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MODELING AND ANALYSIS OF TEAMS OF INTERACTING DECISIONMAKERS
WITH BOUNDED RATIONALITY¹

by

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ABSTRACT

A methodology for analyzing and evaluating alternative organizational structures is presented. An information theoretic framework is used in which each team member is described by a two-stage model consisting of situation assessment and response selection stages as well as interconnections with the rest of the organization. The information processing and decisionmaking load of each team member and the measure of organizational performance are depicted in the performance-workload space as implicit functions of the decision strategies of each individual member. The approach to evaluating organizational structures is illustrated through the detailed analysis of an organization consisting of two decisionmakers with bounded rationality.

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I. INTRODUCTION

A basic model of an interacting decisionmaker, appropriate for the study of command and control organizations was introduced by Boettcher and Levis (1982). In subsequent work, Levis and Boettcher (1982) considered the modeling of organizations consisting of several decisionmakers who form a team. In this paper, emphasis is placed on the modeling and evaluation of organizational structures using the methodology for analysis already developed.

An organization is formed in order to perform a set of tasks that individuals cannot perform alone. The tasks to be performed by the organizations being considered consist of receiving signals or inputs from one or more sources, processing them, and producing outputs which can be actions or signals. A single decisionmaker cannot perform these tasks because of the large amount of information processing required and because of the fast tempo of operations. The latter reflects the rate at which tasks are assigned to the organization for execution.

The analytical framework used for modeling both the internal processing within an organization member and of the organization as a whole is that of n-dimensional information theory (McGill, 1954; Conant, 1976). The basic departure from previous information theoretic models of a decisionmaker (for a review, see Sheridan and Ferrell, 1974) is that in addition to information transmission, the internal generation of information, blockage, and the coordination of the information processing and decisionmaking functions are also modeled. Consequently, the limitations of humans as information processors and problem solvers are modeled as bounds on the total information processing activity. To avoid overload, the total processing activity associated with the tasks assigned to each team member must remain within the bound. This model represents one interpretation of the hypothesis that decisionmakers exhibit bounded rationality (March, 1978).

The inputs to the organization may be such that different signals are received by different team members. It has been shown by Stabile, Levis, and

Hall (1982) that the general case can be modeled by a single vector source and a set of partitioning matrices that distribute components of the vector signal to the appropriate decisionmakers within the organization. This model is shown in Figure 1, where the input vector is denoted by X and takes values from a finite alphabet $\mathbf{X}$. The partitions x^i may be disjoint, overlapping or, on occasion, identical.

Fig. 1 Information structures for organizations.

In addition to defining the structure of the organizations, it is necessary that the protocols, i.e., the rules that govern the interactions between organization members, be specified. The types of interactions allowed between team members are shown in Figure 2. The protocols are such that the resulting information structures can be represented by acyclical graphs (Levis and Boettcher, 1982). In the model shown in Figure 2, each team member is assigned a specific task, whether it consists of processing inputs received from the external environment or from other team members, for which he is well trained and which he performs again and again for successively arriving inputs. First, he processes the signals from the environment in the situation assessment (SA) stage to determine or select a particular value of the variable z that denotes the situation. He may communicate his assessment of the situation to other members and he may

receive their assessments in return. This supplementary information may be used to modify his assessment, i.e., it may lead to a different value of z . Possible alternatives of action are evaluated in the response selection (RS) stage. The outcome of this process is the selection of a local action or decision response y that may be communicated to other team members or may form all or part of the organization's response. A command input from other decisionmakers may affect the selection process.

Figure. 2 Allowable team interactions.

In the model of the organization developed in the following sections, internal decision strategies for each decisionmaker are introduced that determine the overall mapping between the stimulus (input) to the organization and its response (output). The total activity of each DM as well as the performance measure for the organization as a whole are expressed then in terms of the internal decision strategies.

