A Gallery for Mathematics

by

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B.S. Architecture
Massachusetts Institute of Technology 2001

Submitted to the Department of Architecture
in Partial Fulfillment of the the Requirements of the Degree of
Master of Architecture

at the

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Abstract

The Gallery for Mathematics seeks to interpret and communicate the character and culture of mathematics through the lens of architectural tectonic. The form and program of the Gallery are driven by three goals for the architectural experience and mathematical journey: *[movement]*, *[interaction]*, and *[solitude]*. By redesigning the traditional museum experience, the Gallery aims not to simply inform its visitors, but instead to incite their curiosity, producing spaces of inquiry rather than spaces of information. Sited on top of the parking garage for Boston's Museum of Science, the Gallery and its garden also serve to reconnect the Boston and Cambridge park systems on opposite banks of the Charles River, embedding the mathematical journey within the urban context.

Thesis Supervisor: Jan Wampler
Title: Professor of Architecture


a gallery for mathematics

"What I cannot create, I do not understand."

-Richard Feynman

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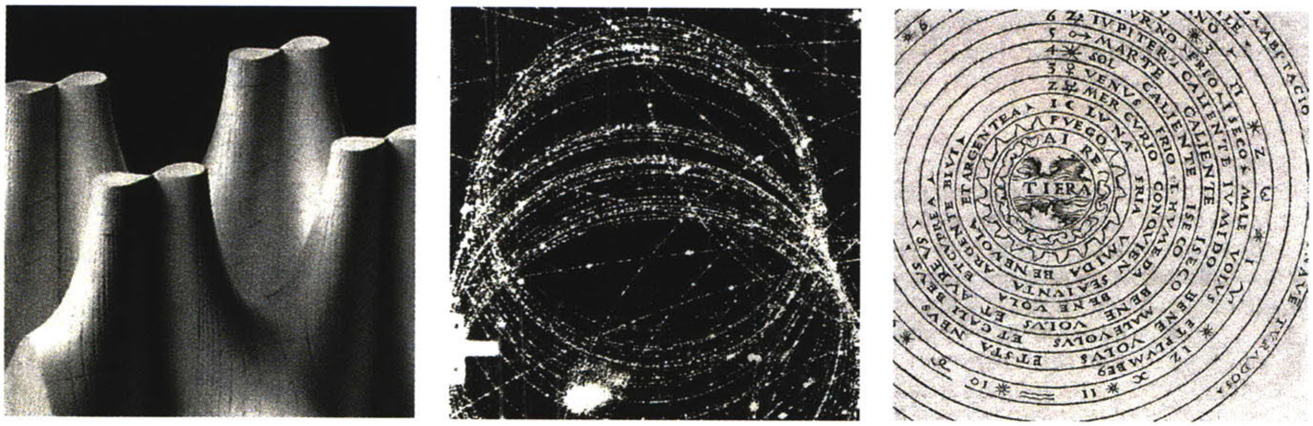


Math as image: (from left) *plaster model of a P-function; Cloud Chamber Atlas; Diagram of the Ptolemaic cosmos, 1551.*

"The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonius way. Beauty is the first test. There is no permanent place in this world for ugly mathematics."

-G.H. Hardy

A Gallery for Mathematics

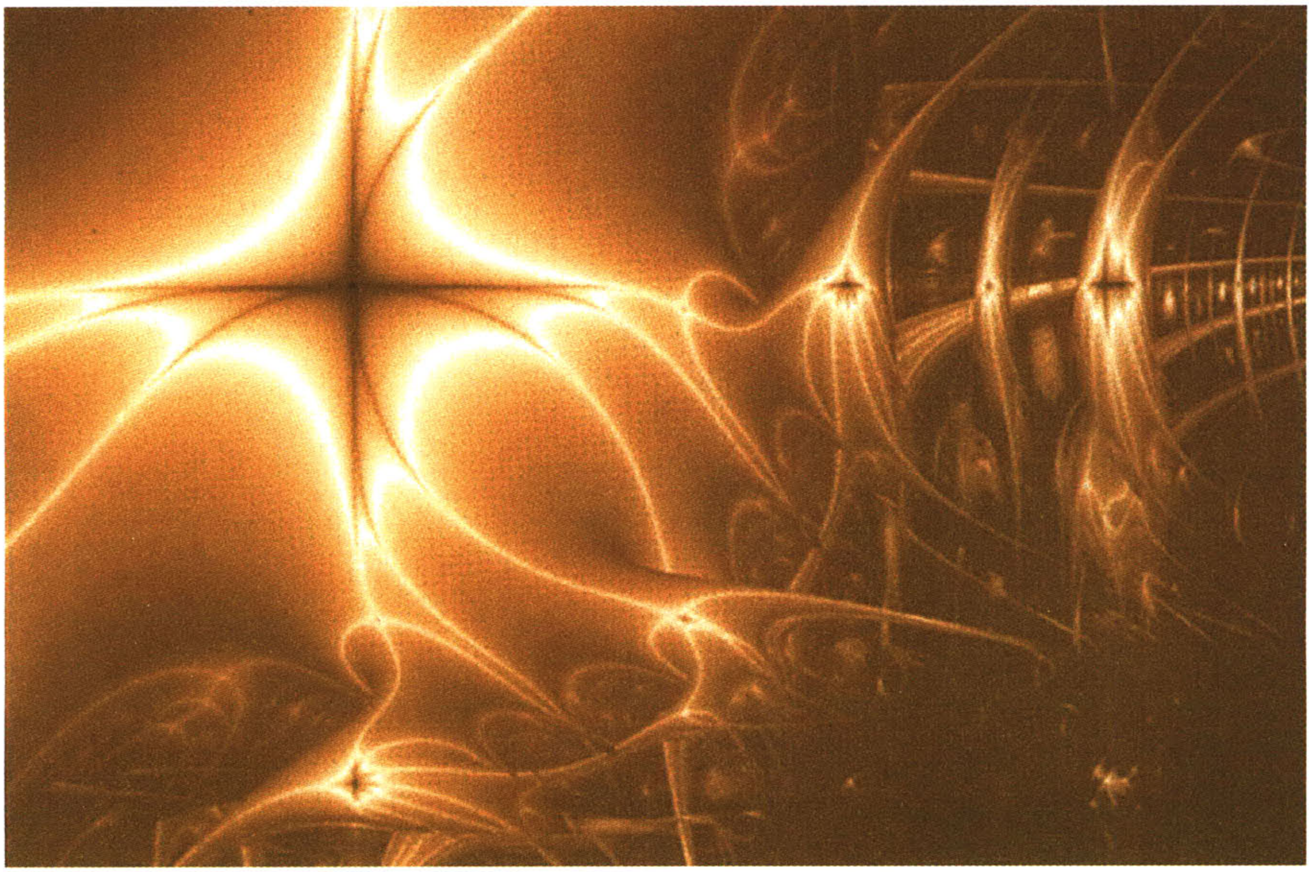
Mathematics is a discipline characterized by rigorous logic and processes of clearly defined operations. Methods of computation arise out of precisely defined numeric systems, which in turn logically determine valid operations and geometric representations. From the initial construction of these systems—the definition of the Real numbers; the selection of a flat, hyperbolic, elliptical, or spherical geometry; or the designation of a logarithmic base—any number of unique, self-consistent mathematical systems can be created. In this sense, mathematicians define their own realms of thought, operating within internally logical, yet necessarily rigid parameters of numbers and space. It is through the rigid framework of number theories that mathematics finds its freedom—the simplification and representation of immense complexities. Mathematics in its purest form seeks not merely solutions, but elegance, simplicity, and beauty. As Paul Dirac famously said, "This result is too beautiful to be false; it is more important to have beauty in one's equations than to have them fit experiment."[1]

This creative and strikingly beautiful side of mathematics is one few people experience, in part due to the deep complexity of higher maths, as well as the generally insular nature of the mathematics community. Yet to limit the appreciation of this work merely to its mathematical machinery is to overlook the beauty of the method itself—the process of constructing a simple and elegant abstract expression to represent the greatest complexity. Einstein once explained, "After a certain high level of technical skill is achieved, science and art tend to coalesce in esthetics, plasticity, and form. The greatest scientists are always artists as well."[2] In this view, mathematics can be understood not only through its numerical gymnastics, but also simply as a method looking at the world—a process which can be experienced and appreciated apart from the quantitative analyses that it governs.

Architecture, too seeks beauty through form and structure,

1. Quoted by Graham Farmelo. *It Must Be Beautiful*, (London: Granata Books, 2003).

2. Remark made in 1923; recalled by Archibald Henderson, *Durham Morning Herald*, August 21, 1955; Einstein Archive 33-257, as quoted by Alice Calaprice, *The Quotable Einstein* (Princeton: The Hebrew



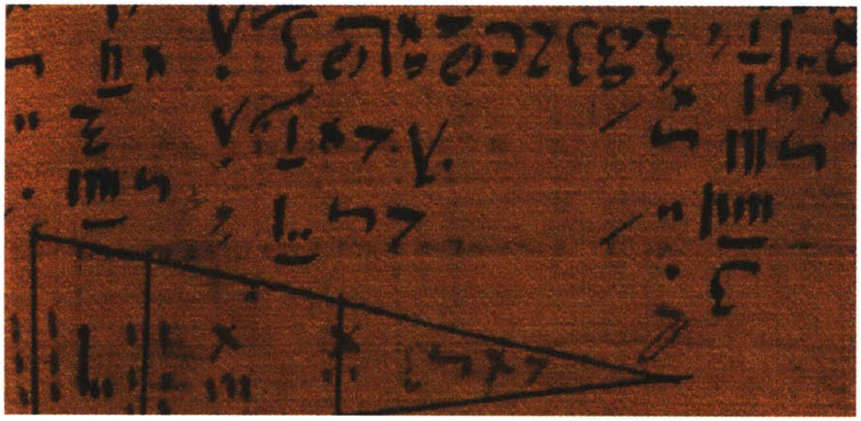
Computer-generated image of the stability of a dynamical system using the Lyapunov exponent.

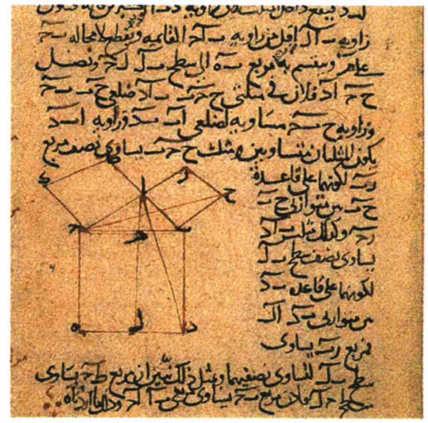
and, in the best instances, conveys through the articulation of its spatial logic a new or altered relationship with its surroundings. This project seeks to address and convey the methods of mathematical understanding through a tectonic experience of light, space, and movement. The gallery for mathematics is conceived not simply as a space for the display of mathematical models and images, but as a place for discovery, contemplation, and repose. It is a place for the teacher and the student, the mathematician and the layman alike, while privileging neither view. To again quote Einstein, the gallery is conceived as a place in which "Imagination is more important than knowledge."[1]

University of Jerusalem and Princeton University Press, 1996), 170.

1. Caprice, 223.

mathematics: *history and culture*





The Rhind Papyrus, c. 1650 BCE; (right) discussion of the Pythagorean Theorem in an Arab text.

"I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. That would be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming ... and a little mad."

-W.H. Auden

The story of mathematics spans nearly 40,000 years of human history, making it nearly impossible to construct a concise overview of even the most notable discoveries. What is perhaps more useful than an accounting of the specifics of mathematical progress is to study the primary methods and views of mathematics throughout the ages. For thousands of years mathematics has represented man's primary tool for understanding the world and the universe. As such, its methods and representations are indicative not just of the progress of human quantitative abilities, but also of man's view of the universe and his place in it.

Counting and Arithmetic

The story of math begins 35,000 years BCE, with the first known evidence of counting. Likely a record of hunting kills, carved bone fragments are proof of prehistoric man's conception of number and quantity. In samples found dating to 20,000 BCE, notches are grouped by fives, in the tally system still used today. By 8,500 BCE, artifacts show that prime numbers were given special significance, and by 5,000 BCE both decimal number systems and fractions were in use in ancient Egypt. Around 2,000 BCE the Babylonians developed a sexigecimal number system used for both theoretical and practical applications, including approximation of square roots and irrational numbers. Pi was known to one decimal place at this time. Due to its sexigesimal base, it is likely that the system was primarily developed to make astrological the calculations necessary in creating an accurate calendar.[1]

By the year 550 BCE, mathematical theory had reached Greece, where Pythagoras set forth his famous theorem, and it was here than Western mathematics began. For Pythagoras and his students, numbers represented not merely quantity or tools for prediction, but

1. Richard Mankiewicz. *The Story of Mathematics*, (Princeton: Princeton University Press, 2000).

it was believed that they held mystical significance. Pythagoras started a school in which numbers were worshipped with religious devotion and coined the term *philosophy* to describe their scholarly pursuits. The Pythagorean sect viewed particular numbers as perfect, and were deeply disturbed by the existence of irrational numbers. As the story goes, Pythagoras sentenced to death by drowning the unlucky student who proved that the square root of two could not be expressed as a fraction.[1]

More than an unfortunate aside in the history of mathematics, the story illustrates the extreme devotion and fanatical belief in the power of numbers in ancient societies. In many cultures, numbers were the mystical tools of the priests, used to foretell the movements of the stars and the arrival of the rains, a critical piece of information for agricultural societies. It was believed that favorable planetary alignments could affect the outcome of battles, and the ability to predict such occurrences could give soldiers an edge in war. Given such critical importance in early civilizations, it is not surprising that numbers were viewed with mystical reverence. Pythagoras believed that the perfection of numbers reflected the beauty of nature, and the existence of irrational, inexpressible numbers implied nothing less than a deeply flawed universe. It was a belief which would resurface many times throughout history, as scientific predictions seemed to threaten the authority of the Church or even God.

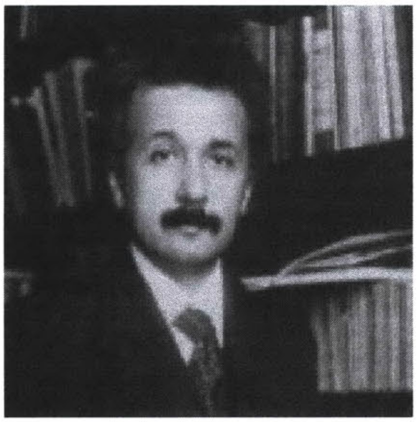
The Advent of Abstraction

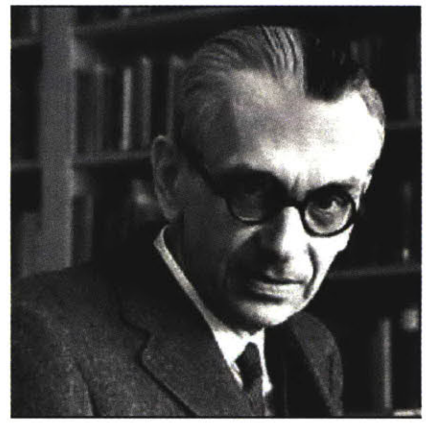
Around the turn of the first century, a major shift occurred in the methods and representation of algebraic calculation. Algebra to this point existed as a graphical method of problem-solving, using units of measurement to represent quantities and geometric constructions for calculation. In 250, Diophantus of Alexandria published *Arithmetica*, the first volume to focus upon numeric, rather than geometric problem-solving. The work represented the beginning of mathematics as a tool of abstraction, separating applied mathematics from the new, purely theoretical method of study. By 500, Western mathematics had been condensed into a comprehensive textbook in use throughout the Roman Empire. Pi at this time was known to seven decimal places. In 529, however, the Emperor Justinian closed all of the pagan philosophical schools throughout the Empire, effectively ending Western dominance in mathematics.

The shift toward abstraction reached another milestone in 969, when Arabic numerals first appeared. Over the next several centuries, algebra, aided by the new standardized number system, advanced to incorporate methods of calculating the quadratic and cubic equations as well as binomial expansion. Much of this progress was fueled by the work of the astronomers seeking to produce more accurate calendars. The measure of time now operated not simply to predict crop cycles, but also represented the authority of the Church and the Empire. The calendar marked feast days and religious holidays, which also determined when feudal farmers delivered their crops to their lords and paid their taxes. The accuracy of the calendar thus represented not only the technical advancement of the Empire, but its harmony with divine authority. Astrological predictions were also needed to produce accurate maritime maps, as the ability to locate position by the stars was growing increasingly critical in the colonial conquests of the nations of Western Europe. Mathematics at this time became the tool of empires, an increasingly intricate and precise field of study, and ultimately, a symbol of control.

Despite the advances in mathematical abstraction and representation, instruction in Renaissance universities was not yet scientific in the modern sense. Mathematics students were taught the Aristotelian view of the universe and instructed in astrology. When in 1543 a young Church canon

1. Simon Singh, *Fermat's Enigma* (New York, Random House, 1997) 49-50.





Three faces of 20th Century mathematics, (from left) *Albert Einstein, Werner Heisenberg, and Kurt Gödel.*

named Nicolaus Copernicus published a work proposing a heliocentric model of the heavens, the role of mathematics in the Holy Roman Empire was violently altered. His work was continued by Brahe, Kepler, and Galileo, and marked the shift from mathematics as priestly pursuit to heretical blasphemy. Although many mathematicians were persecuted for their studies, the split did serve to liberate mathematics from the practical concerns of the Empire.

The 1600's saw a return to the purely abstract pursuits of mathematics largely ignored by Western mathematicians for almost a thousand years. Although much of the work still centered around astrological and physical predictions, the major advances in mathematical thought during this century established the discipline as a discrete field of study, distinct from the sciences. The invention of the slide rule facilitated rapid calculation for the first time and in 1640, Rene Descartes' *La Géométrie* introduced modern algebraic convention and established the graphical, rather than geometric expression of polynomials. The 1680's saw the development of calculus by Liebniz and Newton, marking the first truly abstract method of calculation.

