OIL SUPPLY FORECASTING USING DISAGGREGATED POOL ANALYSIS

By

Supply Analysis Group

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OIL SUPPLY FORECASTING USING DISAGGREGATED POOL ANALYSIS

As a part of a larger project to analyze the world oil market, effort is being devoted to the development of methods for forecasting oil supply from key exporter regions. For purposes of this study, oil suppliers are divided into two rough categories:

(1) The Cartel Core. There is a small group of Persian Gulf nations who form the core of the oil cartel, and who are the "price-makers" in the sense that they determine the price through their own ability and willingness to balance total supply (from inside and outside the cartel core) with world oil demand. The group includes Saudi Arabia, Kuwait, and others in the Gulf; under some definitions is also may include Libya, Iran, and Venezuela.

(2) "Price-Taker" Suppliers. This is the group of petroleum exporters who appear to act as "price-takers" in the sense that each takes the world price (which is being set by others) as given, and makes supply decisions according to his own parochial interest. This group includes various non-OPEC sources such as the producers of the North Sea, Mexico, the USSR, and China. It also may include members of OPEC itself, such as Algeria, Iraq, Indonesia, and Nigeria.

The purpose of this part of the world oil study is to develop ways to analyze and forecast the behavior of oil supply from the latter group of nations. What is needed, in the context of the overall market analysis,[1] is a set of relations that can be used to simulate supplies from price-taker regions given an hypothesis of likely price-setting behavior on the part of the cartel core.

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[1] For an overview of the research design, see the World Oil Project's six-month report to NSF [12].

1. OVERVIEW OF THE ANALYTIC METHOD

1.1 Methods Used By Others

Several types of analysis have been used in attempts to explain and forecast the supply of petroleum from regions that are responsive to the oil price. One approach that has gained acceptance in recent years, particularly in the United States, is the use of econometric techniques.[1] Econometrics has been applied in circumstances where hundreds of fields, each containing a number of reservoirs, have given the productive systems the stability of large numbers, and where the depletion effect (tending to raise costs as less of a reserve remains) has to a large extent been offset by new discoveries and by improvements in technology.

Unfortunately, in studying supply from many areas of the world, the conditions for econometric analysis are less favorable than in the U.S. In many countries the oil fields are both fewer and younger, and even the short histories are poorly documented. Another factor limiting the use of econometrics in studies of oil supply is the fact that the price series are so fragmentary and untrustworthy. The so called "posted prices" of the past were rendered meaningless around 1960, when they became artifacts used for the calculation of taxes. Moreover, data on arms-length sales of crude oil are insufficient and are ridden with too many errors to serve as a basis for econometric investigation. Because of these limitations, an orderly summation of the past (which is what an econometric model is) is of limited help in forecasting future relations between prices and outputs.

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[1] For an excellent example, see the work of MacAvoy and Pindyck [11].

Another approach to supply forecasting, which also involves an "orderly summation of the past," is that typified by the work of the National Petroleum Council [13], and subsequently applied by the Federal Energy Administration in its Project Independence Analysis [17, 18, 19]. Under this approach, the experience of past exploratory drilling is summarized, and a trend in the finding of reserves per foot drilled is established. Based on estimates of the costs of exploration and development, calculations can be made of the relative attractiveness of exploratory activity, conditioned on some assumption about the price of oil. Given an estimate of exploratory drilling, the forecast of barrels added per foot drilled, a reserve-to-production ratio, and hoped-for stability in reserve expansion in old "fully-developed" fields, it is possible to forecast supply into the future.

Unfortunately, many of the shortcomings of the econometric approach apply as well to the NPC-type format. In many areas of the world the exploratory histories are poorly documented, and several of the relationships which are required for this approach may be estimated only very approximately. This is because many important supply areas of the world are relatively new, and the experience that makes the NPC method believable simply does not exist. Moreover, the more productive potential areas in the world often are located in offshore or otherwise inaccessible areas, and the cost of production of particular resources weighs very heavily in the supply relation, as opposed to the phenomenon of exploration and finding which is emphasized in the NPC method.

Still another approach involves a combination of simple extrapolations of past experience and judgments about likely developments in the future. The simulation model of Odell and Rosing [14] demonstrates one way in which the judgmental assessments regarding the discovery phenomenon and the reservoir development process can be translated into a long range forecast of petroleum supply. In that study, detailed predictions of annual drilling and finding rates are employed in conjunction with an estimate of the size distribution of discoveries to generate an expected time-profile of additions to recoverable reserves. Reservoir development and production out of reserves proceeds according to an assumed time schedule.

To capture the influence of geological variations and economic uncertainty the authors impose random disturbances at each step in the procession from discoveries to final production. They are then able to determine not only the supply forecast that is implied by a typical simulation of the system, but also the extremities that are attained by the coincidence of most unlikely events.

A basic concern regarding the Odell-Rosing approach is that all of the basic behavioral relationships (e.g., discovery, development, and production) are formulated without explicit reference to economic variables or motivations. As a consequence, there is nothing to prevent simulated behavior from departing from what might be regarded as economically rational. In a footnote [note #10, page 32] the authors mention one anomoly of this kind: their forecast has the annual drilling effort rising to a sustained peak at just the time (1982-83) when production is outrunning the hypothesized demand for North Sea oil. In such a circumstance, one would want to have a feedback loop from the demand/supply imbalance back to a revision in exploration and development activities. However, because there is no explicit economic formulation of these relationships, it is impossible to reach a consistent forecast by iteration.

A parallel concern is that the insensitivity of the Odell-Rosing supply forecast to changing economic conditions makes it difficult to apply to many questions of public policy. The prevailing degree of uncertainty in the world petroleum market requires the flexibility to conduct analyses under alternative price/cost/tax scenarios. Indeed, the economic implications of changing conditions in the energy markets constitute the focal point of public energy policy.

Finally, there are the methods of resource estimation used by oil companies in evaluating prospective areas, and in constructing global estimates of regional or world resources. These methods, which draw on detailed geologic and geophysical data as well as on past drilling experience, seem to be rarely used for supply estimation work of the type being carried out here. They do, nonetheless, contain important components of concept, information and analytic method, and use is made of these approaches below.

1.2 The Basin Development Model

The approach we have adopted for analysis of price-taker suppliers is to develop a simulation of the process of exploration, discovery, and production in major oil-producing basins of the world. The method attempts to make use of geological data, engineering cost estimation, and analysis of government and industry development decisions. This is an ambitious undertaking, and the analysis cannot be applied in full detail to more than a limited number of basins. This limitation is not too stringent, fortunately, for the overall pattern of supply in the oil market is heavily influenced by a relatively small number of price-taker areas. Little is lost be estimating the supply from other areas by far simpler approximations, by judgmental forecasts, or by borrowing from the analyses of others.

The process which we are attempting to analyze is presented in Figure 1. Shown there are the processes of exploration, discovery, and production--looked at from the viewpoint of a particular region or basin with its collection of reservoirs. When a new reservoir is discovered, a determination must be made as to whether it is economic to develop it. This process is shown in the right-hand side of the figure. A new reservoir may be put into production immediately, or it may be held as part of an "inventory," awaiting more favorable external economic conditions (such as a higher oil price, lower costs, or relaxed tax rules).

Whether a reservoir is scheduled for development or rejected on economic grounds depends greatly on the characteristics of the reservoir itself, and on its location. In easily-accessible low-cost onshore areas almost any reservoir which produces oil is developed. In high-cost offshore areas, on the other hand, a reservoir may have to be extremely large by onshore standards (perhaps as large as several hundred million barrels of recoverable reserves) before it is economical to develop it. Moreover, the estimates of likely reservoir economics given any set of in-the-ground resources are strongly dependent on costs and tax rules. (This is one reason why supply analysis based on simple extrapolation, as discussed above, is so severely limited.)

Naturally, the resources that are available for possible development are the result of some exploratory effort, and the next stage in the analysis, moving leftward in Figure 1, is concerned with the process of exploration and finding. Assuming some level of exploratory effort (which has gone past the geological-geophysical phase and is now measurable in the number of wells

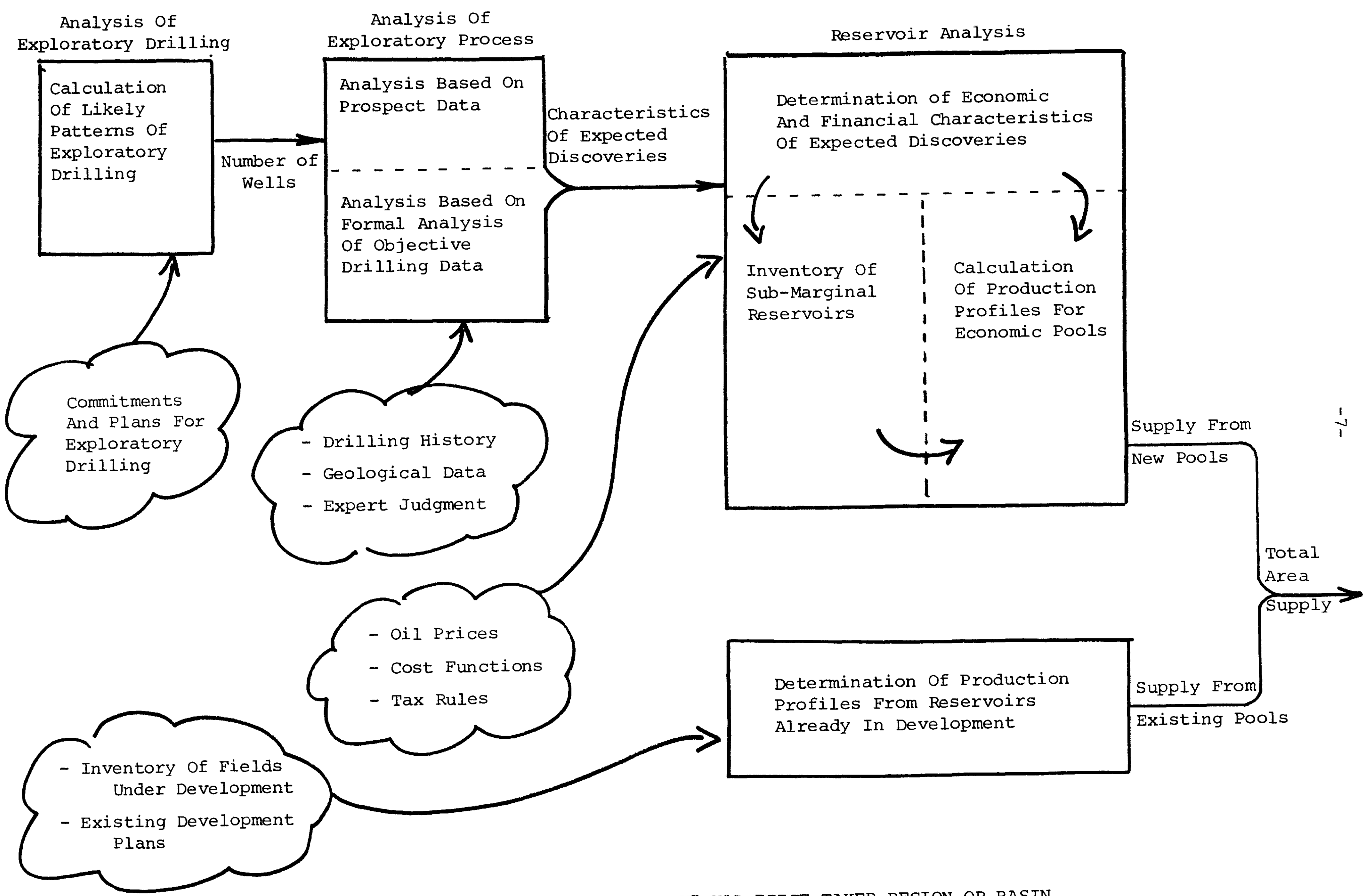

FIGURE 1. SUPPLY ANALYSIS FOR INDIVIDUAL PRICE TAKER REGION OR BASIN

drilled per year), one needs a method of forecasting what will be found. This type of analysis draws on drilling histories, on detailed geological data, and on expert judgment. The wide variation among supply forecasts often is the result of differences in results at this particular stage of the analysis.

Finally, in order to understand the ultimate supply from a basin, one must have some way of forecasting what the level of exploratory drilling will be in the future. This then is the first box in the sequence shown in Figure 1.

Thus, what Figure 1 shows is a simple summary of the process of discovery, development, and production; as the figure indicates, the outcome is influenced by a host of factors. Resource constraints are a critical determinant; each stage in the process results from the interaction of the <u>costs</u> of development and production, the <u>expected price</u> and the local tax policies of the government concerned.

In the sections that follow, we develop the various parts of this analysis. We begin with the reservoir analysis shown in the right-hand side of Figure 1 and focus on the construction of a sub-model of the supply of oil out of reservoirs, where the characteristics of those reservoirs (in terms of recoverable reserves, water depth, etc.) are known. As indicated in the figure, such reserves fall into a spectrum, which we divide into the following rough categories:

(1) Fields that are fully developed and absorb no additional investment.

(2) Fields where development drilling is already under way or planned. At one extreme, this includes strict "infill" drilling to moderate the decline rate in old fields. At the other, it includes fields not yet producing.

(3) Areas where reservoir capabilities are known, but where decisions about their economic viability have not yet been made.

In the first two cases, the estimates of capacity and supply can sometimes be based on relatively good data, some of which are in the public domain. Estimates of supply from these existing regions are the most accurate of all the estimates contributing to the calculation of future supply. The analysis of the third category, potential new fields, requires a simulation of the behavior of governments and private corporations, and the development of a model to do this is the subject of Section 2 below.

Section 3 discusses the analysis of the exploratory process, given a particular level of exploratory effort in a region. Two lines of approach are discussed in Section 3: One is based on the extensive use of geological data and judgment about likely finding; the other attempts to introduce statistical analysis of the exploration for oil.

The analysis of exploratory drilling itself--that is, the calculation of the likely rate of drilling in future years--is an important aspect of a simulation of the supply from a region. For purposes of this paper, the focus is on the exploratory process and on reservoir analysis, and the level of exploratory drilling is a forecast based on existing plans for exploration. Naturally, decisions to explore are determined by the economics of subsequent steps in the exploitation process, and in Figure 1 there needs to be a grand feedback loop from total supply and the economics of supply back to the rate of exploratory drilling itself. That feedback, and the work on forecasting exploratory drilling, are beyond the scope of this particular paper. For the purposes of this discussion, and the North Sea example developed below, the level of exploratory drilling is based on existing plans. This means that the time horizon over which one can forecast is limited to two or three years of exploratory effort, plus the years that it takes to carry out delineation and development drilling. Thus, with the particular analysis of exploratory drilling now used in this analysis, the model may be thought of as capable of forecasting up to 10 years for offshore regions and, perhaps, to five or six years for onshore regions.

In Section 4 below, we present a sample application of the analysis to the North Sea. This is probably one of the most important price-taker regions and provides a laboratory for the testing of the analytical tools being developed.

2. THE RESERVOIR DEVELOPMENT MODEL

The reservoir submodel is designed to analyze the economic viability of individual reservoirs and to determine the physical characteristics of the marginal reservoir depending upon the prevailing level of economic incentives. By extension, the model demonstrates the sensitivity of reservoir development to changing economic conditions.

There are usually two parties to the determination of economic viability of a reservoir: the government and the industry. Government decisions are represented in the reservoir development model by a set of fiscal and non-fiscal regulatory parameters. These regulatory decisions are exogenous to the model. The development decisions of industry, or of the government if it is also the operator (e.g., Mexico and China), are based on the net present value of the reservoir development venture. If the net present value of a reservoir is positive, the reservoir will be produced, otherwise it will not. Reservoirs that are presently being produced, or for which development plans have been announced, are assumed to be produced according to the announced production schedules.

2.1 Reservoir Definition

A reservoir is characterized for our purposes by a set of physical attributes (Table 1). Notice that the level of recoverable reserves is defined as a reservoir characteristic. In addition, the rate of development and extraction are exogenous to the reservoir submodel, and are assumed to be a known constant for all reservoirs belonging to a specific reservoir category in a given area.[1] These physical characteristics in conjunction with economic variables determine, in a simplified way, the economic viability of the reservoir.

TABLE 1

Reservoir Characteristics

- recoverable reserves (oil and/or gas)
- average well productivity (oil and/or gas)
- reservoir depth
- water depth
- distance to shore (terminal)

Our concept of a reservoir should be distinguished from the engineering concept of a reservoir. Reservoir engineers usually conceive of a reservoir as a set of geological conditions that could sustain various levels and rates of petroleum production, depending upon the level of investment, the chosen production profile, and other factors. The traditional engineering development model is designed to optimize several of the quantities we have taken as fixed reservoir attributes. The M.I.T. offshore development model [10] is an example of such an analytical framework.

[1] A "reservoir category" consists simply of those reservoirs thought to share common characteristics, when measured along the dimensions enumerated in Table 1.

Although our simplified definition of the reservoir prevents fine-tuning of the rate of extraction from economic reserves, there are several compelling arguments for its use. To go beyond our idealized concept of a reservoir would require detailed information on hydrocarbons in place and on the geologic variables that determine the recovery factor and the rate of development and extraction (e.g., permeability, porosity, formation thickness, initial pressure, temperature, etc.). Unfortunately, we have no basis for predicting how each of these variables will behave in the exploration-discovery process. There is, however, as described in Section 3, a substantial amount of work on how the hydrocarbons in place and (with a fixed recovery factor) the recoverable reserves change as an area is drilled up. As our focus is on the intermediate to the longer-term future, the analysis includes the discovery and development of new reservoirs. It does not make sense to try to analyze a reservoir along more dimensions than can be predicted with reasonable confidence.

In addition, even if we had the information and resources to carry out such a detailed analysis for all the relevant oil-producing areas, endogenizing the recovery factor and the rate of recovery results in a reservoir model which is too detailed to be incorporated in an aggregate supply model of the type being developed here. Our simplifying assumptions greatly reduce the complexity of model structure without being very restrictive when evaluating the reservoirs of particular interest--those on the borderline of economic viability.

The cost categories that can be explained using predictive reservoir characteristics (Table 1) are also more aggregative than those of a traditional reservoir development model. Total development and extraction costs are divided into the categories of Table 2. From the point of view of data collection, this level of cost-disaggregation seems appropriate.

TABLE 2

Cost Categories

- development drilling
- platform (or Arctic pad) structures and their installation
- platform equipment
- pipelines and terminals/tankers and offshore loading facilities
- operating costs, platforms, and equipment
- operating costs, pipelines and terminals/tankers and offshore loading facilities

The fiscal cost, or the share of total revenues that is being paid to the government, is a highly significant determinant of the attractiveness to a private operator of an individual reservoir. For this reason the rules and regulations that determine the government take are represented in a detailed fashion in the reservoir development submodel. The rules and regulations that are included in the model are listed in Table 3. The lack of homogeneity and the degree of "ad hocism" in tax systems across the world necessitates a separate set of equations for each national jurisdiction.[1]

In addition, the degree of administrative freedom that is built into tax systems varies across nations. Consequently it has been difficult in some cases to determine how the various rules and regulations would actually be interpreted. Most petroleum producing countries have been in the process of changing their petroleum legislation since the 1973 embargo, and there are few cases on which to base judgments as to the interpretation of rules and regulations.

[1]The ideal tax system removes only economic rent, and hence leaves investment unaffected at the margin. But actual tax systems depart so far from the ideal that we must try to capture the effects of that divergence.

TABLE 3

Fiscal and Non-Fiscal Regulations

- the definition of the tax reference price (posted price or market price)
- the royalty rate (fixed or sliding scale)
- the number and level of tax rates (income tax, special taxes, dividend source tax, capital tax, etc.)
- the depreciation rule used for each tax category
- production and capital allowances, tax-free income
- rules regarding deductibility of interest cost
- rules regarding the carrying forward of losses
- rules regarding dividend distribution
- timing of tax payments
- definition of tangible and intangible investments
- ring fence provisions (definition of areas for taxation purposes)
- definition of taxable income for each tax category
- carried interest/participation/production-sharing arrangements (buy-back price, participation in cash or in kind, financing, and operating responsibilities of the private/foreign partner under the arrangements)
- non-fiscal regulations, quota-systems, local purchase priorities

2.2 Functional Relationships

The reservoir development submodel is essentially a net present value calculator. It uses the reservoir characteristics of Table 1 as explanatory variables in a set of functional relationships that determine the development and extraction costs associated with a given reservoir. A set of rules of thumb and tax regulations is then applied to determine the cash flow for the reservoir. If, and only if, the net present value of this cash flow is positive, then the recoverable aggregate production is added to the pool of recoverable reserves. In this sense the reservoir submodel checks the economic viability of a discovered field.

