.

CHAPTER 7

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APPENDIX A

THE RELATIVE MAGNITUDE OF CONDUCTION AND CONVECTION RESISTANCES

The equation which defines the resistance per unit length to heat transfer is the following:

$$q = \frac{\Delta T}{R} , \tag{A.1}$$

where q is the total heat flow per unit axial length, ΔT is the temperature difference driving the heat flow, and R is the resistance per unit length to heat transfer.

The heat flow per unit length due to convection from the outer surface of the insulation to the oil is [10]

$$q = 2\pi r_o h(\Delta T) , \tag{A.2}$$

where h is the convective film coefficient, and r_o is the outer radius of the insulation. The convective resistance per unit length is therefore

$$R_h = \frac{1}{2\pi r_o h} . \tag{A.3}$$

The heat flow per unit length due to conduction in the cable insulation, which is assumed to be one-dimensional in this type of calculation, depends not only on ΔT, but also on the distributed dielectric loss. Because of this additional dependence on the dielectric loss, it is not analytically possible to model the conduction path as a simple resistance. However, a conduction resistance may be obtained numerically, by substituting appropriate values into Equation A.1. The (ΔT) for a given set of losses (q) is available from the one-dimensional solution presented in Appendix G. A typical set of values for (q) and (ΔT) yields

$$R_k = \frac{\Delta T}{(q)} = 0.93 \ \frac{\text{hr-ft-°F}}{\text{Btu}} \ , \qquad (A.4)$$

where ΔT is the temperature drop across the insulation, and (q) is the heat flow per unit length emanating from within the insulation (the conductor and dielectric losses).

The corresponding value for R_h from Equation A.3 is

$$R_h = \frac{1}{2\pi r_o h} = 0.18 \ \frac{\text{hr-ft-°F}}{\text{Btu}} \ , \qquad (A.3)$$

where the most conservative value for the natural convection film coefficient (5.0 Btu/hr-ft^2-°F) was used [2]. The relative magnitude of the two resistances is therefore

$$\frac{R_h}{R_k} = \frac{0.18}{0.93} \ \frac{\text{(hr-ft-°F/Btu)}}{\text{(hr-ft-°F/Btu)}} = 0.19 \ . \qquad (A.5)$$

Thus the natural convection resistance, which is always larger than the combined forced and natural convection resistance, is small when compared to the conduction resistance, and the latter is the limiting resistance to heat transfer.

APPENDIX B

INVESTIGATION OF THE CABLE-CONDUIT BOUNDARY CONDITION

In order to examine the cable-conduit boundary condition described in Chapter 2, a portion of the conduit wall is thermally modelled as a fin. The geometry from which to determine a fin length is shown in Figure B.1. It is based on a cradled configuration of cables, since that configuration produces the largest conduit loss. Of the two possible lengths L_1 and L_2, the latter is chosen so as to maximize the temperature drop through the fin. L_2 is found from the following relations:

$$\sin \beta_1 = \frac{r_2 + \left(\frac{r_3 - r_2}{2}\right)}{R_p - r_3} = 0.660 , \quad \text{(B.1)}$$

from which

$$\beta_1 = \sin^{-1}(0.660) = 41.3° . \quad \text{(B.2)}$$

Then

$$\beta_2 = 180 - 2\beta_1 = 97.4° , \quad \text{(B.3)}$$

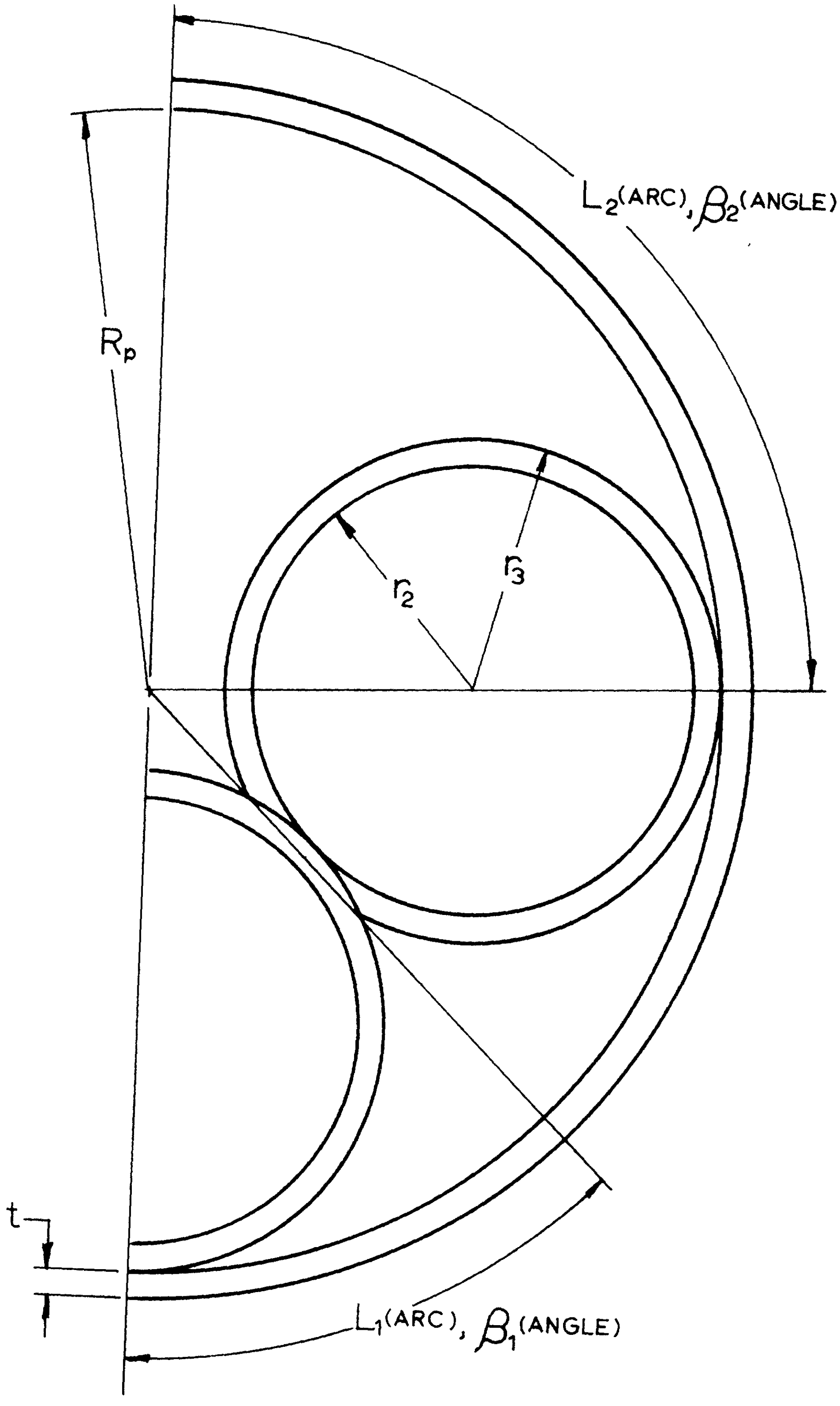

FIGURE B.1

Fin Geometry for the Cable-Conduit Boundary Condition and

$$L_2 = \beta_2\left(R_p + \frac{t}{2}\right) = 8.92" \ . \qquad (B.4)$$

One end of this fin is insulated by symmetry, and as a conservative approximation, the side of the fin which is adjacent to the earth is likewise taken to be insulated. Furthermore, the pipe loss is taken to be concentrated at the end of the fin which touches the cable (again to maximize the temperature drop through the fin). This thermal model is illustrated in Figure B.2. Considering the heat flow to be one-dimensional, the fin temperature is governed according to [11]

$$\frac{d^2T}{dz^2} - \frac{h_pP}{k_pA}\left(T-T_{oil}\right) = 0 \ , \qquad (B.5)$$

where P is the convective perimeter, and A is the cross-sectional area. Using the dimensionless variables

$$\eta = \frac{z}{L_2} \ , \qquad \theta = \frac{T - T_{oil}}{W_p/k_p} \ , \qquad (B.6)$$

Equation B.5 becomes the following:

$$\frac{d^2\theta}{d\eta^2} - L_2^2\,\frac{h_pP}{k_pA}\,\theta = 0 \ . \qquad (B.7)$$

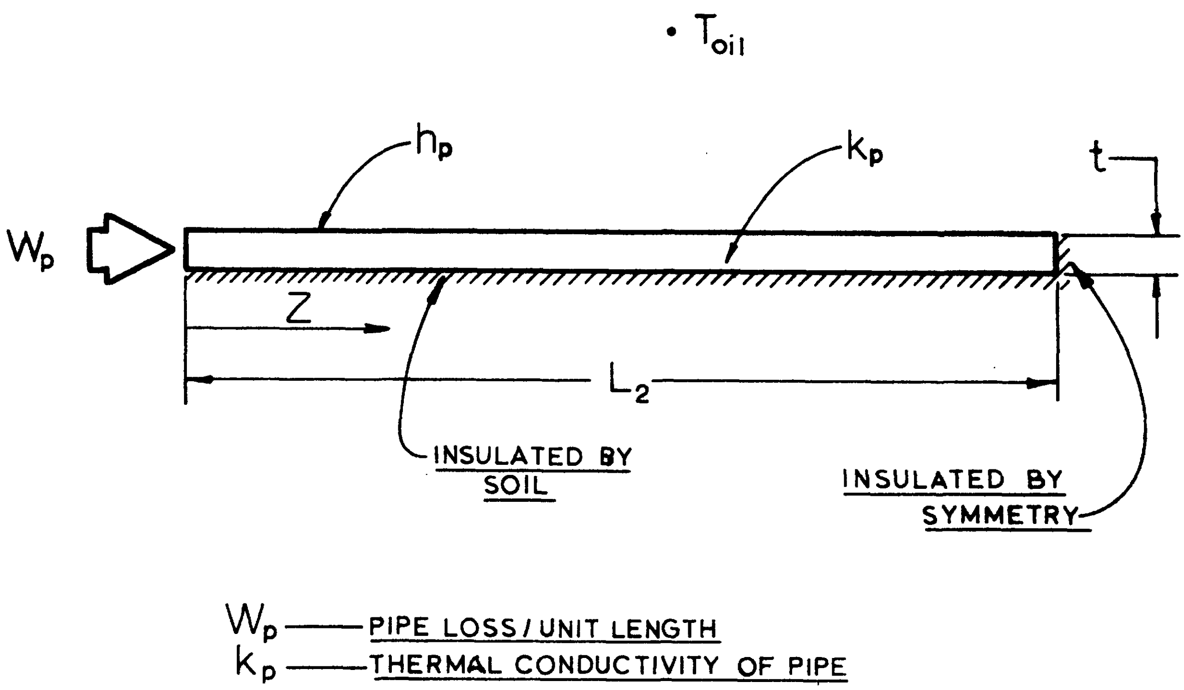

FIGURE B.2

Thermal Model of the Conduit Wall

Making the substitution

$$m^2 = L_2^2 \frac{h_p P}{k_p A} , \tag{B.8}$$

the governing equation is just

$$\frac{d^2\theta}{d\eta^2} - m^2\theta = 0 . \tag{B.9}$$

The boundary conditions are

$$-k_p t \left.\frac{dT}{dz}\right|_{z=0} = W_p , \tag{B.10}$$

and

$$\left.\frac{dT}{dz}\right|_{z=L_2} = 0 . \tag{B.11}$$

These are rendered dimensionless to give

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = - \frac{L_2}{t} . \tag{B.12}$$

and

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=1} = 0 . \tag{B.13}$$

The general solution of Equation B.9 is

$$\theta(\xi) = C_1 e^{-m\eta} + C_2 e^{m\eta} \tag{B.14}$$

The two arbitrary constants C_1 and C_2 are found from the boundary conditions to be

$$C_1 = \frac{L_2}{tm(1-e^{-2m})}, \quad C_2 = \frac{L_2}{tm(e^{2m}-1)} . \tag{B.15}$$

When these are substituted into the general solution, Equation B.14 can be manipulated into the following form:

$$\theta(\eta) = \frac{L_2}{tm} \frac{\cosh[m(1-\eta)]}{\sinh(m)} . \tag{B.16}$$

The following values are then taken:

$$\begin{aligned} h_p &= 3.3 \text{ Btu/hr-ft}^2\text{-°F} \quad \text{(conservative, based on [2])} \\ P &= 1 \text{ ft} \\ k_p &= 25 \text{ Btu/hr-ft-°F} \quad \text{(1\% C steel)} \\ A &= 0.0208 \text{ ft}^2 \end{aligned} \tag{B.17}$$

Equation B.16 finally becomes

$$\theta(\eta) = 5.73 \cosh[1.92(1-\eta)] . \tag{B.18}$$

It is desired to know the temperature drop from the fin base to the oil, so it is now convenient to return to the form

$$T(\eta) - T_{oil} = 5.73 \frac{W_p}{k_p} \cosh[1.92(1-\eta)] \ . \qquad (B.19)$$

The desired quantity is obtained by evaluating Equation B.19 at $\eta = 0$ and by substituting $W_p = \frac{1}{6}$ (8.85 watts/system-ft). The quantity in parentheses is a typical loss value, which is divided by 6 to account for the 3 cables and for the splitting of the heat flow to either side at the point of cable contact. The temperature drop is then

$$T(0)-T_{oil} = 5.73 \frac{\frac{1}{6}\ (8.85\ \text{watts/ft})}{(25\ \text{Btu/hr-ft-}^\circ\text{F})} \frac{(3.413\ \text{Btu})}{(\text{watt-hr})} \cosh(1.92)$$

$$= 3.9^\circ\text{F} \ . \qquad (B.20)$$

It is now desired to compare this temperature drop to the drop across the cable insulation. Using once again the one-dimensional solution for a cable with all losses (Appendix G), the temperature distribution in the insulation is given by

$$\theta(\xi) = \frac{W_d}{4\pi W(\ln \xi_1)} (\ln \xi)^2 - \frac{(W_d+W_c)}{2\pi W} \ln \xi + \frac{(W_c+W_d+W_s)}{2\pi Whr_2} k \ , \qquad (G.16)$$

where W_d, W_c, and W_s are the dielectric, conductor, and sheath losses per unit length, respectively, and W is an arbitrary loss per unit length. The dimensionless temperature drop across the insulation is given by

$$\theta(\xi_1) - \theta(1) = -\frac{\ln \xi_1}{2\pi W}\left(\frac{W_d}{2} + W_c\right) . \qquad (B.21)$$

Returning to the dimensional temperature by means of Equation G.6, this result becomes:

$$T(\xi_1) - T(1) = -\frac{\ln \xi_1}{2\pi k}\left(\frac{W_d}{2} + W_c\right) . \qquad (B.22)$$

The dielectric and conductor losses corresponding to the W_p used previously are 3.18 and 5.66 (watts/conductor-ft), respectively. These values, together with $k = 0.1153$ (Btu/hr-ft-°F) give

$$T(\xi_1) - T(1) = 31.1°F . \qquad (B.23)$$

The relative magnitude of the two temperature drops is therefore

$$\frac{(\Delta T)_{fin}}{(\Delta T)_{insulation}} = \frac{3.9°F}{31.1°F} = 12.5\% . \qquad (B.24)$$

It may be inferred from this result, which represents the most conservative comparison of the two effects, that contact between the cables and the steel conduit does not significantly alter the overall temperature distribution. The cable conduit boundary should thus be modelled as a convective one, with at most a slightly modified film coefficient.

APPENDIX C

THE SOLUTION FOR MAXIMUM CURRENT

The Superposition Method for Solution 2

In Solution 2 it is necessary that current-produced losses be treated separately from voltage-produced losses. This makes it possible to distinguish the variable component of the temperature distribution from the stationary component, and subsequently to adjust the variable component so as to maximize the current. When the losses are so separated, there are two complete problems, each one having three components and resembling the problem presented in Chapter 3. In fact, the solution in Chapter 3 for $\theta(\underset{\sim}{x})$ may be taken as one component of Solution 2, provided that the forcing term $f(\underset{\sim}{x})$ is clearly identified, either with stationary losses or with variable losses. Accordingly, let $f(\underset{\sim}{x})$ describe all forcing effects in the domain which are attributable to current. The solution $\theta(\underset{\sim}{x})$ then denotes that part of the total temperature distribution which is current-dependent.

It is now necessary to determine the stationary portion of the temperature, that portion which depends on voltage. Let this part of the total temperature solution be called $\theta D(\underset{\sim}{x})$. The solution $\theta D(\underset{\sim}{x})$ satisfies the governing equation

$$\nabla^2 \theta D(\underset{\sim}{x}) = g(\underset{\sim}{x}) \ , \qquad \text{(C.1)}$$

where $g(\underset{\sim}{x})$ describes all forcing effects in the domain which are attributable to voltage. $\theta D(\underset{\sim}{x})$ also satisfies the following boundary conditions (using the notation introduced in Chapter 3):

$$\theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} = \theta D_{01} , \tag{C.2}$$

where θD_{01} is some unknown dimensionless temperature.

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_2}\right|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} \tag{C.3}$$

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} = -\frac{hr_2}{k}\,\theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_3(\underset{\sim}{x})} \tag{C.4}$$

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x}\in C_4(\underset{\sim}{x})} = 0 \tag{C.5}$$

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x}\in C_5(\underset{\sim}{x})} = 0 \tag{C.6}$$

$$\theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_6(\underset{\sim}{x})} = \theta D_{02} , \tag{C.7}$$

where θD_{02} is some unknown dimensionless temperature.

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} = -\frac{h\rho_2}{k}\, \theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} \tag{C.8}$$

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} = -\frac{2Dh}{k}\, \theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} \tag{C.9}$$

$$\left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} = -\frac{2Dh}{k}\, \theta D(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} \tag{C.10}$$

As before, $\theta D(\underset{\sim}{x})$ may be decomposed into three component problems. However, it is not necessary to introduce three new components, for the solutions $\theta A(\underset{\sim}{x})$ and $\theta B(\underset{\sim}{x})$ of Chapter 3 already describe the homogeneous components required. So only one additional component is needed, and let it be referred to as $\theta E(\underset{\sim}{x})$. This component satisfies the nonhomogeneous governing equation

$$\nabla^2 \theta E(\underset{\sim}{x}) = g(\underset{\sim}{x}) \ . \tag{C.11}$$

The boundary conditions satisfied by $\theta E(\underset{\sim}{x})$ are identical to those satisfied by $\theta C(\underset{\sim}{x})$, Equations 3.32 - 3.40.

Once again the three component solutions are linearly combined according to

$$\theta D(\underset{\sim}{x}) = b_1 \theta A(\underset{\sim}{x}) + b_2 \theta B(\underset{\sim}{x}) + \theta E(\underset{\sim}{x}) , \qquad (C.12)$$

where b_1 and b_2 are two new arbitrary constants. The validity of Equation C.12 is readily established by direct substitution into the appropriate governing equation and boundary conditions, Equations C.1 - C.10. Since this procedure is identical to that followed in Chapter 3 (see Equations 3.42 - 3.51), it is not repeated here. The two constants b_1 and b_2 are again determined from a knowledge of the losses at the conductors. By analogy with Equations 3.56 and 3.57,

$$\int_{C_1(\underset{\sim}{x})} \left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_1}\right|_{\underset{\sim}{x} \in C_1(\underset{\sim}{x})} dC_1(\underset{\sim}{x}) = 0 , \qquad (C.13)$$

and

$$\int_{C_6(\underset{\sim}{x})} \left.\frac{\partial \theta D(\underset{\sim}{x})}{\partial n_6}\right|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} dC_6(\underset{\sim}{x}) = 0 . \qquad (C.14)$$

The zero right-hand sides of these equations reflect that there are no voltage-produced conductor losses. When

Equation C.12 is substituted into Equations C.13 and C.14, two simultaneous algebraic equations result which uniquely determine b_1 and b_2. The stationary part of the total temperature solution, $\theta D(\underset{\sim}{x})$, is therefore available.

Attention is now turned to the total solution, $\theta I(\underset{\sim}{x})$, which includes both stationary and variable losses. $\theta I(\underset{\sim}{x})$ satisfies the nonhomogeneous governing equation

$$\nabla^2 \theta I(\underset{\sim}{x}) = f(\underset{\sim}{x}) + g(\underset{\sim}{x}) \ , \tag{C.15}$$

as well as the following boundary conditions:

$$\theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_1(\underset{\sim}{x})} = \theta I_{01} \ , \tag{C.16}$$

where θI_{01} is some unknown dimensionless temperature.

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_2}\right|_{\underset{\sim}{x} \in C_2(\underset{\sim}{x})} = -\frac{hr_2}{k} \ \theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_2(\underset{\sim}{x})} \tag{C.17}$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} = -\frac{hr_2}{k} \ \theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} \tag{C.18}$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x} \in C_4(\underset{\sim}{x})} = 0 \tag{C.19}$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x} \in C_5(\underset{\sim}{x})} = 0 \tag{C.20}$$

$$\theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} = \theta I_{02} \ , \tag{C.21}$$

where θI_{02} is some unknown dimensionless temperature.

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} = -\frac{h\rho_2}{k}\ \theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_7(\underset{\sim}{x})} \tag{C.22}$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} = -\frac{2Dh}{k}\ \theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_8(\underset{\sim}{x})} \tag{C.23}$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} = -\frac{2Dh}{k}\ \theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_9(\underset{\sim}{x})} \tag{C.24}$$

The total solution $\theta I(\underset{\sim}{x})$ is obtained as a simple sum of the particular solutions $\theta(\underset{\sim}{x})$ and $\theta D(\underset{\sim}{x})$:

$$\theta I(\underset{\sim}{x}) = \theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x}) \ . \tag{C.25}$$

The validity of C.25 is established by direct substitution into Equations C.15 - C.24:

$$\nabla^2 \theta I(\underset{\sim}{x}) = \nabla^2[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})] = f(\underset{\sim}{x}) + g(\underset{\sim}{x}) \quad \text{Check} \tag{C.26}$$

$$\theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} = [\theta(\underset{\sim}{x})+\theta D(\underset{\sim}{x})]\Big|_{\underset{\sim}{x}\in C_1(\underset{\sim}{x})} = \theta_{01} + \theta D_{01} \quad \text{Check ,} \tag{C.27}$$

provided $\theta I_{01} = \theta_{01} + \theta D_{01} = (a_1+b_1)A_o$. This result follows directly from the linearity of the problem: a_1 and b_1 were determined by the variable and stationary components, respectively, of the loss at the conductors. Since the variable and stationary losses may be added to give the total loss at the conductors, the temperatures a_1A_o and b_1A_o may likewise be added to give the true conductor temperature.

$$\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_2}\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} = \frac{\partial}{\partial n_2}[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})]\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})]\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}\,\theta I(\underset{\sim}{x})\Bigg|_{\underset{\sim}{x}\in C_2(\underset{\sim}{x})} \quad \text{Check} \tag{C.28}$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_3}\right|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} = \frac{\partial}{\partial n_3}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})}$$

$$= -\frac{hr_2}{k}\, \theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_3(\underset{\sim}{x})} \quad \text{Check} \qquad (C.29)$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_4}\right|_{\underset{\sim}{x} \in C_4(\underset{\sim}{x})} = \frac{\partial}{\partial n_4}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x} \in C_4(\underset{\sim}{x})}$$

$$= 0 \quad \text{Check} \qquad (C.30)$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_5}\right|_{\underset{\sim}{x} \in C_5(\underset{\sim}{x})} = \frac{\partial}{\partial n_5}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x} \in C_5(\underset{\sim}{x})}$$

$$= 0 \quad \text{Check} \qquad (C.31)$$

$$\theta I(\underset{\sim}{x})\Big|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} = \left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Big|_{\underset{\sim}{x} \in C_6(\underset{\sim}{x})} = \theta_{02} + \theta D_{02} \quad \text{Check,} \qquad (C.32)$$

provided $\theta I_{02} = \theta_{02} + \theta D_{02} = (a_2+b_2)B_o$. This result is analogous to Equation C.27, again following from linearity.