Proof and Uncertainty

If modern mathematics can be characterized by a single pursuit, it has been the quest to fully define and prove the fundamental basis of mathematics. Much of the noteworthy work of 18th century mathematics dealt with systematically proving and defining many of the foundations of the discipline, including the classification of curves, a rigorous definition of logarithms and complex numbers, defining a theory of functions, and proving the Fundamental Theorem of Algebra. While many of these theorems had long been accepted as self-evident, the exercise of constructing rigorous proofs led to several new discoveries. Until 1827 it had been assumed that Euclidean geometry was the only possible self-consistent geometric system, an idea which was disproved when Nikolai Ivanovich Lobachevsky's developed his hyperbolic geometry.[1] When Riemann redefined geometry as the study of manifolds in 1854, space was no longer limited to three dimensions. By 1900 several other non-Euclidean geometries had been defined, as had the first non-commutative

1. Eric W. Weisstein. "Non-Euclidean Geometry." From *Math World* -- A Wolfram Web Resource. available from http://mathworld.wolfram.com/Non-EuclideanGeometry.html [Cited January 7, 2005].

algebra.

At the turn of the century, theoretical mathematics had been pushed far beyond the confines of the physical universe, while advanced maths were suggesting physical phenomena very different from traditional Newtonian model. Albert Einstein produced his paper on Special Relativity in 1905, followed by the Theory of General Relativity in 1916. Game theory, which deals with analysis of complex systems, was introduced in 1926, with applications from Go boards to the stock market. With the development of counting machines and the ever increasing breadth of mathematical understanding, it seemed that eventually everything would be describable as mathematical functions.

The first blow to the scientific certainty of the first quarter century came in 1926 when Werner Heisenberg published his paper on the Uncertainty Principle. Just as the scientific community was confronted with the limits of observation and measurement, the mathematical community faced a similar dilemma in 1931. Kurt Gödel's Incompleteness Theorems showed that every axiomatic system must contain certain propositions which cannot be proven true or false, and no system can prove its own consistency.[1] The Incompleteness Theorems did not invalidate mathematical results, however it did demonstrate the limits of mathematical proof.

The mathematics of the latter half of the 20th century was profoundly affected by the capabilities of the computer. The fields of chaos theory, fractal geometry, and nonlinear dynamics would have been nearly unworkable with traditional methods of calculation. Still, the devotees of pencil-and-paper proofs have lain to rest several centuries' old problems in the last decade. The most famous of these was Andrew Wiles' proof of Fermat's Last Theorem in 1994, but significant progress has been made on at least three other major conjectures in recent years as well.

In the 17th century Galileo wrote:

> The universe stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth.[2]

The mathematics which governs the universe has not changed in the 8 billion years of its existence, or in the 40,000 years that we have been struggling to comprehend it through numbers. Our methods, our understanding, and our outlook have remained in constant flux, mapping our changing insight and relationship with the world as we see it.

1. Weisstein, 2005.

2. Galileo Galilei, as quoted by Theoni Pappas, *The Joy of Mathematics*. (San Carlos, CA: Wide World Publishing, 2002).

<< Prehistory

Bronze Age >>

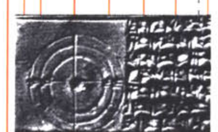
Roman Empire >>

2000 BCE | 1000 BCE | 500 BCE | 400 BCE | 300 BCE | 200 BCE

+ **35,000 BCE** Earliest record of counting.

+ **[2,000 BCE]** Babylonians develop sexagesimal number system used for both theoretical and practical applications, including algebra, approximation of square roots, and knowledge of irrational numbers.

+ **[1850 BCE]** Egyptians invent fractional notation.



+ **[800 BCE]** Indian texts show rudimentary understanding of the Pythagorean Theorem.

+ **[550 BCE]** Pythagoras sets forth the Pythagorean Theorem along with a numerical analysis of music.

+ **[450 BCE]** Chinese develop graphical method for calculation of Pythagorean triples.

+ Zeno in Greece poses problems on the infinite.

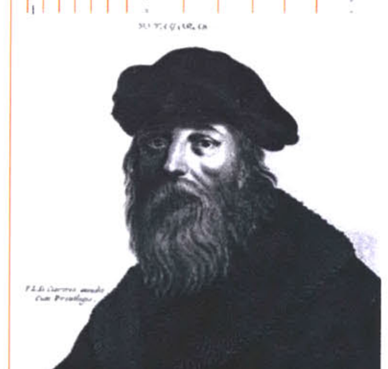
+ **[400 BCE]** Mayans develop vigesimal number system including zero.

+ **[300 BCE]** Aristrachus of Greece first suggests heliocentric solar system.

+ Euclid writes *The Elements*, defining plane and solid geometry, number theory, and the theory of irrational numbers, for the first time with rigorous proofs.

+ **[200 BCE]** Apollonius produces first advanced work on conic sections.

known digits of pi 3.1 3.141



Dark Ages >>

100 BCE | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000

+ **[100 BCE]** Hipparchus founds modern astronomy on geometric principles, dividing the circle into 360 degrees as the basis for trigonometry.

+ **[100]** *Nine Chapters*, the first standardized mathematic textbook written in China, including 246 problems and their solutions.

+ **[130]** Ptolemy writes *Syntaxis* (*The Mathematical Collection*).

+ **[250]** Diophantus of Alexandria writes *Arithmetica*, a fundamental work on algebra which focuses on numeric, rather than geometric problem-solving.

+ **[400]** Hypatia, the first female mathematician recorded, becomes head of the Platonic school at Alexandria. She was later murdered by fanatic Christian monks.

+ **[520]** Boethius condenses the major works of Greek mathematics into *Arithmetic*, the standard textbook of schools in the Roman Empire.

+ [529] The Emperor Justinian closes all pagan philosophical schools in the Roman Empire, effectively ending Western dominance in mathematics.

+ **[662]** Hindu numerals, the precursor to the Arabic numbers, first appear in Western mathematics.

+ [820] Abu Jafar Muhammad ibn Musa al-Khwarizmi writes *Hisab al-jabr w'al-muqabala, Calculation by Restoration and Reduction*, on the solution of linear and quadratic equations. *(The English words for algebra and algorithm are derived from his name.)*

+ **[969]** Arabic numerals first appear.

+ **[990]** Al-Karaji of Baghdad writes the *al-Fakhri*, a treatise on algebra which includes definitions of the higher powers and their reciprocals, polynomials, and binomial expansion.

3.14159

3.1415926

Crusades >> | Renaissance >> | Baroque Era >> | Enlightenment >>

1100 | 1200 | 1300 | 1400 | 1500 | 1550 | 1600 | 1625 | 1650 | 1675 | 1700 | 1725

+ **[1074]** Omar Khayyam writes *Algebra*, which includes a geometric method for solving cubic equations. He goes on to develop a solar calendar accurate to 1 day in 3770 years, better than the Gregorian calendar, accurate to 1 day in 3330 years.

+ **[1126]** Adelard of Bath translates al-Khwarizmi's astronomical tables and Euclid's *Elements* into Latin.

+ **[1202]** Fibonacci publishes *Liber abbaci, Book of the Abbacus.*

+ **[1230]** Roger Grosseteste publishes work on optics and proposes a theory for the creation of the universe similar to that of the Big Bang. His work is continued by his student at Oxford, Roger Bacon.

+ **[1290]** Mechanical clock invented.

+ **[1339]** William of Ockham writes several attacks on Aristotelian thought, believing that "natural philosophy" should be based upon empirical observation. He also sets forth "Ockham's Razor."

+ **[1400]** Arabic numerals appear in their modern form.

+ [1415-1420] Brunelleschi develops the measured perspective.

+ **[1494]** Luca Pacioli writes *Summa*, a guide to modern accounting.

+ **[1523]** Albrecht Dürer completes his *Treatise on Proportions*, and two years later, *Treatise on Measurement*, the first mathematical texts to be published in German. The works introduce the subject of perspectival geometry.

+ **[1535]** Tartaglia produces the first exact solution for the cubic equation.

3.14159265358 | 3.14159265358 | 3.14159265358979323846264338327950288 | 3.14159265358979 3

+ **[1543]** Robert Recorde writes the first English mathematic textbook, *The Ground of Artes*.

+ [1543] Copernicus publishes *On the Revolutions of the Heavenly Spheres*, reintroducing the idea of a heliocentric solar system.

+ **[1557]** First use of the modern equals sign.

+ **[1569]** Mercator develops Mercator projection method for maps.

+ **[1593]** Chinese abacus in its modern form first appears.

+ **[1596]** Johannes Kepler publishes *Mysterium cosmographicum*, bringing public attention to the sun-centered solar system model.

+ **[1609]** Kepler publishes first two laws of planetary motion in *Astronomia nova*.

+ **[1614]** John Napier of Scotland writes *A Description of the Marvelous Rule of Logarithms*, defining the logarithmic function as well as calculation methods and tables, including a table of natural logs.

+ **[1618]** Kepler publishes his third law of planetary motion in *Harmonice mundi*.

+ **[1624]** William Oughtred invents the slide rule.

+ [1632] Galileo publishes his *Dialogue Concerning the Two Chief World Systems*.

+ **[1634]** Galileo writes *Discourses and Mathematical Demonstrations* in which he lays out the principles of gravitational motion.

+ [1640] Rene Descartes publishes *La Géométrie*, the first text on analytic geometry. The work introduces modern algebraic convention and shifts representation of polynomials from a geometric expression to a graphical one.

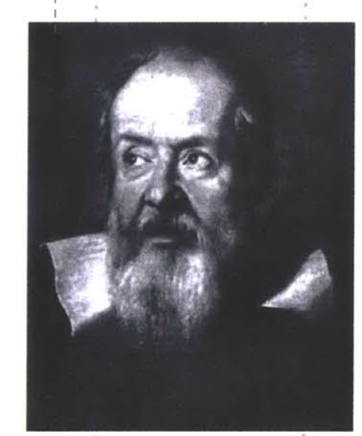
+ **[1670]** Fermat's Last Theorem is published posthumously by his son.

+ [1684] Leibniz publishes the fundamental theorem of calculus with proofs based on infinite series.

+ [1687] Newton publishes his *Philosophiae naturalis principia mathematica*, which includes his three Laws of Motion and Universal Gravitation and introduces the principles of calculus with geometric proofs.

+ **[1706]** Greek letter π first introduced to represent the ratio of circumference to diameter.

+ **[1718]** De Moivre publishes *Doctrine of Chances*, a seminal work on probability.



Romantic Period >>

1725 | 1750 | 1775 | 1800 | 1825 | 1850 | 1860 | 1870 | 1880 | 1890 | 1900

+ [1727] Euler introduces the symbol e as the base of natural logarithms.

+ [1735] Euler introduces the notation $f(x)$.

+ [1737] Euler publishes *Mechanica*, representing Newtonian mechanics in mathematical form.

+ [1750] Cramer publishes *Introduction à l'analyse des lignes courbes algébraiqu*, an investigation and classification of curves, including "Cramer's rule.

+ [1751] Euler publishes his work on the logarithms of complex numbers.

+ [1806] Legendre develops the method of least squares as the method of determining the best approximation of data.

+ [1807] Fourier publishes *On the Propagation of Heat in Solid Bodies*, which includes his method of representing any continuous function as the sum of a series of trigonomic functions.

+ [1817] Bessel discovers a class of integral functions now known as "Bessel Functions" while studying Kepler's Three-Body Problem.

+ [1827] Lobachevsky publishes *On the Principles of Geometry*, the first non-Euclidian geometric system.

+ [1851] Joseph Liouville proves pi to be a transcendental number.

+ [1852] Four color theorem proposed by Francis Guthrie.

+ [1853] William Rowan Hamilton introduces the first non-commutative algebra.

+ [1854] Riemann delivers a lecture defining geometry as a study of manifolds.

+ [1854] Boole publishes *Investigation of the Laws of Thought*, introducing algebraic logic now known as Boolean algebra.

+ [1865] Plücker mathematically defines four-dimensional space.

+ [1872] Dadekind publishes a formal construction of the Real numbers, including the first rigorous definition of an integer.

+ [1873] James Clerk Maxwell publishes his *Treatise on Electricity and Magnetism* which includes his field equations on electromagnetism.

+ [1874] Cantor publishes the first paper introducing set theory.

+ [1881] Venn introduces "Venn Diagrams" for the study of Set Theory.

+ [1882] Lindemann proves that Pi is transcendental.

+ [1883] Poincaré publishes a paper introducing the field of analytic functions of complex variables.

+ [1895] Henri Poincaré publishes his *Analysis situs*, an early treatment of algebraic topology which includes the Poincaré Conjecture.

+ [1895] Cantor publishes the first of two works on transinfinite arithmetic.

+ [1896] Hadamard and de la Vallée-Poussin independently prove the Prime Number Theory, showing that the number of primes less than a number, n goes to infinity as $n/\log n$.

462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282

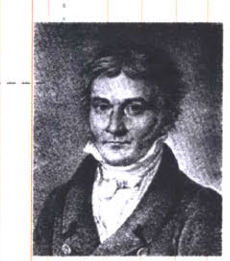
+ [1761] Lambert proves that Pi is irrational.

+ [1777] Euler introduces i to represent the square root of -1.

+ [1788] Lagrange publishes *Mécanique analytique (Analytical Mechanics)*, summarizing mechanics since Newton. The work includes a theory of differential equations, which establishes mechanics as a field of mathematical analysis.

+ [1797] Lagrange publishes *Théorie des functions anlytiques (Theory of Analytical Functions)*, the first work on the theory of functions of a real variable. In it Lagrange first introduces the notation dy/dx for derivatives.

+ [1797] Gauss proves the Fundamental Theorem of Algebra.



+ [1829] Bolyai develops an "absolute science of space," another treatise on non-Euclidean geometry.

+ [1834] Charles Babbage designs the "Analytical Engine," the first calculating machine, but lacks funding for production.

+ [1844] Edwin Abbot publishes *Flatland*, a novel about two-dimensional beings confronted by a three-dimensional object.

+ [1847] Bolzano first defines the infinite set.

+ [1847] George Boole publishes *The Mathematical Analysis of Logic*, the first work on the subject.

+ [1868] Beltrami publishes his *Essay on Interpretation of Non-Euclidean Geometry*, which includes models for the geometries of Lobachevsky and Bolyai.

+ [1887] Levi-Civita develops tensor calculus.

WWI

WWII

1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010 2020

+ **[1900]** Poincaré shows that there is no solution to the n-body problem, proving that the universe is a chaotic system.

+ **[1901]** Poincaré publishes a paper on the Diophantine problem, a major contribution to number theory.

+ **[1910]** Steinitz gives the first abstract definition of a field.

+ **[1913]** Rumanujan moves to Cambridge and begins his work on Number Theory with Hardy.

+ **[1916] Einstein publishes his paper on General Relativity.**

+ **[1919]** Russell publishes *Introduction to Mathematical Philosophy*.

+ **[1921]** Borel publishes the first in a series of papers on game theory, becoming the first to define games of strategy mathematically.

+ **[1931] Kurt Gödel publishes his Incompleteness Theorems, which show that every axiomatic system must contain propositions which cannot be proven true or false, and no system can prove its own consistency.**

+ **[1932]** Von Neumann completes *Grundlagen der Quantenmechanik* on quantum mechanics.

+ **[1936]** Alan Turing publishes his theory of computability.

+ **[1940] The first automatic calculating machine built at Harvard and funded by IBM.**

+ **[1950]** John Forbes Nash writes the paper on game theory which would win a Nobel Prize in Economics in 1994.

+ **[1955]** Nicholas Metropolis introduces the idea of simulated annealing algorithms in global optimization problems.

+ **[1955]** Taniyama poses a conjecture on elliptic curves which is later fundamental in the proof of Fermat's Last Theorem.

+ **[1955]** Enrico Fermi and John Pasta study a nonlinear spring model for heat conduction and discover solitary wave type behavior.

+ **[1970]** John Conway invents "the game of life," a cellular automata model which produces a variety of structures in dynamic equillibria.

+ **[1975] James Yorke coins the term "chaos" in a paper with T.Y. Li in describing the behavior of non-deterministic systems.**

+ **[1975] Benoit Mandelbrot publishes paper on recursive geometry in which he coins the term *fractal*.**

+ **[1975]** Feigenbaum discovers a new constant, approximately 4.6692016609..., related to period-doubling bifurcations, which proves integral in chaos theory.

+ **[1990]** Alaine Connes and John Lott develop non-commutative geometry.

+ **[1994] Andrew Wiles proves part of the Taniyama-Shimura conjecture, proving Fermat's Last Theorem.**

+ **[1994]** Mattel's releases Teen Talk Barbie, whose phrases include "Math class is tough."

+ **[1998]** Thomas Hales (almost certainly) proves the Kepler Conjecture, regarding the density of sphere-packing.

+ **[1999]** Conrad and Taylor complete the proof of the Taniyama-Shimura Conjecture.

3.14159265358979323846264338327950288419716939937510582097494459230781

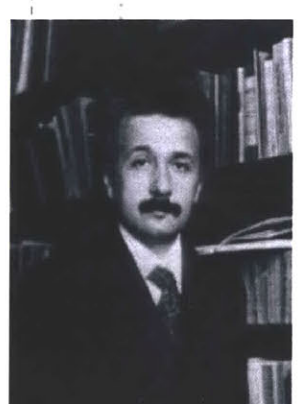
+ **[1901]** Dickenson publishes *Linear Groups*, which introduces the Galois field theory.

+ **[1905] Albert Einstein publishes the Special Theory of Relativity.**

+ **[1906]** Fréchet formulates the abstract theory of compactness in his doctoral dissertation.

+ **[1907]** Brouwer attacks the logical foundations of mathematics and marks the beginning of the Institutionist School.

+ **[1908]** Hardy and Weinberg study proportions of dominant and recessive traits in populations, establishing the mathematical basis for population genetics.