There is no theory that can help us specify the functional relationships between the reservoir characteristics of Table 1 and the cost categories of Table 2. Very little econometric work has been done at this level of detail, using the reservoir definition of this study. There is, of course, a wealth of experience in the petroleum industry (as well as common sense) that can be exploited in specifying the general form of the cost relationships. Such a general form is developed below, and the estimation of specific cost relationships, using the North Sea as an example, is reported in Section 4.1.

2.2.1 Cost of Development Drilling. Drilling cost depends on the size and location of the reservoir, and on the average well productivity. If the reservoir is offshore, drilling costs depend heavily on water depth and distance to shore. The level of recoverable reserves and the average well productivity determine the total number of development wells. Reservoir depth, water depth, and distance to shore determine the cost of drilling the development wells.

Thus the cost of development drilling, DC, is an increasing function of reserve size, R, reservoir depth, D, water depth, W, and distance to shore, D. It is a decreasing function of average well productivity, WP. To separate offshore and onshore areas a dummy variable, DV, may be inserted. The cost equation in its general form is

$$CD = CD(R,D,W,S,WP,DV). \quad (1)$$

2.2.2 Cost of Production Platforms. The most important component of development cost of an offshore reservoir is the production platforms. Platform cost, CP, is determined by the total number of platforms required times the cost per platform. The number of platforms depends on the total number of production wells and the location of these wells vis-a-vis each other (that is, the configuration of the reservoir). In an optimizing reservoir model, the number of production wells and the number of platforms would be determined simultaneously. As the shape of a reservoir is not a datum being exploited in this simplified model, it is assumed that the reserve level and the average well productivity determine the number of production platforms. The water-depth explains the cost per platform. The dummy variable separating onshore and offshore areas is included to take care of a possible discontinuity at zero water-depth. This cost-relationship in its general form, then, is

$$CP = CP(R,WP,W,DV). \tag{2}$$

2.2.3 Cost of Platform Equipment. Equipment cost depends on the size of the reservoir, the amount of associated gas, and the type of drive applied. Among these explanatory variables, the size of the reservoir is the only datum exploited in this model. The cost of platform equipment, CPE, is consequently represented as

$$CPE = CPE(R). \tag{3}$$

2.2.4 Transportation or Disposal Equipment Expenditures. The transport cost for hydrocarbons being extracted from a particular reservoir (or a group of reservoirs close enough to use a single pipeline) depends on the distance to a disposal system with available capacity, or on the distance to shore (or to the nearest terminal) in case of an "isolated" reservoir. There are a number of transportation methods available for offshore reservoirs [7], the major choice being between sub-sea pipelines and tanker transport loading at a buoy. Buoy loading is often applied in the early stage of larger fields, and on a permanent basis for smaller oil fields. The cost of tanker-cum-buoy transport, TT, serves as a ceiling on the cost of transportation. The cost of pipeline(s) in general depends on the quantities being transported, the length of the pipeline, and the hostility of the environment as measured by the water-depth and sea bottom conditions offshore, or by difficulty of terrain onshore, TO . The general form of the cost of transportation equipment, CT, is thus

$$CT = CT(R,S,W,TO,TT,DV). \qquad (4)$$

2.2.5 Total Capital Expenditures. From the equations 1 to 4 total capital expenditures, CE, may be derived. A markup factor, M, is inserted to capture miscellaneous expenditures--i.e.,

$$CE = M * (CD + CP + CPE + CT). \qquad (5)$$

2.2.6 Annual Operating Costs. Operating costs, CO, could conceivably be separated into a fixed and a variable component, the variable component being dependent on the quantity produced. Experience indicates, however, that annual operating costs are fairly insensitive to the level of production once production has started, and that the level of operating costs is explained well by total capital expenditures. One significant exception is fields not connected to pipelines, where capital cost may be saved, but only at greatly increased operating costs. We have consequently formulated a simple relationship,

$$CO = CO(R,\ Q_p) \tag{6}$$

where the peak output, Q_p, becomes relevant in the cast of non-pipeline-connected fields.

Equations (5) and (6) represent total development expenditures and annual operating costs as perceived at the time the development decision is made. For the cash flow analysis the left hand side of these two equations has to be transformed into an annualized cash flow. Based upon the rate of investment in a "typical" reservoir of a given reservoir category, a rule of thumb has been formulated to allocate total capital expenditures over time. The investment outlay in period t, I_t, is thus some exogenously determined share, δ_t, of total expenditures, multiplied by the rate of inflation, IN, to which this share has been subject.

$$I_t = (\delta_t * CE) * (1 + IN)^t \tag{7}$$

$$\sum_{t=0}^{T} \delta_t = 1 \tag{8}$$

The time horizon of a reservoir is denoted by T. The annualized operating costs are likewise determined as demonstrated in equation (9).

$$CO_t = CO(1 + IN)^t \tag{9}$$

2.2.7 Production Profile From a Reservoir. The production profile, Q_t, is a complex function of geologic, economic, and regulatory variables, as is indicated by the complex nature of optimizing reservoir models. As the reservoir model developed here does not include the detail required to make the production profile endogenous, a simple rule of thumb is applied to allocate recoverable reserves to the various production periods. Such a rule of thumb assumes that the build-up period, the number of periods of peak production, and the decline period for the production from a reservoir in a particular area are homogenous across reservoirs in each reservoir category.

$$Q_t = \lambda_t R \tag{10}$$

$$\sum_{t=0}^{T} \lambda_t = 1 \tag{11}$$

For analysis of an individual reservoir, the expected market price of the hydrocarbons produced, P_t, is exogenously determined. This price, plus the 11 equations described above, are sufficient to determine the real cash flow associated with a reservoir if developed. When the government itself is the operator, the present value of the real cash flow determines the economic viability of a reservoir (assuming price-taker behavior).

2.2.8 Fiscal Regimes. Even in circumstances where development decisions are made by private corporations, government policies are an extremely important determinant of the viability of a reservoir. These regulatory regimes may differ substantially among countries; there is, however, a common set of regulatory instruments that are being used to influence the development/ production activity. They fall into two basic categories:

(1) Instruments used for fiscal purposes, to capture oil rents and generate income to the government.

(2) Instruments used for direct regulation of oil exploitation.

The level and distribution of income from petroleum activities are important determinants of the fiscal policy of a country. The local employment situation and "limits to growth" considerations also have been used as arguments for direct regulations. In this model the fiscal- and non-fiscal regime are treated as exogenously determined.

Direct regulation of the production from a reservoir may be represented by a policy-determined production profile, $Q_t = Q_t^*$ where

$$Q_t^* = \lambda^*_t R_t, \tag{12}$$

and

$$\sum_{t=0}^{T} \lambda^*_t = 1. \tag{13}$$

The investment profile may also be determined in this fashion.

The royalty rate in a particular period, RY_t, is usually either a flat rate or a sliding scale rate dependent on the level of production in that period:

$$RY_t = RY(Q_t). \tag{14}$$

Usually the total tax payments for a field will depend on a number of particular tax provisions like a general corporate tax, a specific petroleum tax, a posted price tax, etc., (see Table 3). For each separate tax, k, there is some revenue flow to the government, TX^k, where

$$TX_t^k = TX^k \; (Q_t, P_t, CE, CO_t) \tag{15}$$

There also may be bonus payments, B^ℓ, for each of ℓ bonus schemes. These are similar to taxes, but because they usually are lump-sum payments dependent on actual or expected production (not on the profit rate), they are dealt with separately:

$$B_t^{\ell} = B^{\ell}(Q_0, \ldots, Q_T, P_t) \tag{16}$$

In general, each tax category presumes a separate depreciation schedule, D^k. Each schedule is related in some fashion to the historic investment pattern of the reservoir,

$$D_t^k = D(I_0, \ldots, I_t) \tag{17}$$

There are a number of additional tax equations that might be defined to capture the range of tax systems around the world, as is obvious from the fiscal regulations listed in Table 3. In Section 4.2.1 the specific formulation of the fiscal equations is indicated for the U.K. and Norway.

The 17 equations above allow us to calculate a net cash flow in each period for a private party, NC_t, and for the government, NCG_t. By introducing a private, r_p, and a social, r_s, discount factor, the net present value of the reservoir development venture can be calculated for the private party, PV, and for the government, PVG,

$$PV = \sum_{t=0}^{T} \frac{NC_t}{(1+r_p)^t} \tag{18}$$

$$PVG = \sum_{t=0}^{T} \frac{NCG_t}{(1+r_s)^t} \tag{19}$$

In the countries where the private party is the operator Q_t will be added to the regional supply level only if PV is positive. In countries where the government is the operator Q_t may be added to regional supply if PVG is positive.

3. THE EXPLORATORY PROCESS SUBMODEL

The purpose of the submodel of the exploratory process is to produce a discovery sequence over time, to explain the process by which the geologist's list of prospects is transformed into an inventory of reservoirs to be developed. This process consists of the following (somewhat simplified) steps:

1. Identification of sedimentary basin--that is, original depressions in the earth's crust that have been filled-in by the slow levelling processes of nature, and in which petroleum generating source rock and potential reservoirs are present.

2. The systemtatic investigation of a basin by geological and geophysical methods (predominantly seismic) to define more precisely those areas where hydrocarbons may have been preserved, and to identify "concepts" or "prospects."

3. The drilling of the prospects that are large enough to contain commercially attractive reserves of hydrocarbons.

4. Delineation of the prospects that turn out to be reservoirs containing an accumulation of hydrocarbons.

5. Addition of the reservoirs that contain commercially attractive reserves of hydrocarbons to the inventory of reservoirs to be developed.

Given these exploratory steps we might discuss different kinds of discoveries--e.g., of basins, concepts or prospects, reservoirs, or commercially-producible reservoirs. In the following we define a "discovery" to mean the discovery of a <u>reservoir</u>.

The relationship between the reservoir and several alternative "units of observation" is indicated in the following chart.

The Units of Analysis

Prospect or Concept:	A geological configuration conceived to have trapped hydrocarbons that forms a target for drilling
Pool or Reservoir:	A continuous containment of hydrocarbons closed on all sides
Field:	A hydrocarbon-bearing reservoir of a collection of contiguous reservoirs
Play:	A group of similar geological configurations conceived or proven to contain hydrocarbons
Basin:	A continuous containment of possible hydrocarbon source rocks
Geopolitical Area:	A (potential) supply area of hydrocarbons which may or may not transcend national borders. In the geopolitical concept is included the total number of basins and national jurisdictions that will determine supply of hydrocarbons from an area of the world.

The reservoir submodel is designed to determine which reservoirs among those generated in the discovery process are commercially attractive. By separating the discovery/development process in this fashion, an attempt is made to separate the geological characteristics of an area from the economic attractiveness of the area.[1]

There is, as stated in Section 2, no basis for predicting how each of the geologic variables that characterize a reservoir will behave as a basin is drilled up. By properly idealizing a basin, however, we may learn how

[1] As is obvious from Step 3 above, this separation is not complete; the selection of the prospects to be drilled is influenced by the expected economics of the reservoirs to be discovered.

certain of these variables behave as the exploration process matures. The most significant geological variable (which is a composite of a number of geological phenomena) is the number and size distribution of the individual reservoirs to be discovered.

It is possible on the basis of geophysical data to subdivide a basin into groups of similar geological configurations conceived to contain hydrocarbons. Such a group "conceived or proven to contain hydrocarbons" is labeled a "play." The location of a play within a basin makes it possible to estimate some of the reservoir characteristics defined in Table 1--namely reservoir depth, water depth, and distance to shore (terminal).

There is a substantial amount of work on the size-frequency distribution of reservoirs at the play level [8], and as noted below we have some basis for predicting the size of discoveries to be made.

The fifth reservoir characteristic defined in Table 1, Average Well Productivity, is a significant determinant of the economic viability of a reservoir in high-cost areas. Experience indicates that the average well productivity per reservoir varies within a fairly narrow range among reservoirs in a given play. It is therefore possible to predict with "reasonable confidence" the average well productivity of the reservoirs to be discovered in a play, once the play has been discovered. For prospects grouped into a potential play, sensitivity analysis with respect to the average well productivity, based upon the geological similarity to known plays, is as close as it is possible to come to "predicting" the behavior of this variable.

The riskiness of oil exploration is a most significant aspect of the discovery process. In this analysis, the risks involved in exploring for hydrocarbons are subdivided into two categories: the geologic risk and the economic risk. The geologic risk results from the lack of a technology to determine, prior to drilling, whether a prospect is in fact a reservoir (i.e., contains hydrocarbons). The geologic risk is represented by the probability of discovering a reservoir of hydrocarbon fluids. The economic risk results from the fact that some of the reservoirs to be discovered may not contain hydrocarbons in commercially attractive quantities. The reservoir submodel of Section 2 determines the size of the economically marginal reservoir, and thus can be used to evaluate the economic risk of the exploration process.

From the above it can be seen that the four most important elements of a discovery model are the following:

(i) The size-frequency distribution of the reservoirs to be found

(ii) The probability of discovering a reservoir

(iii) The sequence of discoveries (i.e., the nature of the "sampling" process)

(iv) The rate of exploratory drilling

In the following discussion we make a distinction between discovery analysis based on geologic and judgmental data, and analysis that makes use of statistical models of the exploratory process. At this stage the distinction refers to the way the size-frequency distribution of the expected reservoirs is estimated, and also to the way in which drilling is assumed to proceed, (i.e., to assumption (iii) above). At a later stage the probability of success--i.e., the probability of making a discovery--may also be estimated based on statistical as well as subjective data. The rate of exploratory drilling is exogenous to the existing version of the Basin Development model, but may be endogenized in a later version.

Although the present version of this paper distinguishes between judgmental and statistical analyses as though the two are mutually exclusive, this need not be true. Part of the ongoing research of the World Oil Project is devoted to the development of a suitable methodology for integrating the two. The theory of Bayesian inference provides a general approach to this problem.

The reservoir and the discovery submodels make it possible to estimate the amount of hydrocarbons that will be extracted from an area within a given time frame under any mix of price and cost structures, licensing and tax policies. All the important aspects of the exploration/production process as seen from the point of view of the operator are explicitly included--that is, the number, location, and size of reservoirs as well as the risks involved in exploring for and producing these reservoirs.

3.1 Analysis Using Geologic-Judgmental Estimates

Analysis with geologic-judgmental data is understood to involve interpretation of the significance of each of the exploratory parameters of Table 4 with respect to the hydrocarbon producing potential of an area. For each combination of these parameters a discovery sequence or time-profile of additions to recoverable reserves may be arrived at. Those interested in a more extensive discussion of the relationship between these exploratory parameters and the hydrocarbon potential of an area, and the estimation of each of the parameters, should consult a publication of the AAPG [1].

TABLE 4

The Hydrocarbon Potential

- The expected number of hydrocarbon prospects by reservoir category
- The expected number of hydrocarbon bearing blocks by reservoir category
- Expected success ratios
- The expected rate of exploratory drilling (delineation not included) over the "years to come"
  - (a) given existing licensing policies
  - (b) without licensing constraints

The geologic-judgmental discovery analysis assumes that a population of prospects has been identified by the use of geophysical and geological analysis and that reservoir engineering equations have been applied to estimate the size of each prospect in the population of prospects. Because the "block" is the administrative unit, and the licensing policy of the government may play an important role, we also assume that the geologist and the petroleum engineer may give us the hydrocarbon potential of an area on a block basis. Consequently, we feel the need to maintain these two "units of observation" in the analysis.

We begin by specifying a size classification to be applied to the hydrocarbon potential of each block. The term S_i denotes the i^{th} size category, with S_I being the largest size class and S_M the smallest. On the basis of geological assessment of hydrocarbon potential, each prospective block is assigned to one of the size categories. We assume there are n_i prospective blocks belonging to the class S_i. Within a size class S_i, each block may contain several individual reservoirs or pools. The average number of pools per block in the i^{th} size class is denoted k_i, and the average individual pool size is given by D_i. Consequently, $b_i = k_i \cdot D_i$ equals the average block size within the S_i size class.

The success ratio, PS, is the judgmentally-determined estimate of the frequency that a prospect (the hydrocarbon potential of a block) represents an actual reservoir. The geologist is assumed to be able to indicate this ratio either on the basis of similarity to other areas, or on the basis of the drilling record of the area, or some combination of the two.

Application of the success ratio to the estimated hydrocarbon potential of an area determines expected recoverable reserves. Table 5 illustrates this relationship for each of the size categories.

An hypothesis regarding the most likely sequence of drilling and discoveries is needed to translate the hydrocarbon potential of Table 5 into a forecast of future increments to the stock of available economic reserves. A convenient assumption is that annual additions to recoverable reserves in an area depend on the rate of exploratory drilling only. A competing hypothesis says that net additions to reserves depend on the cumulative number of wells drilled, as well as the rate of drilling. That is, an exploration decline curve may be hypothesized. In the remainder of this section we discuss two simple drilling hypotheses that illustrate both notions. It is desirable, and eventually will become possible, to work with more elaborate versions of the hypotheses used in this paper. At present, our goal is to make clear the most critical assumptions underlying intermediate and longer-term judgmental supply analysis.

For the immediate future (2 to 3 years) it is reasonable to treat the rate of exploratory drilling as exogenous. This is particularly so in areas where the concessions include working program provisions and the incentive to scale up the activity beyond this level is non-existent. If there are no licensing constraints, a rate of exploratory drilling may be estimated from

TABLE 5

Expected Recoverable Reserves

Block Size Category	Number Of Blocks	Mean Size Within Block Category	Estimated Total Recoverable Reserves in the Size Category
S_1	N_1	$k_1 * D_1$	$PS * n_1 * k_1 * D_1 = TR_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	n_i	$k_i * D_i$	$PS * n_i * k_i * D_i = TR_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_M	n_M	$k_M * D_M$	$PS * n_M * k_M * D_M = TR_M$
Total	N_B	$\sum_{i=1}^{M} k_i * D_i$	$\sum_{i=1}^{M} TR_i = TR$

Notes:

(1) Estimated Recoverable Reserves from each productive block in the i^{th} size category:

$$\bar{b}_i = \frac{TR_i}{n_i * PS} = k_i * D_i$$

(2) Estimated Recoverable Reserves from each productive block, overall average:

$$\bar{b} = \frac{TR}{PS * \sum_{i=1}^{M} n_i} = \frac{\sum_{i=1}^{M} n_i * k_i * D_i}{\sum_{i=1}^{M} n_i} = \sum_{i=1}^{M} p_i * k_i * D_i$$

... where $p_i = \frac{n_i}{\sum_{i=1}^{M} n_i}$ = The portion of all prospective blocks falling within the i^{th} size class.

the availability of drillable prospects, the expected level of delineation drilling, and the availability of rigs.

The drilling effort for the next, say, three years is represented as $[(W_{11}, W_{12}, \ldots, W_{1j_1}), (W_{21}, W_{22}, \ldots, W_{2j_2}), (W_{31}, W_{32}, W_{3j_3})]$. That is, exactly j_i wells are drilled in the i^{th} year. Of these j_i, a portion $PS \cdot j_i$ are assumed to discover actual reservoirs. Notice that k_i of these successful wells are needed (on average) to establish the hydrocarbon potential of a single block in the S_i size category.

As drilling proceeds, some blocks will prove to contain an amount of reserves below that which is economical to develop. We call this cutoff point the minimum economic block size (MEBS). The judgmentally determined probability that the reserves of a productive block within the i^{th} size class will not exceed MEBS is written:

$$\text{Prob [Block } \varepsilon S_i < \text{MEBS]} = F_i(\text{MEBS}).$$ [1]

[1] If a judgmental density function, f(R), is determined which describes the distribution of block size within the ith size class as a random variable, we then have:

$$F_i(\text{MEBS}) = \int_{o}^{\text{MEBS}} f_i(R)\, dR$$

I.e., F_i(MEBS) is the MEBS-fractile of block size within the ith size category.

At the present time, we implement this principle in the following way:

The first exploration hypothesis (H1) treats all blocks as indistinguishable and drilling is assumed to proceed randomly from the entire population. Essentially, there is a single size class, whose mean size equals the population mean size. F(MEBS) is then taken to be the MEBS-fractile computed from the actual North Sea prospect portfolio (page 69).