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_7}\right|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})} = \frac{\partial}{\partial n_7}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Bigg|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})}$$

$$= -\frac{h\rho_2}{k}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Bigg|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})}$$

$$= -\frac{h\rho_2}{k}\,\theta I(\underset{\sim}{x})\Bigg|_{\underset{\sim}{x}\in C_7(\underset{\sim}{x})} \qquad \text{Check} \qquad (C.33)$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_8}\right|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} = \frac{\partial}{\partial n_8}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Bigg|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Bigg|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\,\theta I(\underset{\sim}{x})\Bigg|_{\underset{\sim}{x}\in C_8(\underset{\sim}{x})} \qquad \text{Check} \qquad (C.34)$$

$$\left.\frac{\partial \theta I(\underset{\sim}{x})}{\partial n_9}\right|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})} = \frac{\partial}{\partial n_9}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Bigg|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\left[\theta(\underset{\sim}{x}) + \theta D(\underset{\sim}{x})\right]\Bigg|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})}$$

$$= -\frac{2Dh}{k}\,\theta I(\underset{\sim}{x})\Bigg|_{\underset{\sim}{x}\in C_9(\underset{\sim}{x})} \qquad \text{Check} \qquad (C.35)$$

It is thus established that the overall solution $\theta I(x)$ is available from its stationary and variable components according to Equation C.25. It now remains to adjust the variable component $\theta(\underset{\sim}{x})$ so as to maximize current with respect to the allowable cable temperature and the oil temperature.

<u>Maximizing Current in Solution 2</u>

The current I is introduced into the temperature solution through the relation

$$\theta(\underset{\sim}{x}) = \gamma_1(\underset{\sim}{x})I^2 , \quad (C.36)$$

where $\gamma_1(\underset{\sim}{x})$ is a constant of proportionality whose magnitude depend on position $\underset{\sim}{x}$. Equation C.36 follows directly from two elementary facts: 1. Because of linearity, the cable temperature is directly proportional to the cable loss. 2. Current-produced losses are directly proportional to I^2. It is now recalled that the solution $\theta(\underset{\sim}{x})$ is available, provided that the current-produced losses (and hence I) have been specified. Accordingly, let I_o be an arbitrary current from which a temperature distribution $\theta_o(\underset{\sim}{x})$ is determined. Then from Equation C.36,

$$\theta_o(\underset{\sim}{x}) = \gamma_1(\underset{\sim}{x})I_o^2 , \quad (C.37)$$

and hence

$$\gamma_1(\underset{\sim}{x}) = \frac{\theta_o(\underset{\sim}{x})}{I_o^2} . \tag{C.38}$$

Equations C.38, C.36, and C.25 may be combined to give the result

$$\theta I(\underset{\sim}{x}) = \theta_o(\underset{\sim}{x}) \frac{I^2}{I_o^2} + \theta D(\underset{\sim}{x}) , \tag{C.39}$$

where $\theta_o(\underset{\sim}{x})$, I_o, and $\theta D(\underset{\sim}{x})$ are all known.

It is desired to have $\theta I(\underset{\sim}{x})$ take on some maximum allowable value, say θ_{max}. Inserting this value into Equation C.39,

$$\theta_{max} = \theta_o(\underset{\sim}{x}) \frac{I^2}{I_o^2} + \theta D(\underset{\sim}{x}) . \tag{C.40}$$

Upon rearrangement this relation yields

$$\frac{I^2}{I_o^2} = \frac{\theta_{max} - \theta D(\underset{\sim}{x})}{\theta_o(\underset{\sim}{x})} \equiv \delta(\underset{\sim}{x}) , \tag{C.41}$$

where the dimensionless scalar $\delta(\underset{\sim}{x})$ has been introduced for brevity. $\delta(\underset{\sim}{x})$, which is known throughout the domain, determines the ratio I^2/I_o^2 which will produce a temperature of θ_{max} at the location $\underset{\sim}{x}$. It is now necessary to choose the particular ratio I^2/I_o^2 (and hence the particular value of $\delta(\underset{\sim}{x})$) which will yield a maximum temperature θ_{max} in the distribution C.39. This is accomplished simply by taking the smallest possible ratio I^2/I_o^2. Let $\bar{\delta}$ denote the minimum over all $\underset{\sim}{x}$ of $\delta(\underset{\sim}{x})$. The desired temperature distribution is then

$$\theta I(\underset{\sim}{x}) = \bar{\delta}\theta_o(\underset{\sim}{x}) + \theta D(\underset{\sim}{x}) \ . \tag{C.42}$$

Proof is as follows: let x_o be the location at which the minimum value of $\delta(\underset{\sim}{x})$ occurs:

$$\bar{\delta} = \delta(x_o) \ . \tag{C.43}$$

It then follows from Equation C.41 that

$$\theta I(x_o) = \theta_{max} \ . \tag{C.44}$$

Now consider any other location in the domain, say x_1. It is known from the definition C.41 of $\delta(\underset{\sim}{x})$ that

$$\delta(x_1)\theta_o(x_1) + \theta D(x_1) = \theta_{max} \ . \tag{C.45}$$

It is also known that

$$\bar{\delta} \leq \delta(x_1) \ , \tag{C.46}$$

since $\bar{\delta}$ is the minimum over the entire domain of $\delta(\underset{\sim}{x})$. It then follows directly from the relations C.45 and C.46 that

$$\bar{\delta}\theta_o(x_1) + \theta D(x_1) = \theta I(x_1) \leq \theta_{max} \ . \tag{C.47}$$

The distribution C.42 is therefore proven to be the correct one, and the maximum allowable current is determined from Equation C.41 with $\delta(\underset{\sim}{x}) = \bar{\delta}$:

$$I = I_o\sqrt{\bar{\delta}} \ . \tag{C.48}$$

APPENDIX D

THE DIFFERENCE FORM OF THE CONDUCTOR BOUNDARY CONDITION

In Chapter 3 the heat flow emanating from the conductor of Cable 1 was given as

$$q_1 = -k \int_0^{\pi} \left.\frac{\partial T}{\partial r}\right|_{r_1,\phi} r_1 d\phi = W_{C1} \ . \tag{3.52}$$

It is now convenient to express this in the dimensionless form

$$\int_0^{\pi} \left.\frac{\partial \theta}{\partial \bar{r}}\right|_{\bar{r}_1,\phi} \bar{r}_1 d\phi = -\frac{W_{C1}}{W} \ , \tag{D.1}$$

where $\bar{r}_1$ denotes the dimensionless inner radius of the insulation. As the discussion of Chapter 4 indicated, a problem is incurred in the discretization of this boundary condition. For if the standard central difference approximation 4.3 is substituted for the derivative at the conductor, a fictitious temperature within the conductor is introduced. Since no governing equation is applied at the conductor, there is no way to eliminate such a fictitious temperature. In approximating the boundary condition D.1, it is therefore necessary to have a difference expression which involves only real temperatures.

There are a number of methods for approximating this boundary condition. As a reasonable compromise between accuracy and simplicity, the following method is chosen, where reference is made to Figure D.1: some central location $\bar{r}_*$ in between $\bar{r}_1$ and $\bar{r}_2$ is sought, at which location a good approximation to the derivative can be achieved. The boundary condition D.1 is then satisfied at the location $\bar{r}_*$, rather than at the conductor:

$$\int_0^{\pi} \left.\frac{\partial\theta}{\partial\bar{r}}\right|_{\bar{r}_*,\phi} \bar{r}_* d\phi \approx -\frac{W_{C1}}{W} . \qquad \text{(D.2)}$$

The difference form of the derivative is constructed according to

$$\left.\frac{\partial\theta}{\partial\bar{r}}\right|_{\bar{r}_*,\phi} \approx \frac{\theta_{1,k1} - \theta_{01}}{\bar{r}_2 - \bar{r}_1} , \qquad \text{(D.3)}$$

where θ_{01} denotes the conductor temperature. In order to ascertain the location $\bar{r}_*$, attention is turned to the corresponding one-dimensional problem. The analytical temperature distribution for the half-cable with prescribed conductor loss W_{C1} is readily found to be

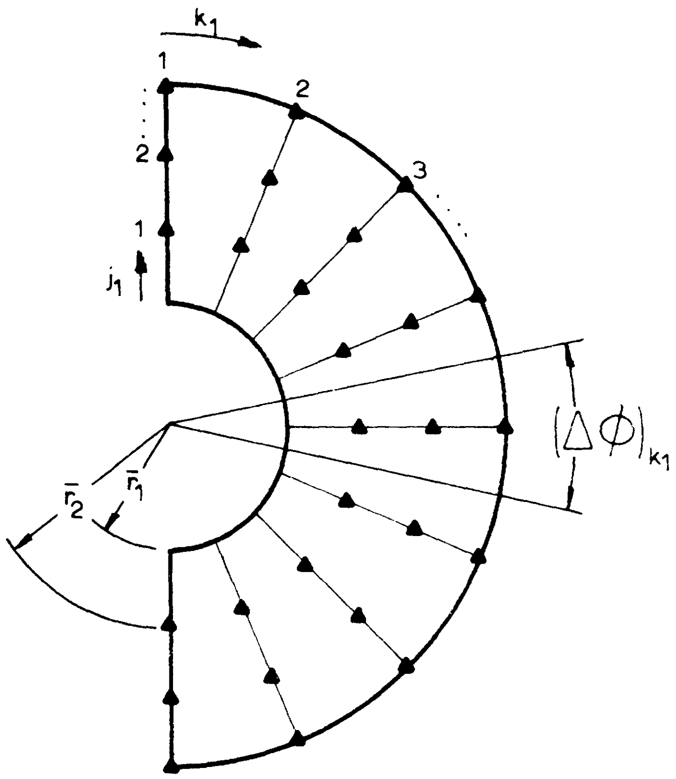

FIGURE D.1

Nomenclature for the Discretized Conductor Boundary Condition

$$\theta(\bar{r}) = \frac{W_{C1}}{\pi W}\left(\frac{k}{hr_2} - \ln \bar{r}\right) . \tag{D.4}$$

With the distribution D.4, a criterion for determining the location $\bar{r}_*$ is available: $\bar{r}_*$ is chosen such that, as the true temperature distribution approaches the one-dimensional solution (which it does in the vicinity of the conductor), the difference form of Equation D.2 becomes exact. This is accomplished by replacing the discrete temperatures of Equation D.3 with their analytical expressions and by then substituting the result into Equation D.2. The difference form of Equation D.2 is

$$\sum_{kl}\left[\left(\frac{\theta_{1,kl}-\theta_{01}}{\bar{r}_2-\bar{r}_1}\right)\bar{r}_*(\Delta\phi)_{kl}\right] = -\frac{W_{C1}}{W} . \tag{D.5}$$

Replacing the discrete temperatures with analytical ones gives

$$\sum_{kl}\left[\left(\frac{\theta(\bar{r}_2)-\theta(\bar{r}_1)}{\bar{r}_2-\bar{r}_1}\right)\bar{r}_*(\Delta\phi)_{kl}\right] = -\frac{W_{C1}}{W} . \tag{D.6}$$

Upon expanding through Equation D.4, this becomes

$$\frac{W_{C1}}{\pi W}\sum_{kl}\left[\left(\frac{\ln \bar{r}_1-\ln \bar{r}_2}{\bar{r}_2-\bar{r}_1}\right)\bar{r}_*(\Delta\phi)_{kl}\right] = -\frac{W_{C1}}{W} . \tag{D.7}$$

Since the term in brackets does not depend on k1, the summation can be carried out. Equation D.7 then yields, upon rearrangement,

$$\bar{r}_* = \frac{\bar{r}_2 - \bar{r}_1}{\ln \bar{r}_2 - \ln \bar{r}_1} \; . \tag{D.8}$$

When this result is substituted back into Equation D.5, the difference form of the conductor boundary condition becomes

$$\sum_{k1} \left[\left(\frac{\theta_{1,k1} - \theta_{01}}{\ln \bar{r}_2 - \ln \bar{r}_1} \right) (\Delta\phi)_{k1} \right] = - \frac{W_{C1}}{W} \; . \tag{D.9}$$

The procedure is of course analogous for Cable 2:

$$\sum_{k2} \left[\left(\frac{\theta_{1,k2} - \theta_{02}}{\ln \bar{\rho}_2 - \ln \bar{\rho}_1} \right) (\Delta\alpha)_{k2} \right] = - \frac{W_{C2}}{W} \; . \tag{D.10}$$

The results D.9 and D.10 need not be weakened by the assumption that the temperature distribution becomes one-dimensional near the conductor. The distribution is always one-dimensional right at the conductor, since it has a uniform temperature. So the only requirement is to choose the radial mesh size so as to place $\bar{r}_*$ out of the range of strong two-dimensional effects. This choice is a matter of judgment, and it depends on the given problem.

APPENDIX E

USER INSTRUCTIONS

Geometry and Mesh Size

In setting up a problem for computer solution, it is necessary to provide information about the region size within the cables and about the number and distribution of mesh points. This section discusses specification of region size, subdivision of regions, special considerations for D_3, and weighting of the mesh so as to have a good expression for the gradient at each conductor.

Reference is now made back to Figure 2.3, where a set of regional divisions is depicted, and to Figure 2.2, which shows the origin of cylindrical coordinates for both cables. The domain D_1 always has four regional divisions. The domain D_2 employs only six regional divisions, since Regions IV and VI are never used simultaneously. The angles included by the various regions are determined from the azimuthal coordinates of their bounding radial lines. For example, the angle ϕ_1 specifies the location of the boundary between Regions I and II in D_1, and it thus determines the size of Region I. The angle ϕ_2 likewise specifies the location of the boundary between Regions II and III of D_1. The angle included by Region II is then $(\phi_2 - \phi_1)$. The sizes of all the regional divisions are therefore specified by three angles ϕ in D_1 and by five angles α in D_2. However, since D_1 and D_2 share a single inter-cable conduction path, only six of the above eight angles are independent. The orientation of the two cables is determined by α_1 and α_2 (or by ϕ_1 and ϕ_2). Specifically, the orientation angle is $(1/2)(\alpha_1+\alpha_2)$. The convention of establishing regional divisions within the cables actually has two purposes. It first of all provides a way to clearly identify a given portion of the cable surface with a given boundary condition. The second purpose of the divisional convention is to provide a mechanism for varying the azimuthal distribution of mesh points, so that they may be concentrated where the largest gradients are expected.

Mesh points inherently exist at all regional boundaries. They are placed inside a given region by specifying the number of subdivisions within that region, both in the radial and in the azimuthal direction. Here the term "subdivision" denotes the smallest element of the region, rather than the act of subdividing. Thus if a region has three azimuthal subdivisions, it is uniformly divided into three sectors by two radial boundaries, and two azimuthal mesh locations within the region are thereby introduced. The number of radial subdivisions does not vary from region to region; it is uniform within a particular domain. Thus a choice of four radial subdivisions in D_2 places four uniformly spaced radially points at every azimuthal location in D_2. Placement of mesh points inside D_3 proceeds in similar fashion, by specifying the number of normal and tangential subdivisions within the domain. In the computer program the various numbers of subdivisions throughout the solution domain are denoted by the variables N(J) and M(J). These are described in Table 7, together with a column containing the minimum allowable value of each variable. It is noted that the variables M(2) and M(11) cannot be chosen to be less than two. This is necessary in order to preserve the basic trapezoidal structure of D_3.

Attention is now returned to the discretized model of D_3. The width of D_3 is taken to be equal to the skid wire thickness, a number supplied directly as input data. The height of D_3 is determined from the outer cable radius and the angle $(\alpha_2-\alpha_1)$, as was shown in Figure 4.2. However, as the discussion of Chapter 4 indicated, the height so designated is only an apparent height and not the effective height. For when the regular form of governing equation is applied at the four corner points of D_3, four effective corner locations are produced which lie outside the corner mesh points. These effective corner locations then define two effective surfaces, as shown in Figure E.1. The effective upper and lower surfaces of D_3 extend halfway to the neighboring mesh points above and below the domain, as suggested by the figure. A conduction resistance based on this extended length is implicitly added in series with the convection resistance for boundary points in the numerical model. Of primary concern to the user is that an appropriate allowance must be made for this extension in specifying the height of D_3. Say, for example, that each

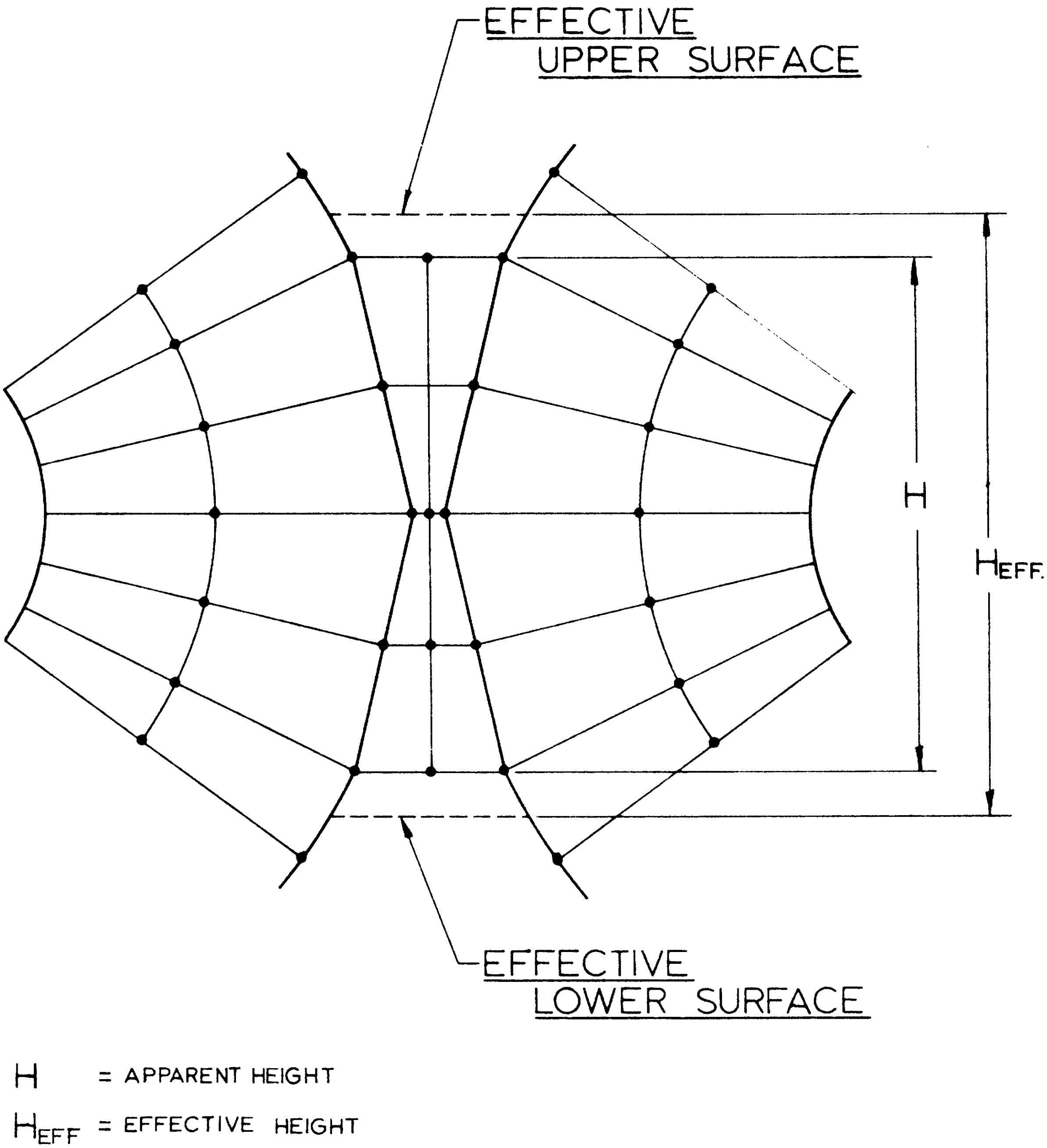

FIGURE E.1

Effective Surfaces of D_3

TABLE 7

SPECIFICATION OF SUBDIVISIONS THROUGHOUT D_1, D_2, AND D_3 IN TERMS OF THE COMPUTER VARIABLES N(J) AND M(J)

Type and Location of Subdivision	Number of Subdivisions	Minimum Allowable Value
Radial -		
D_1	N(1)	1
D_2	N(2)	1
Azimuthal -		
D_1:		
Region I	M(1)	1
Region II	M(2)	2
Region III	M(3)	1
Region IV	M(4)	1
D_2:		
Region I	M(5)	1
Region II	M(2)	2
Region III	M(6)	1
Region IV	M(7)	1
Region V	M(8)	1
Region VI	M(9)	1
Region VII	M(10)	1
Normal - D_3	M(11)	2
Tangential - D_3	M(2)	2

azimuthal subdivision in Figure E.1 happens to be 10° in size. The effective height of D_3 is then based on an included angle of 50°, whereas the apparent included angle is only 40°. So in order to achieve this true included angle of 50°, the apparent angle $(\alpha_2-\alpha_1) = 40°$ would have been specified, and the mesh points above and below D_3 would have been chosen so as to place neighboring points at an azimuthal distance of 10°. An additional consideration is that the four surface mesh points in D_1 and in D_2 which are adjacent to the corner mesh points of D_3 should be reasonably symmetrical about the y-axis of D_3. This is necessary so that the effective surfaces of D_3 remain parallel or nearly parallel to the normal axis. Some degree of foresight is therefore required in laying out the regional divisions and in choosing appropriate numbers of subdivisions.

The final topic of this section concerns the conductor boundary condition. It is recalled that the discrete form of this boundary condition involves a summation of temperatures around the innermost discrete ring of mesh points. In the summation each temperature is weighted according to the azimuthal sector associated with the given mesh point. Attention is now called to the physical circumstance that Regions II of D_1 and D_2 are regions of elevated temperature, owing to the presence of the inter-cable conduction path. The temperature trails off rapidly on either side of these regions, tending toward the one-dimensional distribution. Since the gradient at the conductor is constructed numerically by means of summing discrete temperature differences around the cable, it is essential that a good sampling of temperatures near Regions II of D_1 and D_2 be taken. This ensures that the elevated temperatures in those locations will not be unduly weighted. Based on comparisons with one-dimensional solutions, the following convention for weighting mesh points has been found to produce a sufficiently accurate numerical expression for the gradient at the conductor: the number of azimuthal subdivisions in Regions I and III is chosen so as to place a minimum of two radial mesh locations adjacent to Regions II, each at an azimuthal spacing equal (or nearly equal) to the azimuthal spacing of points within Regions II. In Figure E.1, for example, this means that there should be a minimum of two 10°-sectors on both sides of both Regions II. This convention should also be followed for all regions whose surfaces are insulated, for the same argument then applies.

Input Variables

The input variables used by the computer program are listed in Table 8, together with a brief description of each variable.

The five angles ALPHA(J) of D_2 are specified sequentially, skipping over any region not present. So if Region IV is used, ALPHA(3) denotes the III - IV boundary

TABLE 8

INPUT VARIABLES FOR THE COMPUTER PROGRAM

Variable Name	Description and Units ( )
ALPHA(J) =	the five angles α which specify the boundaries of the six regions of D_2. (degrees)
FILMP(J) =	the variable film coefficients for the surface mesh points of D_2. (Btu/hr-ft^2-°F)
FILMR(J) =	the variable film coefficients for the surface mesh points of D_1. (Btu/hr-ft^2-°F)
FILMX3(J) =	the variable film coefficients for the mesh points on the upper surface (+y) of D_3. (Btu/hr-ft^2-°F)
FILMX4(J) =	the variable film coefficients for the mesh points on the lower surface (-y) of D_3. (Btu/hr-ft^2-°F)
IPARAM =	1 or 2: 1 denotes that Region VI of D_2 is present; 2 denotes that Region IV of D_2 is present. (unitless)
IPROB =	1, 2, or 3, corresponding to Solution 1 (for maximum cable temperature), Solution 2 (for maximum current), or Solution 3 (for maximum oil temperature). (unitless)
M(J) =	various numbers of subdivisions, as per Table 7. (unitless)
N(J) =	various numbers of subdivisions, as per Table 7. (unitless)
PHI(3) =	the angle in D_1 which specifies the boundary between Regions III and IV. (degrees)
RAD1 =	the inner radius of the insulation of Cable 1. (inches)

TABLE 8

(Continued)

Variable Name	Description and Units ( )
RAD2 =	the outer radius of the insulation of Cable 1. (inches)
REST1 =	the DC resistance of the conductor of Cable 1. (μΩ/ft)
REST2 =	the DC resistance of the conductor of Cable 2. (μΩ/ft)
RHO1 =	the inner radius of the cable insulation of Cable 2. (inches)
RHO2 =	the outer radius of the cable insulation of Cable 2. (inches)
SKID =	the skid wire thickness. (inches)
TMAX =	the maximum allowable temperature in the cable system. (°F)
TOIL =	the oil temperature outside the convective boundary layer. (°F)
WD1 =	the total dielectric loss per unit length in Cable 1. (watts/ft)
WD2 =	the total dielectric loss per unit length in Cable 2. (watts/ft)
XHFILM =	the thermal film coefficient for the one-dimensional solution. (Btu/hr-ft^2-°F)
XI1 =	the current in Cable 1. (k-amps)
XI2 =	the current in Cable 2. (k-amps)
XI2OI1 =	the ratio of the current in Cable 2 to the current in Cable 1. (unitless)
XK =	the thermal conductivity of the insulation. (Btu/hr-ft-°F)

TABLE 8

(Continued)

Variable Name		Description and Units ( )
YC1	=	the AC/DC ratio at the conductor of Cable 1. (unitless)
YC2	=	the AC/DC ratio at the conductor of Cable 2. (unitless)
YS1	=	the AC/DC ratio at the sheath of Cable 1. (unitless)
YS2	=	the AC/DC ratio at the sheath of Cable 2. (unitless)
ICUTAP	=	0 denotes that no tape is present; any other integer indicates that tape is wrapped around either Cable (unitless)
THICK1	=	thickness of tape wrapped around Cable 1 (in)
THICK2	=	thickness of tape wrapped around Cable 2 (in)
XKCU1	=	conductivity of tape wrapped around Cable 1 (BTU/hr-ft°F)
XKCU2	=	conductivity of tape wrapped around Cable 2 (BTU/hr-ft°F)

line. All angles ALPHA(J) are measured from the vertical, as was shown in Figure 2.2.