+ **[1922]** Emmy Nether publishes a paper fundamental to the development of abstract algebra.

+ **[1926]** John von Neumann writes his doctoral thesis on game theory.

+ **[1928]** Von Neumann proves the minimax theorem of game theory.

+ **[1944]** Von Neumann and Morgenstern publish their *Theory of Games and Economic Behaviour*.

+ **[1946]** The RAND Corporation is founded as the first think tank to study strategies of nuclear war.

+ **[1949] The first stored-program computer, known as EDSAC, is built in the UK.**

+ **[1962]** Daniel Schanks and John Wrench compute Pi to 100,000 decimal places using the IMB 7090 computer.

+ **[1963] Edward Lorenz of MIT publishes a paper on weather modeling identifying the "Butterfly Effect," a fundamental theorem of chaos theory.**

+ **[1963]** Cohen proves the independence of the axiom of choice and the continuum hypothesis.

+ **[1964]** Gordon Moore, founder of Intel, outlines Moore's Law, stating that the complexity of integrated circuits doubles every 18 months.

+ **[1967]** Texas Instruments introduces the first hand-held calculator.

+ **[1968]** Michael Atiyah and Isadore Singer prove the Atiyah-Singer index theorem, which unifies topology and analysis in the mathematics of manifolds and differential operators.

+ **[1976]** Kenneth Appel and Wolfgang Haken prove the Four-Color Theorem using a computer.

+ **[1976]** Apple Computer introduces the Mac I.

+ **[1977]** Adelman, Rivest, and Shamir introduce public-key codes, a system for encoding messages using large primes and a key which can be published.

+ **[1979]** Atari's 8-bit computer is released.

+ **[1982]** Mandelbrot publishes *The Fractal Geometry of Nature*, further developing his work on fractals.

+ **[1983]** The classification of finite simple groups, a 30-year collaborative effort of over 100 mathematicians, is completed.

+ **[1984]** George Cowan founds the Rio Grande Institute, a think tank probing areas of non-linear dynamics and complexity theory.

+ **[1987]** Pi is computed to 134,000,000 decimal places.

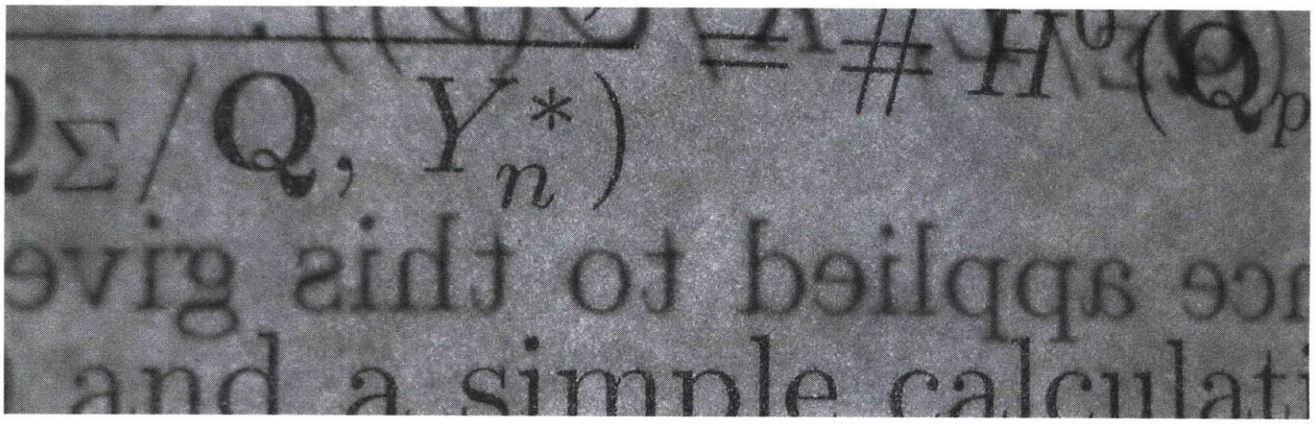
+ **[1988]** Elkies finds a counterexample to Euler's Conjecture.

+ **[2000]** The Clay Mathematics Institutes establishes the Millennium Prize Problems, seven classic unsolved problems in mathematics.

+ **[2002]** Manindira Agrawal, Nitin Saxena, and Neeraj Kayal create an unconditional deterministic polynomial time algorithm to determine whether a given number is prime.

+ **[2002]** Pi is calculated to 1,241,000,000 digits.

+ **[2004]** Dr. Grigori Perelman announces he has proven the Poincaré Conjecture.



An image from Wiles' proof, Modular Elliptic Curves and Fermat's Last Theorem, *from the May 1995 issue of* Annals of Mathematics.

"A mathematician is a device for turning coffee into theorems."

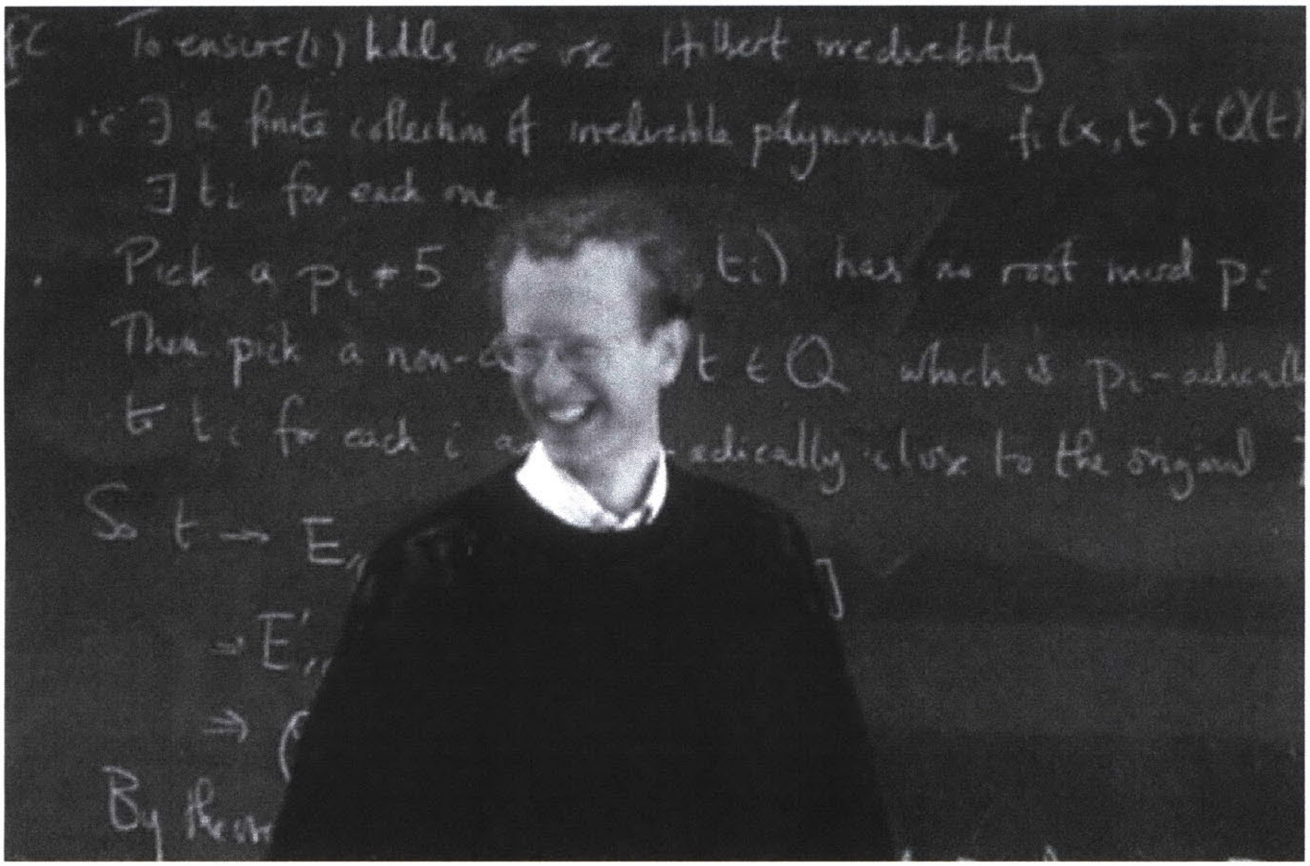
-Paul Erdös

Papers and Personalities

When Andrew Wiles delivered his historic lecture on "Modular Forms, Elliptic Curves, and Galois Representations" at Cambridge University in 1993, only a handful of the mathematicians in his audience could even hope to follow the proof he presented. Despite the extremely esoteric nature of his work, Wiles soon became an international celebrity, the subject of hundreds of interviews, articles, and books, as well as an episode of Nova on PBS. What captured the public's imagination was not Fermat's Last Theorem or its proof, but the dramatic story of Wiles' obsession with it. The Princeton professor seemed to perfectly embody the popular notion of the mathematician—a man consumed by a single equation, ultimately cloistering himself in his attic for eight years until finally finding his Holy Grail.

Indeed, there are probably many mathematicians like Wiles who prefer to work in solitude and avoid the distractions of normal life. This popular notion fails to capture the collaborative nature of the mathematical community itself, however. The community is, for the most part, a small and insular one. Like Wiles, most mathematicians work within highly specialized areas of study, their research groups often limited to a few dozen colleagues scattered throughout the world. Despite the appearance of intellectual isolation, however, mathematics is the one of the most highly collaborative fields of research. Since mathematical breakthroughs rarely involve patents or intellectual property rights, research is much less heavily guarded than in other disciplines. Far freer collaboration exists in math than within the scientific community, and although there is certainly an element of competition within the field, rivalries rarely produce any real secrecy surrounding techniques or results. Much of the richness of the discipline grows out of the balance between the work of individuals or small research groups, and the free sharing of those results within the close-knit mathematics community.

In fact, the very image of the mathematician sitting alone at a desk turns out to be something



Andrew Wiles unveiling his proof of Fermat's Last Theorem at Cambridge University in 1993.

of a fallacy. One of the most freeing aspects of mathematics, as compared to research in the sciences, is that it requires no equipment, no laboratory, and no specific physical environment. Many mathematicians describe their best work happening not at desks or chalkboards in controlled environments, but while walking, driving, or even looking at art. Even Wiles described the peculiar wanderlust that often accompanies mathematical thought:

> If there was one particular thing buzzing in my mind then I didn't need anything to write with or any desk to work at, so instead I would go for a walk down by the lake. When I walk I find I can concentrate my mind on the one very particular aspect of a problem, focusing on it completely. I'd always have a pencil and paper ready, so if


Paul Erdös, a prolific number theorist famous for his wanderlust.

> I had an idea I could sit down at a bench and start scribbling away.[1]

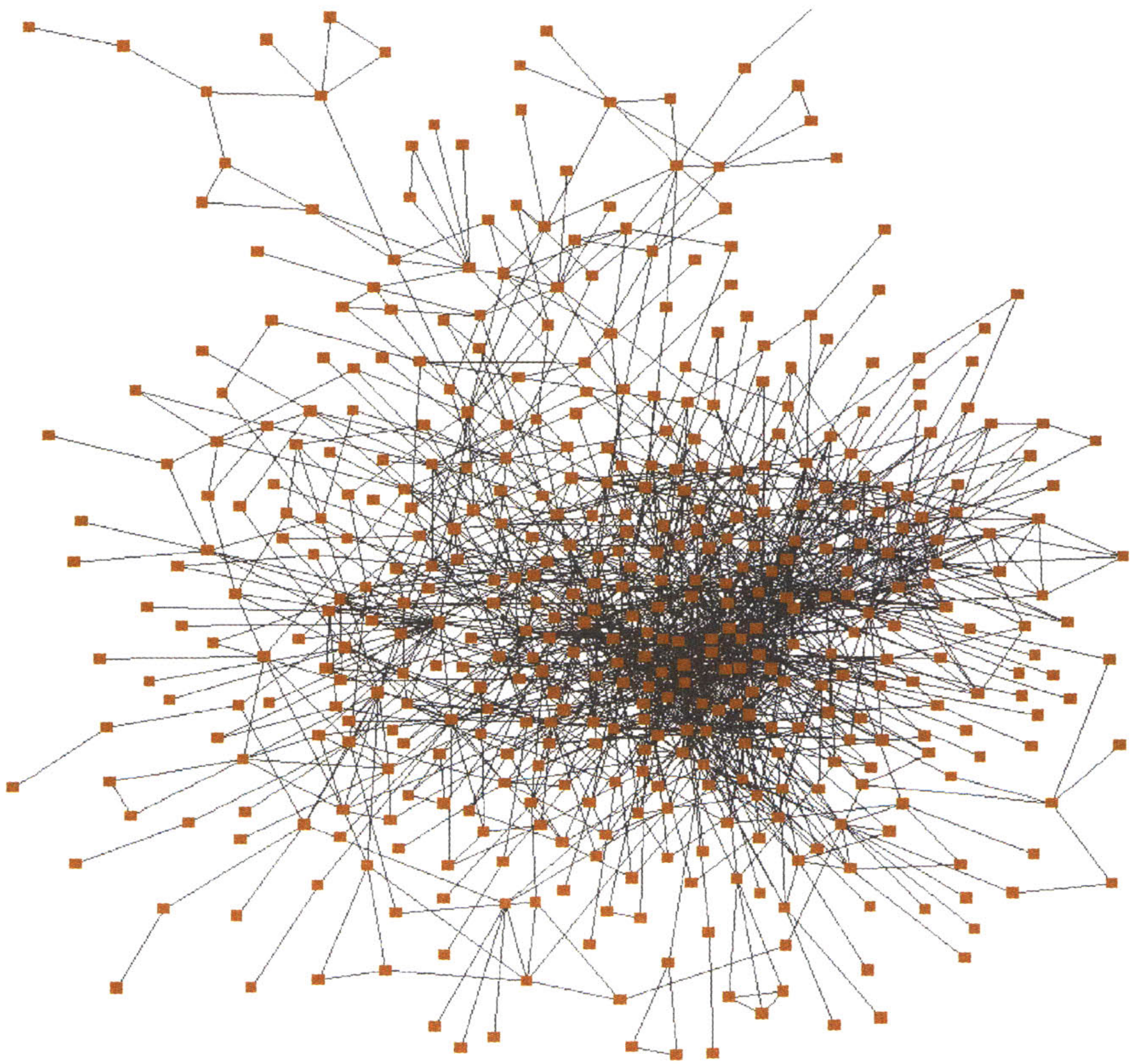
Another 20th century mathematician, Paul Erdös, built his career on his inability to stay in one place, living, as the story goes, out of two suitcases—one for clothes and the other for his math books. Erdös was the most prolific mathematician in modern history, publishing more than 1,500 papers in collaboration with 458 of his colleagues all over the world.[2] Erdös had no permanent address, traveling internationally and staying with colleagues and friends for most of his adult life. His wanderings allowed him to work with the best mathematicians and scientists of the century, creating a dense network of international collaboration.

Erdös died in 1996, but his influence lives on, not only in the papers he published, but in the community he created through his work. Mathematicians talk about their "Erdös Number," which refers to their degree of connection with him. An Erdös Number of one indicates a person who has published a paper directly with Erdös, a number two designates that someone has published with someone who published with Erdös, and so on. Although Erdös represents an extreme case, his life illustrates the fertile paradox of mathematical research—the solitary, placeless, intensely personal experience of inspiration and insight, combining with the fruitful collaboration between mathematicians that defines the community academically and socially.

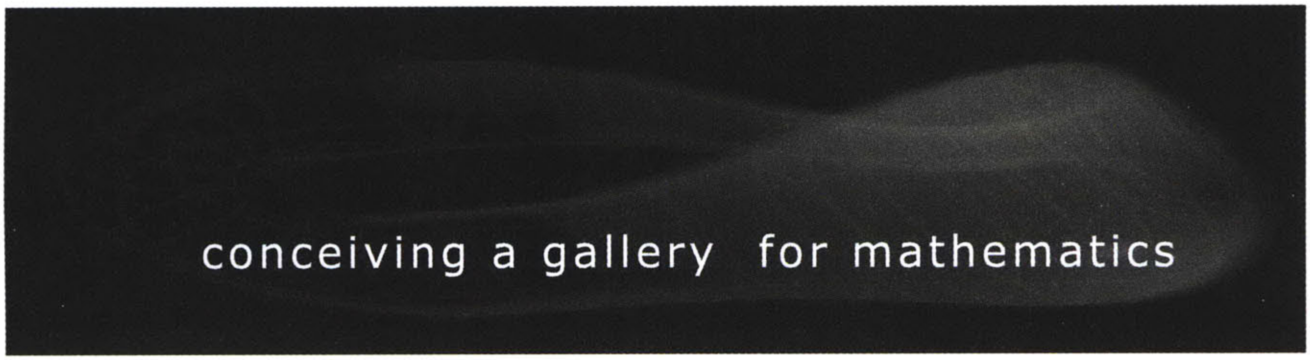
In reality, mathematical culture exists somewhere between the extremes of the cloistered genius and the traveling professor. The goal and high for mathematicians is the eureka moment, the instant when a solution becomes clear and a new piece of mathematical truth is revealed. This moment is a solitary, private revelation, even in the context of collaborative work. Still, these insights are nearly always inspired by and built upon the work of colleagues and the larger math community. What is so tantalizing about mathematical culture are the results of this intellectual symbiosis.

1. Simon Singh, 1997.

2. Paul Hoffman, *The Man Who Loved Only Numbers*, (New York: Hyperion, 1998).



A mapping of "Erdös Numbers," illustrating the network of collaboration built over his long career.


conceiving a gallery for mathematics

Lake Carnegie, the site of Andrew Wiles' many thoughtful walks.

"It seems to me now that mathematics is capable of an artistic excellence as great as that of any music, perhaps greater; not because the pleasure it gives (although very pure) is comparable, either in intensity or in the number of people who feel it, to that of music, but because it gives in absolute perfection that combination, characteristic of great art, of godlike freedom, with the sense of inevitable destiny; because, in fact, it constructs an ideal world where everything is perfect and true."