The second exploration hypothesis distinguishes prospects on the basis of block size. Only those prospects which fall in size classes exceeding the MEBS are drilled. We then assume that expectations are realized, i.e., that the expected reservoir size is realized. Consequently, F_i(MEBS) = 0 for all prospects drilled. This is an assumption of convenience, which permits us to temporarily avoid the difficult task of assessing judgmental size distributions within individual size classes. [The statistical treatment of discoveries in the next section provides a solution to this problem.]

We then assume that the proportion of productive blocks from the i^{th} size class which prove to be economic to develop is given by $\pi_i(MEBS) = 1-F_i(MEBS)$.

The expected additions to economically recoverable reserves can now be determined from the particular hypothesis governing the drilling sequence. In Table 6 two extremes are illustrated. Hypothesis 1 (H1) says that additions to reserves depend upon the rate of drilling (and the prevailing MEBS), and that each block category has an equal probability of being discovered in any period. Hypothesis 2 (H2) states that larger blocks are found before the smaller ones; that is, drilling proceeds deterministically according to ranked size.

The supply profile of production of hydrocarbons is very sensitive to the kind of discovery process hypothesized, as can easily be verified by inserting real numbers into Tables 5 and 6. Nevertheless, the logic of this subjective, deterministic analysis is simple and intuitively appealing. The major issues in developing a discovery framework are included. The subjective analysis is well conceived for preliminary analysis of an area, as well as for the development of a more realistic and thus also more complicated discovery apparatus.

One point that will demand further elaboration in the future is the treatment of judgmental uncertainty regarding the size potential of individual prospects. This problem has thus far been treated only lightly. In the illustration (Table 6) we assume that fractiles of the predictive density of reservoir size are known for each prospect in question. This is a strong assumption, but it appears necessary in order to translate the sequence of discoveries into a sequence of additions to economic reserves in a meaningful way.

TABLE 6

Expected Additions To Economically Recoverable Reserves

Discovery Hypothesis	Year 1	Year 2	Year 3
H1:	$PS \cdot j_1 \cdot \bar{b} \cdot \frac{\pi(MEBS)}{\bar{k}}$	$PS \cdot j_2 \cdot \bar{b} \cdot \frac{\pi(MEBS)}{\bar{k}}$	$PS \cdot j_3 \cdot \bar{b} \cdot \frac{\pi(MEBS)}{\bar{k}}$
	... where $\bar{k} = \frac{\sum_{i=1}^{M} n_i k_i}{\sum_{i=1}^{M} n_i} = \sum_{i=1}^{n} p_i \cdot k_i$ = avg. number pools/block, overall.		
H2:	$PS \cdot j_1 \cdot \sum_{i=1}^{M} g_i^1 \cdot \pi_i (MEBS) \cdot D_i$	$PS \cdot j_2 \cdot \sum_{i=1}^{M} g_i^2 \cdot \pi_i (MEBS) . D_i$	$PS . j_3 \cdot \sum_{i=1}^{M} g_i^3 \cdot \pi_i (MEBS) . D_i$
	... where it is understood that g_i^t represents the proportion of total wells of year t that are drilled in blocks of the i^{th} size class; when drilling is assumed to proceed according to ranked order by size.		

There are several alternative means of incorporating pool size uncertainty into the analysis. We briefly describe several to indicate the range of assumptions and results that are possible.

(1) Description: Prospective blocks are partitioned into size classes on the basis of perfect knowledge. Drilling is strictly limited to those blocks exceeding the MEBS criterion, and drilling proceeds according to ranked order by size.

Comments: This model makes no pretense at size risk, and consequently is optimistic. There is nothing in the forecast that reflects the measure of judgmental uncertainty inherent in the size classification; and the translation of discoveries into reserves is immediate. The role of a price elasticity of supply is clearly defined in this model, (a price rise causes the domain of ranked order drilling to extend to smaller pools), but does not seem very realistic. Notice that until previous drilling has exhausted all available prospects exceeding MEBS, price increases have no effect--drilling proceeds at the top of the ranked order, while submarginal prospects occur at the bottom. Consequently, if we choose to work with this version it would be helpful to relate the drilling rate itself to price incentives.

(2) Description: Perhaps the simplest way of accounting for size risk is to perform a sensitivity analysis on a completely deterministic model, as in (1). In this way supply forecasts under very optimistic assumptions can be compared with less favorable results.

Comments: The principal weakness of this alternative is that it gives no sense of the likelihood that extreme results will in fact be realized. Also, it is not clear how the short run (off-the-shelf) elasticity of supply can be restored to a meaningful role via sensitivity analysis.

(3) Description: The most elementary method of introducing size risk quantitatively would follow from the assumption that drilling proceeds randomly from the set of all prospective blocks, which are indistinguishable. The discovery size of each block is a random variable, characterized by (m, s^2), where m represents the expected discovery size, and s^2 is its variance.

Comments: Total discoveries are found as the summation of individual drilling efforts. The feature that differentiates this from the previous models is that we can express our uncertainty about the volume of total discoveries quantitatively. The variance of the sum is easily computable, and can take account of positive correlations that may exist in the judgmental estimates of separate prospects. In the case where each judgment is independent of the others, the variance of our forecast of total discoveries is simply $n \cdot s^2$, where n represents the number of successful wells sunk during the forecast period.

This alternative clearly has its drawbacks. There is no response to changing price incentives--for this we would need to endogenize the drilling rate. The critical pitfall, however, is that although size risk is acknowledged and quantified to a certain extent, there is no means of judging the probability that a discovery will fall below the MEBS criterion. Consequently, we cannot construct a consistent model of the inventory of submarginal prospects, nor can we assess the short run "off-the shelf" price elasticity.

(4) Description: We generalize the preceding model substantially by assuming the n prospective blocks are distinguishable, with the i^{th} block characterized as (m_i, s_i^2). We assume ranked-order drilling with m_i being the criterion.

Comments: Model (4) is superior to (3) in that it introduces a more realistic dose of geologic insight regarding the size distribution of blocks, while still providing a quantitative measure of uncertainty. Nevertheless, the characteristics of the forecast are not much changed. The supply forecast is still insensitive to changing price incentives (unless we endogenize the drilling rate), and there is yet no way to use the quantitative measure of size risk to distinguish economic reserves from discoveries.

(5) Description: The simplest judgmental model we can envision which corrects the difficulties of (1)--(4) is the following: the prospective blocks are assumed to be distinguishable, with the size of reserves in the i^{th} block regarded as a random variable with distribution function $h(\cdot \mid m_i, s_i^2)$. Drilling proceeds in ranked order of the m_i.

Comments: The key assumption in Model (5) is that in addition to the first two moments of the size distribution, the form of the judgmental distribution itself is known. As before, we can calculate the variance of the discovery forecast (incorporating correlations which may appear among prospects). In addition, we can quite easily construct a model to predict the rate at which discoveries accrue as economic reserves. (The method of partial expectations discussed in Section 3.2. can be adapted straightforwardly to the assumptions of Model (5). In turn, we can discuss estimates of the off-the-shelf supply elasticity implied by our geological-judgmental data, just as we discuss the elasticity in the context of the statistical modeling effort, (see next section).

In summary, the workable range of alternative geologic-judgmental models is quite broad: The most significant distinction among the models is the level of sophistication in the judgmental assessments of size risk. The simplest and least tenable assessment is that there is no size risk. Expressing this risk in the form of a standard deviation is more satisfactory; but specification of the complete judgmental distribution of discovery size for each prospect is required if we are to speak meaningfully about the rate at which economic reserves accrue during periods of changing price incentives.

3.2 Statistical Analysis of Pool Size

In this section we describe a method of forecasting the supply of economic reserves generated by new petroleum discoveries that incorporates formal statistical methods of inference and analysis.

Two points of special interest characterize the method we propose, and distinguish it from other models of the discovery process. First, the influence of resource depletion (discovery decline curve) is incorporated explicitly in a manner that consistently reflects the physical and geologic phenomenon. Second, the sequence in which discoveries accrue as economic reserves does not necessarily correspond to the sequence of discoveries, itself, (i.e., many pools are too small to be developed under the economic incentives prevailing at the time of discovery). Consequently, the forecast of future increments to reserves must reflect not only the ongoing exploration process, but also a reappraisal of the running inventory of sub-economic reserves, consistent with changing economic conditions.

3.2.1 The Sequence of Petroleum Discoveries. We begin with a postulated level of exploratory effort within the geographical region of a "play." This exploratory effort, which is measured in terms of the drilling rate over a period of years, is estimated in accordance with anticipated economic and geologic conditions. However, the drilling rate is exogenous to the main body of our model; it is determined judgmentally, rather than in a formal manner.

In addition, we stipulate the "dry hole risk" expected to prevail during the forecast period. This risk factor also reflects judgmental estimates of the relevant factors. The drilling rate and dry hole risk jointly determine the expected number of "geologically successful" wells sunk during the forecast period. The rate of geological success, PS , is determined as the excess of unity above dry hole risk. Corresponding to a drilling effort of W wells per unit of time, we expect to observe N = PS * W geologically successful wells. Nothing is implied concerning the pool size of each of the N discoveries; the geological success rate signifies only the expectation that they are not dry holes.[1]

The simple hypothesis of N discoveries is expanded into a detailed sequence of predictive distributions of respective discovery sizes by the technique due to Kaufman [9]. We assume the deposition of pools by volume in the region follows a known probability law, f(D) where D represents potentially recoverable reserves found in an individual pool. Then we imagine that exploratory drilling proceeds such that the probability of each discovery size is random and proportional to size, and computed without replacement. In this way it is possible to derive from f(D) a sequence of distributions $f_1(\cdot)$, $f_2(\cdot)$, ..., $f_N(\cdot)$, describing the outcome of the N successive wells.

[1]Eventually we will contrast the notion of a geologically successful well with that of an "economically successful" well, where there is an implication concerning pool size.

It is customary to assume that f(D) takes the form of the lognormal distribution, $f_L(D|\mu, \sigma^2)$. We follow this practice and estimate the parameters μ and σ^2 from observations on discoveries in the play occurring prior to the forecast period.[1] Equipped with estimates of μ and σ^2, it is possible to obtain the sequence of predictive densities, $f_1(.)$, $f_2(.)$, ..., $f_N(.)$ in either of two ways--via Monte Carlo sampling experiments, or using analytical methods. The complexity of the analytical method reduces our estimates of the $f_j(.)$ to approximations. Actually, several degrees of accuracy are available in the results with widely varying costs of computation. Unfortunately, we have little experience with approximations of this type and can make only provisional judgments of their desirability [again, the reader is referred to [2] for a description of the methods used.) However, the alternative of resorting to Monte Carlo methods to derive the form of the $f_j(.)$ appears to be prohibitively expensive and unnecessary for our purposes. We will proceed using the notation $f_j(.)$ for the predictive densities of the sequence of discoveries and $\hat{f}_j(.)$ for the approximations arrived at via numerical analysis.

3.2.2 Translation of Discoveries into Economic Reserves. It is implicit in the discovery model that some of the N pools will not constitute economic reserves immediately upon their discovery. Because the geological success rate (which determines N) does not reflect economic characteristics of pools, many pools discovered will fall below the minimum economic pool size (MEPS) which warrants development. These sub-economic

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[1] It is not correct to simply fit observed discoveries to a lognormal curve because the observations do not constitute a random sample, and are made without replacement. The correct method of estimation is too complex to be reported adequately here. Details of the procedure may be found in [2].

pools reside in an inventory of undeveloped reserves until such time as a change in economic incentives draws them out. In the remainder of this section, we describe the way the model deals with this phenomenon.

By the process already described, we postulate the sequence of N successful exploratory wells;

$$W_1, W_2, \ldots, W_N$$

The associated sequence of discovery volumes is denoted:

$$D_1, D_2, \ldots, D_N$$

The discovery volume, D_j, is treated as a random variable following the density $f_j(D)$, (or the approximation, $\hat{f}_j(D)$, developed in [2]).

The expected volume of the jth discovery is given by:

$$E(D_j) = \int_0^{\infty} D * f_j(D)\, dD \tag{20}$$

It is important to notice that $E(D_i) \neq E(D_j)$, for $i \neq j$. The difference is accounted for by the effect of depletion.

We may partition the range of feasible pool sizes, D, into an arbitrary number of cells. Consider for illustrative purposes a three-cell partition:

S_I: Size Class I $= \{D | D \geq 200\}$

S_{II}: Size Class II $= \{D | 100 \leq D < 200\}$

S_{III}: Size Class III $= \{D | D < 100\}$

To reiterate, D represents the volume of a single pool discovered by any of our N exploratory wells.

The purpose of the size classification is to embody our notion of a minimum economic pool size. As economic incentives change, various size classes come in and out of play, (e.g., as the price of crude oil rises progressively smaller size classes become economic).

Associated with each successful well, w_j, there is a sequence of the volume of economic reserves originating with that well, and accruing in each ensuing year:

$$X_j^1, X_j^2, \ldots, X_j^t$$

Normally, only one element of this sequence will be non-zero. The whole of the reserves, D_j, will accrue in the first period when D_j exceeds minimum pool size. However, at the time of our forecast, the amount D_j is not known with certainty, so each element of the sequence may have a non-zero expectation.

Finally, there is a sequence of annual increments to economic reserves:

$$R_1, R_2, \ldots, R_t,$$

where t indexes the year. By definition,

$$R_t = {}_j\Sigma X_j^t$$

where the summation is taken over the set of wells, w_j, drilled before the end of year t. While we are ultimately interested in forecasting the sequence of R_t, it is clear that to do so we must focus on the sequence of:

$$X_j^t, \; j = 1, \ldots, N$$

$$t = 1, \ldots, T$$

To describe the distribution of X_j^t, we need to identify the size class that becomes economic for the first time in year t since the jth well was drilled, and denote this class of pool size by the symbol, θ_j^t. Depending on the time path of economic variables, θ_j^t will include some union of the disjoint sets: S_I, S_{II}, S_{III}, and ϕ. Specifically, the set θ_j^t is constructed as:

$$\theta_j^t = [\Delta_{jt}^I \cap S_I] \cup [\Delta_{jt}^{II} \cap S_{II}] \cup [\Delta_{jt}^{III} \cap S_{III}] \qquad (21)$$

where Δ_{jt}^i is the set-theory counterpart to a "Kronecker delta." That is:

Δ_{jt}^i = the universe, if S_i becomes economic for the first time in year t since the drilling of well j,

$\Delta_{jt}^i = \phi$, otherwise.

Thus, $[\Delta_{jt}^i \cap S_i] = S_i$ or ϕ, depending on the condition.

The volume of reserves originating with the jth well and accruing in year t, denoted X_j^t, takes on the following values:

$$\begin{aligned} X_j^t &= D_j \quad &&, \text{ if } D_j \varepsilon \theta_j^t \\ X_j^t &= 0 &&, \text{ otherwise} \end{aligned} \qquad (22)$$

Moreover, the probability density of values, X_j^t, falling in the θ_j^t class is given simply by $f_j(D)$. The density of the zero value of X_j^t is simply the probability that D_j does not fall in the class θ_j^t, that is, the integral:

$$\int_{D \not\varepsilon \theta_j^t} f_j(D)\ dD$$

This description of the behavior of X_j^t is sufficient to permit the calculation of its expectation. For an arbitrary well (j) and year (t), the expected volume of reserves accruing is given by:

$$E(X_j^t) = \int_{D\varepsilon\theta_j^t} D*f_j(D)\,dD + \int_{D\not\varepsilon\theta_j^t} 0*f_j(D)\,dD \tag{23}$$

$$= \int_{D\varepsilon\theta_j^t} D*f_j(D)\,dD$$

$$= [\delta_{jt}^{I} * \int_{D\varepsilon S_I} D*f_j(D)\,dD] + [\delta_{jt}^{II} * \int_{D\varepsilon S_{II}} D*f_j(D)\,dD] + [\delta_{jt}^{III} * \int_{D\varepsilon S_{III}} D*f_j(D)\,dD] \tag{24}$$

where δ_{jt}^{i} is the Kronecker delta corresponding to the Δ_{jt}^{i} defined earlier. That is:

$$\delta_{jt}^{i} = 1 \quad \text{if} \quad \Delta_{jt}^{i} = \text{the universe}$$

$$\delta_{jt}^{i} = 0 \quad \text{if} \quad \Delta_{jt}^{i} = \phi.$$

Then it results that

$$E(X_j^t) = \delta_{jt}^{I} * P_j^{I} + \delta_{jt}^{II} * P_j^{II} + \delta_{jt}^{III} * P_j^{III} \tag{25}$$

$$= \sum_{i=I}^{III} \delta_{jt}^{i} * P_j^{i}$$

In step (24) we use the definition of $\theta_j^t = \bigcup_{i=I}^{III} [\Delta_{jt}^i \cap S_i]$ to split the integral into individual pieces corresponding to the specified size classes: S_I, S_{II}, and S_{III}. The Kronecker deltas in (24) then serve to annihilate those pieces of the integral corresponding to size classes not represented in θ_j^t.

In step (25) we implicitly define the partial expectation of reserves within the i^{th} size class discovered by the j^{th} well:

$$P_j^i = \int_{D \in S_i} D * f_j(D) dD \quad , \qquad i = I, II, III.$$

To summarize briefly, we have shown in (25) that the expected volume of reserves originating with the jth well and accruing in the t^{th} year can be written as the inner product of two vectors; the first vector (δ_{jt}^i) being determined solely from the postulated time-trend of minimum pool size, and the second vector (P_j^i) being computed as the set of partial expectations of the function, $f_j(D)$, corresponding to the postulated size classification. Our choice of the size classification being arbitrary, we may generalize (25) directly to:

$$E(X_j^t) = \sum_{i=I}^{M} \delta_{jt}^i * P_j^i \qquad (26)$$

where we have specified M size classes, S_I, S_{II}, ..., S_M.

The total increment to economic reserves accruing in year t is simply the sum of reserves accruing from each individual well then existing:

$$R_t = \sum_j X_j^t$$

where the summation is taken for values of j occurring by the end of year t.

Taking the expectation:

$$E(R_t) = E(\sum_j X_j^t)$$

$$= \sum_j E(X_j^t)$$

and using (26) above

$$= \sum_j \sum_{i=I}^{M} \delta_{jt}^i \cdot P_j^i \qquad (27)$$

We can now arrange our calculations in a way that facilitates computation of an unbiased estimate of future additions to reserves. The following table is to be constructed, showing a column for each hypothesized discovery and a row for each size class.

size class \ sequence of discovery	W_1	...	W_N
S_I	P_1^I	...	P_N^I
S_{II}	P_1^{II}	...	P_N^{II}
...	...	...	...
S_M	P_1^M	...	P_N^M

where $P_j^i = \int_{D \varepsilon S_i} D * f_j(D)\, dD = i^{th}$ partial expectation of $f_j(D)$.

In addition, the vector (δ^i_{jt}) is determined exactly from the postulated time-trend of minimum economic pool size. There are no stochastic elements in the construction of the vector (δ^i_{jt}). Finally, we form the forecast volume of reserves accruing in year t, $\hat{R}_t$:

$$\hat{R}_t = \sum_j \sum_{i=I}^{M} \delta^i_{jt} P^i_j \tag{28}$$

The expected value of the forecast error is immediately seen to be zero.[1]

One interesting aspect of the forecast method just described is that the expected contribution of each well drilled is partitioned into mutually exclusive categories and appears to be "smeared" over time. Thus the individual pool represented by each successful well is not treated as a unified increment to economic reserves, but rather a sequence of staggered partial increments determined by fluctuations in economic variables. It might be thought that this conception departs from the underlying physical process. In reality the pool is either economic or it is not; either the entire pool is an increment to economic reserves at the time of its discovery, or the entire pool must await favorable economic developments before it adds to available supplies. One might ask whether the forecast results are sensitive to this disparity.

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[1] In the text we are representing the random variable D_j by its true distribution function, $f_j(D)$. This formulation leads to the conclusion that the forecast is unbiased. In fact, our forecast is based on approximations to the function $f_j(D)$, so the results do not apply directly. Further work is required to assess the damage inflicted on the forecast as a result of using the approximation, as opposed, say, to the forecast generated by Monte Carlo simulations of the model. The most definite results on this question are to be found in [2].