For all the variable film coefficients FILMP(J), FILMR(J), FILMX3(J), and FILMX4(J), a separate value is specified for each convective boundary point. The total number of values specified in each case is thus determined by the total number of mesh points on the respective surfaces. The sequence for specifying the various coefficients is as follows: FILMR(J) starts at $\phi = 0$ and proceeds clockwise around D_1; FILMP(J) starts at $\alpha = 0$ and proceeds clockwise around D_2; FILMX3(J) starts at (-A,D) and ends at (+A,D); FILMX4(J) starts at (-A,-D) and ends at (+A,-D). The inter-cable conduction path is merely skipped over in specifying FILMR(J) and FILMP(J).

The variable IPARAM specifies whether Region IV or Region VI of D_2 is present. It is convenient to use IPARAM = 1 for the cradled configuration and IPARAM = 2 for the equilateral configuration, even though those boundary conditions have not been implicitly programmed. For the open and equilateral-pipe configurations the choice is arbitrary, because the strict correspondence between regions and boundary conditions then no longer applies.

Of the 11 variables M(J), only ten are specified as input. The omitted region of D_2 is skipped over, and the program subsequently assigns a value of zero subdivisions for the region not present.

Of the three regional angles ϕ in D_1, it is only necessary to specify PHI(3). PHI(1) and PHI(2) are determined automatically from ALPHA(1), ALPHA(2), and the outer radii of the two cables.

It is noted that the variables REST1 and WD1 describe only half a cable. So if Cable 1 and Cable 2 had identical properties and losses, REST1 and WD1 would be exactly half of REST2 and WD2, respectively. The variables XI1, YC1, and YS1 are not affected by this distinction.

Should it be desired to compute the total dielectric loss per unit length W_d rather than to specify it directly, the following integrated-out form is available [12]:

$$W_d = \frac{V_{\ell\ell}^2}{3} \frac{\omega(7.354)(10^{-12})(SIC)(df)}{\log_{10}\left(\frac{D}{d}\right)} \frac{\text{watts}}{\text{conductor-ft}}, \qquad (E.1)$$

where

$V_{\ell\ell}$ = line-to-line voltage (volts)

ω = $2\pi f$ = frequency (Hz)

(SIC) = specific inductive capacitance, or relative dielectric constant

(df) = dissipation factor

d = inner radius of insulation

D = outer radius of insulation.

The conductor and sheath losses are computed according to

$$W_c = I^2 R Y_c \tag{E.2}$$

and

$$W_s = I^2 R (Y_s - Y_c) \ , \tag{E.3}$$

respectively, where R is the DC resistance per unit length of the conductor, Y_c is the AC/DC ratio at the conductor, and Y_s is the AC/DC ratio at the sheath.

Output Variables

A listing and brief description of the output variables from the computer program are presented in Table 9.

In the computer printout six values of ALPHA(J) are written rather than five. However, two of the six are always equal, reflecting that one of the regions in D_2 (either Region IV or Region VI) has an included angle of zero degrees.

Because of the matrix scaling method employed in the program, it is expected that the determinant of the coefficient matrix will never attain an unwieldy order of magnitude. The variable DETERM is nevertheless printed out so that its magnitude may be monitored for each problem. The user need not be concerned with this variable so long as it lies in the general range 10^{-50} to 10^{+50}. However, if it takes on values significantly outside this range, another matrix scaling procedure may be required in order to avoid an underflow or overflow. If the value of DETERM is ever

TABLE 9

OUTPUT VARIABLES FROM THE COMPUTER PROGRAM

Variable Name		Description and Units ( )
ALPHA(J)	=	the regional angles of D_2, as per Table 8. (degrees)
DETERM	=	the determinant of the coefficient matrix. (unitless)
IERR	=	0, 1, or 2. This is a condition code from the matrix inversion subroutine. IERR = 0 denotes no difficulties encountered in the inversion. IERR = 1 denotes that the matrix is not dimensioned correctly or that the subroutine is not called correctly. IERR = 2 denotes a singular matrix. (unitless)
M(J)	=	various numbers of subdivisions, as per Table 7. (unitless)
N(J)	=	various numbers of subdivisions, as per Table 7. (unitless)
PHI(J)	=	the regional angles of D_1, as per Table 8. (degrees)
TAMAX1	=	the maximum (conductor) temperature from the one-dimensional solution, based on the properties of Cable 1 — Solutions 1 and 3 only. (°F)
TAMAX2	=	the maximum (conductor) temperature from the one-dimensional solution, based on the properties of Cable 2 — Solutions 1 and 3 only. (°F)
TANAL1(J1)	=	the analytical temperature distribution from the one-dimensional solution, based on the properties of Cable 1. (°F)

TABLE 9

(Continued)

Variable Name		Description and Units ( )
TANAL2(J2)	=	the analytical temperature distribution from the one-dimensional solution, based on the properties of Cable 2. (°F)
TCON1I	=	the conductor temperature of Cable 1 — Solution 2 only. (°F)
TCON2I	=	the conductor temperature of Cable 2 — Solution 2 only. (°F)
TCOND1	=	the conductor temperature of Cable 1 — Solutions 1 and 3 only. (°F)
TCOND2	=	the conductor temperature of Cable 2 — Solutions 1 and 3 only. (°F)
THETA(J)	=	the temperature distribution for the entire solution domain — Solutions 1 and 3 only. (°F)
THETAI(J)	=	the temperature distribution for the entire solution domain — Solution 2 only. (°F)
TOIL	=	the maximum allowable oil temperature — Solution 3 only. (°F)
XANAV1	=	the maximum allowable current from the one-dimensional solution, based on the properties of Cable 1 — Solution 2 only. (k-amps)
XANAV2	=	the maximum allowable current from the one-dimensional solution, based on the properties of Cable 2 — Solution 2 only. (k-amps)

TABLE 9

(Continued)

Variable Name	Description and Units ( )
XI1MAX =	the maximum allowable current in Cable 1 — Solution 2 only. (k-amps)
XI2MAX =	the maximum allowable current in Cable 2 — Solution 2 only. (k-amps)

identically zero, the matrix is then singular. Provided no underflow has occurred, the probable cause is either that the matrix has been dimensioned incorrectly, or that the calling statement for RMINV is not correct.

The printout of the M(J) includes all 11 values, with a null value inserted for the region not present.

All three regional angles PHI(J) of D_1 are printed out.

The analytical temperatures TANAL1(J1) and TANAL2(J2) are printed out for each radial mesh point in D_1 and D_2, respectively. They are written sequentially, moving radially outward; the first temperature in each sequence is the conductor temperature.

The complete temperature distributions THETA(J) and THETAI(J) are printed out in the following sequence: starting with the mesh point nearest to the origin of coordinates, all temperatures in D_1 are written, the azimuthal index moving through its entire range for each increment of the radial index; the identical procedure is then followed for all temperatures in D_2; finally, all temperatures in D_3 are printed, beginning in the lower left-hand corner of the domain (-x,-y) and moving through the entire range of the normal index for each increment of the tangential index.

Array Dimensions

A number of subscripted variables, or arrays, are used in the computer program. These arrays and the variables which determine the size are listed in Table 10, together with their dimensions in the present program. For brevity the following computer variables have been used in the table:

$$M14 = M(1) + M(2) + M(3) + M(4) \; ; \qquad (E.4)$$

$$M5210 = M(5) + M(2) + M(6) + M(7) + M(8) + M(9) + M(10) \; ; \qquad (E.5)$$

$$NM3 = N(1) \times [M14 + 1] + N(2) \times M5210 + [M(2) + 1] \times [M(11) - 1] \; . \qquad (E.6)$$

Arrays or portions of arrays which have no variable dimension listed in the table have been permanently dimensioned at their present size.

Data Card Assembly

Instructions for assembling data cards for the computer program are listed in Table 11. While most of this table is self-explanatory, a few additional remarks are offered here.

Attention is called to the integer variables N(J) and M(J), which employ the I-format for their input. It is necessary that all these entries be right-justified to their respective columns.

Values of the variable film coefficients FILMR(J), FILMP(J), FILMX3(J), and FILMX4(J) begin on card 9, as the table indicates. The total number of cards required for these variables depends on the mesh size chosen for the

TABLE 10

ARRAY DIMENSIONS

Name of Array	Variable Dimension(s)	Present Dimension(s)
ALFNT(J)	—	J = 7
ALFSQ(J)	—	J = 7
ALFTN(J)	—	J = 7
ALPHA(J)	—	J = 6
C1FRAC(J)	J = M14 + 2	J = 20
C2FRAC(J)	J = M5210	J = 36
COEFF(J,J)	J = NM3	J = 168
FACTOR(J)	J = NM3	J = 168
FILMP(J)	J = M5210 - M(2) - 1	J = 33
FILMR(J)	J = M14 - M(2)	J = 16
FILMX3(J)	J = M(11) - 1	J = 1
FILMX4(J)	J = M(11) - 1	J = 1
IWORK(J,K)	J = NM3	J = 168
	—	K = 2
M(J)	—	J = 11
N(J)	—	J = 2
P(J)	J = N(2) + 1	J = 5
P1(J)	J = N(2) + 1	J = 5
P2(J)	J = N(2) + 1	J = 5
P1HX(J)	J = M(2) + 1	J = 3
P2HX(J)	J = M(2) + 1	J = 3
P3HX(J)	J = M(2) + 1	J = 3
P4HX(J)	J = M(2) + 1	J = 3
PALF(J,K)	J = N(2) + 1	J = 5
	—	K = 7
PALFNT(J,K)	J = N(2) + 1	J = 5
	—	K = 7
PHI(J)	—	J = 3
PHINT(J)	—	J = 3
PHISQ(J)	—	J = 4
PHITN(J)	—	J = 3

TABLE 10
(Continued)

Name of Array	Variable Dimension(s)	Present Dimension(s)
PMID(J)	J = N(2)	J = 4
R(J)	J = N(1) + 1	J = 5
R1(J)	J = N(1) + 1	J = 5
R2(J)	J = N(1) + 1	J = 5
R1FRAC(J)	J = N(1) + 1	J = 5
R2FRAC(J)	J = N(2) + 1	J = 5
R1HX(J)	J = M(2) + 1	J = 3
R2HX(J)	J = M(2) + 1	J = 3
R3HX(J)	J = M(2) + 1	J = 3
R4HX(J)	J = M(2) + 1	J = 3
RATIO1(J)	J = N(1) + 1	J = 5
RATIO2(J)	J = N(2) + 1	J = 5
RMID(J)	J = N(1)	J = 4
RPHI(J,K)	J = N(1) + 1	J = 5
	—	K = 4
RPHINT(J,K)	J = N(1) + 1	J = 5
	—	K = 3
TANAL1(J)	J = N(1) + 1	J = 5
TANAL2(J)	J = N(2) + 1	J = 5
THET1(J)	J = NM3	J = 168
THET2(J)	J = NM3	J = 168
THET3(J)	J = NM3	J = 168
THET4(J)	J = NM3	J = 168
THETA(J)	J = NM3	J = 168
THETAD(J)	J = NM3	J = 168
THETAI(J)	J = NM3	J = 168
VECTR1(J)	J = NM3	J = 168
VECTR2(J)	J = NM3	J = 168
VECTR3(J)	J = NM3	J = 168
VECTR4(J)	J = NM3	J = 168
XHALF(J)	—	J = 7
XHPHI(J)	—	J = 4

TABLE 10

(Continued)

Name of Array	Variable Dimension(s)	Present Dimension(s)
XHX(J)	J = M(2) + 1	J = 3
XHXHY(J)	J = M(2) + 1	J = 3
XHXSQ(J)	J = M(2) + 1	J = 3
XIVAR(J)	J = NM3	J = 168
XM(J)	—	J = 11
XN(J)	—	J = 2

TABLE 11

DATA CARD ASSEMBLY

Card(s)	Column(s)	Variable	Format
1	1	IPROB	I
	2	IPARAM	I
2	1 - 10	SKID	F
	11 - 20	RAD1	F
	21 - 30	RAD2	F
	31 - 40	RHO1	F
	41 - 50	RHO2	F
	51 - 60	XK	F
	61 - 70	REST1	F
	71 - 80	REST2	F
3	1 - 10	YC1	F
	11 - 20	YC2	F
	21 - 30	YS1	F
	31 - 40	YS2	F
	41 - 50	WD1	F
	51 - 60	WD2	F
	61 - 70	XHFILM	F
4, Solution 1	1 - 10	XI1	F
	11 - 20	XI2	F
	21 - 30	TOIL	F
4, Solution 2	1 - 10	TMAX	F
	11 - 20	XI2OI1	F
	21 - 30	TOIL	F
4, Solution 3	1 - 10	XI1	F
	11 - 20	XI2	F
	21 - 30	TMAX	F

TABLE 11

(Continued)

Card(s)	Column(s)	Variable	Format
5	1 - 10	ALPHA (1)	F
	11 - 20	ALPHA (2)	F
	21 - 30	ALPHA (3)	F
	31 - 40	ALPHA (4)	F
	41 - 50	ALPHA (5)	F
6	1 - 10	PHI (3)	F
7	1 - 5	N (1)	I
	6 - 10	N (2)	I
8	1 - 5	M (1)	I
	6 - 10	M (2)	I
	11 - 15	M (3)	I
	16 - 20	M (4)	I
	21 - 25	M (5)	I
	26 - 30	M (6)	I
	31 - 35	M (7)or(8)	I
	36 - 40	M (8)or(9)	I
	41 - 45	M (10)	I
	46 - 50	M (11)	I
9-(10)	-	FILMR (J)	F
(11-13)	-	FLIMP (J)	F
(14)	-	FLIMX3(J)	F
(15)	-	FLIMX4(J)	F
(16)	1 - 10	ICUT AP	I
	11 - 20	XKCU1	F
	21 - 30	XKCU2	F
	31 - 40	THICK1	F
	41 - 50	THICK2	F

particular problem. The individual coefficients are entered sequentially across each data card, with each value occupying ten columns. When all the values of a particular variable have been specified, the next variable begins on a new data card. The numbers in parentheses in the Card(s)-column are typical for common mesh sizes.

If it is desired to run more than one problem at a time, data decks may be assembled in series. Each separate deck should be arranged according to Table 11.

A final data card having a zero in columns one and two must always be placed at the end of the overall data deck. This double-zero card tells the program that there is no more data to be transmitted.

Example Problem

This section illustrates the solution of a particular cable problem using the computer program. For the example problem it is desired to know the maximum allowable oil temperature for an equilateral-pipe configuration of System 1 cables, given their current and the maximum allowable system temperature. In particular let the current in each cable be 942 amperes, and say that the maximum allowable system temperature is 185°F. The thermal conductivity of the insulation is taken to be 0.1153 Btu/hr-ft-°F, and the film coefficient on convective surfaces is taken as 5.0 Btu/hr-ft^2-°F. Also the conservative assumption of a thermally nonconducting conduit is made. A complete set of input data for this problem is listed in Table 12, and the resulting discrete model is depicted in Figure E.2.

A number of observations are made about the discrete model used in this problem. The effective included angle in either cable associated with the inter-cable conduction path is seen to be 30°, a slightly conservative angle. An insulated arc of this size is centered about the point of cable-conduit contact which, from elementary geometrical calculations, is found to occur at $\alpha = 220°$. Also it is seen that there is no particular association between regions and boundary conditions: Region III of D_2 is primarily concerned with the cable-cable effect, while Region V is involved with the cable-conduit interaction. This breakdown of convention is necessary in modelling the equilateral-pipe configuration, because in that configuration there are too many separate effects operating around D_2 for the available number of regions. (It is noted, however, that the thermal model is equally effective.) The angle PHI(3) = 170° is likewise arbitrary in this problem, since there is no cable-conduit contact on the surface of D_1. The radial mesh size of four subdivisions has been found from experience to be sufficiently fine to produce an accurate solution; use of a finer radial mesh does not significantly alter the temperature distribution. It is finally noted that the azimuthal distribution of mesh points adjacent to insulated regions follows the convention outlined in the first section of this appendix; such a

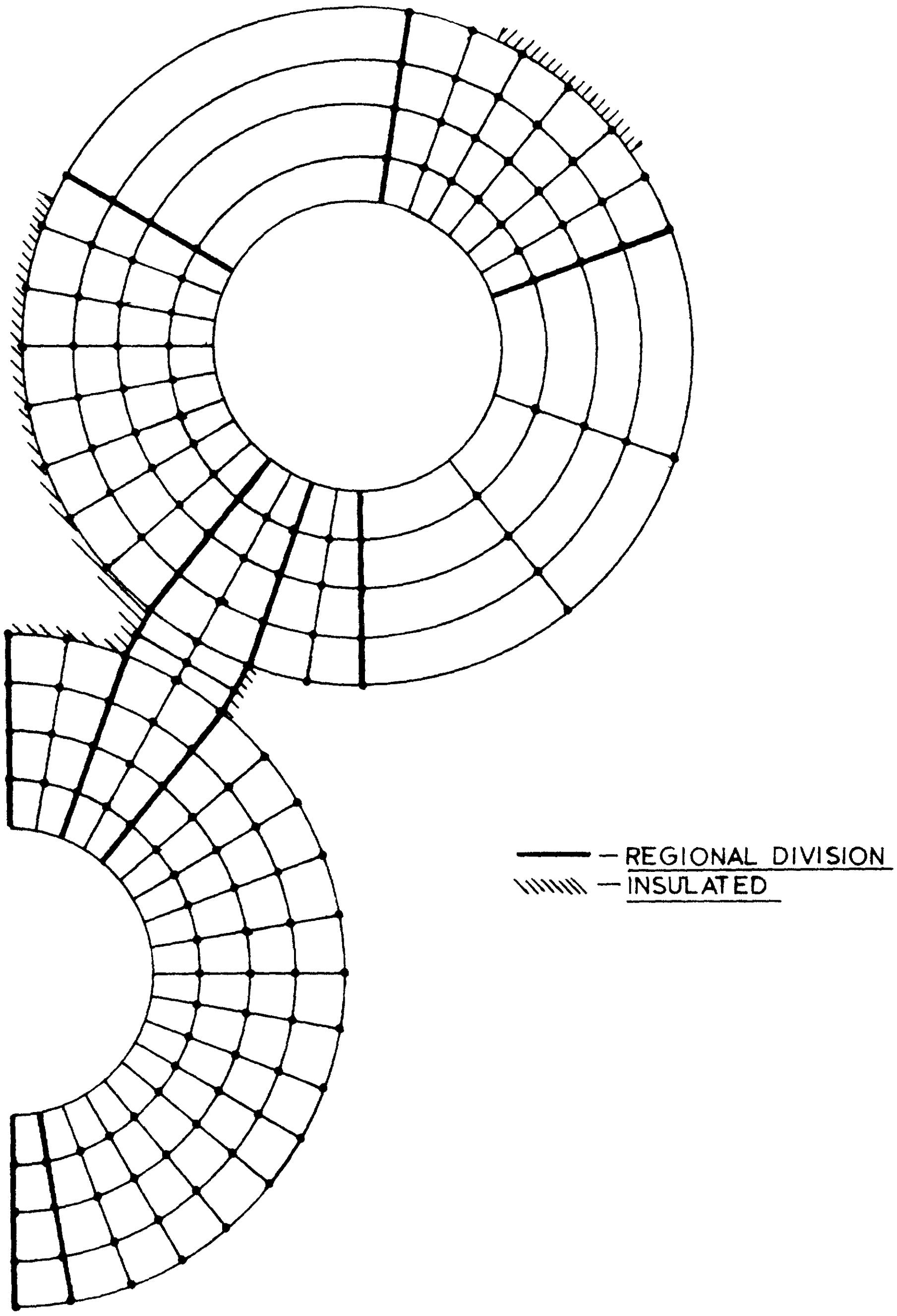

FIGURE E.2

A Discrete Model of the Equilateral-Pipe Configuration

TABLE 12

INPUT DATA FOR EXAMPLE PROBLEM

Card	Column(s)	Data
1	1	3
	2	2
2	1 - 10	0.10
	11 - 20	0.9125
	21 - 30	2.0675
	31 - 40	0.9125
	41 - 50	2.0675
	51 - 60	0.1153
	61 - 70	2.68
	71 - 80	5.36
3	1 - 10	1.19
	11 - 20	1.19
	21 - 30	1.24
	31 - 40	1.24
	41 - 50	1.59
	51 - 60	3.18
	61 - 70	5.0
4	1 - 10	0.942
	11 - 20	0.942
	21 - 30	185.0
5	1 - 10	20.0
	11 - 20	40.0
	21 - 30	120.0
	31 - 40	190.0
	41 - 50	250.0
6	1 - 10	170.0

TABLE 12
(Continued)

Card	Column(s)	Data
7	5	4
	10	4
8	5	2
	10	2
	14,15	13
	20	1
	25	2
	30	8
	35	1
	40	6
	45	3
	50	2
9	1 - 10	0.0
	11 - 20	0.0
	21 - 30	5.0
	31 - 40	5.0
	41 - 50	5.0
	51 - 60	5.0
	61 - 70	5.0
	71 - 80	5.0
10	1 - 10	5.0
	11 - 20	5.0
	21 - 30	5.0
	31 - 40	5.0
	41 - 50	5.0
	51 - 60	5.0
	61 - 70	5.0
	71 - 80	5.0

TABLE 12
(Continued)

Card	Column(s)	Data
11	1 - 10	5.0
	11 - 20	5.0
	21 - 30	0.0
	31 - 40	0.0
	41 - 50	0.0
	51 - 60	0.0
	61 - 70	0.0
	71 - 80	0.0
12	1 - 10	5.0
	11 - 20	5.0
	21 - 30	5.0
	31 - 40	5.0
	41 - 50	0.0
	51 - 60	0.0
	61 - 70	0.0
	71 - 80	5.0
13	1 - 10	5.0
	11 - 20	5.0
	21 - 30	5.0
14	1 - 10	0.0
15	1 - 10	0.0
16	5	0
17	1,2	00

distribution should produce an accurate numerical expression for the gradient at each conductor.

The computer solution for this problem is shown in Figure E.3, where the desired oil temperature 140.3°F is printed. The present version of the program requires approximately 220 K of core memory for execution. It requires 310 K of core memory for compilation on the FORTRAN IV G1-compiler; it is too large to permit optimization on the FORTRAN IV H-compiler.