-Bertrand Russell

As one studies the role of mathematics throughout history, what becomes particularly clear are the ways in which math has always shaped human understanding of the world, both through physics and as a cultural lens. If the essence of mathematical thought can be generalized, it is the drive to question the complex—to define, categorize, and simplify, representing conclusions in the most clear and elegant means possible. While mathematics represents the most purely objective method of abstract analysis, the interpretations of results and ways in which mathematics has been used have varied widely. In this sense, the ways in which man has interpreted and represented mathematical knowledge reveals as much about his society as his art and literature. It is this premise which drives the proposal for a gallery of mathematics. The gallery is conceived of not simply as a museum, which promotes only a passive engagement with its subject, but as a space which encourages the same interrogative view of the world that its subject requires.

What is the nature of a space for mathematical discovery? Mathematics is portable, existing primarily in the space of the mind. Yet over and over mathematicians describe the habit of taking walks as they work through difficult problems, the immersion in the physical environment somehow enabling a deeper internal focus. The image of placeless, spaceless intellectual journeys mapping out a physical path on the environment is incredibly evocative, as if these most abstract and immaterial intellectual processes in some way demand a physical space. The idea that the space of mathematics is the space of the city—a journey simultaneously immersed in and removed from its physicality drives the conception of the gallery for mathematics.

Where many museums seek to remove the visitor entirely from his or her urban environment to be completely immersed in the collection, the mathematics gallery must continually reconnect and reorient its visitors with the city.[1] In addition to establishing this relationship, the path of the mathematical journey must be choreographed to allow for alternating periods contemplation, interaction, and repose. As a space intended to evoke, rather than to instruct, the gallery must pull back at points, giving space for reflection rather than presenting a constant barrage of information. Andrew Wiles stresses this need for periods of respite following extreme concentration:

> When you've reached a real impasse, when there's a real problem that you want to overcome, then the routine kind of mathematical thinking is of no use to you. Leading up to that kind of new idea there has to be a long period of tremendous focus on the problem without any distraction. You have to really think about nothing but that problem—just concentrate on it. Then you stop. Afterwards there seems to be a kind of period of relaxation during which the subconscious appears to take over, and it's during that time that some new insight comes.[2]

With new insight comes an altered perception. Such breakthroughs may not yet represent a grasp of the final solution, but a crucial reorientation which provides a new view of the problem. It is these changing perspectives that characterize the interrogative method of mathematics and carry critical spatial implications for the formal qualities of the gallery.

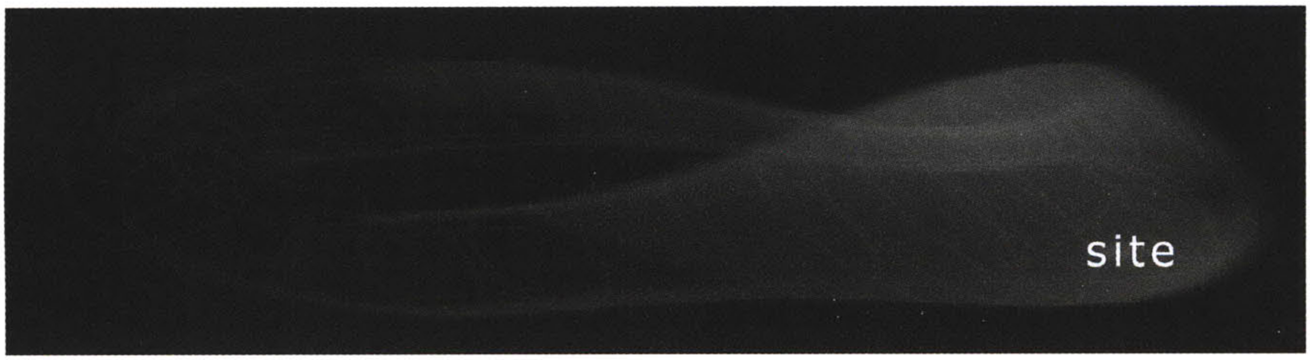
Given the complex relationship between the individual mathematician and the larger math community, the social choreography of the gallery could be as evocative as the physical form. Just as alternating between states of concentration and relaxation contributes to the intellectual experience, so too does the interplay between the individual and social experience of the space. In a gallery setting, characterizing spaces as being intended for individual or group interaction can be ambiguous, if not impossible; however, choreographing activities which might preference varying levels of internal or social experience is quite feasible.

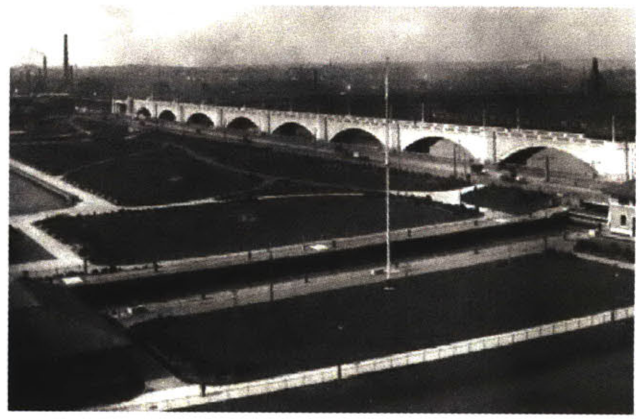
It is important to frame the discussion as the distinction between internal and shared experiences, rather than simply understanding it to mean solitary or group activities. As anyone who has walked through an art gallery with a friend can attest, it is entirely possible to have a very personal experience in the company of another person. As any mathematician can attest, some of the most social events in the discipline occur not physically in a group setting, but in the publishing of a paper. Like the traces left on a blackboard, mathematics texts provide a means of interaction across distance and time as valuable to most researchers as conferences and lectures. The gallery, if it is to truly capture the essence of the discipline, must provide spaces and means for all of these types of social interaction.

Keeping all of these points in mind, the gallery for mathematics requires three distinguishing attributes. First, it demands a path designed to reconnect and reorient the visitor with the physical environment. ***[movement]*** Second, it requires spaces for focused concentration, punctuated with moments of repose. ***[contemplation]*** Third, the gallery must engage its visitors in shared experiences which foster an exchange of ideas as well as the framing of new questions. ***[interaction]***

1. The mathematical journey shares some characteristics with the Situationists' *derive*, which Guy Debord defined in his essay, *Theory of the Dérive*, as "a technique of rapid passage through varied ambiances [which] involve playful-constructive behavior and awareness of psychogeographical effects." The mathematician's journey through the city, however, focuses not on one's own perceptions of the urban environment, but on a particular problem, making it a far less analytically self-aware activity.

2. Singh, 1997.


site



The estuary before construction of the dam in 1910 (left), *and in 1913 upon completion of the dam and rail viaduct* (right).

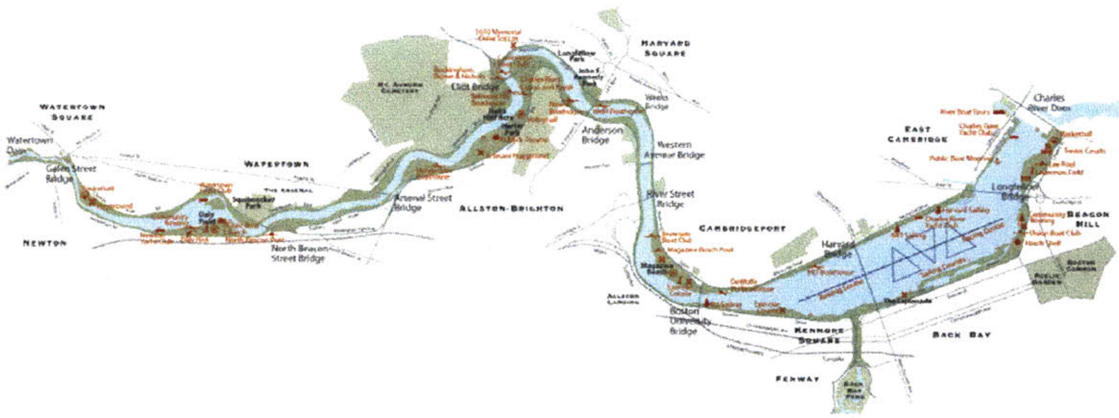
Given the requirement for a direct but controlled urban connection, siting for the gallery for mathematics is crucial. An ideal site would allow for immediate interaction with the city, while still providing opportunities to remove oneself from the distractions encountered in an urban setting. Additionally, the issue of movement along a continuous path suggests the need for direct and safe pedestrian access to the site. Given these criteria, the Museum of Science in Boston emerges as an intriguing location for the gallery. Built on the Charles River Dam, the MoS occupies a completely unique site within the urban fabric with panoramic views of Boston, Cambridge, and the Charles. The parcel was originally part of Olmstead's Emerald Necklace, acting as the point of connection between the Boston and Cambridge park systems. Although there is little buildable space left on the site at ground level, the roof deck of the parking garage offers nearly 40,000 square feet and dramatic views of the river and skylines. The site offers the additional benefit of offering a symbiotic relationship between the MoS and the gallery, creating a stronger cultural node within the city.

Site History

Rapid development of Boston's Back Bay in the 1890's left the Charles River Basin dangerously polluted by the turn of the century. An additional problem for maritime transport on the river was the nine-foot tide, which left the mudflats exposed at low tide. In 1910 Henry Pritchett, then president of MIT, proposed damming the river. Architect Guy Lowell was commissioned to design a park for the site based on a plan by Charles Eliot, a disciple of Frederick Law Olmstead. Lowell developed the plans for the Charles River Reservation, a seventeen-mile linear park stretching from Watertown to the Craigie Bridge and connecting with the Esplanade on the Boston side. Eliot described the dam site as the "Central Court of Honor"[1] in Boston and the centerpiece of the park system. The dam was completed in 1913, along with the rail viaduct, which screened the park from the industrial sites to the east.

The Museum of Science was originally founded in 1830 as the Boston Society of Natural History, and found its first permanent home in Back Bay in 1864. By the end of World War II, the Boston Museum of Natural History, as

2. The Metropolitan District Commission in *Charles River Basin: Characterand History*, [online], availableathttp://www.mass.gov/mdc/CRBasinContents.htm, [cited January 13, 2005].



The current Charles River Park system, disconnected at the point of the Museum of Science.

it was then known, had outgrown its facilities. In 1951 the Museum negotiated a deal with the cities of Boston and Cambridge to build Science Park on the Charles River Dam. Originally only a modest presence on the park space, the Museum undertook its first expansion project in 1958 with the construction of the planetarium, followed by completion of the West Wing in the early 1970's. Construction of the parking garage in 1972 cut off access between the north and south banks of the Charles and completely obscured the original locks of the dam. Two more major expansions in the 1980's produced several popular attractions, but left little of the original park visible on the site.[1] Today the only remaining trace of Eliot's "Central Court of Honor" is the waterfront pavilion now used for Duck Boat launches.

Current Issues

The Museum of Science has become one of the city's most popular attractions with over 1.6 million visitors annually, and now enjoys increased visibility to tourists as the location for the Boston's famous Duck Tours. Unfortunately, the rapid growth of the Museum on the site has disrupted not only connectivity of the Boston and Cambridge park systems, but also pedestrian flow across the site and access to the water. The waterfront parks on both sides of the river are used year-round by pedestrians, joggers, and bicyclists and become particularly crowded in the summer months. At present, the only point of connection across the river is through the MoS parking garage and across a six-foot-wide bridge, a path traversed by more than 300 people every hour at peak season.[2] These accessibility issues impede not just the flow of local pedestrian traffic, but also affect visitors to the MoS, who experience little connection with the waterfront or green spaces surrounding the museum. The confusing network of paths, buildings, bridges, and vehicle lanes on the site make casual crossing of the river nearly impossible at a point where a connection would be particularly advantageous.

Before construction of the garage in 1972, limited parking facilities meant that most visitors reached the museum via the Green Line train from the Science Park stop 200 yards from the museum's main entrance. The five-level, 40,000 square

1. *History of the Museum* [online]. Boston: Museum of Science. [cited January 10, 2005] Available from http://www.mos.org/doc/1082.

2. Metropolitan District Commission, 2005. [cited January 13, 2005].

Views from the parking garage.

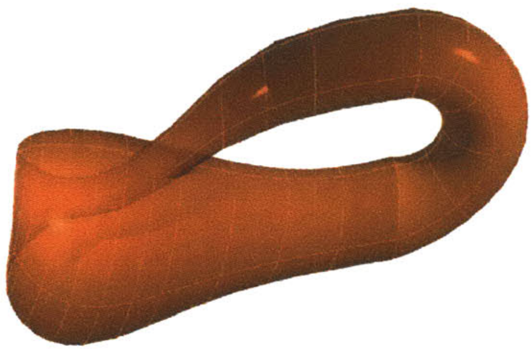
foot garage facilitated far easier access to the museum by car and increased Museum revenue dramatically, but it also created major circulation problems of its own. The primary point of entrance to the museum shifted from the front door to the garage entrance on the first floor, while traffic congestion between the garage entrance and the drop-off lane made pedestrian navigation from the T stop dangerous.

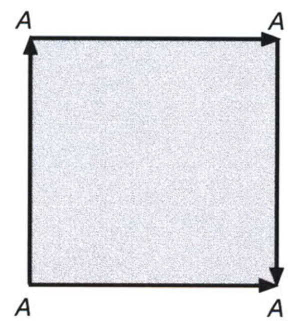
The top level of the garage offers a surprising and spectacular view of the city. Unobstructed views of the Boston skyline, punctuated by the Hancock and Prudential towers, activity on the Esplanade, the Charles Gate Yacht Club, MIT, and the Longfellow and Zakim Bridges offer a unique perspective on the city and its landmarks. Since the only access to the Museum of Science is from the first floor of the garage and the top level is open to the elements, this deck is largely underused, particularly in the winter when snow removal is impossible. This roofdeck therefore has potential as site for building and also as an opportunity to restore pedestrian access to the waterfront and park system.

A lack of accessibility and poor planning have resulted in an underutilized and neglected waterfront.

Site Strategy

The siting of the gallery for mathematics on top the parking garage creates an ideal relationship between the gallery and the urban environment. The site suggests the opportunity for a multi-level public garden on the garage which would tie into and link the Cambridge and Boston park systems along the Charles. This roof garden will provide not only a much-needed pedestrian connection between the riverbanks, but also situate the node of the mathematical journey within an existing and well-established pedestrian path in the city. The panoramic views from the garage position the gallery within the urban environment, while simultaneously establishing a tension created by the immateriality of the site—on top of a garage on a dam in the middle of the river. By choreographing movement and view,





A three-dimensional model of the Klein bottle, along with the topological diagram characterizing its connectivity.

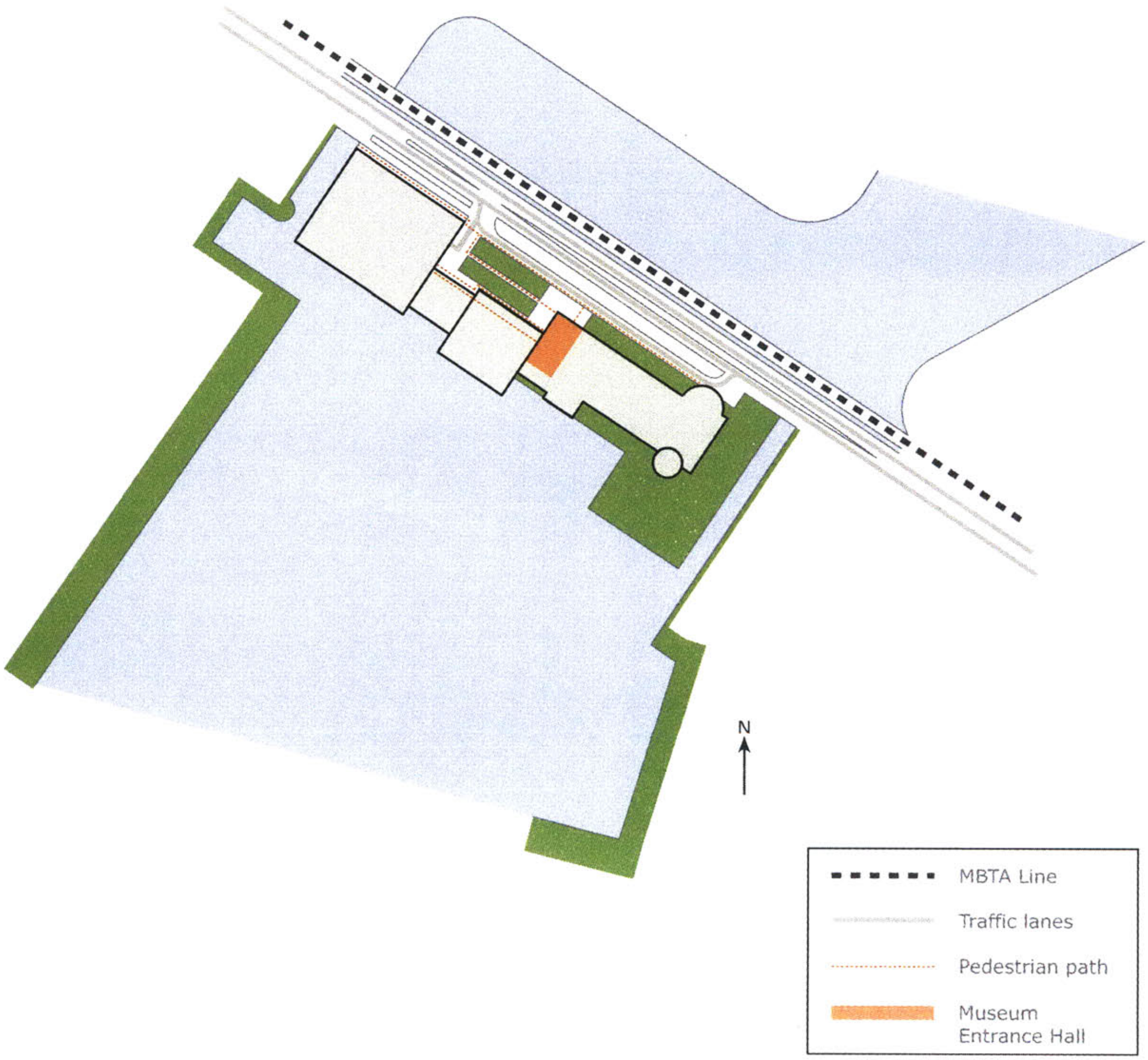
the gallery in this setting can operate to constantly reorient the visitor against the city, creating new relationships with familiar landmarks.