In fact, this is not a valid objection to the method. The apparent phasing-in of discoveries is an illusion created by the operation of statistical expectation. The actual definition of the random variable X_j^t indicates that if we were to simulate the stochastic process we have described, (as in Monte Carlo sampling experiments), the reserves of each pool would be treated as an indivisible unit, entering the category of economic reserves all at once. (This is shown by the fact that X_j^t takes on only two values: zero and D_j, the value D_j occuring at only one point in time.) Thus, the forecast drawn from the method of phasing-in partial expectations is implied by the distribution of a random variable that correctly represents the underlying physical process.[1]

3.2.3 The Role of Size Class. One advantage of dealing with the size distribution of discoveries is that it facilitates treatment of the running inventory of sub-economic pools, and the supply of reserves which accrues from this inventory over time. In addition, the size of each discovery is of interest as a matter of public policy because the tax provisionswhich apply to development and production are often determined as a function of this size.

[1]Again, the fact that our forecasts involve approximations to $f_j(D)$ violates the statement in the text. It would be informative to perform an extended Monte Carlo experiment where pools are sampled from the parent population, $f_L(D_i;\mu,\sigma^2)$ and subjected to a first passage test keyed to the level of minimum pool size. This would be a direct simulation of the process described in the text, circumventing the approximations to the functions, $f_j(D)$. A comparison of results from the two methods would indicate the loss from using $\hat{f}_j(D)$ in the method of partial expectations.

The framework we have laid out thus far is not adequate for the purpose of forecasting the expected number of discoveries from among the next N that fall within any particular size class. The fact that the reserves of each pool are statistically phased-in prevents us from simply adding up a running count of individual pools through time. However, this problem is easily handled by an extension of our line of reasoning.

Consider the j^{th} discovery chosen arbitrarily from among the next N. The size of discovered reserves is denoted D_j and follows the distribution $f_j(D)$. If D_j exceeds the lowest level of minimum economic pool size (MEPS) in effect during the forecast period, then the number of "economic discoveries" produced by the j^{th} well is clearly unity; otherwise the number is zero. We let Q_j^{MEPS} represent the number of economic discoveries produced by the j^{th} well.[1]

$$Q_j^{MEPS} = 1 \text{ with probability } = \int_{MEPS}^{\infty} f_j(D)\,dD$$

$$Q_j^{MEPS} = 0 \text{ with probability } = \int_{0}^{MEPS} f_j(D)\,dD$$

The <u>expected</u> number of economic discoveries generated by the j^{th} well is seen to be:

$$E(Q_j^{MEPS}) = 1 * \int_{MEPS}^{\infty} f_j(D)\,dD + 0 * \int_{0}^{MEPS} f_j(D)\,dD \qquad (29)$$

$$= \int_{MEPS}^{\infty} f_j(D)\,dD$$

[1] The superscript "MEPS" is attached to remind one of the fact that the number of "economic discoveries" depends parameterically upon the level of MEPS.

Thus, $E(Q_j^{MEPS}) = 1 - F_j(MEPS)$

...where $F_j(MEPS)$ = the MEPS-fractile of the distribution of f_j.

The expected total number of economically successful discoveries from among the next N (conditional on the relevant MEPS) is then calculated as:

$$\begin{aligned} Q(MEPS) &= \sum_{j=1}^{N} E(Q_j^{MEPS}) \\ &= \sum_{j=1}^{N} \int_{MEPS}^{\infty} f_j(D)\,dD \\ &= \sum_{j=1}^{N} [1-F_j(MEPS)] \\ &= N - \sum_{j=1}^{N} F_j(MEPS) \end{aligned} \tag{30}$$

The rate of economic success[1], σ_e, is defined as:

$$\sigma_e = \frac{Q(MEPS)}{N} = 1 - \frac{1}{N} \sum_{j=1}^{N} F_j(MEPS) \tag{31}$$

The formulation is easily generalized to give the expected number of pools falling within any restricted size class. We denote this number $Q(D_L, D_U)$ where D_L and D_U are respectively the lower and upper bounds of the size class.

[1]The economic success rate (which varies with the level of minimum economic pool size) should be compared with the geological success rate (PS) defined above, which is not affected by economic variables within the model.

Following the method used above, we obtain:

$$Q(D_L, D_U) = \sum_{j=1}^{N} \int_{D_L}^{D_U} f_j(D)\,dD$$

$$= \sum_{j=1}^{N} F_j(D_U) - \sum_{j=1}^{N} F_j(D_L) \qquad (32)$$

$$= \sum_{j=1}^{N} [F_j(D_U) - F_j(D_L)]$$

where again, $F_j(.)$ is the fractile of the distribution of $f_j(D)$.

To facilitate the computation in (32) we construct a second table in the same format as earlier:

size class \ sequence of discovery	W_1	W_2	...	W_N
S_I	G_1^I	G_2^I	...	G_N^I
S_{II}	G_1^{II}	G_2^{II}	...	G_N^{II}
...	...	...	...	...
S_M	G_1^M	G_2^M	...	G_N^M

The size classes, S_i, define our restricted pool range, and the G_j^i are computed as the probability mass of $f_j(D)$ within this range: $G_j^i = \int_{D \in S_i} f_j(D)\,dD$. The expected number of pools falling within the ith size class from among the next N wells is given by

$$Q(S_i) = \sum_{j=1}^{N} G_j^i$$

4. NORTH SEA EXAMPLE

We have applied the methodology presented above to the estimation of future crude oil production in the North Sea. In this area we include the British and the Norwegian sectors between 56° and 62° North latitude, the boundary between Norway and Denmark, and the zero paleocene depth contour. This area covers the "oil area" of the North Sea as seen by most industry writers [3].

In Section 4.1 the estimation of the relationships between the reservoir characteristics of Table 1 and the cost categories of Table 2 is discussed. These cost relationships and a set of investment, production, and price profile assumptions are inserted into the reservoir development model along with a detailed representation of the British and the Norwegian tax systems in order to determine the characteristics of the marginal reservoir for each sector. This minimum pool size calculation is discussed in Section 4.2.

In Section 4.3 the results of the geologic-judgmental and the statistical discovery analyses are presented. The implications of the analysis for the future level of crude oil production, and for income to the public and the private sectors, are summarized in Section 4.4.

4.1 Cost Data and Cost Relationships

The investment/production history of the North Sea is not substantial enough to estimate cost relationships on the basis of actual investment experience. Only four fields are currently being produced - Ekofisk, Argyll, Forties and Auk--all of which are at an early stage in the build-up period. We therefore had to base our estimation of cost relationships on estimates of the itemized investment and operating expenditures of individual fields to be developed. Wood, Mackenzie & Co. (WM) investment advisers of Edinburgh, Scotland, have been computing and updating appraisals of North Sea fields since early 1973. Our data base for the North Sea consists of their set of capital and operating expenditures for the individual fields. The WM data consists of actual capital expenditures in current dollars prior to 1976, and reflect an assumed rate of inflation for capital expenditures of 25% in 1976, of 20% in 1977, and of 10% thereafter. To adjust the pre-1976 capital expenditures to January 1976 dollars, a rate of cost inflation of 30% was assumed for 1972, of 40% for 1973 and 1974, and of 30% for 1975. These historic inflation rates are consistent with the revisions of expected capital expenditures made over time by some of the companies, and consequently are only rough estimates of a true inflation index.

The WM data covers 17 actual and potential crude oil producing fields. The Ekofisk complex is treated as one field. Among the fields of particular interest to this study--i.e., small fields that may be close to or beyond the limit of economic viability--there are three categories: (1) the average isolated field, (2) the special tanker offtake/high peak-ratio fields, and (3) the field discovered close to a large field with available transportation capacity.

Fields in the first category typify the usual situation in the North Sea province--the field promises average productivity, but requires the complete build-up of supporting infrastructure. The second category, as exemplified by Argyll and Auk, consists of fields which can achieve peak production through the substitution of variable expenses for the high fixed capital costs associated with permanent infrastructure. The third category is comprised of fields that are favorably situated and able to take advantage of existing infrastructure --thus avoiding both high capital costs and high variable costs.

When estimating the cost relationships as discussed below we focus on the fields in category 1. Categories 2 and 3 have to be considered special low-cost cases. The fields of these two categories will be analyzed separately at a later stage. By excluding such fields we bias the minimum field size upwards and the level of ultimate recoverable reserves downwards, even if only slightly so.

Although the WM sample reflects considerable variation in the size of recoverable reserves, the current sample size is deficiently small. This turns out to be consequential in the estimation attempts, because of our inability to reliably estimate nonlinearities that are inherent in the cost functions.

Scale economics in reservoir development would generally be expected to lead to a cost relationship similar to the curved line in Figure 2, below. The S-shape of the curve indicates that the unit cost of reservoir development first decreases with size, and later stabilizes or increases again as the source of scale economies is exhausted. Unfortunately, the WM data provide too few observations on the left-most segment of the cost curve. The available data fall mainly in the intermediate, nearly-linear segment. Consequently, the best linear fit to the data points implies negative intercepts and strong diseconomies of scale (represented by the

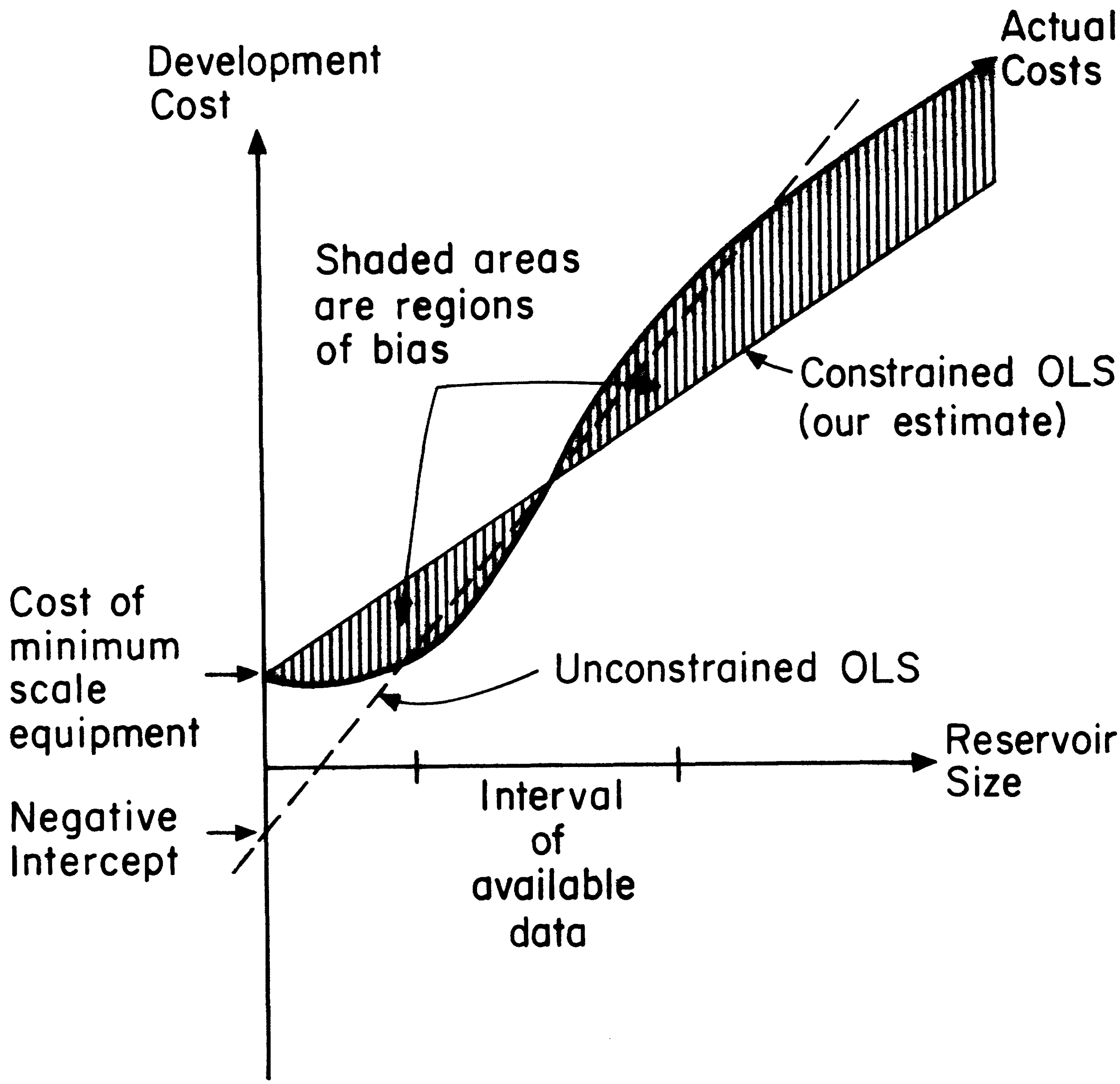

FIGURE 2 DIAGRAMMATICS OF COST
ESTIMATION PROBLEMS

dotted line). We believe this to be a statistical artifact rather than a valid cost relationship.

To mitigate the problem of scale diseconomies we elected to constrain the intercepts to positive values indicated by rough engineering-type minimum cost analyses. With this constraint, the data is then fitted to the best linear function of the explanatory variable. The estimated cost relationship would appear as the solid straight line in the figure. This is obviously not the best approach to estimation, but for the present time we are prohibited from more complex and sophisticated specifications by the lack of data.

In addition, the small number of observations and the homogeneity of the North Sea with respect to non-size characteristics made the coefficients of non-size characteristics when included as explanatory variables in the cost relationships turn out to be not significant. Although engineering type analysis clearly points out the significance of flow rate and water depth as determinants of total capital expenditures, our sample did not allow us to verify this.

The units of recoverable reserves, R, and the cost categories are measured in millions of barrels of oil and in millions of January 1976 dollars respectively for the purpose of estimation. The statistic usually reported to indicate the goodness of fit, R^2, is not meaningful when the intercept is constrained. To indicate the explanatory power of the relationship we have calculated the simple correlation, r, between the true and fitted values for each cost category. The results of our estimating effort can then be presented as follows:

1. Development drilling, CD. The intercept constrained to 50.

   $CD = 50 + 3.88 \sqrt{R}$ $(r=0.96)$ (4.1)

2. The platform structures and their installation, CP. The intercept constrained to 137.5.

   $CP = 137.5 + 0.28\ R\ (r=0.84)$ (4.2)

3. The platform equipment, CPE. The intercept constrained to 100.

   $CPE = 100 + 0.09\ R\ (r=0.91)$ (4.3)

4. Pipelines and transportation facilities except terminal, CT. The intercept constrained to 25.

   $CT = 25 + 7.16\ \sqrt{R}\ (r=0.85)$ (4.4)

5. Terminal expenditures, CTER. The intercept constrained to 20.

   $CTER = 20 + 0.083\ R\ (r=0.94)$ (4.5)

6. Miscellaneous expenditures, CMISC. The intercept constrained to 20.

   $CMISC = 20 + 1.12\ \sqrt{R}\ (r=0.59)$ (4.6)

7. Operating cost platform and installations, COP. The intercept constrained to 20.

   $COP = 20 + 0.024\ R\ (r=0.83)$ (4.7)

8. Operating cost transportation and terminal facilities, COT. The intercept constrained to 10.

   $COT = 10 + 0.255\ \sqrt{R}\ (r=0.62)$ (4.8)

To indicate the explanatory power of recoverable reserves (R) for total development expenditures, (CE) the following equation was estimated in an unconstrained form:

$$CE = 320 + 0.785\ R\ \ (R^2 = 0.8163) \quad (4.9)$$

We emphasize that these estimates are very preliminary and should be interpreted as nothing more than approximate summary relationships useful mainly for the pedagogical purpose of illustrating the methodology of the Basin Development Model. We are still at an early stage in our data gathering effort and hope to improve on the results presented above.

For the present, it would be useful to recount the sources of bias that probably afflict our estimates. Recall (p. 54) that our sample excludes special low-cost fields altogether (categories 2 and 3). This causes us to overstate the minimum economic pool size in the North Sea area. In addition, the schematic presentation of cost curves (Figure 2) suggests that our "linearization" of the function would cause the costs of small fields to again be overestimated.[1] Thus, areas for additonal empirical work are clearly indicated.

4.2 Analysis of Minimum Reservoir Size

The Reservoir Development Sub-Model is a cash-flow model. To distribute the total capitalized investment and operating expenditures as determined by equations (4.1) to (4.8) over time we have to make assumptions about investment and production profiles. The WM data allow us to calculate an average investment and production profile for fields in different size categories. These profiles are listed in Table 7 below.

Our point of reference as far as time is concerned is the year of discovery. For cash-flow and tax purposes it is necessary to include the

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[1] Not all cost equations were estimated in a pure linear form. The square root transformation preserves the curvature of several of our estimates.

TABLE 7

Fraction of Total Exploration/Delineation Expenditures, Investment Expenditures, and Recoverable Reserves Occurring In Each Year Following Discovery

Year	Exploration/ Delineation Profile All Fields	Fields < 300 MM Bbl		300 MM Bbl ≤ Fields ≤ 1500 MM Bbl		1500 MM Bbl < Fields	
		Investment Profile	Production Profile	Investment Profile	Production Profile	Investment Profile	Production Profile
1	0.1	0	0	0	0	0	0
2	0.2	0	0	0	0	0	0
3	0.2	.04	0	.04	0	.04	0
4	0.2	.44	0	.12	0	.12	0
5	0.2	.27	.09	.20	.03	.20	.01
6	0.1	.11	.13	.24	.08	.24	.04
7		.08	.15	.16	.10	.16	.06
8		.06	.13	.07	.10	.07	.09
9			.13	.06	.10	.06	.10
10			.11	.06	.10	.06	.10
11			.08	.05	.10	.05	.10
12			.07		.10		.10
13			.06		.08		.10
14			.05		.06		.08
15					.05		.07
16					.04		.05
17					.03		.03
18					.03		.03
19							.02
20							.01
21							.01

time profile of exploration and delineation expenditures. Even if there are substantial differences a reasonable average is an expenditure of $600,000 for geophysics and geology along with an average of 6 exploration/delineation wells at $4.6 million per well, totalling an average exploration expenditure per field of approximately $30 million dollars. The time-profile for exploration/delineation expenditures is also included in Table 7.

4.2.1 Taxes As we indicated earlier (p. 14 note), the British and Norwegian tax systems do more than simply extract economic rent from the operating companies. The various non-profit-related production fees, therefore, impinge on the net present value of each reservoir, postponing development of some reservoire that would otherwise have been produced. As a result, the minimum economic pool size based on a calculation of real social costs is smaller than the minimum size of an economic reservoir based on the costs incurred by the industry. A description of the fiscal regimes of the UK and Norway, as included in the Reservoir Development Model is given below.[1]

United Kingdom:

The current tax laws became effective in November of 1974. Goverment revenues are comprised of the following components: (1) royalty payments, (2) petroleum revenue tax, (P.R.T.), and (3) corporate tax.

Royalty is calculated as a percentage of gross oil production on a per field basis. The current rate is 12.5 percent. The Energy Ministry has the power to refund the royalty wholly or in part.

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[1] This discussion of tax regimes appears in Beall [3].

The petroleum revenue tax considers that an investment fence extends around each field, including pipelines and terminal facilities allocated to that field. A "field" includes all areas within 5000 meters of the field boundary. Exploration or delineation costs, even if abortive, are allowed as expense if within this fence, with "uplift" on investment.[1]

Field by field computation of P.R.T. is required, thus current losses on one field cannot be offset against profits on another field. P.R.T. is payable at 45 percent of corporate taxable income which itself reflects the following deductions:

(1) Investment is multiplied by an "uplift" factor of 1.75 for the purpose of calculating taxable income.

(2) The operator receives an oil allowance or the cash equivalent of 7.3 million barrels of oil per year of production subject to (a) 73 million barrels maximum over the field life, (b) a carry-forward of unused amounts, but still subject to 7.3 million barrels per year maximum deduction, and (c) the allowance does not start until uplifted investments have been recovered.

(3) The maximum P.R.T. liability in any year is 80 percent of the difference between the taxable income for P.R.T. before oil allowance, and 30 percent of investment.

(4) Interest costs are not allowed as expense for P.R.T. calculations.

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[1]See the next paragraph for the definition of "uplift".

(5) P.R.T. is not payable on gas fields with signed contracts to the British Gas Corporation as of June 30, 1975.

(6) Although not in the legislation, it is apparently the government's intention that the rate of P.R.T. can/will be changed if crude prices change substantially in real terms. P.R.T. can thus be construed as an excess profits tax.

The corporate tax computation is relatively straight-forward and payable at a rate of 52 percent subject to the following deductions.