For each set of admissible internal decision strategies, one for each DM, a point is defined in the performance-workload space. The locus of all such points is characteristic of the organizational structure. For particular bounded rationality and performance constraints applied in this space, the

effectiveness of a given organizational structure can be assessed and then compared to alternative structures.

In the next section, the model of the interacting organization member is reviewed. In the third section, a specific two-decisionmaker organization is considered and the performance-workload locus is constructed and analyzed. In the fourth section, a method for comparing alternative organizational structures is presented and applied to two variations of the organization considered in Section III.

II. MODEL OF THE ORGANIZATION MEMBER

The overall decisionmaking process is modeled as shown in Figure 3. The presentation of the model given here is brief; for a more detailed discussion, see Boettcher and Levis (1982).

Figure 3. Single interacting decisionmaker model.

The DM receives a possibly noisy measurement x of his environment x' , which is in turn a subset of the organization's input X' . The vector x takes values from a known finite alphabet according to the probability distribution $p(x)$. Two information theoretic quantities that describe the input and its

subsequent processing by the DM are entropy, defined for a variable x as and measured in bits per symbol, and conditional entropy

$$H_x(z) = - \sum_x p(x) \sum_z p(z|x) \log_2 p(z|x) \quad (2)$$

It is assumed that successive inputs are independent and that no learning takes place as a sequence of inputs is processed. Therefore, the model is memoryless¹, and all information theoretic expressions are on a per symbol basis. The mean symbol interarrival time is τ seconds; hence τ becomes a description of the tempo of operations (Lawson, 1981).

Each arriving input is processed first by one of the U algorithms or procedures f_i . The selection of f_i is made through the specification of the variable u in accordance with the situation assessment (SA) strategy $p(u)$. Each algorithm f_i is deterministic, which implies that once the input value is known, then all the variables, including the output z , are determined uniquely. The deterministic algorithm A completes the SA stage processing by combining z with the supplementary situation $\underline{z}'$ received from other organization members. The modified situation assessment is denoted by $\bar{z}$.

In the response selection stage, the DM again makes a selection; in this case an algorithm h_j is chosen according to the response selection strategy $p(v|\bar{z})$. However, a command input vector $\underline{v}'$ may modify the choice v into $\bar{v}$ according to a specified protocol. This is represented by the (deterministic) algorithm B . The result of the RS processing is y , the output of the decisionmaker.

Four aggregate information theoretic quantities characterize the decisionmaking process. First, the mutual information or transmission or throughput between inputs $x, \underline{z}'$, and $\underline{v}'$ and outputs y and $\underline{z}$, denoted by

¹This assumption has been relaxed by S.A. Hall (1982) through the introduction of memory.

$T(x, \underline{z}', \underline{v}; y, \underline{z})$ is a description of the input-output relationship of the DM model and expresses the amount by which the outputs are related to the inputs:

$$G_t = T(x, \underline{z}', \underline{v}; y, \underline{z}) = H(x, \underline{z}', \underline{v}) + H(y, \underline{z}) - H(x, \underline{z}', \underline{v}, y, \underline{z}) \quad (3)$$

Second, a quantity complementary to the throughput G_t is that part of the input entropy which is not transmitted by the system. It is called blockage and is defined as

$$G_b = H(x, \underline{z}', \underline{v}') - G_t \quad (4)$$

In this case, inputs not received or rejected by the system are not taken into account. A third quantity derives from the concept of noise present in transmission, i.e., uncertainty in the output when the input is known. Generalizing this notion to include the total system uncertainty which remains when the input is known gives the quantity G_n :

$$G_n = H_{x, \underline{z}', \underline{v}'}(u, W^1, \dots, W^{U+V}, W^A, W^B, \underline{z}, \bar{z}, z, \bar{v}, y) \quad (5)$$

where W^i is the set of internal variables of algorithm i ; let a_i be the number of elements in the set. In the present context, G_n is not necessarily undesirable noise; rather it is given the more general interpretation of internally generated information.