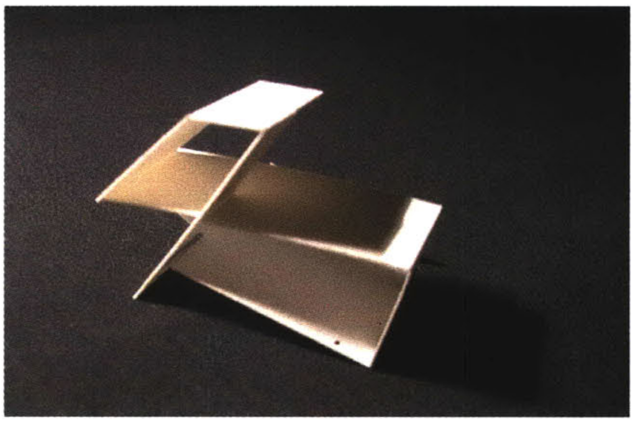
The strategy for circulation becomes crucial at this point, as the movement over the site will determine both the ease of accessibility and the visitors' experience of the city. To take a mathematically inspired approach, topology, the study of curves and surfaces, including knots and manifolds, offers an intriguing point of departure. Topology is concerned with "the properties that are preserved through deformations, twistings, and stretchings of objects,"[1] and therefore characterizes the connectivity of objects without regard to their detailed forms. The sphere, the torus, the tube, the möbius strip, and the Klein bottle are examples of distinct topological spaces. All of these spaces, with the exception of the Klein bottle, can exist quite happily in three-dimensional space. The Klein bottle, however, is actually a four-dimensional form, described by its discoverer as what one gets by sewing two möbius strips together. Analogous the möbius strip, which is a three-dimensional shape with only one side, the Klein bottle is four-dimensional object with one side. The space twists on itself, creating a condition in which the inside of the bottle becomes the outside in a single continuous surface.[2]

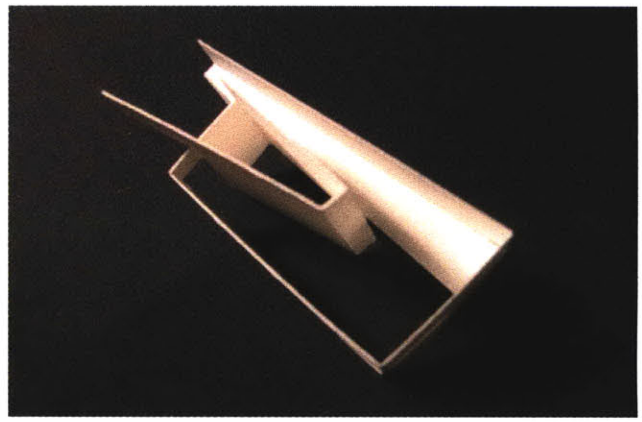
The Klein bottle represents an interesting topological model for the gallery in that a section taken along its length produces a surface that forms a continuous loop with interior and exterior continuity. The result is a continuous volume which folds the exterior into its interior and twits on itself, producing spaces which flow into one another with constantly shifting orientations. An architectural translation of the Klein bottle must find ways of adapting a four-dimensional object into a three-dimensional form while mitigating the transition from interior to exterior. These conditions offer unique opportunities to explore what happens to space, circulation, light, and view at these particular moments in the building.

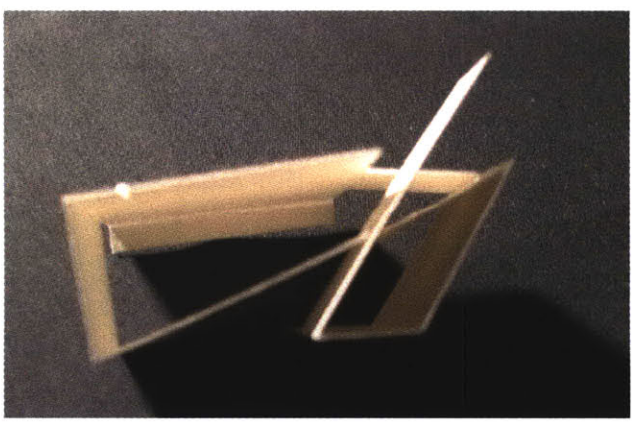
1. Weisstein, 2005.

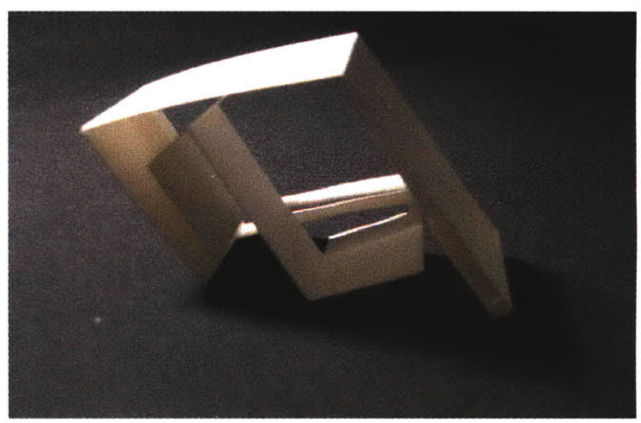
2. Diagrams like the one above are used to characterize this connectivity, with the letters indicating the same point in space, and arrows showing parallel lines which join in the orientation specified. In the case of the Klein bottle, the diagram illustrates a connectivity equivalent to gluing the opposite edges of a rectangle together and giving one of those edge-pairs a half-twist.


N
MBTA Line
Traffic lanes
Pedestrian path
Museum
Entrance Hall

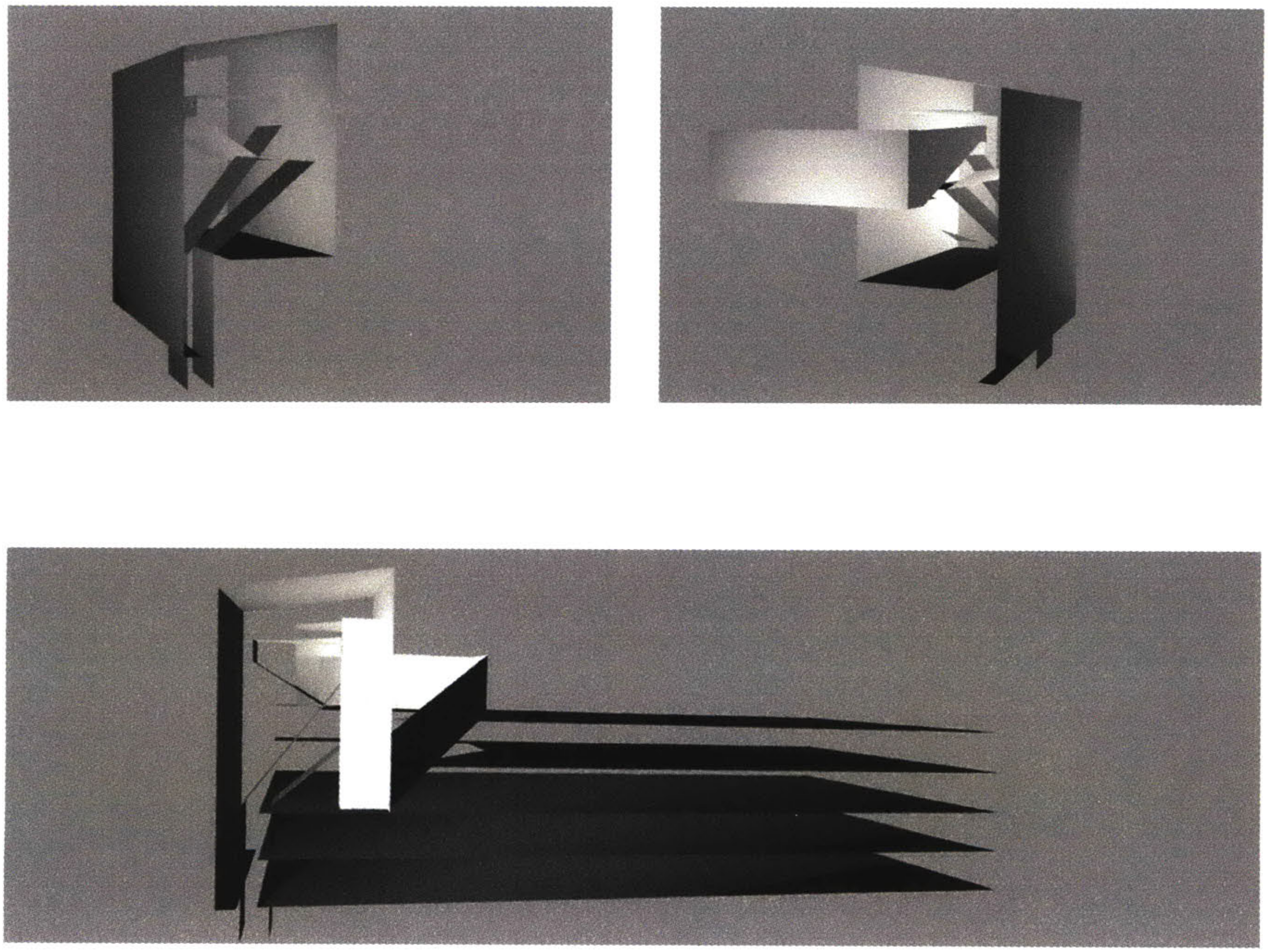




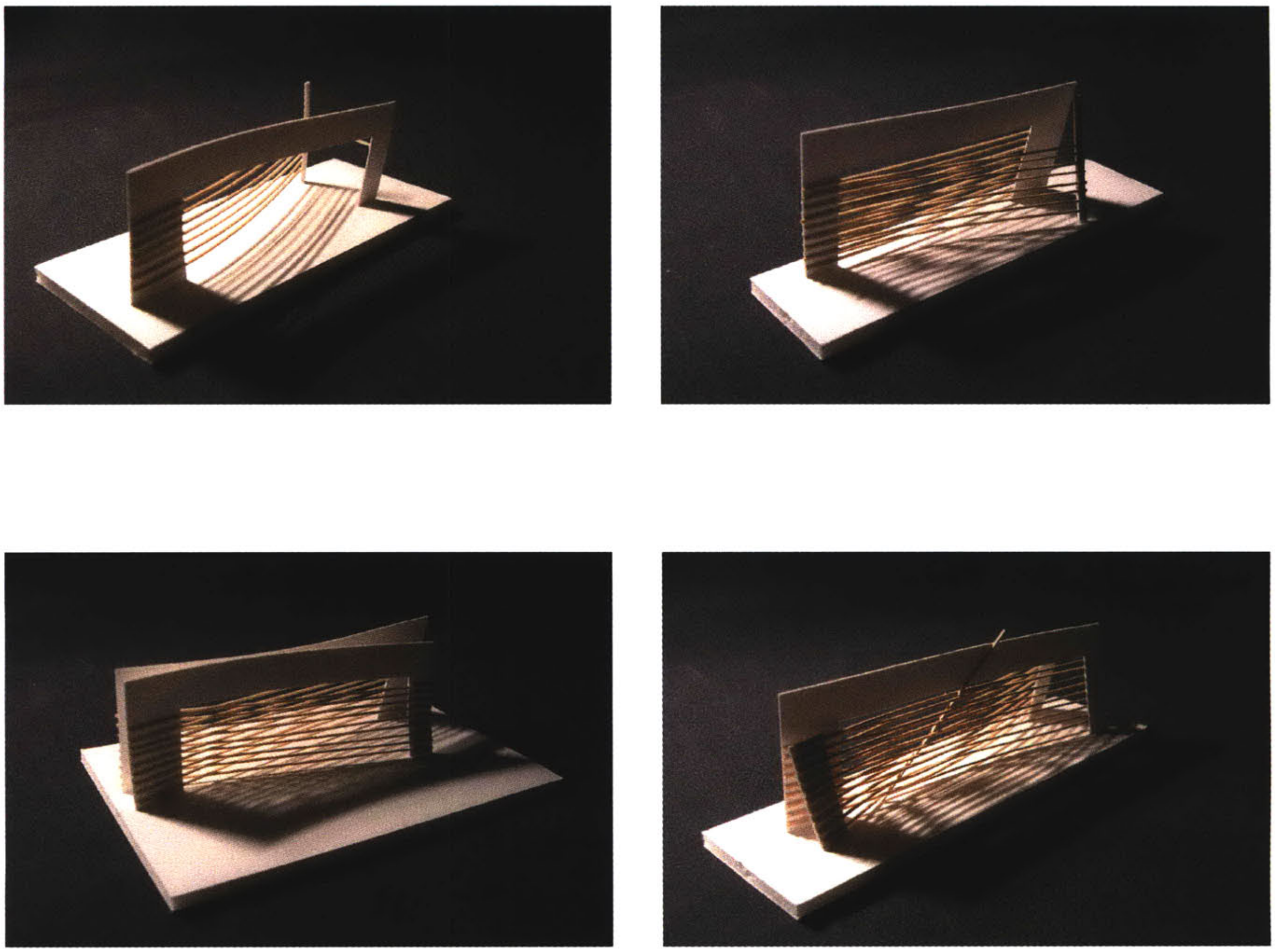




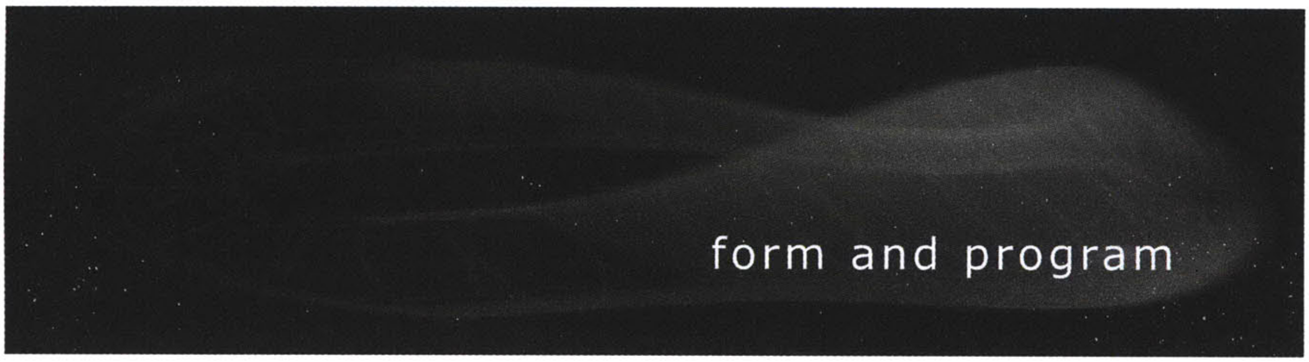
Sketch models exploring possible Klein bottle diagrams.



Computer models of possible Klein sections and interaction with the existing structure.



Study models of moiré patterns in screening and shadows.


