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AN INERTIALLY GUIDED MISSILE WITH MID-COURSE RE-ALIGNMENT

by

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by

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Submitted to the Department of Aeronautics and Astronautics on May 19, 1961, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

A weapons system has been designed which would employ an inertially guided air-to-surface missile against tactical targets. Target information is obtained from radar carried in the launching aircraft. This thesis proposes a modified system, where target information is supplied by a ground observer near enough to the target to acquire accurate target information. The geometric alignment between the observer and missile is critical because accurate data transfer must be obtained. A method for data transfer and one with data transfer plus missile navigation reference system re-alignment are formulated and compared.

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The presentation of this thesis does not constitute approval by the Bureau of Weapons nor the Instrumentation Laboratory of the findings or conclusions contained therein. It is presented only for the exchange and stimulation of ideas.

TABLE OF CONTENTS

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OBJECT

To investigate the accuracy of an inertially-guided air-to-surface missile using target data provided during flight by a forward observer.

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CHAPTER I

INTRODUCTION

1.1 Proposed Weapons System

In the use of strategic missiles against targets such as enemy cities, railway centers, shipping harbors, etc., the position of the target is fixed and is known at the launch point. However, in the tactical use of missiles, such as air-to-surface missiles, against battle line targets the situation is quite different. In this case, the targets may be small, moveable, and often difficult to locate. Obviously, accurate target information is necessary, since the missile can certainly be no more accurate than the target positional data that it possesses.

Conventionally, this information is usually obtained with the use of equipment carried in the launching aircraft or the missile. Frequently though, it is difficult to obtain target information in this manner with sufficient accuracy to be acceptable. Enemy jamming procedures and camouflage techniques, atmospheric conditions, or terrain conditions could introduce large errors in the determination of the target's position. However, situations may exist when this information could be obtained by a forward observer near the target area.

It is proposed that the target information, after it is obtained by the observer, be passed to the missile in flight, after its

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release from the launching aircraft, and be used by the missile to proceed to the target.

However, the orientation of the missile's navigation coordinates with respect to the observer's reference coordinates must be known to accomplish data transfer. It is evident that the missile's navigation coordinates and the observer's reference coordinates should initially be aligned to the same reference frame. Then at any time during the relatively short time of flight of the missile, the misalignment between the two coordinate systems will be small.

The misalignment between the two coordinate systems consists of two parts; the initial misalignment and the misalignment due to the drift of the missile's navigation reference coordinates. The initial misalignment is from errors which result from instrumentation in an attempt to align one coordinate system with a reference coordinate system, misalignment due to reference coordinate drift is self explanatory.

The measure of success of this proposed weapons system, or of any weapons system, is the accuracy with which it impacts the target.

1.2 General Description of the Weapons System

The three major components of the proposed system are the missile, the observer, and the launching aircraft. A pictorial description is shown in figure 1-1.

The missile is launched possessing only the approximate position of the target. Hence at this time, its direction of flight is only approximately towards the target.

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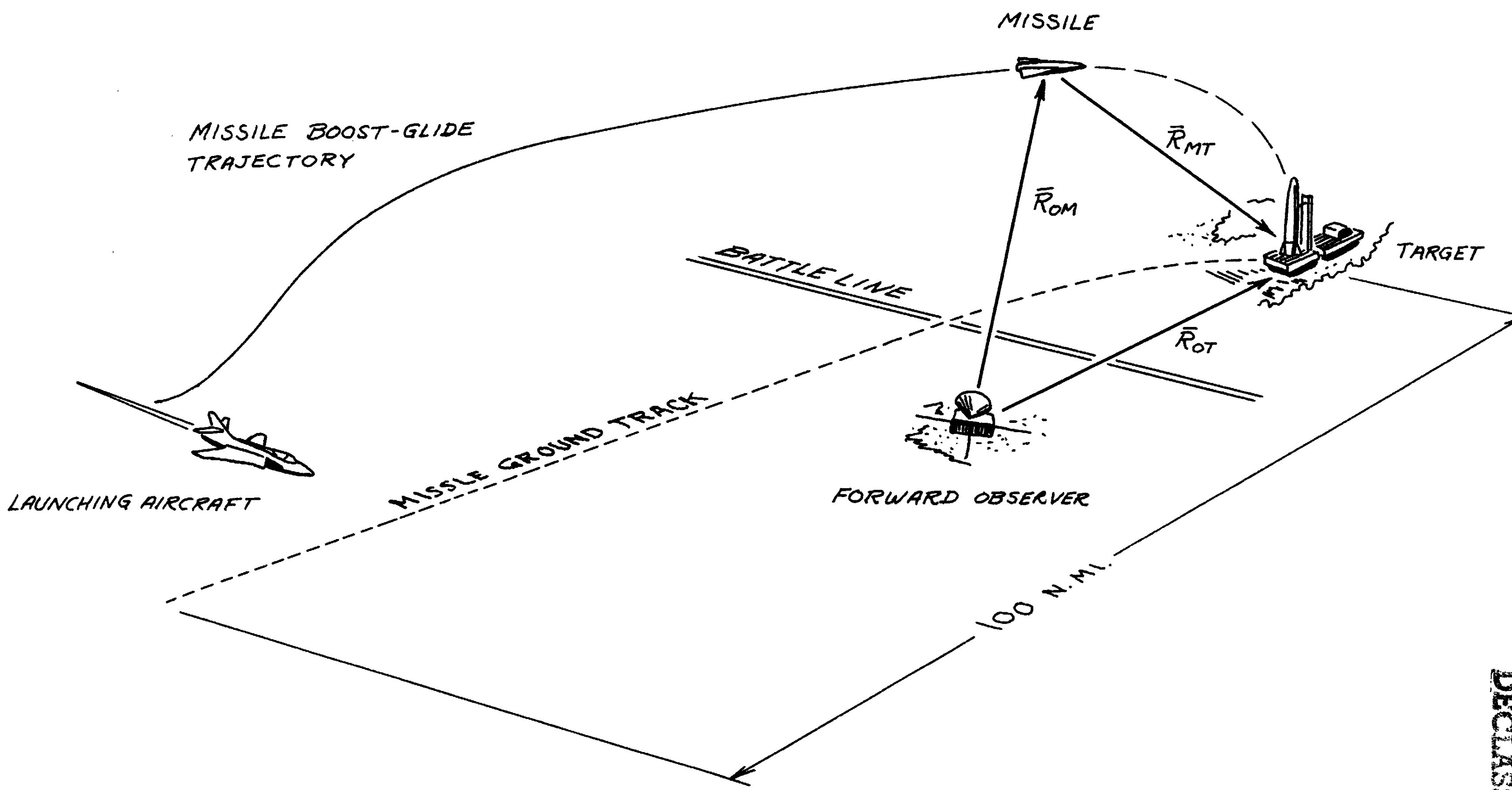

Fig. 1-1. Pictorial sketch of proposed weapons system

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Prior to missile launch, the observer has obtained R_{OT}, which is the vector from his position to the target. The observer obtains $\bar{R}_{OM}$, the vector from the observer to the missile, by tracking the missile continuously as it proceeds toward the target area. Then the observer computes $\bar{R}_{MT}$, the vector from the missile to the target, ($\bar{R}_{MT} = \bar{R}_{OT} - \bar{R}_{OM}$) and sends it to the missile in a form that can be used by the missile's navigation system.

1.3 Conditions for Data Transfer

As stated previously, the geometric alignment between the observer's reference coordinates and the missile's navigation reference coordinates must be known to accurately transfer the vector $\bar{R}_{MT}$ from the observer to the missile. If the two reference coordinates are not parallel and there is no compensation to account for this angular difference, the missile will not receive the true vector $\bar{R}_{MT}$. Some technique must be used to compare the orientation of one coordinate system with the other, if the errors introduced by data transfer are to be minimized.

1.4 Modes of Operation

Two different modes of operation will be explained in Chapter II and the results will be offered in Chapter III, showing circular impact error versus range.

In one mode, called the "Initial Alignment Mode", the observer will send the vector $\bar{R}_{MT}$ to the missile with no attempt to measure or correct for the misalignment existing between the two coordinate systems.

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The second mode of operation is called the "Mid-Course Re-alignment Mode". This operation involves obtaining the angular difference between the two coordinate systems, then re-aligning the missile's navigation coordinates with the observer's reference coordinates before transmitting the vector $\bar{R}_{MT}$.

1.5 Missile Description

The type of missile suggested for use in the proposed weapons system is an inertially guided air-to-surface missile, which is instrumented to accept navigational information from an external source, such as the proposed observer.

An inertial navigation system possesses several favorable characteristics which make it desirable for use in the missile. It has an all-weather capability and is immune to electronic countermeasures directed against it.

The function of the navigation system is to determine the instantaneous position of the missile with respect to some reference point, and to generate signals that will make the missile fly some desired trajectory to the target. The trajectory is discussed in Appendix C.

1.6 Observer Description

The observer may be a man or a group of men near the main line of resistance, and he must have the mobility required to operate in battlefield situations.

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The functions of the observer are to obtain the target information and to transmit it to the missile in a meaningful form. The equipment associated with the observer is described in Chapter V.

The methods the observer may use to gather target information will not be discussed; however, there is considerable literature covering this subject.[2,3,4,5] It is assumed for this thesis that the observer knows the target position accurately with respect to his coordinate system.

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CHAPTER II

SYSTEM DESIGN

2.1 Introduction

The system as proposed includes three basic elements: the launching aircraft, the missile, and the observer. These three elements must function together so as to cause the missile to impact at the target as accurately as possible.

Four fundamental assumptions form the basis for the design of the system:

1. Only the observer knows the exact location of the target.
2. The function of the launching aircraft in the system is completed when the missile is launched.
3. The missile is inertially guided.
4. The amount of communications equipment carried by the missile is to be minimized.

In order to determine the vector range from the missile to the target, at any instant, the basic system vector triangle of figure 2-1 must be solved. It is assumed that the observer knows the vector $\bar{R}_{OT}$; he must track the missile in three dimensions in order to determine $\bar{R}_{OM}$. The solution for the vector range $\bar{R}_{MT}$ is then simply:

$$[\bar{R}_{MT}]_o = [\bar{R}_{OT}]_o - [\bar{R}_{OM}]_o \qquad (2\text{-}1)$$

Equation (2-1) will be solved in the observer's reference coordinate system, which will not, in general, be the same as the missile's. Therefore the missile-to-target vector must be transformed into the missile's reference coordinate system:

$$[\bar{R}_{MT}]_p = T_{p,o} [\bar{R}_{MT}]_o \qquad (2\text{-}2)$$

where the transformation $T_{p,o}$ rotates the observer's reference coordinate system into the missile's reference coordinate system. The vector $[\bar{R}_{MT}]_p$ is then telemetered to the missile where it provides the final condition for the missile navigation computer.

The observer's and missile's reference coordinate systems must be chosen so that the transformation $T_{p,o}$ can be computed from information available to the observer. This transformation must be very accurate, for even a small angular error in specifying $[\bar{R}_{MT}]_p$ could result in a large impact error at the target. While it is possible to compute a transformation from the observer's <u>true</u>* reference coordinate system to the missile's <u>true</u> reference coordinate system, it is not possible to account in this way for the instrumentation errors in the missile's and the observer's <u>indicated</u> reference coordinates. These errors can be large in terms of the resulting missile impact error at the target.

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*The terms "true reference coordinates" and "indicated reference coordinates" refer respectively to the mathematically defined ideal coordinates in which the solution to the problem is formulated, and to the instrumented reference frame in which measurements are made.

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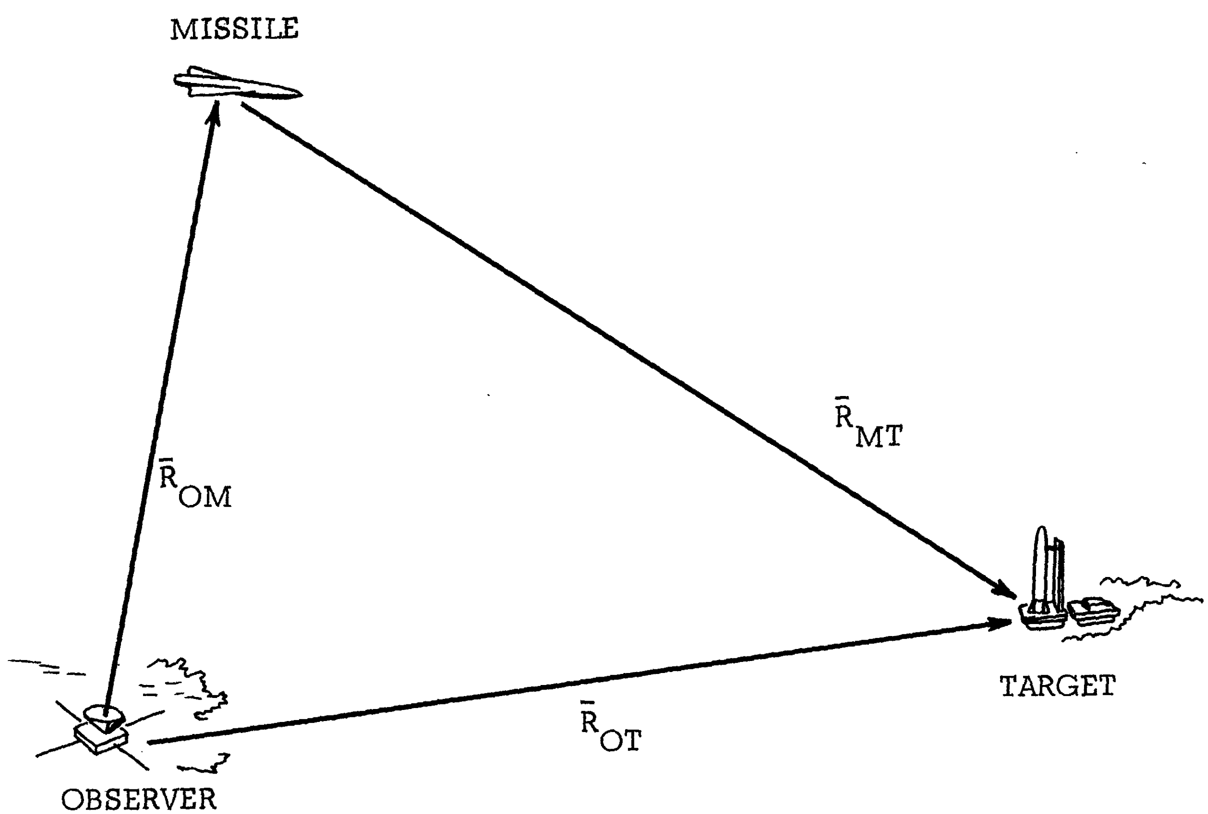

Fig. 2-1. Basic vector triangle

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If the use of a forward observer to provide target data is to be feasible, this resulting impact error must be kept as small as possible. One obvious method of doing so is aligning the observer's and missile's indicated reference coordinates as accurately as possible with their respective true coordinates, and accepting the remaining error. This will be referred to as the "Initial Alignment" mode of operation of the system.

The error can be further reduced if a direct alignment compatison is made between the missile's and the observer's indicated reference coordinate systems, as the missile comes under the control of the observer. A correction can then be made which will reduce, though of course, not entirely eliminate, the instrumentation errors in the alignment of one coordinate system with respect to the other. The circumstances in which this comparison can be performed, and the procedure to be used, will be discussed in later sections. This will be called the "Mid-Course Re-alignment" mode of operation of the system.

2.2 Reference Coordinate Systems

Fundamental to the design of any fire control system is the choice of reference coordinates. While numerous coordinate systems are available, it is desirable to choose that system into which the problem most naturally fits.[6] For the present problem, the coordinate system should be one which is meaningful to, and readily indicated by, both the observer and the missile.

It is apparent that a good choice of reference coordinate system is geographic, with the three axes aligned with, respectively, true North, East, and local vertical. This system can

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be readily instrumented by the observer, who is fixed on the surface of the Earth. The missile can be provided with this coordinate system in its initial erection and alignment, and after launch its inertial navigation system can continuously track local vertical and compute true North. The directions (in inertial space) of local vertical and true North at the missile will differ, in general, from those at the observer. Knowledge of their relative geographic positions, which will be obtained from the tracking link, will enable the observer to compute the necessary transformation. Hence, geographic coordinates appear to be a logical reference frame for this system. Vectors which are referred to local geographic coordinates at the observer will be given the subscript "o"; vectors referred to local geographic coordinates at the missile will be given the subscript "p".

It is necessary for the analysis of the system to distinguish between <u>true</u> and <u>indicated</u> coordinates in each case; therefore, vectors referred to the observer's indicated reference coordinates will be given the subscript "o_i", and vectors referred to the missile's indicated reference coordinates will be given the subscript "p_i".

2.3 Coordinate Transformations

The geographic position transformation $T_{p,o}$ which transforms a vector from the observer's true geographic coordinates to the missile's true geographic coordinates is derived in Appendix B. This transformation applied to a vector in the observer's <u>indicated</u> reference coordinates rotates that vector into the

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observer's indicated missile coordinates, denoted by the subscript "p_{o_i}". This new coordinate frame is a local geographic frame with its origin in the missile; but the instrumentation errors by which its axes differ from true North, East, and local vertical are those of the observer's equipment, not the missile's. Any further errors due to the transformation are small enough to be negligible. Thus, the coordinate system "p_{o_i}", "observer's indicated missile coordinates", is defined by the equality:

$$T_{p_{o_i}, o_i} = T_{p,o} \tag{2-3}$$

An additional transformation is desired to rotate observer's indicated missile coordinates into missile's indicated reference coordinates. This transformation is denoted by $T_{p_i, p_{o_i}}$. The two transformations applied successively will rotate a vector from the observer's indicated reference coordinates to the missile's indicated reference coordinates:

$$[\bar{R}_{MT}]_{p_i} = T_{p_i, p_{o_i}} \, T_{p_{o_i}, o_i} \, [\bar{R}_{MT}]_{o_i} \tag{2-4}$$

Determining the elements of and applying the second transformation $T_{p_i, p_{o_i}}$ is the essence of Mid-Course Re-alignment. The rotation angles are small, and reference (7) shows that the transformation between two nearly parallel coordinate systems, using small angle approximations, takes the form:

$$T_{p_i, p_{o_i}} = \begin{pmatrix} 1 & C_z & -C_y \\ -C_z & 1 & C_x \\ C_y & -C_x & 1 \end{pmatrix} \tag{2-5}$$

[redacted]

where C_x, C_y, C_z are the small-angle rotations about the three axes, required to bring "p_{o_i}" coordinates into coincidence with "p_i" coordinates.

