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INFORMATION STRUCTURES

by

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ABSTRACT

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DECISIONMAKING ORGANIZATIONS WITH ACYCLICAL INFORMATION STRUCTURES*

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Summary

An analytical model of a team of well-trained human decisionmakers executing a well-defined decisionmaking task is presented. Each team member is described by a two-stage model consisting of a situation assessment and a response selection stage. An information theoretic framework is used in which bounded rationality is modeled as a constraint on the total rate of internal processing by each decisionmaker. Optimizing and satisfying strategies are derived and their properties analyzed in terms of organizational performance and individual workload. The results are applied to the analysis and evaluation of two three-person organizational designs.

I. Introduction

A command control and communications (C^3) system is defined as the collection of equipment and procedures used by commanders and their staff to process information, arrive at decisions, and communicate these decisions to the appropriate units in the organization in a timely manner. Implicit in this definition is the notion that the role of the human decisionmaker is central to the design of organizations and the C^3 systems that support them. A basic model of an interacting decisionmaker, appropriate for a narrow but important class of problems, was introduced by Boettcher and Levis [1]. In a second paper, Levis and Boettcher [2] considered the modeling of organizations consisting of two decisionmakers that form a team. In this paper, the methodology is extended to the analysis and evaluation of teams with acyclical information structures. Two three-person organizations are used to illustrate the approach.

The basic assumption in designing organizations is that a given task, or set of tasks, cannot be carried out by a single decisionmaker because of the large amount of information processing required and the severe time constraints present in a tactical situation. In designing an organizational structure for a team of decisionmakers, two issues need to be resolved: who receives what information and who is assigned to carry out which decisions. The resolution of these issues depends on the limited information processing rate of individual decisionmakers and the tempo of operations. The latter reflects the rate at which tasks are assigned to the organization and the interval allowed for their execution.

An information theoretic framework is used for both the modeling of the individual decisionmaker and of the organization. Information theoretic approaches to modeling human decisionmakers have a long history [3]. The basic departure from previous models is in

the modeling of the internal processing of the inputs to produce outputs. This processing includes not only transmission (or throughput) but also internal coordination, blockage, and internally generated information. Consequently, the limitations of humans as processors of information and problem solvers are modeled as a constraint to the total processing activity. This constraint represents one interpretation of the hypothesis that decisionmakers exhibit bounded rationality [4].

The task of the organization is modeled as receiving signals from one or many sources, processing them, and producing outputs. The outputs could be signals or actions. The input signals that describe the environment may come from different sources and, in general, portions of the signals may be received by different members of the organizations. It has been shown [5] that the general case can be modeled by a single vector source and a set of partitioning matrices that distribute components of the vector signal to the appropriate decisionmakers within the organization.

Consideration in this paper will be restricted to structures that result when a specific set of interactions is allowed between team members: each team member is assigned a specific task, whether it consists of processing inputs received from the external environment or from other team members, *for which he is well trained and which he performs again and again for successively arriving inputs*. In general, a member of the organization can be represented by a two-stage model as shown in Fig. 1. First, he may receive signals from the environment that he processes in the situation assessment (SA) stage to determine or select a particular value of the variable z that denotes the situation. He may communicate his assessment of the situation to other members and he may receive their assessments in return. This supplementary information may be used to modify his assessment, i.e., it may lead to a different value of z . Possible alternatives of action are evaluated in the response selection (RS) stage. The outcome of this process is the selection of a local action or decision response y that may be communicated to other team members or may form all or part of the organization's response. A command input from other decisionmakers may affect the selection process. A further restriction is introduced in that the information structures be acyclical.

The overall mapping between the stimulus (input) to the organization and its response (output) is determined by the internal decision strategies of each decisionmaker. The total activity of each DM as well as the performance measure for the organization as a whole are expressed then in terms of these internal decision strategies. For each set of admissible internal decision strategies, one for each DM, a point is defined in the performance-workload space. The locus of all

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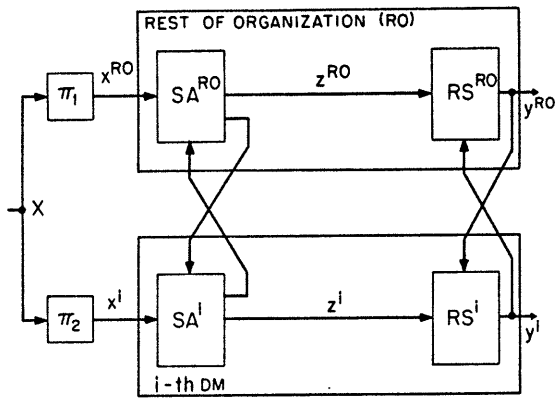


Fig. 1 Allowable team interactions

such points is characteristic of the organizational structure. Once the locus has been constructed, it is then possible to analyze the effects of the bounded rationality constraints on the organization's performance when either optimizing or satisficing behavior is assumed.

In the next section, the model of the interacting organization member is reviewed. In the third section the model of a team with acyclical information structures is described analytically. In the fourth section, the optimal and the satisficing decision strategies for the two three-person organizations are obtained and analyzed.

II. Model of the Organization Member

The complete realization of the model for a decisionmaker (DM) who is interacting with other organization members and with the environment is shown in Fig. 2. The detailed description and analysis of this model, as well as its relationship to previous work, notably that of Drenick [6] and Froyd and Bailey [7], has been presented in [1]. Therefore, only concepts and results needed to model the organization are described in this section. The presentation is similar to that in [2].