Capabilities and Limitations of the Computer Program

The present computer program has two notable capabilities which have not yet been specifically mentioned. The first is that Cables 1 and 2 need not be the same size. In the case of unequal cable radii, the included angle associated with the inter-cable conduction path is still designated by $(\alpha_2-\alpha_1)$; the angle $(\phi_2-\phi_1)$ is then automatically adjusted so as to equalize the lengths of the $D_1 - D_3$ and $D_2 - D_3$ interfaces. The second capability not yet mentioned is that there is no restriction to alternating current; either one or both of the two cables may carry direct current. This is handled by merely setting to zero the appropriate AC/DC ratios and dielectric losses. In addition to these two features, it is noted that, while only certain orientations of the two cables are physically realizable, the computer program permits arbitrary configurations. Finally, attention is called to two automatic tests which will facilitate the location of certain input

FIGURE E.3

Computer Solution Printout for Example Problem

```
PHI(J)    =    20.000    40.000   170.000
ALPHA(J)  =    20.000    40.000   120.000   190.000   250.000   250.000
N(J) =     4     4
M(J) =     2     2    13     1     2     8     1     6     0     3     2

DETERM =                                  -0.0000000000000253212855

IERR =0

TOIL (DEG F) =    140.300

THETA(J) (DEG F) =   181.995   181.746   180.989   179.718   178.023   176.270   174.867   173.895
                     173.275   172.858   172.674   172.544   172.469   172.426   172.402   172.388
                     172.381   172.377   172.376   181.108   180.680   179.341   176.918   173.272
                     169.452   166.807   165.208   164.286   163.762   163.466   163.298   163.204
                     163.150   163.120   163.103   163.094   163.089   163.088   180.709   180.198
                     178.572   175.359   168.925   161.890   158.368   156.666   155.807   155.355
                     155.110   154.975   154.900   154.857   154.833   154.820   154.813   154.810
                     154.808   180.740   180.235   178.739   176.165   165.264   152.204   149.293
                     148.284   147.829   147.597   147.474   147.406   147.368   147.347   147.336
                     147.329   147.325   147.324   147.323   175.998   177.525   179.382   181.154
                     182.476   183.254   183.491   183.198   182.377   181.040   179.274   177.327
                     175.628   174.883   175.630   176.365   176.675   176.403   175.710   175.008
                     174.084   174.224   167.221   170.148   174.163   177.948   180.465   181.850
                     182.253   181.727   180.240   177.710   174.131   169.935   166.643   165.626
                     167.340   169.351   170.176   169.414   161.481   165.884   164.426   164.605
                     158.301   162.220   169.496   176.104   179.453   181.171   181.648   180.967
                     179.022   175.520   165.962   162.203   157.732   156.854   159.553   164.280
                     165.979   164.346   159.699   157.166   155.709   155.853   149.156   152.349
                     165.617   176.698   179.450   181.137   181.615   180.884   178.788   174.854
                     167.735   152.552   148.842   148.386   150.866   161.946   164.632   162.002
                     150.947   148.576   147.759   147.831   164.972   176.357   178.962

TCOND1 (DEG F) =    182.989
TCOND2 (DEG F) =    185.000

TANAL1(J1) (DEG F) =    173.961   165.937   158.684   152.082   146.024

TANAL2(J2) (DEG F) =    173.961   165.937   158.684   152.082   146.024

TAMAX1 (DEG F) =    173.961
TAMAX2 (DEG F) =    173.961
```

errors. Should the input specify that the maximum allowable system temperature is less than or equal to the oil temperature, an appropriate error statement is then printed, and the program passes to the next problem. A similar procedure is followed if the maximum allowable system temperature is too small for the given dielectric loss.

Two limitations of the computer program are also called to the attention of the user. The first is that there are constraints on the admissible size of D_3. In particular, the thickness of D_3 must be non-zero, and its included angle in D_1, $(\phi_2 - \phi_1)$, must be less than two radians. Secondly, the included angles of the various regions must all be non-zero. Included angles of 2° have been used successfully by the author, but regions smaller than this are not recommended.

Program Modifications

Brief suggestions for effecting modifications in the present computer program are offered in this section. The modifications to be considered are the following: provision for simultaneous use of Regions IV and VI or D_2, and simplification of I/O procedures.

A modification which would permit simultaneous use of both Regions IV and VI of D_2 could be effected without much difficulty. However, before making such a change, consideration should be given to the handling of boundary conditions. In the present program all boundary conditions are specified by means of a variable film coefficient. This was done in order to retain the greatest amount of flexibility in treating boundaries. If it is desired to preserve this feature, then there is no substantial advantage in performing the above modification. For when boundary conditions are specified separately by means of the variable coefficient, regional divisions are important only in varying the azimuthal distribution of mesh points; there need be no correlation between the various regions and specific boundary conditions (as the example problem illustrated). If, on the other hand, it is desired to treat some or all of the boundary conditions implicitly, then the modification under consideration may be necessary. Boundary conditions may be implicitly written into the program by inserting the appropriate values of the various film coefficients directly into the matrix-generation portion of the program. This would be convenient for boundary conditions which never change from problem to problem. For example, an insulated surface might be identified with a particular region (such as D_2, Region IV). Then upon specifying the size and location of that region, the appropriate boundary would be inherently insulated. The number of such permanent kinds of boundaries depends on the particular problems the user elects to solve with the program. This method for handling boundaries, though, would make it necessary to have seven available regions in D_2 for the equilateral-pipe configuration. The modification required for this involves the input format for ALPHA(J) and the matrix-generation statements for Region III through VII of D_2. Provisions for generating the variables associated with all seven regions of D_2 presently exist; certain statements are merely bypassed at IPARAM-type decision branches. It would probably be convenient to introduce a third category, IPARAM = 3, for a new branching criterion. This criterion could then be used in the matrix generator to choose such branches from the (IMARAM = 1)-type and (IPARAM = 2)-type tests so as to move sequentially through all the regions of D_2. The (IPARAM =3)-test could likewise signal a special input format for ALPHA(J), indicating that six rather than five angles are to be read. The program so modified would be most convenient, provided that boundary conditons were treated implicitly.

Finally, some brief suggestions are given for simplifying the I/O procedures of the computer program. Concerning input, two primary areas could be improved: specification of the variable film coefficients and specification of the effective size of D_3. The former is bothersome because so many values must be entered, and because of the need to keep track of all the individual surface mesh points. Implicit treatment of boundary conditions would completely eliminate this inconvenience. If explicit specification is retained, a provision might at least be written to simplify the input. For example, it might be desirable to merely specify a single coefficient and direct that it apply for all the mesh points of a given surface (when appropriate). Or since most of the surface points are convective, it might be more convenient to read in just the non-standard coefficients. Specification of the height of D_3 by means of the apparent angle ($\alpha_2 - \alpha_1$) is akward; much foresight is required in order to end up with the desired effective height. It would be much more convenient to work with ($\alpha_2 - \alpha_1$) as the effective included angle, referring the associated aximuthal adjustments in D_1 and D_2 to the computer. Concerning output, three suggested improvements are mentioned. First of all, in its present version the program prints out very little of the input data. Such procedures as solution identification, error location, and output analysis would be facilitated if more of the original data were written. Secondly, it would be a simple matter to include the conservative approximate solutions in the program. These could be written alongside the one-dimensional solutions, thereby making the upper and lower bounds for system performance immediately available. Lastly, the overall temperature distribution is not very descriptive in its present format. Separating the temperatures in the print-out according to the three domains D_1, D_2, and D_3 would present no problem, and this would help considerably in identifying features of the distribution. Also effective use could be made of plotter routines for illustrating the temperature distribution graphically.

APPENDIX F

LISTING OF THE SOURCE PROGRAM

```
      REAL*4    CCEFF(168,168), CETERM
      INTEGER*4    IWCRK(168,2), IERR
      DIMENSION PHI(3), ALPHA(6), N(2), M(11), XHPHI(4), XHALF(7),
     $XN(2), XM(11), PHISC(4), PHINT(3), PHITN(3), ALFSQ(7), ALFNT(7),
     $ALFTN(7)
      DIMENSION R(5), P(5), XHX(3), R1(5), R2(5), RPHI(5,4),
     $RPHINT(5,3), P1(5), P2(5), PALF(5,7), PALFNT(5,7), XHXSQ(3),
     $XHXHY(3), R1HX(3), R2HX(3), R3HX(3), R4HX(3), P1HX(3), P2HX(3),
     $P3HX(3), P4HX(3), FILMR(16), FILMP(33), FILMX3(1), FILMX4(1)
      DIMENSICN VECTR1(168), VECTR2(168), VECTR3(168), VECTR4(168),
     $THET1(168), THET2(168), THET3(168), THET4(168), THETA(168),
     $THETAD(168), THETAI(168), RMID(4), PMID(4), R1FRAC(5), R2FRAC(5),
     $RATIC1(5), RATIC2(5), XIVAR(168), TANAL1(5), TANAL2(5),
     $C1FRAC(20), C2FRAC(36), FACTCR(168)
    1 READ (5,36) IPRCB, IPARAM
      IF(IPROB.EQ.0) GO TO 2350
      READ (5,2) SKIC, RAD1, RAD2, RHO1, RHO2, XK, REST1, REST2,
     $YC1, YC2, YS1, YS2, WD1, WD2, XHFILM
    2 FORMAT ((8F10.5))
      YS1=YS1-YC1
      YS2=YS2-YC2
      YC1=YC1-1.0
      YC2=YC2-1.0
      IF(IPRCB.EQ.1) READ (5,2) XI1, XI2, TCIL
      IF(IPRCB.EQ.2) GC TC 3
      IF(IPROB.EQ.3) GO TO 4
      GC TC 5
    3 READ (5,2) TMAX, XI2OI1, TCIL
      IF(TMAX.LE.TOIL) GO TO 2321
      XIO1=1.0
      XIO2=XI2CI1
      GO TO 5
    4 READ (5,2) XI1, XI2, TMAX
      TOIL=140.0
    5 CONTINUE
      CON1=1.0
```

```
      CCN2=1.0
      PI=3.1415926
      RAD1=RAC1/12.
      RAC2=RAC2/12.
      RHC1=RHC1/12.
      RHO2=RHO2/12.
      REAC (5,6) (ALPHA(J),J=1,5)
    6 FORMAT (5F10.5)
      IF(IPARAM.EQ.1) GC TC 7
      IF(IPARAM.EQ.2) GO TO 9
    7 DC 8 J=3,5
      K=9-J
      KM1=K-1
    8 ALPHA(K)=ALPHA(KM1)
      GC TC 10
    9 ALPHA(6)=ALPHA(5)
   10 CONTINUE
   11 READ(5,12) PHI(3)
   12 FORMAT (F10.5)
      PHI(1)=ALPHA(1)/2.*(1.+RHC2/[illegible]2)+ALPHA(2)/2.*(1.-RHO2/RAD2)
      PHI(2)=ALPHA(1)/2.*(1.-RHC2/[illegible]2)+ALPHA(2)/2.*(1.+RHO2/RAD2)
      WRITE (6,13) PHI, ALPHA
   13 FORMAT ('1',4X,'PHI(J)   =',[illegible].3/5X,'ALPHA(J) =',6F1C.3)
      DC 15 I=1,3
   15 PHI(I)=PHI(I)*PI/180.
      DO 20 I=1,6
   20 ALPHA(I)=ALPHA(I)*PI/180.
      READ (5,21) N
   21 FORMAT (2I5)
      REAC (5,22) (M(J),J=1,10)
   22 FCRMAT (10I5)
      IF(IPARAM.EQ.1) GC TO 23
      IF(IPARAM.EQ.2) GO TO 25
   23 DC 24 J=7,10
      K=18-J
      KM1=K-1
```

```
24   M(K)=M(KM1)
     M(7)=0
     GO TO 27
25   DO 26 J=9,10
     K=20-J
     KM1=K-1
26   M(K)=M(KM1)
     M(9)=0
27   CONTINUE
28   DO 30 I=1,2
30   XN(I)=N(I)
     DO 35 I=1,11
35   XM(I)=M(I)
36   FORMAT (2I1)
     NSUB1=N(1)
     NSUB2=N(2)
     MSUB1=M(1)
     MSUB5=M(5)
     M12=M(1)+M(2)
     M13=M12+M(3)
     M14=M13+M(4)
     M52=M(5)+M(2)
     M526=M52+M(6)
     M527=M526+M(7)
     M528=M527+M(8)
     M529=M528+M(9)
     M5210=M529+M(10)
     N1M1=N(1)-1
     N1M2=N(1)-2
     N1P1=N(1)+1
     N2M1=N(2)-1
     N2M2=N(2)-2
     N2P1=N(2)+1
     M1P1=M(1)+1
     M1P2=M(1)+2
     M1P3=M(1)+3
```

```
M2P1=M(2)+1
M2P2=M(2)+2
M2P3=M(2)+3
M12P1=M12+1
M12P2=M12+2
M12P3=M12+3
M12P4=M12+4
M13P1=M13+1
M13P2=M13+2
M13P3=M13+3
M14P1=M14+1
M14P2=M14+2
M14P3=M14+3
M5P1=M(5)+1
M5P2=M(5)+2
M52P1=M52+1
M52P2=M52+2
M56P1=M526+1
M56P2=M526+2
M57P1=M527+1
M57P2=M527+2
M58P1=M528+1
M58P2=M528+2
M59P1=M529+1
M59P2=M529+2
M510M1=M5210-1
M2M1=M(2)-1
M3M1=M(3)-1
M5M1=M(5)-1
M6M1=M(6)-1
M7M1=M(7)-1
M8M1=M(8)-1
M9M1=M(9)-1
M10M1=M(10)-1
M11M1=M(11)-1
M11M2=M(11)-2
```

```
      M11M3=M(11)-3
      M2O2P2=M(2)/2+2
      M2C2P3=M(2)/2+3
      NM1=NSUB1*M14P1
      NM2=NM1+NSUB2*M521C
      NM3=NM2+M2P1*M11M1
      MFILM1=M14-M(2)
      MFILM2=M5210-M2P1
      A=SKID/24.0
      B=RAD2*(PHI(2)-PHI(1))/2.
      D=RAD2*SIN(B/RAD2)
      C=RAD2*(1.-COS(B/RAD2))
C     PHI(2)-PHI(1) MUST BE < 2. RADIANS.
      XHRAD=(1.-RAD1/RAD2)/XN(1)
      XHRHO=(1.-RHO1/RHO2)/XN(2)
      XHPHI(1)=PHI(1)/XM(1)
      DO 40 I=2,3
      J=I-1
   40 XHPHI(I)=(PHI(I)-PHI(J))/XM(I)
      XHPHI(4)=(PI-PHI(3))/XM(4)
      XHALF(1)=ALPHA(1)/XM(5)
      XHALF(2)=(ALPHA(2)-ALPHA(1))/XM(2)
      IF (IPARAM.EQ.1) GO TO 41
      IF (IPARAM.EQ.2) GO TO 44
   41 DO 42 I=3,6
      IF (I.EQ.4) GO TO 42
      J=I-1
      K=I+3
      XHALF(I)=(ALPHA(I)-ALPHA(J))/XM(K)
   42 CONTINUE
      XHALF(4)=0.0
      GO TO 46
   44 DO 45 I=3,5
      J=I-1
      K=I+3
   45 XHALF(I)=(ALPHA(I)-ALPHA(J))/XM(K)
```

```
      XHALF(6)=C.C
 46   CONTINUE
      XHALF(7)=(2.*PI-ALPHA(6))/XM(1C)
      XHXC=(1.+C/A)/XM(11)
      DELHX=XHXC/(1.+A/C)*2./XM(2)
      CO 50 J=2,M2O2P2
      K=J-2
      AK=K
      JM1=J-1
 50   XHX(JM1)=XHXC-AK*DELHX
      DC 55 J=M2C2P3,M2P2
      K=M2P3-J
      JM1=J-1
 55   XHX(JM1)=XHX(K)
      XHY=(1.+B/D)/XM(2)
      XHMEAN=(PHI(2)-PHI(1)+1.+B/D)/(2.*XM(2))
      DO 60 J=1,N1P1
      K=J-1
      AK=K
 60   R(J)=RAD1/RAD2+AK*XHRAD
      DC 62 J=1,NSUB1
 62   RMID(J)=R(J)+XHRAD/2.
      DO 65 J=1,N2P1
      K=J-1
      AK=K
 65   P(J)=RHO1/RHO2+AK*XHRHO
      DC 67 J=1,NSUB2
 67   PMID(J)=P(J)+XHRHC/2.
      READ (5,2) (FILMR(J),J=1,MFILM1)
      REAC (5,2) (FILMP(J),J=1,MFILM2)
      READ (5,2) (FILMX3(J),J=1,M11M1)
      READ (5,2) (FILMX4(J),J=1,M11M1)
      WRITE (6,115) N, M
115   FORMAT (5X,'N(J) =',2I5/5X,'M(J) =',11I5)
      DO 170 J1=1,N1P1
170   R1(J1)=R(J1)/XHRAD*(R(J1)/XHRAD-0.50)
```

```
      DO 180 I=1,4
      DO 175 J1=1,N1P1
175   RPHI(J1,I)=-2.*(R(J1)**2/XHRAD**2+1./XHPHI(I)**2)
180   CONTINUE
      DO 185 J1=1,N1P1
185   R2(J1)=R(J1)/XHRAD*(R(J1)/XHRAD+0.50)
      DO 190 I=1,4
190   PHISQ(I)=1./XHPHI(I)**2
      DO 200 I=2,4
      IM1=I-1
      DO 195 J1=1,N1P1
195   RPHINT(J1,IM1)=-2.*(R(J1)**2/XHRAD**2+1./(XHPHI(IM1)*XHPHI(I)))
200   CONTINUE
      DO 205 I=2,4
      IM1=I-1
205   PHINT(IM1)=2./(XHPHI(IM1)*(XHPHI(IM1)+XHPHI(I)))
      DO 210 I=2,4
      IM1=I-1
210   PHITN(IM1)=2./(XHPHI(I)*(XHPHI(IM1)+XHPHI(I)))
      DO 215 J2=1,N2P1
215   P1(J2)=P(J2)/XHRHO*(P(J2)/XHRHO-0.50)
      IF (IPARAM.EQ.1) GO TO 216
      IF (IPARAM.EQ.2) GO TO 220
216   DO 218 I=1,7
      IF (I.EQ.4) GO TO 218
      DO 217 J2=1,N2P1
217   PALF(J2,I)=-2.*(P(J2)**2/XHRHO**2+1./XHALF(I)**2)
218   CONTINUE
      DO 219 J2=1,N2P1
219   PALF(J2,4)=0.0
      GO TO 224
220   DO 222 I=1,7
      IF (I.EQ.6) GO TO 222
      DO 221 J2=1,N2P1
221   PALF(J2,I)=-2.*(P(J2)**2/XHRHO**2+1./XHALF(I)**2)
222   CONTINUE
```

```
      DC 223 J2=1,N2P1
223   PALF(J2,6)=0.0
224   CONTINLE
      CO 230 J2=1,N2P1
230   P2(J2)=P(J2)/XHRHO*(P(J2)/XHRHO+0.50)
      IF (IPARAM.EQ.1) GC TC 231
      IF (IPARAM.EQ.2) GO TO 233
231   DC 232 I=1,7
      IF (I.EQ.4) GC TC 232
      ALFSQ(I)=1./XHALF(I)**2
232   CONTINUE
      ALFSQ(4)=0.0
      GO TO 235
233   CO 234 I=1,7
      IF (I.EQ.6) GO TO 234
      ALFSQ(I)=1./XHALF(I)**2
234   CONTINUE
      ALFSQ(6)=0.0
235   CCNTINUE
      IF (IPARAM.EQ.1) GO TC 236
      IF (IPARAM.EQ.2) GO TO 241
236   DC 238 I=2,7
      IF (I.EQ.4) GO TC 238
      IF (I.EQ.5) GO TO 238
      IM1=I-1
      DC 237 J2=1,N2P1
237   PALFNT(J2,I)=-2.*(P(J2)**2/XHRHO**2+1./(XHALF(IM1)*XHALF(I)))
238   CCNTINUE
      DC 239 J2=1,N2P1
239   PALFNT(J2,4)=C.C
      DO 240 J2=1,N2P1
240   PALFNT(J2,5)=-2.*(P(J2)**2/XHRHO**2+1./(XHALF(3)*XHALF(5)))
      GO TO 246
241   CO 243 I=2,5
      IM1=I-1
      DC 242 J2=1,N2P1
```

```
242   PALFNT(J2,I)=-2.*(P(J2)**2/XHRHO**2+1./(XHALF(IM1)*XHALF(I)))
243   CONTINUE
      DO 244 J2=1,N2P1
244   PALFNT(J2,6)=0.0
      DO 245 J2=1,N2P1
245   PALFNT(J2,7)=-2.*(P(J2)**2/XHRHO**2+1./(XHALF(5)*XHALF(7)))
246   CONTINUE
      DO 248 J2=1,N2P1
248   PALFNT(J2,1)=-2.*(P(J2)**2/XHRHO**2+1./(XHALF(7)*XHALF(1)))
      IF (IPARAM.EQ.1) GO TO 249
      IF (IPARAM.EQ.2) GO TO 251
249   DO 250 I=2,7
      IF (I.EQ.4) GO TO 250
      IF (I.EQ.5) GO TO 250
      IM1=I-1
      ALFNT(I)=2./(XHALF(IM1)*(XHALF(IM1)+XHALF(I)))
250   CONTINUE
      ALFNT(4)=2./(XHALF(3)*(XHALF(3)+XHALF(5)))
      ALFNT(5)=0.0
      GO TO 253
251   DO 252 I=2,5
      IM1=I-1
252   ALFNT(I)=2./(XHALF(IM1)*(XHALF(IM1)+XHALF(I)))
      ALFNT(6)=2./(XHALF(5)*(XHALF(5)+XHALF(7)))
      ALFNT(7)=0.0
253   CONTINUE
      ALFNT(1)=2./(XHALF(7)*(XHALF(1)+XHALF(7)))
      IF (IPARAM.EQ.1) GO TO 254
      IF (IPARAM.EQ.2) GO TO 256
254   DO 255 I=2,7
      IF (I.EQ.4) GO TO 255
      IF (I.EQ.5) GO TO 255
      IM1=I-1
      ALFTN(I)=2./(XHALF(I)*(XHALF(IM1)+XHALF(I)))
255   CONTINUE
      ALFTN(4)=0.0
```

```
      ALFTN(5)=2./(XHALF(5)*(XHALF(3)+XHALF(5)))
      GO TO 258
  256 DO 257 I=2,5
      IM1=I-1
  257 ALFTN(I)=2./(XHALF(I)*(XHALF(IM1)+XHALF(I)))
      ALFTN(6)=0.0
      ALFTN(7)=2./(XHALF(7)*(XHALF(5)+XHALF(7)))
  258 CONTINUE
      ALFTN(1)=2./(XHALF(1)*(XHALF(1)+XHALF(7)))
  259 DO 260 K3=2,M2O2P2
      K3M1=K3-1
  260 XHXSQ(K3M1)=1./XHX(K3M1)**2
      DO 265 K3=M2O2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
  265 XHXSQ(K3M1)=XHXSQ(I3)
      XHYCON=4.*A**2/(C**2*XHY**2)
      DO 270 K3=2,M2O2P2
      K3M1=K3-1
  270 XHXHY(K3M1)=-2.*(XHXSQ(K3M1)+XHYCON)
      DO 275 K3=M2O2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
  275 XHXHY(K3M1)=XHXHY(I3)
      XMNSQ=1./XHMEAN**2
      DO 280 K3=2,M2O2P2
      K3M1=K3-1
  280 R1HX(K3M1)=(2.*R(N1P1)**2-XHX(K3M1)*R(N1P1))/(XHRAD*
     $(XHRAD+XHX(K3M1)))
      DO 285 K3=M2O2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
  285 R1HX(K3M1)=R1HX(I3)
      DO 290 K3=2,M2O2P2
      K3M1=K3-1
  290 P1HX(K3M1)=(2.*P(N2P1)**2-XHX(K3M1)*P(N2P1))/(XHRHC*
```

```fortran
     $(XHRHO+XHX(K3M1)))
      DO 295 K3=M2C2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
295   P1HX(K3M1)=P1HX(I3)
      DO 300 K3=2,M2C2P2
      K3M1=K3-1
300   R2HX(K3M1)=(R(N1P1)*(XHX(K3M1)-XHRAD)-2.*R(N1P1)**2)/
     $(XHRAD*XHX(K3M1))
      DO 305 K3=M2D2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
305   R2HX(K3M1)=R2HX(I3)
      DO 310 K3=2,M2D2P2
      K3M1=K3-1
310   R3HX(K3M1)=R2HX(K3M1)-2.*XMNSQ
      DO 315 K3=M2D2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
315   R3HX(K3M1)=R3HX(I3)
      DO 320 K3=2,M2D2P2
      K3M1=K3-1
320   P2HX(K3M1)=(P(N2P1)*(XHX(K3M1)-XHRHO)-2.*P(N2P1)**2)/
     $(XHRHO*XHX(K3M1))
      DO 325 K3=M2C2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
325   P2HX(K3M1)=P2HX(I3)
      DO 330 K3=2,M2C2P2
      K3M1=K3-1
330   P3HX(K3M1)=P2HX(K3M1)-2.*XMNSQ
      DO 335 K3=M2C2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
335   P3HX(K3M1)=P3HX(I3)
      DO 340 K3=2,M2C2P2
```

```
      K3M1=K3-1
340   R4HX(K3M1)=(2.*R(N1P1)**2+XHRAD*R(N1P1))/(XHX(K3M1)*
     $(XHRAD+XHX(K3M1)))
      DO 345 K3=M2O2P3,M2P2
      K3M1=K3-1
      I3=M2P3-K3
345   R4HX(K3M1)=R4HX(I3)
      DO 350 K3=2,M2O2P2
      K3M1=K3-1
350   P4HX(K3M1)=(2.*P(N2P1)**2+XHRHO*P(N2P1))/(XHX(K3M1)*
     $(XHRHO+XHX(K3M1)))
      DO 355 K3=M2O2P3,M2P2
      K3M1=K3-1
355   P4HX(K3M1)=P4HX(I3)
      R5HX=R2HX(M2P1)-2./(XHPHI(1)*XHPHI(2))
      R6HX=R2HX(1)-2./(XHPHI(2)*XHPHI(3))
      P5HX=P2HX(1)-2./(XHALF(1)*XHALF(2))
      P6HX=P2HX(M2P1)-2./(XHALF(2)*XHALF(3))
      FLMK=2.*XHRAD*RAD2/XK
      DO 435 J=1,MFILM1
435   FILMR(J)=FLMK*FILMR(J)
      FLMJ=2.*XHRHO*RHO2/XK
      DO 445 J=1,MFILM2
445   FILMP(J)=FLMJ*FILMP(J)
      FLMI=2.*XHY*D/XK
      DO 465 J=1,M11M1
465   FILMX3(J)=FLMI*FILMX3(J)/(1.+FILMX3(J)/XK*D/XM(2))
      DO 470 J=1,M11M1
470   FILMX4(J)=-1.*FLMI*FILMX4(J)/(1.+FILMX4(J)/XK*D/XM(2))
      DO 495 J=1,NM3
      DO 490 K=1,NM3
490   COEFF(J,K)=0.0
495   CONTINUE
      IF (NSUB1.LT.2) GO TO 516
      IF (NSUB1.LT.3) GO TO 506
  496 DO 505 J1=3,NSUB1
```

```
      DC 500 K1=2,M1P1
      ICOL=(J1-3)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
500   CCEFF(IRCW,ICCL)=COEFF(IRCW,ICCL)+R1(J1)
5C5   CCNTINLE
 506  DO 5C7 K1=2,M1P1
      ICCL=N1M2*M14P1+K1-1
      IRCW=N1M1*M14P1+K1-1
 507  COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+R1(N1P1)+R2(N1P1)
      DO 515 J1=2,NSUB1
      DC 510 K1=2,M1P1
      ICOL=(J1-2)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
510   CCEFF(IRCW,ICOL)=COEFF(IROW,ICCL)+RPHI(J1,1)
515   CCNTINLE
516   DO 517 K1=2,M1P1
      K1M1=K1-1
      ICCL=N1M1*M14P1+K1-1
      IROW=N1M1*M14P1+K1-1
 517  CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+RPHI(N1P1,1)-R2(N1P1)*
     $FILMR(K1M1)
      IF (NSLB1.LT.2) GC TC 527
      DO 525 J1=2,NSUB1
      DC 520 K1=2,M1P1
      ICOL=(J1-1)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
520   CCEFF(IRCW,ICCL)=COEFF(IROW,ICCL)+R2(J1)
525   CCNTINLE
527   DO 531 J1=2,N1P1
      ICCL=(J1-2)*M14P1+2
      IRCW=(J1-2)*M14P1+1
 531  COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*PHISQ(1)
      IF (NSUB1.LT.2) GO TO 551
      DC 540 J1=2,N1P1
      DO 535 K1=3,M1P1
      ICOL=(J1-2)*M14P1+K1-2
```

```
      IROW=(J1-2)*M14P1+K1-1
  535 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(1)
  540 CONTINUE
      DO 550 J1=2,N1P1
      DO 545 K1=3,M1P1
      ICOL=(J1-2)*M14P1+K1
      IROW=(J1-2)*M14P1+K1-1
  545 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(1)
  550 CONTINUE
  551 CONTINUE
C     STATEMENTS 496-550 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D1,
C     REGION I.
      IF (NSUB1.LT.2) GO TO 576
      IF (NSUB1.LT.3) GO TO 556
  552 DO 555 J1=3,NSUB1
      ICOL=(J1-3)*M14P1+M1P1
      IROW=(J1-2)*M14P1+M1P1
  555 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(J1)
  556 DO 560 J1=2,NSUB1
      ICOL=(J1-2)*M14P1+M1P1
      IROW=(J1-2)*M14P1+M1P1
  560 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHINT(J1,1)
      DO 565 J1=2,NSUB1
      ICOL=(J1-1)*M14P1+M1P1
      IROW=(J1-2)*M14P1+M1P1
  565 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R2(J1)
      DO 570 J1=2,NSUB1
      ICOL=(J1-2)*M14P1+M(1)
      IROW=(J1-2)*M14P1+M1P1
  570 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHINT(1)
      DO 575 J1=2,NSUB1
      ICOL=(J1-2)*M14P1+M1P2
      IROW=(J1-2)*M14P1+M1P1
  575 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHIIN(1)
  576 CONTINUE
C     STATEMENTS 552-575 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D1,
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C     I-II INTERFACE.
      IF (NSUB1.LT.2) GO TO 626
      IF (NSUB1.LT.3) GO TO 586
  577 DO 585 J1=3,NSUB1
      DO 580 K1=M1P3,M12P1
      ICOL=(J1-3)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
  580 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(J1)
  585 CONTINUE
  586 DO 595 J1=2,NSUB1
      DO 590 K1=M1P3,M12P1
      ICOL=(J1-2)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
  590 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHI(J1,2)
  595 CONTINUE
      DO 605 J1=2,NSUB1
      DO 600 K1=M1P3,M12P1
      ICOL=(J1-1)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
  600 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R2(J1)
  605 CONTINUE
      DO 615 J1=2,NSUB1
      DO 610 K1=M1P3,M12P1
      ICOL=(J1-2)*M14P1+K1-2
      IROW=(J1-2)*M14P1+K1-1
  610 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(2)
  615 CONTINUE
      DO 625 J1=2,NSUB1
      DO 620 K1=M1P3,M12P1
      ICOL=(J1-2)*M14P1+K1
      IROW=(J1-2)*M14P1+K1-1
  620 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(2)
  625 CONTINUE
  626 CONTINUE
C     STATEMENTS 577-625 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D1,
C     REGION II.
```