It is the function of the alignment comparison mentioned in section 2.1 to evaluate C_x, C_y, and C_z. While it is possible, using the principles discussed in this thesis, to instrument a system which will evaluate all three rotation angles, it has been decided to investigate a simplified approach in which only C_z, the azimuth error angle, is evaluated. This simplification is justified on the grounds that C_z is three to four times larger than C_x or C_y, a fact which is verified in the system error analysis, Appendix C (specifically, figures C-6 and C-7). With this simplification, the transformation $T p_i, p_{o_i}$ takes the form:

$$T p_i, p_{o_i} = \begin{pmatrix} 1 & C_z \\ -C_z & 1 \end{pmatrix} \qquad (2\text{-}6)$$

Methods for evaluating C_z are discussed in the following section.

2.4 Mid-Course Re-alignment

There are, in general, two methods of comparing the alignment of one coordinate system with another. These are described in reference (8) as Direct Copying and Physical Vector Matching.

2.4.1 Direct Copying

This includes mechanical and optical techniques for aligning systems located in close proximity to each other, and also RF Interferometer techniques[9] which are useful over greater distances.

The use of an RF interferometer involves antenna problems, and places restrictions on the relative orientations and locations of the missile and the observer. Consequently, the use of Direct Copying is not feasible in this system.

2.4.2 Physical Vector Matching

In this technique, a physical vector which can be readily tracked is chosen as a basis for azimuth alignment. This vector is tracked in both coordinate systems and the apparent orientations compared. From knowledge of the geometry of the situation, the orientation of one coordinate system with respect to the other about one axis can be derived. The precision of this method is limited by the accuracy with which the vector can be tracked in the two coordinate systems.

2.4.3 Choice of the Physical Vector

A number of vectors present themselves as a possible choice for alignment comparison. Several criteria can be established to aid in making a choice. These are:

1. Readily measured by instrumentation available to the observer and to the missile.
2. Direction of the vector must be indicated accurately by both the missile and at the observer, in their respective coordinate systems.
3. The vector should be as nearly horizontal as possible.

Some of the physical vectors available are:

1. Missile Velocity. This vector is available to the missile in missile indicated reference coordinates as an output from its inertial navigation computer, with

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accuracy limited by initial conditions, accelerometer errors, and integrator errors. It can be obtained by the observer in observer's indicated reference coordinates by a process of smoothing and differentiating the missile's position vector, with accuracy determined by the nature of the tracking equipment.

2. Missile Acceleration. This vector is available to the missile with high accuracy, for it is free from initial condition errors and is limited only by the performance of the missile accelerometers. However, it is a small quantity throughout most of the missile's trajectory, and difficult for the observer to measure accurately. This might be avoided by having the missile perform a maneuver, say, a large angle turn, when alignment comparison is to be performed.

3. Missile Position Between Two Successive Fixes. This vector can be determined accurately by the observer, but the accuracy with which it can be computed in the missile is limited by initial velocity error and the errors introduced by the accelerometers and two integrations.

4. Observer-to-Missile Vector. The vector $\bar{R}_{OM}$ is necessarily indicated by the observer as part of the solution to the fire control problem. However, in order to indicate this vector in the missile, three-dimensional tracking equipment, such as an automatic tracking radar would have to be installed in the missile.

In this thesis, only the use of the missile velocity vector as the basis for alignment comparison is investigated. This vector has

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the following advantages, which makes it a logical choice:

1. It is available to the missile without requiring any additional equipment.

2. It is obtained by the missile and by the observer in their respective indicated reference coordinate systems with about the same degree of accuracy.

3. It does not change very rapidly with time, during most of the trajectory.

4. It is nearly horizontal during most of the trajectory, except for the missile's final dive to the target, during which an azimuth alignment correction is not really of value.

2.4.5 Instrumentation of Mid-Course Re-alignment

The use of only the missile velocity vector as a basis for alignment comparison provides a correction only for azimuth misalignment between the missile's and observer's indicated reference coordinates. This correction can be instrumented in the following manner: The observer tracks the missile, and computes missile velocity $[V_m]_{p_{o_i}}$ in observer's indicated missile coordinates, as seen in observer's indicated reference coordinates, by smoothing, differentiating, and transforming the missile position vector $[\bar{R}_{OM}]_{o_i}$. The horizontal component of this missile velocity vector is telemetered to the missile, where, in a special section of the missile computer, it is compared with the horizontal component of missile velocity in the missile's indicated reference coordinates, as computed by the missile navigation computer. Within the accuracy with which the vectors

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can be tracked, the angular difference between these two vectors is the azimuth alignment difference between the missile's and the observer's indicated reference coordinates. This angle can be applied as a correction to the missile's coordinate system to bring it into azimuth agreement with the observer's coordinate system. Assuming the correction is small (as it will be), the angle can be computed by calculating the cross product of unit vectors in the directions of the two velocity vectors:

$$\bar{C}_{z_{p_i, p_{o_i}}} = [\bar{1}\, V_{mh}]_{p_i} \times [\bar{1}\, V_{mh}]_{p_{o_i}} \qquad (2\text{-}7)$$

2.5 Missile-Observer Coordination

Having chosen the system computational reference coordinate system, defined the necessary coordinate transformations, and described the procedure for mid-course re-alignment, it is now possible to summarize in equation form the coordination required between the missile and the observer in order to provide the missile with the target vector.

The observer is assumed to know the target location in his indicated reference coordinate system. As the missile comes within range of his tracking equipment, the observer determines the missile position and velocity vectors and solves the basic vector triangle to obtain the missile-to-target vector:

$$[\bar{R}_{MT}]_{o_i} = [\bar{R}_{OT}]_{o_i} - [\bar{R}_{OM}]_{o_i} \qquad (2\text{-}8)$$

[redacted]

This vector and the horizontal missile velocity are transformed by the geographic position transformation:

$$[\bar{R}_{MT}]_{p_{o_i}} = T_{p_{o_i}, o_i} [\bar{R}_{MT}]_{o_i} \tag{2-9}$$

$$[\bar{V}_{mh}]_{p_{o_i}} = T_{p_{o_i}, o_i} [\bar{V}_{mh}]_{o_i} \tag{2-10}$$

These two vectors are then telemetered to the missile, where $[\bar{R}_{MT}]_{p_{o_i}}$ is entered in the missile navigation computer as the required final condition. A special section of the missile computer performs the calculation of equation (2-7). The reference coordinates of the missile navigation system are then re-aligned in azimuth using the transformation of equation (2-6):

$$[\bar{1}_x]_{p_{o_i}} = T^{-1}_{p_i\, p_{o_i}} [\bar{1}_x]_{p_i} \tag{2-11}$$

2.6 Functions of Elements of the System

Although the problems of mid-course re-alignment, as discussed in section 2.4, and missile-observer coordination, discussed in section 2.5, are the central ideas of the thesis, it is necessary to describe the weapons system as a whole in order to make a reasonable estimate of the system performance. Many assumptions are made, based on existing design data, for components which do not directly affect missile-observer coordination or mid-course re-alignment.

The three basic elements of the system are the launching aircraft, the missile, and the observer. Figure 2-2 is a block

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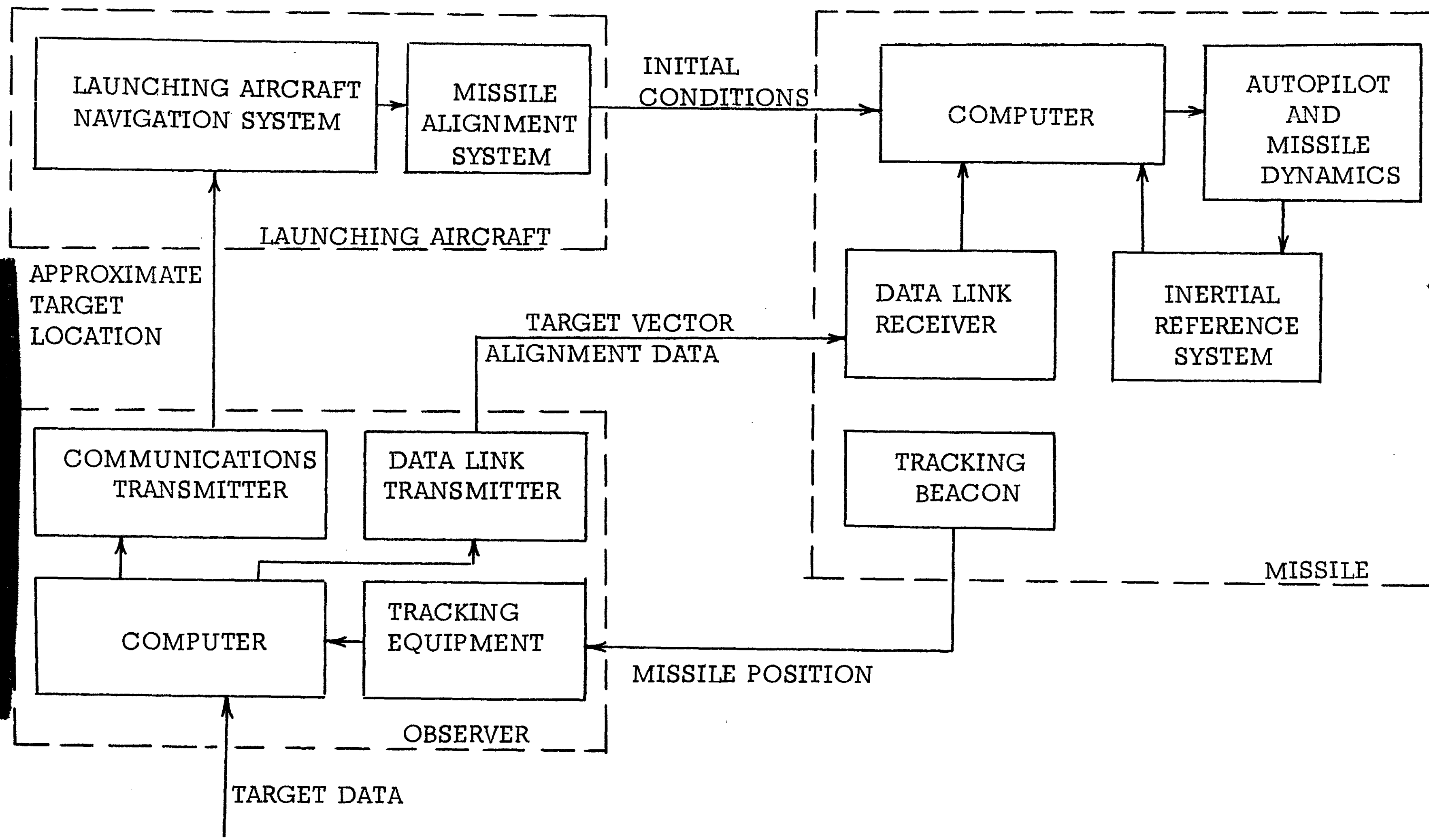

Figure 2-2. System information flow diagram

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diagram showing the components of interest in each of the three elements and the information flow in the system. In the following sections, the role of each of the three elements, and its equipment requirements, will be discussed in more detail.

2.6.1 The Launching Aircraft

The launching aircraft erects and aligns the missile inertial guidance system, provides initial conditions and the course to the approximate position of the target, and launches the missile with the desired launch conditions. To perform these functions, the launching aircraft must be equipped with a means of accurately indicating true North and its own velocity in geographic coordinates. This can be obtained from an inertial navigation system. Missile alignment equipment must be provided, plus communications for coordinating missile launch time and position with the observer. Reference (1) discusses in detail the expected performance capabilities of a master inertial navigation system and the missile alignment equipment. The perfromance data presented therein are assumed for this thesis.

2.6.2 The Missile

The missile accepts initial conditions and alignment information from the Launching Aircraft. After launch it follows a programmed trajectory toward the launching aircraft's estimated target position. When the observer initiates command transmissions, the missile accepts the target and alignment vectors, computes and performs coordinate system re-alignment. This is to be a continuous process, during the time that the observer is able to accurately track the missile. When command transmissions are stopped (this will be referred [redacted] "release time"), the missile will continue

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its flight in accordance with the last [redacted]ved. The missile navigation computer must generate steering signals to make the horizontal component of missile velocity parallel to the horizontal component of the target vector, but will follow the programmed trajectory in the vertical plane until the Guidance Vector (to be defined subsequently), becomes tangent to the trajectory. The missile then follows the Guidance Vector to the impact point.

The Guidance Vector concept, which is taken from reference (1) is a means of providing a vertical terminal dive to the target. The vertical terminal dive is desirable in order to minimize impact errors due to terrain clearance, uncertainty in target height, and instability in inertial navigation along the vertical. If the missile to target vector is expressed in component form as:

$$[\bar{R}_{MT}]_p = \bar{I}_{x_p} X_{MT} + \bar{I}_{y_p} Y_{MT} + \bar{I}_{z_p} Z_{MT} \qquad (2\text{-}12)$$

Then the Guidance Vector is defined as:

$$[\bar{R}_G]_p = \bar{I}_{x_p} X_{MT} + \bar{I}_{y_p} Y_{MT} + \bar{I}_{z_p} \left(Z_{MT} - \sqrt{X_{MT}^2 + Y_{MT}^2}\right) \qquad (2\text{-}13)$$

The Guidance Vector lies in the same vertical plane as the target vector, but has a smaller vertical component, thus directing the flight path above the target vector at all times, until the missile is directly above the target, in a vertical terminal dive. Figure 2-3 shows the resulting trajectory. $\bar{R}_G$ is initially above the horizontal; when it becomes tangent to the flight path, the missile leaves its programmed trajectory and follows $\bar{R}_G$.

[redacted]

In addition to an inertial reference system and a digital computer, the missile must be equipped with a data link receiver, and a beacon transmitter to facilitate tracking by the observer.

The physical characteristics of the missile are assumed to be those of the EAGLE missile proposal of reference (10). For purposes of this thesis, it is necessary to specify only the characteristics of the missile trajectory, and these have been taken directly from reference (10).

2.6.3 The Observer

The observer is assumed to have located the target in his indicated reference coordinates. He tracks the missile to determine its position, and provides the missile with target data and alignment information. The observer must have three dimensional tracking equipment of high accuracy. Two types of tracking equipment are considered in this thesis: Fire Control Radar, and Continuous Wave Phase Comparison techniques. Both types of tracking are discussed in detail in Chapter V.

It will be shown that Fire Control Radar cannot be expected to track the missile's velocity vector with sufficient accuracy to permit its use for alignment comparison. Therefore, if the observer must use radar tracking (e.g. in a submarine), the system can function in the "Initial Alignment" mode only.

CW Phase Comparison is inherently very accurate, but it requires a crossed-baseline antenna array, and its accuracy can be realized only if the baselines are accurately surveyed. The problems under field conditions are obvious. Providing these

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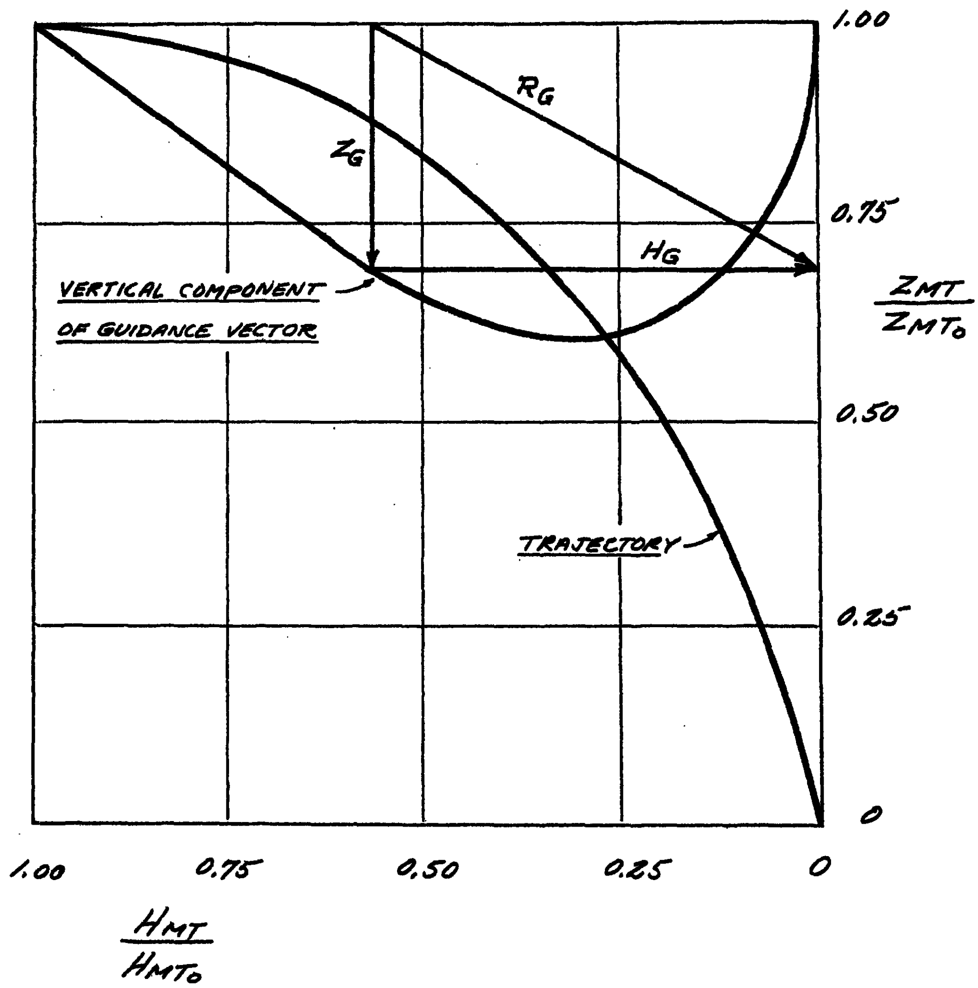

Figure 2-3. Guidance Vector and terminal trajectory

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difficulties can be overcome, the observer can obtain missile position and velocity with high accuracy, and operation in the "Mid-Course Alignment" mode is possible.

In addition to tracking equipment, the observer must have a computer, a data link transmitter, voice communications with the launching aircraft, and equipment for indicating the direction of his reference coordinates.

