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FUEL CYCLE OPTIMIZATION OF THORIUM AND URANIUM FUELED PWR SYSTEMS

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ABSTRACT

The burnup neutronics of uniform PWR lattices are examined with respect to reduction of uranium ore requirements with an emphasis on variation of the fuel-to-moderator ratio (lattice pitch at constant fuel pin diameter) and the use of thorium. Fuel cycles using all combinations of the major fissile (U-235, U-233, Pu) and fertile (U-238, Th) species are examined.

The LEOPARD code and prescriptions developed from a linear reactivity model are used to determine initial core and annual makeup fissile requirements for input into an in-house, simple, systems model, MASFLO-2, which calculates ore (and separative work) requirements per GWeyr for growing, declining, or finite-life nuclear electric systems. For low growth scenarios drier lattices are favored, and the thorium fuel cycle requires as much as 23% less ore than a comparably optimized uranium cycle with full recycle. For unmodified lattices, the thorium fuel cycle with full recycle exhibits long term uranium ore savings of 17% over the comparable uranium cycle with full recycle. For rapidly growing systems, drier lattices, and those using thorium, are less attractive because of their high startup inventories. Thus the introduction of thorium may increase ore and separative work requirements in the short term but will more than repay the ore investment in the very long term.

Very little improvement was achieved by varying fuel pin diameter at a given fuel-to-moderator ratio, but itwas found to be slightly advantageous to recycle plutonium (or U-233) into dedicated reactors having individually optimized lattices: a strategy which may also be attractive for safeguards purposes.

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CHAPTER 1

INTRODUCTION

1.1 Foreword

At present the commercial power reactors being deployed in the U.S. and elsewhere operate on the uranium fuel cycle. Moreover, most reactors are of the light water moderated and cooled type (see Table 1.1), which require more uranium ore to start and sustain than many other concepts. The high ore consumption rates of LWR's have been accepted, in part, because such reactors were viewed as the precursor of fast breeder reactors, which would increase the energy extractable from a given mass of U_3O_8 by a factor of around thirty.

However, recent developments in the U.S. in particular have focused new attention on the efficiency of ore utilization by LWR's:

(a) concerns over nuclear weapons proliferation, diversion or sabotage by terrorist groups, plutonium toxicity and waste disposal have led to deferral of fuel reprocessing and plutonium recycle, thereby locking LWR's into their least efficient mode of operation -- the "once through" fuel cycle.

(b) the above concerns plus cost over-runs in the development program have led to a hiatus of indefinite duration in the liquid metal fast breeder reactor development program.

(c) yellowcake (U_3O_8) prices have escalated rapidly in the past few years, a fact which may presage an earlier than anticipated exhaustion of moderate cost resources; moreover fore-

TABLE 1.1

NUCLEAR POWER PLANTS: Ref. (N-3)

(Operable, Under Construction, or on Order ($\geq$ 30MWe), as of 6/30/77

TYPE	Number of Units U.S.	WORLD
PWR	136 (66.3%)	271 (53.8%)
BWR	66 (32.2%)	124 (24.6%)
PHWR (CANDU)		30
LWCHWR		2
HWBLWR		2
GCHWR		2 (7.1%)
GCR		36
ACR		11
LGR	1	14
HTGR	1	1
THTR		1 (12.5%)
LMFBR	1	8
UNDESIGNATED		2
TOTAL UNITS	205	504
TOTAL GWE	200	392
TOTAL OPERABLE	63	183
GWE OPERABLE	45	92

KEY: PWR =Pressurized Water Reactor
BWR =Boiling Water Reactor
PHWR =Pressurized Heavy Water Moderated and Cooled Reactor
LWCHWR =Light Water Cooled, Heavy Water Moderated Reactor
HWBLWR =Heavy Water Moderated Boiling Light Water Cooled Reactor
GCHWR =Gas Cooled Heavy Water Moderated Reactor
GCR =Gas Cooled Reactor
AGR =Advanced Gas-Cooled Reactor
LGR =Light Water Cooled, Graphite Moderated Reactor
HTGR =High Temperature Gas Cooled Reactor
THTR =Thorium High Temperature Reactor
LMFBR =Liquid Metal Cooled Fast Breeder Reactor

casts of remaining reserves differ vastly - the lower estimates are barely enough to sustain the already committed LWR's through their design lifetime.

(d) work in the U.S. on advanced converter reactors had by and large ceased by 1977: the AEC terminated work on heavy water moderated reactors in the late 1960's and molten salt reactors in the early 1970's; General Atomic withdrew the HTGR from the commercial market in 1976.

In view of the preceding developments it became clear that improvements in the LWR fuel cycle deserved greater attention: the use of thorium appeared particularly interesting in this regard. These concerns led ERDA to establish a thorium assessment program in 1976, one component of which was the present study, funded by ERDA via the M.I.T. Energy Laboratory for work in the M.I.T. Nuclear Engineering Department to evaluate the potential for improved ore utilization by LWR's, with a strong emphasis on the use of thorium. The work reported here focuses on one important aspect of the overall assessment: determination of the optimum fuel-to-moderator volume ratio in terms of ore (and separative work) requirements of PWR's fueled by various combinations of fissile and fertile species.

Subsidiary objectives include:

(a) the development of a simple systems model to predict the ore and SWU requirements of various reactor systems, including breeder reactors.

(b) the investigation of the effect of fuel pin diameter on ore and separative work requirements.

Work is currently underway by other investigators at M.I.T. (F-2),(A-6),(G-1),(C-4) to examine other factors such as power flattening, burnup and fuel management optimization, assessment of the thermal/hydraulic effects of lattice redesign, and finally, the fuel cycle economics of such systems.

Other ERDA and EPRI supported groups are currently working on related topics. The work at General Electric on the use of thorium in PWR's is of particular interest: that and the fact that PWR's account for one-half to two-thirds of capacity-on-order (see Table 1.1) justifies the limitation of the present work to the PWR.

1.2 Background

In this section, a selective review of prior work pertinent to the present research will be presented. It is assumed that the reactor physics of current PWR designs is familiar to the reader. Hence the more elementary considerations will be dispensed with.

1.2.1 The Fuel Cycles

Rather than reviewing the fundamentals of the uranium and thorium fuel cycles, which are well known and documented (K-1), (O-2), the present section will examine the relative performance of these fuel cycles with respect to ore utilization as it follows from nuclear properties.

As will be seen in the chapters to come, the lifetime ore requirements of a given reactor concept are the sum of two

components (1) ore required to provide the initial fissile inventory for startup, and (2) ore required to provide the net yearly fissile makeup.

The largest single factor affecting the initial inventory is the critical mass, CM, the amount of fissile fuel required to achieve criticality. This quantity varies as:

$$CM \propto \frac{1}{(\eta-1)\sigma_a} \qquad (1.1)$$

Where

$\eta = \frac{\nu}{(1+\alpha)}$, number of neutrons created per neutron absorbed.

ν: number of neutrons per fission.

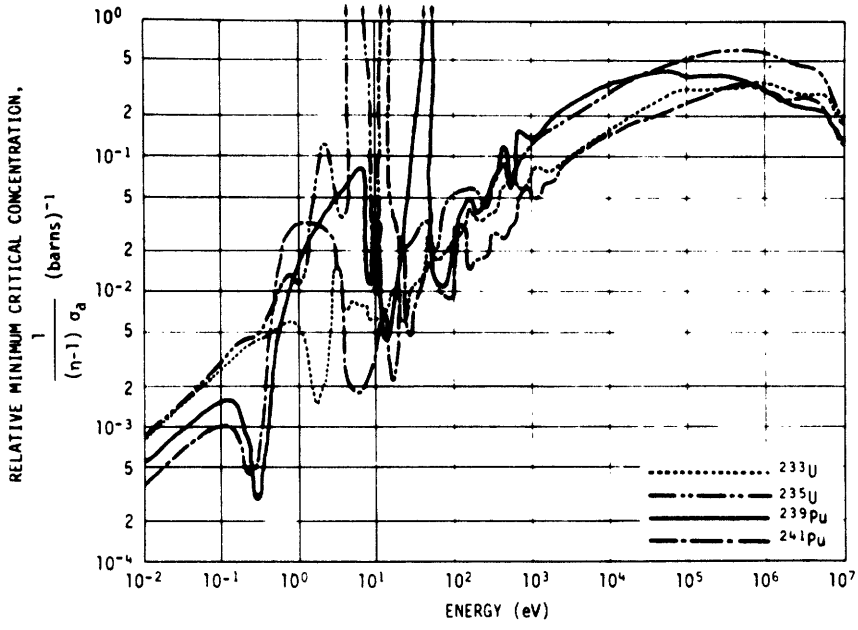
α: ratio of neutron captures to neutron-induced fissions.

σ_a: the microscopic absorption cross section of the isotope.

The upper half of Fig. 1.1 (K-2) shows the relative variation of $1/(\eta-1)\sigma_a$ as a function of neutron energy for the major fissile isotopes.

Although a broad spectrum of neutron energies exists in a typical PWR, most fissile fissions are induced by thermal neutrons. Thus the portion of the curves below 0.5 ev is of greatest current interest. Also of significance is the average epithermal behavior (between 0.5 ev and 1 kev), since as lattice

A)



B)

	Standard Spectra				Typical Reactors	
	0.025 ev	Thermal (Maxwellian) (20°C)	Epithermal (1/E)	Fast (Fission)	Thermal (LWR)	Fast (LMFBR)
U-233	2.29	2.28	2.14	2.64	2.2	2.2→2.4
U-235	2.07	2.05	1.63	2.46	2.0	2.0→2.2
Pu-239	2.09	1.96	1.76	3.03	1.9	2.4→2.7

Fig. 1.1 A) Relative Minimum Critical Concentration for the Fissile Fuels as a Function of Neutron Energy

B) Variation of η with Neutron Energy

pitch is tightened the spectrum hardens and the ratio of epithermal to thermal flux increases. From these curves one can infer that in highly thermalized lattices U-233 and U-235 will have nearly the same critical masses while that of plutonium (primarily Pu-239) will be smaller (same fertile species used in all cases); in epithermal systems the U-235 and Pu-239 critical masses will exceed those of U-233. Thus a PWR first core operatingon the $^{235}UO_2/ThO_2$ cycle would be expected to experience larger net reactivity changes (due to ^{235}U destruction and ^{233}U production) than the corresponding changes in a first core on the $^{235}UO_2/^{238}UO_2$ cycle (^{235}U destruction, ^{239}Pu, ^{241}Pu production) because of the lower critical masses associated with Pu-239 and Pu-241. However, this assumes that both cycles generate the same amounts of bred fuel. As will be seen below the thorium cycle generates more bred fuel than the uranium cycle; thus, the above critical mass advantage of the uranium cycle is to a considerable extent negated by the lower mass increment required to compensate for reactivity losses incurred due to net fissile depletion in the thorium cycle.

A final point here is that the curves in Fig. 1.1 make no distinction with respect to the fertile isotopes U-238 or Th-232. The variation of the fertile capture cross section with energy directly affects the fissile loading of the reactor (indirectly affecting the conversion ratio). Although the infinitely dilute resonance integral of Th-232 is about one third that of U-238, the lumping of the fuel into rods

in a LWR lattice, reduces the effective resonance capture through self-shielding of the resonances, resulting in effective resonance capture integrals for Th-232 and U-238 which are comparable. However, the thermal capture cross-section of Th-232 is about three times that of U-238: this contributes to higher initial fissile requirements in the thorium cycle.

The second mass-determining component affecting the initial requirements, and as will be seen later the annual requirements of a reactor, is the conversion ratio, the ratio of the average rate of fissile atom production to the average rate of fissile-atom consumption.

The conversion ratio C, can be expressed in the form (S-6):

$$C = \eta \left[1 + \frac{\bar{\nu} - 1}{\bar{\nu}} \delta - a(1+\delta)\right] - 1 \qquad (1.2)$$

where

η is the neutrons produced by fissile fissions per fissile absorption;

δ = ratio of fertile to fissile fissions;

$\bar{\nu}$ = mean number of neutrons per fissile and fertile fission;

a = parasitic absorptions and neutron losses per fission neutron (in a current design PWR 5% of the neutrons produced through fission are capture in the control poison, 3 to 4% are lost in light-water coolant capture, 12 to 13% are lost to fission product

absorption, and 4% to leakage from the reactor).

The lower half of Fig. 1.1 shows the spectrum-averaged values of η for typical PWR's and Fast Reactors for the three dominant fissile species in the uranium and thorium fuel cycles. As can be seen, in the thermal energy range U-233 has the highest η value; it also has a higher η value in the epithermal region than Pu-239 or U-235 (the consequences of a higher eta value in the epithermal region will be noted in the next section, where the effect of fuel-to-coolant volume ratio is discussed). Since the potential conversion ratio, ignoring losses and fertile isotope effects is $\eta - 1$, U-233 (i.e., the thorium cycle) has an inherent advantage with respect to conversion ratio in all but fast reactors.

However, the nuclear properties of the fertile elements U-238 and Th-232 (see Eq. (1.2)), both influence the conversion ratio through the fertile to fissile fission ratio, δ . Table 1.2 shows values of δ (and η) for various uranium and thorium fuel cycles (0-1). As can be seen, δ is about 0.07 for U-238 versus 0.02 for Th-232 for all the cycles investigated, giving the uranium cycle a small compensatory advantage. Overall, however, for LWR's the thorium cycle has a higher net conversion ratio than does the uranium cycle. Since a higher conversion ratio implies a lower value of the initial fissile loading increment to compensate for reactivity losses, the higher conversion ratio of the thorium cycle is a compensating factor. Overall, however, the thorium cycle exhibits higher fissile loadings.

The second component of the lifetime ore requirements

TABLE 1.2

VALUES OF η AND δ FOR VARIOUS FUEL CYCLES: Ref. (0-1)

Fuel Cycle*	$^{235}UO_2$	PuO_2/UO_2	$^{235}UO_2/ThO_2$	$^{235}U/Th$	$^{233}UO_2/ThO_2$	$^{233}U/Th$
$\overline{\eta}$	1.93	1.87	2.05	2.05	2.22	2.24
δ	0.07	0.07	0.02	0.02	0.02	0.02

*Values calculated for the 600 MW(e) Angra dos Reis PWR

of a reactor, the annual requirements, M_a STU_3O_8Gwe yr (rated), can be related to the conversion ratio through a simple approximate expression (A-10):

$$M_a = \frac{(\theta)(0.365)\ LZ(1+\bar{\alpha})\ (1-C)}{10^3 \cdot y} \qquad (1.3)$$

where

θ = a conversion factor (including units and product/feed scaling factors for enrichment);

L = the capacity factor of the plant;

Z = grams fissile material fissioned per MWD;

$\bar{\alpha}$ = the average fissile capture/fission ratio;

and

y = thermal efficiency of the plant.

Thus, Eq. (1.3) shows that the higher conversion ratio achieved in the thorium cycle will imply a smaller annual make-up requirement for this cycle than for the uranium cycle.

In summary, both the initial fissile mass and the annual makeup are related to the conversion ratio, C, of the reactor: the initial mass is dominated by a $(\eta - 1)^{-1} \propto C^{-1}$ dependence, while the makeup is $\propto (1-C)$. Thus the thorium fuel cycles tend to have lower annual requirements but higher inventories than the comparable uranium cycles.

Finally, any attempt to improve the resource utilization of either cycle, will mean increasing the conversion ratio, im-

plying a better neutron economy. In the next section it will be shown that the most attractive means to do this is by hardening the neutron spectrum by varying the reactor fuel-to-moderator ratio.