The final quantity to be considered reflects all system variable interactions and can be interpreted as the coordination required among the system variables to accomplish the processing of the inputs to obtain the output. It is defined by

$$G_c = T(u: w_1^1: \dots: w_{a_V}^{U+V}: w_1^A: \dots: w_{a_B}^B: z: \bar{z}: \bar{v}: \underline{z}: y) \quad (6)$$

The Partition Law of Information (Conant, 1976) states that the sum of the four quantities G_t , G_b , G_n , and G_c is equal to the sum of the marginal

entropies of all the system variables (both internal and output variables), i.e.,

$$G = G_t + G_b + G_n + G_c \quad (7)$$

where

When the definitions for internally generated information G_n and coordination

$$G = \sum_{i,j} H(w_i^j) + H(u) + H(\underline{z}) + H(\bar{z}) + H(z) + H(\bar{v}) + H(y) \quad (8)$$

G_c are applied to the specific model of the decisionmaking process shown in Figure 3 they become

$$G_n = H(u) + H_{\bar{z}}(v) \quad (9)$$

and

$$\begin{aligned} G = & \sum_{i=1}^U [p_i g_c^i(x) + a_i H(p_i)] + H(z) + g_c^A(p(z)) + g_c^B(p(\bar{z})) \\ & + \sum_{j=1}^V [p_j g_c^{U+j}(p(z|v=j)) + a'_j H(p_j)] + H(y) \\ & + H(z) + H(\bar{z}) + H(\bar{v}, \bar{z}) + T_z(x; \underline{z}') + T_{\bar{z}}(x', \underline{z}'; \underline{v}') \end{aligned} \quad (10)$$

In expression (10), which defines the system coordination, p_i is the probability that algorithm f_i has been selected for processing the input x and p_j is the probability that algorithm h_j has been selected, i.e., $u=i$ and $v=j$. The quantities g_c represent the internal coordination of the corresponding algorithms and depend on the distribution of their respective inputs. The quantity H is the entropy of a random variable that can take one of two values with probability p (Shannon, 1949):

$$H(p) = -p \log_2 p - (1-p) \log_2 (1-p) \quad (11)$$

The quantity G may be interpreted as the total information processing activity of the system and, therefore, it can serve as a measure of the workload of the organization member in carrying out his information processing and decisionmaking task.

III. TEAMS OF DECISIONMAKERS

In the previous section, the information theoretic model of a decisionmaker interacting with other members of his organization was given. A general discussion of the extension of the framework to the modeling of organizational structures has been presented in Levis and Boettcher (1982). A basic requirement, in order for the methodology to be valid, is that the interactions between DMs are acyclical. To review the construction and characteristics of the organization's locus in performance workload space, and also to provide a simple illustration of the method of organization evaluation presented in the next section, a specific two decisionmaker structure is considered in this section. By varying a particular parameter, two distinct organizations are obtained which can then be compared.

The organizational structure is shown in Figure 4. Both decisionmakers receive synchronized signals x^1 and x^2 from the organization's environment. Each DM processes the external input using his respective situation assessment algorithms; DM^1 may choose between two f 's. A portion of DM^2 's assessment is then passed to DM^1 to be combined with the assessment z^1 to obtain a final assessment $\bar{z}^1$. The first decisionmaker then selects a response which is, in this case, a command input to the second DM. The latter receives the command input v' and, on the basis of that and his situation assessment z^2 , selects an algorithm h_j , $j=1,2$. His output y^2 is the output of the organization.

This particular configuration can be interpreted as follows. The second DM receives detailed observations about a small portion of the environment on which he has to act. He sends his estimate of the situation to the first DM who has a broader view of the situation. DM^1 then determines an overall plan

Figure 4 Organization structure.

and communicates that to DM^2 . This signal, v' , restricts the options of DM^2 to be consistent with the overall plan. Finally, DM^2 generates a response to his (local) situation which has, in general, been affected by the information he has passed to, and in turn received from, DM^1 .