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CHAPTER III

SYSTEM PERFORMANCE

3.1 Introduction

The estimated performance of the proposed weapons system is presented in figures 3-1 through 3-4, which appear at the end of this chapter. The various curves show the estimated CEP versus horizontal distance from missile to target, at the time of release from observer's control. Time of release refers to the time the missile receives its last transmission from the observer, and proceeds independently to the target.

Briefly, the missile is launched within 100 n.m. from the target, and from a point such that the missile will pass within 17 n.m. slant range from the observer. This requires the missile to be within a horizontal range of 10 n.m. from the observer at some point on its trajectory to the target, since the trajectory height is approximately 15 n.m. In this sense, the abscissas of the figures also represent the approximate distance from the observer to the target.

Figures 3-1 through 3-3 show the effects of the major errors that contribute to the CEP. These errors have been placed into three groups: missile navigation system errors, errors due to misalignment between the missile's indicated navigation reference coordinates and the observer's indicated coordinates, and errors

[redacted]

due to the observer's missile [redacted] error groups will be referred to as Navigation System errors, Coordinate Systems Misalignment errors, and Tracking errors, respectively. Figure 3-1 and 3-2 represent the Initial Alignment mode, where Figure 3-1 corresponds to radar tracking, and Figure 3-2 corresponds to phase comparison tracking.* Figure 3-3 shows the errors associated with the Mid-Course Re-alignment mode. Figure 3-4 compares the total CEP for Initial Alignment and Mid-Course Re-alignment.

For example on interpreting the figures, if the missile's range to the target is 20 n.m., the probable CEP, using Mid-Course Re-alignment, would be 240 feet. For a 10 n.m. range, the CEP would be 160 feet. These results are obtained from figure 3-4.

3.2 Initial Alignment Mode

This refers to the mode of operation where no re-alignment is accomplished; only the vector $\bar{R}_{MT}$ is sent to the missile.

One can observe that the estimated CEP for longer ranges is about equal whether using radar or DME-COTAR for tracking the missile. This is because at longer ranges the Tracking error becomes small when compared to the errors caused by the Navigation System inaccuracies and the errors due to coordinate systems misalignment.

However, at ranges less than 20 n.m., tracking with DME-COTAR is significantly superior, and with decreasing range from 20 n.m. it begins to compare favorably with the results obtained with Mid-Course Re-alignment.

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* Subsequently, will be referred to as DME-COTAR, a highly accurate phase-comparison tracking system.

[redacted]

When the missile is tracked [redacted] the theoretical CEP at zero range is 5 feet. However, noise factors and other system disturbances would prevent this high degree of accuracy. Zero range infers that the observer tracks the missile all of the way to impact. This, of course, would be difficult, but it reveals that extremely accurate results can be obtained when the "release of the missile" is accomplished at short missile to target ranges. Obviously, the battle line situation will determine the proximity within which this can be accomplished.

3.3 Mid-Course Re-alignment Mode

This refers to the mode of operation where the missile's indicated reference system is re-aligned prior to receiving the vector $\bar{R}_{MT}$.

Radar can not be used for tracking the missile in conjunction with Mid-course Re-alignment, since with radar, the missile's velocity vector can not be determined accurately enough to be acceptable. However, the use of DME-COTAR produces very satisfactory results. When re-alignment is accomplished, the error due to Coordinate System Mis-alignment is reduced substantially, and is no longer the dominant error source. For example, for a range of 50 n.m., the estimated CEP due to Coordinate Systems Misalignment is reduced from 525 feet to 240 feet. The dominant error source is now from the missile navigation system.

It is noted that the main component of the missile navigation system error results from the error in the initial velocity that is given to the missile. The missile navigation system error is represented in figure C-3. Hence, if Mid-course Re-alignment

[redacted]

[redacted] is accomplished, the next logical step to reduce the overall CEP would be to improve the accuracy with which the missile receives its initial velocity.

3.4 Conclusions

It is important to realize that all the errors due to the misalignment between the missile's navigation coordinate system and the observer's reference coordinates can not be eliminated by Mid-Course Re-alignment, even to within the precision to which the missile's velocity vector can be determined. The error derivations in Appendix C show that the major portion of the misalignment errors can be eliminated, but as section C.6.5 reveals, there remains an angular difference uncompensated for. This is the angle e shown in figure C-4.

The four figures, 3-1 through 3-4, represent the estimated capabilities of the proposed weapon system using an air-to-surface missile in conjunction with the proposed observer. For clarity, the results have been presented in a brief form. The complete derivations are included in Appendix C.

[redacted]

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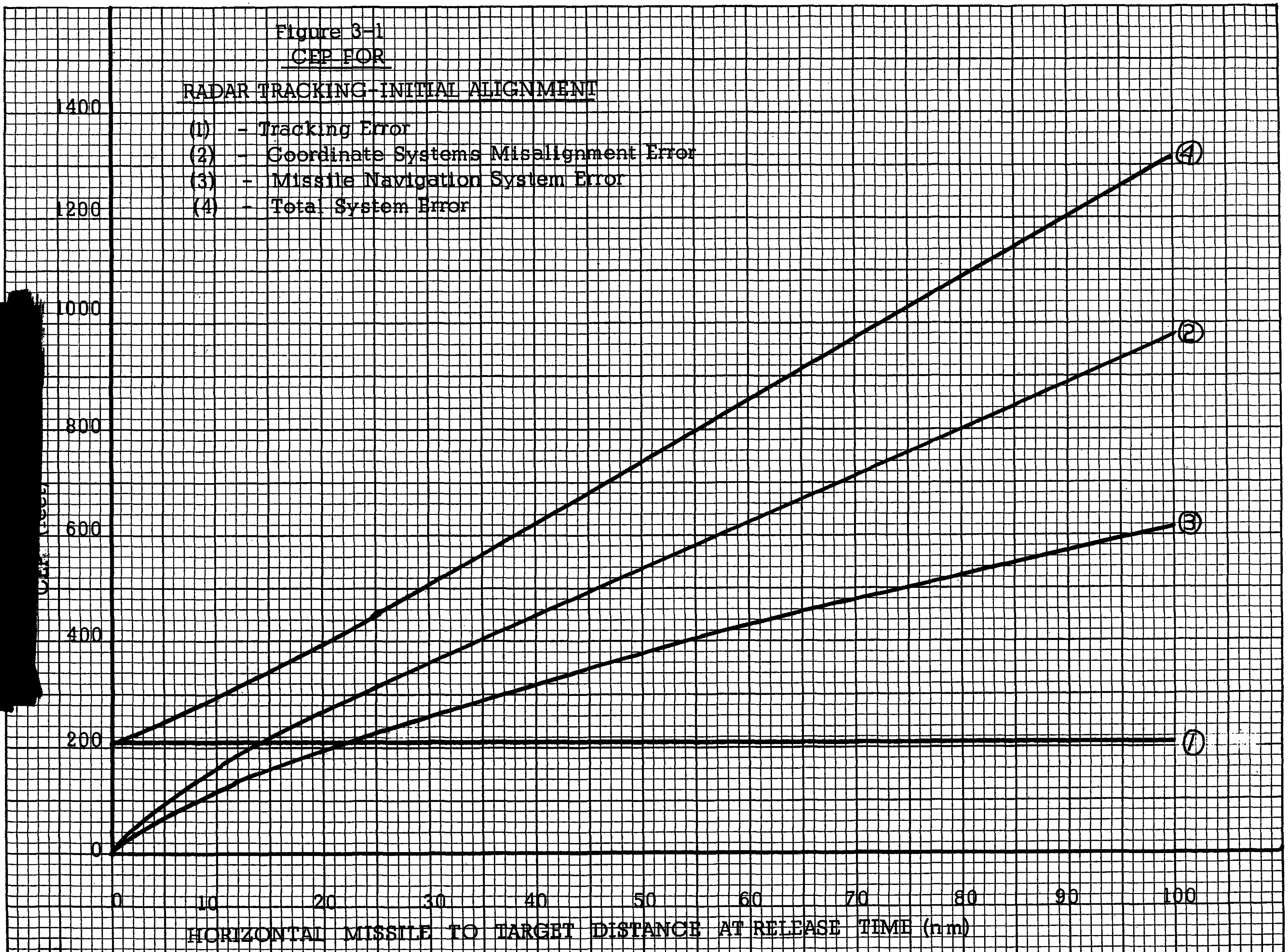
K+E 5 X 5 TO THE CENTIMETER KEUFFEL & ESSER CO. 358-15K MADE IN U.S.A.



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K+E 5 X 5 TO THE CENTIMETER 358-15K
KEUFFEL & ESSER CO. MADE IN U.S.A.

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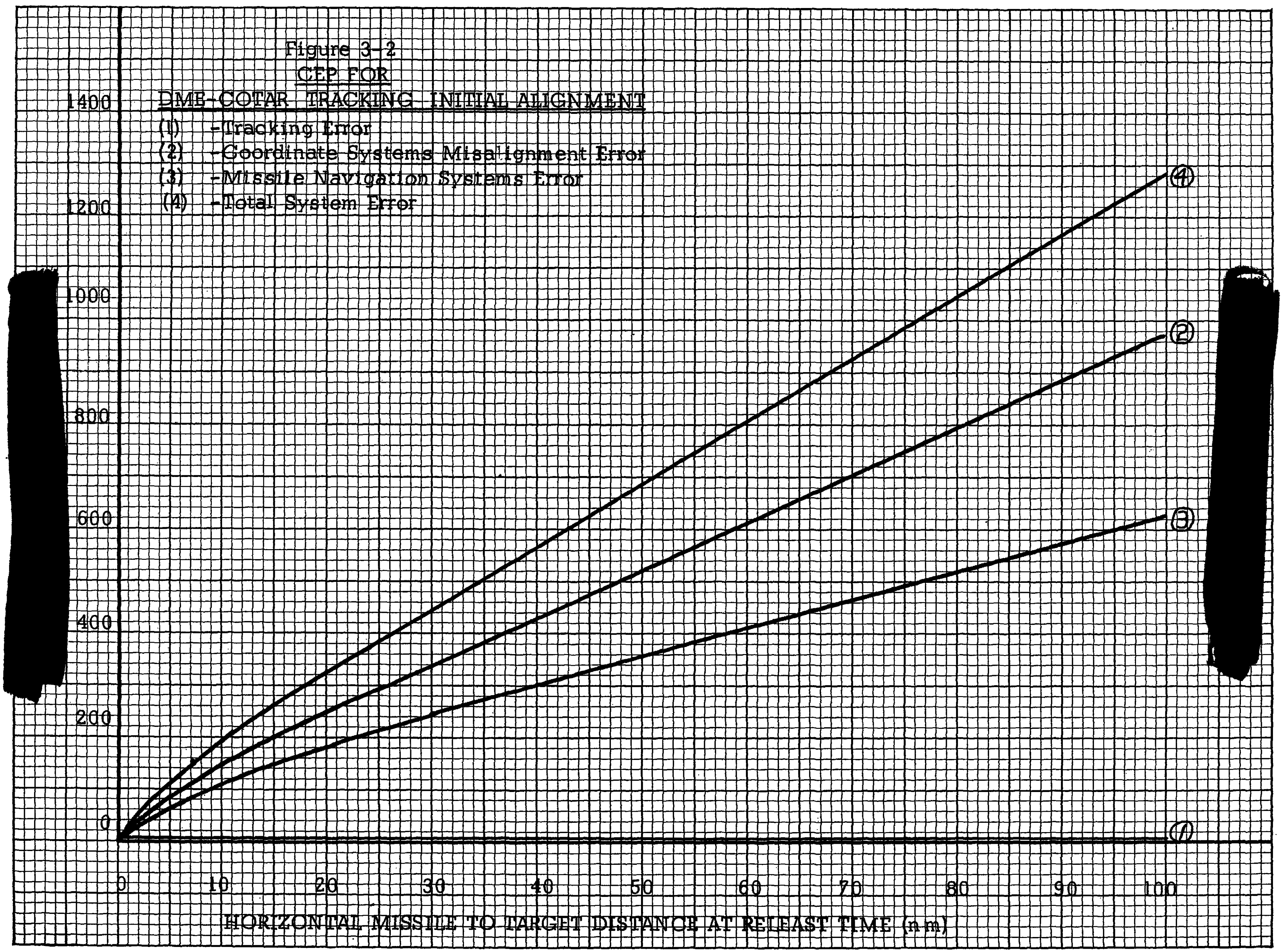

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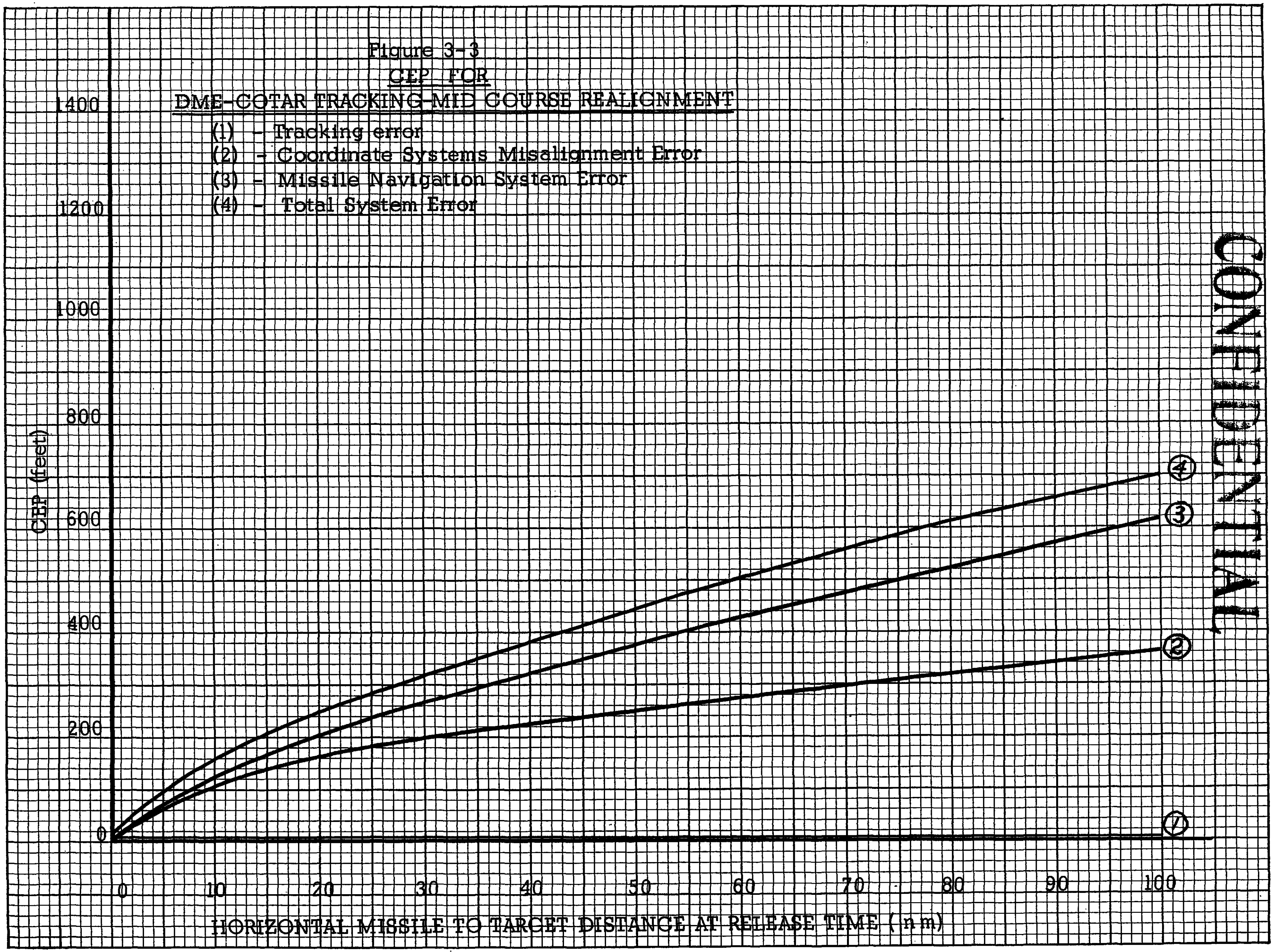

Figure 3-3
CEP FOR
DME-COTAR TRACKING-MID COURSE REALIGNMENT

[redacted]

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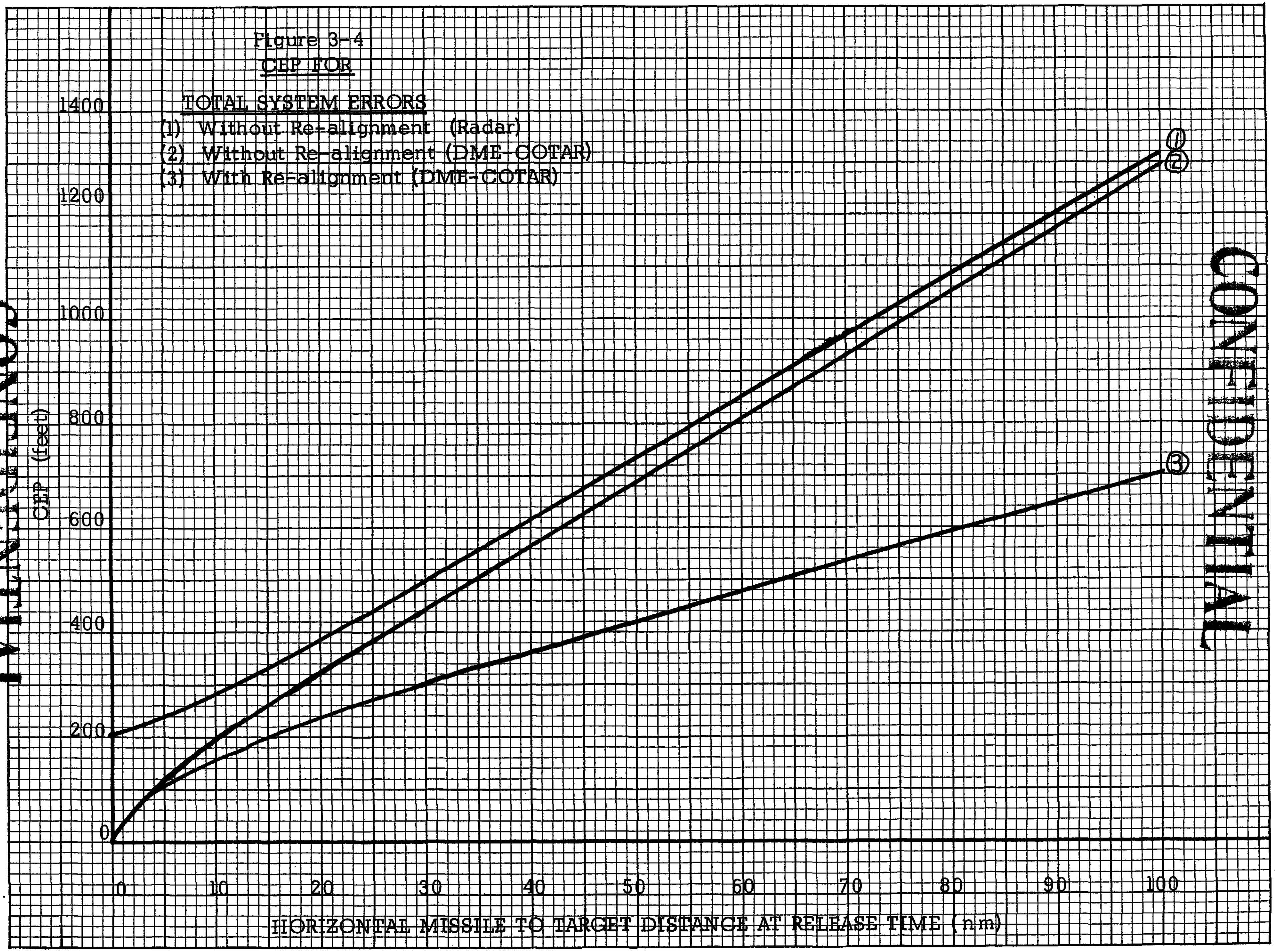

Figure 3-4
CEP FOR

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[redacted]

CHAPTER IV

MISSILE AND LAUNCHER EQUIPMENT

4.1 Missile Equipment

The missile components which will be discussed include only those that are directly associated with the design of the proposed weapons system. They are:

1. Inertial reference system,
2. Digital computer,
3. Data link receiver,
4. Alignment comparison system,
5. CW transmitter and/or transponder.