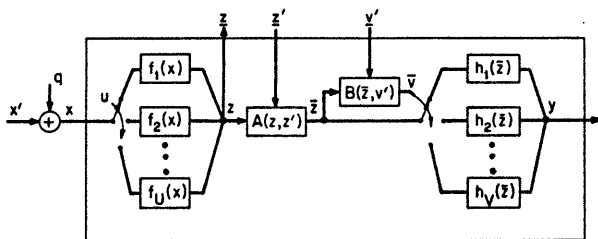


Fig. 2 Single interacting decisionmaker model

Let the organization receive from the environment a vector of symbols, X' . The DM receives x which is a noisy measurement of a portion, x' , of X' . The vector x takes values from a known finite alphabet according to the probability distribution $p(x)$. The quantity

$$H(x) = - \sum_x p(x) \log_2 p(x) \quad (1)$$

is defined to be the entropy of the input [8] measured in bits per symbol generated. The quantity $H(x)$ can also be interpreted as the uncertainty regarding which value the random variable x will take. If input symbols are generated every τ seconds on the average, then τ , the mean symbol interarrival time, is a description of the tempo of operations [9]. The conditional entropy is defined as

$$H_x(z) = - \sum_x p(x) \sum_z p(z|x) \log_2 p(z|x) \quad (2)$$

The situation assessment stage consists of a finite number U of procedures or algorithms f_i that the DM can choose from to process the measurement x and obtain the assessed situation z . The internal decisionmaking in this stage is the choice of algorithm f_i to process x . Therefore, each algorithm is considered to be active or inactive, depending on the internal decision u . In this paper, it is assumed that the algorithms f_i are deterministic. This implies that once the input is known and the algorithm choice is made, all other variables in the first part of the SA stage are known. Furthermore, because no learning takes place during the performance of a sequence of tasks, the successive values taken by the variables of the model are uncorrelated, i.e., the model is memoryless. Hence, all information theoretic expressions appearing in this paper are on a per symbol basis.

The vector variable z' , the supplementary situation assessment received from other members of the organization, combines with the elements of z to produce \bar{z} . The variables z and \bar{z} are of the same dimension and take values from the same alphabet. The integration of the situation assessments is accomplished by the subsystems S^2 which contains the deterministic algorithm A .

If there is no command input vector y' from other organization members, then the response selection strategy $p(v|\bar{z})$ specifies the selection of one of the algorithms h_j that map \bar{z} into the output y . The existence of a command input y' modifies the decisionmaker's choice v . A final choice \bar{v} is obtained from the function $b(v, y')$. The latter defines a protocol according to which the command is used, i.e., the values of \bar{v} determined by $b(v, y')$ reflect the degree of option restriction effected by the command. The overall process of mapping the assessed situation \bar{z} and the command input y' onto the final choice \bar{v} is depicted by subsystem S^3 in Fig. 2. The result of this process is a response selection strategy $p(\bar{v}|\bar{z}, y')$ in place of $p(v|\bar{z})$.

This model of the decisionmaking process may be viewed as a system S consisting of four subsystems: S^1 , the first part of the SA stage; S^2 ; S^3 ; and S^4 , the second part of the RS stage. The inputs to this system S are x, z' , and y' and the outputs are y and the situation assessment transmitted to other DMs. The second output consists of a set of z_i vectors, one for each interacting DM. For notation simplicity, these vectors will be denoted by a single vector \bar{z} consisting of the concatenation of the z_i 's. Furthermore, let each algorithm f_i contain α_i variables denoted by

$$W^i = \{w_1^i, w_2^i, \dots, w_{\alpha_i}^i\} \quad i = 1, 2, \dots, U \quad (3)$$

and let each algorithm h_j contain α_j' variables denoted by

$$W^{U+j} = \{w_1^{U+j}, \dots, w_{\alpha_j'}^{U+j}\} \quad j = 1, 2, \dots, V \quad (4)$$

It is assumed that each algorithm has a self-contained set of variables and that when one algorithm is active, all others are inactive. Consequently,

$$W^i \cap W^j = \emptyset \quad \text{for } i \neq j$$

$$\forall i, j \in \{1, 2, \dots, U\} \text{ or } \{1, 2, \dots, V\} \quad (5)$$

The subsystem S^1 is described by a set of variables

$$S^1 = \{u, w_1^1, \dots, w_{\alpha_1}^1, z, \bar{z}\};$$

subsystem S^2 by

$$S^2 = \{w_1^A, \bar{z}\};$$

subsystem S^3 by

$$S^3 = \{w_1^B, \bar{v}\};$$

subsystem S^4 by

$$S^4 = \{w_1^{U+1}, \dots, w_{\alpha_{U+V}}^{U+V}, y\}.$$

The mutual information or transmission or throughput between inputs x, z' , and \bar{v}' and output y and \bar{z} , denoted by $T(x, z', \bar{v}'; y, \bar{z})$ is a description of the input-output relationship of the DM model and expresses the amount by which the outputs are related to the inputs:

$$G_t = T(x, z', \bar{v}'; y, \bar{z}) = H(x, z', \bar{v}') + H(y, \bar{z}) - H(x, z', \bar{v}', y, \bar{z})$$

$$= H(\bar{z}, y) - H_{x, z', \bar{v}'}(\bar{z}, y) \quad (6)$$

A quantity complementary to the throughput G_t is that part of the input information which is not transmitted by the system S . It is called blockage and is defined as

$$G_b = H(x, z', \bar{v}') - G_t \quad (7)$$

In this case, inputs not received or rejected by the system are not taken into account.