The expressions for the total activity of each decisionmaker can be derived by specializing eqs.(3) to (10).

Decisionmaker 1

$$G_t^1 = T(x^1, z^{21}; v') \quad (12)$$

$$G_b^1 = H(x^1, z^{21}) - G_t^1 \quad (13)$$

$$G_n^1 = H(u^1) \quad (14)$$

$$\begin{aligned} G_c^1 = & \sum_{i=1}^2 [p_i g_c^i(p(x^1)) + \alpha_i H(p_i)] \\ & + H(z^1) + g_c^A(p(z^1, z^{12})) + g_c^h(p(\bar{z}^{-1})) \\ & + H(z^1) + H(\bar{z}^{-1}) + H(v') + T_z 1(x^1; z^{21}) \end{aligned} \quad (15)$$

Decisionmaker 2

$$G_t^2 = T(x^2, v'; z^{21}, y) \quad (16)$$

$$G_b^2 = H(x^2, v') - G_t^2 \quad (17)$$

$$G_n^2 = H_z 2(v^2) \quad (18)$$

$$\begin{aligned} G_c^2 = & g_c^f(p(x^2)) + H(z^2, z^{21}) + g_c^B(p(z^2), p(v')) \\ & + \sum_{j=1}^2 [p_j g_c^j(p(z^2|v^2)) + \alpha_j H(p_j)] \\ & + H(z^2) + H(z^2, \bar{v}^{-2}) + H(y) + T_z 2(x^2; v') \end{aligned} \quad (19)$$

It is clear that each decisionmaker's workload is dependent on the actions of the other team member. Furthermore, in this specific example, the total activity of DM² will vary with DM¹'s choice of algorithm f_i .

Bounded Rationality and Performance Evaluation

The individual limitations of human decisionmakers in processing information are modeled as constraints on the total activity G of each DM. A maximum processing rate F^r in bits/sec is assumed which, together with the mean symbol interarrival time τ (sec/symbol), yields the constraint

$$G^r = G_t^r + G_b^r + G_n^r + G_c^r \leq F^r \tau \quad r = 1, 2 \quad (20)$$

For a detailed discussion of this particular model of bounded rationality see Boettcher and Levis (1982).

The performance of an organization in accomplishing its task is evaluated using the approach shown in Figure 5. The organization designer has a function or table $L(X)$ which specifies a desired response Y for each input X . The organization's actual response y can be compared to the one desired and a cost assigned using a function $d(y, Y)$. The expected value of the cost serves as a performance index J . In the example considered here, $d(y, Y)$ is chosen such that J is the probability of error in decisionmaking.

Figure 5. Performance evaluation of an organization.

Decision Strategies

For a given organization structure, the actual values of the total processing activity G for each decisionmaker and the value of the organization's measure of performance J are functions of the internal decision strategies selected by each and every decisionmaker. A pure internal decision strategy of the r^{th} DM is one for which both the situation assessment strategy $p(u)$ and the response selection strategy $p(v|\bar{z})$ are pure, i.e., one of the algorithms f_i is selected with probability one and one of the algorithms h_j is selected with probability one when the situation is assessed as being $\bar{z}$. Therefore,

$$D_k^r = \{p(u=i) = 1; p(v=j|\bar{z}=\bar{z}_m)\} \quad (21)$$

for some i , some j , and each $\bar{z}_m \in \bar{Z}$. For the r^{th} decisionmaker, there are n_r possible pure internal strategies

$$n_r = U \cdot V^M \quad (22)$$

where U , V , and M are the number of algorithms f , the number of algorithms h , and the dimension of the set Z , respectively. All other internal strategies are mixed (Owen, 1968) and are obtained as convex combinations of pure strategies:

where

$$D^r(p_k) = \sum_{k=1}^{n_r} p_k D_k^r \quad (23)$$

$$\sum_{k=1}^{n_r} p_k = 1 ; \quad p_k \geq 0 \quad \forall k \quad (24)$$

Therefore, the possible strategies for an individual DM are elements of a closed convex polyhedron of dimension $n_r - 1$ whose vertices are unit vectors and correspond to pure strategies D_k^r , i.e., corresponding to each $D^r(p_k)$ is a point in the simplex defined by eq.(23).