4.2 Missile Inertial Reference System

The inertial reference system supplies an inertially fixed member upon which are mounted three accelerometers which measure the orthogonal components of missile specific force. Three single-degree-of-freedom floated integrating gyros, with their input axes arranged to be mutually orthogonal, are mounted on the stable member. These gyros function to maintain the platform non-rotating with respect to inertial space. The initial orientation, within instrumentation error, will be North, East and along the local vertical.

A "phantom vertical" indicating system[11] is used, allowing the platform to be free of torquing devices. The phantom directions of the reference frame are stored in the digital computer. This system possesses the same properties as a physical (torqued platform) indicating system. Yet it has the inherent advantages of being smaller, more accurate, and more flexible.

The platform is immune to the motions of the missile through instrumenting four base-motion-isolation gimbal mounts.

The vertical direction must be stored in the computer, as a set of direction cosines or other reference coordinates, giving the direction of vertical with respect to the frame which does exist in the equipment, namely that of the stable platform. The stable platform, in turn, represents the original reference directions in inertial space.

The phantom vertical indicating system properties oscillate with the 84 minute "Schuler-tuned" period. This property is a necessity for a device to track the local vertical from a moving base (12).

The required performance of the components of the inertial system is summarized in Table 4-1. It is believed that these performance requirements are realistic, and are obtainable with components in use at the present time.

It is interesting to note that the performance obtained even with the best inertial components today, may be far from the ultimate degree of accuracy obtainable. In future years, accuracies several orders of magnitude better than those attained today, may be realizable.

[redacted]

Gyro drift - Fixed	0.25 deg/hr
Mass unbalance	0.40 deg/hr/g
Anisoelasticity	0.015 deg/hr/g^2
Accelerometer bias	0.0025 ft/sec^2
Accelerometer uncertainty	0.0025 ft/sec^2
Accelerometer scale factor	0.01%

Table 4-1

Performance Data for Inertial Components

4.3 Missile Digital Computer

The computer must perform all navigation and guidance computations for the missile. It is the information center of the missile system.

The flexibility that digital computers offer, makes it possible to use the system just described. It allows the complexity of the problem to be taken off the gimbals and to be put into the computer. The capabilities of a digital computer are practically unlimited. They can generate all manner of functions, can make decisions to perform one type of operation (such as a certain trajectory) if a given set of conditions exist, or another type of operation if a different set exists. They can integrate with respect to any variable, and can perform non-linear operations without difficulty. Their accuracy is limited only by the size, weight and number of elements of the computer package. Theoretically, any desired accuracy could be attained.

[redacted]

For this thesis study, the computer could well be the limiting factor on the degree of accuracy obtained. Since the missile position measuring device is very accurate, the accuracy of the digital computer would have to be of the same order of magnitude, at least.

Table 4-2 shows the size computer necessary to attain respective degree of accuracies in azimuth and range computations for a tracking range of 50 n.m.

	Accuracy		
Size (bits)	% of Measurement	Azimuth (MR)	Range (ft)
10	1/1023	6.13	300.0
11	1/2047	3.06	146.0
12	1/4095	1.53	73.5
13	1/8191	0.767	36.8
14	1/16,383	0.384	18.4
15	1/32,767	0.192	9.18
16	1/65,535	0.096	4.6
17	1/131,071	0.048	2.3

Table 4-2

Computer Size and Accuracy

A detailed discussion of the instrumentation of a digital computer is given in References (11) and (13).

4.4 Missile Data Link Receiver

Its function is to receive the information that is sent from the observer into the proper missile components.

4.5 Missile Alignment Comparison System

The orientation comparison of the missile's indicated reference system with the observer's indicated reference system will be done by the computer. The computer receives the observer's reference orientation in the manner described in Chapter II. It compares this orientation with the missile's indicated reference system.

Physical re-alignment of the missile's indicated reference system does not occur. Instead, the computer "remembers" the angular difference between the two reference systems and applies a correction to the data received to compensate for this angular difference. It accomplishes the same effect as physical re-alignment would.

4.6 Missile CW Transmitter or Transponder

If the missile is to be tracked by the observer using DME-COTAR, (see section 5.2.2) a transmitter and transponder will be carried by the missile. The complete package will occupy slightly less than one cubic foot and will weigh fifteen pounds. It is transistorized as much as possible, making it rugged and reliable. Only a radar transponder will be used if the missile is to be tracked by radar. The use of a transponder to aid radar tracking improves the accuracy considerably. A possible type, called a traveling wave tube amplifier weighs about ten pounds and occupies 200 cubic inches. It receives, amplifies (about 25 db) and re-transmits the signal received from the observer. Average power input is 30 watts. It is capable of frequency agility to combat possible enemy jamming procedures.

4.7 Launcher Equipment

The launching aircraft must have the following systems:

1. Master Navigation System,
2. Alignment System,
3. Communications System.

Only a brief description of each system will follow, since the proposed system is independent of the method of launch.

4.7.1 Master Navigation System

The purpose of the launching aircraft's navigation system is to provide highly accurate initial conditions for the missile's navigation system prior to launch. This includes velocity and azimuth information. Table 4-3 contains the assumed performance for this system.

Error Source	Assumed Performance
Indicated velocity	1 fps
Indicated vertical	1 MR
Indicated azimuth	1 MR

Table 4-3

Performance of Master Navigation System

4.7.2 Launcher Alignment System

Alignment consists of erecting the missile's navigation reference platform to the local vertical and aligning it to the Master system in azimuth, which will be indicating true North. This will be done prior to launch. The alignment errors are summarized in Table 4-4.

Component	Error
Vertical alignment	1 MR
Azimuth alignment	2 MR

Table 4-4

Alignment System Performance

The erection of the missile's platform to the vertical is independent of the launcher's vertical indication, since the missile performs this function itself.

The azimuth alignment of the two systems is complicated by the distance separating the two systems and the non-rigidity of the aircraft structure. The two systems can have the same orientation with respect to their bases, and still be misaligned because the bases themselves are misaligned. Base misalignment could occur because of missile installation error, or because of aircraft structure motion due to aerodynamic loads.

4.7.3 Communications System

There must exist a communications system between the launching aircraft and the observer for coordination purposes. The launching aircraft must know the approximate position of the target prior to launching the missile. The observer must know the approximate position and time of the launch to aid him in tracking the missile. Where feasible, the observer could designate the approximate launch point for the launching aircraft, a launch point that would allow the observer to gather more accurate missile tracking data as the missile traverses its flight trajectory.

[redacted]

CHAPTER V

OBSERVER EQUIPMENT

5.1 Introduction

The observer must have the following equipment at his disposal:

1. Missile Tracking System,
2. Reference Direction Indicating System,
3. Computer,
4. Communications System,
5. Equipment of Techniques for acquiring target data.

5.2 Missile Tracking System

As was stated in Chapter II, there are two different methods proposed for tracking the missile: Fire Control Radar and DME-COTAR. A radar missile tracking system is more flexible but less accurate than the DME-COTAR tracking system.

5.2.1 Fire Control Radar

The equipment comprising a radar tracking system could be transported by a land vehicle, such as a truck, allowing a certain degree of mobility. Since radar principles are well known, only the performance characteristics will be presented. This is done in Table 5-1.

[redacted]

Antenna diameter	5 ft.
Beam width	1.1°
Gain	40 db
Carrier frequency	1000 Mcps
Peak Power	500 KW
Pulse width Δ	0.5 usec
P.R.F.	150/sec
Receiver Noise Figure	5 db
System loss	4 db
Accuracy - Range	25' or .2% R
Azimuth	1 MR

Table 5-1

Tracking Radar Performance Characteristics

The range and azimuth accuracy figures were obtained in the following manner:

$$\text{Range accuracy (sec)} = \frac{\Delta}{10} = \frac{.5 \times 10^{-6}}{10} = .05 \times 10^{-6} \text{ sec}$$

$$R_{min} = \frac{C}{2} \Delta = \left(\frac{9.84 \times 10^{8}}{2}\right) (.05 \times 10^{-6}) \approx 25 \text{ ft. (for short ranges)}$$

The tracking range will usually be in excess of twenty miles, therefore the figure .2% R was used for all radar tracking accuracies.

<u>Azimuth Accuracies</u> - The parameter which determines azimuth accuracy is the width of the radar beam, which is usually specified as the beam width (BW) between half-power points. This is the angle between lines on opposite sides of the main beam

[redacted]

axis, along which the power density is half as great as it is on the main axis.

$$BW = 57\,\lambda/D \quad \text{(for paraboloidal dish) } D = \text{dish diameter}$$

$$BW = (57/5)\left(\frac{9.8 \times 10^{8}}{10^{9}}\right) \approx 1.1^{\circ}$$

$$\text{Azimuth accuracy} = \frac{BW}{20} = \frac{1.1^{\circ}}{20} \approx 1 \text{ MR}$$

The radar accuracy cited is believed to be realistic, rather than optimistic, since the missile is a friendly target equipped with a transponder. The observer will have ample time for "smoothing" the tracking information.

CEP errors arising from radar tracking inaccuracies are given in Chapter III, and the derivations are included in the Appendices.

5.2.2 DME-COTAR Tracking System

This tracking system is capable of high tracking accuracies in both range and azimuth. It also possesses the following requirements:

1. Operates from a single site,
2. Measures the missile's spatial coordinates in real time,
3. Has a small data reduction time.

Company sources (14) indicate that a field system has been tested with highly satisfactory results. The entire system can be carried by a small truck. It is represented in the block diagram, figure 5-1.

The DME equipment uses the time of transit from the ground transmitter to the missile transponder, and return, to determine

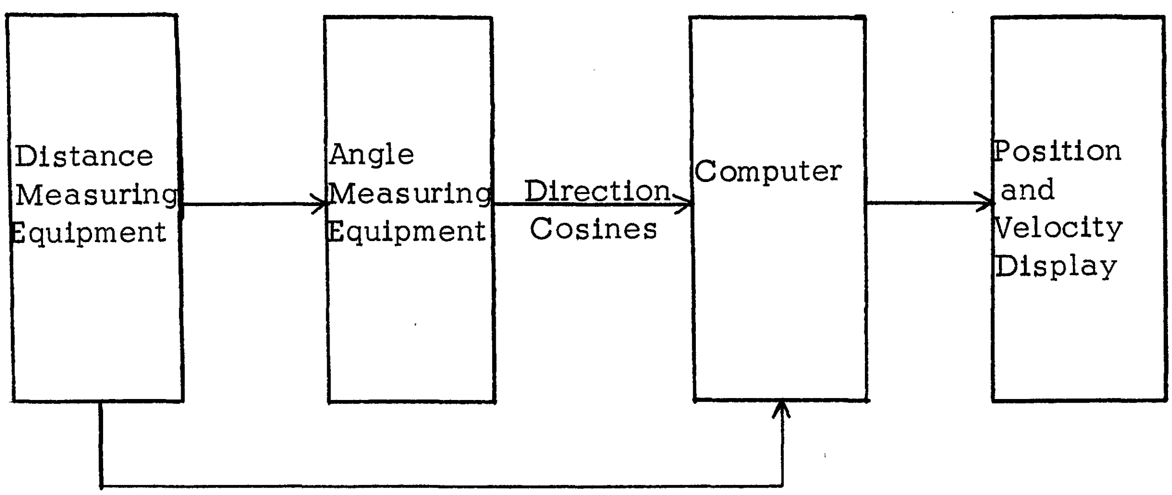


Figure 5-1. DME-COTAR functional block diagram

the slant range between them. The phase of the return signal is compared with that of the transmitted signal, and the resultant time delay measurement is calibrated to read the slant range directly.

The "fine" range measurement is made with a FM sub-carrier of 491.76 KC corresponding to a wave length of 2000 feet. If the radial range to the transponder changes by 1000 feet, the total path length changes by twice that or 2000 feet. Therefore, 1000 feet of range is represented per cycle of phase data. Present day electrical-mechanical servoed phase meters allow a measurement to be made to an accuracy of between one half and one degree of phase data. Hence:

$$\frac{1000 \text{ ft}}{360^{\circ}} = 2.78 \text{ ft/deg of phase data}$$

A conservative figure of a 3 foot range error was used for the error analysis, which is contained in the appendices.

As can be seen, a sub carrier of 491.76 KC provides high precision, but the data it provides cycles with each 1000 foot change in radial range. To resolve the ambiguities that exist initially, sub carriers of lower frequencies are used first, then sub carriers of increasing frequency are used in succession until a frequency is reached which will provide the desired accuracy. A frequency of .815 KC allows a non-ambiguous positional determination to a range of 100 n.m.

Figure 5-2 shows the geometry associated with the Angle Measuring Equipment (AME). The ground equipment consists of a central ground station located between two separated receiving antennas, A and B. The phase delay (ϕ) of the signal received at antenna A with respect to the signal received at antenna B, is a meaaure of the distance, d. The distance S (which is the antenna separation) is precisely known, so the measurement of the phase delay can be calibrated to read directly in the direction cosine value, which describes the transmitter's position.

The "fine" cosine measurement is made at the carrier frequency with an antenna separation of 50 wave lengths. The carrier frequency is 221 Mcps, corresponding to an antenna separation of 220 feet. Each cycle of phase difference corresponds to a change in direction cosine value of $\frac{1}{50}$ or .02. Since measurements can be made within one half to one degree, the direction cosines measurements are made to a precision of between 28 to 56 x 10^{-6}. This results in knowing the direction of the missile velocity vector within a maximum error of .5 MR, using the equations derived in Appendix B.

The ambiguities that result in the data measurement are solved using a less precise measurement corresponding to an antenna separation of 5 wave lengths. This again is resolved with a third even less precise measurement corresponding to an antenna separation of 1/2 wave length. An actual 1/2 wave length separation is not used, as mutually coupling between antennas of this spacing would cause large phase perturbations. The problem is solved electronically, combining data from a 4 1/2 wave length spacing with the 5 wave length spacing.

The associated circuitry for the DME-COTAR is given in Reference (15).

The parameters measured with the AME combine with the slant range measured with the DME to provide the three direction cosine values needed for determining the spatial position of the missile. The direction cosines are defined as ℓ , m, n, where $\ell = x/R$, $m = y/R$, $n = z/R$. The axis system and the associated equations are shown in figure 5-3.

5.3 Reference Direction Indicating System

The two directions the observer must know to align his reference system are true north and the local vertical. Nominal values of 1 MR have been chosen as the accuracy within which he can align to the true values of these directions.

Since the observer is operating from a non-moving base, and has adequate time to determine these directions, he can easily attain this precision. In fact, accuracies on an order of magnitude better than 1 MR may be accomplished.

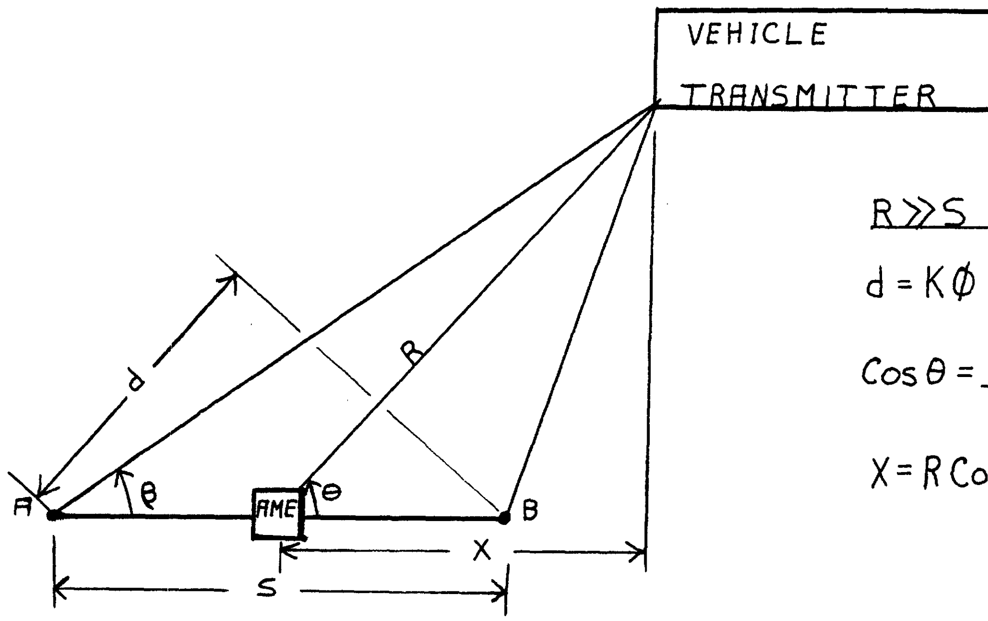
The direction of north may be obtained in several ways.