In contrast to blockage is a quantity that describes the uncertainty in the output when the input is known. It may represent noise in the output generated within S or it may represent information in the output produced by the system. It is defined as the entropy of the system variables conditioned on the input, i.e.,

$$G_n = H_{x, z', \bar{v}'}(u, w_1^1, \dots, w_{\alpha_{U+V}}^{U+V}, w_1^A, w_1^B, z, \bar{z}, \bar{v}, y) \quad (8)$$

The final quantity to be considered reflects all system variable interactions and can be interpreted as the coordination required among the system variables to accomplish the processing of the inputs to obtain the output. It is defined by

$$G_c = T(u: w_1^1: \dots: w_{\alpha_{U+V}}^{U+V}: w_1^A: \dots: w_{\alpha_B}^B: z: \bar{z}: \bar{v}: y) \quad (9)$$

The Partition Law of Information [10] states that the sum of the four quantities G_t , G_b , G_n , and G_c is equal to the sum of the marginal entropies of all the system variables (internal and output variables):

$$G = G_t + G_b + G_n + G_c \quad (10)$$

where

$$G = \sum_{i,j} H(w_i^j) + H(u) + H(z) + H(\bar{z}) + H(\bar{v}) + H(y) \quad (11)$$

When the definitions for internally generated information G_n and coordination G_c are applied to the specific model n of the decisionmaking process shown in Fig. 3 they become

$$G_n = H(u) + H_{\bar{z}}(v) \quad (12)$$

and

$$G_c = \sum_{i=1}^U [p_i g_c^i(p(x)) + \alpha_i \mathcal{H}(p_i)] + H(z)$$

$$+ g_c^A(p(z)) + g_c^B(p(\bar{z}))$$

$$+ \sum_{j=1}^V [p_j g_c^{U+j}(p(\bar{z}|\bar{v}=j)) + \alpha_j' \mathcal{H}(p_j)] + H(y)$$

$$+ H(z) + H(\bar{z}) + H(\bar{z}, \bar{v}) + T_z(x': z')$$

$$+ T_{\bar{z}}(x', z': \bar{v}') \quad (13)$$

The expression for G_n shows that it depends on the two internal strategies $p(u)$ and $p(v|\bar{z})$ even though a command input may exist. This implies that the command input \bar{v}' modifies the DM's internal decision after $p(v|\bar{z})$ has been determined.

In the expressions defining the system coordination, p_i is the probability that algorithm f_i has been selected for processing the input x and p_i is the probability that algorithm h_i has been selected, i.e., $u=i$ and $\bar{v}=j$. The quantities g_c represent the internal coordination of the corresponding algorithms and depend on the distribution of their respective inputs. The quantity \mathcal{H} is the entropy of a random variable that can take one of two values with probability p :

$$\mathcal{H}(p) = -p \log p - (1-p) \log(1-p) \quad (14)$$

If there is no switching, i.e., if for example $p(u=i)=1$ for some i , then \mathcal{H} will be identically zero for all p_i and the only non-zero term in the first sum will be

$$g_c^1(p(x))$$

Similarly, the only non-zero term in the second sum will be

$$g_c^{U+j}(p(\bar{z}|\bar{v}=j))$$

The quantity G may be interpreted as the total information processing activity of system S and, therefore, it can serve as a measure of the workload of the organization member in carrying out his decisionmaking task.

III. Teams of Decisionmakers

In order to define an organizational structure, it is necessary to specify exactly the interactions of

each decisionmaker. A decisionmaker is said to interact with the environment when he receives inputs directly from sources or when he produces outputs that are all or part of the organization's output. The internal interactions consist of receiving inputs from other DMs, sharing situation assessments, receiving command inputs, and producing outputs that are either inputs or commands to other DMs. If these interactions are shown graphically in the form of a directed graph, then the organizational forms being considered have directed graphs which do not contain any cycles or loops. The resulting decisionmaking organizations are defined as having *acyclical information structures*. This restriction in the structure of the organizations is introduced to avoid deadlock and also messages circulating within the organization. It prohibits a DM from sending commands to other DMs from which he is receiving command inputs. However, simultaneous sharing of situation assessment information is allowed.

The types of information-processing and decision-making organizations that can be modeled and analyzed are exemplified by the two three-person organizations A and B shown in Figs. 3 and 4, respectively. Three-person organizations were chosen because they require relatively simple notation. The approach applies to n-person organizations, however. Let the three decisionmakers be denoted by DM^1 , DM^2 , and DM^3 . Their corresponding variables are superscripted 1, 2, and 3, respectively. The notation z^{12} indicates that variable z is generated by DM^1 and is received by DM^2 .

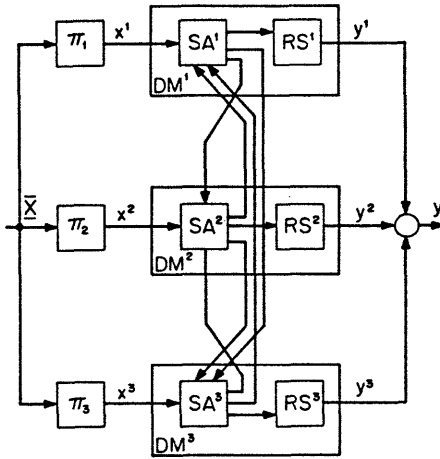


Fig. 3 Three person organization
(A: Parallel Structure)

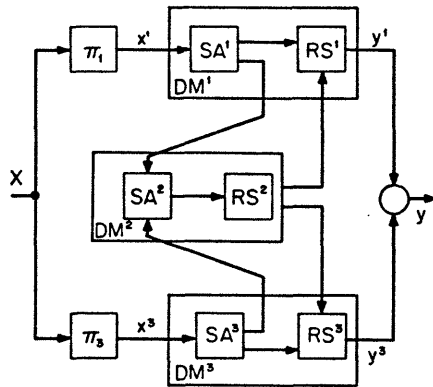


Fig. 4 Three person organization
(B: Hierarchical Structure)

In the first case, A, all three decisionmakers receive signals from the environment, process them to assess the situation as perceived by each one and then share their situation assessments. Each revises his assessment and proceeds to select a response. There are no command inputs; the organizational output is the combined outputs of the three DMs. This is a pure parallel structure: the task has been divided into three subtasks done in parallel. However, there are lateral links - the sharing of situation assessment information - between the three DMs that constitute a single echelon.