1.2.2 Effect of Fuel-to-Moderator Ratio

Without losses, Eq. (1.2), (with values of η,ν and δ appropriate to any LWR fuel combination), would give conversion ratios for PWR's exceeding unity. However, as has been seen, this is not the case, since losses to control poison (5% of fission neutrons), light water coolant capture (3 to 4%), fission product absorption (12 to 13%) and leakage (4%), account for non-productive losses as high as 26% of all fission generated neutrons.

The effect of fuel-to-moderator ratio on neutron economy, and ore utilization, is readily explained. When the fuel-to-moderator ratio is increased the first effect noted in a calculated neutron balance is decreased losses to the light water coolant. Furthermore, the reduction in the light water content, and therefore hydrogen content, of the core results in less moderation and a harder spectrum results, leading to less parasitic losses due to fission product capture (the fission product Xe has a very high thermal absorption cross-section). More neutrons become available for productive capture in fertile material. This leads to a higher conversion ratio and a lower reactivity swing, which contributes to a further increase in the conversion ratio, since less control poison is required. All of these

effects result in increased conversion ratios and improved ore utilization.

However, the very factors which contribute to improved conversion ratios also lead to increased fissile loadings. As the spectrum hardens the η of all the fissile isotopes, except Pu-241 decreases. This, combined with other effects, such as increased Th-232 resonance capture and decreased spectrum-averaged fission cross-sections, increases the fissile loadings required to start up these drier lattices.

Thus, in going to increased fuel to-coolant-volume ratios, there is a tradeoff between better neutron economy, hence fuel utilization, and increased fuel inventory. Figure 1.2 is an event tree showing the effects of tightening the lattice of a LWR.

Although low inventory cores might appear to have superficially attractive features, there are practical limits on how far one could go in the direction of wetter lattice designs. Decreasing the fuel-to-moderator ratio leads to undesirable effects, such as increased neutron capture in the moderator and more importantly, positive temperature coefficients and larger cores (or smaller fuel pins).

Thus this study will attempt to answer the question (among others) of which fuel cycle, and what fuel-to-moderator ratio, will lead to minimum system ore and separative work requirements.

1.3 Previous Work

Previous work in the area of present interest can be

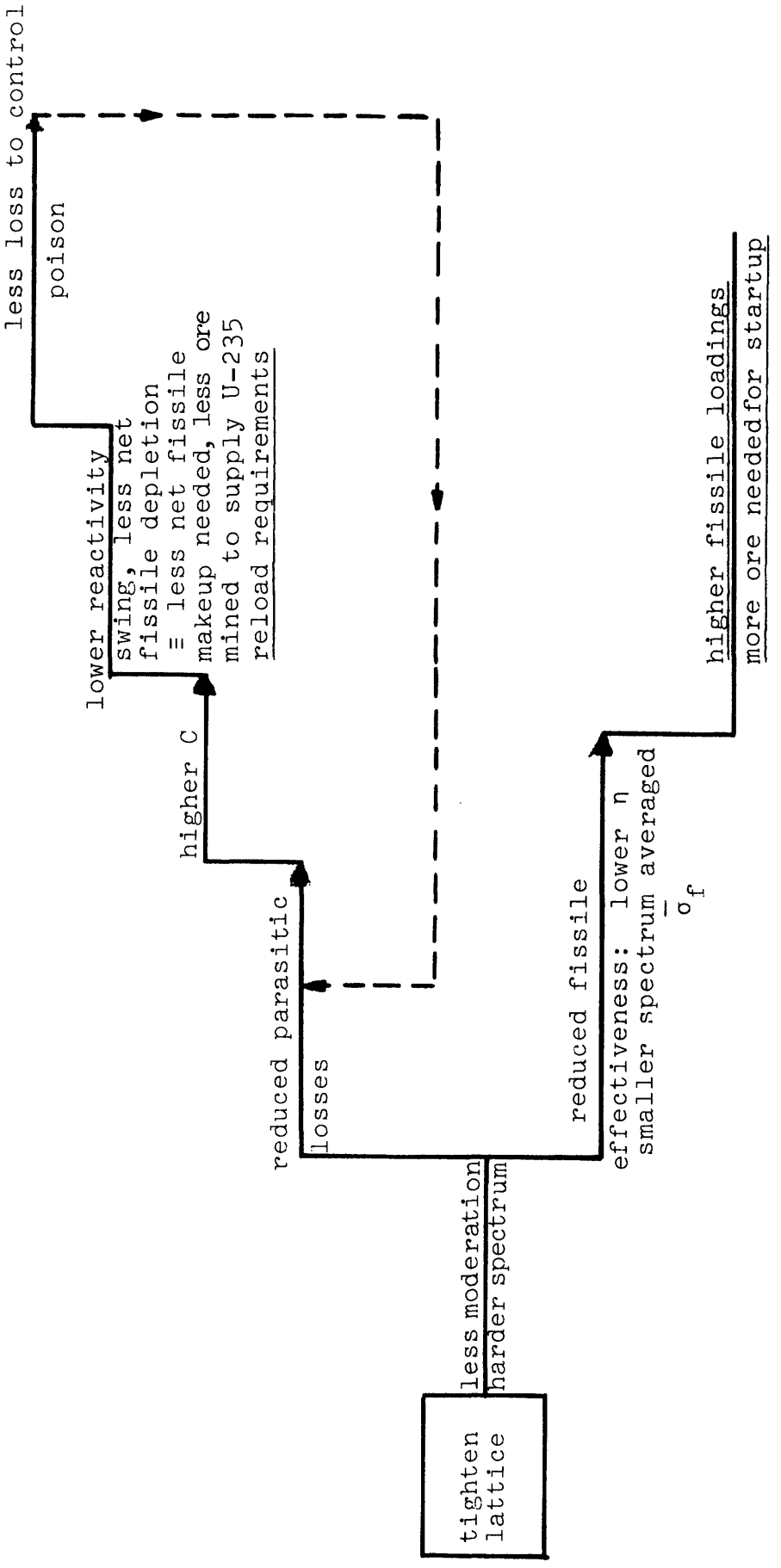


FIG. 1.2. EFFECT OF INCREASED FUEL-TO-MODERATOR RATIO IN A LWR

summarized by referring to three major studies, the first of which was recently completed by Combustion Engineering (CE) Power Systems, (S-1), for EPRI on the utilization of the thorium fuel cycle in PWR's. Although the primary effort in the CE study was to compare various thorium fuel cycles with the conventional uranium cycle in their standard System 80^{TM} plant at constant fuel-to-coolant volume ratio, a brief investigation of the effects of varying the fuel-to-coolant volume ratio was also carried out.

Table 1.3 indicates the 30-year requirements of U_3O_8 and SWU for various uranium-based and thorium-based fuel cycles, on a per-installed-GWe basis, using results extracted from the CE study. The numbers shown are for self-generated recycle: "homogeneous" refers to recycle of bred fissile material uniformly dispersed in all reload assemblies; "segregated" recycle implies confinement of recycled fuel to certain assemblies; "crossed-progeny" means using the bred uranium from the thorium-based fuel as the fissile material for the recycle uranium based fuel, while the bred plutonium from the uranium-based fuel is used as the fissile material for the thorium-based fuel; and finally, "single pass" refers to only one recycle of bred material (here plutonium).

One of the major conclusions of the CE study was that thorium cycles employing highly enriched uranium can increase the energy generated per mined ton of uranium ore on the order of 18 to 34 percent in the long term, with the fully enriched U-235/ThO_2 segregated recycle option being the most attractive case in terms of ore utilization (see Table 1.3). They also found, however, that there is a penalty in terms of increased

TABLE 1.3

30-YEAR U_3O_8 AND SEPARATIVE WORK REQUIREMENTS FOR PWR'S[1]: FROM REF (S-1)

Fuel Cycle	U_3O_8 (Short Tons/GW(e))	SWU Metric Tonnes/GW(e))
(1) UO_2 (No fuel recycle)	5989	3555
(2) UO_2 (U & Pu recycle)	4089	2696
(3) ThO_2 (93% U-235) (Homogeneous U-recycle)	3483	3467
(4) UO_2/ThO_2 (93% U-235)(Segregated U-recycle)	3453	3436
(5) UO_2/ThO_2 (Single Pu pass, homogeneous U & Pu recycle)	4137	2867
(6) UO_2/ThO_2 (Single Pu pass, segregated U & Pu recycle)	4010	2773
(7) UO_2/ThO_2 (segregated U & Pu recycle)	3976	2749
(8) Crossed-Progeny (Single pass Pu recycle)	4194	2897
(9) Crossed-Progeny (Full recycle)	4002	2755

[1] 75% Capacity Factor (Annual Refueling); 0.2 w% Diffusion Plant Tails Assay.

ore and separative work units required for startup.

Another important conclusion of the CE report was that the worth of plutonium as a fuel can be improved by recycling it in thorium rather than in uranium: i.e., compare the U_3O_8 requirement of cycle 2 versus that of cycle 7 in table 1.3.

Finally, another important conclusion (again, see Table 1.3) is that homogeneous recycle and heterogeneous recycle are essentially equivalent in terms of ore and separative work requirements (compare cycles 3 and 4).

The CE study looked briefly at the effect of the fuel-to-coolant volume ratio on the performance of two attractive reactor types, the UO_2 (93% enriched U-235)/ThO_2 reactor and the PuO_2/ThO_2 reactor. Their results indicated that for higher fuel-to-coolant volume ratios the UO_2 (93% enriched U-235)/ThO_2 reactor demonstrated superior performance. In both cycles, furthermore, increased fissile loadings and increased fuel utilization with burnup were observed with increasing fuel-to-coolant volume ratio.

Using the superior reactor at higher fuel-to-coolant volume ratio, i.e., the UO_2 (93% enriched U-235)/ThO_2 reactor, they compared its ore requirements at the standard System 80^{TM} fuel-to-coolant volume ratio, Vf/Vm, to that at Vf/Vm = 1. It was found that the tighter lattice required a 30% higher fissile inventory for startup. However, due to the higher conversion ratio, it was found that a 15% savings in annual ore consumption was achieved by the tighter lattice in the equilibrium cycles. Although their thirty year ore requirements were approximately

the same, the drier lattice system had 30% more fuel at the end-of-reactor-life. Recycle of this fuel to the next-generation of reactors would avoid further startup penalties, and still maintain the 15% annual savings.

The work by Oosterkamp and Correa (O-1) is another recent major study done on thorium utilization in LWR's. In this study the Westinghouse designed, 600 Mw(e) Angra dos Reis PWR in Brazil was used as the base reactor. Table 1.4 shows some of the results of their calculations. The table demonstrates the higher conversion ratio of the thorium cycles (compare the $^{233}UO_2$-ThO_2 reactor to that of any of the uranium based cycles).

Oosterkamp concluded that a 30% savings in the equilibrium cycle (including startup requirements) would result by going to a UO_2 (93% enriched U-235) /ThO_2 system.

Oosterkamp and Correa also looked briefly at optimizing the fuel-to-coolant volume ratio (O-1),(C-2). Their results show an optimum for all fuel cycles considered in the Vf/Vm range of 0.67 to 1.0.

The final work germane to some of that done here has been carried out under the Light Water Breeder Reactor (LWBR) Program (E-4), started in 1965 with the goal of using the thorium/uranium-233 fuel cycle to significantly improve the fuel utilization of existing and future light water reactors. Although thorium has been used in other LWR's, (the first core of the Indian Point I reactor was fueled with thoria-urania (UO_2 - ThO_2) pellets, with 93% enriched uranium), the LWBR is the first PWR designed specifically for thorium utilization. At present the Shipping-

TABLE 1.4

SUMMARY OF THORIUM STUDY BY OOSTERKAMP AND CORREA: Ref. (0-1)

	$^{235}UO_2$	PuO_2-UO_2	$^{235}UO_2$-ThO_2	^{235}U-Th	$^{233}UO_2$-ThO_2	^{233}U-Th
Conversion Ratio	0.63	0.71	0.70	0.70	0.85	0.86
Fissile[1] Inventory (kg)	1250	1232	1480	1900	1130	1440

[1]Based on the 600 MW(e) Angra dos Reis PWR

port reactor is being equipped to employ a demonstration core of this design.

Due to the existence of well documented material on this program (E-4), the treatment here will only be qualitative.

In order to produce the U-233 fuel needed for the LWBR, the Light Water Breeder Program advocates the choice of three prebreeder alternatives. Alternative A1 uses slightly enriched uranium oxide segregated from thorium oxide pellets. Alternative A2 uses a mixture of slightly enriched uranium segregated from rods containing a PuO /ThO_2 mixture, a feature which combined with its tighter lattice (inner core) distinguishes it from the other prebreeder options. Finally, alternative A3 is basically the same as alternative A1, except that the uranium fuel is at 93% enrichment and is in the form of $^{235}UO_2$-ZrO_2 pellets. As will be seen, some of the lattices examined in the present work can be considered for service as prebreeder lattices. In chapter 4, the LWBR, a break even breeder (i.e. Breeding Ratio $\approx$ 1.0), will be looked at from a resource utilization point of view.

Thus, as this section has demonstrated, prior work on the topic of this study, i.e., determination of the optimum fuel-to-moderator volume ratio, Vf/Vm in terms of ore (and separative work) requirements of PWR's fueled by various combinations of fissile and fertile species, is not entirely adequate:

(a) all combinations of interest have not been examined;

(b) there has been less emphasis on fuel-to-moderator optimization;

(c) changes in fuel to moderator ratio are usually con-

fused with effects due to pin diameter variation, as will be discussed in Chapters 2 and 4;

(d) previous studies focus on single reactors, not the more realistic picture of a growing system.

1.4 Outline of Present Work

The work reported here is organized as follows. Chapter 2 will be concerned with the choice of a depletion code and the associated calculational methodology for determining the required reactor parameters, (primarily fissile masses charged to and discharged from a given reactor), including benchmarking of the code against critical and exponential experiments. Chapter 2 will also deal with the choice of a base-case reactor, and definition of the various reactor types examined in this study.

In Chapter 3, a simple model is developed to enable the calculation of ore and separative work requirements of coupled systems of light water reactors as a function of system growth rate, using as input the data developed by the calculations of Chapter 2.

In Chapter 4, the results of the calculations carried out using the simple model developed in Chapter 3 and the data developed in Chapter 2 will be presented. The application of the model to LWR-Breeder systems will also be discussed and results presented.

Finally, in Chapter 5, the results and conclusions of the study will be summarized, together with recommendations for future work. Several appendices are included containing subsidi-

ary analyses and data supporting the work reported in the main text.

CHAPTER 2

DETERMINATION OF REACTOR FUEL CYCLE PARAMETERS

2.1 Introduction

The reactor core parameters of interest in a study such as this, consist of the initial fissile, fertile (and where applicable) the separative work unit (SWU) requirements for the startup and the annual sustenance of a given reactor type. These parameters can in turn be used to examine nuclear systems consisting of combinations of several reactor types. In order to characterize the reactor types of interest several steps are called for: choice of a depletion code, benchmarking the code against experimental lattice data, and then defining a consistent methodology for the calculation of the required fuel cycle parameters. The above topics are the major areas to be discussed in this chapter. First, however, prior applicable work will be reviewed.

2.2 Previous Work

In this section the calculational methodology of previous work will be discussed. While the use of thorium in LWR's has a fairly long history - the Indian Point I PWR-core used UO_2/Tho_2 fuel - we will confine our discussion to the most recent work.