1. Use of a gyro compass,

2. Celestial bodies.

The gyro compass would allow an all weather capability, and would provide the desired performance. It could easily be transported with the rest of the observer's equipment. A magnetic compass probably could not give accuracies better than 1/2 - 1 degree, even when corrected for deviation and local variation.

The use of celestial bodies could be used to determine north, if the observer knew his position and the correct time. This method would require less equipment and would be lighter, and



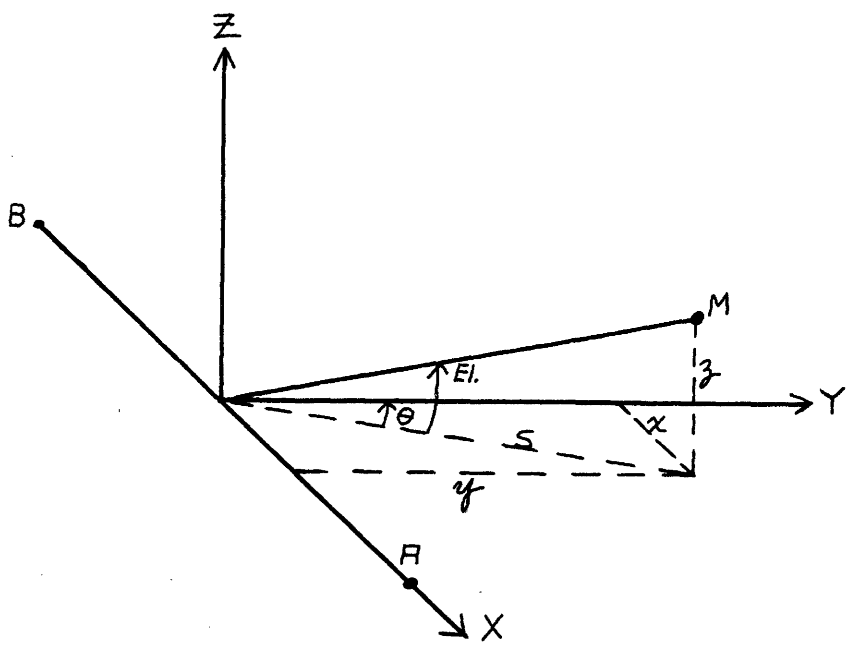
$$R \gg S$$

$$d = K\phi = S\cos\theta$$

$$\cos\theta = \frac{K}{S}\phi\,,\quad \theta \approx \beta$$

$$X = R\cos\theta$$

Figure 5-2. Geometry of A.M.E.



$$\cos\theta = \frac{y}{S}\,,\quad \sin\theta = \frac{x}{S}$$

$$\cos El. = \frac{S}{R}$$

$$m = \frac{y}{R} = \cos\theta\cos El.$$

$$l = \frac{x}{R} = \sin\theta\cos El$$

$$n = \frac{z}{R} = \sqrt{1-l^2-m^2}$$

Figure 5-3. Geometry associated with determining missile position.

easier to transport than a gyro compass. However, it is limited by atmospheric conditions.

The direction of the vertical from a non-moving base can be accurately determined using any accurate leveling device, such as spirit levels.

5.4 Computer

A computer would be required to perform the computations necessary at the observer's site. Since digital computers exhibit a high degree of flexibility, a specially programmed computer could be built that would accomplish this.

5.5 Communications System

Any voice communications system compatible with that of the launching aircraft would be sufficient for coordination between the observer and the launching aircraft.

The information that is passed to the missile will be in digital form. Hence, a data link converter and transmitter will be required as part of the observer's equipment.

5.6 Equipment or Techniques for Gathering Target Data

Literature describing the methods and required equipment for acquiring this information is available.[2,3,4,5] It is sufficient to say that it can be done with a high degree of accuracy.

[redacted]

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CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

Coordination between a surface observer and an inertially guided missile, in the manner described in this thesis, appears to be feasible and might be employed to reduce the missile's CEP, providing the surface observer is capable of accurately fixing the target position.

In addition to providing a new mode of collecting and utilizing target data, the use of a forward observer reduces, as the observer's position is moved closer to the target, the impact error due to inaccuracies in the missile's inertial navigation system.

Along with these advantages, two new errors are introduced: the error with which the observer fixes the missile's position, and an error due to misalignment at the missile's and observer's indicated reference coordinates. Unless the observer is fairly close to the target, the latter error is the largest. It has been shown that this error can be substantially reduced by re-aligning the missile's indicated coordinates to parallel, as closely as possible, the observer's. The alignment procedure is one of Physical Vector Matching. Since radar tracking does not provide the accuracy necessary for this procedure, it is necessary to

[redacted]

[redacted]

[redacted]

employ a CW Phase Comparison type of tracking equipment. This type of tracking reduces the position error in missile tracking to an almost negligible value and if the missile can be tracked until it is close to the target, the CEP would be quite small.

The proposed system is complex. The ground observer must have elaborate tracking and computing equipment, and still be near enough to the enemy to collect target information. Under field conditions it might be very difficult to lay out and accurately survey the crossed-baseline antenna array required for DME-COTAR (Phase Comparison) tracking. In addition, the communications between the missile and the observer may be subject to jamming. The extent to which these practical considerations will degrade the performance of the system has not been investigated.

6.2 Recommendations for Further Investigation

The central idea investigated in this thesis is the mid-course re-alignment of the missile's indicated reference coordinates, on the basis of an alignment comparison with a master reference system, in this case the observer's indicated reference coordinates. It is not claimed that the method employed or the results obtained are optimum. Other systems are possible. In particular, the use of missile acceleration as a physical vector for alignment comparison by Physical Vector Matching presents interesting possibilities. This vector is available in missile indicated reference coordinates with high accuracy, for it is free of initial condition or integration errors.

[redacted]

[redacted]

[redacted]

If some means can be found by which the observer could track this vector with similar accuracy, very precise re-alignment should be possible. It is also possible that by tracking the missile's acceleration vector in response to some commanded maneuver, the observer could deduce the orientation of the missile's indicated reference coordinates with respect to his own, and transform the missile-to-target vector accordingly.

Finally, the observer could command a sequence of maneuvers which would direct the missile's acceleration vector first horizontally, then vertically, thus making possible complete alignment comparison, rather than just azimuth alignment comparison, as was considered in this thesis.

[redacted]

APPENDIX A

GLOSSARY*

$A_{G_x}, A_{G_y}, A_{G_z}$	=	Gyroscopic drift angle about indicated axes.
a_x, a_y, a_z	=	Indicated acceleration along indicated axes.
(C)	=	correction to quantity following symbol.
E_q	=	Error in indicated quantity.
(E)	=	Error in quantity following symbol.
e	=	Angular error in computed missile velocity, due to rotation of stable platform.
G_x, G_y, G_z	=	Element of Geographic Transformation Matrix.
$\bar{g}$	=	Earth's gravity vector.
H_{OM}	=	Horizontal range from observer to missile.
H_{MT}	=	Horizontal range from missile to target.

* Any deviations in the following symbol definitions are explained in the text.

L	=	Geographic Latitude
M_x, M_y, M_z	=	Element of differential angular misalignment matrix.
MR	=	Milliradians
$\bar{R}_{OT}$	=	Vector range, observer to target
$\bar{R}_{OM}$	=	Vector range, observer to missile
$\bar{R}_{MT}$	=	Vector range, missile to target
$\bar{R}_{LM}$	=	Vector range, launch point to missile
$\bar{R}_G$	=	Guidance vector
t_f	=	time of flight (launch time as zero reference)
t_r	=	time of missile release from observer's control (launch time as zero reference)
$\bar{V}_m$	=	Missile velocity vector
$\bar{V}_{mh}$	=	Horizontal component of missile velocity vector
w	=	Angular rate
X_{MT}	=	X-component of missile-to-target vector
Y_{MT}	=	Y-component of missile-to-target vector
Z_{MT}	=	Z-component of missile-to-target vector

α	=	Azimuth angle, measured from true North
δ	=	Increment in quantity following symbol
θ	=	Elevation angle, from horizontal plane
σ_x	=	Standard deviation in down range error
σ_y	=	Standard deviation in cross range error
ϕ_1, ϕ_2	=	Direction angles for $\bar{R}_{OM}$ (DME-COTAR tracking).

Subscripts

i	=	Indicated
h	=	Horizontal component
m	=	Missile
o	=	Observer
(0)	=	Zero time (launch time)
o_i	=	observer's indicated reference coordinates
p	=	Missile computational reference coordinates
p_i	=	Missile indicated reference coordinates
p_{o_i}	=	Observer's indicated missile reference coordinates
RD	=	Radar dish coordinates

[redacted]

Coordinate System Transformations

$T\ p,o$	= Transformation from observer's true reference coordinates to missile's true reference coordinates
$T\ p_{o_i}, o_i$	= Transformation from observer's indicated reference coordinates to observer's indicated missile reference coordinates
$T\ p_i, p_{o_i}$	= Transformation from observer's indicated missile reference coordinates to missile's indicated reference coordinates.
T_{o_i}, RD	= Transformation from radar dish coordinates to observer's indicated reference coordinates

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Name	Subscript	Type	Origin Location	Orientation
Observer's true Reference Coordinates	o	earth fixed local geographic	Observer	$\bar{1}_{x_o}$ True North $\bar{1}_{y_o}$ East $\bar{1}_{z_o}$ Local Vertical
Observer's Indicated Reference Coordinates	o_i	earth fixed indicated local geographic	Observer	$\bar{1}_{x_{o_i}}$ Indicated North $\bar{1}_{y_{o_i}}$ Forms orthogonal right hand set $\bar{1}_{z_{o_i}}$ Indicated local vertical
Missile's true Reference Coordinates	p	vehicle fixed local geographic	Missile	$\bar{1}_{x_p}$ True North $\bar{1}_{y_p}$ East $\bar{1}_{z_p}$ Local Vertical
Missile's Indicated Reference Coordinates	p_i	Vehicle fixed indicated local geographic	Missile	$\bar{1}_{x_{p_i}}$ Indicated North $\bar{1}_{y_{p_i}}$ Form orthogonal right hand set $\bar{1}_{z_{p_i}}$ Indicated local vertical
Radar Dish Coordinates	RD	Fixed in Radar Tracking Antenna	Observer	$\bar{1}_R$ Along antenna tracking line $\bar{1}_\theta$ Along antenna elevation axis $\bar{1}_\psi$ Forms orthogonal right hand set

Table A-1 Reference Coordinate Frames

APPENDIX B

OBSERVER'S TRACKING EQUATIONS

B.1 Introduction

The purpose of this Appendix is to present the equations which must be solved by the observer's computing equipment in order to provide target data and alignment data to the missile, in the missile's reference coordinates. Three computations are involved:

1. Computation of missile to target vector in observer's indicated reference coordinates.
2. Computation of missile velocity vector in observer's indicated reference coordinates.
3. Transformation of both vectors from observer's indicated reference coordinates to observer's indicated missile reference coordinates.

B.2 Computation of Missile-to-Target Vector

The solution to the basic vector triangle of figure 2-3 in observer's indicated reference coordinates is simply:

$$[\bar{R}_{MT}]_{o_i} = [\bar{R}_{OT}]_{o_i} - [\bar{R}_{OM}]_{o_i} \qquad \text{(B-1)}$$

The target vector $[\bar{R}_{OT}]_{o_i}$ is assumed to be known with suitable accuracy. The tracking vector is derived from radar tracking information by the following transformation (referring to figure B-1):

$$\bar{I}_R = \bar{I}_{x_o} \cos\theta \cos\alpha + \bar{I}_{y_o} \cos\theta \sin\alpha - \bar{I}_{z_o} \sin\theta$$

$$\bar{I}_\theta = -\bar{I}_{x_o} \sin\alpha + \bar{I}_{y_o} \cos\alpha$$

$$\bar{I}_\psi = \bar{I}_{x_o} \sin\theta \cos\alpha + \bar{I}_{y_o} \sin\theta \sin\alpha + \bar{I}_{z_o} \cos\theta$$

The transformation from Radar Dish to Observer's indicated reference coordinates is:

$$T\, o_1, RD = \begin{pmatrix} \cos\theta\cos\alpha & -\sin\alpha & \sin\theta\cos\alpha \\ \cos\theta\sin\alpha & \cos\alpha & \sin\theta\sin\alpha \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{B-2}$$

Since $[\bar{R}_{OM}]_{RD} = R\,\bar{I}_R$, we have, in observer's indicated reference coordinates,

$$[\bar{R}_{OM}]_{o_1} = X_{OM}\,\bar{I}_{x_{o_1}} + Y_{OM}\,\bar{I}_{y_{o_1}} + Z_{OM}\,\bar{I}_{z_{o_1}}$$

$$X_{OM} = \cos\theta \cos\alpha\, R_{OM}$$

$$Y_{OM} = \cos\theta \sin\alpha\, R_{OM}$$

$$Z_{OM} = -\sin\theta\, R_{OM} \tag{B-2a}$$

If DME-COTAR tracking is employed, with orthogonal baselines, as shown in figure B-2, the transformation, as presented in reference (14) is:

$$
\begin{aligned}
X_{OM} &= R_{OM} \cos \phi_1 \\
Y_{OM} &= R_{OM} \cos \phi_2 \\
Z_{OM} &= -R_{OM} \sqrt{1 - \cos^2 \phi_1 - \cos^2 \phi_2}
\end{aligned} \qquad \text{(B-3)}
$$

B.3 Computation of the Missile Velocity Vector

Operation in the Mid-Course Alignment mode is feasible only when using phase-comparison tracking techniques. Only the horizontal components of the velocity vector are desired. These are given by

$$
\begin{aligned}
\dot{X}_{OM} &= R_{OM} \frac{d}{dt} \cos \phi_1 + \cos \phi_1 \frac{d}{dt} R_{OM} \\
\dot{Y}_{OM} &= R_{OM} \frac{d}{dt} \cos \phi_2 + \cos \phi_2 \frac{d}{dt} R_{OM}
\end{aligned} \qquad \text{(B-4)}
$$

Then

$$
[\bar{V}_{mh}]_{o_i} = \dot{X}_{OM} \bar{1}_{x_{o_i}} + \dot{Y}_{OM} \bar{1}_{y_{o_i}} \qquad \text{(B-5)}
$$

$$
[\bar{1} V_{mh}]_{o_i} = \frac{\dot{X}_{OM}}{\sqrt{\dot{X}^2_{OM} + \dot{Y}^2_{OM}}} \bar{1}_{x_{o_i}} + \frac{\dot{Y}_{OM}}{\sqrt{\dot{X}^2_{OM} + \dot{Y}^2_{OM}}} \bar{1}_{y_{o_i}}
$$

(B-6)

B.4 Transformation from Observer's Indicated Reference Coordinates to Observer's Indicated Missile Reference Coordinates

While both reference coordinate systems are geographic, their respective axes, due to their different geographic locations, will not be parallel. Because the horizontal distance from the observer to the missile is small (on the order of ten miles) the correction angles are small, and the Earth may be assumed spherical. Reference (7) gives the transformation between two nearly-coincident orthogonal coordinate systems:

$$Tp,o = \begin{pmatrix} 1 & G_z & -G_y \\ -G_z & 1 & G_x \\ G_y & -G_x & 1 \end{pmatrix} \tag{B-7}$$

The G's are rotations about the respective axes, and can be evaluated by inspection of figure B-3:

$$G_x = \frac{H_{OM} \sin a}{R_E} \tag{B-8a}$$

$$G_y = \frac{-H_{OM} \cos a}{R_E} \tag{B-8b}$$

$$G_z = \frac{H_{OM} \cos a \tan L}{R_E} \tag{B-8c}$$

The transformation of equation (B-7) rotates geographic coordinates at the observer's geographic position into geographic coordinates at the missile's geographic position. In chapter II, section 2.3, the observer's indicated missile reference coordinate system was defined so that:

$$Tp_{o_1}, o_1 = Tp,o \tag{B-9}$$

The transformation (B-9) is applied to both vectors $[\bar{R}_{MT}]_{o_1}$ and $[\bar{V}_{mh}]_{o_1}$ before they are telemetered to the missile:

$$[\bar{R}_{MT}]_{p_{o_1}} = Tp_{o_1}, o_1 \; [\bar{R}_{MT}]_{o_1} \tag{B-10}$$

$$[\bar{V}_{mh}]_{p_{o_1}} = Tp_{o_1}, o_1 \; [\bar{V}_{mh}]_{o_1} \tag{B-11}$$

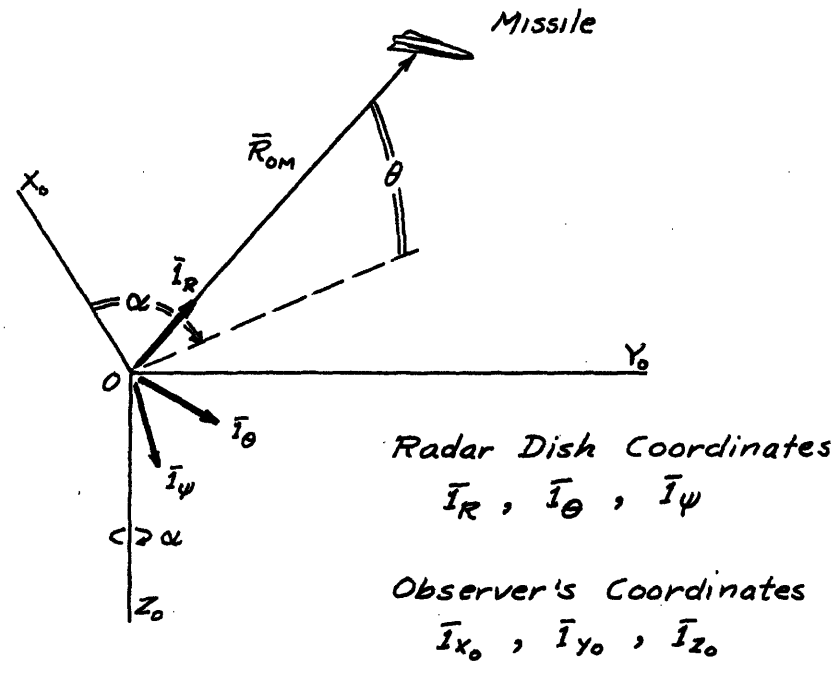

Figure B-1. Transformation from Radar Dish to Observer's Coordinates

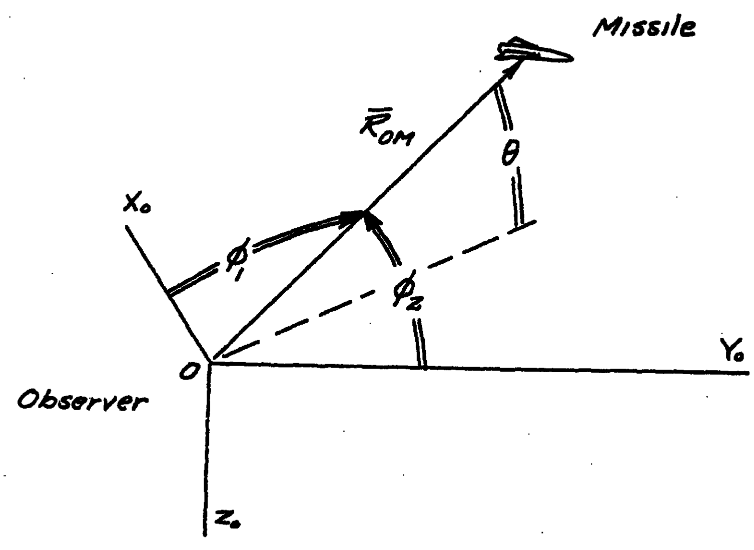

Figure B-2. DME-COTAR tracking

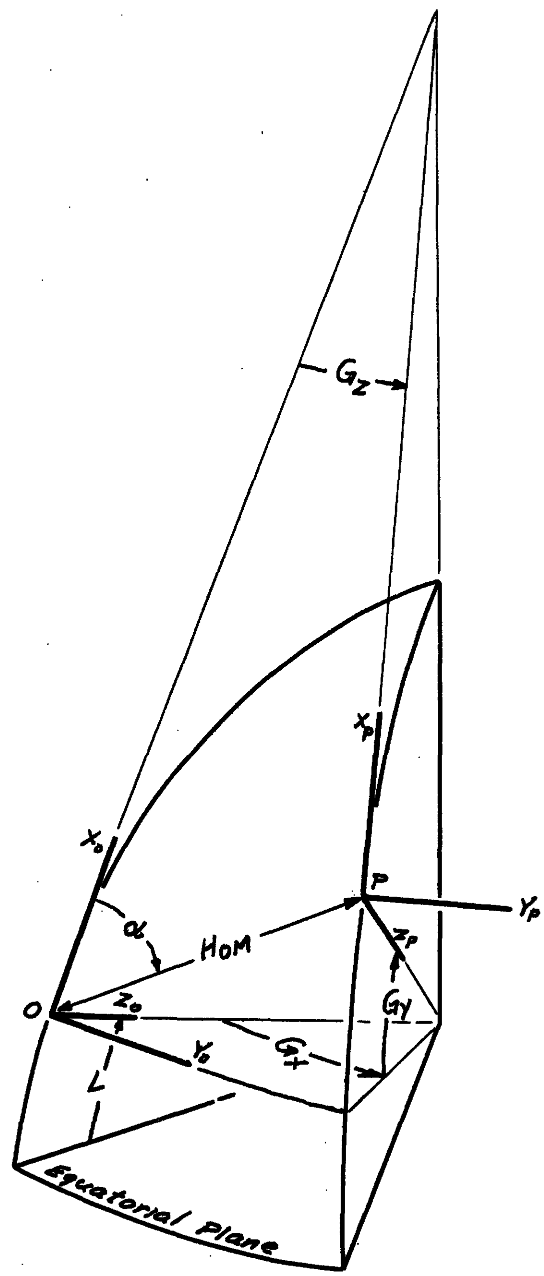

Figure B-3. Geographic Transformation from Observer's to Missile's true reference coordinates.

APPENDIX C

SYSTEM ERROR ANALYSIS

C.1 Introduction

The measure of system performance is the Circular Probable Error, or CEP, of missile impact at the target. Evaluating the system performance involves making estimates of all uncertainties in the system, and evaluating their effects on the CEP. In this process certain basic assumptions have been made. These are:

1. All errors are independent, with normal, or Gaussian distribution. This permits errors to be combined by the root-sum-square procedure.

2. The tactical situation is as shown in figure 1-1. The fundamental parameter for system performance is the horizontal missile-to-target distance when the missile is released from the observer's control. The precise position of the observer along the missile's track need not be specified, but it is assumed that his slant range is 17 nautical miles or less. This requires the missile's track to pass within 10 nautical miles of the observer's position.

3. The missile's trajectory is taken from reference (10) and is shown in figure C-1. Launch range is 100 miles and time of flight is 240 seconds. The missile could be launched closer to the target, and in a lower trajectory; this would result in a somewhat smaller CEP. The impact

error presented in this chapter is for the approximate maximum range of the system.

4. Errors in the observer's target information are not included in this analysis.

C.2 Sources of Error

The sources of error are summarized as follows:

1. From the launching aircraft: initial missile velocity and initial missile indicated reference coordinates alignment errors.

2. From the missile navigation system: inertial reference system drift, and inaccuracies in missile accelerometers.

3. From the observer: indicated reference coordinates alignment errors.

4. From Missile-Observer coordination: tracking errors in $[\bar{R}_{OM}]_{o_i}$ and $[\bar{V}_m]_{o_i}$; coordinate transformation errors; missile indicated reference coordinate system re-alignment errors, and computation errors.

C.3 Errors from Launching Aircraft Equipment

The performance of the launching aircraft's inertial navigation and missile alignment systems is discussed in section 4.8. The effects of these errors are summarized in table C-1.

[redacted]

[redacted]

Component and Output	Assumed Performance	Initial Missile Velocity Error	Initial Missile Reference Alignment Error	
			Azimuth	Vertical
Inertial Navigation System				
indicated velocity	1 fps	1 fps	0	0
indicated vertical	1 MR	negligible	0	0
indicated azimuth	1 MR	0	1 MR	0
Alignment System				
vertical alignment	1 MR	negligible	0	1 MR
azimuth alignment	2 MR	1.6 fps	2 MR	0
RSS Total		1.9 fps	2.2 MR	1 MR

Table C-1

Launching Aircraft Equipment Errors

C.4 Missile Navigation System Errors

The following analysis is derived primarily from references (1) and (13). Two two major sources of error in the missile navigation system are:

1. Drift of missile gyros.
2. Inaccuracies in missile accelerometers.

All other sources of error, such as integrator non-linearity and sensitivity errors, noise, guidance system dynamics, and computation errors are considered negligible in this analysis.

C.4.1 Drift of Missile Gyros

Drift of the missile gyros results in a rotation of the missile's reference coordinate frame, [redacted]nt error with

[redacted]

respect to geographic coordinates. The drift angles about axes located along true North, East, and local vertical, at launch time, are given by the following equations:

$$(A_G)_x = \int_o^{t_f} (\dot{A}_G)_x \, dt \tag{C-1a}$$

$$(A_G)_y = \int_o^{t_f} (\dot{A}_G)_y \, dt \tag{C-1b}$$

$$(A_G)_z = \int_o^{t_f} (\dot{A}_G)_z \, dt \tag{C-1c}$$

The drift rates $(\dot{A}_G)_x$, $(\dot{A}_G)_y$, and $(\dot{A}_G)_z$ have both constant and acceleration sensitive components. In order to estimate the magnitude of the drift angle, the conservative assumption has been made that the platform drifts isotropically as though the full missile acceleration were applied along each of the three axes. The result is the same for all three equations, and is plotted as a function of time of flight in figure C-2, using gyro performance data from table 4-1.

Drift of the missile gyros also causes a position error, which is given, in components parallel to North, East, and vertical at the launch point by:

$$(E)X_{LM_G} = \int_o^{t_f}\int_o^{t_f} a_y (A_G)_z \, dt \, dt + \int_o^{t_f}\int_o^{t_f} a_z (A_G)_y \, dt \, dt \tag{C-2a}$$

$$(E)Y_{LM_G} = \int_o^{t_f}\int_o^{t_f} a_x (A_G)_z \, dt \, dt + \int_o^{t_f}\int_o^{t_f} a_z (A_G)_x \, dt \, dt \tag{C-2b}$$

$$(E)Z_{LM_G} = \int_o^{t_f}\int_o^{t_f} a_y (A_G)_x \, dt \, dt + \int_o^{t_f}\int_o^{t_f} a_x (A_G)_y \, dt \, dt \tag{C-2c}$$

where a_x, a_y, a_z = components of missile acceleration parallel to North, East, and vertical at the launch point.

Position errors given by equations (C-2) are included in figure C-3.

C.4.2 Inaccuracies in Missile Accelerometers

Missile accelerometer inaccuracies cause a position error, with reference to the launch point, given by:

$$(E)X_{LM_A} = \int_o^{t_f}\int_o^{tf} (EA)_x \, dt \, dt \qquad \text{(C-3a)}$$

$$(E)Y_{LM_A} = \int_o^{t_f}\int_o^{t_f} (EA)_y \, dt \, dt \qquad \text{(C-3b)}$$

$$(E)Z_{LM_A} = \int_o^{t_f}\int_o^{t_f} (EA)_z \, dt \, dt \qquad \text{(C-3c)}$$

where $(E_A)_x$, $(E_A)_y$, $(E_A)_z$ are the errors in indication of missile acceleration along the three coordinate directions. The position error given by equations (C-3) is included in figure C-3, using accelerometer performance data from table 4-1.

C.5 Errors from Observer's Equipment

It is assumed that the observer will be able to determine the directions of true North and local vertical to within 1 MR. This results in a coordinate systems misalignment error in the Initial Alignment Mode. In the Mid-Course Re-alignment mode, the observer's indicated azimuth becomes the azimuth reference, and the error due to the observer's indication of true North is eliminated.

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C.6 Missile-Observer Coordination Errors

C.6.1 Tracking Error in Missile Position

The error with which the observer locates the missile in his indicated reference coordinate system depends upon the type of tracking employed. The two types, radar and DME-COTAR are discussed in the following sections.

C.6.2 Radar Tracking Error

In terms of the "Radar Dish" coordinates of figure B-1, the error in the missile position vector is:

$$[(E)\bar{R}_{OM}]_{RD} = E_R \ \bar{I}_R + R \cos\theta \ E_\alpha \ \bar{I}_\theta - R \ E_\theta \ \bar{I}_\psi \qquad \text{(C-4)}$$

where

E_R = range error

E_θ = elevation angle error

E_α = azimuth angle error

In the Observer's indicated reference coordinates

$$[(\bar{E})R_{OM}]_{o_i} = T_{o_i,RD} \ [(\bar{E})R_{OM}]_{RD} \qquad \text{(C-5)}$$

The transformation $T_{o_i,RD}$ is given by equation (B-2). The desired position error, in the horizontal plane, is given by the X_o and Y_o components of the expansion of equation (C-5):

$$\begin{aligned}(\bar{E})H_{OM} = {} & (\cos\theta \cos\alpha \ E_R - R\cos\theta \ \sin\alpha \ E_\alpha - R\sin\theta \ \cos\alpha \ E_\theta) \ \bar{I}_{x_{o_i}} \\ & + (\cos\theta \ \sin\alpha \ E_R + R\cos\theta \ \cos\alpha \ E_\alpha - R\sin\theta \ \sin\alpha \ E_\theta) \bar{I}_{y_{o_i}}\end{aligned} \qquad \text{(C-6)}$$

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The magnitude of this error is:

$$(E)H_{OM} = R\left[\left(\frac{E_R}{R}\cos\theta - \sin\theta\, E_\theta\right)^2 + \cos^2\theta\, E_\alpha^2\right]^{1/2}$$

It is assumed, for simplicity, that the angular errors in azimuth and elevation are equal:

$$E_\alpha = E_\theta = E_A$$

and that for maximum error, E_A and E_R are opposite in algebraic sign:

$$(E)H_{OM} = R\left[\left(\frac{E_R}{R}\right)^2 \cos^2\theta + 2\frac{E_R}{R} E_A \cos\theta + E_A^2\right]^{1/2}$$

For the assumed tactical situation, the slant range R is 17 miles, or 1.03×10^5 feet, and the missile elevation angle is 55 degrees. Under these conditions,

$$(E)H_{OM} = 1.03 \times 10^5\left[0.33\left(\frac{E_R}{R}\right)^2 + 0.94\frac{E_R}{R} E_A + E_A^2\right]^{1/2} \qquad \text{(C-7)}$$

Equation (C-7) has been evaluated with the equipment performance data given in chapter V. Regarding $(E)H_{OM}$ as the standard deviation of a circular error in the missile's position, the tracking error causes equal down range and cross range errors given by:

$$\sigma_{x_i} = \sigma_{y_i} = (E)H_{OM} \qquad \text{(C-7a)}$$

The results from (C-7) and (C-7a) are given in table C-2.

C.6.3 DME-COTAR Tracking Error

The horizontal error in locating the missile using DME-COTAR can be evaluated by taking the differential of equations (B-3):

$$(E)X_{OM} = (E)R_{OM} \cos \phi_1 + R_{OM}(E) \cos \phi_1$$

$$(E)Y_{OM} = (E)R_{OM} \cos \phi_2 + R_{OM}(E) \cos \phi_2$$

$$(E)H_{OM} = \sqrt{(E)^2 X_{OM} + (E)^2 Y_{OM}} \qquad \text{(C-8)}$$

The accuracy of DME-COTAR tracking can be estimated conservatively by assuming

$$\cos \phi_1 = \cos \phi_2 = 1.0$$

$$(E)\cos \phi_1 = (E) \cos \phi_2$$

and the equipment performance data given in chapter V. The results are given in table C-2.

Tracking Equipment	Assumed Performance	Impact Error	
		Down range (σ_{x_1})	Cross range (σ_{y_1})
Fire Control Radar	Range: .002 R Bearing: 1 MR	180	180
DME-COTAR	Range: 3 feet Direction cosines 56×10^{-6}	5	5

Table C-2

Inpact Error Due to Observer's Tracking Error

C.6.4 Error in Geographic Transformation Matrix

The Observer's error in H_{OM} causes an error in the matrix (B-9) which in turn causes an impact error through its effect on $[\bar{R}_{MT}]_{poi}$ However, numerical analysis shows that this error

[redacted]

[redacted]

[redacted]

is quite negligible in comparison with the other errors in the system.

C.6.5 Error in Missile Indicated Reference Coordinate System Re-alignment

The accuracy of the re-alignment procedure described in section 2.4 depends upon the accuracy with which the missile and the observer can determine the missile's velocity vector in their respective indicated reference coordinates. The error remaining, after the correction obtained from equation (2-6) is applied to the missile's indicated coordinates, will be the root-sum-square of the errors in the velocity vector azimuth angles αp and αo.

The errors in αp (computed by the missile) are due to initial conditions, accelerometer uncertainty, and platform drift. Assuming an initial velocity of 800 fps, initial velocity error of 1.9 fps (from table C-1), a glide velocity of 3000 fps, a maximum t_r of 240 seconds, and accelerometer performance from table 4-1, the various errors in αp are found to be:

Initial Conditions

$$(E)\ \alpha p_{ic} = \frac{(E)V_{m_i}}{V_m(T_r)} = \frac{1.9}{3000} = 0.63 \text{ MR} \qquad \text{(C-9)}$$

Accelerometer Uncertainty

$$(E)\ \alpha p_{au} = \frac{\int_0^{t_r} E_A\, dt}{V_m(t_r)} = \frac{.0037 \times 240}{3000} = 0.3 \text{ MR} \qquad \text{(C-10)}$$

[redacted]

[redacted]

Platform Drift The error due to platform drift is less easily evaluated. Referring to figure C-4 and assuming, for the moment, all other errors zero, the desired azimuth correction is

$$\text{Des (C) } \alpha p = M_{z_o} - A_{G_z}(t)$$

where M_{z_o} is the initial misalignment between the missile and the observer, and $A_{G_z}(t)$ is the platform drift angle as a function of time. The actual correction is:

$$\text{Act (C) } \alpha p = \alpha p(t) - \alpha o$$

$$= \alpha p(o) - e - \alpha o$$

$$= M_{z_o} - e$$

The error in the correction is

$$\text{(E) } \alpha p_{pd} = \text{Des (C)} \alpha p - \text{Act (C) } \alpha p = e - A_{G_z}(t) \qquad \text{(C-11)}$$

The velocity vector computed by the missile is

$$[\bar{V}_m]_{p_i} = [\bar{V}_{m\ (o)}]_{p_i} + \int_o^t [\bar{a}_m]_{p_i}\, dt$$

$$= [\bar{V}_{mo}]_{p_i} + \int_o^t T_{p_i\,,p}\ [\bar{a}_m]_p\ dt$$

where the transformation $T_{p_i, p}$

$$T_{p(o),p} = \begin{pmatrix} 1 & -A_{G_z}(t) \\ A_{G_z}(t) & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -A_{G_z}(t) \\ A_{G_z}(t) & 0 \end{pmatrix}$$

[redacted]

With only horizontal components considered.

$$[\bar{a}_m]_p = a_x \bar{I}_{x_p} + a_y \bar{I}_{y_p}$$

The velocity error vector is:

$$(E)\bar{V}(t) = \int_0^t \begin{pmatrix} 0 & -A_{G_z}(t) \\ A_{G_z}(t) & 0 \end{pmatrix} \begin{pmatrix} a_x \bar{I}_{x_p} \\ a_y \bar{I}_{y_p} \end{pmatrix} dt$$

$$= \int_0^t A_{G_z}(t)\,[\bar{a}_m]_p \times \bar{I}_{z_p} \quad dt \qquad \text{(C-12)}$$

And the angular error e is

$$e = \frac{1}{V(t)} \left| (E)\,\bar{V}\,(t) \right| \qquad \text{(C-13)}$$

Data for the solution of (C-12), (C-13) and (C-11) are taken from figures C-1, C-3, and C-4. Equation (C-11), the error due to platform drift, is plotted in figure C-5. Two cases are presented; unidirectional platform drift, and drift which reverses when the acceleration becomes negative. The latter curve has been chosen as more nearly representing the expected actual drift. It is recognized that this assumption may not be valid in every case; however, the error is too small to justify a more detailed analysis.