The second organizational structure, Fig. 4, is more complex. The task is divided into two subtasks. The first and third DMs receive the external inputs and assess the situation. They transmit the assessments to the second DM who processes them and generates commands that he then transmits to the other two DMs. These commands restrict the options in selecting responses by DM^1 and DM^3 . The two produce the outputs which constitute the organization's output. The second decisionmaker has, clearly, a supervisory role, even though he is in the same echelon.

The four quantities that characterize the total information-processing and decisionmaking activity G of each DM in organizations A and B are obtained directly by specializing equations (6), (7), (12) and (13). In organization A, all decisionmakers have an identical structure although the specific algorithms f_i and h_i in the SA and RS stages, respectively, may differ. The expressions are presented for DM^1 ; the expressions for DM^2 and DM^3 are identical in form but with the appropriate superscripts.

Organization A: Decisionmaker 1 (or 2, or 3)

$$G_t^1 = T(x^1, z^{121}, z^{131}; y^1) \quad (15)$$

$$G_b^1 = H(x^1, z^{121}, z^{131}) - G_t^1 \quad (16)$$

$$G_n^1 = H(u^1) + H_{\bar{z}}^1(v^1) \quad (17)$$

$$\begin{aligned} G_c^1 = & \sum_{i=1}^{U^1} [p_i^1 g_c^1(p(x^1)) + \alpha_i^1 \mathcal{H}(p_i^1)] \\ & + H(z^1) + H(z^{12}) + H(z^{13}) \\ & + g_c^{1A}(p(z^1, z^{121}, z^{131})) \\ & + g_c^{1B}(p(\bar{z}^1)) \\ & + \sum_{j=1}^{V^1} [p_j^1 g_c^{1j}(p(\bar{z}^1 | \bar{v}^1 = j)) + \alpha_j^1 \mathcal{H}(p_j^1)] \\ & + H(y^1) + H(z^1) + H(\bar{z}^1) + H(\bar{z}^1, \bar{v}^1) \\ & + T_{\bar{z}}^1(x^1; z^{121}, z^{131}) \end{aligned} \quad (18)$$

In organization B, decisionmakers DM^1 and DM^3 serve identical roles and, therefore, the expressions for the four terms are similar. Only those for DM^1 are presented; those for DM^3 are obtained by substituting the appropriate superscripts. The second decisionmaker acts as a coordinator and supervisor, and does not receive inputs directly from the environment. This is reflected in the expression for coordination.

Organization B: Decisionmaker 1 (or 3)

$$G_t^1 = T(x^1, v^{121}; z^{12}, y^1) \quad (19)$$

$$G_b^1 = H(x^1, v^{121}) - G_t^1 \quad (20)$$

$$G_n^1 = H(u^1) + H_{z^1}(v^1) \quad (21)$$

$$G_c^1 = \sum_{i=1}^{U^1} [p_i^1 g_c^{1i}(p(x^1)) + \alpha_i^1 \mathcal{H}(p_i^1)] \\ + H(\bar{z}^1) + H(z^{112}) + g_c^{1B}(p(\bar{z}^1, v^{121})) \\ + \sum_{j=1}^{V^1} [p_j^1 g_c^{1j}(p(\bar{z}^1 | \bar{v}^1=j)) + \alpha_j^1 \mathcal{H}(p_j^1)] \\ + H(y^1) + H(\bar{z}^1) + T_{z^1}(z^{112}, v^{121}) \quad (22)$$

Organization B: Decisionmaker 2

$$G_t^2 = T(z^{112}, z^{132}; y^{21}, y^{23}) \quad (23)$$

$$G_b^2 = H(z^{112}, z^{132}) - G_t^2 \quad (24)$$

$$G_n^2 = H(u^2) + H_{z^2}(\bar{v}^2) \quad (25)$$

$$G_c^2 = g_c^{2A}(p(z^{112}, z^{132})) + g_c^{2B}(p(\bar{z}^2)) \\ + \sum_{j=1}^{V^2} [p_j^2 g_c^{2j}(p(\bar{z}^2 | \bar{v}^2=j)) + \alpha_j^2 \mathcal{H}(p_j^2)] \\ + H(v^{121}) + H(v^{123}) + H(\bar{z}^2) + H(\bar{z}^2, \bar{v}^2) \quad (26)$$

It follows from expressions (15) to (26) that the interactions affect the total activity G of each DM. At the same time these interactions model the control that is exerted by the DMs on each other. These controls are exerted either directly through the command inputs v' or indirectly through the shared situation assessments z' .

All decisionmakers in Fig. 3 are subject to indirect control. The supplementary situation assessments z' modify the assessments z to produce the final assessment z . Since z affects the choice of output, it follows that each DM is influenced by the assessments of the other DMs.

Direct control is exerted in organization B, Fig. 4, through the command inputs from DM² to the other two members. The variables v' modify the response selection strategies $p(v|z)$ of DM¹ and DM³. Note that both types of controls, direct (v') and indirect (z'), can improve the performance of a decisionmaker, but can also degrade it.

The value of the total processing activity G , of each decisionmaker depends on the choice of the internal decision strategies adopted by him, but also on those of the other members of the organization with whom he interacts directly or indirectly.