The C.E. Power System's study on the Thorium Fuel Cycle in PWR's for EPRI (S-1) discussed in Chapter 1, utilized as their base-case reactor the CE-System 80^{TM}. In their study, due to the large number of calculations required to evaluate

the various fuel cycle options, the calculational methodology was based on a relatively simple reactor model to reduce analysis time and expense. The code used was CEPAK, a one-dimensional lattice code employing a synthesis of a number of computer codes including FORM(M-1), THERMOS (H-1), and CINDER (E-1). The code represents the entire neutron spectrum by 83 groups, with FORM calculating the non-thermal ($\geq$0.625 ev. thermal cutoff) energy spectrum for a homogeneous cell employing 54 groups. Resonance shielding and Doppler Broadening are corrected for with input L factors. The THERMOS module calculates the thermal spectrum for a one dimensional unit cell using the remaining 29 groups.

For use in thorium systems several modifications were made to CEPAK, including the use of updated Th-232 and U-233 resonance absorption treatments and the updating of the cross-section library to include ENDF/B-IV values. The latter modifications will be commented on later in this chapter.

Although CEPAK was adequate for estimating fissile loading requirements, cycle lengths and material inventories, determination of power distributions, core operating characteristics and their effects on achievable cycle lengths, required that spatial calculations using PDQ (C-1) be performed.

Ideally, in using CEPAK, a single enrichment fuel batch is analyzed, with its exposure followed throughout its life in the core to a desired cycle length and fuel burnup. The environment in a critical core is simulated by inputting into CEPAK soluble boron concentrations and energy-dependent neutron leakages provided by preliminary spatial core calculations done with PDQ.

These parameters are used to adjust the neutron spectrum so that the depletion is performed in a critical environment; by a process of iteration, the desired level of precision is achieved.

However, due to the fact that the results of spatial calculations would not be available for many of the fuel cycles examined, C.E. initially employed a simplified approach. Here, critical boron levels and batch dependent leakages as a function of fuel burnup were obtained from spatial calculations for system-80^{TM} in the UO_2(U-235) equilibrium cycle. As these values would only be approximate for other cycles, a critical soluble boron search was performed in CEPAK at each burnup step, to obtain, with the given leakages a K_{eff} of 1.0 for the batch.

C.E. also assigned the core - average power density to the fuel batch throughout its lifetime, implying a constant average fuel batch temperature and equal incremental cycle burnups. Here they assert that this assumption leads to end-of-life nuclide concentrations and reactivity lifetimes which are essentially the same as those predicted in more detailed calculations.

Thus, in the CE study the reactivity lifetime of the fuel batch was determined from the relation:

$$\bar{K}_\infty = \sum_j^J f_j \; k_\infty(E_j) + \Delta k \qquad (2.1)$$

where f_j is the fraction of the in-core fuel which is resident in the reactor for j cycles; $k_\infty (E_j)$ is the infinite medium multiplication factor of a batch with a burnup of E_j at the end of cycle j; and Δk is a reactivity increment accounting for

leakage and biases. Typically, $f_j = 1/3$, $J = 3$ and $\bar{E}_j = j \times E$, where E is the average burnup per batch per cycle. The reactivity bias, $\Delta k \approx 4\%$, was determined by comparing the CEPAK - predicted $\bar{K}_\infty$ for a System 80^{TM} UO_2(U-235) equilibrium batch and the end-of-cycle K_{eff} predicted by a detailed PDQ spatial core calculation.

As a final note, the discharge burnups of thorium based and uranium based fuels examined were 33,400 MWD/MT and 30,400 MWD/MT, respectively.

The work done by Oosterkamp (O-1), utilized the reactor characteristics of the 600 MW(e), Westinghouse-designed, Angra dos Reis PWR scheduled for operation in Brazil in 1978.

This study of the thorium and uranium fuel cycles was composed of two parts, namely, cell calculations and full core spatial calculations.

In the cell calculations, the HAMMER cell code (S-2) was utilized with modifications made to the code to enable the handling of fuel burnup. The main subprograms of HAMMER are THERMOS (H-1), HAMLET (S-2), and FLOG (S-2), which is a modification of FOG (F-6). Using the above subprograms HAMMER calculates infinite lattice parameters using multigroup transport theory, and composite reactor parameters using few group diffusion theory.

For the cell calculations, the initial enrichment was varied so that k_∞ was 1.00 at a fuel burnup of 15,000 MWD/MT, not taking into account leakage or control poison. Here the linear reactivity model, which will be discussed later was

assumed. Thus, the final "critical enrichments determined here were those of a critical infinite reactor without control poison with a final fuel burnup of 30,000 MWD/MT.

The full core calculations were made to include the effects of isotope buildup and control poison using the spatial code CITATION (F-1) for three reactor types, the UO_2(U-235) reactor, the UO_2(U-235) with plutonium recycle reactor, and the UO_2 (U-235(93% enriched))/ThO_2 fuel recycle reactor, assuming spontaneous recycle of bred fuel where pertinent. In these reactors the U-235 enrichment was varied to obtain the excess reactivity needed for 300 effective full power days.

The CITATION calculation involved an R-Z model with three radial zones of equal volumes. Initially, a HAMMER calculation was done to determine nuclide densities and cross-sections for the UO_2(U-235), and UO_2(93% enriched U-235)/ThO_2 cores. These were then input to CITATION. Fourteen burnup steps including the first two, which were two days each to account for xenon and samarium buildup), were then carried out. After 300 effective full power days, the bred fissile material from the above two reactors, i.e., the UO_2(235) and UO_2 (93% enriched U-235)/ThO_2 reactors, was then recycled to their respective reactors and the same calculational procedure repeated.

Ideally the cross-sections should have been updated after each CITATION burnup step using HAMMER. However, due to the large amount of calculational time that would result, the cross-sections were updated after the end of six full cycles (i.e., 64 CITATION steps or 5 recycles). After the update a further

six cycles were run using the same calculational methodology.

As a final note, some work was done on the effect of fuel-to-coolant volume ratio on resource utilization. Here the methodology employed was similar to that done by CE(S-1), i.e., the fuel-to-coolant volume ratio was varied by varying the fuel pin diameter. In contrast, however, here the final burnup was kept constant, whereas the CE study maintained the total energy of each core constant, i.e., the burnup of the fuel varied with the total core heavy metal loading.

F. Correa (C-2) redid the calculations presented in reference (O-1). The improvements included addition of natural boron in the HAMMER cell calculations, and adoption of a more consistent basis for the full core calculations.

With the perspective provided by this earlier work, one is now prepared to examine the methods adopted for the present study.

2.3 Reactor Lattices Investigated

An essential prerequisite to any study of this scope is a delimitation of the vast array of possible variables, such as the selection of clad type and thickness, fuel pin diameter and linear power rating. In this regard it is particularly convenient to select a given reactor design as a base case. In the present work the Maine Yankee PWR was chosen for this role. Reasons for this choice included the fact that Maine Yankee is a large modern PWR, is an operating system, designed by Combustion Engineering Inc., a participant in the ERDA/EPRI thorium assess-

ment program, and is operated by Yankee Atomic Electric Co. who graciously consented to provide M.I.T. with supplementary information to facilitate this study. Hence the first part of this section will consider the pertinent reactor parameters of Maine Yankee.

2.3.1 The Maine Yankee PWR

Maine Yankee is a 2,440 MW(TH) PWR using a Combustion Engineering Nuclear Steam Supply System, located in Wiscasset Maine. Table 2.1 (M-3), (Y-1) briefly summarizes the reactor's key core parameters. Tables G.1 and G.2 list the dimensional, thermodynamic and reactor physics parameters required to describe this reactor in a LEOPARD supercell calculation, the supercell being the unit cell consisting of fuel, clad and moderator regions plus an "extra" region to account for non-unit cell constituents of the reactor, i.e., water slots, control rod followers etc.

2.3.2 Reactor Types

In Table 2.2 the types of reactors to be investigated are listed, including for each a brief description of their fuel types. For the PuO_2/ThO_2 and PuO_2/UO_2 cores the fissile species charged is plutonium whose isotopic makeup is characteristic of the discharge plutonium from the base case UO_2(slightly enriched U-235) reactor.

For the $^{233}UO_2/ThO_2$ reactor the fuel fissile isotopic

TABLE 2.1

MAINE YANKEE CORE PARAMETERS[1]

Parameters	Value	Units
Core Thermal Power	2440	MWTh
Nominal Electric Output	790	MW(e)
Nominal Thermal Efficiency	32.5%	
Core Heavy Metal Loading	87	MTU
Fuel Management	3 Batch, Mixed Central Zone	
Equilibrium Discharge Burnup	33,000	MWD/MTHM
Power Density	75.016	KW/LITER
Nominal System Pressure	2100.0	psia
Nominal Fuel Temperature (Average)	1209.5	°F
Nominal Coolant Bulk Temperature	562.5	°F
Number of Fuel Assemblies	217	
Fuel Rod Array	14x14	
Fuel Rod Pitch	0.580	inches
Fuel Pellet Diameter	0.390	inches
Clad Material	Zircaloy 2	
Clad Thickness, Nominal	0.031	inches
Diametrical Gap, Nominal	0.008	inches
UO_2/H_2O Volume Ratio, Vf/Vm:		
Supercell	0.4816	
Unit Cell	0.621	
Fuel Density, % Theoretical	92%	

[1] Data available as of June, 1976 (subject to design changes).

TABLE 2.2

REACTOR LATTICES AND CORRESPONDING FUEL TYPES INVESTIGATED IN THE PRESENT STUDY

Lattice Type	Fuel Type Charged
1) $^{235}UO_2/^{238}UO_2$	Slightly enriched uranium (on the order of 3%)
2) $^{235}UO_2/ThO_2$	93% enriched uranium oxide mixed with ThO_2
3) PuO_2/ThO_2	Plutonium-isotopic mix characteristic of LWR (base case) discharge mixed with ThO_2
4) PuO_2/UO_2	Same Pu mix as in #3; Uranium is tails at 0.2 w% U-235
5) $^{233}UO_2/ThO_2$	Uranium isotopic mix characteristic of discharge from PuO_2/ThO_2 at standard Vf/Vm

composition was selected to be characteristic of that discharged from the PuO_2/ThO_2 fueled reactor at the standard fuel-to-coolant volume ratio of Maine Yankee. While the U-234/U-233 ratio for fuel discharged from the UO_2 (Fully enriched U-235)/ThO_2 reactor is about 30 percent higher than that of the PuO_2/ThO_2 reactor; the actual mass of U-234 is small (less than 12 percent of the total U-233 plus U-234 for the UO_2 (Fully enriched U-235)/ThO_2 reactor) and thus this difference is not especially important.

Table 2.3 summarizes the isotopic composition of the fissile masses used in these studies.

All reactors chosen for this study are of the "dedicated" type, i.e., each reactor accepts as feed, fuel containing only one of the major fissile species: U-235, U-233(with U-234 and U-235) and fissile plutonium (Pu-239, Pu-241 with Pu-240 and Pu-242). The way in which these reactors combine to form various fuel recycle systems will be discussed in Chapter 3.

The reactor types were primarily chosen to permit examination of all fissile and fertile combinations possible. This will become clearer in the discussion in Chapter 3, where dedicated reactors are coupled to provide this capability. The slightly enriched uranium system was chosen on the basis that any study of thorium utilization in LWR's must utilize the present uranium system as a basis for comparison.

The choice of the fully enriched UO_2/ThO_2 reactor type was based on several factors. Correa (C-3) has shown that a fully enriched reactor of this type is preferable to lower enriched systems (i.e., those containing more U-238 and less Th-232), a

TABLE 2.3

ISOTOPIC COMPOSITION OF FUEL CHARGED TO CONSUMER REACTORS[1]

Fuel Type	Isotope	Weight Fraction
1). Plutonium	Pu-239	0.542196
	Pu-240	0.259582
	Pu-241	0.139364
	Pu-242	0.058858
2). ^{233}Uranium[2]	U-233	0.907158
	U-234	0.0803653
	U-235	0.0124766

[1] Consumer reactors- those whose fissile makeup is bred by other reactor types.

[2] U-236 is neglected due to its small concentration (<0.06%).

conclusion supported by CE findings (S-1). Oosterkamp (O-2) has also concluded that losses in fuel performance and increased ore requirements would result if the enrichment in such a reactor type were limited to 20 percent. In addition, the prebreeder phase of the LWBR (E-4) program has concentrated on this option.

The PuO_2/ThO_2 reactor type was chosen for a number of reasons. As was discussed in Chapter 1, in such a reactor one is effectively trading plutonium for U-233, which is a far superior fuel in LWR's. Secondly, in terms of proliferation related problems, U-233 may be preferable because its U-232 content enhances detectability. Furthermore, it can be denatured with U-238 to reduce its suitability for weapons use, and it is less hazardous as a radiological weapon than plutonium.

The all plutonium/U-238 reactor type (APR) was chosen due to the extensive studies done on this concept and the recent interest in its use in an energy park to reduce the risks of diversion. Although multiple plutonium recycle in a LWR results in degradation of the plutonium (evolution of an unfavorable isotopic composition), storage of the degraded material for use in fast breeder reactors, where the non-fissile isotopes would not be as great a disadvantage, could be an attractive option.

The $^{233}UO_2/ThO_2$ reactor type is attractive in that it has the best overall breeding capability of all thermal reactor fuel cycles: it is, for example, the fuel cycle used in the LWBR. Like the plutonium burner it is also subject to fuel degradation due to U-234 buildup.

2.4. The Burnup Code

In any study similar to the present, due to the large number of calculations required to evaluate the various fuel cycle options, the calculational methodology has to be based on a relatively simple reactor model in order to reduce the cost of, and time required for, the analyses. In the present work an EPRI version of the LEOPARD code containing an up-to-date cross-section library derived from ENDF/B-IV, was used for depletion calculations.

Briefly, LEOPARD is a spectrum dependent zero-dimensional depletion code. From lattice parameters such as geometry, material composition and thermal data, it determines fast and thermal spectra and spectrum-averaged cross sections. The code optionally computes fuel depletion effects for a dimensionless reactor. The spectra and spectrum averaged cross-sections are recomputed for each discrete burnup step. The consistent B-1 MUFT-IV model (B-2) is used to obtain the spectrum and spectrum averaged cross-sections in the fast and epithermal regions. This model is modified to treat U-238 resonances, as described by Strawbridge et al. (S-3). The SOFOCATE model (A-2) featuring the Wigner-Wilkins spectrum calculation is employed in the thermal region.

The group structure in LEOPARD consists of 172 groups in the thermal region treated by the SOFOCATE model, and 54 groups in the fast and epithermal regions treated by MUFT.

This section will deal with two topics (1) the results of benchmarking the depletion code and its cross section set against 110

critical and exponential experiments (of which 25 are exponentials) (2) a brief discussion of the calculational methodology of LEOPARD and the adequacy of the ENDF/B-IV cross-section library.

2.4.1. Benchmarking Against Critical and Exponential Experiments

In order to justify confidence in a computational tool, testing against experimental results is highly desirable. In this study, the EPRI-version of LEOPARD and its companion cross-section library was tested by comparing calculated effective multiplication constants, K_{eff}, against actual critical and exponential experiments.

The first set of benchmark calculations were against criticals and exponentials of slightly enriched uranium (U-235/U-238) light water lattices.