The error in αo (computed by the observer) depends on the accuracy with which the missile can be tracked, and the time available for smoothing the tracking data. Chapter V discusses radar and DME-COTAR tracking equipment, giving performance data for range and bearing accuracy that are about the best that can be expected. These data are given in sections 5.2.1 and 5.2.2. Reference (15) provides a method for estimating velocity error from position tracking data:

$$\delta \dot{R} = \frac{2\sqrt{3}}{t\sqrt{n}}\,\delta R \qquad \text{(C-14a)}$$

$$\delta \omega = \frac{2\sqrt{3}}{t\sqrt{n}}\,\delta\theta \qquad \text{(C-14b)}$$

where
- $\delta \dot{R}$ = error in range rate
- δR = error in range
- t = tracking time
- n = number of position measurements
- $\delta \omega$ = error in angular rate
- $\delta \theta$ = error in angular position

Assuming the following values:

- $R = 10^5$ feet
- $\delta R = .002\ R$
- $\delta \theta = 1$ MR
- $t = 10$ seconds
- $n = 50$ (five samples per second)

then

- $\delta \dot{R} = 10$ feet per second
- $\delta\omega = .05$ MR per second

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finally

$$\delta \bar{v} = \delta \dot{R}\ \bar{1}_R + \bar{\omega} \times \delta \bar{R} + \delta \bar{\omega} \times \bar{R} \qquad \text{(C-15)}$$

$$= 10\ \bar{1}_R + 11\ \bar{1}_\omega \times \bar{1}_R$$

$$|\delta \bar{v}| = 15 \text{ fps}$$

The maximum error in αo is

$$\text{(E) } \alpha o = \frac{|\delta \bar{v}|}{V_m} = \frac{15}{3000} = 5 \text{ MR}$$

This error is larger than the expected azimuth error in the Initial Alignment mode. Hence, mid-course alignment is not considered useful when radar tracking equipment must be used.

For DME-COTAR tracking the position tracking error is given in chapter V as $\delta X_{OM} = \delta Y_{OM} \cong 5$ feet. Applying equation (C-14a),

$$\delta \dot{X}_{OM} = \delta \dot{Y}_{OM} = \frac{2\sqrt{3}}{t\sqrt{n}}\ \delta X_{OM} = 0.25 \text{ feet per second}$$

for ten seconds tracking at 5 samples per second. Then using equation (B-5),

$$\delta \bar{V}_{mh} = \delta \dot{X}_{OM}\ \bar{1}_{x_o} + \delta \dot{Y}_{OM} \bar{1}_{y_o}$$

$$|\delta V_{mh}| = 0.35 \text{ fps}$$

The error in αo, for an average missile velocity of 3000 fps, is

$$\text{(E)} \alpha o = \frac{0.35}{3000} = 0.12 \text{ MR} \qquad \text{(C-16)}$$

[redacted]

C.6.6 Computation Errors

The precision of measurement indicated by equation (C-16) can be utilized only if computations within the system are performed with similar accuracy. Table 5-2 shows that 16 bits are required to obtain an accuracy of one-tenth of a milliradian. For this thesis it is assumed that 16 bit computers are used, and it is noted that if fewer than 16 bits were used, computation errors would be a limiting factor on system accuracy.

C.7 Error Summary

The index of system performance is the Circular Probable Error, or CEP, which is defined as the radius of the circle within which 50% of all missile impacts are expected to occur. If the down range and cross range errors are independent, normally distributed, and have equal, or nearly equal standard deviations, the CEP is defined in reference (16) as:

$$\text{CEP} = 1.177 \quad \sigma_x \sigma_y \tag{C-17}$$

or, for highly elliptical distributions,

$$\text{CEP} = \frac{1.177}{2} \left[\sigma_x + \sigma_y \right] \tag{C-18}$$

where σ_x = standard deviation of down range error

σ_y = standard deviation of cross range error.

It is assumed that all contributing errors are independent and normally distributed, and therefore can be combined by a root-sum-square procedure:

$$\sigma_x^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2 \quad \cdots\cdots \tag{C-19}$$

$$\sigma_y^2 = \sigma_{y_1}^2 + \sigma_{y_2}^2 + \sigma_{y_3}^2 \quad \cdots\cdots \tag{C-20}$$

There are three contributions to the down range and cross range errors:

1. σ_{x_1} = Down range error due to observer's tracking inaccuracy in fixing the missile's position.

   σ_{y_1} = Cross range error due to observer's tracking inaccuracy in fixing missile's position.

2. σ_{x_2} = Down range error due to coordinate system misalignment (missile with respect to observer) at time of release from observer's control.

   σ_{y_2} = Cross range error due to coordinate system misalignment.

3. σ_{x_3} = Down range error due to drift of missile's navigation system after release.

   σ_{x_3} = Cross range error due to drift of missile's navigation system after release.

C.7.1 <u>Observer's Tracking Error</u>

This error depends on the tracking equipment being used. For the assumed tactical situation, the down range and cross-range errors (σ_{x_1} and σ_{y_1}) for both radar and phase comparison tracking are given in table C-2.

C.7.2 <u>Coordinate System Misalignment</u>

The alignment error of the missile's coordinates with respect to the observer's results in the target vector being incorrectly set into the missile's computer. The amount of misalignment depends on the time of flight, and whether or not Mid-Course Alignment is utilized. The alignment errors are summarized in table C-3, and plotted in figures C-6 and C-7. The resulting impact errors are found as follows:

$$[\bar{R}_{MT}]_{p_i} = T\,p_i,o_i\,[\bar{R}_{MT}]_{o_i} \tag{C-21}$$

$$T\,p_i,o_i = T\,po_i,o_i + (M)T\,p_i,o_i \tag{C-22}$$

$$(M)\,T\,p_i,o_i = \begin{pmatrix} 0 & M_z & -M_y \\ -M_z & 0 & M_x \\ M_y & -M_x & 0 \end{pmatrix}$$

$(M)T\,p_i,o_i$ is a differential matrix, the elements of which are the small angle misalignment errors between the missile's and the observer's coordinates. Substituting (C-22) into (C-21), the error in $[\bar{R}_{MT}]_p$ is:

$$(E)[\bar{R}_{MT}]_{p_i} = (M)\,T_{p_i},o_i\,[\bar{R}_{MT}]_{o_i} \tag{C-23}$$

$$\begin{aligned} &= (M_z Y_{MT} - M_y Z_{MT})\,\bar{I}x_{o_i} \\ &\quad + (M_x Z_{MT} - M_z X_{MT})\,\bar{I}y_{o_i} \\ &\quad + (M_y X_{MT} - M_x Y_{MT})\,\bar{I}z_{o_i} \end{aligned} \tag{C-24}$$

The horizontal components of (C-24) may be rewritten as

$$\begin{aligned} (E)_{hor}[\bar{R}_{MT}]_{p_i} &= M_z\,(Y_{MT}\,\bar{I}x_{o_i} - Y_{MT}\,\bar{I}y_{o_i}) \\ &\quad + Z_{MT}(M_x\bar{I}y_{o_i} - M_y\,\bar{I}x_{o_i}) \\ &= M_z\bar{I}z_{o_i}\ \mathrm{x}\ [\bar{R}_{MT}]_{o_i} + Z_{MT}(M_x\bar{I}y_{o_i} - M_y\bar{I}x_{o_i}) \end{aligned} \tag{C-25}$$

The first term of (C-25) is a cross range error, while the second contributed to both cross range and down range errors, depending on the orientation of the target vector. To evaluate the magnitude of the second term, it is reasonable to assume $M_x = M_y$. Then under the assumption of normal distribution, the cross range and down range contributions of this term are equal and given by

$$\delta \sigma_{x_2} = \delta \sigma_{y_2} = |M_x| Z_{MT} \tag{C-26}$$

Therefore the magnitude of the down range and cross range errors resulting from coordinate system misalignment are:

Down range: $$\sigma_{x_2} = |M_x| Z_{MT} \tag{C-27}$$

Cross range: $$\sigma_{y_2} = \left\{ [\, M_x Z_{MT}]^2 + [\, M_z R_{MT}]^2 \right\}^{1/2} \tag{C-28}$$

C.7.3 <u>Drift of Missile Inertial Guidance System</u>

Position errors developed in the missile inertial guidance system result in an impact error. While errors in position computation begin with launch, the action of the observer in assigning the target vector at release time "resets" the system, eliminating position errors up to t_r. The impact error is found by reading from figure C-3 the position error from t_r to t_f. It is again assumed that the position error is normally distributed and the down range error (σ_{x_3}) and cross range error (σ_{y_3}) are equal.

Error Source	Pre-alignment	Mid-Course Alignment
Vertical Alignment		
Launcher Navigation System	0	0
Initial Missile Alignment	1 MR	1 MR
Missile Navigation System	figure C-2	figure C-2
Observer Alignment	1 MR	1
RSS Total	figure C-6	figure C-6
Azimuth Alignment		
Launcher Navigation System	1 MR	0
Launcher-to-Missile Alignment	2 MR	0
Missile Navigation System	figure C-2	0
Observer Alignment	1 MR	0
Missile-to-Observer Alignment		
Initial Conditions		0.63
Missile Accelerometers		0.30
Platform drift		figure C-5
Observer's Tracking Errors		0.12
RSS Total	figure C-7	figure C-7

Table C-3

Coordinate System Alignment Errors

[redacted]

C.7.4 Circular Probable Error

In order to show the relative importance of the three major sources of error, individual CEP's are calculated, defined as follows:

CEP_1 = Circular Probable Error due to observer's tracking inaccuracy in fixing missile's position. The distribution of this error is circular, and equation (C-17) applies:

$$CEP_1 = 1.177\sqrt{\sigma_{x_1}\ \sigma_{y_1}} \qquad (C-29)$$

CEP_2 = Circular Probable Error due to coordinate system misalignment (missile with respect to observer) at time of release from observer's control. The distribution of this error is quite elliptical, and equation (C-18) applies:

$$CEP_2 = 1.177 \left[\frac{\sigma_{x_2} + \sigma_{y_2}}{2} \right] \qquad (C-30)$$

CEP_3 = Circular Probable Error due to missile navigation system drift after release from observer's control. The distribution of this error is circular:

$$CEP_3 = 1.177\sqrt{\sigma_{x_3}\ \sigma_{y_3}} \qquad (C-31)$$

Finally, the overall system CEP is calculated. The distribution is elliptical and equation (C-18) is used:

$$CEP = 1.177\left[\frac{\sigma_x + \sigma_y}{2}\right] \qquad (C-32)$$

$$\sigma_x = \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2}$$

$$\sigma_y = \sqrt{\sigma_{y_1}^2 + \sigma_{y_2}^2 + \sigma_{y_3}^2}$$

Equations (C-29) through (C-32) are the final results of the error analysis, and are plotted in figures 3-1 through 3-4 as functions of missile-to-target distance at the time of release from the observer's control.

[redacted]