Because of the possible interactions among organization members, the value of the workload G^r depends, in general, on the internal decision strategies of all decisionmakers. Define a pure organizational strategy for a two person organization to be a pair of pure strategies, one for each DM:

$$\Delta_{ij} = \{D_i^1, D_j^2\} \quad (25)$$

Since each DM is assumed to select his strategy independently, the strategy space of the organization S^0 is determined as the direct sum of the individual strategy spaces:

$$\begin{aligned} S^0 &= S^1 + S^2 \\ \dim S^0 &= (n_1 - 1) + (n_2 - 1) \end{aligned} \quad (26)$$

The strategies of each DM, whether pure or mixed, induce a behavioral strategy (Owen, 1968) for the organization:

$$\Delta = \sum_{i,j} p_i p_j \Delta_{ij} \quad (27)$$

where p_i and p_j are the probabilities of using D_i^1 and D_j^2 , respectively.

Each decisionmaker in the organization of Figure 4 possesses two pure strategies. They are denoted as D_1^1 and D_2^1 for the first decisionmaker and correspond to selecting the first situation assessment algorithm ($u=1$) or the second one ($u=2$), respectively. The pure strategies for the second decisionmaker are:

$$D_1^2 : \begin{cases} v^2 = 1 & \text{if } z^2 \in \tilde{Z} \\ v^2 = 2 & \text{otherwise} \end{cases}$$

$$D_2^2 : v^2 = 2$$

where $\tilde{Z}$ is a subset of the alphabet of z^2 , Z^2 . Therefore, the choice of response selection algorithm depends on the value of the assessed situation z^2 . By varying the subset $\tilde{Z}$, different operating procedures can be implemented and, consequently, different organizational performance will be observed. Two operating procedures will be considered. In the first one, referred to as Organization A, $\tilde{Z}$ is a strict subset of Z^2 ; the situation assessment values are partitioned into two sets with each set being processed by a different response selection algorithm h . In the second case, Organization B, the choice of h is independent of z^2 , i.e., $\tilde{Z}$ is the set Z^2 .

Since the number of pure strategies of both DM^1 and DM^2 is two, it follows from eq.(26) that the dimension of S^0 is also two; the organization's strategy space is the unit square. All the strategies of each decisionmaker can be expressed as a convex combination of two pure strategies:

$$D^r(p_k) = D^r(\delta_r) = (1 - \delta_r) D_1^r + \delta_r D_2^r \quad (28)$$

$$r = 1, 2 \quad \delta_r \in [0, 1]$$

Therefore, the set of all strategies of the two person organization, eq.(27),

can be expressed as

$$\Delta = \begin{bmatrix} (1 - \delta_1) & \delta_1 \end{bmatrix} \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \begin{bmatrix} (1 - \delta_2) \\ \delta_2 \end{bmatrix} \quad (29)$$

The strategy matrix has as elements the four pure strategies of the organization; they are also the vertices of the unit square strategy space S^0 .