The upper half of Fig. 2.1 shows a histogram of the effective multiplication constants, K_{eff}, calculated using EPRI-LEOPARD for 63 slightly-enriched-uranium (U-235/U-238) light water lattices, (See Appendix A for the identification of each assembly employed and the calculated value of K_{eff}.) The histogram demonstrates that for slightly enriched (U-235/U-238) lattices, EPRI-LEOPARD results are not significantly positively or negatively biased: the average K_{eff} was 1.00257 for the 63 cases. Furthermore, with the exception of one case, the variations from criticality are less than 3%.

The code was then tested against 42 plutonium-enriched uranium oxide light water critical and exponential experiments. The lower half of Fig. 2.1 is a histogram of the results. It

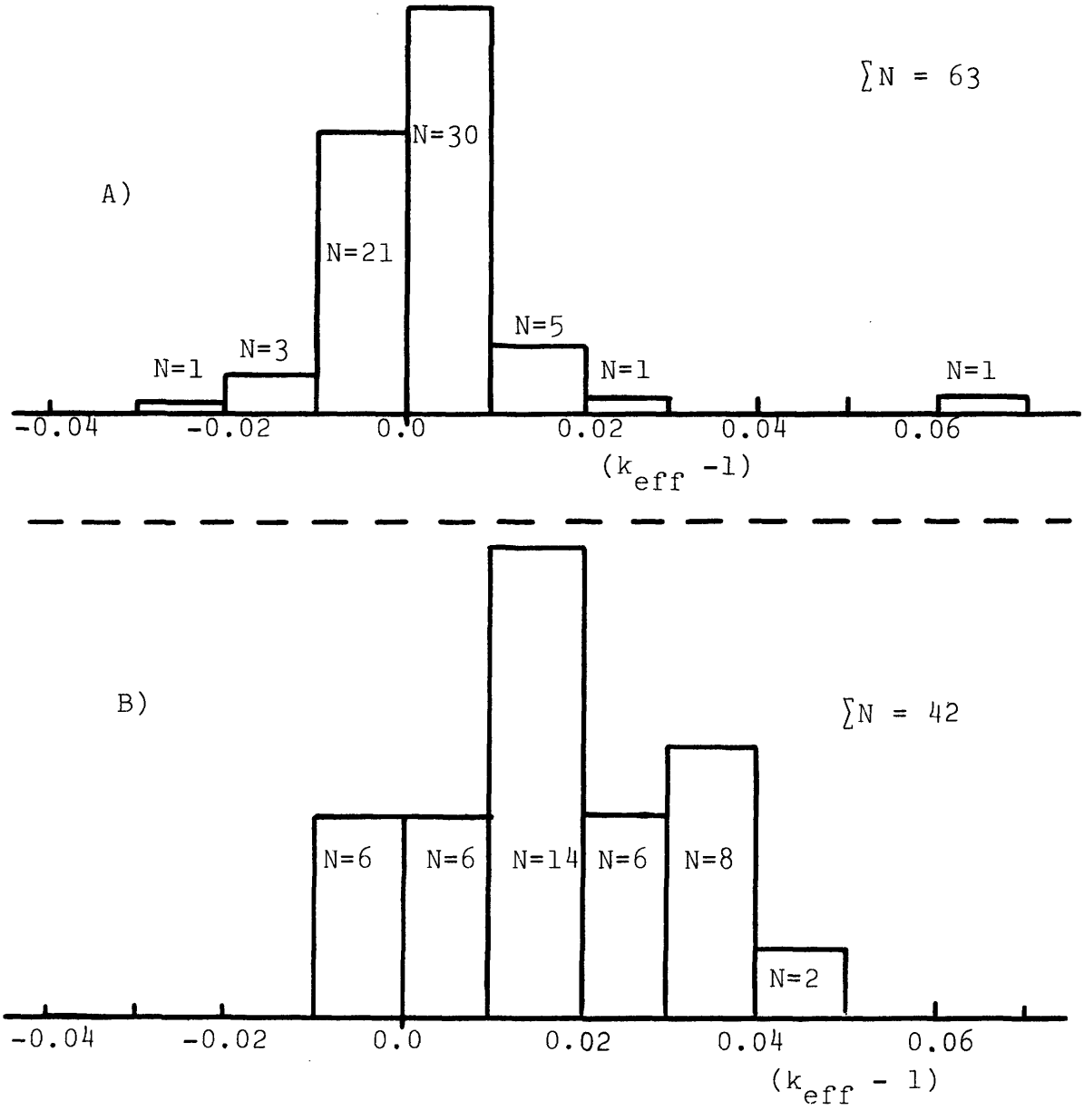


FIG. 2.1. DISTRIBUTION OF CALCULATED k_{eff} FOR:
(A) Slightly enriched Uranium Light Water Lattices
(B) Plutonium enriched PuO_2/UO_2 Light Water Lattices

demonstrates a definite positive bias, with a significant number of cases having a +4% deviation. The mean K_{eff} of 1.01783 also demonstrates this positive bias.

Finally, EPRI-LEOPARD was tested against five ^{233}U enriched thorium oxide light water exponential experiments. Fig. 2.2 is a plot of K_{eff} calculated using EPRI-LEOPARD versus the fuel-to-coolant volume ratio.

A number of observations can be made regarding the results of Fig. 2.2: (1) the effective multiplication constant calculated by EPRI LEOPARD has a definite positive bias (the average K_{eff} is 1.0103), (2) this positive bias decreases with increasing fuel-to-coolant volume ratio. However, such a conclusion should be tempered by the fact that the sample number here is small (n=5), all are from one set of experiments, and that these benchmarks are light water exponential experiments which are usually not as accurate as actual critical experiments. The detailed results are presented in Appendix A.

Table 2.4 presents the overall summary of the benchmarks checked in this study. As can be seen the range of fuel-to-coolant volume ratios tested covers the values investigated in the present study with the important exception of the highest value, Vf/Vm = 1.5.

From the calculated mean values of K_{eff}, one can see a definite positive bias, which is the largest for the plutonium lattices.

Overprediction of the effective multiplication constant results in an underprediction of the critical enrichments. The

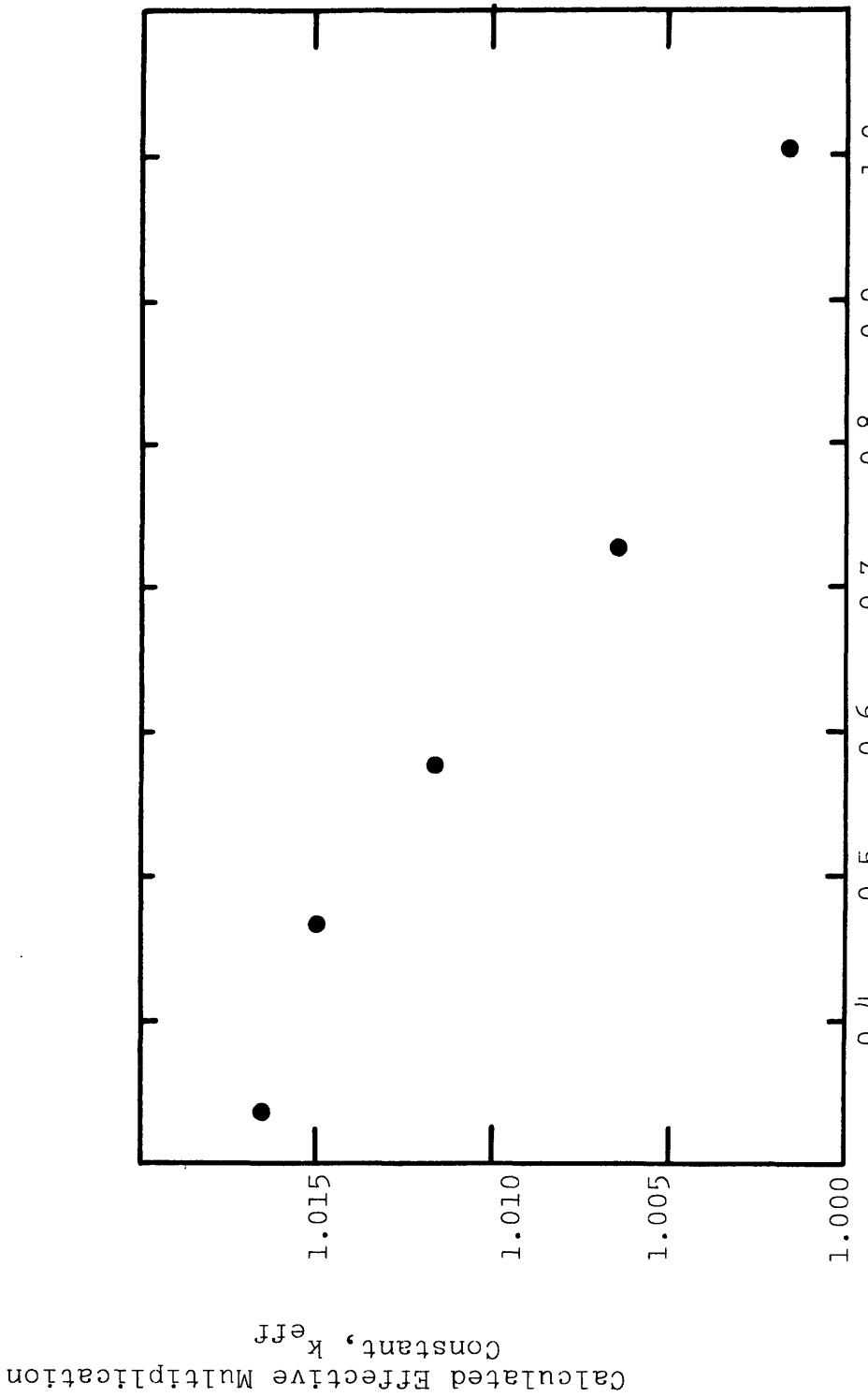


FIG. 2.2. CALCULATED k_{eff} VERSUS Vf/Vm FOR $^{233}UO_2/ThO_2$ LATTICES

TABLE 2.4

SUMMARY OF BENCHMARK COMPARISONS

Fuel Composition	$^{235}UO_2/^{238}UO_2$	PuO_2/UO_2	$^{233}UO_2/ThO_2$
Number of cases	63	42	5
Lattice type*	35S,28H	12S,30H	5H
Fissile Enrichment Range, W%	1.3-4.1	1.5-6.6 (Puf)	2.63
Fuel-to-Coolant Volume Ratio Range	0.10-1.282	0.111-1.127	0.33-1.00
Mean Value of k_{eff}	1.00257	1.01783	1.0103

* S = Square, H = Hexagonal

approximate relation (See Appendix C):

$$\left[\frac{\Delta X}{X}\right] \approx -2 \left[\frac{\Delta k}{k}\right] \qquad (2.6)$$

provides an estimate of this effect, where X is the enrichment, and K the multiplication constant. Using the above prescription, a +1% bias in K will lead to a -2% bias in fissile enrichment. (i.e., 3.00% Pu would be computed as 2.94%). This is well within the precision required of scoping studies of the present type.

Thus we conclude that the calculational method is suitable for present purposes, with the further reservations noted in the following section. More detailed information on each benchmark lattice including the calculated K effective, lattice parameters and references are presented in Appendix A.

2.4.2. Comments on LEOPARD and the ENDF/B - IV Cross Section Library

In this section several observations on the calculational methodology employed in LEOPARD will be made. This will be followed by some comments on the ENDF/B-IV cross-section library.

The particular comments to be made here on LEOPARD, concern its treatment of plutonium-fueled lattices. In general, neutronic analyses of LWR's fueled with plutonium have been found to be less accurate than analyses of LWR's fueled with enriched uranium. This is due primarily to the existence in the fuels of higher concentrations of the isotopes of plutonium; the neutron cross-sections of the plutonium isotopes have signifi-

cant resonances at thermal and near thermal energies. Of particular interest is the large, sharp Pu-240 resonance at 1.056 ev. $\pm$ 0.002 ev(M-2) with a capture width of 0.031 $\pm$ 0.003 ev and a total width of 0.0333 $\pm$ o.003 ev, which leads to strong self-shielding. Plutonium-240 has been widely recognized as a significant problem in calculations of plutonium fueled cores (C-5)(P-1).

In discussing LEOPARD's treatment of plutonium fueled lattices, it is best to compare it to another depletion code LASER (P-1). Both LEOPARD and LASER utilize a unit cell which is assumed to be in an inifinite sea of like cells (since the boundary-conditions are zero net current) and the fast flux is assumed to be spatially flat across the cell, with leakage accounted for by an input geometric buckling, B_g^2, which is used in the calculation of K_{eff}.

It has long been established that LASER is superior to LEOPARD in the calculation of plutonium-fueled lattices. Since both codes use MUFT (B-2) in the epithermal and fast regions, the difference in the accuracy of their treatment of plutonium fueled lattices is in the thermal region. In the thermal region LASER calculates, using integral transport theory, the neutron spectrum at up to 14 space points in the cylindrical cell, thereby calculating a spatial distribution (1 dimensional) of the various nuclides as the cell is burned.

LEOPARD on the other hand performs a zero dimensional calculation, employing an approximate method of treating space-energy effects by means of multi-group disadvantage factors.

The disadvantage factors are computed using the model of Amouyal, Benoist and Horowitz (the ABH method) (A-9) for each of the 172 thermal energy groups.

However, plutonium isotopes exhibit strong spatial dependences with burnup. (Celnik et al (C-10) has shown that variations with burnup of the number densities of plutonium isotopes are as high as 110% across the fuel pin). This implies that LEOPARD is at a disadvantage. However, Celnik et al (C-10) also found that the errors in representing spatial depletion tend to be cancelled by the softer spectrum resulting from the zero dimensional calculation of LEOPARD, with both LEOPARD and LASER giving essentially identical reactivity data as a function of burnup. Although this does not imply that the end-of-life nuclide concentrations agree, previous work (M-5) has shown that for the fissile isotopes of plutonium (which are the main interest in this work) LEOPARD's end-of-life nuclide concentrations vary at most 4% from comparable LASER calculated values, with the variations having cancelling effects.

Another advantage of LASER lies in its thermal energy cutoff of 1.855 ev which was done explicitly (P-1) to include the 1.05 ev resonance of Pu-240; hence allowing for this resonance to be treated in much more detail by the space energy transport equation used by LASER in the thermal region. LEOPARD's disadvantage here lies in its 0.625 ev cutoff, leading to a poorer treatment of the Pu-240 resonance (P-1).

In spite of its advantages over LEOPARD, LASER was not used in this study due to: (1) the outdated cross section library

of the version available at MIT which does not include thorium, (2) the expense involved in running this code and (3) the fact that benchmark studies and other previous work (M-5)(S-5) show that LEOPARD is sufficient for a scoping study such as this. Future work, (C-4), however is planned to use LASER as a check on the present study. Apart from the theoretical and numerical treatment used by the code, the ENDF/B-IV cross-section library may not be entirely adequate for the task at hand. In a recent communication from Argonne National Laboratory (B-3), it was pointed out that thorium cross-sections have not been re-evaluated in 10 years. Thus even though reactor analysts performing calculations for the thorium assessment program state that they are using the latest version of ENDF(ENDF/B-IV), the thorium cross-sections in this file are in fact those carried over from the earliest versions of ENDF/B.

Furthermore, it was pointed out that recent work done by B.R. Leonard (Battelle N.W.)(to be published in a forthcoming EPRI report), on thorium cross sections below 50 kev. has indicated that:

1. The thorium capture cross-sections and resonance parameters for the first few lowest levels are highly uncertain;

2. The thorium fission cross section and the total cross-sections are highly uncertain;

3. Above 50 kev nothing can be trusted and fast reactor calculations with the current evaluation must be viewed as suspect.