Figure C-1

Missile Trajectory

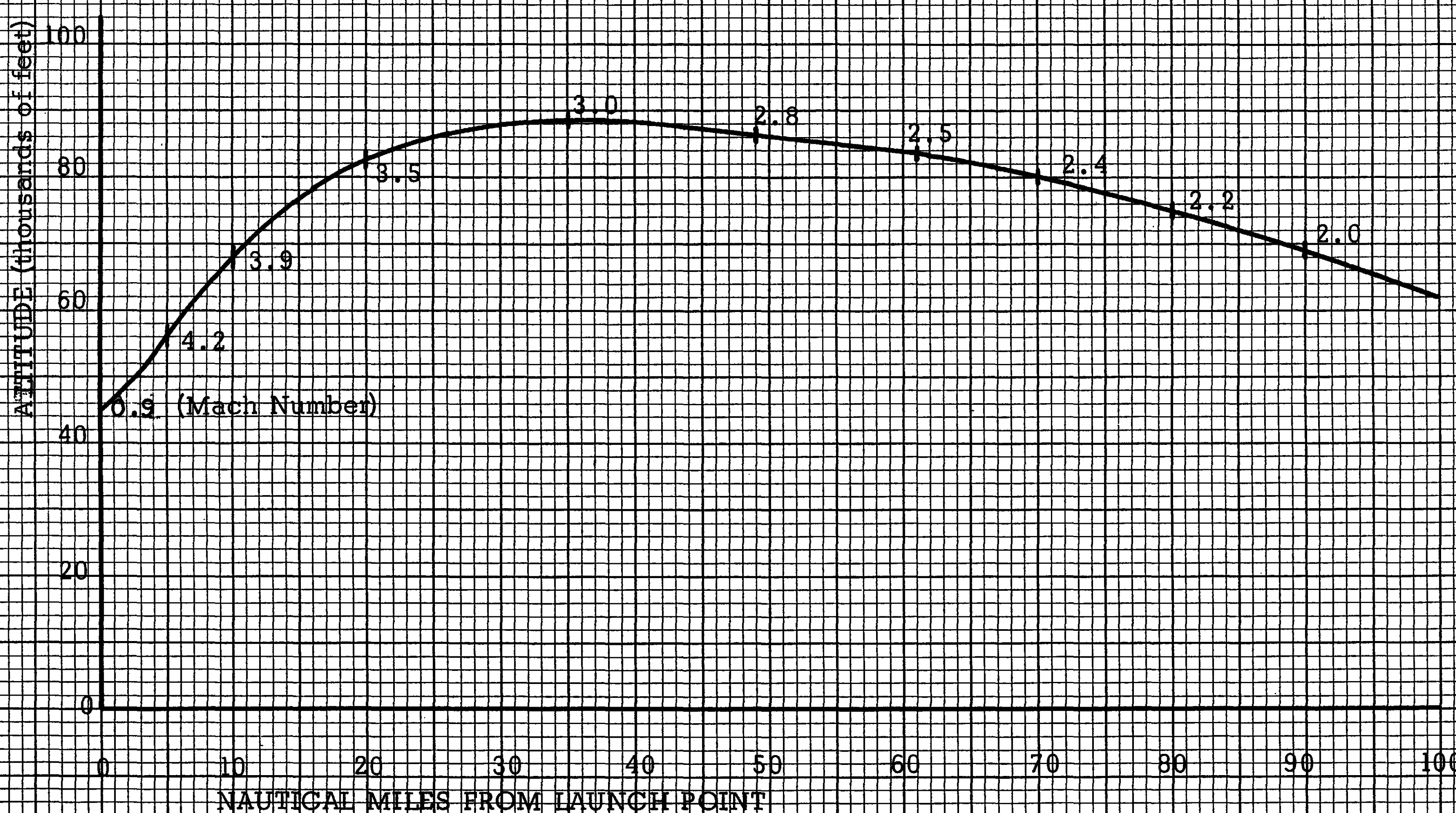


Figure C-2

STABLE REFERENCE PLATFORM DRIFT

VS. TIME OF FLIGHT

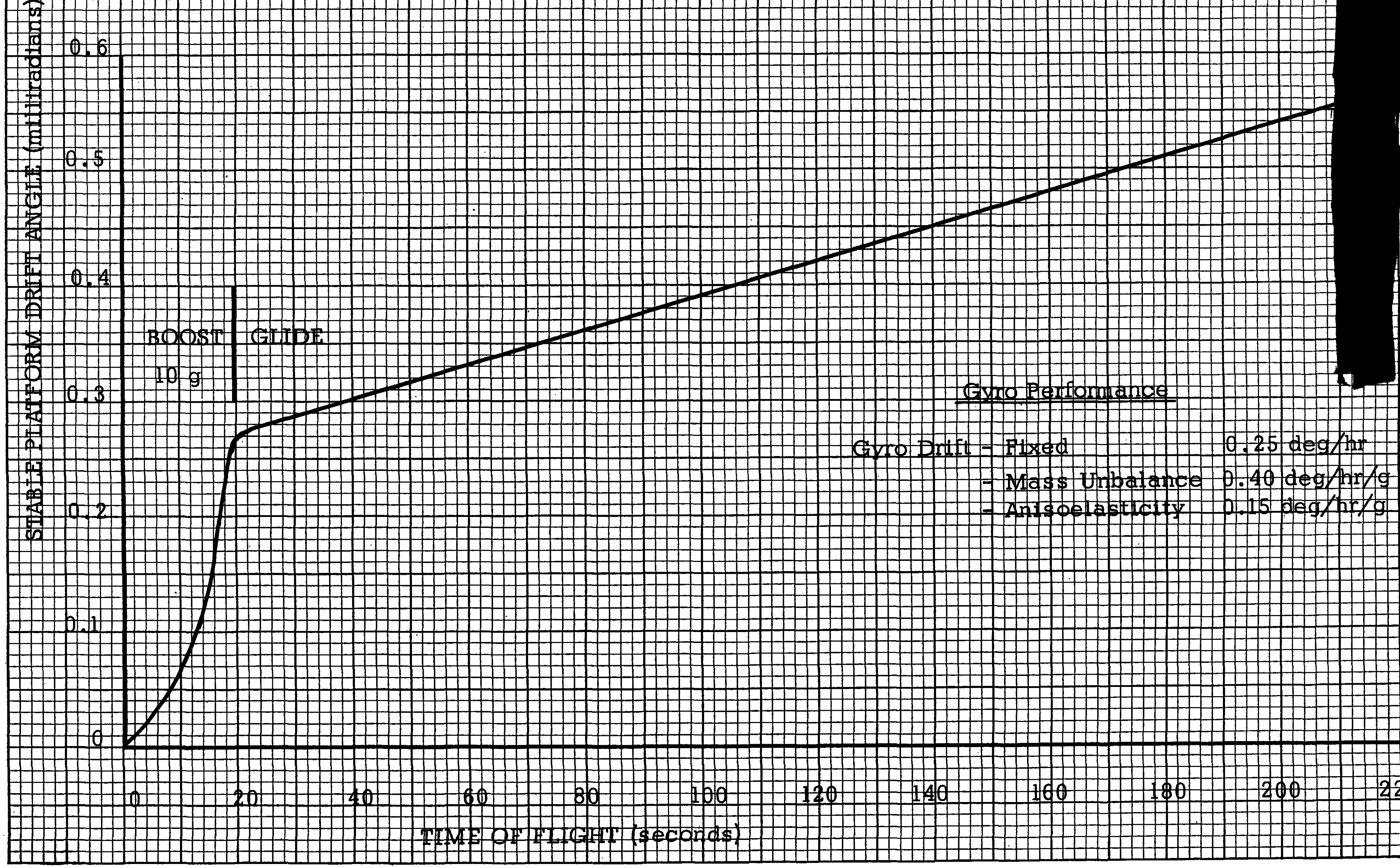
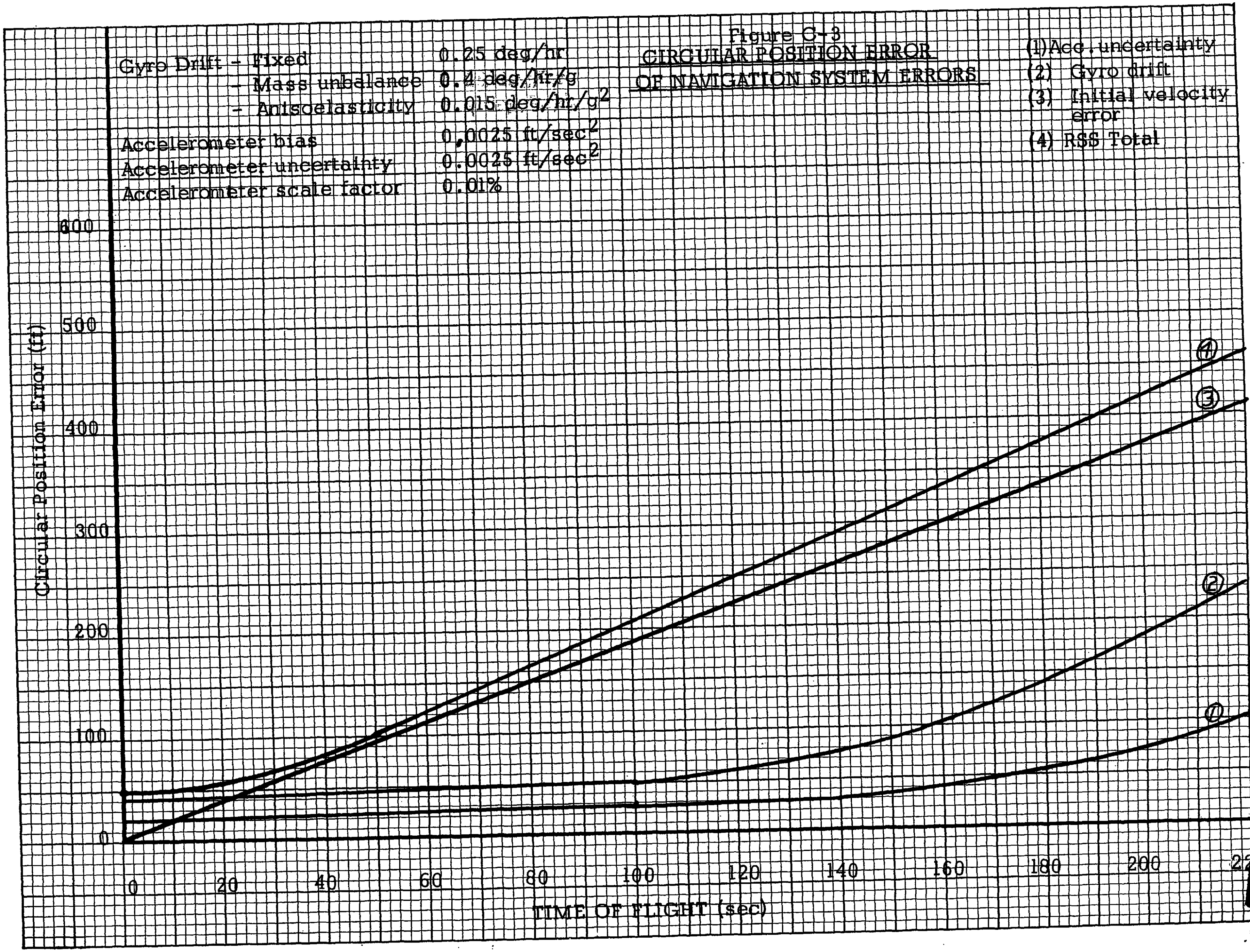


Figure C-3
CIRCULAR POSITION ERROR
OF NAVIGATION SYSTEM ERRORS



 [redacted]

 [redacted]

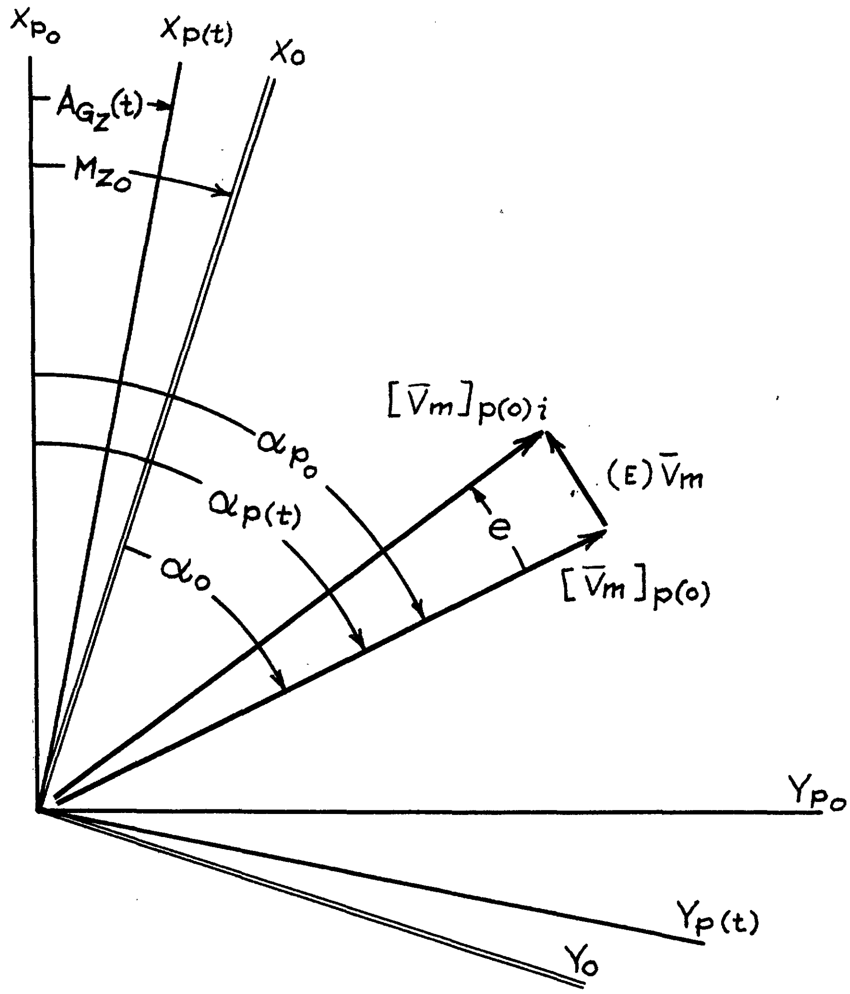


Figure C-4. Re-alignment error due to Platform Drift

$[\bar{V}_m]_{p(o)}$ = Actual missile velocity

$[\bar{V}_m]_{p(o)_i}$ = Indicated missile velocity

$(E)\bar{V}_m$ = Missile velocity error due to rotation of stable platform.

K+E 5 X 5 TO THE CENTIMETER KEUFFEL & ESSER CO. 358-15K MADE IN U.S.A.

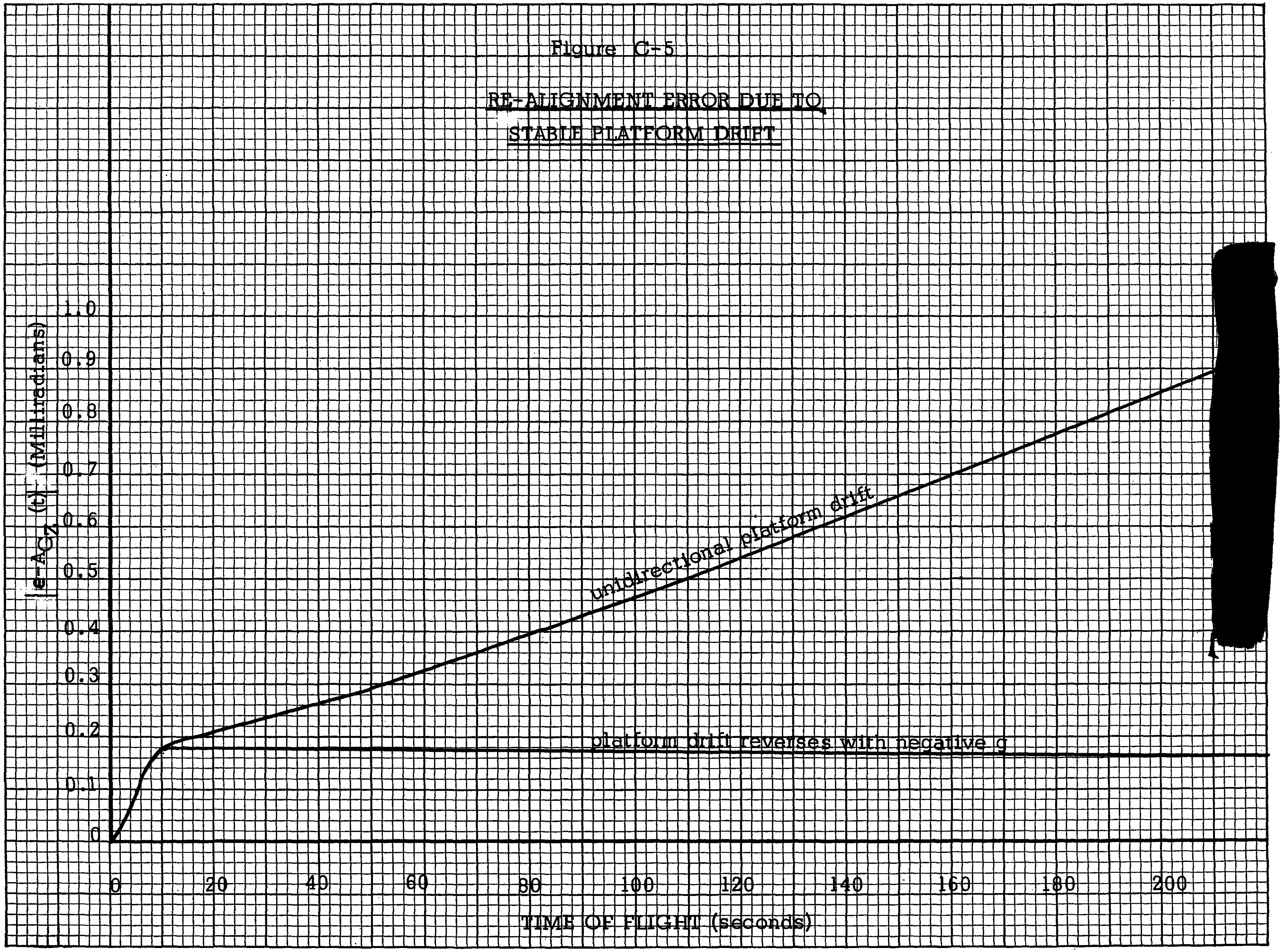


Figure C-5

RE-ALIGNMENT ERROR DUE TO STABLE PLATFORM DRIFT

[redacted]

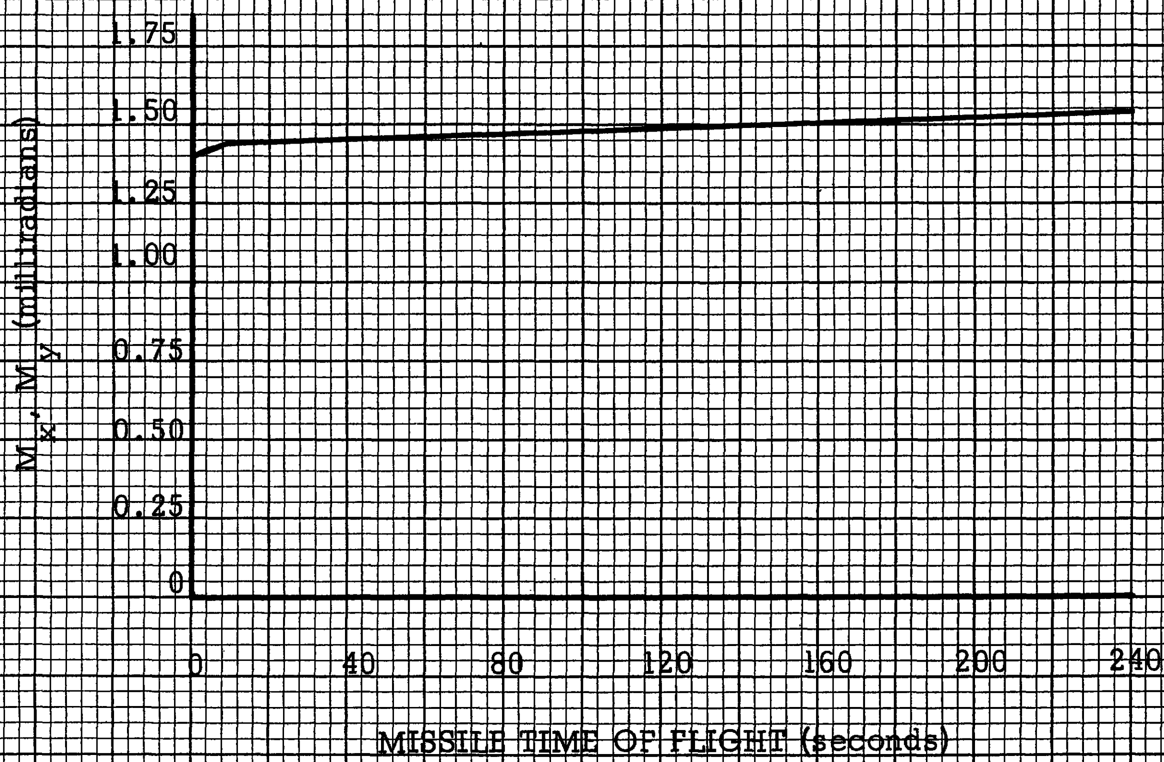
[redacted] DECLASSIFIED

[redacted]

Figure C-6

COORDINATE SYSTEM VERTICAL ALIGNMENT ERROR

missile with respect to observer



[redacted]

[redacted]

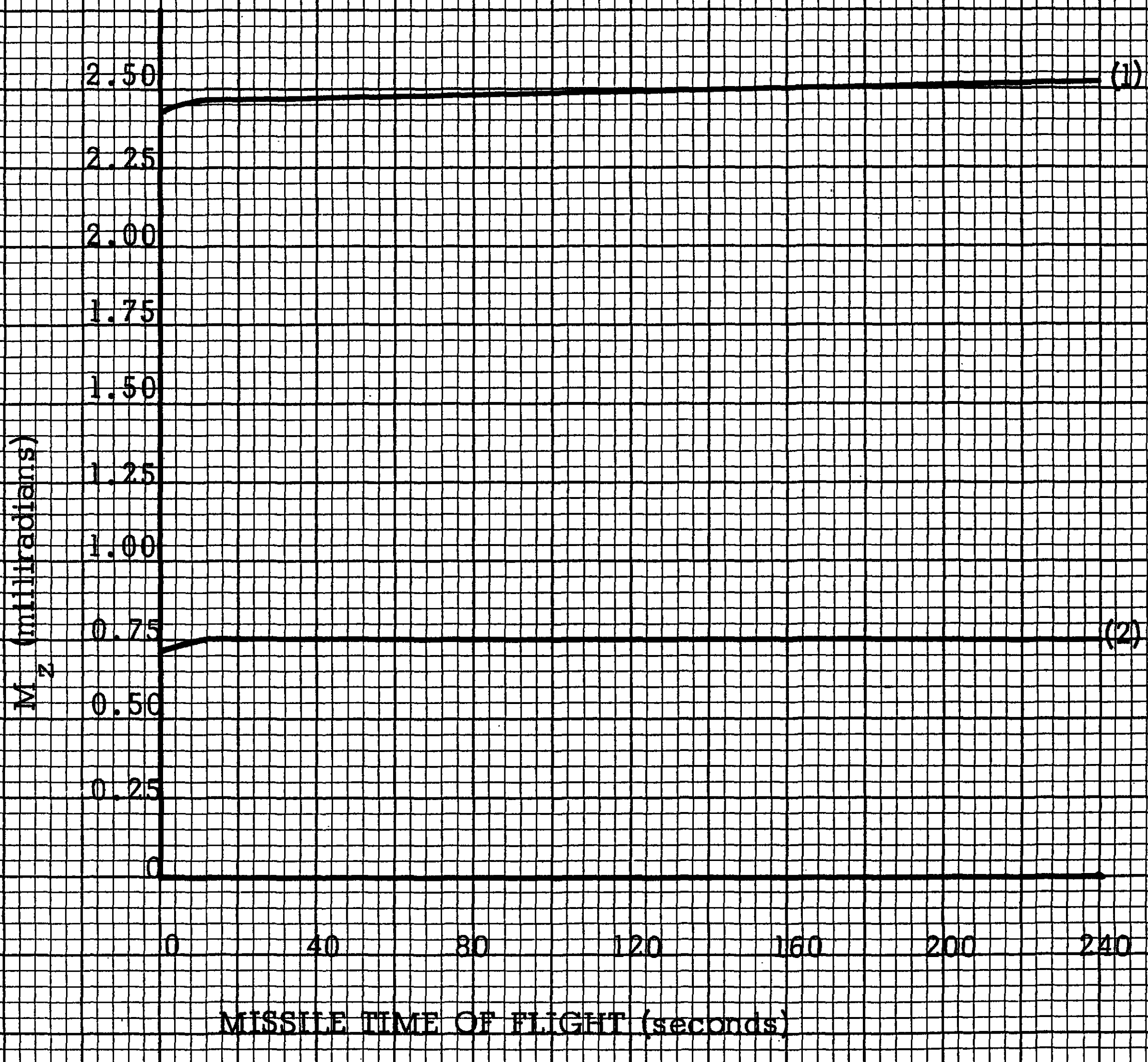
[redacted]

[redacted]

Figure C-7

COORDINATE SYSTEM AZIMUTH ALIGNMENT ERROR

missile with respect to observer



(1) Initial Alignment mode, or prior to Mid-Course Re-alignment

(2) Immediately after Mid-Course Re-alignment procedure.

[redacted]

APPENDIX D

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