(1) Operating costs, royalty payments, interest costs, and P.R.T. are fully deductible from revenue.

(2) Depreciation is fully deductible and can be written off as incurred if a tangible investment. Intangible investment is written off over the project life.

(3) Loss carry-forward is deductible and written off as fast as income is available.

(4) Deficits anywhere in the U.K. North Sea can be applied against income in the North Sea, but not against onshore income. Deficits onshore can be applied against North Sea income. Corporate tax payment lags by one year whereas P.R.T. is paid as accrued. The tax-reference price of crude, (the Norm Price), will probably be set equal to the average U.K. North Sea realized price.

Norway:

Under the new petroleum tax law which became effective in January of 1975, government revenues are comprised of the following: (1) royalty payments, (2) corporation tax, (3) state tax, (4) local tax, (5) special tax, (6) withholding (source) tax on distributed dividends, and (7) capital tax. All taxes are deferred one year.

Royalty is calculated as a percentage of gross oil production on a per field basis. For blocks allocated in the first licensing round, the royalty is fixed at 10 percent. For all subsequently licensed blocks, royalty is computed on the basis of production rates as follows:

40,000 barrels/day or less	= 8 percent
40,000 to 100,000 bbls/day	= 10 percent
100,000 to 225,000	= 12 percent
225,000 to 250,000	= 14 percent
350,000 and greater	= 16 percent

Once the royalty rate reaches 12 percent, it does not decline with subsequently lower production levels.

The corporation tax is payable at a rate of 50.8 percent on the basis of revenue less operating costs, royalty, depreciation, loss carry-forward, interest costs, and distributed dividends. Payment is deferred one year. Deductions are explained as follows:

(1) Depreciation of production and transportation facilities will be linear over a period of six years from the year the plant was taken into ordinary use, or when petroleum is produced.

(2) Carried-forward losses can be deducted provided they arise from offshore operations during the past 15 years. The losses must be spread over a 3 year period on a straight-line basis. All offshore losses can be offset against

other company profits derived from Norwegian activities. However, only 50 percent of losses derived from other Norwegian activities can be offset against offshore profits. For purposes of calculation, we must ignore the possibility of external losses in this study.

(3) Interest costs may be deducted from taxable income whether it arises from a parent company loan or a third-party loan. Interest is deductible for both corporate and special tax.

State tax is payable at a rate of 26.5 percent of net taxable income less distributed dividends. Distributed dividends are available earnings less tax liability, and will probably vary between 30 and 60 percent of net taxable income.

Local tax is computed as 24.3 percent of net taxable income.

The special tax can be essentially construed as an excess profits tax. The special tax is computed at a rate of 25 percent of taxable revenue less operating cost, royalty, intangibles expensed, depreciation, interest, losses carried-forward, and taxfree income. Tax free income is ten percent of tangible investment that has been put into operation in the preceding 15 years but purchased prior to the end of the preceding year. The unused portion may be carried forward.

The withholding (source) tax is computed on the basis of 10 percent of distributed dividends. Payment is deferred one year.

The capital tax is calculated at a rate of 0.7 percent of the net capital, (i.e. after depreciation), carried on the company's books. Taxable capital includes production, transport and storage facilites as well as other equipment used in the company's activities. The same applies to stocks of products produced, securities and bank deposits. The capital tax is not regarded as deductible in the assessment of other taxes.

4.2.2. Participation and Government Policy. In Norwegian waters, current government participation varies from 5 to 50 percent on selected blocks. The Ministry of Industry is now composing a standard contract with active government participation for future awards. The government's share will vary from 20 to 50 percent, and will be exercised per discovery. Statoil will not share in costs until a commercial discovery is made, and will take it's share in kind. The private participants will, in turn, have to market Statoil's share if this is desirable. Current developments in Norway indicate that Statoil intends to become an internationally integrated oil company as rapidly as possible.

British intentions regarding participation have been considerably less aggressive as compared to Norway. Agreements reached to date primarily involve loan guarantees on the part of the government in return for agreements on an option to purchase a significant share of production on a per field basis. The government has repeatedly emphasized that private companies would be no better or no worse off than before signing participation agreements. One might infer that the main thrust of the participation is the pronouncement that any new licenses will be issued on the stipulation of majority U.K. government participation in all discoveries.

To separate the issue of fiscal effects from the participation effects, the participation issue will not be included in the discussion below.

4.2.3 Sample Estimates. The minimum reservoir size is the size of the reservoir that makes the net present value of the cash-flow to the operating company resulting from delineating, developing and producing the reservoir equal to zero. In our examples the discovery year is 1976. In line with the WM estimates[1] we have assumed a rate of capital expenditure inflation of 25% in 1976, of 20% in 1977, and of 10% thereafter. We assumed that both price and operating expenditures would stay constant in current dollars and that the general worldwide rate of inflation would be 8% over the production horizon of the reservoir. The discount factor applied was 18%; 10% for time preference and risk, and 8% for general inflation.

To indicate the price-sensitivity of the minimum reservoir size we ran the reservoir model assuming three different price scenarios. In the first case the oil price was assumed to stay constant in current dollars at $12, in real terms the price starts at $12 and then erodes with the general rate of inflation. This price scenario corresponds approximately to an average real price over the time-horizon of a reservoir (assuming the shortest production profile of Table 7) of $7 (in January 1976 dollars). This oil price scenario is identical to the WM price assumptions. Under this price scenario and a debt-ratio of 80%, a six-year repayment schedule, and a rate of interest of 12%, the reservoir size that results in a net present value of the cash-flow to the private operator of approximately zero is 250 million barrels of oil under the British as well as the Norwegian tax regime.

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[1] The Petroleum Economist, December 1975 pp. 458-460.

The mix of tax instruments used in the British and the Norwegian tax regimes is different. The identical effect of the two regimes on a reservoir produced under the set of assumptions described above seem to indicate that the negotiations between the oil industry and the two governments centered around the fisal effects on such a reservoir. Because of the different mix of instruments used in the two regimes the fiscal effects will, however, differ for reservoirs developed and produced under a different set of circumstances than those of what we might label the "base case" as described above. A more detailed discussion of the two tax regimes is beyond the scope of this discussion, but will be the topic of a separate working paper.

We also calculated the minimum reservoir size assuming an average real price of $9 and $12 (in January 1976 dollars). These price assumptions are also well within the range of likely price-paths to be observed in the international petroleum market [5]. The net present value of the after-tax cash-flow in the British and the Norwegian sectors is not identical under these two price scenarios. The two net present value figures are, however, sufficiently close to zero and to each other, to allow us to determine a common minimum reservoir size for the two North Sea sectors. An average real price of $9 results in a minimum reservoir size of approximately 200 million barrels, whereas a $12 average price makes the minimum reservoir size drop to about 90 million barrels of recoverable reserves. These reservoir calculations are summarized in Table 8.

TABLE 8

Minimum Reservoir Size: Sample Calculations

REAL PRICE 01/01/76	RECOVERABLE RESERVES (mm bls.)
$7	250
$9	200
$12	90

The price assumptions above refer to the landed price of crude oil. To get to the wellhead price which is the tax-reference price for royalty payments a unitized transportation cost was estimated. Administrative practice seems to be to divide total capitalized transportation expenditures by total recoverable reserves rather than the net present value of the production profile. We followed this "administrative practice" when estimating unitized transportation costs.

If the cost inflation is higher and/or the price inflation is lower than the average worldwide rate of inflation, which is the case in the North Sea, according to the WM assumptions, then the minimum reservoir size will increase over time. For the purpose of illustrating the link between the reservoir and the discovery submodels we choose to disregard this aspect of the discovery/development process. The significance of this aspect is indicated in section 4.3. The issue of the time-profile of the minimum reservoir size will be included in the next version of this paper.

4.3 North Sea Discoveries.

As shown in Figure 3, the area of study[1], hereafter called the Area, has been delimited on the basis of exploration parameters which consist of: (1) selected depth contours on the base of the Paleocene, (2) the primary structural elements of the North Sea, (3) discussions with explorationists and data from the literature, and (4) geopolitical considerations. The Area is defined between 56° and 62° North latitude, the boundary of Norway and Denmark, and the zero Paleocene depth contour. Areas outside these approximate boundaries are considered poorly to non-prospective for the purposes of this investigation.

The various depth contours and structural elements define a central North Sea graben or down-faulted trough which generally contains the thickest sedimentary section, particularly of post-Jurassic sediments. It is this sedimentary section which contains most of the currently-known reserves. Production presently derives from three main horizons/intervals: (1) Tertiary Paleocene sands, (e.g. Forties and Frigg), (2) Danian reservoirs, (Ekofisk complex), and (3) the major Jurassic producing horizon of the North Sea, (e.g. Statfjord, Brent, and Piper).
it is considered impractical, for the purposes of this study, to attempt to identify separate potential fairways within the Area. Of the three horizons, the Ekofisk-type production appears to be limited most specifically to the deeply buried central basin. The reader is therefore advised that this latter region has the greatest Danian potential although statistical treatment to follow does not differentiate.

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[1] The area of study is discussed in greater detail in Beall [3].

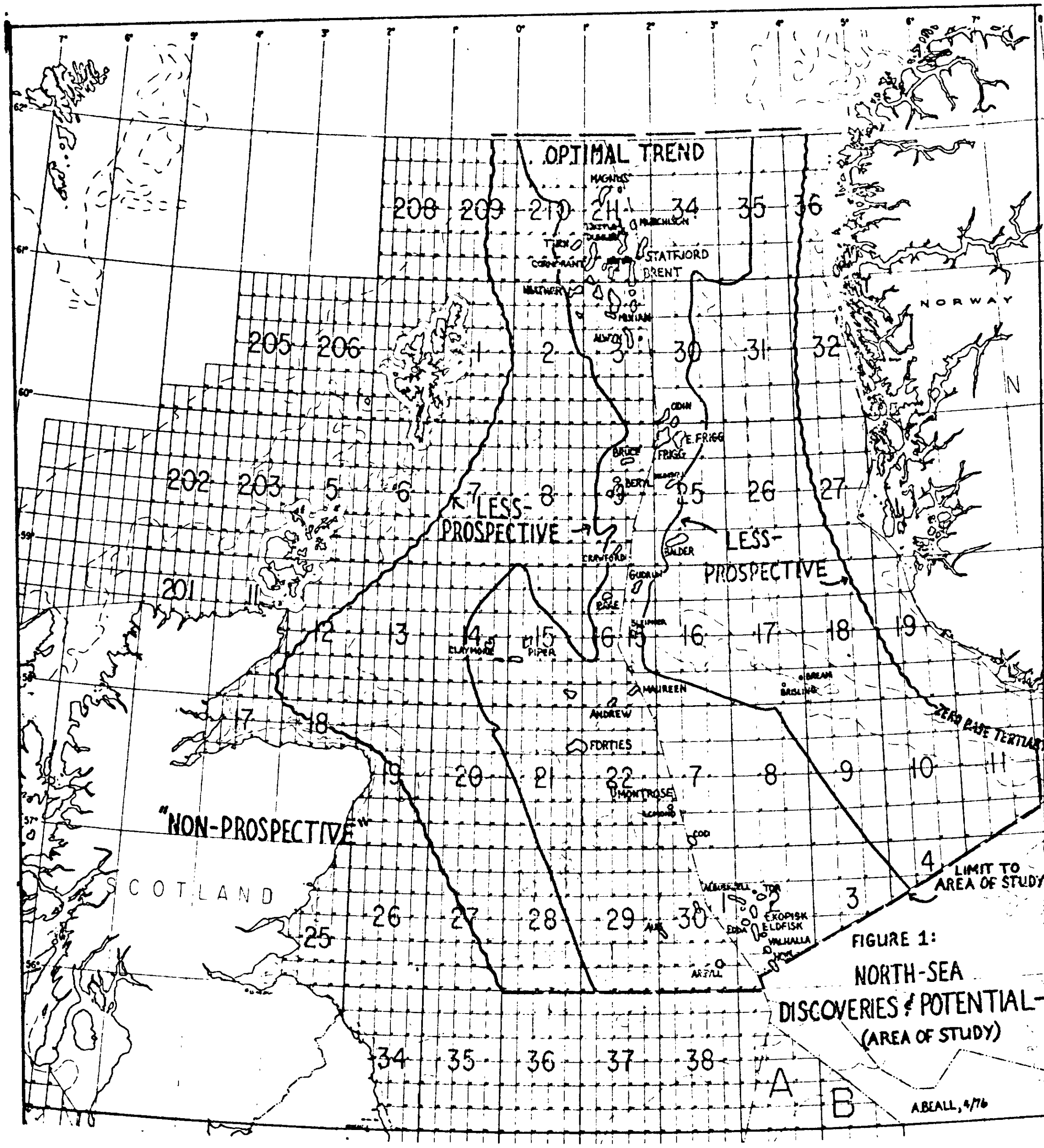

FIGURE 3

North Sea Discoveries and Potential - (Area of Study)

Source: Beall [2], p. 14.

The Area, (Figure 3), has been further differentiated into an "optimal or prime trend" and a "less prospective trend". While a discussion of the geologic basis for this differentiation is beyond the scope of this paper, it can be noted that the discovery rate within the prime trend (24 percent) is substantially better than the rate (6 percent) within the less prospective trend. Finally, the boundary between the two trends is rather arbitrary in the southern part of the area of study, and is placed on the basis of the -3000 feet contour on the base of the Paleocene.

At this stage of exploration, the northern North Sea has reached an intermediate stage of exploration evaluation. Considerable amounts of seismic data of post-1970 vintage are now available over the entire area of interest. This data, in conjunction with geologic data derived from boreholes and field studies, comprises the main body of data on which new prospects are generated.

Within the prime trend, approximately 51 discoveries were made with a wildcat effort of 210 wells. The less prospective area yielded only 3 discoveries out of 49 attempts. The average success rate is 21 percent for all 259 wildcats. An independent assessment by CONOCO personnel estimates 31 "commercial" discoveries out of 139 attempts, for a success rate of 22 percent. As we will focus primarily on the area within the prime trend a geological success rate of 24% will be assumed in the following.

The rate of exploratory drilling is to a large extent determined by the concession agreements. Based upon existing concessions and past experience the offshore division of the Norwegian shipbroker company R.S. Platou A/S[1] estimated the exploratory activity over the next three

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[1] R.S. Platou A/S: OFFSHORE NEWSLETTER January 9, 1976; p.2.

years to be an accumulation of 200 wells drilled in the British sector and 65 wells drilled in the Norwegian sector. If we assume that on the average four delineation wells will be required to determine the reserves of each discovery, that the geologic success ratio and the rate of exploratory drilling will stay constant over the three-year period, then there will be made 8 discoveries in the British sector and 3 discoveries in the Norwegian sector each year in the three years to come, namely 1976, 1977, and 1978. As this simplified estimation of the rate of geologic discoveries is valid only if the working commitments of the companies can be determined with reasonable confidence, we will limit ourselves to the next three years and the undiscovered potential of industry-held acreage.

4.3.1 Geologic-Judgmental Discovery Analysis. An inventory of acreage within the designated Area has been assembled in Table 9. Note that the Norwegian blocks have been converted to U.K. size for purposes of analysis. The total number of blocks, (U.K. size), is 995. Within the prime area of exploration, there are 431 blocks. Industry has held some approximately 308 blocks. Within the less prospective area, there are approximately 564 blocks. Industry has held approximately 153 blocks, with subsequent relinquishment of 51 blocks. Of the blocks currently held, 75 percent are in the prime area and 25 percent in the less prospective area, a significant change from the original holding of 70 percent and 30 percent respectively. This trend will continue, as most of the prime acreage in U.K. waters is held by industry. Norway, by way of contrast, still has some 62 blocks considered to lie in the prime area which have never been awarded. An additional 12 blocks of the industry sector are held by the Norwegian national oil company, Statoil.

TABLE 9

North Sea Acreage

(Blocks Converted to UK Size Equivalent)

	UK		Norway	
	Prime Area	Less Prospective Area	Prime Area	Less Prospective Area
Blocks Never Awarded	11	222	62	189
Blocks Relinquished	13	12	37	39
Blocks Retained by Industry	215	48	81	54
Blocks Reserved For Statoil	-	-	12	-
Total	239	282	192	282

In order to estimate the undiscovered potential of industry-held acreage, it has been necessary to establish the percentage of acreage that is considered "prospective" under current industry interpretation.

We have not yet had access to company estimates of the potential of all the blocks in the North Sea. We did, however, get access to company estimates of the potential of about 6% of the blocks in the prime area. This sample of blocks was considered representative of the prime area. The distribution of prospects and of prospects per block as indicated by the sample was consequently scaled up to serve as a distribution of the prospect and block potential of the prime area. The results are summarized in Tables 10 and 11. The size categories are designed to match the minimum reservoir categories of Table 8 as well as to indicate the prospect and block potential arrived at. A prospect version (Table 10) as well as a block version (Table 11) of Table 5 (Section 3) are included to indicate the significance of the chosen unit of analysis. If the prospect potential of each block is considered one potential reservoir for minimum block size purposes, the fact that two or more submarginal prospects may add up to one economic block potential will add to total recoverable potential as indicated by the difference between Tables 10 and 11. There is an average of 1.636 prospects per prospective block in our block portfolio. 60% of the blocks are non-prospective.

In Table 12, North Sea discovery scenarios are carried out under the three minimum pool size/price expectation assumptions and the two discovery process hypothesis of Table 6 (Section 3). Under the first hypothesis (H1) it was assumed that each projected discovery would result in additions to reserve equal to the average of all prospective blocks (prospects) reduced in proportion to the fraction of total prospective reserves that was

TABLE 10

Expected Recoverable Reserves (Prospect Potential)
On Industry Held Acreage In Prime Area (MM Bbl)
(Geological Success Rate = 0.24)

Prospect Size Category	Number Of Prospects	Mean Size Within Prospect Category	Recoverable Reserves
$450 \leq S_1$	49	550	6512
$250 \leq S_2 < 450$	132	350	11051
$200 \leq S_3 < 250$	33	210	1663
$90 \leq S_4 < 200$	82	115	2263
Total	296	300	21489

TABLE 11

Expected Recoverable Reserves (Block Potential)
On Industry Held Acreage In Prime Area (MM Bbl)
(Geological Success Rate 0.24)

Block Size Category	Number Of Blocks	Mean Size Within Block Category	Recoverable Reserves
$450 \leq S_1$	82	816	16059
$250 \leq S_2 < 450$	49	317	3728
$200 \leq S_3 < 250$	16	220	845
$90 \leq S_4 < 200$	33	135	1069
Sum	180	500	21701

TABLE 12

Geologic-Judgmental North Sea Discovery Scenarios

(Million Barrels of Recoverable Barrels of Oil)

		H1						H2					
		250		200		90		250		200		90	
		P*	B*	P	B	P	B	P	B	P	B	P	B
1976	UK	1960	2230	2145	2325	2400	2445	4400	3990	4400	3990	4400	3990
	N	735	836	800	872	900	917	1650	1497	1650	1497	1650	1497
1977	UK	1960	2230	2145	2325	2400	2445	2911	3990	2911	3990	2911	3990
	N	735	836	800	872	900	917	1091	1497	1091	1497	1091	1497
1978	UK	1960	2230	2145	2325	2400	2445	2800	3811	2800	3811	2800	3811
	N	735	836	800	872	900	917	1050	1430	1050	1430	1050	1430

*P indicates analysis on the "prospect" unit of observation;

B indicates analysis on the "block" unit of observation.

considered economically submarginal. This is a random drilling hypothesis. The first entry in Table 12, U.K. discoveries in 1976 under hypothesis H1 based on the prospect data and assuming a minimum economic size of 250 million barrels is thus arrived at in the following way. The mean prospect size is 300 million barrels of recoverable oil (MMBO). Potentially recoverable reserves in reservoirs smaller than 250 MMBO are 1663 plus 2263 MMBO out of a total of 21489 MMBO. As discussed above we assume that there will be made 8 discoveries in the British sector in 1976. These assumptions are combined as follows to produce the first entry of Table 12:

$$\left(1 - \frac{1663 + 2263}{21489}\right) * 300 * 8 \approx 1960.$$

Under the second hypothesis (H2) drilling proceeds deterministically according to ranked size. Because the largest prospects are drilled first, the minimum reservoir size will affect the termination of the discovery process but not the discovery rate at an early stage in the sequence. According to H2 (Table 12), 3990 MMBO will be found in the British sector of the North Sea in 1976 if the block is the relevant unit of analysis. As stated above there is an average of 1.636 prospects per prospective block. According to Table 11, the mean size of the largest block size category is 816 MMBO. Again assuming 8 discoveries in 1976, the 1976 contribution to oil reserves is produced as follows:

$$816 * 8 * \left(\frac{1}{1.636}\right) = 3990.$$

The Norwegian reserve additions are produced by multiplying the British estimates by 3/8, which is the ratio between the exploration effort in the two sectors of the North Sea.