Performance - Workload Loci

A useful way of characterizing an organization is to consider the locus of possible values of individual workload and the organization's measure of performance as the organization's strategy Δ takes all possible values in S^0 . For a two decisionmaker organization, the locus is contained in the three dimensional space (J, G^1, G^2) . The total activity G of each DM is a parametric function of the two δ 's, i.e.,

$$G^T(\Delta) = G^T(\delta_1, \delta_2) \quad (30)$$

and the organization's measure of performance J can be expressed as

$$J(\Delta) = \begin{bmatrix} (1 - \delta_1) & \delta_1 \end{bmatrix} \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} (1 - \delta_2) \\ \delta_2 \end{bmatrix} \quad (31)$$

where J_{ij} is the performance corresponding to pure strategy Δ_{ij} . The locus of all admissible (J, G^1, G^2) triples is obtained by first assigning to one DM a pure internal strategy and then considering the binary variation between the second DM's two pure strategies. The complete locus is obtained in a similar manner by fixing δ_i and varying δ_j from zero to one, where δ_i also

takes all values from zero to one. The resulting loci for the two organizational forms, A and B, are shown in Figures 6 and 7. The individual performance-workload loci are shown as projections of the locus on the (J, G^i) planes.

Figure 6. Organizational performance versus individual workload for Organization A.

It is clear from Figures 6 and 7 that the range of workload G for each DM does not vary significantly between the two organizational forms. This is not too surprising since the only difference between A and B is in the size of the subset $\tilde{Z}$; the structure and basic operating procedures of both organizations are the same. Consequently, the main change in the characteristics of the locus comes from changes in the value of J due to change in the subset $\tilde{Z}$. This difference becomes significant in the presence of binding bounded rationality constraints and satisficing constraints. For

example, let the maximum admissible probability of error be 0.32, i.e., the performance of the organization is satisficing if J is less than or equal to 0.32. This satisficing constraint can be represented as a horizontal plane in the (J, G^1, G^2) space that intersects the J axis at 0.32. It also intersects the loci of both organizations A and B. Since a larger portion of organization A's locus satisfies the constraint than does B's locus, one may deduce that A is the preferred design. Similarly, the bounded rationality

Figure 7. Organizational performance versus individual workload for Organization B.

constraints can be represented by a plane orthogonal to the corresponding G^r axis. These planes may also partition the performance-workload loci. The bounded rationality constraints are shown in Figures 6 and 7 as lines on the projections of the performance-workload locus on each (J, G^r) plane. Thus, a qualitative comparison of two organizational forms can be made by comparing the portions of the performance-workload loci that satisfy the bounded rationality constraints of each individual member and the satisficing bound on the organization's performance. The results of the comparison may change as the tempo of operations changes and as the performance threshold $\bar{J}$

changes. In the next section, a quantitative method for carrying out this comparison is presented.

IV. ASSESSING AND COMPARING ALTERNATIVE ORGANIZATION DESIGNS

From the viewpoint of the organization designer, specification of a structure means the allocation of information processing and decisionmaking tasks to the organization's members so that the overall task is performed without anyone being overloaded. In the implementation of a designed structure, however, individual decisionmakers select their own internal decision strategies independently of all other organization members. For given constraints on processing load and performance, a particular structure can yield a broad range of performance depending on the actual strategies chosen by the decisionmakers. The designer must therefore also assess the likelihood that strategies which are organizationally acceptable will be selected, i.e., it must be insured that individual decision strategies are mutually consistent.

Organization design begins with a set of specifications to be met, a task to be performed. With the present framework, the designer proposes a particular structure and specifies the protocols and sets of procedures to be used by individual organization members. The selection of a specific procedure as the organization operates is left as a free variable, the organization decision strategy. To determine whether the design will meet the specifications, the designer must consider whether the possible combinations of individual member decisions which may arise will be consistent, on the whole, with design goals. For the present case, the design specifications include a performance threshold $\bar{J}$, i.e., performance must be at least as good as $\bar{J}$, and a maximum tempo of operations, i.e., minimum τ , with which the organization must be able to cope.