form and program

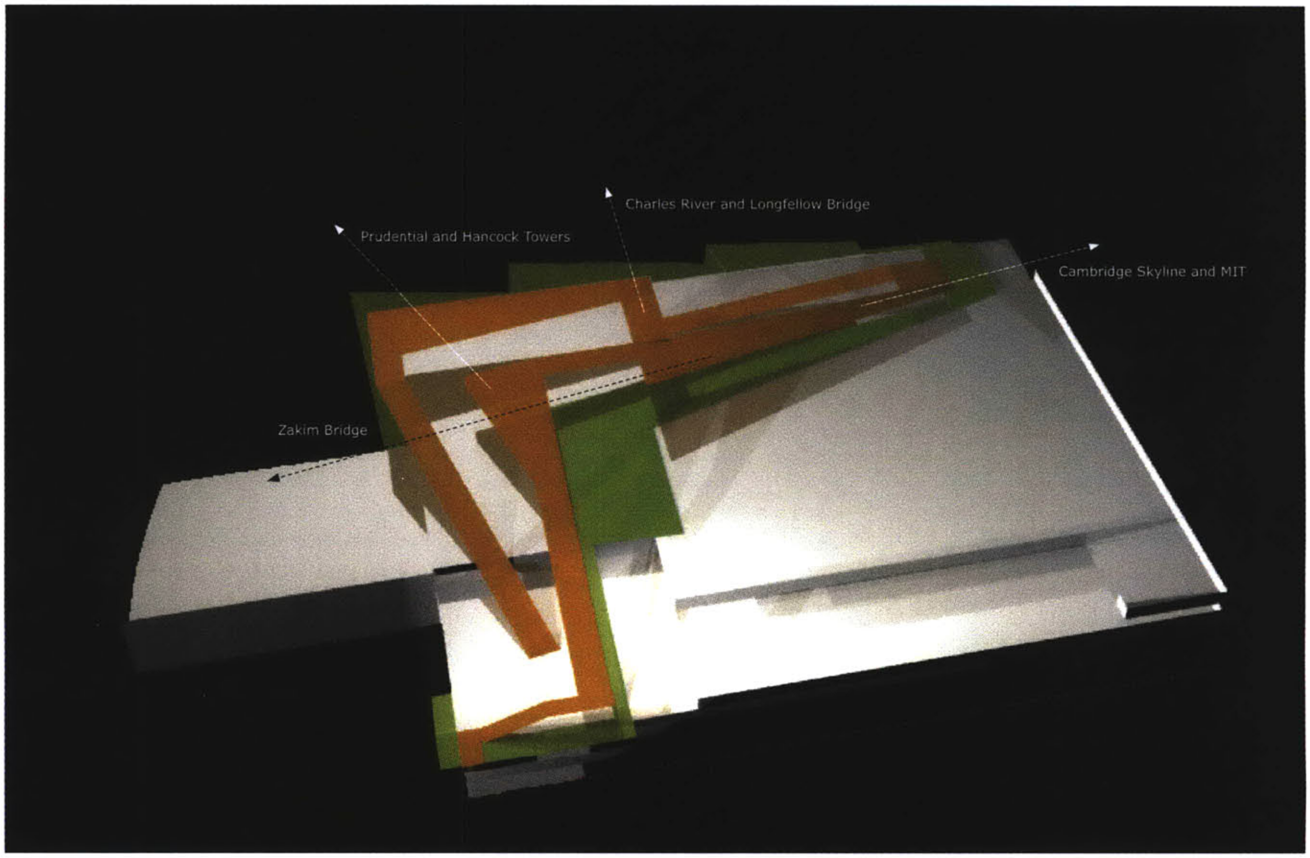
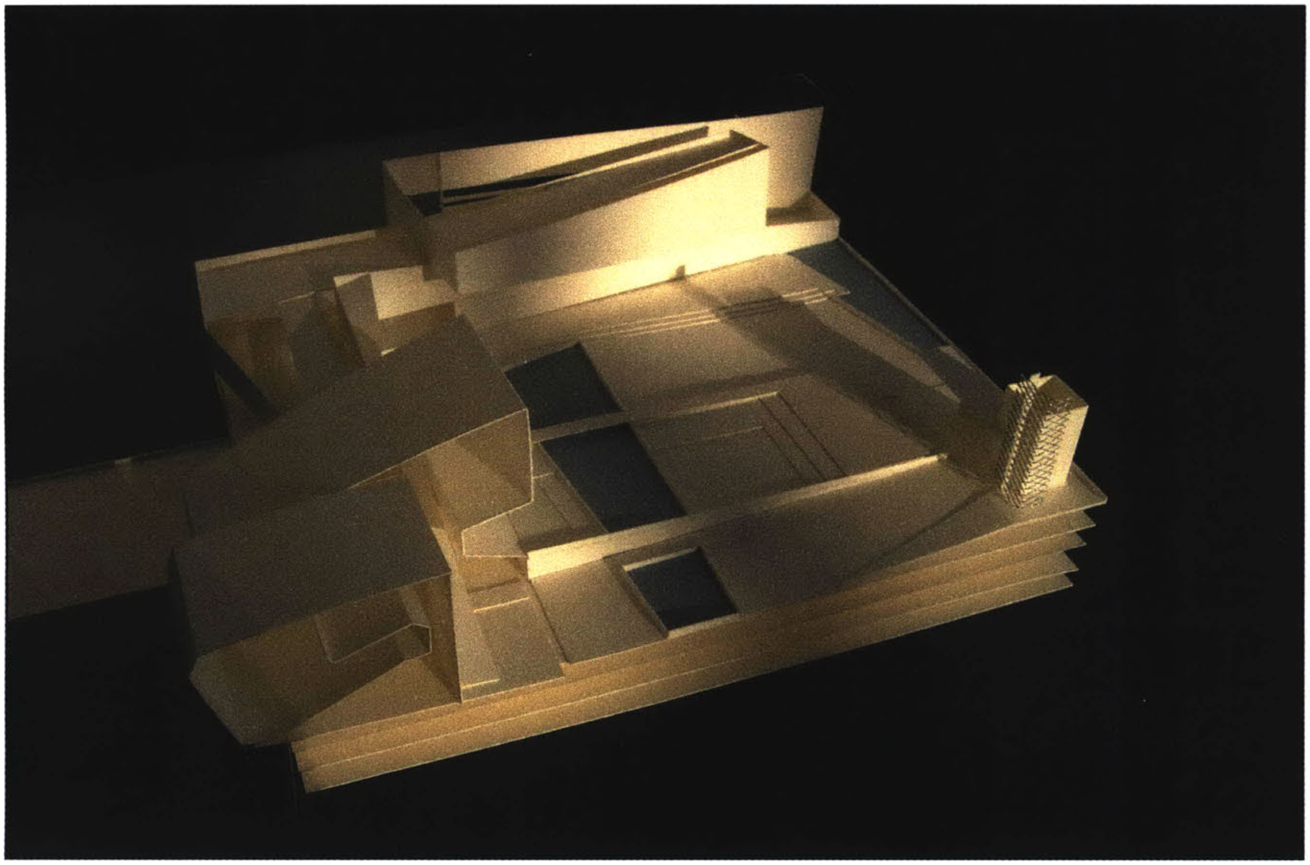


Diagram showing the public garden path (green), the gallery circulation (orange), along with views.

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty - a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."

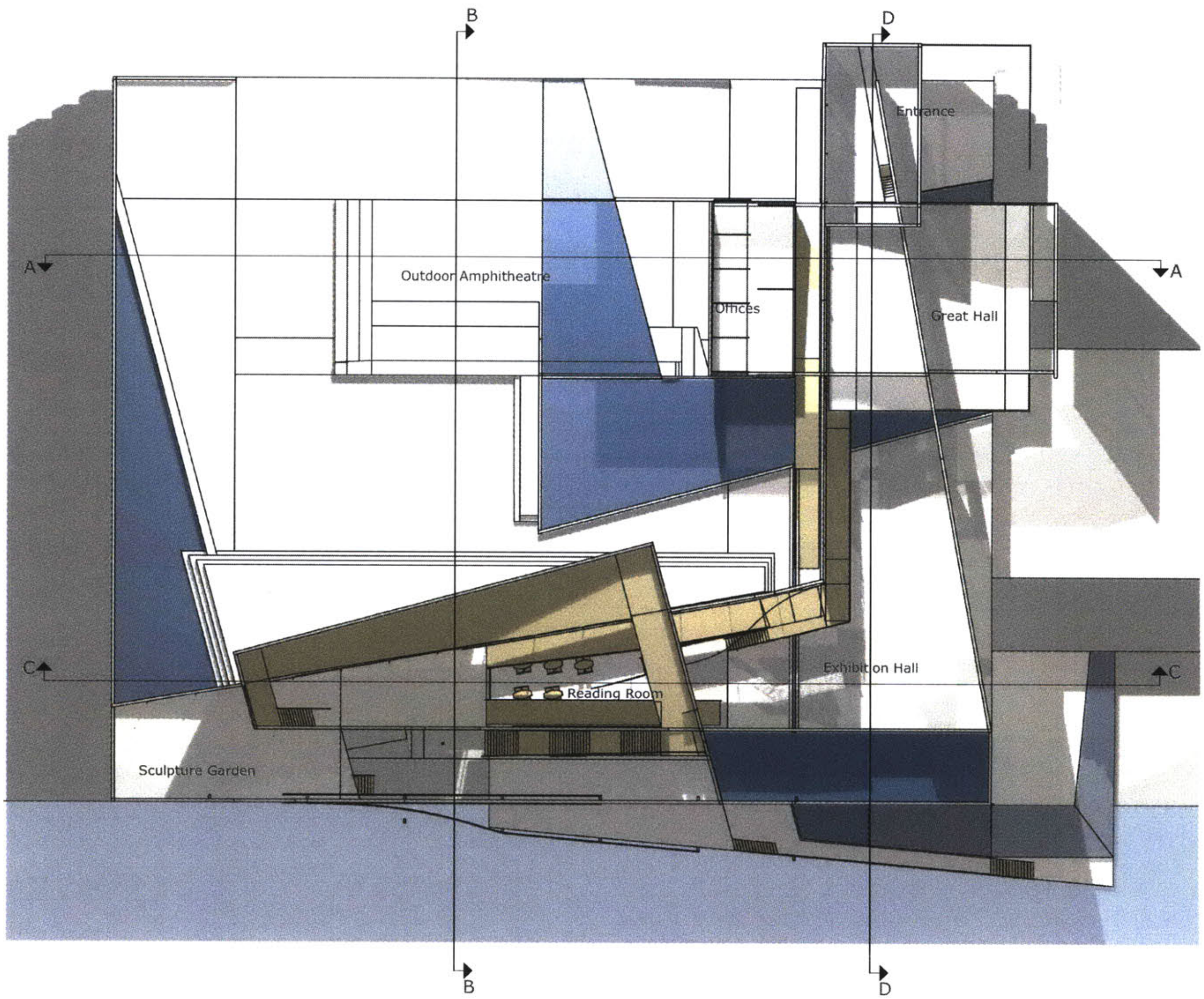
-Bertrand Russell

The proposed program for the project includes a large gallery in the form of a continuous serpentine path which will house the permanent collection, which is envisioned to be image-based. Circulation through this gallery space represents ***[movement]***. The Gallery for Mathematics will also include a theatre for lectures and films, as well as a large exhibition hall, intended to accommodate models and participatory exhibits. ***[interaction]*** The final component of the project is a reading room, where chalkboards, workspaces, wireless internet,

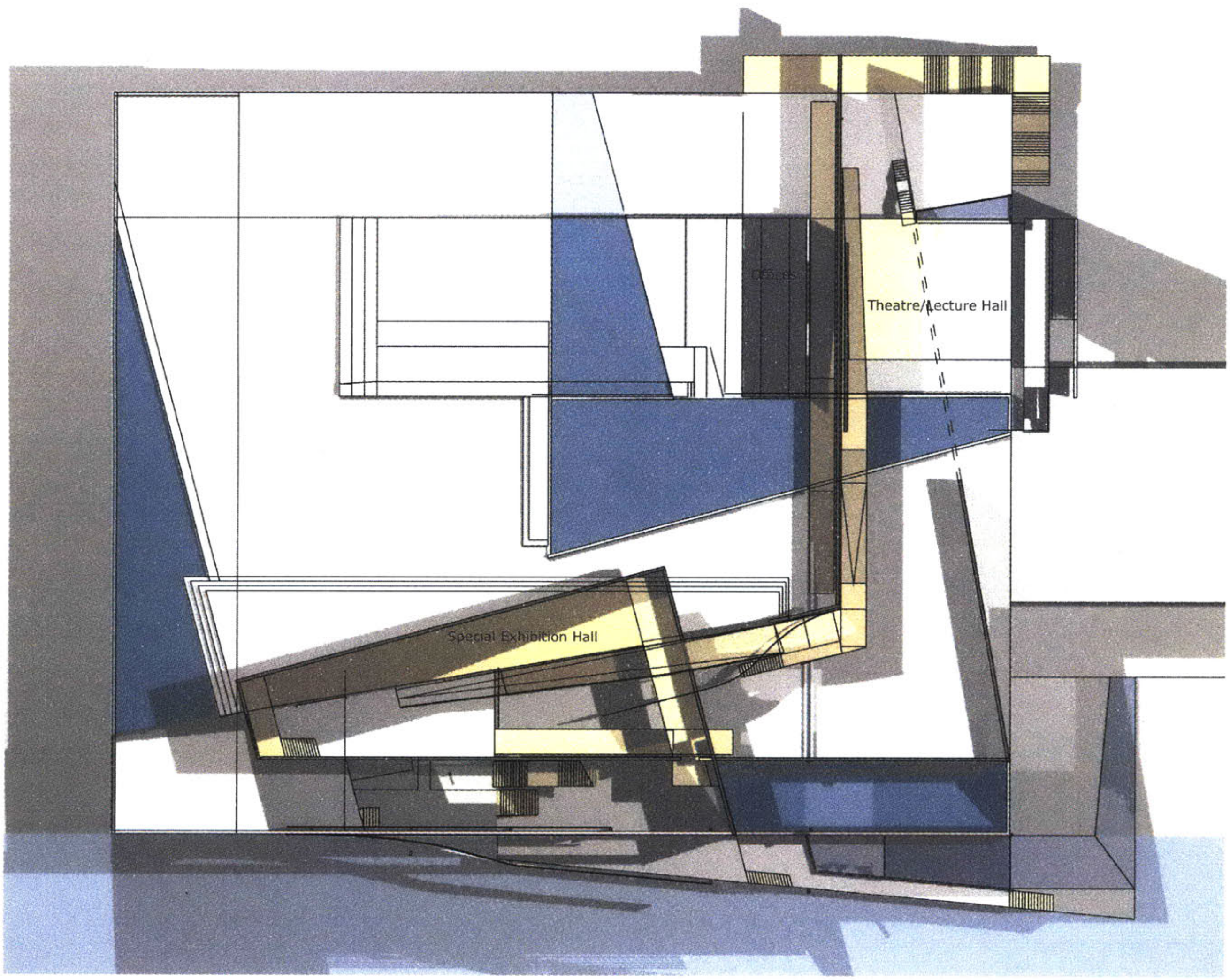


Model at 1/32"=1', showing massing and public space.

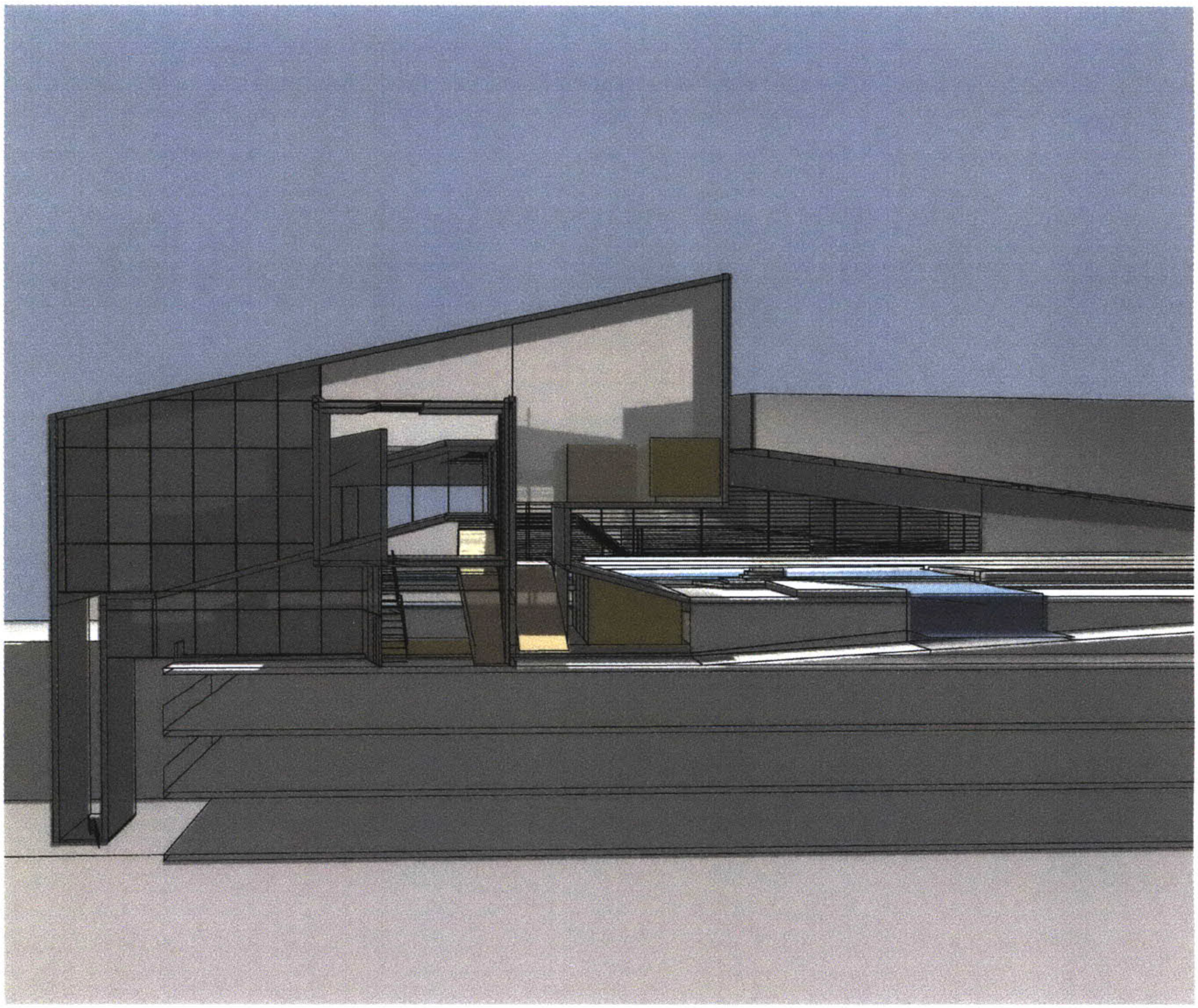
and a small library are available to visitors. The space is conceived simply as a place of respite and stillness within—but separated from—the movement of the exhibit space. ***[solitude]***