4.3.2 Statistical Discovery Analysis. The statistical approach is based upon an analysis of the discovery history of the North Sea. Table 13 shows current assessment of recoverable reserves in the area of study, along with order of discovery, field names, spud date, and number of wildcats spudded up to that time. Gas reserves have been converted to oil-equivalent values using a conversion factor of 1 trillion cubic feet of gas equal to 178 million barrels of oil.

Examination of the table would seem to indicate a more or less random distribution of large-reserve discoveries. It should be noted that the record for 1975 is somewhat incomplete. Revision should not greatly affect the conclusions drawn herein. Classification of an announced discovery as "significant" is highly subjective during the early phases of evaluation in most instances. Table 13 is complicated by inclusion of some discoveries which undoubtedly are not commercial in themselves. At the same time, in order to fully evaluate the amount of discovered hydrocarbons currently known as well as to be discovered, it appears important to assess the amount present in accumulations down to 50 million barrels in size. Current proven reserves are estimated at 29.369 billion barrels oil equivalent, of which 22.648 billion barrels, or 77.1 percent, is oil. Of the 59 discoveries, 8 can be classified as true gas accumulations with very little associated liquid.

Figure 4 illustrates the reserve data plotted cumulatively in terms of reserves and in terms of discovery size class.[1] Note that both

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[1]The difference between the two curves is explained by the fact that the fraction of total reserves in small reservoirs is smaller than the fraction of the total number of discoveries that hit small reservoirs.

TABLE 13

Northern North Sea Discoveries

Order of Discovery	Field Name or Location	Spud Date	Cumulative Wilcats	Recoverable Reserves Oil Equiv.	(oil)
1	Cod	2/68	16	159	(25)
2	Montrose	4/69	31	200	(200)
3	Ekofisk	9/69	46	1932	(1060)
4	Josephine	6/70	52	250	(250)
5	Tor	8/70	55	245	(150)
6	Eldfisk	8/70	56	927	(500)
7	Forties	8/70	58	1800	(1800)
8	W. Ekofisk	8/70	60	706	(350)
9	Auk	9/70	64	50	(50)
10	Frigg	4/71	70	1264	(0)
11	Brent	5/71	72	2375	(1750)
12	Argyll	6/71	74	75	(75)
13	Bream	12/71	89	75	(75)
14	Lomond	2/72	95	500	(500)
15	S.E.Tor	4/72	96	34	(25)
16	Beryl	5/72	100	550	(550)
17	Cormorant	6/72	103	400?	(400)?
18	Edda	6/72	104	126	(55)
19	Heimdal	7/72	107	414	(23)
20	Albuskjell	7/72	109	560	(150)
21	Thistle	7/72	111	450	(450)
22	Piper	11/72	123	800	(800)
23	Maureen	11/72	124	500	(500)
24	Dunlin	4/73	138	400	(400)
25	3/15-2	4/73	141	150	(150)
26	Hutton	7/73	153	300	(300)
27	Alwyn	7/73	154	500	(500)
28	E. Frigg	8/73	157	623	(0)
29	Heather	8/73	159	150	(150)
30	Brisling	8/73	160	75	(75)
31	Ninian	9/73	163	1200	(1200)
32	Statfjord	12/73	178	4595	(3900)
33	Odin	12/73	181	178	(0)
34	Bruce	3/74	188	450	(450)
35	Magnus	4/74	190	1080	(1080)
36	N.E.Frigg	4/74	191	71	(0)
37	Balder	4/74	193	100	(100)
38	Andrew	4/74	195	?	?
39	Claymore	4/74	196	400	(400)
40	E. Magnus	6/74	208	250	(250)

TABLE 13--(Continued)

Northern North Sea Discoveries

Order of Discovery	Field Name or Location	Spud Date	Cumulative Wildcats	Recoverable Reserves Oil Equiv.	(oil)
41	9/13-4	6/74	210	220	(220)
42	15/6-1	9/74	223	150	(150)
43	Brae	9/74	226	185	(185)
44	Sleipner	9/74	227	50	(0)
45	Hod	11/74	237	75	(75)
46	211/27-3	11/74	238	450	(450)
47	Gudrun	11/74	239	450	(0)
48	2/10-1	11/74	240	100	(100)
49	3/4-4	12/74	244	100	(100)
50	14/20-1	1/75	245	75	(75)
51	Crawford	1/75	246	150	(150)
52	9/13-7	1/75	247	350	(350)
53	3/8-3	1/75	248	100	(100)
54	Tern	2/75	249	175	(175)
55	21/2-1	2/75	254	175	(175)
56	3/2-1A	3/75	260	200	(200)
57	Valhalla	4/75	264	50	(50)
58	3/4-6&3/9-1			200	(200)
59	15/13-2			200	(200)
60	211/26-4			175	(175)

Source: Beall [3], pp. 17-18

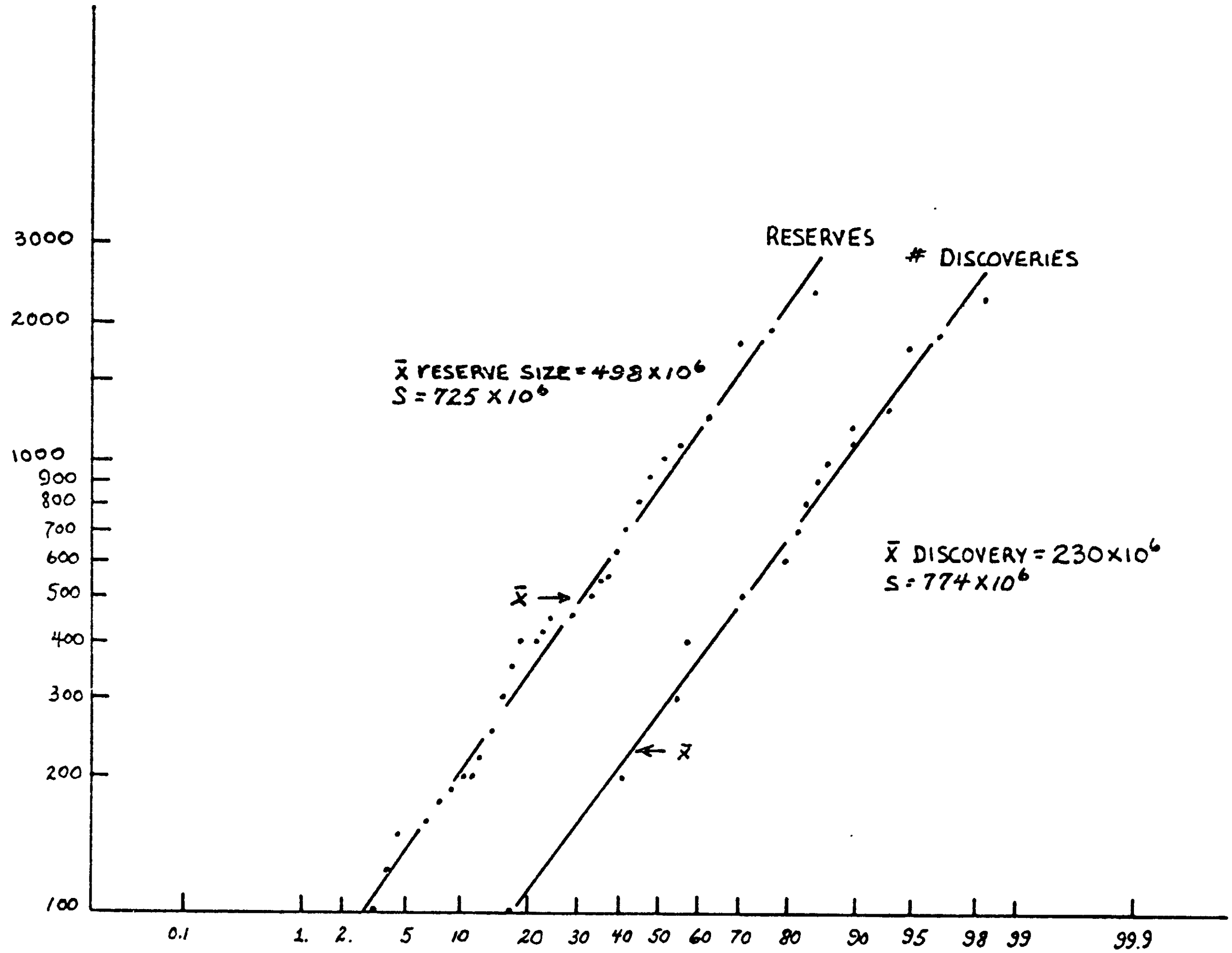

FIGURE 4

Cumulative Distribution, North Sea Oil and Gas Reserves and Discoveries

Source: Beall [3], p. 20

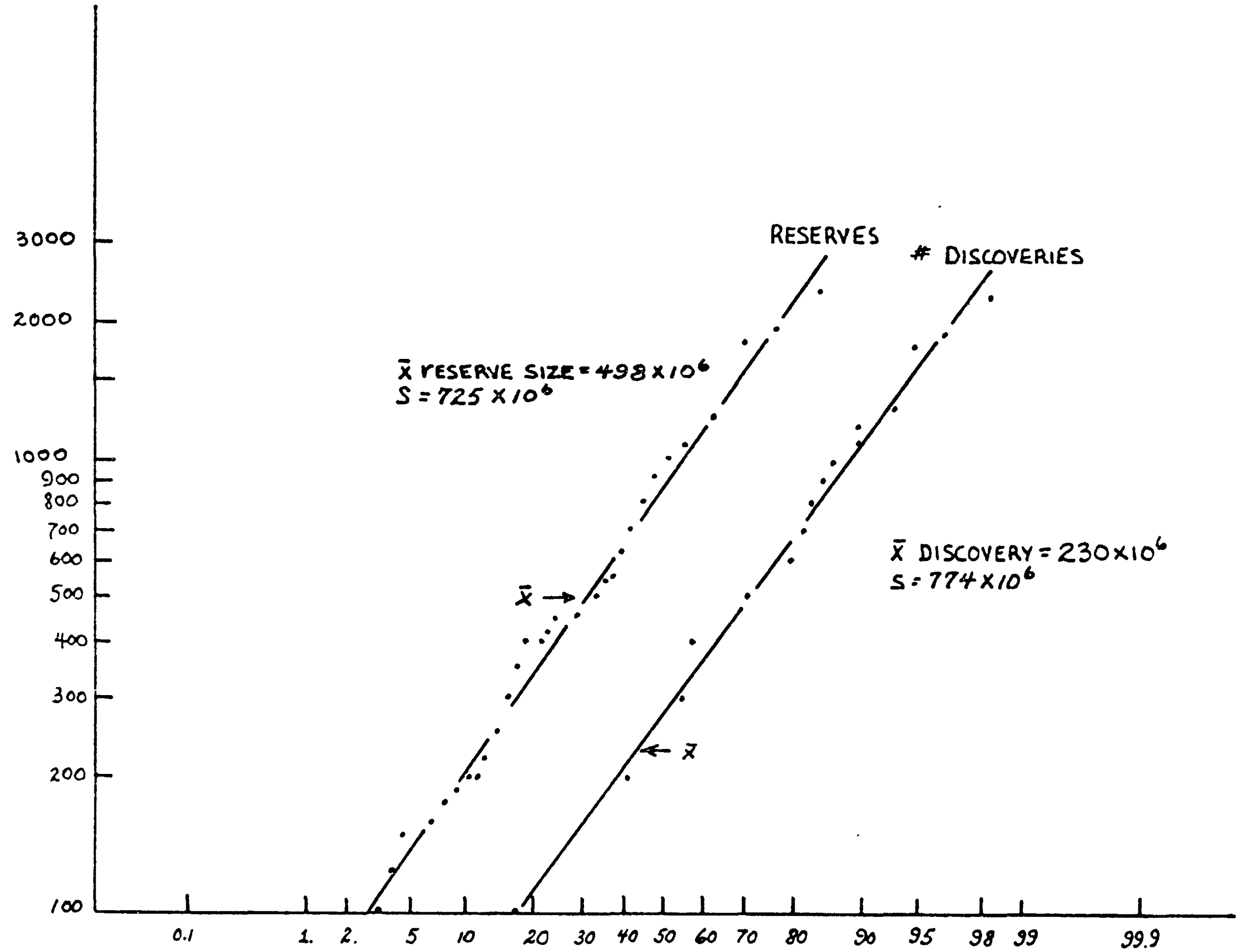

FIGURE 4

Cumulative Distribution, North Sea Oil and Gas Reserves and Discoveries

Source: Beall [3], p. 20

distributions are good approximations of a log-normal distribution, as would be predicted by Kaufman. The mean discovery size is 230 million barrels recoverable whereas the mean reserve size is significantly larger at 498 million barrels. These data are illustrated more graphically in Figure 5. Some 37 percent of the discoveries contain 64 percent of the total reserves. The largest discovery, Statfjord, represents over 15 percent of the total North Sea reserves. From these data one could estimate that the probability of discovery of another Ekofisk is very low (less than 5 percent), whereas the probability of encountering fields in the 500 million to 1 billion barrel class is relatively high.

Application of the estimation procedure cited earlier [2] to the data in Table 13 generates a log-normal distribution of reservoirs with mean size 645.5 MM barrels, and standard deviation 1,113.3. This distribution represents our best estimate of the variation in the size of reservoirs in the ground prior to the depletion due to exploratory drilling.[1]

On the assumption that the first 59 discoveries represent sampling "without replacement and proportional to size", it is then possible to derive the predictive distribution of the 60th discovery; and conditional upon the 60th, the 61st; etc. [2]. The sequence of predictive distributions for discoveries #60 through #96 is reported in Table 14. The table shows the expected value of reserves for each successive discovery, and also the partition of this total according to the respective size classes

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[1] At the time of estimation we did not have an estimate of reserves of of the Andrew field (discovery number 18 in Table 13). This field was consequently ommitted, and the parameter estimates reflect the remaining 59 historical oil and gas discoveries.

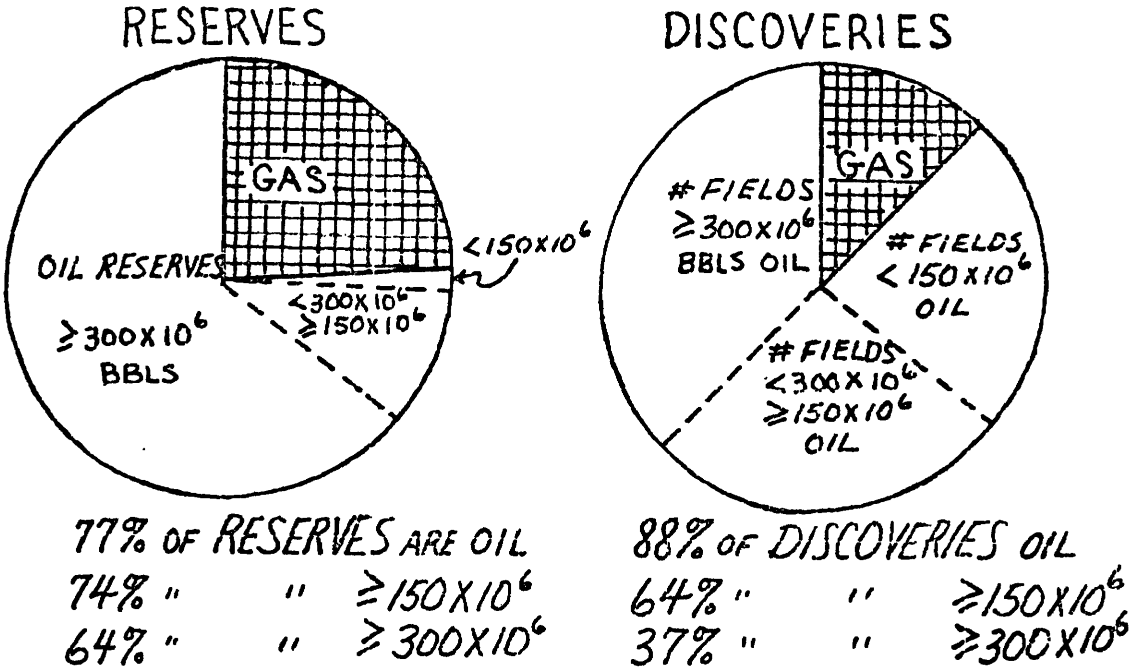

FIGURE 5

North Sea Reserves and Discoveries Summary Comparison

Source: Beall [3], p. 22

TABLE 14

Predictive Discovery Distributions

(Showing Partial Expectations, In MM Bbl Oil Equivalent)

Order Of Discovery

Size Category	60	61	62	63	64	65	66	67	68	69	70	71
$500 < S_4$	371	365	360	354	349	344	339	334	330	326	321	317
$375 < S_3 \leq 500$	56	56	55	55	54	54	53	53	52	52	52	51
$250 < S_2 \leq 375$	48	48	47	47	46	46	45	45	44	44	44	43
$125 < S_1 \leq 250$	36	35	35	35	34	34	33	33	33	32	32	32
Expected Value Of I^{th} Discovery (Total Expectation)	525	518	510	503	496	490	483	477	471	466	460	455
Size Category	72	73	74	75	76	77	78	79	80	81	82	83
$500 < S_4$	314	310	307	303	300	297	294	291	288	285	283	280
$375 < S_3 \leq 500$	51	50	50	50	49	49	49	48	48	48	48	47
$250 < S_2 \leq 375$	43	43	42	42	42	41	41	41	40	40	40	40
$125 < S_1 \leq 250$	31	31	31	30	30	30	30	29	29	29	29	29
Expected Value of I^{th} Discovery (Total Expectation)	450	445	441	436	432	428	424	420	416	413	409	406
Size Category	84	85	86	87	88	89	90	91	92	93	94	95
$500 < S_4$	278	276	273	271	269	267	264	262	260	258	257	255
$375 < S_3 \leq 500$	47	47	47	46	46	46	46	45	45	45	45	45
$250 < S_2 \leq 375$	39	39	39	39	38	38	38	38	38	37	37	37
$125 < S_1 \leq 250$	28	28	28	28	28	27	27	27	27	27	27	26
Expected Value of I^{th} Discovery (Total Expectation)	403	400	397	394	391	388	385	382	380	377	374	372

(i.e. the partial expectations discussed in Section 3, p. 46). Figure 6 indicates how the sequence of predictive distributions might appear graphically.

In order to calculate annual additions to economic reserves, a total discovery rate of 11 discoveries per year is allocated between the British and Norwegian sectors as before (8:3). The actual contribution of each discovery is calculated as the sum of partial expectations for the discovery corresponding to size classes exceeding the prevailing minimum economic pool size (see Section 3, p. 46).[1] The results of this calculation appear in Table 15.

The discovery scenarios of Table 15 assume that minimum economic pool size remains constant through time; i.e., that the real price of crude oil is unchanging. However, for this to be true, the current price of oil must increase substantially through time to compensate for general inflation. For example, the cost forecasts implicit in the WM data base show dramatic increase in development costs, far in excess of the general worldwide inflation rate. If in addition oil prices stay constant in current dollars then minimum economic pool size would increase by 40 percent, from 250 MMBO in 1976 to 360 MMBO by 1978.

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[1]The predictive densities of Table 14 represent oil and gas in barrels of oil equivalents. For the purpose of applying the minimum economic pool size, we assumed the oil content alone (taken as 77.1% of the total reserve) must exceed the cutoff point. Thus, the minimum pool size of 125 million barrels stated in oil and gas equivalents (Table 14) corresponds to a minimum discovery size of the oil component alone equal to approximately 90 million barrels. Recall that this criterion is consistent with a $7 real price scenario (p. 68). Similarly, the 250 million barrel cutoff point (Table 14) corresponds to a $9 real price scenario. The third scenario, $12 real price, is consistent with a cutoff point falling in the middle of the 250-375 million barrel category, stated in terms of oil and gas equivalent. We have had to allocate reserves in this category above and below the actual cutoff point by means of linear interpolation. The size classes will be appropriately modified in the future.