Let an internal decision strategy for a given decisionmaker be defined as pure, if both the situation assessment strategy $p(u)$ and the response selection strategy $p(v|z)$ are pure, i.e., an algorithm f_h is selected with probability one and an algorithm f_s is selected also with probability one when the situation is assessed as being \hat{z} :

$$D_k = \{p(u=r) = 1; p(v=s|\bar{z}=\hat{z}) = 1\} \quad (27)$$

for some r , some s , and for each \hat{z} element of the alphabet \bar{Z} . There are n possible pure internal strategies,

$$n = U \cdot V^M \quad (28)$$

where U is the number of f_i algorithms in the SA stage, V the number of h_j algorithms in the RS stage and M the dimension of the set \bar{Z} . All other internal strategies are mixed [11] and are obtained as convex combinations of pure strategies:

$$D(p_k) = \sum_{k=1}^n p_k D_k \quad (29)$$

where the weighting coefficients are probabilities.

A triplet of pure strategies, one for each DM, defines a pure strategy for the organization:

$$\Delta_{k,\ell,m} = \{D_k^1, D_\ell^2, D_m^3\} \quad (30)$$

Independent internal decision strategies for each DM, whether pure or mixed, induce a behavioral strategy [11] for the organization.

$$\Delta = \{D^1(p_k), D^2(p_\ell), D^3(p_m)\} \quad (31)$$

Given such a behavioral strategy, it is then possible to compute the total processing activity G for each DM:

$$G^1 = G^1(\Delta); G^2 = G^2(\Delta); G^3 = G^3(\Delta) \quad (32)$$

This interpretation of the expressions for the total activity is particularly useful in modeling the bounded rationality constraint for each decisionmaker and in analyzing the organization's performance in the performance-workload space.

VI. Bounded Rationality and Performance Evaluation

The qualitative notion that the rationality of a human decisionmaker is not perfect, but is bounded, has been modeled as a constraint on the total activity G :

$$G^1 = G_t^1 + G_b^1 + G_n^1 + G_c^1 \leq F^1 \tau \quad (33)$$

where τ is the mean symbol interarrival time and F the maximum rate of information processing that characterizes decisionmaker i . This constraint implies that the decisionmaker must process his inputs at a rate that is at least equal to the rate with which they arrive. For a detailed discussion of this particular model of bounded rationality see Boettcher and Levis [1].

As stated earlier, the task of the organization has been modeled as receiving inputs X' and producing outputs y . Now, let Y be the desired response to the input X' and let $L(X')$ be a function or a table that associates a Y with each member of the input X' . The organization's actual response y can be compared to the desired response Y using a function $d(y, Y)$. The expected value of the cost can be obtained by averaging over all possible inputs. This value, computed as a function of the organization's decision strategy Δ , can serve as a performance index J . For example, if the function $d(y, Y)$ takes the value of zero when the actual response matches the desired response and the value of unity otherwise, then

$$J(\Delta) = E\{d(y, Y)\} = p(y \neq Y) \quad (34)$$

which represents the probability of the organization making the wrong decision in response to inputs x ; i.e., the probability of error. The procedure for evaluating the performance of an organization is shown in Fig. 5.

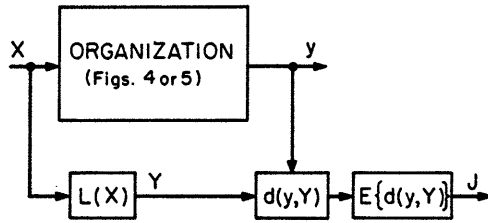


Fig. 5 Performance evaluation of an organization

The information obtained from evaluating the performance of a specific organizational structure and the associated decision strategies can be used by the designer in defining and allocating tasks (selecting the partitioning matrices π_i), in changing the number and contents of the situation assessment and response selection algorithms and in redesigning the interaction between the DMs. In order to do this, the designer can formulate and solve two problems: (a) the determination of the strategies that minimize J and (b) the determination of the set of strategies for which $J \leq \bar{J}$.

The first is an optimization problem while the latter is formulated so as to obtain satisficing strategies with respect to a performance threshold \bar{J} . Since the bounded rationality constraint for all DMs depends on τ , the internal decision strategies of each DM will also depend on the tempo of operations. The unconstrained case can be thought of as the limiting case when $\tau \rightarrow \infty$.

The solutions of the optimization and satisficing problems can be depicted graphically in the $N+1$ dimensional performance-workload space $(J, G^1, G^2, \dots, G^N)$. The locus of the admissible $(N+1)$ -tuples is determined by analyzing the functional dependence of the organizational performance J and the total activity G^1 of each decisionmaker i on the organization's strategy Δ .

For organization A and B the performance workload space is four dimensional, namely (J, G^1, G^2, G^3) . The G^1 of each decisionmaker is a convex function of the Δ eq. (31), in the sense that

$$G^1(\Delta) \geq \sum_{k,l,m} G^1(\Delta_{k,l,m}) p_k p_l p_m \quad (35)$$

where $\Delta_{k,l,m}$ is defined in eq. (30). Note that an alternate representation of Δ can be obtained from eqs. (30) and (31):

$$\Delta = \sum_{k,l,m} \Delta_{k,l,m} p_k p_l p_m \quad (36)$$

The result in eq. (35) follows from the definition of G^1 as the sum of the marginal entropies of each system variable, eq. (11), and the fact that the possible distributions $p(w)$, where w is any system variable, are elements of convex distribution space determined by the organization decision strategies, i.e.,

$$p(w) \in \{p(w) | p(w) = \sum_{k,l,m} p(w|\Delta_{k,l,m}) p_k p_l p_m\} \quad (37)$$