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      IF (NSLB1.LT.2) GC TC 651
      IF (NSUB1.LT.3) GC TO 631
  627 DC 630 J1=3,NSUB1
      ICOL=(J1-3)*M14P1+M12P1
      IROW=(J1-2)*M14P1+M12P1
 630  CCEFF(IRCW,ICOL)=COEFF(IROW,ICOL)+R1(J1)
  631 DC 635 J1=2,NSUB1
      ICOL=(J1-2)*M14P1+M12P1
      IRCW=(J1-2)*M14P1+M12P1
 635  CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+RPHINT(J1,2)
      DO 640 J1=2,NSLB1
      ICCL=(J1-1)*M14P1+M12P1
      IRCW=(J1-2)*M14P1+M12P1
 64C  COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+R2(J1)
      CO 645 J1=2,NSUB1
      ICCL=(J1-2)*M14P1+M12
      IROW=(J1-2)*M14P1+M12P1
 645  COEFF(IROW,ICOL)=COEFF(IRCW,ICOL)+PHINT(2)
      DC 650 J1=2,NSUB1
      ICOL=(J1-2)*M14P1+M12P2
      IROW=(J1-2)*M14P1+M12P1
 650  CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+PHITN(2)
 651  CCNTINUE
C     STATEMENTS 627-650 GENERATE ELEMENTS CF THE CCEFF MATRIX FCR D1,
C     II-III INTERFACE.
      IF (M13P1.LT.M12P3) GC TC 706
      IF (NSLB1.LT.2) GC TC 671
      IF (NSUB1.LT.3) GO TO 661
  652 DC 660 J1=3,NSUB1
      DO 655 K1=M12P3,M13P1
      ICOL=(J1-3)*M14P1+K1-1
      IRCW=(J1-2)*M14P1+K1-1
 655  CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+R1(J1)
 66C  CONTINUE
  661 DO 662 K1=M12P3,M13P1
      ICCL=N1M2*M14P1+K1-1
```

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      IROW=N1M1*M14P1+K1-1
  662 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(N1P1)+R2(N1P1)
      DO 670 J1=2,NSUB1
      DO 665 K1=M12P3,M13P1
      ICOL=(J1-2)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
  665 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHI(J1,3)
  670 CONTINUE
  671 DO 672 K1=M12P3,M13P1
      J=K1-M2P2
      ICOL=N1M1*M14P1+K1-1
      IROW=N1M1*M14P1+K1-1
  672 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHI(N1P1,3)-R2(N1P1)*FILMR(J)
      IF (NSUB1.LT.2) GO TO 682
      DO 680 J1=2,NSUB1
      DO 675 K1=M12P3,M13P1
      ICOL=(J1-1)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
  675 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R2(J1)
  680 CONTINUE
  682 DO 695 J1=2,N1P1
      DO 690 K1=M12P3,M13P1
      ICOL=(J1-2)*M14P1+K1-2
      IROW=(J1-2)*M14P1+K1-1
  690 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(3)
  695 CONTINUE
      DO 705 J1=2,N1P1
      DO 700 K1=M12P3,M13P1
      ICOL=(J1-2)*M14P1+K1
      IROW=(J1-2)*M14P1+K1-1
  700 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(3)
  705 CONTINUE
  706 CONTINUE
C     STATEMENTS 652-705 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D1,
C     REGION III.
      IF (NSUB1.LT.2) GO TO 716
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      IF (NSUB1.LT.3) GO TO 711
  707 DO 710 J1=3,NSUB1
      ICOL=(J1-3)*M14P1+M13P1
      IROW=(J1-2)*M14P1+M13P1
  710 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(J1)
  711 DO 712 J1=N1P1,N1P1
      ICOL=(J1-3)*M14P1+M13P1
      IROW=(J1-2)*M14P1+M13P1
  712 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(N1P1)+R2(N1P1)
      DO 715 J1=2,NSUB1
      ICOL=(J1-2)*M14P1+M13P1
      IROW=(J1-2)*M14P1+M13P1
  715 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHINT(J1,3)
  716 DO 717 K1=M13P2,M13P2
      J=K1-M2P2
      ICOL=N1M1*M14P1+K1-1
      IROW=N1M1*M14P1+K1-1
  717 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHINT(N1P1,3)-R2(N1P1)*FILMR(J)
      IF (NSUB1.LT.2) GO TO 722
      DO 720 J1=2,NSUB1
      ICOL=(J1-1)*M14P1+M13P1
      IROW=(J1-2)*M14P1+M13P1
  720 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R2(J1)
  722 DO 730 J1=2,N1P1
      ICOL=(J1-2)*M14P1+M13
      IROW=(J1-2)*M14P1+M13P1
  730 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHINT(3)
      DO 735 J1=2,N1P1
      ICOL=(J1-2)*M14P1+M13P2
      IROW=(J1-2)*M14P1+M13P1
  735 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHITN(3)
C     STATEMENTS 707-735 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D1,
C     III-IV INTERFACE.
      IF (NSUB1.LT.2) GO TO 756
      IF (NSUB1.LT.3) GO TO 746
  736 DO 745 J1=3,NSUB1
```

```
      DO 740 K1=M13P3,M14P2
      ICOL=(J1-3)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
740   COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(J1)
745   CONTINUE
 746  DO 747 K1=M13P3,M14P2
      ICOL=N1M2*M14P1+K1-1
      IROW=N1M1*M14P1+K1-1
 747  COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R1(N1P1)+R2(N1P1)
      DO 755 J1=2,NSUB1
      DO 750 K1=M13P3,M14P2
      ICOL=(J1-2)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
750   COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHI(J1,4)
755   CONTINUE
756   DO 757 K1=M13P3,M14P2
      J=K1-M2P2
      ICOL=N1M1*M14P1+K1-1
      IROW=N1M1*M14P1+K1-1
757   COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+RPHI(N1P1,4)-R2(N1P1)*FILMR(J)
      IF (NSUB1.LT.2) GO TO 767
      DO 765 J1=2,NSUB1
      DO 760 K1=M13P3,M14P2
      ICOL=(J1-1)*M14P1+K1-1
      IROW=(J1-2)*M14P1+K1-1
760   COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R2(J1)
765   CONTINUE
767   CONTINUE
      IF (M14P1.LT.M13P3) GO TO 790
      DO 780 J1=2,N1P1
      DO 775 K1=M13P3,M14P1
      ICOL=(J1-2)*M14P1+K1-2
      IROW=(J1-2)*M14P1+K1-1
775   COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(4)
780   CONTINUE
      DO 789 J1=2,N1P1
```

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      DO 785 K1=M13P3,M14P1
      ICOL=(J1-2)*M14P1+K1
      IROW=(J1-2)*M14P1+K1-1
  785 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHISQ(4)
  789 CONTINUE
  790 DO 791 J1=2,N1P1
      ICOL=(J1-2)*M14P1+M14
      IROW=(J1-1)*M14P1
  791 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*PHISQ(4)
C     STATEMENTS 736-791 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D1,
C     REGION IV.
      IF (MSUB5.LT.2) GO TO 846
      IF (NSUB2.LT.2) GO TO 811
      IF (NSUB2.LT.3) GO TO 801
  792 DO 800 J2=3,NSUB2
      DO 795 K2=2,MSUB5
      ICOL=(J2-3)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
  795 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
  800 CONTINUE
  801 DO 802 K2=2,MSUB5
      ICOL=N2M2*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
  802 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
      DO 810 J2=2,NSUB2
      DO 805 K2=2,MSUB5
      ICOL=(J2-2)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
  805 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(J2,1)
  810 CONTINUE
  811 DO 812 K2=2,MSUB5
      ICOL=N2M1*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
  812 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(N2P1,1)-P2(N2P1)*FILMP(K2)
      IF (NSUB2.LT.2) GO TO 822
      DO 820 J2=2,NSUB2
```

```
      DC 815 K2=2,MSUB5
      ICCL=(J2-1)*M521C+K2+NM1
      IROW=(J2-2)*M521C+K2+NM1
 815  CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+P2(J2)
 82C  CCNTINLE
 822  DO 835 J2=2,N2P1
      DC 830 K2=2,MSUB5
      ICCL=(J2-2)*M5210+K2+NM1-1
      IROW=(J2-2)*M5210+K2+NM1
 830  COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(1)
 835  CONTINUE
      DO 845 J2=2,N2P1
      DC 840 K2=2,MSUB5
      ICCL=(J2-2)*M5210+K2+NM1+1
      IROW=(J2-2)*M521C+K2+NM1
 840  CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICOL)+ALFSQ(1)
 845  CCNTINLE
 846  CONTINLE
C     STATEMENTS 792-845 GENERATE ELEMENTS CF THE CCEFF MATRIX FCR D2,
C     REGICN I.
      IF (NSUB2.LT.2) GC TC 871
      IF (NSUB2.LT.3) GO TO 851
  847 DC 850 J2=3,NSUB2
      ICCL=(J2-3)*M5210+M5P1+NM1
      IROW=(J2-2)*M521C+M5P1+NM1
 850  COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
  851 DC 855 J2=2,NSUB2
      ICOL=(J2-2)*M5210+M5P1+NM1
      IRCW=(J2-2)*M5210+M5P1+NM1
 855  COEFF(IROW,ICOL)=CCEFF(IRCW,ICCL)+PALFNT(J2,2)
      DO 860 J2=2,NSUB2
      ICCL=(J2-1)*M5210+M5P1+NM1
      IROW=(J2-2)*M5210+M5P1+NM1
 860  COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
      DO 865 J2=2,NSUB2
      ICOL=(J2-2)*M5210+MSUB5+NM1
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      IRCW=(J2-2)*M5210+M5P1+NM1
  865 COEFF(IROW,ICOL)=COEFF(IRCW,ICOL)+ALFNT(2)
      DO 870 J2=2,NSUB2
      ICOL=(J2-2)*M5210+M5P2+NM1
      IROW=(J2-2)*M5210+M5P1+NM1
  870 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFTN(2)
  871 CONTINUE
C     STATEMENTS 847-870 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     I-II INTERFACE.
      IF (NSUB2.LT.2) GO TO 921
      IF (NSUB2.LT.3) GO TO 881
  872 DO 880 J2=3,NSUB2
      DO 875 K2=M5P2,M52
      ICOL=(J2-3)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
  875 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
  880 CONTINUE
  881 DO 890 J2=2,NSUB2
      DO 885 K2=M5P2,M52
      ICOL=(J2-2)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
  885 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(J2,2)
  890 CONTINUE
      DO 900 J2=2,NSUB2
      DO 895 K2=M5P2,M52
      ICOL=(J2-1)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
  895 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
  900 CONTINUE
      DO 910 J2=2,NSUB2
      DO 905 K2=M5P2,M52
      ICOL=(J2-2)*M5210+K2+NM1-1
      IROW=(J2-2)*M5210+K2+NM1
  905 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSC(2)
  910 CONTINUE
      DO 920 J2=2,NSUB2
```