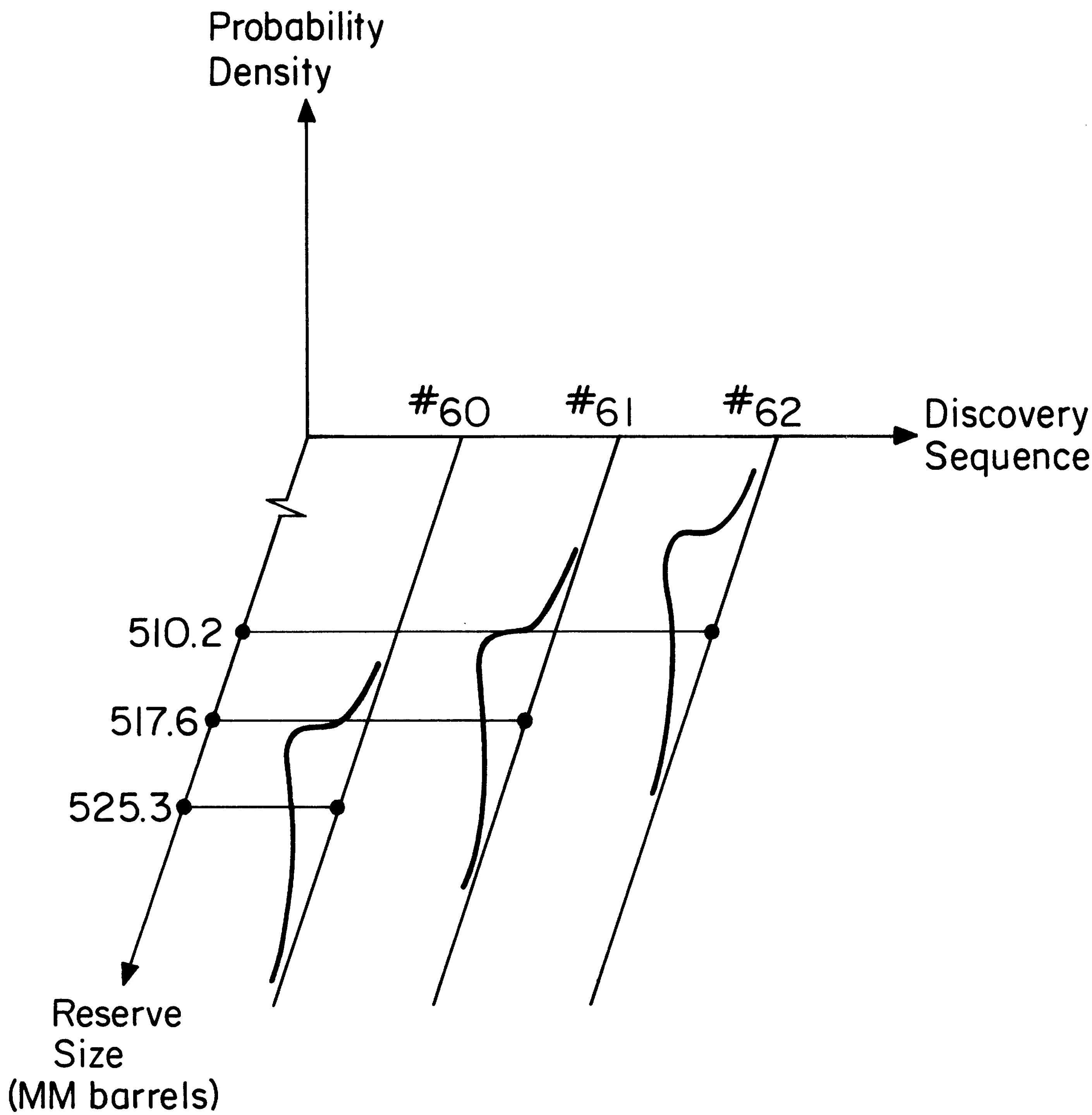

FIGURE 6 THE SEQUENCE OF PREDICTIVE DISCOVERY DISTRIBUTIONS

TABLE 15

Statistically Determined North Sea Discovery Scenarios

(Million Barrels Of Recoverable Oil Reserves)

		Minimum Pool Size		
		250	200	90
1976	UK	2585	2747	2956
	Norway	969	1030	1108
1977	UK	2264	2415	2601
	Norway	849	905	975
1978	UK	2057	2197	2369
	Norway	771	824	888

A more extensive treatment of the time profile of prices is clearly needed to reveal the sensitivity of supply forecasts to price projections. A broader discussion and analysis of these effects will be included in the next version of this paper.

Finally, when comparing the discovery forecasts in Tables 12 and 15 we see that neither approaches the recoverable reserves potential of the North Sea (as indicated by Tables 10 and 11). This suggests that the unregulated rate of exploratory drilling would have been greater than that which we observe today. If prospects as small as 75 million barrels of recoverable reserves are included in our prospect analysis, as well as acreage currently held by governments, we can derive a total grand ultimate potential for the North Sea of 60.2 billion barrels oil and gas or 46.3 billion barrels of oil reserves.

4.4 North Sea Forecasts

In the following section the implications of the three minimum pool size/price expectation scenarios for the level of production activity in the North Sea will be discussed. The three scenarios are those of Table 8. The discovery scenarios of Table 15 are considered a reasonable representation of Tables 12 and 15, and will be used in determining production forecasts.

We have not discussed the contribution of fields for which development plans have been announced as well as of the fields that have been recently discovered, but for which no development plans have yet been announced. We have assumed that existing fields will contribute to North Sea supplies as indicated by WM under all three price scenarios. Table 16 summarizes the contribution of these fields to the supply of North Sea oil

TABLE 16

Production Of Oil From Existing North Sea Fields

(Thousands Of Barrels Per Day)

Name \ Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
U.K.:												
Montrose	10	35	50	50	50	50	50	43	36	51	26	22
Forties	175	320	400	400	400	380	360	324	292	262	236	213
Auk	30	40	40	11	11							
Brent	20	50	170	250	350	470	480	470	370	300	280	260
Argyll	35	31	29	27	24	22	20					
Cormorant	-	10	25	45	45	45	45	40	36	33	30	27
Beryl	60	80	80	80	80	80	80	70	65	60	55	50
Thistle	-	20	100	180	180	160	125	100	90	80	70	60
Piper	80	170	220	220	220	200	180	160	140	120	100	90
Dunlin	-	10	40	60	80	90	100	100	110	100	90	80
Hutton	-	-	-	25	70	100	100	100	85	61	52	44
Alwyn	-	-	25	75	100	100	100	100	100	90	81	73
Heather	-	-	25	50	50	50	50	43	36	31	26	22
Ninian	-	-	50	150	275	300	300	275	235	200	170	145
Claymore	-	40	90	110	110	110	100	90	75	70	65	60
Statfjord (U.K.)	-	-	-	10	35	55	70	80	80	80	70	60
Sum - U.K.	410	806	1344	1743	2080	2212	2160	1995	1750	1538	1351	1206
Norway:												
Ekofisk Complex	400	600	800	800	800	800	800	720	650	580	525	470
Statfjord (N)	0	10	80	170	315	495	630	720	720	720	610	520
Sum - Norway	400	610	880	970	1115	1295	1430	1440	1370	1300	1135	990

Source: Wood, Mackenzie & Company, North Sea Reports

TABLE 16 (Continued)

Production of Oil From Existing North Sea Fields

(Thousands of Barrels Per Day)

Name / Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
U.K.:												
Montrose	19	16	14	12								
Forties	191	172	153	139	126	113	102					
Auk												
Brent	230	200	160	130	100	75	60	50				
Argyll												
Cormorant	24	20	15									
Beryl	45	40	35	30	30	25						
Thistle	40	30										
Piper	85	75	60	40								
Dunlin	60	60	40	40	35							
Hutton	38	32	27	23	20							
Alwyn	66	59	53	48	43	39	35	31	28	25		
Heather	19	16										
Ninian	120	105	90	75	65	55	45					
Claymore	55	50	40	30								
Statfjord (U.K.)	50	45	40	35	30	20	20	15	10	10	5	
Sum - U.K.	1042	920	727	602	449	327	262	96	38	35	5	
Norway:												
Ekofisk Complex	425	380	345	310	275	250	215					
Statfjord (N)	440	375	315	265	225	200	165	145	125	90	55	
Sum - Norway	865	755	660	575	500	450	380	145	125	90	55	

in the years to come. By comparing Tables 13 and 16 it is apparent that there are a number of oil discoveries in the North Sea for which no decision about the economic viability has yet been made. For the purpose of estimating the supply potential of these fields, we assume that fields larger than the respective minimum pool sizes will be produced according to the production profiles of Table 7, with initial production in 1979. I.e., this set of fields was treated as if discovered in 1975. Our reserve assumptions are listed in Table 17 (in MMBO). The reserve estimates of this set of fields have been subject to frequent revisions. The estimate for the Andrew field has recently been reduced to 300 MMB. The Brae field is expected to yield between 900 and 1200 MMBO rather than the 185 MMBO reported in Table 17. All the fields are located in the British sector. The Norwegian oil discoveries for which development plans have not yet been made are all smaller than 90 MMBO except for Balder which may contain 100 MMBO. The recent discoveries in the Norwegian sector have consequently been ommitted.

The subtotals in Table 17 correspond to the three minimum reservoir sizes of Table 8: 250, 200, and 90 MMBO. Table 17 is a summary of our assumptions with respect to the recent discoveries. There is a great deal of uncertainty surrounding the estimates and revisions are expected to be made.

In Table 18 is summarized the contribution of the three field types--existing fields (Table 16), recent discoveries (Table 17), and the 76-78 discoveries (Table 15)--to the level of production in the British sector under the three minimum reservoir size (MRS) scenarios. 76-78 discoveries are assumed to be produced according to the 300 to 1500 MMBO production profile of Table 7. Assuming a MRS of 90 MMBO, production is estimated to peak

TABLE 17

Reserve Assumptions For Fields Discovered For Which No Development Plans Have Been Announced

(Millions of Barrels of Oil)

Name	Reserves	Totals
Magnus	1080	
Lamond	500	
Andrew	500	
Maureen	500	
211/27 - 3	450	
Bruce	450	
9/13 - 7	350	
Josephine	250	
East Magnus	250	4330 (250)
9/13	220	
15/13 - 2	200	
3/4 - 6 + 3/9 - 1	200	
3/2 - 1A	200	5150 (200)
Brae	185	
21/2 - 1	175	
211/26 - 4	175	
Tern	175	
3/15 - 2	150	
15/6 - 1	150	
Crawford	150	
2/10 - 1	100	
3/4 - 4	100	
3/8 - 3	100	6435 (90)
14/20 - 1	75	6510

TABLE 18

Production Of Oil In The British Sector

(Millions Of Barrels Per Day)

Year / Field Category:	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
Existing Fields	.41	.81	1.34	1.74	2.08	2.21	2.16	1.99	1.75	1.54	1.35	1.21
Recent Discoveries	0	0	0	.44	1.02	1.25	1.23	1.23	1.2	1.16	1.14	.92
1976-78 Discoveries	0	0	0	0	.21	.75	1.37	1.78	1.89	1.89	1.89	1.89
Total (MRS = 250)	.41	.81	1.34	2.18	3.31	4.22	4.76	5.00	4.84	4.59	4.39	4.02
Existing Fields	.41	.81	1.34	1.74	2.08	2.21	2.16	1.99	1.75	1.54	1.35	1.21
Recent Discoveries	0	0	0	.64	1.31	1.59	1.52	1.52	1.45	1.34	1.30	1.06
1976-78 Discoveries	0	0	0	0	.23	.80	1.46	1.90	2.02	2.02	2.02	2.02
Total (MRS = 200)	.41	.81	1.34	2.38	3.61	4.60	5.14	5.41	5.21	4.89	4.67	4.29
Existing Fields	.41	.81	1.34	1.74	2.08	2.21	2.16	1.99	1.75	1.54	1.35	1.21
Recent Discoveries	0	0	0	1.00	1.83	2.19	2.04	2.04	1.89	1.66	1.58	1.30
1976-78 Discoveries	0	0	0	0	.24	.86	1.57	2.04	2.17	2.17	2.17	2.17
Total (MRS = 90)	.41	.81	1.34	2.74	4.15	5.26	5.77	6.07	5.81	5.37	5.10	4.67

TABLE 18 (Continued)

Production Of Oil In The British Sector

(Millions Of Barrels Per Day)

Field Category: / Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Existing Fields	1.04	.92	.73	.60	.45	.33	.26	.10	.04	.04	0	0
Recent Discoveries	.70	.52	.42	.31	.31	0	0	0	0	0	0	0
1976-78 Discoveries	1.75	1.48	1.18	.93	.74	.62	0	0	0	0	0	0
Total (MRS = 250)	3.49	2.93	2.32	1.85	1.51	.95	.26	.10	.04	.04	0	0
Existing Fields	1.04	.92	.73	.60	.45	.33	.26	.10	.04	.04	0	0
Recent Discoveries	.81	.52	.42	.31	.31	0	0	0	0	0	0	0
1976-78 Discoveries	1.87	1.58	1.25	.99	.79	.66	0	0	0	0	0	0
Total (MRS = 200)	3.72	3.03	2.40	1.91	1.55	.99	.26	.10	.04	.04	0	0
Existing Fields	1.04	.92	.73	.60	.45	.33	.26	.10	.04	.04	0	0
Recent Discoveries	1.01	.52	.42	.31	.31	0	0	0	0	0	0	0
1976-78 Discoveries	2.01	1.70	1.35	1.07	.85	.72	0	0	0	0	0	0
Total (MRS = 90)	4.06	3.15	2.50	1.99	1.62	1.04	.26	.10	.04	.04	0	0

at about 6 MMBO/D in 1983. Each field category is then contributing about one third of the total.

Table 19 summarizes future production in the Norwegian sector. Assuming an MRS of 90 MMBO, production in the Norwegian sector is also peaking in 1983 at about 2.2 MMBO/D, two-thirds of which comes from existing fields. The Norwegian government has indicated a production ceiling of 1.8 MMB/D of oil and gas in oil equivalents. Assuming a gas-ratio of 23% this implies that the maximum rate of oil production would be about 1.4 MMB/D or equal to the 1983 contribution from existing fields alone. The production ceiling might be changed over time to make it consistent with the present level of exploration activity. If we add the likely contribution of discoveries made beyond 1978, the infeasibility of the Norwegian constraint seems even more obvious, barring a dramatic cut in the level of exploratory drilling. This does not, however, seem to be the intention of the Norwegian government.

Production from the British and the Norwegian sectors is totalled in Table 20. The MRS scenario of 90 MMBO produces an aggregate peak of 8.28 MMBO/D in 1983. Existing fields contribute only about 42% of this total. Table 20 can be summarized by the fact that the 1985 price-responsiveness of North Sea supply corresponds to an elasticity of supply of about 0.2.

The cash-flows associated with the exploration, development, and production of North Sea oil are impressive. We have summarized the cash-flows associated with existing fields, as indicated by the WM data, in Tables 21 to 23. The recent upward adjustment of Ekofisk development work (approximately 17%) is not included. The price and cost assumptions are those of WM and are consistent with a MRS of 250 MMBO. The debt-ratio is assumed to be 80%, the rate of interest 12%, and the repayment period 6 years. Table 21 indicates the net cash-flow to the private operators of the

TABLE 19

Production Of Oil In The Norwegian Sector

(Millions Of Barrels Per Day)

Year / Field Category:	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
Existing Fields	.4	.61	.88	.97	1.11	1.29	1.43	1.44	1.37	1.3	1.13	.99
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	0	0	.08	.28	.51	.67	.71	.71	.71	.71
Total (MRS = 250)	.4	.61	.88	.97	1.19	1.58	1.94	2.11	2.08	2.01	1.84	1.70
Existing Fields	.4	.61	.88	.97	1.11	1.29	1.43	1.44	1.37	1.3	1.13	.99
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	0	0	.08	.30	.55	.71	.76	.76	.76	.76
Total (MRS = 200)	.4	.61	.88	.97	1.20	1.59	1.98	2.15	2.13	2.06	1.89	1.75
Existing Fields	.4	.61	.88	.97	1.11	1.30	1.43	1.44	1.37	1.3	1.13	.99
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	0	0	.09	.32	.59	.76	.81	.81	.81	.81
Total (MRS = 90)	.4	.61	.88	.97	1.21	1.62	2.02	2.20	2.18	2.11	1.95	1.80

TABLE 19 (Continued)

Production Of Oil In The Norwegian Sector

(Millions Of Barrels Per Day)

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Field Category:												
Existing Fields	.86	.75	.66	.57	.50	.45	.38	.14	.12	.09	.05	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	.66	.56	.44	.35	.28	.23	0	0	0	0	0	0
Total (MRS = 250)	1.52	1.31	1.10	.92	.78	.68	.38	.14	.12	.09	.05	0
Existing Fields	.86	.75	.66	.57	.50	.45	.38	.14	.12	.09	.05	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	.70	.59	.47	.37	.30	.25	0	0	0	0	0	0
Total (MRS = 200)	1.56	1.35	1.13	.95	.80	.70	.38	.14	.12	.09	.05	0
Existing Fields	.86	.75	.66	.57	.5	.45	.38	.14	.12	.09	.05	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	.75	.64	.51	.40	.32	.27	0	0	0	0	0	0
Total (MRS = 90)	1.62	1.39	1.17	.98	.82	.72	.38	.14	.12	.09	.05	0

TABLE 20

Production Of Oil In The North Sea

(Millions Of Barrels Per Day)

Field Category: / Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
Existing Fields	.81	1.42	2.22	2.71	3.19	3.51	3.59	3.43	3.12	2.84	2.49	2.20
Recent Discoveries	0	0	0	.44	1.02	1.25	1.23	1.23	1.2	1.16	1.14	.92
1976-78 Discoveries	0	0	0	0	.29	1.03	1.89	2.45	2.60	2.60	2.60	2.60
Total (MRS = 250)	.81	1.42	2.22	3.15	4.50	5.80	6.70	7.11	6.92	6.60	6.23	5.72
Existing Fields	.81	1.42	2.22	2.71	3.19	3.51	3.59	3.43	3.12	2.84	2.49	2.20
Recent Discoveries	0	0	0	.64	1.31	1.10	2.01	2.61	2.77	2.77	1.30	1.06
1976-78 Discoveries	0	0	0	0	.31	1.10	2.01	2.61	2.77	2.77	2.77	2.77
Total (MRS = 200)	.81	1.42	2.22	3.35	4.81	6.20	7.12	7.56	7.34	6.95	6.56	6.02
Existing Fields	.81	1.42	2.22	2.71	3.19	3.51	3.59	3.43	3.12	2.84	2.49	2.20
Recent Discoveries	0	0	0	1.00	1.83	2.19	2.04	2.04	1.89	1.66	1.58	1.30
1976-78 Discoveries	0	0	0	0	.33	1.18	2.16	2.81	2.98	2.98	2.98	2.98
Total (MRS = 90)	.81	1.42	2.22	3.71	5.36	6.88	7.79	8.28	7.99	7.48	7.05	6.48

TABLE 20 (Continued)

Production Of Oil In The North Sea

(Millions Of Barrels Per Day)

Year / Field Category:	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Existing Fields	1.91	1.67	1.39	1.18	.95	.78	.64	.24	.16	.12	.06	0
Recent Discoveries	.70	.52	.42	.31	.31	0	0	0	0	0	0	0
1976-78 Discoveries	2.41	2.04	1.62	1.28	1.02	.86	0	0	0	0	0	0
Total (MRS = 250)	5.01	4.24	3.43	2.77	2.28	1.63	.64	.24	.16	.12	.06	0
Existing Fields	1.91	1.67	1.39	1.18	.95	.78	.64	.24	.16	.12	.06	0
Recent Discoveries	.81	.52	.42	.31	.31	0	0	0	0	0	0	0
1976-78 Discoveries	2.56	2.18	1.72	1.36	1.09	.91	0	0	0	0	0	0
Total (MRS = 200)	5.28	4.38	3.53	2.86	2.35	1.69	.64	.24	.16	.12	.06	0
Existing Fields	1.91	1.67	1.39	1.18	.95	.78	.64	.24	.16	.12	.06	0
Recent Discoveries	1.01	.52	.42	.31	.31	0	0	0	0	0	0	0
1976-78 Discoveries	2.76	2.34	1.86	1.47	1.17	.98	0	0	0	0	0	0
Total (MRS = 90)	5.68	4.54	3.67	2.96	2.43	1.76	.64	.24	.16	.12	.06	0