The performance index of the organization can also be obtained as a function of Δ . Corresponding to each

$\Delta_{k,l,m}$ is a value $J_{k,l,m}$ of the performance index. Since any organization strategy being considered is a weighted sum of pure strategies, eq. (36), the organization's performance can be expressed as

$$J(\Delta) = \sum_{k,l,m} J_{k,l,m} p_k p_l p_m \quad (38)$$

Equations (35) and (38) are parametric in the probabilities p_k, p_l , and p_m . The locus of all admissible (J, G^1, G^2, G^3) quadruplets can be obtained by constructing first all binary variations between pure strategies; each binary variation defines a line in the four dimensional space (J, G^1, G^2, G^3) . Then successive binary combinations of mixed strategies are considered until all possible strategies are accounted for. The resulting locus can be projected on the two-dimensional spaces (J, G^1) as shown in [1] in order to analyze the performance of a single decisionmaker. For organization A and B, projection of the locus on the three dimensional space (J, G^1, G^2) is practical and convenient because in both cases the properties of DM^3 are analogous to those of DM^1 .

The bounded rationality constraints, eq. (33), can be realized in the performance-workload space by constructing planes of constant G for each DM. For example, the constraint for DM^1 is defined by a plane that is normal to the G^1 axis and intersects it at $G^1_r = F^1_r \tau$. For fixed values of F^1_r , the bounded rationality constraint is proportional to the tempo of operations. As the tempo becomes faster, i.e., the interarrival time τ becomes shorter, the G^1_r becomes smaller and, consequently, a smaller part of the locus satisfies the constraint.

The solutions of the satisficing problem can be characterized as the subset of feasible solutions for which the performance measure $J(\Delta)$ is less than or equal to a threshold value \bar{J} . This condition also defines a plane in the performance-workload space that is normal to the J axis and intersects it at \bar{J} . All points on the locus on or below this plane which also satisfy the bounded rationality constraint for each decisionmaker in the organization are satisficing solutions.

The method of analysis presented thus far is illustrated in the next section through a simple example in which the two organizations forms, A and B are compared.

Example

A simple example has been constructed based on aspects of the problem of organizing batteries of surface to air missiles. Let a trajectory of a target be defined by an ordered pair of points located in a rectangle that represents a two-dimensional (flat) sector of airspace. From the ordered pair, the speed and direction of flight of the target can be determined. On the basis of that information, the organization should respond by firing either a slow or a fast surface-to-air missile or by not firing at all. The size of the sector and the frequency of the arrival of targets is such that three units are needed.

The first organization structure, corresponding to Organization A, is defined as follows. The rectangular sector is divided into three equal subsectors and a decisionmaker is assigned to each one. Each DM is capable of observing only the points that appear in his subsector. He can assess the situation, i.e., estimate the trajectory, and select the response, i.e., which weapons to fire, for targets with trajectories totally within his subsector. This is the case when both points

that define the target are within his subsector. Since it is possible for trajectories to "straddle" the subsector boundaries, it is necessary that situation assessment information be shared. Thus, DM^1 and DM^2 share information that relates to their common boundary. Similarly, DM^2 and DM^3 share information that relates to targets that cross their common boundary. To keep the computational effort small and the resulting loci uncomplicated, the situation assessment stages of DM^1 and DM^3 are assumed to contain a single algorithm f ; that of DM^2 contains two algorithms, f_1^2 and f_2^2 . In contrast, the response selection stage of DM^2 contains a single algorithm h , while the RS stages of DM^1 and DM^3 contain two algorithms h_1^i and h_2^i , $i=1,3$. Therefore, the internal decision strategies are $p(u^2)$, $p(v^1|\bar{z}^1)$ and $p(v^3|\bar{z}^3)$. The detailed structure of this organization is shown in Figure 6.

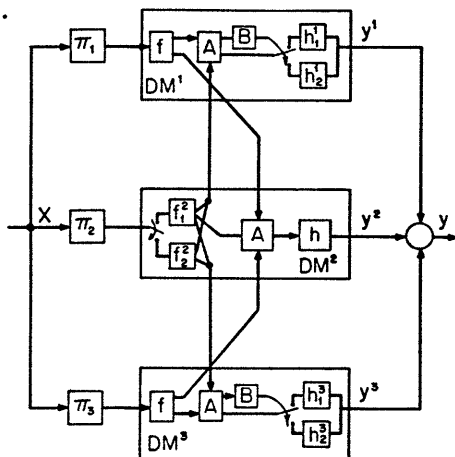


Fig. 6 Organization A in example

The second organizational structure, corresponding to Organization B, is defined as follows. The rectangular sector is divided into two equal subsectors for which DM^1 and DM^3 are responsible for assessing the situation and selecting a response. The two DMs do not share situation assessment between themselves; however, data from the area adjacent to the boundary between DM^1 and DM^3 is transmitted to the coordinator or supervisor DM^2 , who resolves conflicts and assigns targets either to DM^1 or to DM^3 , as appropriate. This is accomplished through command inputs $v^{1,2}$ and $v^{3,2}$ from the coordinator to the two commanders. They, in turn, exercise their response y^1 and y^3 , respectively. Again, for computational simplicity, it is assumed that DM^1 and DM^3 have a single algorithm f for their SA stage and two algorithms h_1^i and h_2^i for the RS stage. The coordinator, DM^2 , has an algorithm A for processing the assessed situations $z^{1,2}$ and $z^{3,2}$ and two algorithms, h_1^2 and h_2^2 , in the RS stage. The internal decision strategies are $p(v^1|\bar{z}^1)$, $p(v^2|\bar{z}^2)$ and $p(v^3|\bar{z}^3)$. The structure of this organization is shown in Figure 7.