```
      DO 915 K2=M5P2,M52
      ICOL=(J2-2)*M5210+K2+NM1+1
      IRCW=(J2-2)*M5210+K2+NM1
  915 COEFF(IROW,ICCL)=CCEFF(IROW,ICOL)+ALFSQ(2)
  920 CCNTINUE
  921 CCNTINUE
C     STATEMENTS 872-920 GENERATE ELEMENTS CF THE CCEFF MATRIX FCR D2,
C     REGION II.
      IF (NSUB2.LT.2) GO TO 946
      IF (NSLB2.LT.3) GC TC 926
  922 DO 925 J2=3,NSLB2
      ICCL=(J2-3)*M5210+M52P1+NM1
      IRCW=(J2-2)*M5210+M52P1+NM1
  925 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+P1(J2)
  926 DO 930 J2=2,NSUB2
      ICCL=(J2-2)*M5210+M52P1+NM1
      IROW=(J2-2)*M5210+M52P1+NM1
  930 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALFNT(J2,3)
      DC 935 J2=2,NSUB2
      ICOL=(J2-1)*M5210+M52P1+NM1
      IROW=(J2-2)*M521C+M52P1+NM1
  935 COEFF(IROW,ICCL)=COEFF(IROW,ICOL)+P2(J2)
      DC 940 J2=2,NSUB2
      ICOL=(J2-2)*M521C+M52+NM1
      IROW=(J2-2)*M5210+M52P1+NM1
  940 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFNT(3)
      DO 945 J2=2,NSLB2
      ICOL=(J2-2)*M5210+M52P2+NM1
      IRCW=(J2-2)*M5210+M52P1+NM1
  945 COEFF(IROW,ICCL)=COEFF(IRCW,ICCL)+ALFIN(3)
  946 CONTINUE
C     STATEMENTS 922-945 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     II-III INTERFACE.
      IF (M526.LT.M52P2) GO TO 1CC1
      IF (NSUB2.LT.2) GO TO 966
      IF (NSLB2.LT.3) GC TC 956
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```
947 DO 955 J2=3,NSUB2
    DO 950 K2=M52P2,M526
    ICOL=(J2-3)*M5210+K2+NM1
    IROW=(J2-2)*M5210+K2+NM1
950 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
955 CONTINUE
956 DO 957 K2=M52P2,M526
    ICOL=N2M2*M5210+K2+NM1
    IROW=N2M1*M5210+K2+NM1
957 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
    DO 965 J2=2,NSUB2
    DO 960 K2=M52P2,M526
    ICOL=(J2-2)*M5210+K2+NM1
    IROW=(J2-2)*M5210+K2+NM1
960 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(J2,3)
965 CONTINUE
966 DO 967 K2=M52P2,M526
    J=K2-M2P1
    ICOL=N2M1*M5210+K2+NM1
    IROW=N2M1*M5210+K2+NM1
967 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(N2P1,3)-P2(N2P1)*FILMF(J)
    IF (NSUB2.LT.2) GO TO 977
    DO 975 J2=2,NSUB2
    DO 970 K2=M52P2,M526
    ICOL=(J2-1)*M5210+K2+NM1
    IROW=(J2-2)*M5210+K2+NM1
970 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
975 CONTINUE
977 DO 990 J2=2,N2P1
    DO 985 K2=M52P2,M526
    ICOL=(J2-2)*M5210+K2+NM1-1
    IROW=(J2-2)*M5210+K2+NM1
985 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(3)
990 CONTINUE
    DO 1000 J2=2,N2P1
    DO 995 K2=M52P2,M526
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```
      ICOL=(J2-2)*M521C+K2+NM1+1
      IRCW=(J2-2)*M5210+K2+NM1
  995 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFSQ(3)
 1000 CONTINUE
 1001 CCNTINUE
C     STATEMENTS 947-1000 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     REGION III.
      IF (IPARAM.EQ.1) GO TO 1086
      IF (NSUB2.LT.2) GC TC 1011
      IF (NSUB2.LT.3) GC TO 1006
 1002 DO 1C05 J2=3,NSUB2
      ICCL=(J2-3)*M5210+M56P1+NM1
      IROW=(J2-2)*M5210+M56P1+NM1
 1005 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
 1006 DC 1007 J2=N2P1,N2P1
      ICOL=(J2-3)*M5210+M56P1+NM1
      IROW=(J2-2)*M521C+M56P1+NM1
 1007 CCEFF(IRCW,ICOL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
      DC 1010 J2=2,NSUB2
      ICOL=(J2-2)*M521C+M56P1+NM1
      IROW=(J2-2)*M5210+M56P1+NM1
 1010 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+PALFNT(J2,4)
 1011 DO 1C12 K2=M56P1,M56P1
      J=K2-M2P1
      ICCL=N2M1*M5210+K2+NM1
      IRCW=N2M1*M521C+K2+NM1
 1012 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALFNT(N2P1,4)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GO TO 1017
      DC 1015 J2=2,NSUB2
      ICOL=(J2-1)*M521C+M56P1+NM1
      IROW=(J2-2)*M5210+M56P1+NM1
 1C15 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICOL)+P2(J2)
 1017 DO 1C25 J2=2,N2P1
      ICOL=(J2-2)*M5210+M526+NM1
      IRCW=(J2-2)*M5210+M56P1+NM1
 1025 COEFF(IROW,ICOL)=CCEFF(IRCW,ICCL)+ALFNT(4)
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```
      DO 103C J2=2,N2P1
      ICCL=(J2-2)*M5210+M56P2+NM1
      IROW=(J2-2)*M521C+M56P1+NM1
 1030 CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+ALFIN(4)
C     STATEMENTS 1002-1030 GENERATE ELEMENTS CF THE COEFF MATRIX FOR D2,
C     III-IV INTERFACE.
      IF (M527.LT.M56P2) GO TO 1C86
      IF (NSUB2.LT.2) GC TC 1051
      IF (NSUB2.LT.3) GC TC 1C41
 1031 DO 1040 J2=3,NSUB2
      DC 1035 K2=M56P2,M527
      ICOL=(J2-3)*M5210+K2+NM1
      IROW=(J2-2)*M521C+K2+NM1
 1035 CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+P1(J2)
 1C40 CCNTINLE
 1041 DO 1042 K2=M56P2,M527
      ICCL=N2M2*M5210+K2+NM1
      IRCW=N2M1*M5210+K2+NM1
 1042 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+P1(N2P1)+P2(N2P1)
      DO 1050 J2=2,NSUB2
      DO 1045 K2=M56P2,M527
      ICOL=(J2-2)*M5210+K2+NM1
      IROW=(J2-2)*M521C+K2+NM1
 1C45 CCEFF(IRCW,ICCL)=COEFF(IROW,ICCL)+PALF(J2,4)
 1C50 CCNTINLE
 1051 DO 1052 K2=M56P2,M527
      J=K2-M2P1
      ICCL=N2M1*M521C+K2+NM1
      IROW=N2M1*M521C+K2+NM1
 1052 CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+PALF(N2P1,4)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GC TC 1062
      DO 106C J2=2,NSUB2
      DO 1055 K2=M56P2,M527
      ICCL=(J2-1)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
 1055 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
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 1060 CONTINUE
 1062 DO 1075 J2=2,N2P1
      DO 1070 K2=M56P2,M527
      ICOL=(J2-2)*M5210+K2+NM1-1
      IROW=(J2-2)*M5210+K2+NM1
 1070 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(4)
 1075 CONTINUE
      DO 1085 J2=2,N2P1
      DO 1080 K2=M56P2,M527
      ICOL=(J2-2)*M5210+K2+NM1+1
      IROW=(J2-2)*M5210+K2+NM1
 1080 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(4)
 1085 CONTINUE
 1086 CONTINUE
C     STATEMENTS 1031-1085 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     REGION IV.
      IF (NSUB2.LT.2) GO TO 1096
      IF (NSUB2.LT.3) GO TO 1091
 1087 DO 1090 J2=3,NSUB2
      ICOL=(J2-3)*M5210+M57P1+NM1
      IROW=(J2-2)*M5210+M57P1+NM1
 1090 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
 1091 DO 1092 J2=N2P1,N2P1
      ICOL=(J2-3)*M5210+M57P1+NM1
      IROW=(J2-2)*M5210+M57P1+NM1
 1092 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
      DO 1095 J2=2,NSUB2
      ICOL=(J2-2)*M5210+M57P1+NM1
      IROW=(J2-2)*M5210+M57P1+NM1
 1095 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALFNT(J2,5)
 1096 DO 1097 K2=M57P1,M57P1
      J=K2-M2P1
      ICOL=N2M1*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1097 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALFNT(N2P1,5)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GO TO 1102
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      CC 1100 J2=2,NSUB2
      ICCL=(J2-1)*M5210+M57P1+NM1
      IROW=(J2-2)*M5210+M57P1+NM1
 1100 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
 11C2 CCNTINLE
      IF (IPARAM.EQ.1) GC TC 1111
      CO 111C J2=2,N2P1
      ICCL=(J2-2)*M5210+M527+NM1
      IRCW=(J2-2)*M5210+M57P1+NM1
 1110 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFNT(5)
      GC TC 1113
 1111 DC 1112 J2=2,N2P1
      ICOL=(J2-2)*M521C+M527+NM1
      IRCW=(J2-2)*M5210+M57P1+NM1
 1112 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFNT(4)
 1113 CONTINLE
      CO 1115 J2=2,N2P1
      ICCL=(J2-2)*M5210+M57P2+NM1
      IRCW=(J2-2)*M5210+M57P1+NM1
 1115 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFIN(5)
C     STATEMENTS 1087-1115 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     IV-V INTERFACE.
      IF (M528.LT.M57P2) GC TC 1171
      IF (NSUB2.LT.2) GO TO 1136
      IF (NSLB2.LT.3) GC TC 1126
 1116 DO 1125 J2=3,NSLB2
      CO 112C K2=M57P2,M528
      ICCL=(J2-3)*M5210+K2+NM1
      IRCW=(J2-2)*M521C+K2+NM1
 1120 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
 1125 CCNTINUE
 1126 DC 1127 K2=M57P2,M528
      ICOL=N2M2*M521C+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1127 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+P1(N2P1)+P2(N2P1)
      DO 1135 J2=2,NSLB2
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      DO 1130 K2=M57P2,M528
      ICCL=(J2-2)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
 1130 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(J2,5)
 1135 CCNTINUE
 1136 DC 1137 K2=M57P2,M528
      J=K2-M2P1
      ICOL=N2M1*M5210+K2+NM1
      IRCW=N2M1*M5210+K2+NM1
 1137 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+PALF(N2P1,5)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GO TO 1147
      DC 1145 J2=2,NSUB2
      DO 114C K2=M57P2,M528
      ICOL=(J2-1)*M521C+K2+NM1
      IRCW=(J2-2)*M5210+K2+NM1
 1140 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+P2(J2)
 1145 CONTINUE
 1147 DC 1160 J2=2,N2P1
      DC 1155 K2=M57P2,M528
      ICOL=(J2-2)*M521C+K2+NM1-1
      IROW=(J2-2)*M5210+K2+NM1
 1155 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFSQ(5)
 1160 CONTINUE
      DC 1170 J2=2,N2P1
      DC 1165 K2=M57P2,M528
      ICOL=(J2-2)*M521C+K2+NM1+1
      IROW=(J2-2)*M5210+K2+NM1
 1165 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFSQ(5)
 1170 CONTINUE
 1171 CONTINUE
C     STATEMENTS 1116-1170 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     REGICN V.
      IF (IPARAM.EQ.2) GO TO 1256
      IF (NSUB2.LT.2) GO TO 1181
      IF (NSUB2.LT.3) GC TO 1176
 1172 DO 1175 J2=3,NSUB2
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      ICCL=(J2-3)*M5210+M58P1+NM1
      IROW=(J2-2)*M521C+M58P1+NM1
 1175 CCEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
 1176 DC 1177 J2=N2P1,N2P1
      ICOL=(J2-3)*M5210+M58P1+NM1
      IROW=(J2-2)*M521C+M58P1+NM1
 1177 CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
      DC 118C J2=2,NSUB2
      ICOL=(J2-2)*M521C+M58P1+NM1
      IRCW=(J2-2)*M5210+M58P1+NM1
 1180 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+PALFNT(J2,6)
 1181 DO 1182 K2=M58P1,M58P1
      J=K2-N2P1
      ICCL=N2M1*M5210+K2+NM1
      IROW=N2M1*M521C+K2+NM1
 1182 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALFNT(N2P1,6)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GC TC 1187
      DO 1185 J2=2,NSUB2
      ICOL=(J2-1)*M521C+M58P1+NM1
      IROW=(J2-2)*M521C+M58P1+NM1
 1185 COEFF(IROW,ICCL)=CCEFF(IROW,ICOL)+P2(J2)
 1187 DC 1195 J2=2,N2P1
      ICOL=(J2-2)*M5210+M528+NM1
      IROW=(J2-2)*M5210+M58P1+NM1
 1195 CCEFF(IRCW,ICCL)=COEFF(IRCW,ICOL)+ALFNT(6)
      DO 120C J2=2,N2P1
      ICOL=(J2-2)*M521C+M58P2+NM1
      IRCW=(J2-2)*M5210+M58P1+NM1
 12C0 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFTN(6)
C     STATEMENTS 1172-12CC GENERATE ELEMENTS CF THE CCEFF MATRIX FCR C2,
C     V-VI INTERFACE.
      IF (M529.LT.M58P2) GO TC 1256
      IF (NSUB2.LT.2) GC TC 1221
      IF (NSUB2.LT.3) GO TO 1211
 12C1 DC 1210 J2=3,NSUB2
      DO 1205 K2=M58P2,M529
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      ICOL=(J2-3)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
 1205 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(J2)
 1210 CONTINUE
 1211 DO 1212 K2=M58P2,M529
      ICOL=N2M2*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1212 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
      DO 1220 J2=2,NSUB2
      DO 1215 K2=M58P2,M529
      ICOL=(J2-2)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
 1215 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(J2,6)
 1220 CONTINUE
 1221 DO 1222 K2=M58P2,M529
      J=K2-M2P1
      ICOL=N2M1*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1222 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(N2P1,6)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GO TO 1232
      DO 1230 J2=2,NSUB2
      DO 1225 K2=M58P2,M529
      ICOL=(J2-1)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
 1225 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
 1230 CONTINUE
 1232 DO 1245 J2=2,N2P1
      DO 1240 K2=M58P2,M529
      ICOL=(J2-2)*M5210+K2+NM1-1
      IROW=(J2-2)*M5210+K2+NM1
 1240 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(6)
 1245 CONTINUE
      DO 1255 J2=2,N2P1
      DO 1250 K2=M58P2,M529
      ICOL=(J2-2)*M5210+K2+NM1+1
      IROW=(J2-2)*M5210+K2+NM1
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 1250 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+ALFSC(6)
 1255 CONTINUE
 1256 CCNTINUE
C      STATEMENTS 1201-1255 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C      REGION VI.
      IF (NSUB2.LT.2) GO TO 1266
      IF (NSUB2.LT.3) GC TC 1261
 1257 DO 126C J2=3,NSUB2
      ICOL=(J2-3)*M521C+M59P1+NM1
      IRCW=(J2-2)*M5210+M59P1+NM1
 1260 COEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+P1(J2)
 1261 DO 1262 J2=N2P1,N2P1
      ICCL=(J2-3)*M5210+M59P1+NM1
      IRCW=(J2-2)*M5210+M59P1+NM1
 1262 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+P1(N2P1)+F2(N2P1)
      CO 1265 J2=2,NSUB2
      ICCL=(J2-2)*M5210+M59P1+NM1
      IROW=(J2-2)*M5210+M59P1+NM1
 1265 COEFF(IROW,ICOL)=COFFF(IROW,ICOL)+PALFNT(J2,7)
 1266 DC 1267 K2=M59P1,M59P1
      J=K2-M2P1
      ICOL=N2M1*M521C+K2+NM1
      IRCW=N2M1*M5210+K2+NM1
 1267 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+PALFNT(N2P1,7)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GO TO 1272
      DC 1270 J2=2,NSUB2
      ICCL=(J2-1)*M5210+M59P1+NM1
      IROW=(J2-2)*M521C+M59P1+NM1
 1270 COEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+P2(J2)
 1272 CCNTINUE
      IF (IPARAM.EQ.2) GC TC 1275
      DC 1274 J2=2,N2P1
      ICOL=(J2-2)*M5210+M529+NM1
      IROW=(J2-2)*M521C+M59P1+NM1
 1274 CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+ALFNT(7)
      GC TC 1277
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 1275 DO 1276 J2=2,N2P1
      ICCL=(J2-2)*M5210+M529+NM1
      IRCW=(J2-2)*M5210+M59P1+NM1
 1276 COEFF(IROW,ICCL)=CCEFF(IROW,ICOL)+ALFNT(6)
 1277 CCNTINUE
      IF (M1CM1.LT.1) GC TC 1282
      DO 1281 J2=2,N2P1
      ICOL=(J2-2)*M5210+M59P2+NM1
      IRCW=(J2-2)*M5210+M59P1+NM1
 1281 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+ALFTN(7)
      GO TO 1285
 1282 DO 1283 J2=2,N2P1
      ICCL=(J2-2)*M5210+1+NM1
      IROW=(J2-2)*M5210+M59P1+NM1
 1283 COEFF(IRCW,ICOL)=COEFF(IROW,ICOL)+ALFIN(7)
 1285 CCNTINUE
C     STATEMENTS 1257-1285 GENERATE ELEMENTS CF THE COEFF MATRIX FCR D2,
C     VI-VII INTERFACE.
      IF (M1CM1.LT.1) GC TC 1346
      IF (NSUB2.LT.2) GC TO 1306
      IF (NSUB2.LT.3) GO TO 1296
 1286 DO 1295 J2=3,NSUB2
      DO 1290 K2=M59P2,M5210
      ICOL=(J2-3)*M5210+K2+NM1
      IRCW=(J2-2)*M5210+K2+NM1
 1290 COEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+P1(J2)
 1295 CONTINUE
 1296 DO 1297 K2=M59P2,M5210
      ICCL=N2M2*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1297 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1(N2P1)+P2(N2P1)
      DO 1305 J2=2,NSUB2
      DO 1300 K2=M59P2,M5210
      ICOL=(J2-2)*M5210+K2+NM1
      IRCW=(J2-2)*M5210+K2+NM1
 1300 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+PALF(J2,7)
```

```
 1305 CONTINUE
 1306 DO 1307 K2=M59P2,M5210
      J=K2-M2P1
      ICOL=N2M1*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1307 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALF(N2P1,7)-P2(N2P1)*FILMP(J)
      IF (NSUB2.LT.2) GO TO 1317
      DO 1315 J2=2,NSUB2
      DO 1310 K2=M59P2,M5210
      ICOL=(J2-1)*M5210+K2+NM1
      IROW=(J2-2)*M5210+K2+NM1
 1310 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P2(J2)
 1315 CONTINUE
 1317 DO 1330 J2=2,N2P1
      DO 1325 K2=M59P2,M5210
      ICOL=(J2-2)*M5210+K2+NM1-1
      IROW=(J2-2)*M5210+K2+NM1
 1325 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(7)
 1330 CONTINUE
      IF (M510M1.LT.M59P2) GO TO 1342
 1331 DO 1340 J2=2,N2P1
      DO 1335 K2=M59P2,M510M1
      ICOL=(J2-2)*M5210+K2+NM1+1
      IROW=(J2-2)*M5210+K2+NM1
 1335 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(7)
 1340 CONTINUE
C     NOTE THAT IN STATEMENTS 1331-1340, K2 DOES NOT TAKE ON THE VALUE
C     M5210.  THIS IS DUE TO THE INDICIAL DISCONTINUITY AT THE VII-I
C     INTERFACE.
 1342 DO 1345 J2=2,N2P1
      ICOL=(J2-2)*M5210+1+NM1
      IROW=(J2-1)*M5210+NM1
 1345 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFSQ(7)
 1346 CONTINUE
C     STATEMENTS 1286-1346 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D2,
C     REGION VII.
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      IF (NSUB2.LT.2) GO TO 1356
      IF (NSUB2.LT.3) GC TO 1351
 1347 DC 135C J2=3,NSUB2
      ICOL=(J2-3)*M521C+1+NM1
      IRCW=(J2-2)*M5210+1+NM1
 1350 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+P1(J2)
 1351 DO 1352 J2=N2P1,N2P1
      ICOL=(J2-3)*M5210+1+NM1
      IRCW=(J2-2)*M5210+1+NM1
 1352 COEFF(IROW,ICOL)=CCEFF(IRCW,ICCL)+P1(N2P1)+P2(N2P1)
      CO 1355 J2=2,NSUB2
      ICCL=(J2-2)*M5210+1+NM1
      IRCW=(J2-2)*M521C+1+NM1
 1355 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PALFNT(J2,1)
 1356 DC 1357 J2=N2P1,N2P1
      ICCL=(J2-2)*M5210+1+NM1
      IROW=(J2-2)*M521C+1+NM1
 1357 COEFF(IROW,ICCL)=COEFF(IROW,ICOL)+PALFNT(N2P1,1)-P2(N2P1)*FILMP(1)
      IF (NSUB2.LT.2) GC TC 1362
      DO 136C J2=2,NSUB2
      ICOL=(J2-1)*M521C+1+NM1
      IRCW=(J2-2)*M5210+1+NM1
 1360 CCEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+P2(J2)
 1362 DO 1365 J2=2,N2P1
      ICCL=(J2-1)*M5210+NM1
      IRCW=(J2-2)*M5210+1+NM1
 1365 COEFF(IROW,ICCL)=CCEFF(IRCW,ICOL)+ALFNT(1)
      DO 1370 J2=2,N2P1
      ICCL=(J2-2)*M5210+2+NM1
      IROW=(J2-2)*M5210+1+NM1
 1370 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFTN(1)
C     STATEMENTS 1347-1370 GENERATE ELEMENTS CF THE COEFF MATRIX FOR D2,
C     VII-I INTERFACE.
      IF (M11M2.LT.2) GO TO 1378
 1371 DC 1377 K3=2,M2P2
      K3M1=K3-1
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```
      DO 1375 J3=2,M11M2
      ICCL=(K3-2)*M11M1+J3+NM2-1
      IRCW=(K3-2)*M11M1+J3+NM2
 1375 COEFF(IROW,ICCL)=CCEFF(IROW,ICOL)+XHXSQ(K3M1)
 1377 CONTINUE
      GC TC 1380
 1378 DO 1379 K3=2,M2P2
      K3M1=K3-1
      K1=M12P4-K3
      ICCL=N1M1*M14P1+K1-1
      IROW=(K3-2)*M11M1+1+NM2
 1379 CCEFF(IROW,ICCL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
 1380 CCNTINUE
      IF (M11M2.LT.2) GO TO 1383
      DO 1382 J3=2,M11M2
      ICCL=J3+NM2
      IROW=J3+NM2
 1382 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(1)+XHYCON*FILMX4(J3)
      GC TC 1385
 1383 DO 1384 J3=1,1
      ICOL=J3+NM2
      IRCW=J3+NM2
 1384 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+XFXFY(1)+XHYCON*FILMX4(J3)
 1385 CONTINUE
      IF (M11M2.LT.2) GO TO 1388
      DC 1387 K3=3,M2P1
      K3M1=K3-1
      DO 1386 J3=2,M11M2
      ICCL=(K3-2)*M11M1+J3+NM2
      IROW=(K3-2)*M11M1+J3+NM2
 1386 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(K3M1)
 1387 CCNTINUE
      GO TO 1390
 1388 DO 1389 K3=3,M2P1
      K3M1=K3-1
      ICCL=(K3-2)*M11M1+1+NM2
```

```
      IROW=(K3-2)*M11M1+1+NM2
 1389 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(K3M1)
 1390 CONTINUE
      IF (M11M2.LT.2) GO TO 1392
      DO 1391 J3=2,M11M2
      ICOL=M(2)*M11M1+J3+NM2
      IROW=M(2)*M11M1+J3+NM2
 1391 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(M2P1)-XHYCON*FILMX3(J3)
      GO TO 1394
 1392 DO 1393 J3=1,1
      ICOL=M(2)*M11M1+1+NM2
      IROW=M(2)*M11M1+1+NM2
 1393 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(M2P1)-XHYCON*FILMX3(J3)
 1394 CONTINUE
      IF (M11M2.LT.2) GO TO 1401
      DO 1400 K3=2,M2P2
      K3M1=K3-1
      DO 1395 J3=2,M11M2
      ICOL=(K3-2)*M11M1+J3+NM2+1
      IROW=(K3-2)*M11M1+J3+NM2
 1395 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
 1400 CONTINUE
      GO TO 1403
 1401 DO 1402 K3=2,M2P2
      K3M1=K3-1
      K2=K3+M5M1
      ICOL=N2M1*M5210+K2+NM1
      IROW=(K3-2)*M11M1+1+NM2
 1402 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
 1403 CONTINUE
      IF (M11M2.LT.2) GO TO 1407
      DO 1406 K3=3,M2P1
      DO 1405 J3=2,M11M2
      ICOL=(K3-3)*M11M1+J3+NM2
      IROW=(K3-2)*M11M1+J3+NM2
 1405 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCON
```

```
 1406 CONTINUE
      GO TO 1409
 1407 DO 1408 K3=3,M2P1
      ICOL=(K3-3)*M11M1+1+NM2
      IROW=(K3-2)*M11M1+1+NM2
 1408 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCON
 1409 CONTINUE
      IF (M11M2.LT.2) GO TO 1411
      DO 1410 J3=2,M11M2
      ICOL=M2M1*M11M1+J3+NM2
      IROW=M(2)*M11M1+J3+NM2
 1410 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*XHYCON
      GO TO 1413
 1411 DO 1412 J3=1,1
      ICOL=M2M1*M11M1+1+NM2
      IROW=M(2)*M11M1+1+NM2
 1412 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*XHYCON
 1413 CONTINUE
      IF (M11M2.LT.2) GO TO 1415
      DO 1414 J3=2,M11M2
      ICOL=M11M1+J3+NM2
      IROW=J3+NM2
 1414 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*XHYCON
      GO TO 1417
 1415 DO 1416 J3=1,1
      ICOL=M11M1+1+NM2
      IROW=1+NM2
 1416 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*XHYCON
 1417 CONTINUE
      IF (M11M2.LT.2) GO TO 1421
      DO 1420 K3=3,M2P1
      DO 1419 J3=2,M11M2
      ICOL=(K3-1)*M11M1+J3+NM2
      IROW=(K3-2)*M11M1+J3+NM2
 1419 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCON
 1420 CONTINUE
```