A more recent communication (P-2) has concluded that a

reevaluation of the $^{232}_{90}$Th cross sections is warranted and future work is planned in this area.

Acceptable computation of the thorium benchmark lattices in the present work, however, suggests that the methods and cross-sections should be adequate for present purposes.

2.5 Calculational Methodology for Reactor Mass Flows

In the interest of consistency and because burnup-optimization is the subject of other studies (E-2), the discharge burnup of the fuel was maintained constant at 33,000 MWD/MT (that of the equilibrium core in Maine Yankee, see table 2.1). Similarly, a three-batch core was employed throughout: i.e., one third of the core is replaced at each refueling. Thus the above conditions are typical of current PWR designs in general and Maine Yankee in particular. A linear reactivity model then defines an equivalent cycle burnup:

$$B_n = \left(\frac{2n}{n+1}\right) \cdot B_o \qquad (2.2)$$

where n is the number of batches per core, B_o corresponds to the fuel burnup at which a one-batch core would be just critical without poison (here 22,000 MWD/MT) and B_n is the final fuel discharge burnup of an n batch core (here 33,000 MWD/MT).

Thus the calculational methodology was to first adjust the fissile concentration in the heavy metal oxide until the computed effective multiplication constant was unity at 22,000 MWD/MT for a control poison free core with leakage.

Natural boron was then added homogeneously to the coolant until the "average" cycle effective multiplication constant was unity. This "average" condition was defined as follows: In a system in which reactivity changes linearly with time (burnup), if one ignores the initial reactivity loss due to xenon and samarium buildup, the core-averaged effective multiplication constant will be achieved at the middle of cycle burnup.

The upper sketch in Fig. 2.3 demonstrates the enrichment adjustment step for the case of a boron-free core. Here, if the initial drop in the multiplication constant due to xenon and samarium buildup is ignored, there is a linear relation between the effective multiplication constant and burnup. The initial fissile enrichment is varied until a value, shown as ξ_3 in the sketch, gives a k_{eff} of unity at 22,000 MWD/mt; ξ_1 and ξ_2 are trial enrichments. The trial and error is facilitated by noting that k_{eff} varies approximately linearly with enrichment at a given fuel burnup.

At this point (see lower sketch of Fig. 2.3) natural boron is added until a boron concentration is achieved (β_2 in Fig. 2.3) for which k_{eff} is unity at midcycle (16,500 MWD/mt). It is important to note that the boron is not depleted, but the concentration remains constant over the entire burnup cycle (B-1). This is also a trial and error procedure; again facilitated by the linearity of k_{eff} with boron concentration at a given fuel burnup. Thus considering only the linear component of the reactivity history one has an average cycle k_{eff} of unity. Previous work has shown that this technique of using a constant

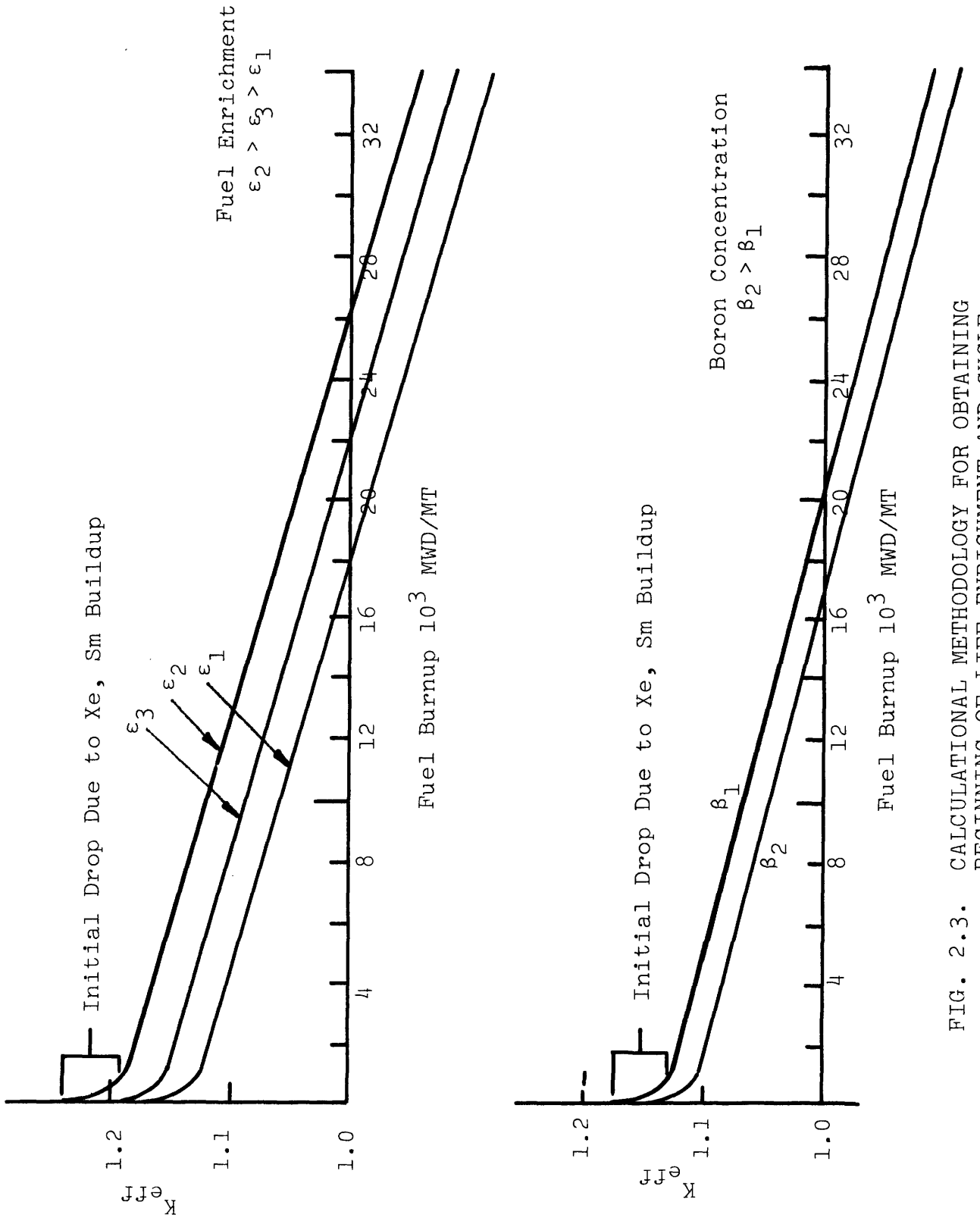


FIG. 2.3. CALCULATIONAL METHODOLOGY FOR OBTAINING BEGINNING-OF-LIFE ENRICHMENT AND CYCLE-AVERAGE BORON CONCENTRATION

cycle average boron concentration gives results very close to a burnup step by burnup step boron variation both in LWR's and LMFBR's (C-2)(W-6).

The fuel discharged from this final calculation (using enrichment ξ_3 and boron concentration β_2), after being extended to a burnup of 33,000 MWD/MT, is then the representative discharged fuel used in this study.

In all cases a fuel density of 92% of theoretical density was used. This reflects the consequences of manufacturing processes, stacking voids, fuel design and safety considerations and is the actual fuel density of Maine Yankee (see Table 2.3). Furthermore, it has been shown in previous work (F-3) that ore and separative work requirements minimize at these higher theoretical densities, at least for the systems studied.

The range of fuel-to-coolant volume ratio, Vf/Vm, considered in this study was 0.34 to 1.5, (Maine Yankee's Vf/Vm value, the base case, is 0.482).

A full discussion of the definition of fuel-to-coolant volume ratio, as used in this study, is given in Appendix G.2.

In deciding on the range of Vf/Vm values to be investigated the following points were considered. A practical lower limit on fuel-to-coolant volume ratio, Vf/Vm, is set by the need to avoid overmoderation and the attendant positive coolant temperature and void reactivity coefficients. In addition, for Vf/Vm values lower than those of current designs, it might be difficult to sustain the full thermal output of the core without going to excessively thin pins.

The upper limit on Vf/Vm is set by manufacturing and thermal hydraulic limits. These limits are not well established at this point, but it should be noted that LMFBR cores of current design, which are rather tightly packed with fuel pins, have Vf/Vm values of approximately 0.75.

Although it was our judgement that Vf/Vm values of 1.5 or less covered the range of interest, it should be noted that Edlund has recently suggested core designs in the range $1.9 \leq Vf/Vm \leq 3.0$ (E-3). A further reason for restricting the upper limit on Vf/Vm in the present work was the fact that tight lattices have a very hard spectrum, and there is some concern that thermal reactor based physics methods may not be fully adequate for such hard spectra. Experimental benchmark data does not exist, as far as a thorough literature search indicated, for Vf/Vm values much in excess of 1.0.

The fuel-to-coolant volume ratio variation was conducted as follows. Although in some previous studies (S-1),(O-1), Vf/Vm was varied by changing the fuel pin diameter, in this study the fuel diameter was held constant and the lattice pitch was varied. Since the effective resonance integrals of the dominant fertile species (U-238 or Th-232) are so strongly dependent on fuel pin diameter, it was felt that this procedure was more appropriate, since it does not confuse the effects of volumetric composition and fuel lump size.

The effect of pin diameter was also independently investigated: the results, which are presented in Chapter 4, confirm the approach taken in this study, i.e., that the fuel pin diameter

is not an important variable with respect to resource utilization and volumetric composition, Vf/Vm, is the major variable.

Thus, in all cases parameters relatively unimportant to ore and SWU requirements, such as clad thickness and the volume occupied by structural material were also kept constant.

A further constraint was that the supercell thermal power was held constant. The supercell power, Q_c, is given by:

$$Q_c = \frac{Q'''\,avg \;.\; p^2 \;.\; H}{(1 - NLF)} \qquad (2\text{-}3)$$

where Q"' avg is the average volumetric power density of the supercell, P is the lattice pitch, NLF is the non-lattice fraction (the ratio of the volume of the "extra" region containing structural material, water not associated with the unit cell, and the like, to that of the entire supercell) and H is the fuel length.

As the fuel length and non-lattice fraction were maintained constant, the corresponding ratios of Q_c for two Vf/Vm values, becomes:

$$\frac{Q_c^{(1)}}{Q_c^{(2)}} = \frac{Q'''\,avg^{(1)}}{Q'''\,avg^{(2)}} \times \frac{P_1^2}{P_2^2} \qquad (2\text{-}4)$$

Thus maintaining the supercell power constant implies that:

$$Q'''avg. \;\; P^2 = \text{constant} \qquad (2\text{-}5)$$

Since Q_{cell} and the fuel pin diameter are kept constant the effective linear heat generation rate Q', and the thermal heat flux Q" are also maintained constant. Variations in thermal

conductivity (the primary materials here, UO_2 and ThO_2, have very similar values), and in heat transfer coefficients (which are a function of coolant flow rate etc.) and other parameters, are assumed to be negligible. Thus the temperature field within the fuel and moderator (which does affect neutronics) is the same for all cases calculated.

The thermal hydraulic consequences (pressure drop, emergency cooling, etc.) arising from these assumptions are of obvious concern, but beyond the scope of the present work. They are, however, the subject of other related research at M.I.T. (C-4), (G-1).

A final point to note here concerns the lumped fission product cross-section used by LEOPARD (B-1). In LEOPARD, all fission products except I-135, Xe-135, Pr-149 and Sm-149 are lumped into one pseudo-element. The lumped fission product cross sections for plutonium fuels are higher than for uranium fuels. In order to account for this difference, the results of Spierling's work (S-5) were employed.

2.6 Results

The detailed results of the supercell calculations of the present study are presented in Appendix A. In this section the discussion will center on some selected results of the present calculations. The primary discussion will center on (1) the "Phoenix Effect" (H-2), and its pertinence to the present work (2) the effect of the fuel-to-coolant volume ratio on the neutron spectrum and (3) the question of the degradation of fuel isotopic

composition.

2.6.1. The Phoenix Effect

The same properties which give rise to analytical problems (as mentioned previously) with plutonium fuels also lead to some unique advantages, one of which is the "Phoenix Effect," which involves the transformation of fertile Pu-240 by neutron capture into fissile Pu-241.

Plutonium discharged from a typical PWR has a large concentration of Pu-240 (here 26% by weight). As was discussed above, Pu-240 has a large, sharp resonance at 1.05 ev and thus becomes, in effect, a self-shielded burnable poison for thermal neutrons. Since neutron capture also results in the production of a high-worth fissile isotope, Pu-241, there is an added reactivity increase above that due to removal of a burnable poison. The term "Phoenix Fuels" has been applied to high concentration Pu-240 fuels, because fissile Pu-241 is "reborn" from Pu-240 "ashes" much like the legendary Phoenix of mythology.

Previous work (H-2), (E-2),(A-3),(A-4),(G-2), has indicated that Phoenix fuel should provide a flatter reactivity swing during fuel depletion than comparable uranium fuel. Thus one can expect reduced control requirements to hold down excess reactivity using this "Phoenix fuel".

As previous studies, (H-2), have found, the reactivity swing of these fuels decreases significantly with increasing initial Pu-240 content. In the same studies it was also found that the reactivity swing for a given fuel isotopic composition

also decreased with fuel-to-coolant volume ratio.

Similar effects were observed in the present study. Figure 2.4 is a plot of k_{eff} versus burnup for 4 lattices (1) UO_2 (slightly enriched U-235) at the base case Vf/Vm = 0.4816, (2) PuO_2/UO_2 with the same initial plutonium concentration as used in this study (see Table 2.3) with a base case Vf/Vm of 0.4816, (3) PuO_2/UO_2 with the same Vf/Vm value with an initial plutonium concentration similar to that of Table 2.3, but with the Pu-240 content replaced by UO_2 and (4) PuO_2/UO_2 with the same plutonium concentration as in (2) but at the highest Vf/Vm value of 1.5.

The figure demonstrates first that the reactivity swing of the plutonium fueled lattices is significantly lower than that of a comparable uranium-fueled lattice. Furthermore the reactivity swing is decreased by increased Pu-240 content in the initial fuel. And finally, the reactivity swing is further decreased with an increased fuel-to-coolant volume ratio. In the present work, the same effect was seen in the PuO_2/ThO_2 lattices.