TABLE 21

Operator Cash-Flow From Oil Production

(Millions of Dollars)

Year / Name	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
U.K.:												
Montrose	13	5	19	121	96	50	61	46	39	35	22	16
Forties	474	146	804	294	412	361	343	287	276	279	271	264
Auk	84	32	55	-39	9	-8						
Brent	-2	7	9	56	292	1512	265	457	321	343	357	348
Argyll	95	50	31	27	20	18	15	-19				
Cormorant	-55	21	5	20	67	146	97	46	51	43	37	32
Beryl	181	39	85	84	86	89	89	68	68	63	64	62
Thistle	-105	-7	20	36	279	338	122	98	102	113	109	105
Piper	49	420	294	237	235	193	172	153	131	112	91	69
Dunlin	-4	-12	3	1	4	174	323	100	135	122	119	115
Hutton	-105	-180	-4	40	3	28	297	308	71	69	62	49
Alwyn	-126	-2	34	10	43	277	147	91	111	94	96	93
Heather	-100	-	50	17	10	106	120	43	36	29	20	14
Ninian	-600	-9	56	13	106	700	987	220	328	318	217	180
Claymore	-	118	44	210	116	120	101	91	71	76	76	76
Statfjord (U.K.)	-	-90	-55	6	55	40	-7	31	159	139	87	86
Sum - U.K.	-201	538	1450	1133	1833	4144	3132	2020	1899	1885	1628	1509
Norway:												
Ekofisk Complex	307	594	1703	1097	1045	959	877	503	450	380	371	319
Statfjord (N)	-266	9	191	182	255	433	811	1269	925	834	347	278
Sum - Norway	41	603	1894	1279	1300	1392	1688	1772	1375	1214	718	597

Source: Wood, Mackenzie & Company, North Sea Reports

TABLE 21 (Continued)

Operator Cash-Flow From Oil Production

(Millions of Dollars)

Name \ Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
U.K.:												
Montrose	12	8	5	3	-6							
Forties	256	250	230	190	168	144	128	-162				
Auk												
Brent	334	286	163	128	73	36	28	20	-42			
Argyll												
Cormorant	27	18	6	-17								
Beryl	59	43	33	24	34	16	-27					
Thistle	74	15	16	-39								
Piper	81	79	72	40	-63							
Dunlin	68	95	18	58	38	-51						
Hutton	41	31	24	19	14	-21						
Alwyn	90	87	67	60	50	45	38	31	27	22	-29	
Heather	10	5	-11									
Ninian	134	126	98	72	62	44	26	-48				
Claymore	73	71	39	19	-41							
Statfjord (U.K.)	58	59	50	40	31	3	23	4	-5	5	-15	
Sum - U.K.	1317	1173	810	597	360	216	216	-155	-20	27	-44	
Norway:												
Ekofisk Complex	306	262	252	202	152	146	80	-576				
Statfjord (N)	223	199	178	136	129	154	65	75	41	-38	-81	-61
Sum - Norway	529	461	430	338	281	300	145	-501	41	-38	-81	-61

TABLE 22

Government Revenues From Oil Production

(Millions of Dollars)

Name \ Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
U.K.:												
Montrose	5	18	26	26	88	134	123	107	84	66	57	45
Forties	94	172	246	1362	1260	1223	1154	1052	923	789	683	589
Auk	15	20	93	62	14	8						
Brent	11	27	91	134	187	412	1727	1492	1190	861	759	681
Argyll	18	46	56	51	45	38	33	19				
Cormorant	-	5	14	24	24	24	75	104	82	77	69	61
Beryl	32	42	52	217	219	216	216	194	172	155	132	112
Thistle	-	11	53	95	355	539	410	296	241	201	162	149
Piper	45	92	635	697	699	653	586	518	452	384	317	295
Dunlin	-	5	21	32	42	48	53	283	312	283	240	200
Hutton	-	-	-	13	37	53	53	94	266	163	131	109
Alwyn	-	-	13	39	53	53	250	307	287	260	219	187
Heather	-	-	13	26	26	26	58	105	82	67	54	42
Ninian	-	-	26	78	143	156	190	905	621	478	448	375
Claymore	-	22	48	59	327	329	302	268	223	196	174	152
Statfjord (U.K.)	-	-	-	5	19	29	37	43	43	128	190	147
Sum - U.K.	220	460	1387	2920	3538	3941	5267	5787	4978	4108	3635	3144
Norway:												
Ekofisk Complex	389	952	1639	2317	2369	2455	2537	2561	2307	2070	1838	1650
Statfjord (N)	1	6	39	95	199	537	1116	1670	2094	2185	2190	1865
Sum - Norway	390	958	1678	2412	2568	2992	3653	4231	4401	4255	4028	3515

Source: Wood, Mackenzie & Company, North Sea Reports

TABLE 22 (Continued)

Government Revenues From Oil Production

(Millions of Dollars)

Year / Name	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
U.K.:												
Montrose	36	27	21	15	6							
Forties	501	423	360	339	304	271	239	162				
Auk												
Brent	563	480	428	331	255	183	125	89	42			
Argyll												
Cormorant	53	45	35	17								
Beryl	93	87	75	62	52	49	27					
Thistle	120	75	39									
Piper	261	220	161	105	63							
Dunlin	160	133	122	82	80	51						
Hutton	90	74	59	47	39	21						
Alwyn	159	131	125	110	100	86	75	65	56	48	29	
Heather	33	25	11									
Ninian	312	254	216	177	143	117	91	48				
Claymore	133	113	101	77	41							
Statfjord (U.K.)	131	108	95	83	70	55	35	32	19	9	7	
Sum - U.K.	2645	2195	1848	1445	1153	833	592	396	117	57	36	
Norway:												
Ekofisk Complex	1466	1312	1169	1066	963	859	772	576				
Statfjord (N)	1569	1309	1067	890	722	587	523	425	372	297	187	61
Sum - Norway	3035	2621	2236	1956	1685	1446	1295	1001	372	297	187	61

TABLE 23

Capital Expenditures For Oil Development

(Millions of Dollars)

Name \ Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
U.K.:												
Montrose	75	55										
Forties	165	135	65									
Auk	25											
Brent	680	290	175	155	130							
Argyll												
Cormorant	125	65	60	45								
Beryl	75	60	35									
Thistle	285	105	75	70	45							
Piper	80	45										
Dunlin	305	85	50	40	30	25	25	20				
Hutton	105	220	315	85	75	-						
Alwyn	150	295	100	75	10	-						
Heather	100	145	30	63	40							
Ninian	200	680	815	240	155	90	70	50				
Claymore	210	100	65	20								
Statfjord (U.K.)	-	90	250	215	65	65	40	-				
Sum - U.K.	2580	2370	2035	1008	550	180	135	70				
Norway:												
Ekofisk Complex	470	110	45									
Statfjord (N)	345	420	655	645	355	85	80	65				
Sum - Norway	815	530	700	645	355	85	80	65				

Source: Wood, Mackenzie & Company, North Sea Reports

existing fields. The numbers are not reduced by the participation share of the respective governments. The cash-flows to the two governments are likewise calculated net of participation (Table 22). The overall government cash-flows are substantially larger than the private cash-flows. However, private cash-flows are larger in the earlier years. The investment expenditures of existing fields in the 1976-1983 period are listed in Table 23. As we are interested in future cash-flows only, the investment expenditures prior to 1976 are not included.

The reservoir model was used to project cash-flows for recent and projected discoveries. As was pointed out above our preliminary cost relationships bias the cost of small fields upward and the cost of large fields downward. The cash-flows should be interpreted accordingly. We ran each recently discovered field (Table 17) through the reservoir model. For cash-flow purposes we assumed an average field size of 325 MMBO in 1976, of 285 MMBO in 1977, and of 260 MMBO in 1978, but assumed the 300 to 1500 MMBO profiles of Table 7 to be consistent with our production forecasts. We are therefore likely to have underestimated the cash-flows to the public and and private sector resulting from the 76-78 discoveries, and to have biased upwards the capital expenditures. The cash-flows from all field categories have been calculated assuming the WM scenario. The price-sensitivity of these cash-flows will not be indicated in this version of the paper.

Table 24 indicates that the U.K. government might receive as much as \$7.9 billion (in 1983) in revenues from the field categories included in this analysis. The U.K. operators would reach a peak of \$4.09 billion in 1981. The operators in Norwegian waters will receive their maximum annual cash-flow as early as 1978 at \$1.8 billion, whereas the Norwegian government should receive as much as \$4.7 billion in 1984 (Table 25). In 1983 the

TABLE 24

Cash Flows From Oil Exploration, Development, And Production In The British Sector

(Millions Of Dollars)

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
To Operator:												
Existing Fields	-201	538	1450	1133	1833	4144	3132	2020	1899	1885	1628	1509
Recent Discoveries	-27	-144	-489	637	897	1541	1428	702	372	-177	1450	504
1976-78 Discoveries	-24	-72	-196	-474	-1308	-597	-958	-897	-896	-1228	-260	808
Total	-252	322	765	1295	1422	5089	3602	1825	1375	481	2818	2821
To Government:												
Existing Fields	220	460	1387	2920	3538	3941	5267	5787	4978	4108	3635	3144
Recent Discoveries	0	0	0	229	533	657	671	1171	1194	1388	1802	2235
1976-78 Discoveries	0	0	0	0	207	553	779	925	984	984	984	887
Total	220	460	1387	2593	4278	5149	6716	7884	7156	6480	6422	6266
Capital Expenditures:												
Existing Fields	2580	2376	2035	1008	550	180	135	70	0	0	0	0
Recent Discoveries	0	449	2175	2881	3241	2396	1233	1192	1081	991	0	0
1976-78 Discoveries	0	0	380	1652	4039	6850	7985	6806	4626	3185	2297	0
Total	2580	2819	4589	5541	7830	9429	9350	8068	5707	4175	2297	0

TABLE 24 (Continued)

Cash Flows From Oil Exploration, Development, And Production In The British Sector

(Millions Of Dollars)

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
To Operator:												
Existing Fields	1255	1173	810	597	360	216	216	-104	-20	27	-44	0
Recent Discoveries	136	161	241	218	418	-453	0	0	0	0	0	0
1976-78 Discoveries	2527	1918	1877	364	1061	856	275	133	0	0	0	0
Total	3918	3252	2928	1180	1839	619	491	29	-20	27	-44	0
To Government:												
Existing Fields	2645	2195	1848	1445	1153	833	592	396	117	57	36	0
Recent Discoveries	1864	1392	1062	818	618	453	0	0	0	0	0	0
1976-78 Discoveries	918	1680	1263	1136	1226	908	643	293	0	0	0	0
Total	5427	5267	4174	3400	2997	2194	1235	689	117	57	36	0
Capital Expenditures:												
Existing Fields	0	0	0	0	0	0	0	0	0	0	0	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 25

Cash Flows From Oil Exploration, Development, And Production In The Norwegian Sector

(Millions Of Dollars)

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
To Operator:												
Existing Fields	41	603	1894	1279	1300	1392	1688	1772	1375	1214	718	597
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	-9	-2.7	-7.4	-178	-625	-647	-479	-271	-266	80	502	829
Total	32	576	1820	1101	675	745	1209	1501	1109	1294	1221	1426
To Government:												
Existing Fields	390	958	1678	2412	2568	2992	3653	4231	4401	4255	4028	3515
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	.104	.459	28	117	225	282	299	299	341	514
Total	390	958	1678	2412	2596	3109	3878	4513	4700	4554	4369	4029
Capital Expenditures:												
Existing Fields	815	530	70	645	355	85	80	65	0	0	0	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	142	619	1515	2569	2994	2552	1735	0	0	0
Total	815	530	212	1264	1870	2654	3074	2617	1735	0	0	0

TABLE 25 (Continued)

Cash Flows From Oil Exploration, Development, And Production In The Norwegian Sector

(Millions Of Dollars)

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
To Operator:												
Existing Fields	529	461	430	338	281	300	145	-501	41	-38	-81	-61
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	956	866	700	441	350	291	87	0	0	0	0	0
Total	1485	1327	1130	779	631	591	232	-501	41	-38	-81	-61
To Government:												
Existing Fields	3035	2621	2236	1956	1685	1446	1295	1001	372	297	187	61
Recent Discoveries		0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	736	883	878	730	508	370	257	0	0	0	0	0
Total	3771	3504	3114	2686	2193	1816	1552	1001	372	297	187	61
Capital Expenditures:												
Existing Fields	0	0	0	0	0	0	0	0	0	0	0	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	0	0	0	0	0	0	0	0	0	0	0

North Sea governments will have a net cash-flow equal to the total capital expenditures in that year of $12.4 billion, which is the peak flow rate for both cash-flow categories. The North Sea operators will receive their maximum in 1988 at $9.5 billion (Table 26).

4.5 The Analysis in Perspective

In this paper we have presented a methodology that allows us to separate and identify the significance of each of a set of geologic, economic, technological, and institutional variables for the rate of resource exploitation. The methodology is applicable to the analysis of supply of any extractive resource. To illustrate how this general methodology may be applied, we have carried through an analysis of supply of crude oil from the Norwegian and the British sectors of the North Sea. We are, however, still at a preliminary stage as far as the North Sea analysis is concerned.

Our cost data sample alone does not allow us to identify the development cost relationships in the North Sea. More data and more engineering-type analysis are needed to identify the relationships between the reservoir characteristics we know determine development costs and the various development cost categories. We have, as discussed in Section 4.1, only considered the average isolated field for the purpose of cost and minimum reservoir size estimation. This fact as well as the oversimplified form of the cost relationships biases the minimum reservoir size upwards as discussed in Section 4.2.3. This bias is, however, partially offset by disregarding the increase in minimum reservoir size over time. The time-profile of the minimum reservoir size will be discussed in a later version of this paper.

TABLE 26

North Sea Cash Flows From Oil Exploration, Development, And Production

(Millions Of Dollars)

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
To Operator:												
Existing Fields	-160	1141	3344	2412	3133	5536	4820	3792	3274	3099	2346	2106
Recent Discoveries	-27	-144	-489	637	897	1541	1428	702	372	-177	1450	504
1976-78 Discoveries	-33	-99	-269	-652	-1933	-1243	-1437	-1168	-1162	-1148	243	1637
Total	-220	898	2585	2396	2096	5834	4810	3326	2484	1775	4039	4247
To Government:												
Existing Fields	610	1418	3065	5332	6106	6933	8920	10018	9379	8363	7663	6659
Recent Discoveries	0	0	0	229	532	657	671	1172	1194	1388	1802	2235
1976-78 Discoveries	0	0	.104	.459	235	670	1004	1207	1282	1283	1324	1401
Total	610	1418	3065	5561	6874	8260	10594	12397	11855	11034	10789	10296
Capital Expenditures:												
Existing Fields	3140	2550	2045	1943	1130	350	240	175	0	0	0	0
Recent Discoveries	0	449	2175	2881	3241	2396	1233	1192	1081	991	0	0
1976-78 Discoveries	0	0	522	2271	5553	9419	10979	9358	6361	3185	2297	0
Total	3140	2997	4742	7095	9925	12165	12452	10725	7442	4175	2297	0

TABLE 26 (Continued)

North Sea Cash Flows From Oil Exploration, Development, And Production

(Millions Of Dollars)

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
To Operator:												
Existing Fields	1784	1634	1240	935	641	516	361	-605	21	-11	-125	-61
Recent Discoveries	136	161	241	218	418	-453	0	0	0	0	0	0
1976-78 Discoveries	3484	2785	2576	806	1412	1147	362	133	0	0	0	0
Total	5404	4579	4057	1959	2471	1211	723	-472	21	-11	-125	-61
To Government:												
Existing Fields	5680	4816	4084	3401	2838	2279	1887	1397	489	354	223	61
Recent Discoveries	1864	1391	1062	818	618	453	0	0	0	0	0	0
1976-78 Discoveries	1654	2564	2142	1866	1733	1278	900	293	0	0	0	0
Total	9198	8771	7288	6085	5189	4010	2787	1690	489	354	223	61
Capital Expenditures:												
Existing Fields	0	0	0	0	0	0	0	0	0	0	0	0
Recent Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
1976-78 Discoveries	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	0	0	0	0	0	0	0	0	0	0	0

Our analysis of the reserve potential of the North Sea is based upon the acreage presently held by the oil industry. By including blocks in the prime area that have not yet been awarded, the prospective reserve potential of Tables 10 and 11 would increase by about 25%. Our estimates of crude oil reserves to be found are deduced from a sample of the blocks held by the oil industry.

This sample covers only about 6% of the relevant area. The characteristics of the sample are, however, thought to be representative of the exploratory history of the North Sea. We are in the process of trying to obtain additional prospect information regarding the North Sea. Better prospect information will also help us determine the bias that might have been introduced by considering the total prime trend area to be one geological play for the purpose of calculating our statistical discovery sequence (Table 14).

In Section 4.3 we explain how we arrived at our assumption regarding the rate of geological discoveries over the 1976-78 period. The sensitivity of the supply forecast to this assumption necessitates a more extensive analysis of the rate of exploratory drilling in the two sectors of the North Sea.

If the production profiles of Table 7 are considered relevant to future discoveries, then the 1985 production potential should include reservoirs discovered as late as 1981. By disregarding post-78 discoveries we are obviously biasing the North Sea production potential downwards. "Drilling-up" scenarios for the North Sea are presently being evaluated. An example of such a scenario is discussed in [3].

A number of different people and institutions have published estimates of the future level of crude oil production in the North Sea. In Table 27 four of the most commonly referenced estimates of North Sea crude oil

TABLE 27

Comparison Of North Sea Oil Production Estimates

(Million Barrels Of Oil Per Day, Prices In 1975 $)

	1980			1985			1990		
	$7	$9	$12	$7	$9	$12	$7	$9	$12
BP	NA[2]	3.46	NA	NA	6.8	NA	NA	NA	NA
OECD[1]	3.9	NA	4.06	5.16	NA	5.30	NA	NA	NA
Euro Economics	NA	4.96	NA	NA	6.79	NA	NA	NA	NA
Odell & Rosing	NA	4	NA	NA	12	NA	NA	16	NA
M.I.T. Sample Calculations	4.5	4.81	5.35	6.59	6.94	7.48	3.45	3.55	3.69

[1] See text for explanation.

[2] NA = Not Applicable

production are compared to our preliminary results. The BP estimates are those given by Dr. Birks at the "North and Celtic Seas Conference" in London in 1973 [4]. The OECD estimates [15] are based on the 1974 level of proven reserves and the production plans for those as well as what "authoritative sources suggest" might be produced in the North Sea at a Persian Gulf crude oil price of \$6 and \$9 per barrel, in 1972 prices. In Table 27, the \$6 and \$9 estimates have been labeled \$7 and \$12, respectively, which is not accurate but still sufficient for the sake of roughly comparing results. Implicit in the estimates of Euro-Economics [6] is the assumption that the net after-tax price paid to the producers, presumably in current dollars, would increase by 5.5% a year, from \$5.72 in 1975 to \$10.24 in 1985 and \$13.06 in 1990. Such a price scenario must be considered extremely optimistic from the producers point of view. The Odell & Rosing estimates [14] were produced by a probabilistic simulation model. A crucial assumption of that study is that "the supply of and demand for North Sea oil was not to be affected by energy price changes". The range of output between 1987 and 1993 was estimated to be from 12 to 19 MMBD from which we chose 16 MMBD to insert in Table 27. Except for the Odell & Rosing estimates, all the estimates are within a fairly narrow range.

It was indicated in Section 4.3 above that the ultimate level of recoverable oil reserves south of 62° North latitude might be as high as 46 billion barrels of oil. This implies that there would be another 15 billion barrels to be discovered after 1978 under the exploratory assumptions of this study. If we assume a combined peak production of 7.5% of these additional reserves we might add another 3 MMB/D in 1990. The result would still be less than half the Odell & Rosing estimates for 1990.

We have not demonstrated the way our supply framework can be applied to identify the most favorable tax regime for all parties involved. This framework is designed to assist the public policy-maker or the private strategist in evaluating a supply area. Before reporting our work on the institutional changes that might increase the welfare of all the participants in the North Sea arena we want to do more extensive work on the cost relationships of Section 4.1. For this purpose we hope to establish improved working relationships with the companies and the government agencies active in the North Sea and elsewhere where we might learn about the exploration, development, and production process for mutual benefit.

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