```
1369       GO TO 1423
1370  1421 DO 1422 K3=3,M2P1
1371       ICOL=(K3-1)*M11M1+1+NM2
1372       IROW=(K3-2)*M11M1+1+NM2
1373  1422 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCON
1374  1423 CONTINUE
1375 C     STATEMENTS 1371-1423 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D3,
1376 C     INTERIOR POINTS.
1377       IF (M11M2.LT.2) GO TO 1471
1378  1424 DO 1425 K3=2,M2P2
1379       K3M1=K3-1
1380       K1=M12P4-K3
1381       ICOL=N1M1*M14P1+K1-1
1382       IROW=(K3-2)*M11M1+1+NM2
1383  1425 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
1384       DO 1427 K3=2,2
1385       ICOL=(K3-2)*M11M1+1+NM2
1386       IROW=(K3-2)*M11M1+1+NM2
1387  1427 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(1)+XHYCON*FILMX4(1)
1388       DO 1430 K3=3,M2P1
1389       K3M1=K3-1
1390       ICOL=(K3-2)*M11M1+1+NM2
1391       IROW=(K3-2)*M11M1+1+NM2
1392  1430 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(K3M1)
1393       DO 1432 K3=M2P2,M2P2
1394       ICOL=(K3-2)*M11M1+1+NM2
1395       IROW=(K3-2)*M11M1+1+NM2
1396  1432 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(M2P1)-XHYCON*FILMX3(1)
1397       DO 1435 K3=2,M2P2
1398       K3M1=K3-1
1399       ICOL=(K3-2)*M11M1+2+NM2
1400       IROW=(K3-2)*M11M1+1+NM2
1401  1435 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
1402       DO 1440 K3=3,M2P1
1403       ICOL=(K3-3)*M11M1+1+NM2
1404       IROW=(K3-2)*M11M1+1+NM2
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 1440 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCON
      DO 1442 K3=M2P2,M2P2
      ICOL=(K3-3)*M11M1+1+NM2
      IROW=(K3-2)*M11M1+1+NM2
 1442 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*XHYCON
      DO 1443 K3=2,2
      ICOL=(K3-1)*M11M1+1+NM2
      IROW=(K3-2)*M11M1+1+NM2
 1443 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+2.*XHYCON
      DO 1445 K3=3,M2P1
      ICOL=(K3-1)*M11M1+1+NM2
      IROW=(K3-2)*M11M1+1+NM2
 1445 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCON
C     STATEMENTS 1424-1445 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D3,
C     POINTS ADJACENT TO THE D1-D3 INTERFACE.
 1446 DO 1450 K3=2,M2P2
      K3M1=K3-1
      ICOL=(K3-1)*M11M1-1+NM2
      IROW=(K3-1)*M11M1+NM2
 1450 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
      DO 1452 K3=2,2
      ICOL=(K3-1)*M11M1+NM2
      IROW=(K3-1)*M11M1+NM2
 1452 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(1)+XHYCON*FILMX4(M11M1)
      DO 1455 K3=3,M2P1
      K3M1=K3-1
      ICOL=(K3-1)*M11M1+NM2
      IROW=(K3-1)*M11M1+NM2
 1455 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(K3M1)
      DO 1457 K3=M2P2,M2P2
      ICOL=(K3-1)*M11M1+NM2
      IROW=(K3-1)*M11M1+NM2
 1457 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHXHY(M2P1)-XHYCON*FILMX3(M11M1)
      DO 1460 K3=2,M2P2
      K3M1=K3-1
      K2=K3+M5M1
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      ICCL=N2M1*M521C+K2+NM1
      IROW=(K3-1)*M11M1+NM2
 1460 CCEFF(IRGW,ICCL)=COEFF(IROW,ICOL)+XHXSQ(K3M1)
      DC 1465 K3=3,M2P1
      ICOL=(K3-2)*M11M1+NM2
      IROW=(K3-1)*M11M1+NM2
 1465 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+XHYCON
      DO 1467 K3=M2P2,M2P2
      ICOL=(K3-2)*M11M1+NM2
      IRCW=(K3-1)*M11M1+NM2
 1467 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+2.*XHYCCN
      DO 1468 K3=2,2
      ICOL=K3*M11M1+NM2
      IRCW=(K3-1)*M11M1+NM2
 1468 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+2.*XHYCCN
      DO 147C K3=3,M2P1
      ICCL=K3*M11M1+NM2
      IRCW=(K3-1)*M11M1+NM2
 1470 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XHYCCN
 1471 CCNTINUE
C     STATEMENTS 1446-1471 GENERATE ELEMENTS OF THE COEFF MATRIX FOR D3,
C     POINTS ADJACENT TC THE D2-D3 INTERFACE.
      IF (NSUB1.LT.2) GO TO 1477
 1472 DC 1475 K1=M1P3,M12P1
      K3M1=M12P3-K1
      ICOL=N1M2*M14P1+K1-1
      IRCW=N1M1*M14P1+K1-1
 1475 COEFF(IROW,ICOL)=CCEFF(IRCW,ICCL)+R1HX(K3M1)
 1477 DO 1480 K1=M1P3,M12P1
      K3M1=M12P3-K1
      ICOL=N1M1*M14P1+K1-1
      IROW=N1M1*M14P1+K1-1
 1480 CCEFF(IRCW,ICOL)=COEFF(IROW,ICOL)+R3HX(K3M1)
      DC 1485 K1=M1P3,M12P1
      K3=M12P4-K1
      K3M1=M12P3-K1
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      ICOL=(K3-2)*M11M1+1+NM2
      IROW=N1M1*M14P1+K1-1
 1485 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+R4FX(K3M1)
      DO 149C K1=M1P3,M12P1
      ICOL=N1M1*M14P1+K1-2
      IRCW=N1M1*M14P1+K1-1
 1490 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+XMNSG
      CO 1495 K1=M1P3,M12P1
      ICCL=N1M1*M14P1+K1
      IRCW=N1M1*M14P1+K1-1
 1495 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+XMNSG
C     STATEMENTS 1472-1495 CENERATE ELEMENTS OF THE COEFF MATRIX FOR THE
C     D1-D3 INTERFACE, EXCEPT CCRNERS.
      IF (NSUB2.LT.2) GO TO 15C2
 1496 DC 1500 K2=M5P2,M52
      K3M1=K2-M(5)
      ICOL=N2M2*M521C+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1500 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICOL)+P1FX(K3M1)
 1502 DO 1505 K2=M5P2,M52
      K3M1=K2-M(5)
      ICCL=N2M1*M5210+K2+NM1
      IROW=N2M1*M521C+K2+NM1
 1505 COEFF(IROW,ICCL)=COEFF(IROW,ICOL)+P3FX(K3M1)
      DC 1510 K2=M5P2,M52
      K3=K2-M5M1
      K3M1=K3-1
      ICCL=(K3-1)*M11M1+NM2
      IRCW=N2M1*M5210+K2+NM1
 1510 COEFF(IROW,ICOL)=CCEFF(IROW,ICCL)+P4FX(K3M1)
      DO 1515 K2=M5P2,M52
      ICCL=N2M1*M5210+K2+NM1-1
      IROW=N2M1*M521C+K2+NM1
 1515 COEFF(IROW,ICOL)=COEFF(IROW,ICCL)+XMNSG
      DO 1520 K2=M5P2,M52
      ICOL=N2M1*M521C+K2+NM1+1
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      IROW=N2M1*M5210+K2+NM1
 1520 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+XMNSQ
C     STATEMENTS 1496-1520 GENERATE ELEMENTS CF THE CCEFF MATRIX FOR THE
C     C2-C3 INTERFACE, EXCEPT CORNERS.
      IF (NSUBL.LT.2) GO TO 1527
 1521 DC 1525 K1=M1P2,M1P2
      ICOL=N1M2*M14P1+K1-1
      IRCW=N1M1*M14P1+K1-1
 1525 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICOL)+R1HX(M2P1)
 1527 DO 153C K1=M1P2,M1P2
      ICOL=N1M1*M14P1+K1-1
      IRCW=N1M1*M14P1+K1-1
 1530 COEFF(IROW,ICCL)=CCEFF(IRCW,ICCL)+R5HX
      DC 1535 K1=M1P2,M1P2
      K3=M12P4-K1
      ICOL=(K3-2)*M11M1+1+NM2
      IROW=N1M1*M14P1+K1-1
 1535 CCEFF(IRCW,ICCL)=COEFF(IROW,ICOL)+R4HX(M2P1)
      DC 1540 K1=M1P2,M1P2
      ICOL=N1M1*M14P1+K1-2
      IRCW=N1M1*M14P1+K1-1
 1540 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+PHINT(1)
      DO 1545 K1=M1P2,M1P2
      ICOL=N1M1*M14P1+K1
      IRCW=N1M1*M14P1+K1-1
 1545 COEFF(IROW,ICOL)=CCEFF(IROW,ICCL)+PHITN(1)
C     ONE CORNER (UPPER LEFT) HAS NOW BEEN COMPLETED.
      IF (NSUBL.LT.2) GO TO 1552
      DC 155C K1=M12P2,M12P2
      ICOL=N1M2*M14P1+K1-1
      IRCW=N1M1*M14P1+K1-1
 1550 CCEFF(IRCW,ICCL)=CCEFF(IRCW,ICCL)+R1HX(1)
 1552 DO 1555 K1=M12P2,M12P2
      ICOL=N1M1*M14P1+K1-1
      IRCW=N1M1*M14P1+K1-1
 1555 COEFF(IROW,ICOL)=CCEFF(IRCW,ICCL)+R6HX
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      DO 1560 K1=M12P2,M12P2
      K3=M12P4-K1
      ICOL=(K3-2)*M11M1+1+NM2
      IROW=N1M1*M14P1+K1-1
 1560 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+R4HX(1)
      DO 1565 K1=M12P2,M12P2
      ICOL=N1M1*M14P1+K1-2
      IROW=N1M1*M14P1+K1-1
 1565 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHINT(2)
      DO 1570 K1=M12P2,M12P2
      ICOL=N1M1*M14P1+K1
      IROW=N1M1*M14P1+K1-1
 1570 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+PHITN(2)
C     THE LOWER LEFT CORNER HAS NOW BEEN COMPLETED.
C     STATEMENTS 1521-1570 GENERATE ELEMENTS OF THE COEFF MATRIX FOR
C     CORNERS OF THE D1-D3 INTERFACE.
      IF (NSUB2.LT.2) GO TO 1577
 1571 DO 1575 K2=M5P1,M5P1
      ICOL=N2M2*M521C+K2+NM1
      IROW=N2M1*M521C+K2+NM1
 1575 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1HX(1)
 1577 DO 1580 K2=M5P1,M5P1
      ICOL=N2M1*M521C+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1580 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P5HX
      DO 1585 K2=M5P1,M5P1
      K3=K2-M5M1
      ICOL=(K3-1)*M11M1+NM2
      IROW=N2M1*M5210+K2+NM1
 1585 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P4HX(1)
      DO 1590 K2=M5P1,M5P1
      ICOL=N2M1*M521C+K2+NM1-1
      IROW=N2M1*M521C+K2+NM1
 1590 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFNT(2)
      DO 1595 K2=M5P1,M5P1
      ICOL=N2M1*M521C+K2+NM1+1
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      IROW=N2M1*M5210+K2+NM1
 1595 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFTN(2)
C     THE LOWER RIGHT CORNER HAS NOW BEEN COMPLETED.
      IF (NSUB2.LT.2) GO TO 1602
      DO 1600 K2=M52P1,M52P1
      ICOL=N2M2*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1600 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P1HX(M2P1)
 1602 DO 1605 K2=M52P1,M52P1
      ICOL=N2M1*M5210+K2+NM1
      IROW=N2M1*M5210+K2+NM1
 1605 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P6HX
      DO 1610 K2=M52P1,M52P1
      K3=K2-M5M1
      ICOL=(K3-1)*M11M1+NM2
      IROW=N2M1*M5210+K2+NM1
 1610 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+P4HX(M2P1)
      DO 1615 K2=M52P1,M52P1
      ICOL=N2M1*M5210+K2+NM1-1
      IROW=N2M1*M5210+K2+NM1
 1615 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFNT(3)
      DO 1620 K2=M52P1,M52P1
      ICOL=N2M1*M5210+K2+NM1+1
      IROW=N2M1*M5210+K2+NM1
 1620 COEFF(IROW,ICOL)=COEFF(IROW,ICOL)+ALFTN(3)
C     THE UPPER RIGHT CORNER HAS NOW BEEN COMPLETED.
C     STATEMENTS 1571-1620 GENERATE ELEMENTS OF THE COEFF MATRIX FOR
C     CORNERS OF THE D2-D3 INTERFACE.
      READ(5,9000) ICUTAP, XKCU1, XKCU2, THICK1, THICK2
 9000 FORMAT(I5,4F10.4)
      IF(ICUTAP .EQ. 0) GO TO 9140
      THICK1 = THICK1 / 12.
      THICK2 = THICK2 / 12.
      XKEFF1=(XKCU1*THICK1+XK*((RAD2-RAD1)/(2.*XN(1))-THICK1))/((RAD2-
     1RAD1)/(2.*XN(1)))
      XKEFF2=(XKCU2*THICK2+XK*((RHO2-RHO1)/(2.*XN(2))-THICK2))/((RHO2-
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 1RHO1)/(2.*XN(2)))
  FACTR1 = XKEFF1 / XK
  FACTR2 = XKEFF2 / XK
  JA = ( M(1) + M(2) + M(3) + M(4) + 1 ) * ( N(1) - 1 ) + 1
  JNN = M(1) + M(2) + M(3) + M(4) + JA
  JM=JNN+(M(5)+M(2)+M(6)+M(7)+M(8)+M(9)+M(10))*(N(2)-1)+1
  JMM= JM + M(5) + M(2) + M(6) + M(7) + M(8) + M(9) + M(10) - 1
  OLD1=COEFF(JA,JA+1)
  COEFF(JA,JA+1)=OLD1*FACTR1
  COEFF(JA,JA)=COEFF(JA,JA)+OLD1-COEFF(JA,JA+1)
  I1=JA+1
  IL=JNN-1
  DO 6010 IR=I1,IL
  OLD1=COEFF(IR,IR-1)
  OLD2=COEFF(IR,IR+1)
  COEFF(IR,IR-1)=OLD1*FACTR1
  COEFF(IR,IR+1)=OLD2*FACTR1
6010 COEFF(IR,IR)=COEFF(IR,IR)+OLD1+OLD2-COEFF(IR,IR-1)-COEFF(IR,IR+1)
  OLD1=COEFF(JNN,JNN-1)
  COEFF(JNN,JNN-1)=OLD1*FACTR1
  COEFF(JNN,JNN)=COEFF(JNN,JNN)+OLD1-COEFF(JNN,JNN-1)
  OLD1=COEFF(JM,JMM)
  OLD2=COEFF(JM,JM+1)
  COEFF(JM,JMM)=OLD1*FACTR2
  COEFF(JM,JM+1)=OLD2*FACTR2
  COEFF(JM,JM)=COEFF(JM,JM)+OLD1+OLD2-COEFF(JM,JMM)-COEFF(JM,JM+1)
  I1=JM+1
  IL=JMM-1
  DO 6040 IR=I1,IL
  OLD1=COEFF(IR,IR-1)
  OLD2=COEFF(IR,IR+1)
  COEFF(IR,IR-1)=OLD1*FACTR2
  COEFF(IR,IR+1)=OLD2*FACTR2
6040 COEFF(IR,IR)=COEFF(IR,IR)+OLD1+OLD2-COEFF(IR,IR-1)-COEFF(IR,IR+1)
  OLD1=COEFF(JMM,JMM-1)
  OLD2=COEFF(JMM,JM)
```

```
      COEFF(JMM,JMM-1)=OLD1*FACTR2
      COEFF(JMM,JM)=OLD2*FACTR2
      COEFF(JMM,JMM)=COEFF(JMM,JMM)+OLD1+OLD2- COEFF(JMM,JMM-1)-COEFF(JM
     1M,JM)
 9140 CONTINUE
      DO 1625 J=1,NM3
 1625 FACTOR(J)=0.0
      DO 1635 J=1,NM3
      DO 1630 K=1,NM3
 1630 FACTOR(J)=AMAX1(FACTOR(J),ABS(COEFF(J,K)))
 1635 CONTINUE
      DO 1813 J=1,NM3
      DO 1812 K=1,NM3
 1812 COEFF(J,K)=COEFF(J,K)/FACTOR(J)
 1813 CONTINUE
      DO 1822 J=1,NM3
 1820 VECTR1(J)=0.0
      DO 1825 J=1,NM3
 1825 VECTR2(J)=0.0
      IF (NSUB1.LT.2) GO TO 1830
      DO 1827 K1=2,M14P2
      K1M1=K1-1
 1827 VECTR1(K1M1)=VECTR1(K1M1)-CON1*R1(2)
      GO TO 1840
 1830 DO 1832 K1=2,M1P1
      K1M1=K1-1
 1832 VECTR1(K1M1)=VECTR1(K1M1)-CON1*(R1(2)+R2(2))
      DO 1835 K1=M1P2,M12P2
      K1M1=K1-1
      K3M1=M12P3-K1
 1835 VECTR1(K1M1)=VECTR1(K1M1)-CON1*R1HX(K3M1)
      DO 1837 K1=M12P3,M14P2
      K1M1=K1-1
 1837 VECTR1(K1M1)=VECTR1(K1M1)-CON1*(R1(2)+R2(2))
 1840 CONTINUE
      IF (NSUB2.LT.2) GO TO 1845
```

```
      DO 1842 K2=1,M5210
      K2P=K2+NM1
 1842 VECTR2(K2P)=VECTR2(K2P)-CON2*P1(2)
      GO TO 1855
 1845 DO 1847 K2=1,MSUB5
      K2P=K2+NM1
 1847 VECTR2(K2P)=VECTR2(K2P)-CON2*(P1(2)+P2(2))
      DO 1850 K2=M5P1,M52P1
      K2P=K2+NM1
      K3M1=K2-MSUB5
 1850 VECTR2(K2P)=VECTR2(K2P)-CON2*P1HX(K3M1)
      DO 1852 K2=M52P2,M5210
      K2P=K2+NM1
 1852 VECTR2(K2P)=VECTR2(K2P)-CON2*(P1(2)+P2(2))
 1855 CONTINUE
      DO 1856 J=1,NM3
 1856 VECTR1(J)=VECTR1(J)/FACTOR(J)
      DO 1857 J=1,NM3
 1857 VECTR2(J)=VECTR2(J)/FACTOR(J)
      CALL RMINV(260, NM3, COEFF, DETERM, IWORK, IERR)
      DO 1869 J=1,NM3
      THET1(J)=0.0
      DO 1868 K=1,NM3
 1868 THET1(J)=THET1(J)+COEFF(J,K)*VECTR1(K)
 1869 CONTINUE
      DO 1872 J=1,NM3
      THET2(J)=0.0
      DO 1871 K=1,NM3
 1871 THET2(J)=THET2(J)+COEFF(J,K)*VECTR2(K)
 1872 CONTINUE
      WRITE (6,1880) DETERM
 1880 FORMAT (//5X,'DETERM =',F90.45)
      WRITE (6,1890) IERR
 1890 FORMAT (//5X,'IERR =',I1)
      TOTAL1=ALOG(RAD2/RAD1)
      RIFRAC(1)=ALOG(RMID(1)/R(1))/TOTAL1
```

```
      IF(NSUB1.LT.2) GO TO 1945
      DO 1940 J=2,NSUB1
      JM1=J-1
1940  R1FRAC(J)=ALOG(RMID(J)/RMID(JM1))/TOTAL1
1945  R1FRAC(N1P1)=ALOG(1./RMID(NSUB1))/TOTAL1
      TOTAL2=ALOG(RHO2/RHO1)
      R2FRAC(1)=ALOG(PMID(1)/P(1))/TOTAL2
      IF(NSUB2.LT.2) GO TO 1955
      DO 1950 J=2,NSUB2
      JM1=J-1
1950  R2FRAC(J)=ALOG(PMID(J)/PMID(JM1))/TOTAL2
1955  R2FRAC(N2P1)=ALOG(1./PMID(NSUB2))/TOTAL2
      AREA1=PI*(RAD2**2-RAD1**2)/2.
      AREA2=PI*(RHO2**2-RHO1**2)
      DENOM1=R(N1P1)**2-R(1)**2
      DENOM2=P(N2P1)**2-P(1)**2
      XNUM11=RMID(1)**2-R(1)**2
      XNUM12=R(N1P1)**2-RMID(NSUB1)**2
      XNUM21=PMID(1)**2-P(1)**2
      XNUM22=P(N2P1)**2-PMID(NSUB2)**2
      RATIO1(1)=XNUM11/DENOM1
      IF(NSUB1.LT.2) GO TO 1957
      DO 1956 J1=2,NSUB1
      J1M1=J1-1
1956  RATIO1(J1)=(RMID(J1)**2-RMID(J1M1)**2)/DENOM1
1957  RATIO1(N1P1)=XNUM12/DENOM1
      RATIO2(1)=XNUM21/DENOM2
      IF(NSUB2.LT.2) GO TO 1959
      DO 1958 J2=2,NSUB2
      J2M1=J2-1
1958  RATIO2(J2)=(PMID(J2)**2-PMID(J2M1)**2)/DENOM2
1959  RATIO2(N2P1)=XNUM22/DENOM2
      C1FRAC(1)=0.0
      C1FRAC(2)=XHPHI(1)/(2.*PI)
      IF(M1P1.LT.3) GO TO 1965
      DO 1960 K1=3,M1P1
```

```
1960 C1FRAC(K1)=XHPHI(1)/PI
1965 C1FRAC(M1P2)=(XHPHI(1)+XHPHI(2))/(2.*PI)
     DO 1970 K1=M1P3,M12P1
1970 C1FRAC(K1)=XHPHI(2)/PI
     C1FRAC(M12P2)=(XHPHI(2)+XHPHI(3))/(2.*PI
     IF(M13P1.LT.M12P3) GO TO 1980
     DO 1975 K1=M12P3,M13P1
1975 C1FRAC(K1)=XHPHI(3)/PI
1980 C1FRAC(M13P2)=(XHPHI(3)+XHPHI(4))/(2.*PI
     IF(M14P1.LT.M13P3) GO TO 1990
     DO 1985 K1=M13P3,M14P1
1985 C1FRAC(K1)=XHPHI(4)/PI
1990 C1FRAC(M14P2)=XHPHI(4)/(2.*PI)
     C2FRAC(1)=(XHALF(1)+XHALF(7))/(4.*PI)
     IF(MSUB5.LT.2) GO TO 2005
     DO 2000 K2=2,MSUB5
2000 C2FRAC(K2)=XHALF(1)/(2.*PI)
2005 C2FRAC(M5P1)=(XHALF(1)+XHALF(2))/(4.*PI)
     DO 2010 K2=M5P2,M52
2010 C2FRAC(K2)=XHALF(2)/(2.*PI)
     C2FRAC(M52P1)=(XHALF(2)+XHALF(3))/(4.*PI)
     IF(M526.LT.M52P2) GO TO 2017
     DO 2015 K2=M52P2,M526
2015 C2FRAC(K2)=XHALF(3)/(2.*PI)
2017 CONTINUE
     IF(IPARAM.EQ.1) GO TO 2020
     C2FRAC(M56P1)=(XHALF(3)+XHALF(4))/(4.*PI)
     GO TO 2025
2020 C2FRAC(M56P1)=0.0
2025 CONTINUE
     IF(M527.LT.M56P2) GO TO 2035
     DO 2030 K2=M56P2,M527
2030 C2FRAC(K2)=XHALF(4)/(2.*PI)
2035 CONTINUE
     IF(IPARAM.EQ.1) GO TO 2040
     C2FRAC(M57P1)=(XHALF(4)+XHALF(5))/(4.*PI)
```

```
      GO TO 2045
 2040 C2FRAC(M57P1)=(XHALF(3)+XHALF(5))/(4.*PI)
 2045 CONTINUE
      IF(M528.LT.M57P2) GO TO 2055
      DO 2050 K2=M57P2,M528
 2050 C2FRAC(K2)=XHALF(5)/(2.*PI)
 2055 CONTINUE
      IF(IPARAM.EQ.2) GO TO 2060
      C2FRAC(M58P1)=(XHALF(5)+XHALF(6))/(4.*PI)
      GO TO 2065
 2060 C2FRAC(M58P1)=0.0
 2065 CONTINUE
      IF(M529.LT.M58P2) GO TO 2075
      DO 2070 K2=M58P2,M529
 2070 C2FRAC(K2)=XHALF(6)/(2.*PI)
 2075 CONTINUE
      IF(IPARAM.EQ.2) GO TO 2080
      C2FRAC(M59P1)=(XHALF(6)+XHALF(7))/(4.*PI)
      GO TO 2085
 2080 C2FRAC(M59P1)=(XHALF(5)+XHALF(7))/(4.*PI)
 2085 CONTINUE
      IF(M10M1.LT.1) GO TO 2095
      DO 2090 K2=M59P2,M5210
 2090 C2FRAC(K2)=XHALF(7)/(2.*PI)
 2095 CONTINUE
      BIOT1=XK/(XHFILM*RAD2)
      BIOT2=XK/(XHFILM*RHO2)
      BIOTR1=2.*BIOT1-ALOG(R(1))
      BIOTP1=2.*BIOT2-ALOG(P(1))
      BIOTR2=BIOT1-ALOG(R(1))
      BIOTP2=BIOT2-ALOG(P(1))
      DO 2166 J=1,NM3
 2166 VECTR3(J)=0.2
      IF(IPROB.EQ.1) GO TO 2167
      IF(IPROB.EQ.2) GO TO 2184
      IF(IPROB.EQ.3) GO TO 2167
```

```
2167 WC1=(XI1**2*REST1*(1.0+YC1)+WD1*R1FRAC(1'))*3.413
     WC2=(XI2**2*REST2*(1.0+YC2)+WD2*R2FRAC(1  ))*3.413
     WS1=XI1**2*REST1*YS1
     WS2=XI2**2*REST2*YS2
     WC1T=XI1**2*REST1*(1.0+YC1)*3.413
     WC2T=XI2**2*REST2*(1.0+YC2)*3.413
     DICON1=WD1*3.413/(PI*WC1)
     DICON2=WD2*3.413/(2.*PI*WC1)
     DO 2168 J1=1,N1P1
2168 TANAL1(J1)=WC1T*(BIOT1-ALOG(R(J1)))/(PI*WC1)+WS1*3.413*BIOT1
    $/(PI*WC1)+DICON1*(ALOG(R(J1))**2*0.5/ALOG(R(1))-ALOG(R(J1))
    $+BIOT1)
     DO 2169 J2=1,N2P1
2169 TANAL2(J2)=WC2T*(BIOT2-ALOG(P(J2)))/(2.*PI*WC1)+WS2*3.413*BIOT2
    $/(2.*PI*WC1)+DICON2*(ALOG(P(J2))**2*0.5/ALOG(P(1))-ALOG(P(J2))
    $+BIOT2)
     TAMAX1=0.0
     DO 2170 J1=1,N1P1
2170 TANAL1(J1)=TANAL1(J1)*WC1/XK+TOIL
     DO 2171 J1=1,N1P1
2171 TAMAX1=AMAX1(TAMAX1,TANAL1(J1))
     TAMAX2=0.0
     DO 2172 J2=1,N2P1
2172 TANAL2(J2)=TANAL2(J2)*WC1/XK+TOIL
     DO 2173 J2=1,N2P1
2173 TAMAX2=AMAX1(TAMAX2,TANAL2(J2))
     IF(NSUB1.LT.2) GO TO 2177
     DO 2176 J1=2,NSUB1
     DO 2175 K1=2,M14P2
     J=(J1-2)*M14P1+K1-1
2175 VECTR3(J)=VECTR3(J)-(R(J1)*RAD2)**2*WD1*3.413*R1FRAC(J1)/
    $(WC1*AREA1*RATIO1(J1))
2176 CONTINUE
2177 DO 2178 K1=2,M14P2
     J=N1M1*M14P1+K1-1
2178 VECTR3(J)=VECTR3(J)-(R(N1P1)*RAD2)**2*(WD1*R1FRAC(N1P1)+WS1)
```

```
      $*3.413/(WC1*AREA1*RATIO1(N1P1))
       IF(NSUB2.LT.2) GO TO 2181
       DO 2180 J2=2,NSUB2
       DO 2179 K2=1,M5210
       J=(J2-2)*M5210+K2+NM1
 2179  VECTR3(J)=VECTR3(J)-(P(J2)*RHO2)**2*WD2*3.413*R2FRAC(J2)/
      $(WC1*AREA2*RATIO2(J2))
 2180  CONTINUE
 2181  DO 2182 K2=1,M5210
       J=N2M1*M5210+K2+NM1
 2182  VECTR3(J)=VECTR3(J)-(P(N2P1)*RHO2)**2*(WD2*R2FRAC(N2P1)+WS2)
      $*3.413/(WC1*AREA2*RATIO2(N2P1))
       DO 2183 J=1,NM3
 2183  VECTR3(J)=VECTR3(J)/FACTOR(J)
       GO TO 2215
 2184  CONTINUE
       WC1=XI01**2*REST1*(1.0+YC1)*3.413
       WC2=XI02**2*REST2*(1.0+YC2)*3.413
       WC1D=WD1*R1FRAC(1)*3.413
       WC2D=WD2*R2FRAC(1)*3.413
       WS1=XI01**2*REST1*YS1
       WS2=XI02**2*REST2*YS2
       DO 2185 K1=2,M14P2
       J=N1M1*M14P1+K1-1
 2185  VECTR3(J)=VECTR3(J)-(R(N1P1)*RAD2)**2*WS1*3.413/(WC1*AREA1
      $*RATIO1(N1P1))
       DO 2186 K2=1,M5210
       J=N2M1*M5210+K2+NM1
 2186  VECTR3(J)=VECTR3(J)-(P(N2P1)*RHO2)**2*WS2*3.413/(WC1*AREA2
      $*RATIO2(N2P1))
       DO 2187 J=1,NM3
 2187  VECTR3(J)=VECTR3(J)/FACTOR(J)
       DO 2188 J=1,NM3
 2188  VECTR4(J)=0.0
       DO 2190 J1=2,N1P1
       DO 2189 K1=2,M14P2
```