2.6.2. Effect of Fuel-To-Coolant Volume Ratio on the Neutron Spectrum

The effect of the fuel-to-coolant volume ratio on the neutron spectrum was investigated using the ratio of "fast" (>0.625 ev) to thermal fluxes output by LEOPARD as a spectral index. In order to present these ratios on a consistent basis, the flux ratios were determined at the middle of the burnup cycle for a critical, borated lattice (i.e. when $k_{eff} = 1.0$). Table 2.5 lists the results of these calculations. The main

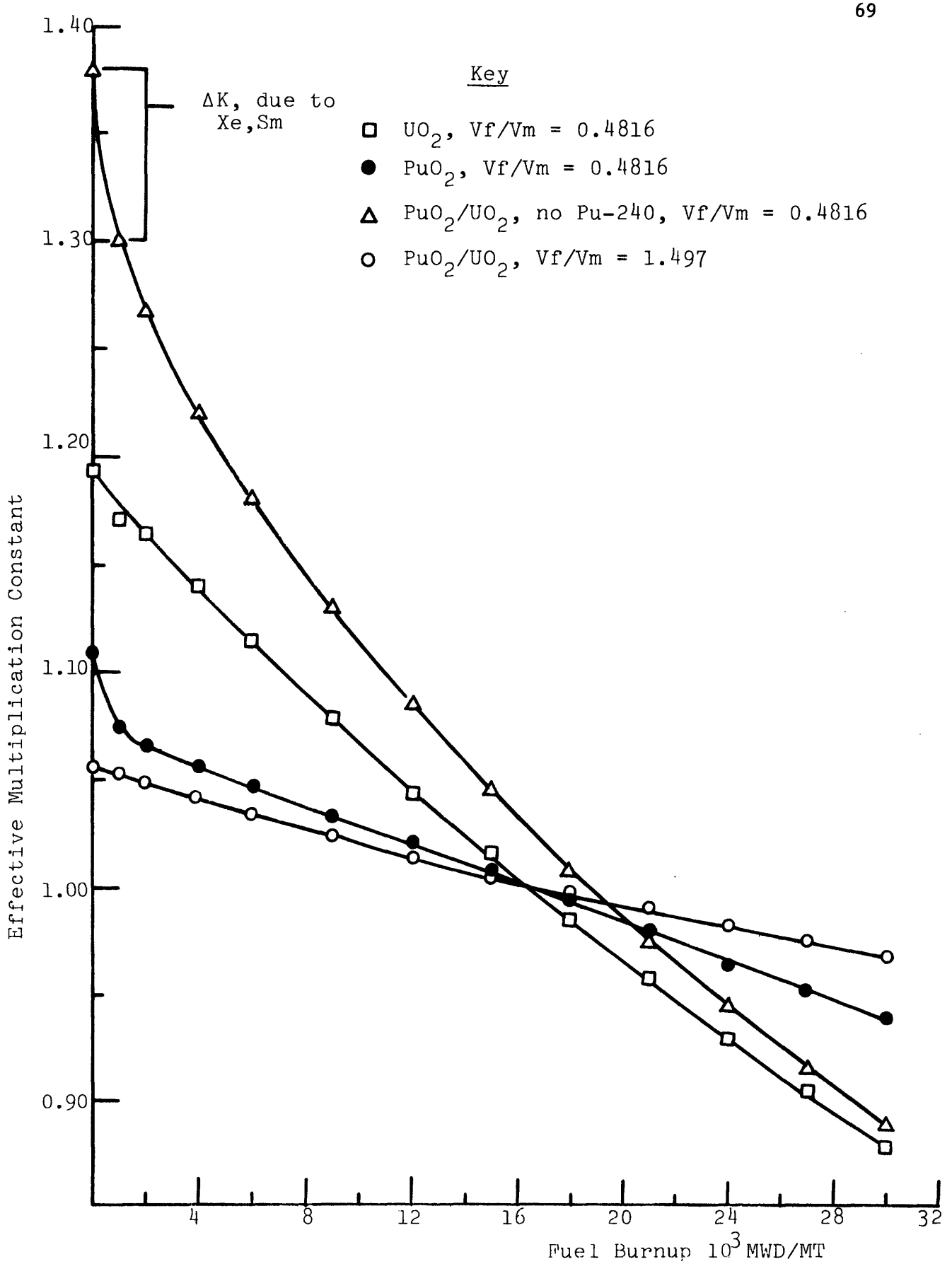


FIG. 2.4. COMPARISON OF REACTIVITY LIFETIME HISTORY FOR VARIOUS CASES

TABLE 2.5

RATIO OF FAST TO THERMAL FLUXES, ϕ_1/ϕ_2 AS A FUNCTION OF LATTICE Vf/Vm[1]

Case	A	B[2]	C	D
Vf/Vm	0.338	0.4816	0.9161	1.497
Lattice Type				
1) UO_2 (slightly enriched U)				
ϕ_1/ϕ_2	3.54	5.30	17.14	60.91
2) UO_2 (Fully enriched U-235/ThO_2				
ϕ_1/ϕ_2	3.55	5.04	12.10	35.64
3) PuO_2/ThO_2				
ϕ_1/ϕ_2	7.43	12.43	144.7	460.5
4) PuO_2/ThO_2				
ϕ_1/ϕ_2	6.39	11.46	137.4	522.8
5) $^{233}UO_2/ThO_2$				
ϕ_1/ϕ_2	3.44	4.93	11.36	28.35

[1]Thermal cutoff (0.625 ev)
[2]Current PWR

point to note is that at high fuel-to-coolant volume ratios, the increase in the flux ratio due to the tightening of the lattices is further magnified for plutonium fueled cores: their values are an order of magnitude larger than the uranium (U-233 or U-235) fueled cores. This enhancement, as discussed before, is due in part to the Pu-240 resonance at 1.056 ev. This was confirmed by comparing these results to lattices which had no initial Pu-240 content. For a PuO_2/ThO_2 lattice with Vf/Vm = 1.497 it was found that by replacing the initial Pu-240 content (see Table 2.3) with ThO_2, the flux ratio ϕ_1/ϕ_2 dropped from the value of 460.5 reported in Table 2.7 to 186.9. At higher fuel-to-coolant volume ratios, the fissile enrichment of plutonium fueled lattices was relatively high (see Appendix B), and therefore the Pu-240 content, was also high. This further hardens the spectrum and causes an enhancement of the "Phoenix Effect".

An important accompanying effect of the high fast-to-thermal flux ratio is a decrease in the xenon caused reactivity decrement (se Fig. 2.4). Since Xe-135 absorbs neutrons primarily in the thermal energy range, spectral hardening greatly reduces parasitic capture in this fission product poison and thereby reduces the excess reactivity (over-enrichment) needed to compensate for full power operation.

2.6.3. Fuel Degradation Due to Multiple Recycle

One of the major problems in a study such as the present one, is proper allowance for the degradation of fissile fuel due to the buildup of higher isotopes with several successive

recycles. Plutonium, for instance, has the problem of Pu-242 buildup. Recent work (H-3), on the effect of multiple recycle of plutonium in all-Pu-fueled-cores at the Vf/Vm values of today's reactors, has shown that the plutonium replacement value with respect to U-235 drops from an initial value of 0.9 to an equilibrium value of 0.6 after about 18 recycles. Of course, after 18 recycles very little of the original plutonium would remain. Table 2.6 shows a comparison of the isotopic composition of the discharge fuels after a 33,000 MWD/MT burnup in the consumer lattices, i.e. PuO_2/ThO_2, PuO_2/UO_2 and $^{233}UO_2/ThO_2$, as well as the initial composition. As the table demonstrates, for the plutonium-fueled lattices the degradation per cycle due to non-fissile isotope buildup decreases significantly as the fuel-to-coolant volume ratio is increased. This is partly due to the enhanced Phoenix Effect observed in these tighter lattices, which leads to enhanced Pu-240 captures, and higher fast-to-thermal flux ratios and therefore a higher Pu-242 fission rate. Table 2.6 also demonstrates that the fuel discharge composition for the $^{233}UO_2/ThO_2$ lattices is comparatively insensitive to the lattice fuel-to-coolant volume ratio.

Table 2.7 shows the ratios of charged and discharged fissile masses of the same consumer lattices. As the table demonstrates, in all lattices the general trend is for the fissile mass loss to decrease with increasing Vf/Vm. This effect is much more apparent in the plutonium fueled cores.

The preceding two tables demonstrate that there are two compensating effects involved in the phenomenon of fuel degrada-

TABLE 2.6

INITIAL AND FINAL FISSILE COMPOSITION OF CONSUMER [1] LATTICES[2], WEIGHT PERCENT

INITIAL COMPOSITION (W%)

Pu	(W%)	^{233}U	(W%)
Pu-239	54.2	U-233	90.7
Pu-240	26.0	U-234	8.0
Pu-241	13.9	U-235	12.5
Pu-242	5.9	U-236	Negligible

Discharge Composition (W%)

Case	A	B	C	D
Vf/Vm =	0.338	0.4816	0.9161	1.497
1) PuO_2/ThO_2				
Pu-239	15.3	19.4	46.1	46.6
Pu-240	36.8	31.2	24.5	26.8
Pu-241	25.3	29.0	21.7	18.9
Pu-242	22.6	20.4	7.7	7.7
2) PuO_2/UO_2				
Pu-239	29.5	37.3	52.2	53.3
Pu-240	32.9	26.7	21.3	23.8
Pu-241	19.1	21.5	19.6	16.4
Pu-242	18.5	14.5	6.9	6.6
3) $^{233}UO_2/ThO_2$				
U-233	73.8	72.8	75.2	77.5
U-234	20.6	19.7	18.0	15.8
U-235	4.7	6.6	5.9	5.9
U-236	0.9	0.9	0.9	0.8

[1] Lattices whose fissile supply is bred by Producer Lattices.

[2] 33,000 MWD/MT discharge burnup.

[3] Does not include Pa-233, which will ultimately decay to U-233.

TABLE 2.7

RATIO OF CHARGE AND DISCHARGE FISSILE MASSES OF CONSUMER[1] LATTICES[2]

Case	A	B	C	D
Vf/Vm =	0.338	0.4816	0.9161	1.497
Lattice Type	Fissile Mass Remaining Percent [4]			
1) PuO_2/ThO_2	25.4	31.1	69.0	71.8
2) PuO_2/UO_2	39.0	57.9	89.8	90.6
3) $^{233}UO_2/ThO_2$ [3]	59.5	65.0	72.5	80.4

[1] Lattices whose fissile supply is bred by Producer Lattices.

[2] 33,000 MWD/MT discharge burnup.

[3] Does not include Pa-233 in discharged fuel, which will ultimately decay to U-233.

[4] Ratio (x100) of primary fissile species discharged to amounts of same fissile species charged (i.e., case (1) does not include bred U-233).

tion, which are greatly enhanced in the plutonium fueled cores: (1) A lower fuel-to-coolant volume ratio lattice suffers severe isotopic degradation but large fissile mass losses. (2) a high fuel-to-coolant volume ratio lattice has large residual fissile masses but smaller isotopic degradation. Thus it may be possible to define a single representative weighting factor, W, between 0 and 1, to account for the effect of fuel degradation due to multiple recycle. This is discussed further in Chapter 3.

Finally, it should be noted that the U-235 cycle also suffers fuel degradation, due to the buildup of U-236 in multiply-recycled U-236. However, since this problem is further complicated by the options of re-enrichment of the spent uranium, or blending it with higher enriched uranium, to produce reload fuel, it is not addressed in this study.

2.7 Discussion and Conclusions

In this chapter we have reviewed the methods used in previous studies of this genre to establish a consensus view of an appropriate compromise between rigor and simplification (hence computation time and cost). In general the approach/computer programs/cross section library used in the present work may be regarded as comparable to these other studies.

In contrast to previous work, where the fuel pin diameter was varied to study the effect of changes in fuel to moderator ratio, in the present study the lattice pitch is varied. This difference should be kept in mind when comparing both the absolute results and the trends.

It has been seen that the benchmarking of the burnup code used in this study, EPRI-LEOPARD and its ENDF/B-IV cross section library, against UO_2, PuO_2/UO_2 and $^{233}UO_2/ThO_2$ light water criticals and exponentials has given results which are quite acceptable for the present work, (average k_{eff} for 110 lattices was 1.00875). However, it must be noted that there may be problems with the thorium cross-sections of the ENDF/B-IV cross section library, which have not been updated in nearly a decade. Furthermore, we have seen that LEOPARD does not treat plutonium fuels as well as some other codes. However, for the present study it appears to be quite acceptable.

A general note of caution was raised in regard to very tight pitch lattices since as one goes beyond a Vf/Vm value of roughly 1.0, the reactor becomes highly epithermal and thermal reactor oriented methods, codes and cross section sets are pushed to their limits of proven applicability. The paucity of experimental benchmarks in this region was also noted.

Work underway at M.I.T. (A-5) is concerned with extending fast reactor methods to these tight lattice systems, which approach (flooded) steam-cooled fast breeder lattices in composition.

Finally, it must be noted that "Phoenix Fuels" as defined previously (E-2) result when lattices low in U-238 content are fueled with plutonium. Others (E-2) have previously noted that the possible augmentation of pure Phoenix fuels by the addition of U-238 or thorium offers many avenues for investigation. In this study, it has been seen that this effect does indeed occur in such cores.

CHAPTER 3

REACTOR SYSTEMS MODEL

3.1 Introduction

In Chapter 2, the methodology for calculating the charge and discharge fissile and fertile masses of the dedicated reactors considered in this study was presented. However, the parameters of importance to a reactor economy are the total requirements for ore and separative work of a given system of reactors per unit of electrical energy produced by that system. In this Chapter, a concise model for obtaining system ore and separative work requirements from the results of Chapter 2 will be developed.

3.2 Previous Work

Previous work in the area of system's models has involved for the most part very involved models, with many degrees of freedom, which consider individual reactor types and reactor systems in great detail, and which also incorporate economic aspects, often involving optimization of future energy scenarios. Since the present interest is in simple approaches, the discussion of these more complex analyses will be kept on a qualitative level: mathematical and other details can be found in the literature referenced.

Table 3.1 lists a number of currently popular system's models. Due to the large number listed, only a representative selection will be reviewed here.

TABLE 3.1

CURRENT ELECTRIC POWER SYSTEM'S MODELS

MODEL	DEVELOPER	REFERENCES
1) SANS	ASEA-ATOM GBRA	(Z-1)(G-3)
2) NEEDS	B.C.L.	(W-1)(M-4)
3) MCFLOW	M. Nagel, R. Cerbone (GA)	(N-1)
4) ALPS	HEDL	(H-4)(H-5)
5) REM	P.L. Joskow, M.L. Baughman	(J-1),(J-2)
6) RES/BESOM	BNL	(C-9),(W-4) (A-8),(N-5)
7) (1)	P.L. Auer, C. Braun	(A-7)
8) (1)	P. Fortescue	(F-4)
9) GAECON	B. Pellaud (GA)	(P-4),(N-2)

1 Model has no formally assigned name.

The SANS code (Z-1), (G-3) was developed originally by ASEA-Atom and later modified and improved by the European Association for the Gas Cooled Breeder Reactor (GBRA), and used for analyzing converter and breeder reactor systems.

In SANS, various types of plant loading (base, peaking) are allowed and both conventional and nuclear capacities are treated. The total system growth is specified as input in the form of a pure or truncated exponential (second or third order polynomial), with the initial installed capacity at time zero being conventional (non-nuclear). The conventional capacity is assumed to increase linearly with time, until only the peaking load is supplied by conventional stations, after which the proportionality between total and conventional capacities is assumed fixed.

The model allows for two different converter reactor types and two different breeder types, representing first and second generation reactor designs. The first generation converter is always introduced in Year 1; the other reactor types may be introduced later at arbitrary times. After the introduction of an advanced reactor, no more first generation reactors (converter or breeder) are installed.

For each converter type, the gross U-235 demand, and the net fissile plutonium discharged must be specified, both for the initial inventory and the replacement fuel. For both the initial inventory and replacement fuels the in-and outgoing uranium enrichments and corresponding equivalent units of separative work are also to be specified.

For each breeder class, the specific fissile plutonium inventory, including the out-of-pile inventory, and the net fissile plutonium production per year are required as program inputs. Finally, the lifetime of each reactor is specified.

The above complete the required input for the model as far as the calculation of ore and separative work requirements of the system. Due to the restricted interest of the present study, both the required inputs for the economic analysis, and the details of the economic analysis itself will not be discussed here.

With the above specified reactor characteristics, and a growth scenario, the cumulative system requirements for ore and separative work units, and the fissile plutonium stockpile are calculated at intervals of one year for the specified period of the study.