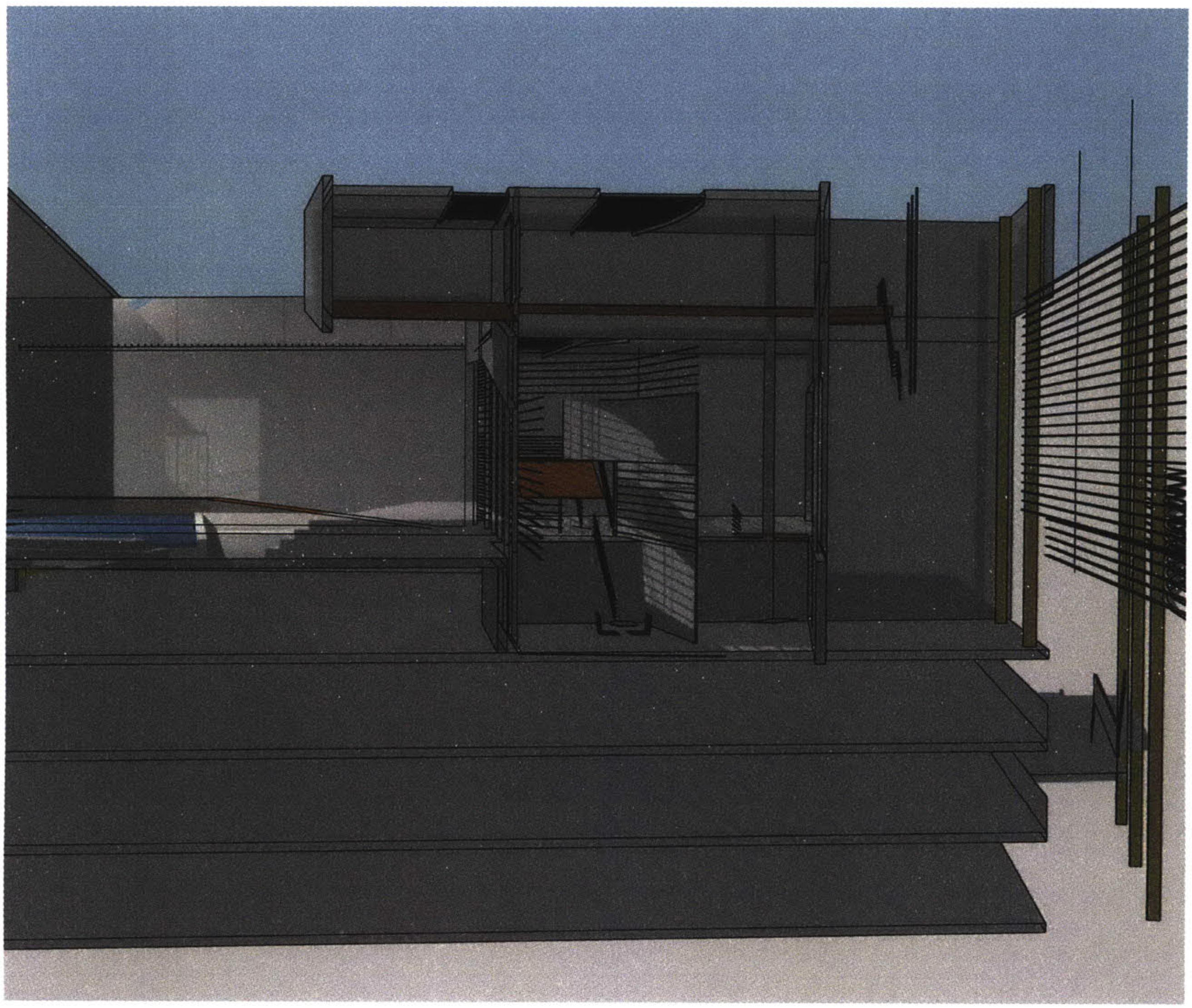
First Floor Plan



Second Floor Plan



Section AA



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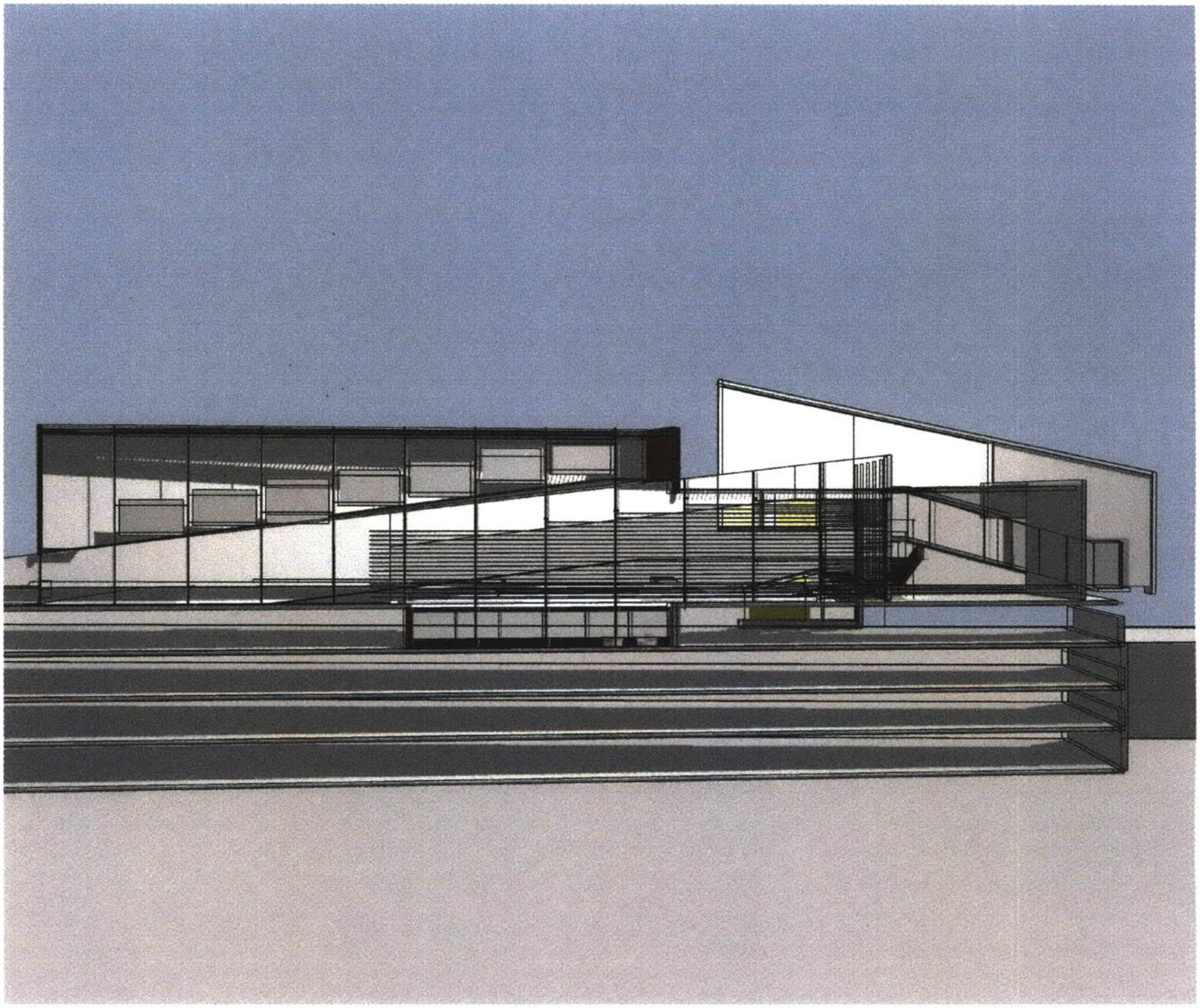

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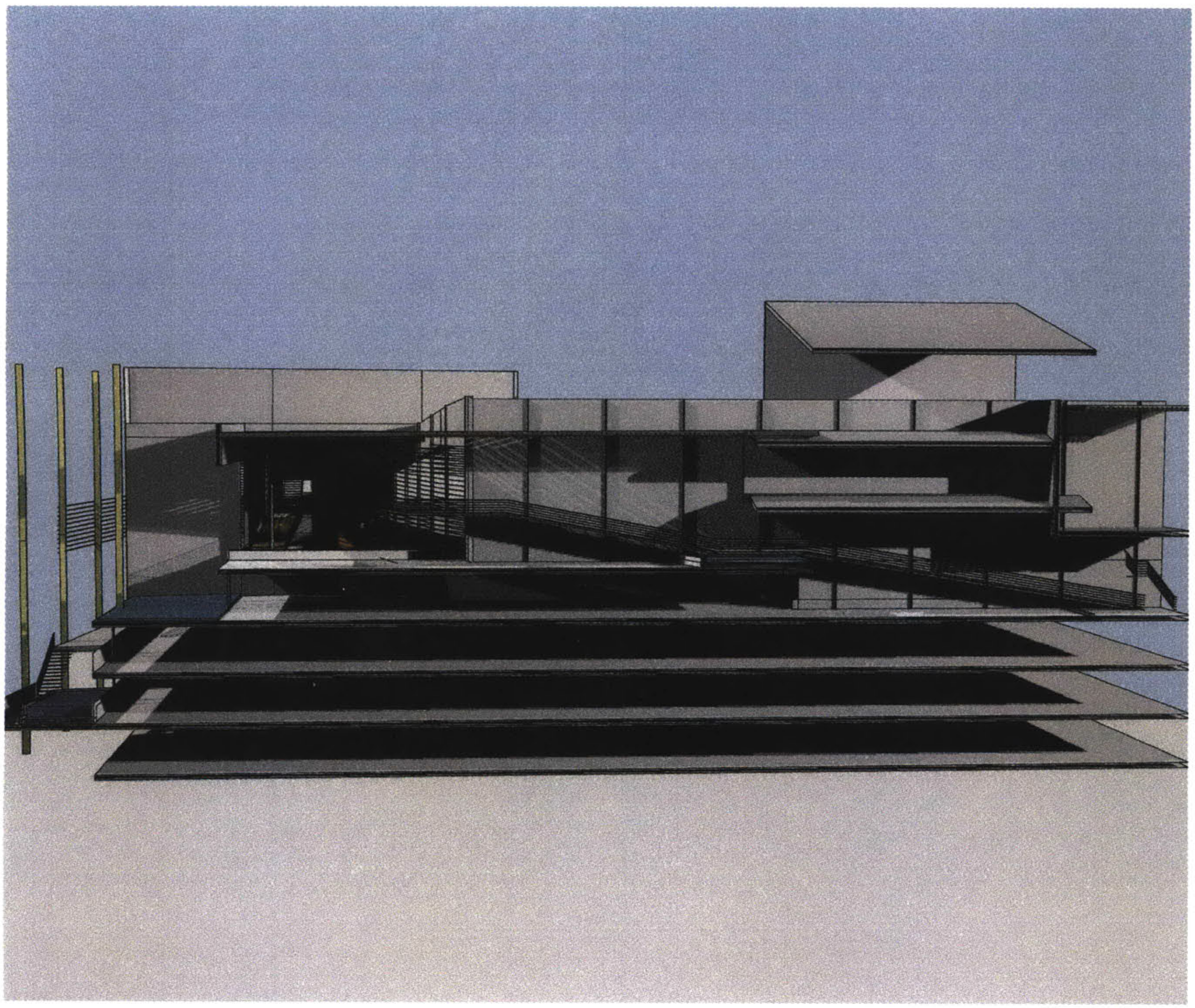
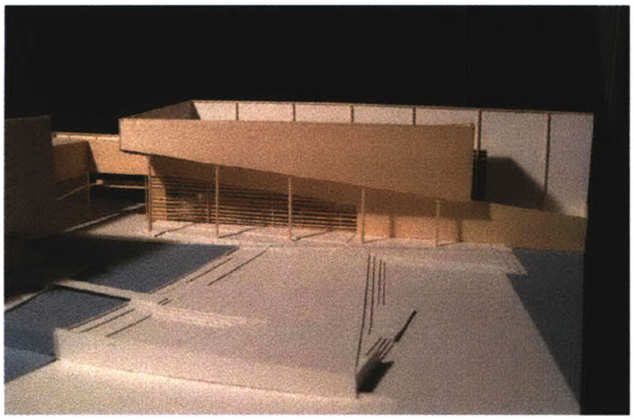
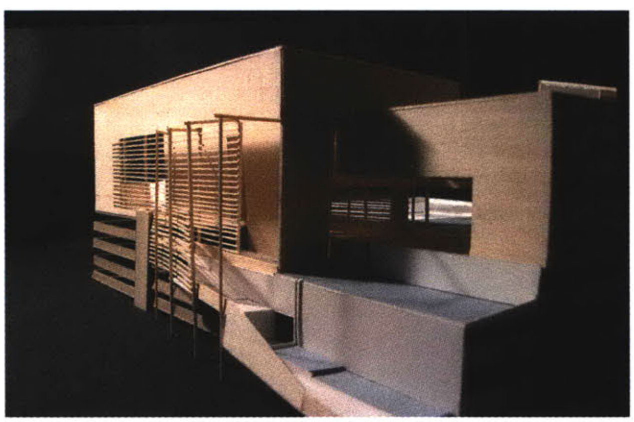
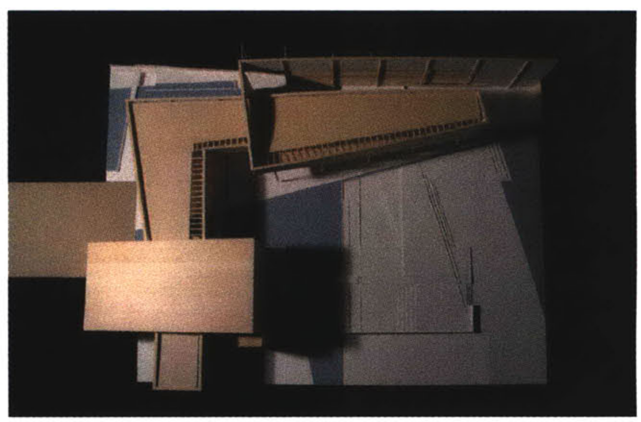


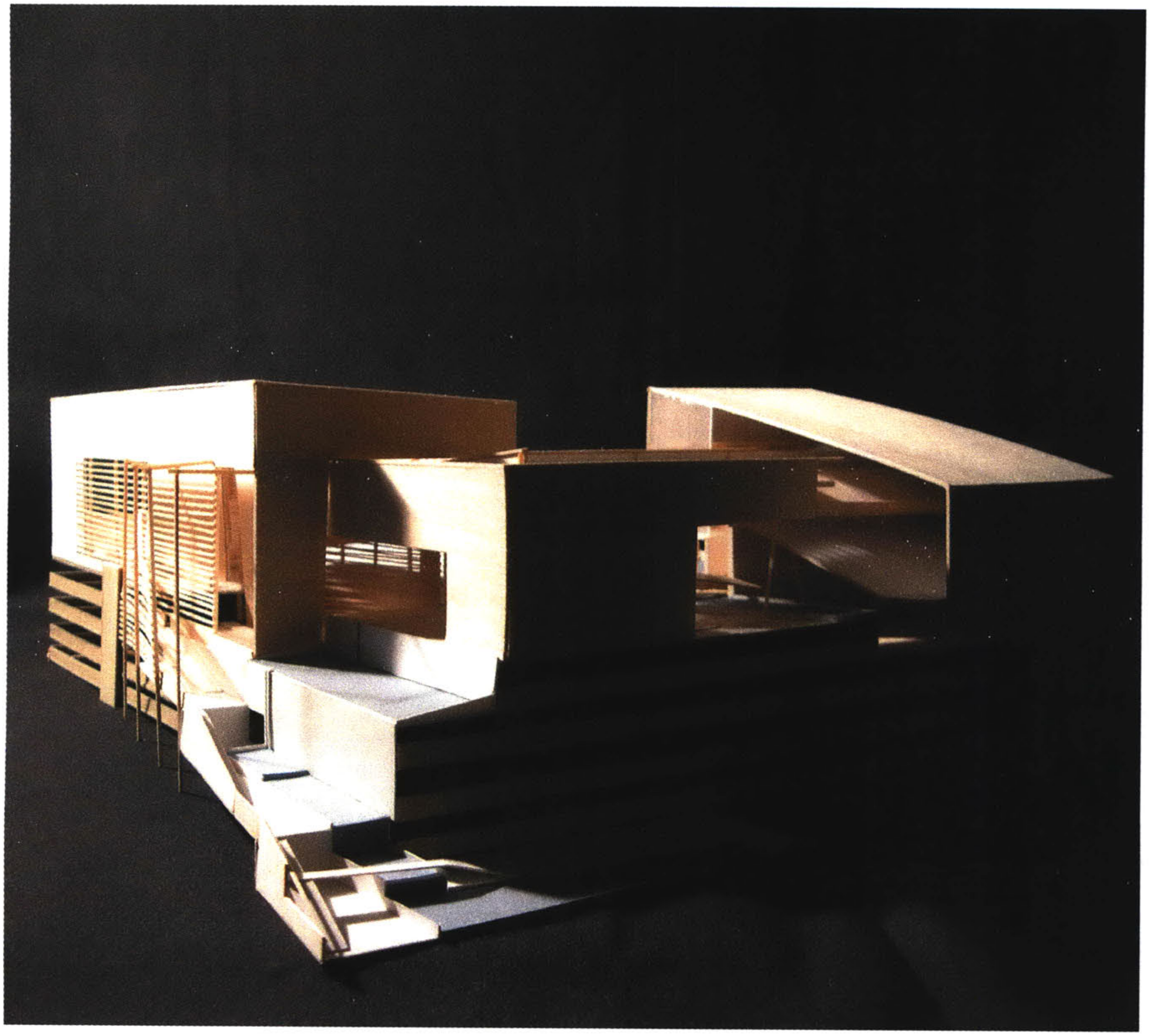
Section DD



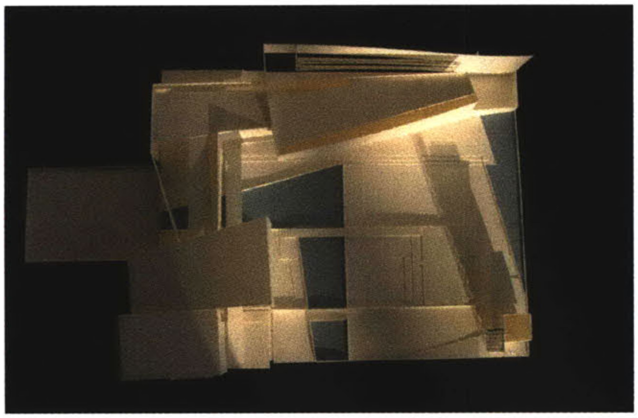


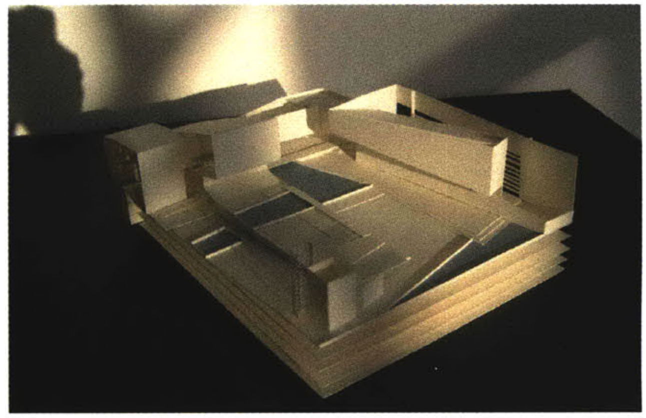


Views and details of the final model.