A possible measure of mutual consistency can be obtained as follows. Design specifications of constraints on performance, $\bar{J}$, and individual

workload, G^I , partition the space of organization strategies into subspaces of feasible strategies. The intersection of such subspaces represents those strategies which are mutually consistent for the given constraints. Comparison of the locus of the feasible strategies with the total locus of the organization strategy space S^0 is an indication of the likelihood that an acceptable organization strategy, eq.(27), will be obtained as a result of the individual choices of organization members. It is therefore an indication of how close the organization may come to satisfying the design specifications.

The problem is to determine, for a two-person organization and for given τ and J , namely,

$$\tau = \tau_0 \quad ; \quad \bar{J} = \bar{J}_0$$

the subspaces R^i of organization strategies which are feasible with respect to the bounded rationality constraint of each DM:

$$R^1 = \{\Delta \mid G^1(\Delta) \leq F^1 \tau_0\}$$

$$R^2 = \{\Delta \mid G^2(\Delta) \leq F^2 \tau_0\}$$

$$R^J = \{\Delta \mid J(\Delta) \leq \bar{J}_0\}$$

$$R^0 = R^1 \cap R^2 \cap R^J$$

(32)

The subspace R^J contains the feasible Δ 's determined by the performance threshold $\bar{J}$; R^0 is the overall feasibility subspace of the organization. The volume of R^0 , denoted by $V(R^0)$, is compared with that of S^0 , $V(S^0)$, to determine the measure of mutual consistency, Q , i.e.,

$$Q = V(R^0)/V(S^0)$$

(33)

The ratio Q is a monotonic function of $\bar{J}$ and τ with minimum zero and maximum one. A null value for Q implies that no combination of strategies of the individual decisionmakers will satisfy the design specifications, while unity implies that all organizational strategies are feasible, i.e., satisfy the bounded rationality constraints and the performance specifications.

Since Q can be expressed as a function of $\bar{J}$ and τ only, it can be plotted in the three-dimensional space $(Q, \bar{J}, \tau)$. The plots of the ratio Q for organizations A and B are shown in Figures 8 and 9, respectively.

The monotonicity of Q with respect to its arguments is shown clearly in both figures. The two surfaces, denoted by Q_A and Q_B , can be used to compare the two organizational forms.

Fig. 8 Mutual consistency measure Q vs. $\bar{J}$ and τ for Organization A.

Let the design specifications be:

- (a) the mean interarrival time τ is 0.95 sec.
- (b) the performance threshold $\bar{J}$ is 0.32.

Fig. 9 Mutual consistency measure Q vs. $\bar{J}$ and τ for Organization B.

These specifications imply that the maximum tolerable probability of error, the measure of the organization's performance, cannot exceed 0.32 and that the maximum tempo of operations that will not lead to overload is $(0.95)^{-1}$ symbols/sec. The values of Q for the two organizational forms are

$$Q_A = 0.73 \quad ; \quad Q_B = 0.56$$

Clearly, Q_A is larger than Q_B . This means that for those design specifications, organizational form A is better than B. If

$$Q_A \geq Q_B$$

for all values of $\bar{J}$ and τ , then the organizational form A would always be superior to B. In general, however, there exist values of $\bar{J}$ and τ for which B is better than A. This is the case for these organizations, too. Indeed, for $\bar{J}$ equal to 0.4 and τ equal to 0.75, Q_A is equal to 0.02 and Q_B is equal

to 0.05. This means, in relative terms, that for these task specifications, a greater percentage of the possible strategies of organization B yield satisfactory performance than those of A. In absolute terms, neither organization is well matched to the task.

V. CONCLUSION

In recent work, an approach to the modeling and evaluation of information processing and decisionmaking organizations has been developed. The emphasis has been on describing an organization in a generalized performance-workload space where the performance refers to organizational performance and workload to the workload of each individual member. In this paper, a quantitative procedure has been presented for modeling and analyzing alternative organizational forms. The comparison is based on an analysis of how well the alternative structures can satisfy the design specifications for a minimum tolerable performance and for maximum tempo of operations. The methodology is illustrated through application to two variants of a two-person organizational structure.

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