Enriched depleted uranium is segregated from enriched mined uranium and thrown away after sufficient depletion to avoid the problem of U-236 buildup. Plutonium is stockpiled during the time before the breeders have been installed, and is then used to fuel the initial breeders. All further net plutonium produced after the introduction of the breeder is transferred to inventories of new breeder reactors, with any extra capacity needed supplied by converter reactors, i.e., the breeders are allowed to grow at the maximum possible rate, constrained only by the supply of fissile plutonium.

MCFLOW (N-1) is another code of this general type, developed at General Atomic to examine the impact of gas-cooled reactor

technology on nuclear reactor implementation studies.

This code requires as input the following reactor parameters:

1. The initial core commitment in terms of fissile plutonium, U-235, and the like, per MW(e) installed;
2. The net yearly consumption of these same constituents in Kg per MW(e) -yr generated at a specified load factor; (net yearly requirements are negative when there is a net production of a given isotope).

Over an assumed 30 year reactor life, the reactor inventory requirements for year N are taken to be the product of [MW(e) installed in year N minus MW(e) retired in year N] and [inventory requirement per MW(e)], an assumption which will be discussed later in this chapter.

MCFLOW also requires as input:

1. A projected installed nuclear electric demand; and,
2. An initial reactor mix, satisfying pre-selected new reactor technology introduction-dates and rates, and meeting the projected electric demand.

Starting with this mix of reactors the user then runs MCFLOW iteratively to determine an optimum reactor mix subject to the following constraints:

a. minimum U_3O_8 ore usage;
b. maximum use of artificially bred materials: U-233 and fissile plutonium (the stockpiles of which are minimized

at the end of the period of study).

Having obtained an optimum mix, MCFLOW is then used to calculate the associate costs of the system.

NEEDS, Nuclear Energy Electrical Demands Simulation, (W-1) (M-4) is a modeling code developed for simulating the nuclear sector of the U.S. Power economy by Battelle Columbus Laboratories and used in a recent study performed for the National Science Foundation (M-4).

In the modeling used in the code, the growth of reactor capacity is approximately continuous with new reactor concepts made available at discrete points in time. Rather than specifying a decision-making policy on the basis of economic criteria, as is commonly done in optimization programs (such as ALPS which will be discussed later), the user specifies the strategic decision rules. In NEEDS, the additions to capacity are made sequentially according to the rules specified in the desired "nuclear mix". The desired "nuclear mix" specifies the reactors included in a particular run plus four parameters for each reactor system in the mix:

1. The reactor priority;
2. the maximum allowable fraction of total additions to capacity;
3. the maximum allowable fraction of the total existing capacity;
4. the maximum permissible capacity in MW(e) of this reactor type.

A reactor priority of 1 implies maximum growth for that reactor. However, the growth rate of all new capacity is arti-

ficially constrained to no more than a doubling of new capacity each year with a first year capacity of less than 2000 MW(e). Reactor parameters input into the model are similar to these for MCFLOW and SANS already discussed.

The dynamics of the model is as follows. At the beginning of year N, after the existing capacity is reduced by that of retired capacity, the capacity to be installed is determined by the difference between the input nuclear demand and the existing capacity. The desired "nuclear mix", as described above, is then used to calculated the new capacity of each reactor type subject to three constraints:

1. N must not be less than the year of introduction of that type reactor;
2. the startup requirements for fissile materials, initial inventory plus replacement core loads until recycle, must not exceed the inventory of available fissile materials (because the model assumes a closed system, apart from mined uranium and thorium);
3. the additional capacity of a reactor type does not exceed twice the existing capacity of that reactor type, (with initial capacity limited to 2.0 GW(e).

At the beginning of each year, using the calculated installed capacity, the startup fuel requirements including fissile, U_3O_8, and ThO_2 requirements and fuel cycle processing requirements are calculated. Plutonium and U-233 inventories are reduced by startup requirements. Fissile materials recovered from retired plants are added to the inventory before computing the additional requirements. At the end of the year, the resource requirements for makeup feed, and the bred fissile

materials are added to these respective inventories.

NEEDS has two features that are of particular interest here.

First, to insure an adequate supply of fissile material, enough fissile fuel is removed from the fissile inventory to keep the reactor in operation until its own recycled fuel is available depending on the out-of-core cycle time. This underestimation of the inventory of fissile materials, however, is counterbalanced by an overestimation of the fissile inventory due to a second feature: the bred fissile materials are assumed to be available immediately upon discharge from the reactor (no cooling or reprocessing time lag).

The final system model to be discussed here is ALPS. A Linear Programming System, developed by Hanford Engineering Development Laboratory (HEDL)(H-4),(H-5), and used in the system's analyses contained in WASH-1535, the Liquid Metal Fast Breeder Reactor Environmental Statement (U-1). ALPS is used to predict the optimum growth patterns in a mix of fossil and nuclear power plants, using linear programming techniques.

ALPS is by far one of the most complicated and extensive of systems models and therefore its calculational methodology can only be briefly summarized here. Its degree of sophistication is seen by the fact that it not only includes fossil plants but distinguishes between coal, oil and gas-fueled plants and even treats fossil plants that consume a mix of coal, oil and gas.

The ALPS fuel cycle model considers nine types of fuels:

1. Th-232;
2. U-233;
3. U-235;
4. U-238
5. Fissile plutonium;
6. Total nuclear fuel
7. coal
8. oil
9. gas

In addition, the model permits any number of independent fuel cycle schemes for each plant, e.g., separate treatment of the core and blanket of a breeder, or each zone in a multi-zoned core.

The inventory of each of the fuel types is an initial condition for nuclear plants. The feed and discharge must be specified on a year-by-year basis; based on a year-by-year load profile for the 30 year lifetime of each plant type. The same convention is also followed for the fossil plants with respect to the coal, gas and oil fuels.

Although mass flows are assumed to occur during the year specified, stockpile changes are displaced in time by input lead or lag times. A growth pattern is assumed for installed electrical capacity and new capacity is installed at the beginning of each year subject to three constraints:

1. the mix of power plants must meet the energy demand;
2. constraints on fuel availability, plant availability, maximum plant growth rate etc. are to be met;
3. the total discounted costs are minimized utilizing linear programming techniques.

The newly installed capacity at the beginning of each step is chosen, under the above three constraints so as to minimize total discounted costs.

The four models discussed above cover two classes of system's models:

1. those employing linear programming techniques (ALPS),
2. those employing simulation techniques (SANS, MCFLOW, NEEDS).

In simulation models the user specifies the rules under which a given reactor system is added to the existing mix of reactors, making it possible to examine a continuous range of reactor scenarios.

In models employing linear programming techniques such as ALPS, the objective function is frequently based on generation costs, with the result that the mix of reactor types chosen by the model is always the most economic, i.e., it calculates the maximum market penetration of the most economic reactor system. This technique has problems stemming from the uncertainty in economic parameters, particularly capital costs of a new reactor type (to say nothing of current LWR's where factor-of-two vari-

ations in installed costs per KW(e) have been experienced). Because of this uncertainty the economic optimum mix of reactors is itself subject to great uncertainty. Deonigi, (D-1), found that a change of $10 per kilowatt was sufficient to completely alter the final mix of reactor types selected by ALPS. This can lead to real or hypothesized biasing: The results developed using ALPS for WASH-1535 (U-1) have been criticized as being biased towards the LMFBR because of the AEC's choice of capital, cost, nuclear demand and uranium resource availability (C-7).

A third classification of system's models, not represented among the four models discussed above, is the "simple model". These models, in spite of their limitations, as regards degrees of freedom, and extensive treatment of given scenarios, provide the user with a convenient, easy tool for examining various nuclear strategies. Moreover, since the prospects for a large number of new reactor types to penetrate the market are diminishing, the need for and pertinence of large programs allowing many degrees of freedom in the selection of future nuclear scenarios is diminished. To a great extent we are moving into a planned rather than a free nuclear economy, where the determinism of the simple model becomes appropriate rather than limiting. Models 7 and 8 (Table 3.1) are good examples of these simple models.

In order to organize the presentation of the results of Chapter 2, it was decided for the reasons presented above to utilize a simple model. To do this, a simple model was developed, tailored to the needs of the present study, using the

most attractive features of the other models referenced in Table 3.1.

In order to validate this model it was necessary to benchmark results against a standard. Due to the extensive detail and rigor of the treatment in ALPS and in view of its use as a standard by ERDA, selected results from ALPS were used to check the approach used in the present work, as others have also done (N-1). The criticism of ALPS regarding its choice of reactor mixes will not affect this benchmark since cases have been selected for comparison which are very restricted in terms of the options allowed. In the sections which follow the development of two versions of this simple model will be presented.

3.3 MASFLO-1, A Simple Systems Model

A reactor can be characterized as a black box into which a mass of material of one composition is charged, and from which another mass of material at a different composition, and electrical energy are discharged.

In a typical reactor, the core is first charged with an initial inventory of fertile and fissile material to start up the reactor. After this startup inventory, the periodic flows of fertile and fissile material are approximately the same: after about six cycles, the typical PWR will be in an equilibrium situation. For the case of annual refueling to a 3-batch-in-core reactor having a working lifetime of 30 years, this continues until the 28th cycle, where the flows are per-

turbed due to a down-grading in reload enrichment in anticipation of the end of reactor life (alternatively, standard batches can be used and discharged in an underburned state). Finally, at the end of reactor life, the core inventory is retrieved.

The MASFLO-1 model approximates the above situation by the treatment illustrated in Fig. 3.1, which shows the U_3O_8 requirements of a reactor in STU_3O_8 per GW(e) installed. The assumption is that the ore usage is the sum of two components: a step increment incurred in the year of startup, plus an annual uniform, steady state requirement, (a constant capacity factor is assumed).

Thus if the installed capacity of reactor type j is $E_j(t)$ in year t, the specific inventory SI_j (STU_3O_8/GW(e) installed) and the annual equilibrium requirements SC_j (STU_3O_8/GW(e)yr at a given capacity factor), then the system requirements $M_j(t)$ in year t are given by

$$M_j(t) = \Delta E_j(t) \cdot SI_j + E_j(t) \cdot SC_j \qquad (3.1)$$

$\frac{ST\ U_3O_8}{yr}$

System Startup Requirements (1) + System Equilibrium Requirements (2)

Where $\Delta E_j(t)$ is the net newly installed capacity in year t. Figure 3.2 illustrates the assumed growth pattern for this model, in which all new net installed capacity is added at the beginning of the year.

Note that term (2) in Eq. 3.1 includes the purchase of

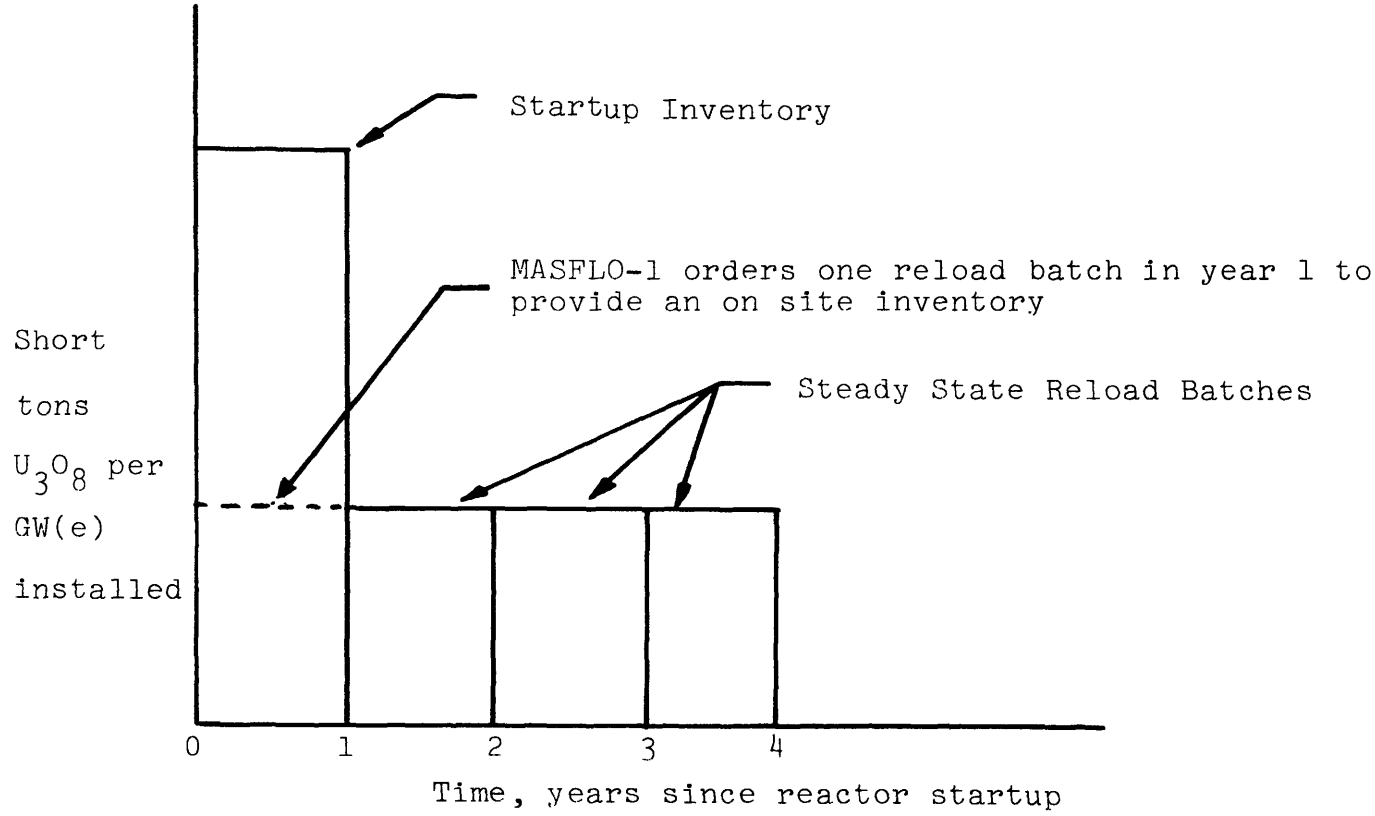


FIG 3.1. SIMPLIFIED MASS FLOW HISTORY FOR A TYPICAL REACTOR(*)