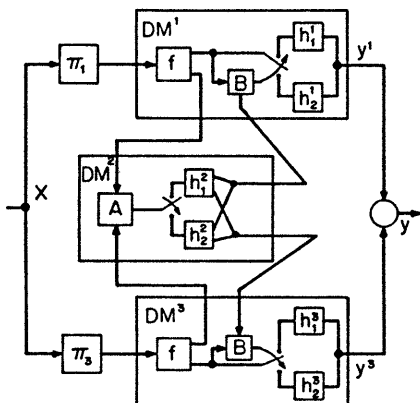


Fig. 7 Organization B in example

In order to compute the performance J of each organization and total activity G^1 of each DM^i , it is necessary to specify the probability distribution of the targets, all the algorithms f and h , the algorithms A and B and a table of correct responses for each possible target. Then, each admissible pure strategy of the organization is identified. The construction technique described in the previous section is used to obtain the locus of all the feasible (J, G^1, G^2, G^3) quadruplets.

Consider first the performance-workload locus for each DM in each one of the two alternative organizational structures. The three loci for each organization are obtained by projecting the (J, G^1, G^2, G^3) locus on each of the three (J, G^i) planes respectively. The results for Organization A of the example are shown in Figures 8-a,b,c; those of Organization B in Figures 9-a,b,c. The index of performance J measures the probability of error and is expressed in percentage. The total activity G^i is measured in bits per symbol. The two sets have been drawn at the same scale to allow for direct comparisons.

In Organization A, the probability that an incorrect response (error) will be made in processing an input ranges from 3.5 percent to 4.6 percent. Decision-makers DM^1 and DM^3 have very similar, but not identical loci. The difference in the loci is due to asymmetries in the input, i.e., $H(x^1) \neq H(x^3)$. Note, however, that their total activity G ranges between 22 to 35 bits per symbol.

The performance-workload locus of DM^2 , however, is quite different: the G ranges from 31 to 51 bits and, for a fixed G , there are, in general, two ranges of possible values of J .

The loci of all three DMs exhibit the properties discussed in [1]. The optimal (minimum error) performance is achieved with a pure strategy when there are no bounded rationality constraints. The existence of such a constraint would be shown by a line of constant G^1 with all feasible loci points to the left (lower G) of the line. If for example, the constraint was the same for all three DMs, namely,

$$G^1 \leq G_r = 40 \text{ bits/symbol}$$

then none of the admissible organization strategies would overload DM^1 and DM^3 ; however, DM^2 would be overloaded for some of the strategies. Therefore, only the organization strategies that do not overload any one of the organization's members are considered feasible.

Comparison for the three loci for the decision-makers in Organization B indicates that their loci are very similar: the organization's probability of error ranges between 2.4 and 4.0 percent. The total activity level for DM^1 and DM^2 is between 30 and 45 bits/symbol. Again, the differences in the two loci are due to asymmetries in the tasks (inputs) assigned to each DM. The coordinator, DM^2 , has a much lower workload: his total activity ranges between 15 and 30 bits per symbol. This is a consequence of not having to process either external inputs (no algorithms f) or command inputs (no algorithm B). In this case, if the bounded rationality constraints are set at $G_r=40$, they will restrict the choice of strategies by DM^1 and DM^3 and hence the organization's strategies.

If the two sets of loci are compared with each other, it becomes apparent that Organization B has the ability to perform better, i.e., make fewer errors, than Organization A. In the absence of bounded rationality constraints, B would be the preferred design. This would be especially true, if there were a satis-