```
      J=(J1-2)*M14P1+K1-1
 2189 VECTR4(J)=VECTR4(J)-(R(J1)*RAD2)**2*WD1*3.413*R1FRAC(J1)/
     $(WC1*AREA1*RATIO1(J1))
 2190 CONTINUE
      DO 2192 J2=2,N2P1
      DO 2191 K2=1,M5210
      J=(J2-2)*M5210+K2+NM1
 2191 VECTR4(J)=VECTR4(J)-(P(J2)*RHO2)**2*WD2*3.413*R2FRAC(J2)/
     $(WC1*AREA2*RATIO2(J2))
 2192 CONTINUE
      DO 2193 J=1,NM3
 2193 VECTR4(J)=VECTR4(J)/FACTOR(J)
      DO 2195 J=1,NM3
      THET4(J)=0.0
      DO 2194 K=1,NM3
 2194 THET4(J)=THET4(J)+COEFF(J,K)*VECTR4(K)
 2195 CONTINUE
      THETMX=(TMAX-TOIL)*XK/WC1
      XANAV1=(THETMX-WD1*3.413*BIOTR1/(2.*PI*WC1))/(BIOTR2/PI
     $+WS1*3.413*BIOT1/(PI*WC1))
      XANAV2=(THETMX-WD2*3.413*BIOTP1/(4.*PI*WC1))/(WC2*BIOTP2/
     $(2.*PI*WC1)+WS2*3.413*BIOT2/(2.*PI*WC1))
      IF(XANAV1.LT.0.0) GO TO 2323
      IF(XANAV2.LT.0.0) GO TO 2323
      DO 2196 J1=1,N1P1
 2196 TANAL1(J1)=WD1*3.413/(PI*WC1)*(ALOG(R(J1))**2*0.5/ALOG(R(1))-
     $ALOG(R(J1))+BIOT1)+XANAV1*BIOT1/PI*(1.-ALOG(R(J1))/BIOT1
     $+WS1*3.413/WC1)
      DO 2197 J1=1,N1P1
 2197 TANAL1(J1)=TANAL1(J1)*WC1/XK+TOIL
      XANAV1=SQRT(XANAV1)
      DO 2198 J2=1,N2P1
 2198 TANAL2(J2)=WD2*3.413/(2.*PI*WC1)*(ALOG(P(J2))**2*0.5/ALOG(P(1))
     $-ALOG(P(J2))+BIOT2)+XANAV2*BIOT2*WC2/(2.*PI*WC1)*(1.-ALOG(P(J2))
     $/BIOT2+WS2*3.413/WC2)
      DO 2199 J2=1,N2P1
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2199 TANAL2(J2)=TANAL2(J2)*WC1/XK+TOIL
     XANAV2=SQRT(XANAV2)
2215 CONTINUE
     DO 2217 J=1,NM3
     THET3(J)=0.0
     DO 2216 K=1,NM3
2216 THET3(J)=THET3(J)+COEFF(J,K)*VECTR3(K)
2217 CONTINUE
     TAVG11=0.0
     DO 2222 K1=2,M14P2
     ICOL=K1-1
2222 TAVG11=TAVG11+THET1(ICOL)*C1FRAC(K1)
     TAVG21=0.0
     DO 2225 K1=2,M14P2
     ICOL=K1-1
2225 TAVG21=TAVG21+THET2(ICOL)*C1FRAC(K1)
     TAVG31=0.0
     DO 2230 K1=2,M14P2
     ICOL=K1-1
2230 TAVG31=TAVG31+THET3(ICOL)*C1FRAC(K1)
     TAVG12=0.0
     DO 2235 K2=1,M5210
     ICOL=K2+NM1
2235 TAVG12=TAVG12+THET1(ICOL)*C2FRAC(K2)
     TAVG22=0.0
     DO 2240 K2=1,M5210
     ICOL=K2+NM1
2240 TAVG22=TAVG22+THET2(ICOL)*C2FRAC(K2)
     TAVG32=0.0
     DO 2245 K2=1,M5210
     ICOL=K2+NM1
2245 TAVG32=TAVG32+THET3(ICOL)*C2FRAC(K2)
2246 RHS1=(ALOG(R(2))-ALOG(R(1)))/PI
     RHS2=(ALOG(P(2))-ALOG(P(1)))*WC2/(2.*PI*WC1)
     C1=((TAVG31+RHS1)*(CON2-TAVG22)+TAVG21*(TAVG32+RHS2))/
    $((CON1-TAVG11)*(CON2-TAVG22)-(TAVG12*TAVG21))
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      C2=(C1*(CON1-TAVG11)-(TAVG31+RHS1))/TAVG21
      DO 2247 J=1,NM3
 2247 THETA(J)=C1*THET1(J)+C2*THET2(J)+THET3(J)
      IF(IPROB.EQ.1) GO TO 2300
      IF(IPROB.EQ.2) GO TO 2248
      IF(IPROB.EQ.3) GO TO 2300
 2248 TAVG41=0.0
      DO 2249 K1=2,M14P2
      ICOL=K1-1
 2249 TAVG41=TAVG41+THET4(ICOL)*C1FRAC(K1)
      TAVG42=0.0
      DO 2250 K2=1,M5210
      ICOL=K2+NM1
 2250 TAVG42=TAVG42+THET4(ICOL)*C2FRAC(K2)
      RHS1D=RHS1*WC1D/WC1
      RHS2D=RHS2*WC2D/WC2
      C1D=((TAVG41+RHS1D)*(CON2-TAVG22)+TAVG21*(TAVG42+RHS2D))/
     $((CON1-TAVG11)*(CON2-TAVG22)-(TAVG12*TAVG21))
      C2D=(C1D*(CON1-TAVG11)-(TAVG41+RHS1D))/TAVG21
 2255 DO 2256 J=1,NM3
 2256 THETAD(J)=C1D*THET1(J)+C2D*THET2(J)+THET4(J)
      DO 2257 J=1,NM3
 2257 XIVAR(J)=(THETMX-THETAD(J))/THETA(J)
      XI1MAX=XIVAR(1)
      DO 2258 J=2,NM3
      XI1MAX=AMIN1(XI1MAX,XIVAR(J))
 2258 CONTINUE
      XIVAR1=(THETMX-CON1*C1D)/(CON1*C1)
      XIVAR2=(THETMX-CON2*C2D)/(CON2*C2)
      XI1MAX=AMIN1(XI1MAX,XIVAR1)
      XI1MAX=AMIN1(XI1MAX,XIVAR2)
      IF(XI1MAX.LT.0.0) GO TO 2323
      DO 2259 J=1,NM3
 2259 THETA(J)=THETA(J)*XI1MAX
      DO 2260 J=1,NM3
 2260 THETAI(J)=THETA(J)+THETAD(J)
```

```
      TCON1I=CON1*(C1*XI1MAX+C1D)
      TCON2I=CON2*(C2*XI1MAX+C2D)
      XI1MAX=SQRT(XI1MAX)
 2288 DO 2289 J=1,NM3
 2289 THETAI(J)=THETAI(J)*WC1/XK+TOIL
      IF(ICUTAP .EQ. 0) GO TO 9003
      WRITE(6,9001)
 9003 CONTINUE
      WRITE (6,2292) (THETAI(J), J=1,NM3)
 2292 FORMAT (///5X,'THETAI(J) (DEG F) =',8F10.3/(24X,8F10.3))
      XI2MAX=XI1MAX*XI2OI1
      TCON1I=TCON1I*WC1/XK+TOIL
      TCON2I=TCON2I*WC1/XK+TOIL
      WRITE (6,2293) XI1MAX, XI2MAX
 2293 FORMAT (///5X,'XI1MAX (K=AMP) =',F10.5/5X,'XI2MAX (K=AMP) =',
     $F10.5)
      WRITE (6,2294) TCON1I, TCON2I
 2294 FORMAT (///5X,'TCON1I (DEG F) =',F10.3/5X,'TCON2I (DEG F) =',
     $F10.3)
      WRITE (6,2311) (TANAL1(J), J=1,N1P1)
      WRITE (6,2312) (TANAL2(J), J=1,N2P1)
      WRITE (6,2297) XANAV1, XANAV2
 2297 FORMAT (///5X,'XANAV1 (K=AMP) =',F10.5/5X,'XANAV2 (K=AMP) =',
     $F10.5)
      GO TO 2335
 2300 CONTINUE
      DO 2301 J=1,NM3
 2301 THETA(J)=THETA(J)*WC1/XK+TOIL
      TCOND1=C1*CON1*WC1/XK+TOIL
      TCOND2=C2*CON2*WC1/XK+TOIL
      IF(IPROB.EQ.1) GO TO 2308
      TEMPMX=0.0
      DO 2302 J=1,NM3
 2302 TEMPMX=AMAX1(TEMPMX, THETA(J))
      TEMPMX=AMAX1(TEMPMX, TCOND1)
      TEMPMX=AMAX1(TEMPMX, TCOND2)
```

```
      DELT=TMAX-TEMPMX
      DO 2303 J=1,NM3
 2303 THETA(J)=THETA(J)+DELT
      TCOND1=TCOND1+DELT
      TCOND2=TCOND2+DELT
      TOIL=TOIL+DELT
      DO 2304 J1=1,N1P1
 2304 TANAL1(J1)=TANAL1(J1)+DELT
 2305 FORMAT (///5X,'THETA(J) (DEG F) =',8F10.3/(23X,8F10.3))
      DO 2306 J2=1,N2P1
 2306 TANAL2(J2)=TANAL2(J2)+DELT
      TAMAX1=TAMAX1+DELT
      TAMAX2=TAMAX2+DELT
      WRITE (6,2307) TOIL
 2307 FORMAT (//5X,'TOIL (DEG F) =',F10.3)
 2308 CONTINUE
      IF(ICUTAP .EQ. 0) GO TO 9002
      WRITE(6,9001)
 9001 FORMAT('+',60X,'**TAPE**')
 9002 CONTINUE
      WRITE (6,2305) (THETA(J), J=1,NM3)
      WRITE (6,2310) TCOND1, TCOND2
 2310 FORMAT (//5X,'TCOND1 (DEG F) =',F10.3/5X,'TCOND2 (DEG F) =',F10.3)
      WRITE (6,2311) (TANAL1(J), J=1,N1P1)
 2311 FORMAT (///5X,'TANAL1(J1) (DEG F) =',8F10.3/(24X,8F10.3))
      WRITE (6,2312) (TANAL2(J), J=1,N2P1)
 2312 FORMAT (///5X,'TANAL2(J2) (DEG F) =',8F10.3/(24X,8F10.3))
      WRITE (6,2313) TAMAX1, TAMAX2
 2313 FORMAT (///5X,'TAMAX1 (DEG F) =',F10.3/5X,'TAMAX2 (DEG F) =',
     $F10.3)
      GO TO 2335
 2321 WRITE (6,2322)
 2322 FORMAT (///5X,'THE MAXIMUM ALLOWABLE TEMPERATURE IS LESS THAN OR E
     $QUAL TO THE OIL TEMPERATURE.')
      GO TO 2335
 2323 WRITE (6,2324)
```

```
2324 FORMAT (///5X,'THE MAXIMUM ALLOWABLE TEMPERATURE IS TOO SMALL FOR
    $THE GIVEN DIELECTRIC LOSS.')
2335 CONTINUE
     GO TO 1
2350 STOP
      END
```

APPENDIX G

ONE-DIMENSIONAL SOLUTIONS FOR TEMPERATURE AND CURRENT

The Temperature Solution

In Chapter 2 the equation which governs the temperature distribution of the cable insulation was given as

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} = -\frac{\dot{q}}{k} . \tag{2.1}$$

Considering now only the radial dependence of the temperature, Equation 2.1 reduces to

$$\frac{1}{r}\frac{dT}{dr} + \frac{d^2 T}{dr^2} = -\frac{\dot{q}}{k} , \tag{G.1}$$

and this may be consolidated to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} . \tag{G.2}$$

Expressing the volumetric heating term $\dot{q}$ in terms of the total dielectric loss per unit length W_d, the governing equation G.2 becomes finally

$$\frac{d}{dr}\left(r\,\frac{dT}{dr}\right) = \frac{W_d}{2\pi rk\,\ln\left(\frac{r_1}{r_2}\right)}\,, \tag{G.3}$$

where r_1 and r_2 denote the inner and outer radii of the insulation, respectively. The boundary condition at the conductor is

$$q_c = -2\pi r_1 k\,\left.\frac{dT}{dr}\right|_{r_1} = W_c\,, \tag{G.4}$$

where W_c is the conductor loss per unit length. As for the boundary condition at the surface, consider for the present that the surface is at some arbitrary uniform temperature:

$$T(r_2) = T_o\,. \tag{G.5}$$

A dimensionless formulation may be obtained by introducing the variables

$$\xi = \frac{r}{r_2}\,, \qquad \theta = \frac{T - T_{oil}}{W/k}\,, \tag{G.6}$$

where W is some arbitrary loss per unit length. The governing equation is then

$$\frac{d}{d\xi}\left(\xi \frac{d\theta}{d\xi}\right) = \frac{W_d}{2\pi\xi W \ln \xi_1} , \tag{G.7}$$

where $\xi_1 = r_1/r_2$, and the boundary conditions are

$$\left.\frac{d\theta}{d\xi}\right|_{\xi_1} = - \frac{W_c}{2\pi W\xi_1} , \tag{G.8}$$

and

$$\theta(1) = \theta_o \equiv \frac{T_o - T_{oil}}{W/k} . \tag{G.9}$$

The general solution to Equation G.7 is

$$\theta(\xi) = \frac{W_d}{4\pi W \ln \xi_1} (\ln \xi)^2 + C_1 \ln \xi + C_2 . \tag{G.10}$$

Substituting this solution into the boundary conditions G.9 and G.10, the arbitrary constants C_1 and C_2 are found to be

$$C_1 = - \frac{1}{2\pi W} (W_d + W_c) , \quad C_2 = \theta_o . \tag{G.11}$$

Putting this result back into the governing equation gives

$$\theta(\xi) = \frac{W_d}{4\pi W \ln \xi_1} (\ln \xi)^2 - \frac{(W_d + W_c)}{2\pi W} \ln \xi + \theta_o . \tag{G.12}$$

Equation G.12 is then the temperature distribution for the case in which the cable surface is at some arbitrary temperature T_o. However, this arbitrary temperature may now be eliminated by applying an energy balance at the cable surface:

$$W_c + W_d + W_s = 2\pi r_2 h (T_o - T_{oil}) \; , \qquad \text{(G.13)}$$

where W_s is the sheath loss per unit length. The nondimensional form of this is

$$\frac{W_c + W_d + W_s}{W/k} = 2\pi r_2 h \theta_o \; , \qquad \text{(G.14)}$$

from which

$$\theta_o = \frac{(W_c + W_d + W_s)}{2\pi W} \left(\frac{k}{hr_2} \right) . \qquad \text{(G.15)}$$

The one-dimensional temperature distribution for the cable is therefore

$$\theta(\xi) = \frac{W_d}{4\pi W \ln \xi_1} (\ln \xi)^2 - \frac{(W_d + W_c)}{2\pi W} \ln \xi + \frac{(W_c + W_d + W_s)}{2\pi W} \left(\frac{k}{hr_2} \right) . \qquad \text{(G.16)}$$

In terms of dimensional temperatures,

$$T(\xi) - T_{oil} = \frac{W_d}{4\pi k \ln \xi_1} (\ln \xi)^2 - \frac{(W_d+W_c)}{2\pi k} \ln \xi + \frac{(W_c+W_d+W_s)}{2\pi r_2 h} \tag{G.17}$$

The Current Solution

In order to obtain the one-dimensional current solution, the stationary and variable components of the temperature distribution are again separated:

$$\theta(\xi) = \theta D(\xi) + \theta C(\xi) , \tag{G.18}$$

where

$$\theta D(\xi) = \frac{W_d}{4\pi W \ln \xi_1} (\ln \xi)^2 - \frac{W_d}{2\pi W} \ln \xi + \frac{W_d}{2\pi W} \left(\frac{k}{hr_2}\right) , \tag{G.19}$$

and

$$\theta C(\xi) = -\frac{W_c}{2\pi W} \ln \xi + \frac{(W_c+W_s)}{2\pi W} \left(\frac{k}{hr_2}\right) . \tag{G.20}$$

The current I is introduced into $\theta C(\xi)$ following the reasoning of Appendix C. By analogy with Equation C.39,

$$\theta C(\xi) = \theta C_o(\xi) \frac{I^2}{I_o^2} , \tag{G.21}$$

where $\theta C_o(\xi)$ is the distribution obtained by using the arbitrary current I_o in Equation G.20. The total solution $\theta(\xi)$ is then

$$\theta(\xi) = \theta D(\xi) + \theta C_o(\xi) \frac{I^2}{I_o^2} . \qquad (G.22)$$

Again it is desired to have $\theta(\xi)$ take on some maximum value θ_{max}. However, in the one-dimensional case, since there are no heat sinks within the cable, θ_{max} must occur at the location $\xi = \xi_1$ (at the conductor). Substituting this information into Equation G.22 gives

$$\theta_{max} = \theta D(\xi_1) + \theta C_o(\xi_1) \frac{I^2}{I_o^2} . \qquad (G.23)$$

Upon rearrangement, Equation G.22 yields the current

$$\left(\frac{I}{I_o}\right)^2 = \frac{\theta_{max} - \theta D(\xi_1)}{\theta C_o(\xi_1)} . \qquad (G.24)$$

It is then only necessary to insert the appropriate values for $\theta D(\xi_1)$ and $\theta C_o(\xi_1)$ from Equations G.19 and G.20:

$$\left(\frac{I}{I_o}\right)^2 = \frac{\theta_{max} - \frac{W_d}{2\pi W}\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{\frac{W_{co}}{2\pi W}\left(\frac{k}{hr_2} - \ln \xi_1\right) + \frac{W_{so}}{2\pi W}\left(\frac{k}{hr_2}\right)} , \qquad (G.25)$$

where W_{co} and W_{so} are the conductor and sheath losses, respectively, produced by the arbitrary current I_o. This expression may be simplified by using the relations

$$W_{co} = I_o^2 R Y_c , \qquad W_{so} = I_o^2 R(Y_s - Y_c) , \tag{G.26}$$

where Y_c and Y_s denote the AC/DC ratios at the conductor and at the sheath, respectively. Making these substitutions, Equation G.25 becomes

$$I^2 = \frac{\theta_{max} - \frac{W_d}{2\pi W}\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{\frac{RY_c}{2\pi W}\left(\frac{k}{hr_2} - \ln \xi_1\right) + \frac{R(Y_s - Y_c)}{2\pi W}\left(\frac{k}{hr_2}\right)} , \tag{G.27}$$

which may be further simplified to

$$I^2 = \frac{2\pi W \theta_{max} - W_d\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{R\left(\frac{Y_s k}{hr_2} - Y_c \ln \xi_1\right)} . \tag{G.28}$$

Finally, in terms of dimensional temperatures,

$$I^2 = \frac{2\pi k (T_{max} - T_{oil}) - W_d\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{R\left(\frac{Y_s k}{hr_2} - Y_c \ln \xi_1\right)} . \tag{G.29}$$

APPENDIX H

CONSERVATIVE APPROXIMATE SOLUTIONS FOR MAXIMUM TEMPERATURE AND CURRENT

General

Conservative approximations for maximum temperature and current in the two-dimensional conduction problem may be achieved from a suitable modification of the one-dimensional solutions presented in Appendix G. The conservative assumption to be employed is that cable-cable and cable-conduit interactions effectively insulate appropriate portions of the cable surface, thereby reducing the perimeter available for heat transfer. Furthermore, it is assumed that the entire cable sector subtended by an insulated arc on the perimeter is also effectively insulated. Any losses which occur in the insulated sector are then referred to the remaining undisturbed portion of the cable. Consider, for example, that a 60°-arc of the cable perimeter is taken to be insulated. The perimeter available for heat transfer is then reduced to $\left(\frac{5}{6}\right)$ its original size, and all losses in the $\left(\frac{5}{6}\right)$-cable must be scaled up by $\left(\frac{6}{5}\right)$ in order to have the same heat flow or temperature as in the original problem. The maximum current or temperature is then computed from the one-dimensional solution, using $\left(\frac{6}{5}\right)$ of the original one-dimensional losses.

The Temperature Solution

In Appendix G the one-dimensional temperature distribution was given as

$$T(\xi) - T_{oil} = \frac{W_d}{4\pi k \ln \xi_1} (\ln \xi)^2 - \frac{(W_d+W_c)}{2\pi k} \ln \xi + \frac{(W_c+W_d+W_s)}{2\pi r_2 h} \tag{G.17}$$

The temperature drop from the conductor to the oil is then

$$T(\xi_1) - T_{oil} \equiv T_o - T_{oil} = -\frac{\ln \xi_1}{4\pi k} (W_d+2W_c) + \frac{(W_c+W_d+W_s)}{2\pi h r_2} , \tag{H.1}$$

where T_o denotes the conductor temperature. It is noted in Equation H.1 that the temperature drop (T_o-T_{oil}) varies linearly with the cable losses. Now let the cable perimeter available for heat transfer take on the value $P' = fP$, where P is the total perimeter, and f is some fraction. The losses in the undisturbed portion of the cable are then scaled up according to $q' = \left(\frac{1}{f}\right)q$. Since the temperature drop (T_o-T_{oil}) varies linearly with loss, it too is scaled up by (1/f), and the conservative expression is given by

$$(T_o-T_{oil})_* = \frac{1}{f} (T_o-T_{oil}) , \tag{H.2}$$

where the temperature drop on the right side is that produced by the one-dimensional solution. Equation H.2 may then be used to conservatively estimate either the maximum allowable oil temperature or the maximum cable temperature in the two-dimensional conduction problem.

The Current Solution

From Appendix G the one-dimensional current solution is

$$I^2 = \frac{2\pi k(T_{max}-T_{oil}) - W_d\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{R\left(\frac{Y_s k}{hr_2} - Y_c \ln \xi_1\right)} . \qquad \text{(G.29)}$$

Again consider that the effective perimeter takes on the value P' = fP, and that the losses are scaled up according to $q' = \left(\frac{1}{f}\right)q$. Since current-produced losses vary as I^2, the latter quantity must itself be linearly scaled, along with the dielectric loss. Inserting this into Equation G.29 gives the result

$$\frac{I_*^2}{f} = \frac{2\pi k(T_{max}-T_{oil}) - \frac{W_d}{f}\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{R\left(\frac{Y_s k}{hr_2} - Y_c \ln \xi_1\right)} , \qquad \text{(H.3)}$$

which may be rearranged to give

$$I_*^2 = \frac{2\pi kf(T_{max}-T_{oil}) - W_d\left(\frac{k}{hr_2} - \frac{\ln \xi_1}{2}\right)}{R\left(\frac{Y_s k}{hr_2} - Y_c \ln \xi_1\right)} . \qquad \text{(H.4)}$$

Equation H.4 is then the conservative approximation for current.

The Effective Perimeter

The size of the inter-cable conduction path is a reasonable guide in selecting the amount by which to reduce the cable perimeter for cable-cable and cable-conduit interactions. This convention was followed in generating the conservative comparisons tabulated in Chapter 6. For those 16 problems the inter-cable conduction path was chosen so as to subtend an angle of 30° on either cable surface. The following cable perimeters were therefore used for the various configurations: open - 330° effective; cradled - 300° effective; equilateral - 270° effective; and equilateral-pipe - 240° effective. It is noted that for an effective perimeter of 360° (f = 1), the one-dimensional solutions are recovered in Equations H.2 and H.4.