View of the final model.



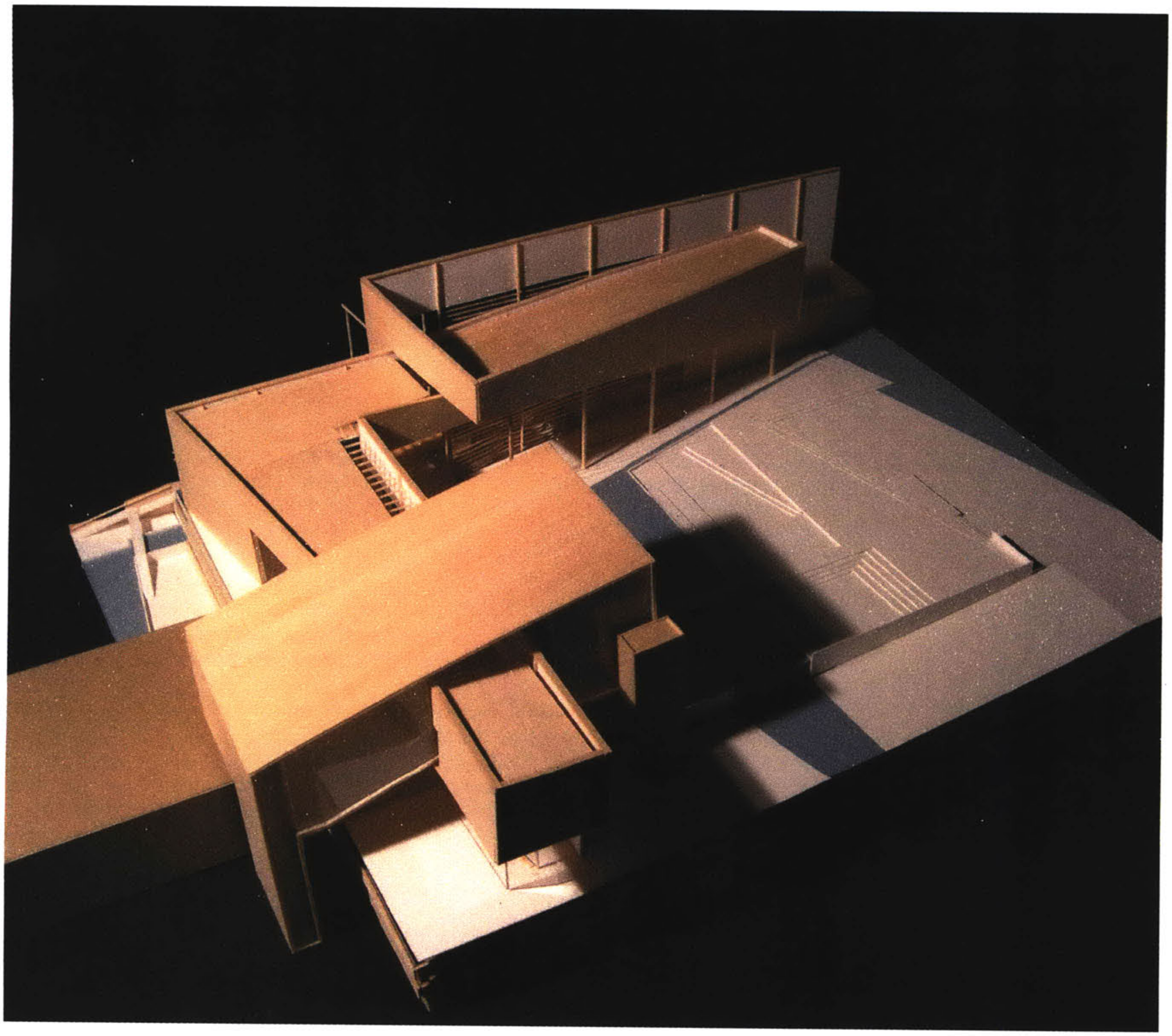


Models showing site plan and massing strategy.

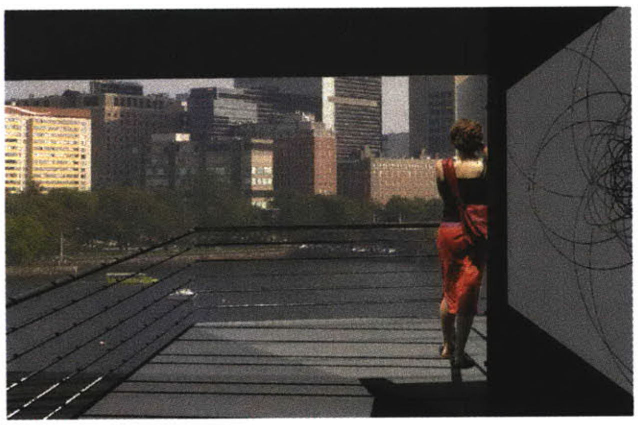
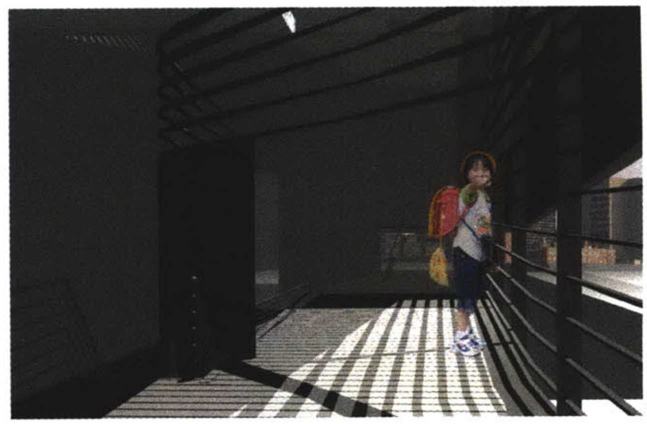
Present patterns of pedestrian, rail and automobile movement make the northeast corner of the garage the most prominent and active area of the site. Visible from Monsignor O'Brien Highway, Green Line trains, and established pedestrian paths around the Museum of Science, this corner becomes a natural point of entrance for accessing the gallery on the roof. This location separates the entrance sequence for the Gallery from that of the Museum of Science, while still allowing easy access between the two. Placing the entrance here also means that those who wish simply to cross from Cambridge to Boston through the garden can do so without having to cross any of the busy traffic patterns around the MoS. For this reason, the public garden occupies most of the northwestern side of the garage, eventually wrapping the southwest corner and descending to the waterfront along the southern façade of the parking structure.

Building massing is concentrated along the southern and eastern edges of the garage, overlapping the facades of the existing structure at points. While the serpentine form of the gallery wraps laterally on itself, the form of the entrance and theatre folds over and around the rest of the structure. The gallery form and path are derived from that of a Klein bottle section, turning and folding on itself in a continuous looping form. The folds in the circulation spine of the gallery correspond to a view of a particular landmark, a change in lighting condition, and indicate a new mathematical perspective in the next section of the gallery. These folds act as pauses in the collection, meant to foster a moment of reflection and reorientation before entering the next section of the gallery. Taken together, the garden and gallery circulations intertwine, wrapping and enclosing the most internally-oriented space, the reading room.

The strategy for brining natural light into the gallery is three-pronged. Where particular views are desired, large windows frame the landmark on the skyline and fill the space with light. In the gallery spaces, skylights wash the display walls with light from above and reinforce the circulation spine through the space. At points where light needs to penetrate deeper into the space, large windows with screens bring in softened daylight while controlling the view outside. The play of



Model showing gallery and garden spaces.

Renderings showing the views from the gallery entry and light studies with screens.

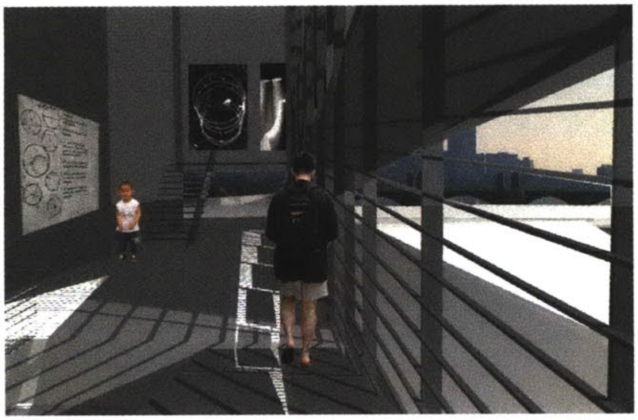
light through overlapping screens creates shifting moiréd shadow patterns, reinforcing the feeling of movement in the space.

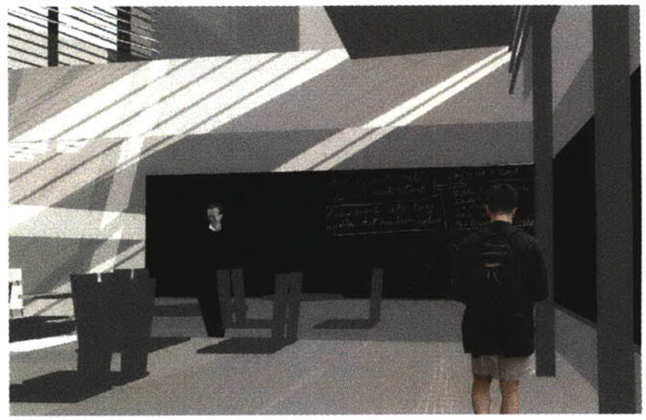
Experiencing the Gallery

Visitors reach the gallery from street level via either the exterior staircase which wraps the corner of the parking structure, or the elevator adjacent to the stairs. An entrance portico leads visitors to the gallery inside, while a path past the building leads to the public garden beyond. The southwest wing of the Gallery houses the Great Hall, designed for large social events or special exhibits; the theatre for film and lecture series; seminar rooms; and offices. From the entrance hall, one can directly access the theatre and seminar rooms above, or take the ramp to the main gallery. The ramp gently ascends through the Great Hall and overlooks the Exhibition Hall, finally opening to a view of the Boston skyline punctuated by the Hancock and Prudential Towers. The path then turns and widens as the wall screen peels apart, creating a pocket for stairs descending to the Exhibition Hall and ultimately, to the reading room. Continuing along the main circulation spine of the gallery, the ramp descends to the main level, from whence one can move to the Exhibition Hall or the Special Exhibition Gallery.

The path to the Special Exhibitions takes the visitor into the "handle" of the Klein bottle, along the exterior wall of the Gallery on the river side. The glass-enclosed staircase overlooks the outdoor sculpture garden and provides a spectacular view of the river and city at the moment where the Klein bottle diagram passes through itself. This is the only point in the circulation loop in which stairs, rather than a ramp, are employed, and the only point at which one actually moves through, rather than along the folding walls of the Gallery. An elevator from the main level provides handicapped accessibility, and a small lobby provides a space for special event ticketing. Designed for flexible use, the Special Exhibition Hall has room for sculptures and large displays as well as images. The hall is at the highest point in the Gallery and overlooks the rest of the spaces, while remaining vertically separated from them. Chalkboard display panels that screen the space from the main gallery can be used for images or as-is.

Light and view study of the gallery's main level.



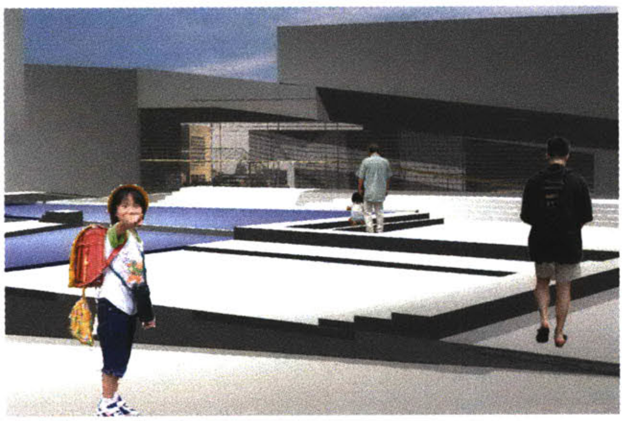


View and light studies descending into the main level of the gallery (left), *and the reading room* (right).

Circulation through the Special Exhibitions brings visitors back to the main level of the Gallery, which is given over to interactive exhibits. Occupying most of the main level, it is envisioned that large displays like the Eames' *Mathematica* or *Powers of Ten* would be housed here. Although circulation through this space still follows the folds of the Klein bottle diagram, movement in and around the exhibits is much freer, encouraging interaction and engagement with the displays and other visitors. This space overlooks the river and gardens below, anchoring visitors back to their first view upon entering the gallery. A ramp from the Exhibition Hall leads visitors back through the Great Hall to the entry, where they can return to street level or enjoy the public garden.

The reading room is the only space in the gallery which is external to the loop of the Klein bottle, and instead is nestled inside the folds of the rest of the gallery. The space represents the inner sanctum of the gallery and as such, is tangential to the circulation at several points without ever directly engaging it. The space is carved into the garage structure, and represents the deepest and most interior space in the gallery. The reading room is equipped with chalkboards and tables, intended not only for mathematics but simply as creative workspaces. A small library of books and math journals would also be available, as well as wireless internet. A three-story height, this is the only vertically-oriented space and the only place which has no direct views to the exterior. Light enters the space from a skylight far above and through the screens on the upper gallery level, creating a dramatic play of light and shadow in the space. In addition to the entrance from the gallery, the reading room also opens onto one of the terraces of the sculpture garden overlooking the river. In this way, the reading room acts as the point of connection between the exterior realm of the gardens and the spaces of the gallery.

Rendering of the reading room.



Renderings of exterior views.

The Public Garden

"To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in."

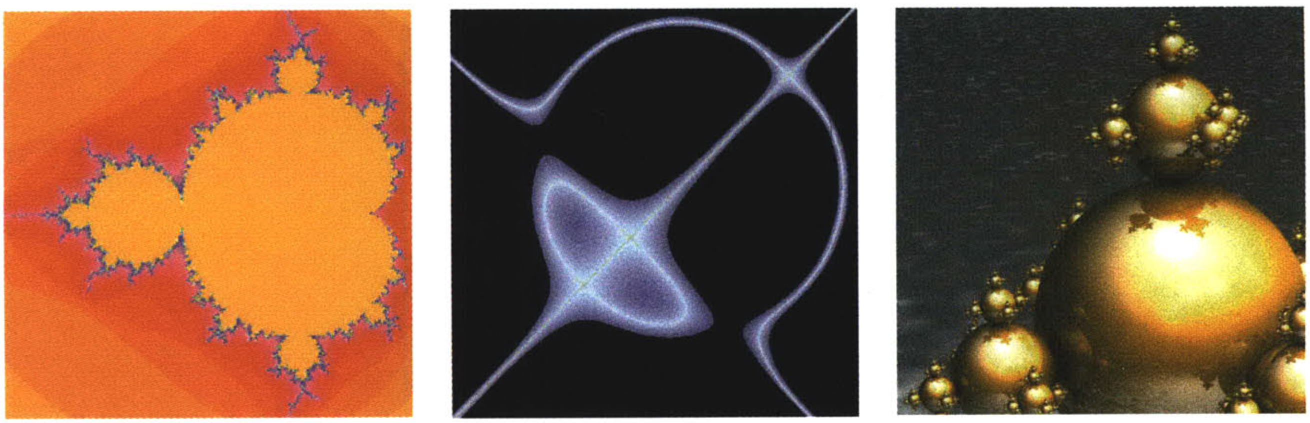
-Richard Feynman

From the stairs at the main entrance, visitors can either enter the gallery or continue past the building to the garden. Here the circuitous movement around the circulation spine in the gallery is echoed on the exterior by a system of water elements. Flowing water and still pools convey similar notions of motion and pause, while also serving as a visual connection between the northeast and southwest sides of the river separated by the dam.

A small outdoor stage with covered seating is tucked into the folds of the theatre form, creating a performance space for musicians. The adjacent amphitheatre is intended as additional seating for concerts, or for evening astronomy lectures and outdoor films, which can be projected onto the back of the theatre. The open and green spaces of the rest of the northeast area of the garden provide varied environments for sitting, alone or in groups, with changing views of the skyline and river at each point.

The sculpture garden is located in front of the main gallery overlooking the river. The garden begins as an exterior space within the folded walls of the Gallery with screened views to the river. As the garden steps down, the screen peels away from the building edge to create a series of terraces along the edge of the façade of the parking structure. Visitors move between levels via ramps and stairs as channels of water flow down the terraces into pools alongside the path. The terraces eventually wrap around the corner of the garage, culminating in a wide platform just above the water. A new footbridge connects the platform with the Museum's existing pavilion, rejoining the Cambridge and Boston park systems.

Rendering of light and view from the sculpture garden.



(from left) *The Mandelbrot Set fractal; The Henon Map fractal; sphere fractal.*

Conclusions

"Mathematics is the archetype of beauty in the world."

-Johannes Kepler

The gallery for mathematics represents the interpretation of the character and culture one discipline through the lens of another. Through form and light, movement and view, the gallery attempts to communicate to its visitors not merely the products of the discipline of mathematics, but also something of the experience of mathematical thinking.

It is naïve to think that a gallery could instruct a layperson on the technical mechanics that produce fractals, or the exquisite beauty of differential geometry, nor would this likely be an interesting process. Instead, the gallery seeks to convey the experience of the mathematical journey in a way that is equally appreciable by both the mathematician and the average person. The process of problem solving and the moment of understanding is a universal experience, and by celebrating this the gallery can relate to the proof of a 300-year-old problem to a child learning long division. The best teachers understand that the secret to meaningful learning is not simply to instruct, but to encourage curiosity. In the same way, this gallery is designed not simply to inform its visitors, but to inspire them.

Transport IV by Eric J. Heller. The image is produced by injected electrons moving through a field of "hills and valleys" created by charged atoms.

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