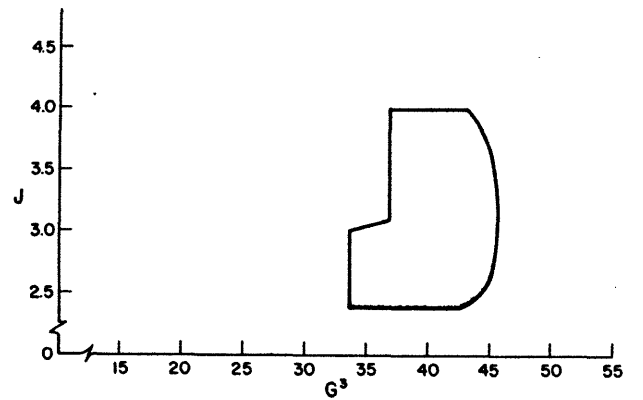
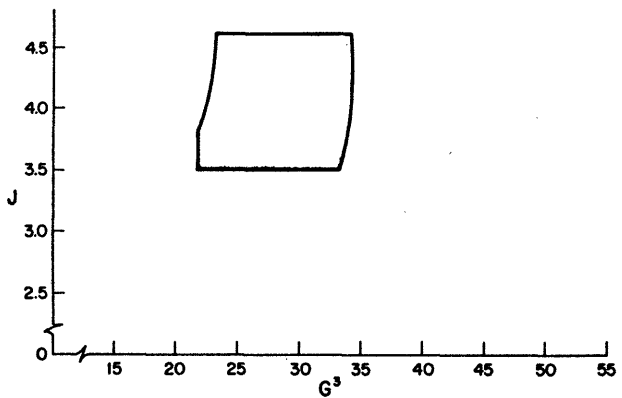
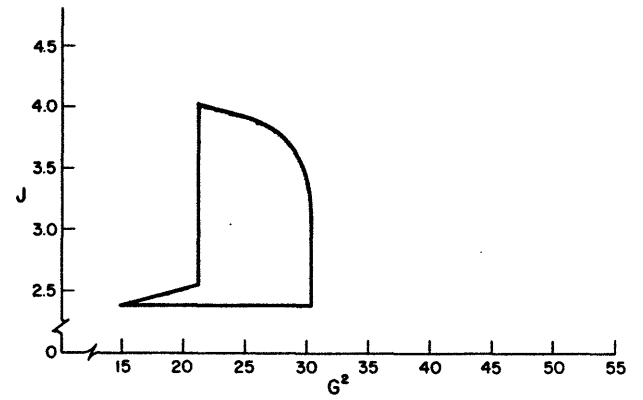
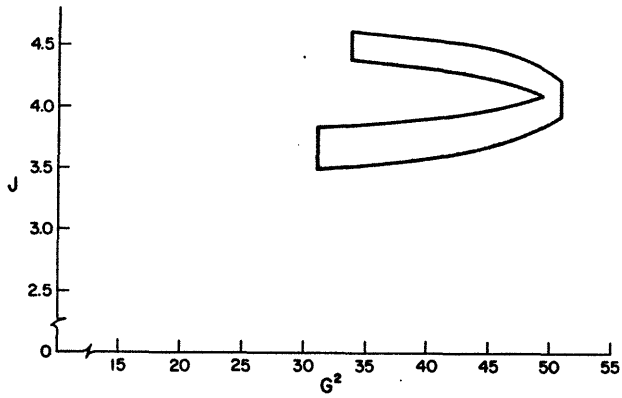
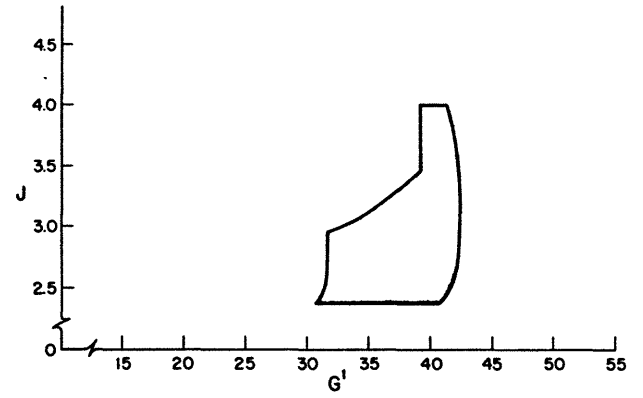
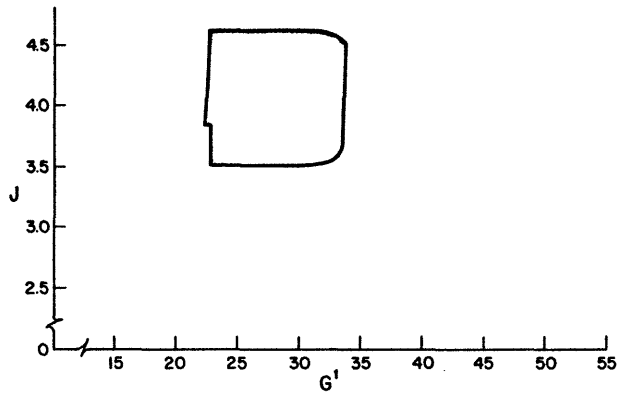


Fig. 8 Performance-Workload projection for DM¹, DM², and DM³, respectively, in Organization A.

Fig. 9 Performance-Workload projection for DM¹, DM², and DM³, respectively, in Organization B.

ficing constraint that required the organization's performance to be such that the error probability be less than a given value, such as three percent.

These results could be seen best by considering the comparison of the two (J, G^1, G^2, G^3) loci and the associated bounded rationality constraints. Since the performance-workload characteristics of DM^1 and DM^3 are essentially the same in each organization, the four-dimensional locus was projected in the (J, G^1, G^3) space. The two loci, L^A and L^B , are shown in Figure 10. The satisficing condition, $J < \bar{J}$ is shown as a plane parallel to the (G^1, G^2) plane intersecting the J axis at 3.0. The bounded rationality constraints for DM^1 and DM^2 are planes parallel to the (J, G^2) and the (J, G^1) plane at 40 bits/symbol.

It is clear from the figure that the choice of preferred organizational structure to carry out the assigned task depends in the values of the bounded rationality constraints and the satisficing threshold \bar{J} . If the satisficing constraint is $\bar{J}=3.0$, then the design represented by Organization A is not an effective one: the organization cannot perform the task. However, there are many strategies that the decisionmakers in Organization B can use to carry out the task without overload.

The evaluation of the two designs has been carried out in a qualitative manner using the geometric relationships between the various loci in the performance-workload space. A quantitative approach to the evaluation and comparison of alternative designs is the subject of current research.

Conclusions

An analytical methodology for modeling and analyzing structures of information-processing and decisionmaking organizations has been presented. The approach was applied to the design of three-person organizations assigned to execute a well-defined task. Implicit in the design of the organizational form is the C^3 system required to support the information processing and decisionmaking activity.

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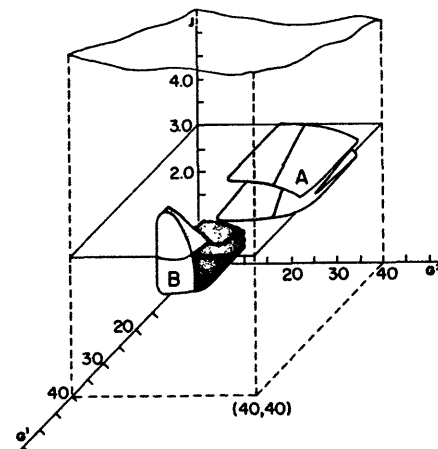


Fig. 10 Organizational Performance versus Individual Workload. Projection of four-dimensional loci in three-dimensional space for Organizations A and B.