* annual refueling

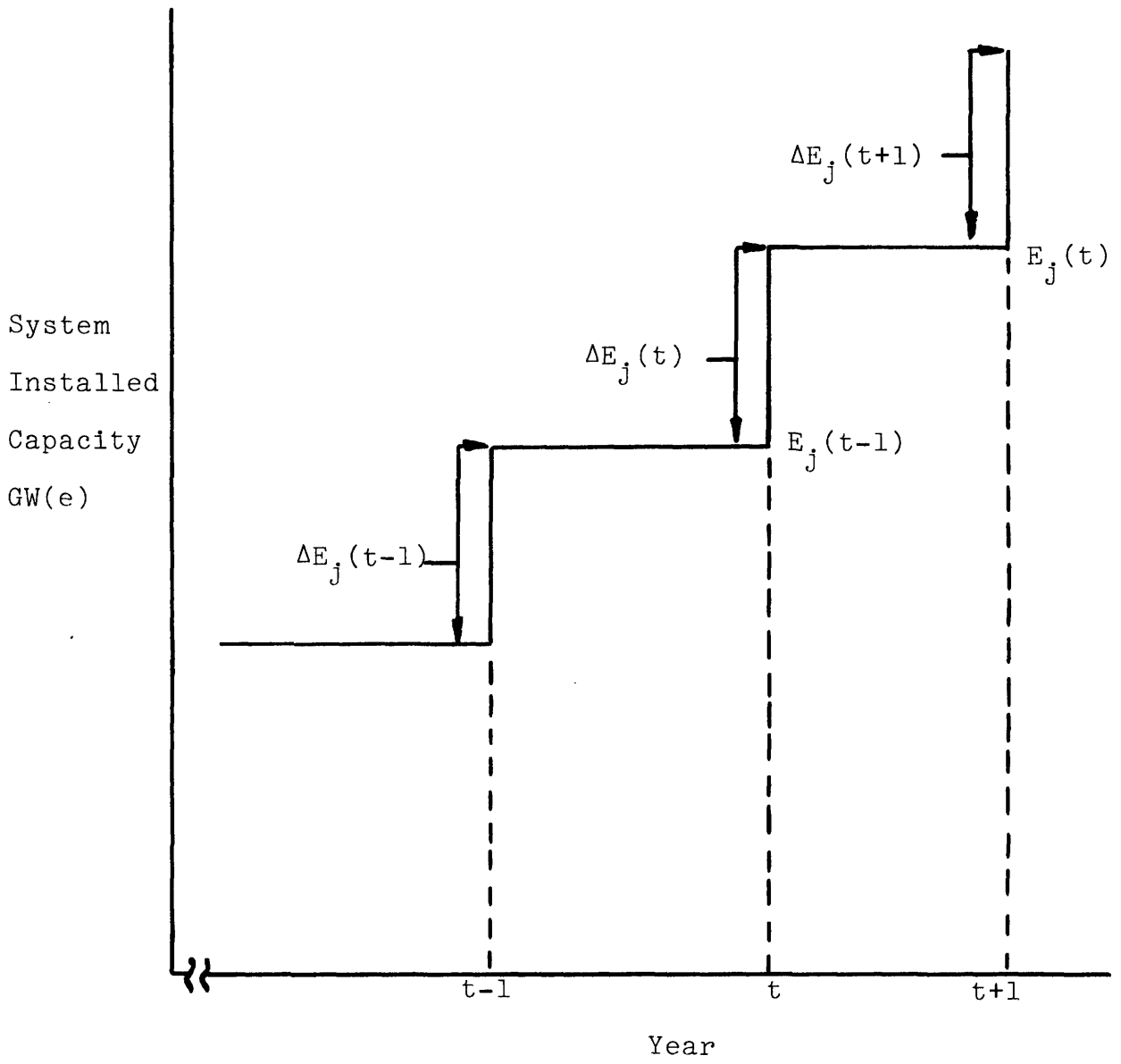


FIG. 3.2. ASSUMED GROWTH PATTERN USED IN PRESENT MODEL

the first reload for the newly installed capacity $\Delta E_j(t)$ in year t. This corresponds to ordering the first and subsequent reloads a year in advance of use; thus we account for on-site inventory in this manner. The means by which other out-of-core off-site inventory (needed to fill the new enrichment and UF_6 production pipelines) can be allowed for, is discussed later in this chapter.

If one assumes that the system growth is exponential, then the system capacity $E_j(t)$ is given by

$$E_j(t) = E_j(0)e^{\frac{rt}{100}} \tag{3.2}$$

where $E_j(0)$ is the installed capacity of reactor type j in year zero and r is the percent per year system growth rate. With the above assumption Eq. 3.1 becomes

$$M_j(t) = \frac{d}{dt}\left(E_j(0)e^{\frac{rt}{100}}\, SI_j + SC_j E_j(0)e^{\frac{rt}{100}}\right) \tag{3.3}$$

or

$$M_j(t) = E_j(0)e^{\frac{rt}{100}}\left(\frac{r}{100}\, SI_j + SC_j\right) \tag{3.4}$$

dividing by $E_j(t)$

$$m_j = \frac{r}{100}\, SI_j + SC_j \tag{3.5}$$

where m_j is the annual specific ore requirement of a system of reactors of type j, growing at a rate r% per year, per GW(e)

of installed capacity. Note that although an exponential growth rate is used here the model can be applied to any growth scenario $E_j(t)$, provided that the growth rate r(t) is defined as:

$$r(t) = \frac{\Delta E_j(t)}{E_j(t)} \tag{3.6}$$

which, when applied to Eq. (3.1), yields Eq. (3.5).

Advantages of the above approach, in addition to simplicity and ease of application, are that it displays results in a format which is independent of system installed capacity and provides a convenient and explicit measure of system performance as a function of the important growth rate variable, r.

In the subject model the fissile inventory discharged at the end of reactor life is dealt with by assuming that this inventory is used in the startup of a replacement reactor, and therefore the term $\Delta E_j(t)$ in Eq. (3.1) represents the net newly installed capacity; other models (N-1) also use this strategy. MASFLO-1 is appropriate for systems of reactor types which do not interact with other reactor types in terms of bred fissile transfers: for example, it is directly applicable to systems which adopt a once-through fuel cycle. The above formulation is also applicable to the computation of system separative work unit requirements, with the SC_j and SI_j terms now representing the corresponding values for separative work units, and m_j the system SWU requirements, MT per GW(e) (installed) year.

In the present study the reactor types examined were dedicated reactors, which utilize only one fissile species and which regularly trade bred fissile fuel among themselves. Thus

MASFLO-2 was developed to deal with these more complicated systems, as described in the section which follows.

3.4 MASFLO-2, A Simple System Analysis Model for Coupled Reactor Systems.

In the last section, it was noted that MASFLO-1 was not capable of handling dedicated, interacting reactors. Before proceeding to remedy this defect, it is appropriate to discuss the reasoning behind the decision to proceed in this manner rather than treating each case as a single reactor employing self-generated recycle.

First of all, limiting the study to dedicated reactors brings about certain safeguard advantages in that the plutonium and U-233 bred in a uranium and thorium economy can be confined to a select group of reactors. The presence of U-233 and plutonium also greatly increase fuel fabrication costs (as much as a factor of 2.8 for U-233 fuels and 2.3 for Pu fuels on a per mass basis, vs. standard $^{235}UO_2$ reactor fuel (K-1)). Hence, the usual decision is to confine these materials to as few assemblies as possible rather than to disperse them through an entire core. By dedicating an entire reactor to the same fuel type (say U-233 or plutonium), the problem of power peaking at the interface between zones of different assembly types is avoided.

In addition, a recent C.E. report (S-1) has shown that for a given reactor type, there is very little difference between segregated and homogeneous recycle. For ThO_2 (93% U-235)

reactors the 30-year ore requirements (STU_3O_8) per GW(e), (S-1), were 3483 STU_3O_8 for homogeneous uranium recycle and 3453 STU_3O_8 for segregated recycle, or a difference of less than 1%. Thus in as far as the overall neutron and mass balances are concerned it makes very little difference whether one recycles assemblies into their parent reactor or into a separate reactor. Hence the present results for a given lattice type may to first order be considered representative of all types of recycle: dispersed, segregated by pin or assembly, or confined to an entire core zone or to a dedicated reactor. Another important factor is that by using only one type of fuel in each reactor, individualized lattice optimization, (eg., fuel pin diameter, pitch) can be carried out to take advantage of the particular nuclear properties of each fissile/fertile combination. A final consideration is that by introducing dedicated reactors, it will be easier to characterize the resulting systems of reactors, with fewer fuel cycle computations, which are more readily benchmarked against experimental data.

With the above advantages in mind, the MASFLO-2 model will now be developed.

Due to the fact that uranium-235 is the only naturally occurring fissile isotope, all coupled systems must use a uranium-235/U-238 mixture in the initial reactor in the chain. Hence the two most important parameters for any coupled system of reactors are uranium ore and separative work (uranium enrichment) requirements.

The first task is to identify the reactor types and

fissile species to be incorporated into the model. Table 3.2 summarizes the classification of reactor types and fissile species to be used.

Before going into the system analysis it is appropriate to make the distinction between consumer and producer reactors. A producer reactor (j=1,2) utilizes enriched uranium as fissile feed to produce the fissile material (U-233 or Pu-239) for a consumer reactor.

Let us now develop the model by first starting with a few appropriate definitions:

XF_{ij}:(1)For i=1 and j=1,2, it is the weight percent of U-235 in charged uranium heavy metal, (2) for all other i and j it is the weight fraction of isotope i in reactor type j in terms of total heavy metal charged.

XD_{ij}: For i=1, j=1,2, it is the weight percent of U-235 with respect to total discharged Uranium-235, Uranium-236 and Uranium-238 heavy metal (See section 3.5). For all other i and j, it is the discharged weight fraction of isotope i from reactor type j with respect to heavy metal initially charged.

SL: loss factor in (1) mining and fabrication and (2) reprocessing and refabrication; the ratio of useful mass out of the above two process steps to mass charged.

r: the system growth rate in percent per year.

TABLE 3.2

CLASSIFICATION OF REACTOR TYPES AND FISSILE SPECIES OF MASFLO-2

Reactor Type - Subscript j

j	Reactor Type
1	UO_2 (slightly enriched U-235)/ U-238
2	UO_2 (93% enriched U-235)/ThO_2
3	PuO_2/ThO_2
4	PuO_2/UO_2
5	$^{233}UO_2/ThO_2$

Fissile Species - Subscript i

i	Fissile Species
1	U-235
2	U-233 (based)*
3	Pu-239 (based)*

* "based" indicates that this is the reference (dominant) fissile isotope in the fuel; in practice a representative isotopic mixture is employed.

E_j: the installed capacity of reactor type j in GW(e).

T_j: refueling interval, in years, of reactor type j: time between post refueling startups.

B_j: the final discharge burnup of the fuel of reactor type j, MW(th)D/MT, in the equilibrium cycle (MT = metric ton of initially charged heavy metal).

η_j: the thermal efficiency of reactor type j, MW(e)/MW(th).

N_j: (1) for i=1, j=1,2; N_j is the equivalent number of batch reloads in the initial startup core for reactor type j, in terms of ore usage(ie STU_3O_8)(2) for all other i and j, N_j is merely the ratio of the fissile loading of isotope i needed to start up reactor type j to the fissile loading of isotope i in an equilibrium reload batch of reactor type j.

P_j: heavy metal loading per batch of reactor type j (MT).

L: time-averaged system capacity factor (assumed constant for all reactor types in each system).

Let us now start with a UO_2 (slightly enriched U-235) reactor in a once through cycle and then develop the model to include a three-tier system recycling all useful discharged fissile material.

Before proceeding further, a very important distinction must be made. There are two alternatives to the treatment of ore and separative work utilization in this model, (1) on the basis of delivered energy, i.e., GW(e)yr (delivered) and (2) on the basis of rated energy output at a given capacity factor, i.e., GW(e)yr at a given capacity/factor, where:

$$GW(e)yr\ (delivered) = GW(e)yr\ (rated).L \quad (3.7)$$

Where L is the capacity factor. The basis chosen for this study is rated energy output at a given capacity factor, due to its general use in the literature (S-1), (C-8). Furthermore the capacity factor of all types of reactors was taken to be a constant value, L, for all reactor types, a simplification tailored to present needs. Ore usage per installed GW(e) at a capacity factor, L, is related to the heavy metal inventory in a reload batch by:

$$F_1 = \frac{1.3.P_1}{SL.E_1} \left(\frac{XF_{11} - X_W}{X_{NAT} - X_W}\right) \frac{STU_3O_8}{GW(e)} \quad (3.8)$$

Where:

F_1 is the yellowcake mined (in STU_3O_8) to provide P_1 MT of heavy metal.

X_W is the weight percent U-235 in the enrichment plant tails, (a value of 0.2% wt. % is used throughout the present

work). X_{NAT} is the weight percent U-235 in natural uranium, (a value of 0.711 wt % is used in the present work). If one assumes that all batches share power equally then the installed capacity E_1 at a capacity factor L is given by:

$$E_1 = \frac{B_1 \eta_1 P_1}{T_1 \ 365250 \ L} \ \underset{\text{(installed)}}{GW(e)} \tag{3.9}$$

(Note that the enrichment of P_1 is chosen to deliver a reactivity limited burnup, B_1.)

Annual ore usage, F_1', by the reactor is given by the product of ore per batch, F_1, and batches per year $\frac{1}{T_1}$ (where T_1 is the refueling interval);

$$F_1' = F_1/T_1 \tag{3.10}$$

Therefore ore usage per GW(e)yr at capacity factor L, by a system of reactors of type j=1 growing at an annual rate of r% per year is

$$F_{S1} = \underset{\substack{\text{Initial Startup of new reactors}}}{\frac{r}{100} \ F_1 N_1} + \underset{\text{reload}}{F_1'} \tag{3.11}$$

or using Eq. (3.10)

$$F_{S1} = \frac{F_1}{T_1} \left(1 + \frac{r}{100} \ N_1 T_1 \right) \tag{3.12}$$

Introducing Eq. (3.8), one obtains for F_{S1} :

$$F_{S1} = \frac{1.3\ P_1}{SL\ E_1 T_1} \left(\frac{XF_{11} - X_W}{X_{NAT} - X_W}\right) \left(1 + \frac{r}{100}\ N_1 T_1\right) \qquad (3.13)$$

If one now introduces uranium recycle, Eq. (3.13) can be shown to become:

$$F_{S1} = \frac{1.3\ P_1}{SL\ E_1 T_1} \left(\frac{XF_{11} - X_W}{X_{NAT} - X_W}\right) \left[1 - (SL)^2 RDI\left(\frac{XD_{11} - X_W}{XF_{11} - X_W}\right)\right.$$

Effect of Uranium Recycle

Once-through cycle

$$\left. + \frac{r}{100}\ N_1 T_1\right] \qquad (3.14)$$

Effect of System growth rate

where RDI is the ratio of the discharged heavy metal uranium to the charged heavy metal uranium for reactor type j, (RDI is several percent less than unity because of the loss of uranium heavy metal due to fission and capture processes).

Using a similar approach, a parallel expression can be derived for j = 2, the UO_2 (93% enriched U-235)/ThO_2 reactor, for the case of U-235 recycle only:

$$F_{S2} = \frac{1.3\ P_2\ \alpha}{SL\ E_2 T_2} \left(\frac{XF_{12} - X_W}{X_{NAT} - X_W}\right) \left[1 - (SL)^2\ \Gamma\left(\frac{XD_{12} - X_W}{XF_{12} - X_W}\right)\right.$$

$$\left. + \frac{r}{100}\ N_2 T_2\right] \qquad (3.15)$$

where α is defined by

$$\alpha = \frac{\text{charged uranium heavy metal (all isotopes)}}{\text{total charged heavy metal}} \qquad (3.16)$$

for reactor j=2, and Γ is defined by

$$\Gamma = \frac{\text{discharged thorium heavy metal (including U-233,U-234)}}{\text{charged uranium heavy metal (all isotopes)}} \qquad (3.17)$$

for reactor type j = 2, and E_2 is the installed capacity of reactor type j = 2, defined similarly to E_1 in Eq. (3.9).

Having examined both producer reactors, it is now appropriate to deal with a three-tier system incorporating consumer reactors. This is the most general case required and readily reduces to the two-tier system appropriate for several of the scenarios to be examined. Figure 3.3 shows a three-tier system in which a UO_2(slightly enriched U-235) reactor, a PUO_2/ThO_2 reactor and a $^{233}UO_2/ThO_2$ reactor are coupled together. The feed streams of the various reactors are also shown.

Reactor type j = 1 produces energy and transfers Pu-based fuels to reactor type j = 3, which produces a certain amount of energy and transfers U-233 based fuels to reactor type j = 5, which in turn produces a certain amount of energy, recycling U-233 to itself.

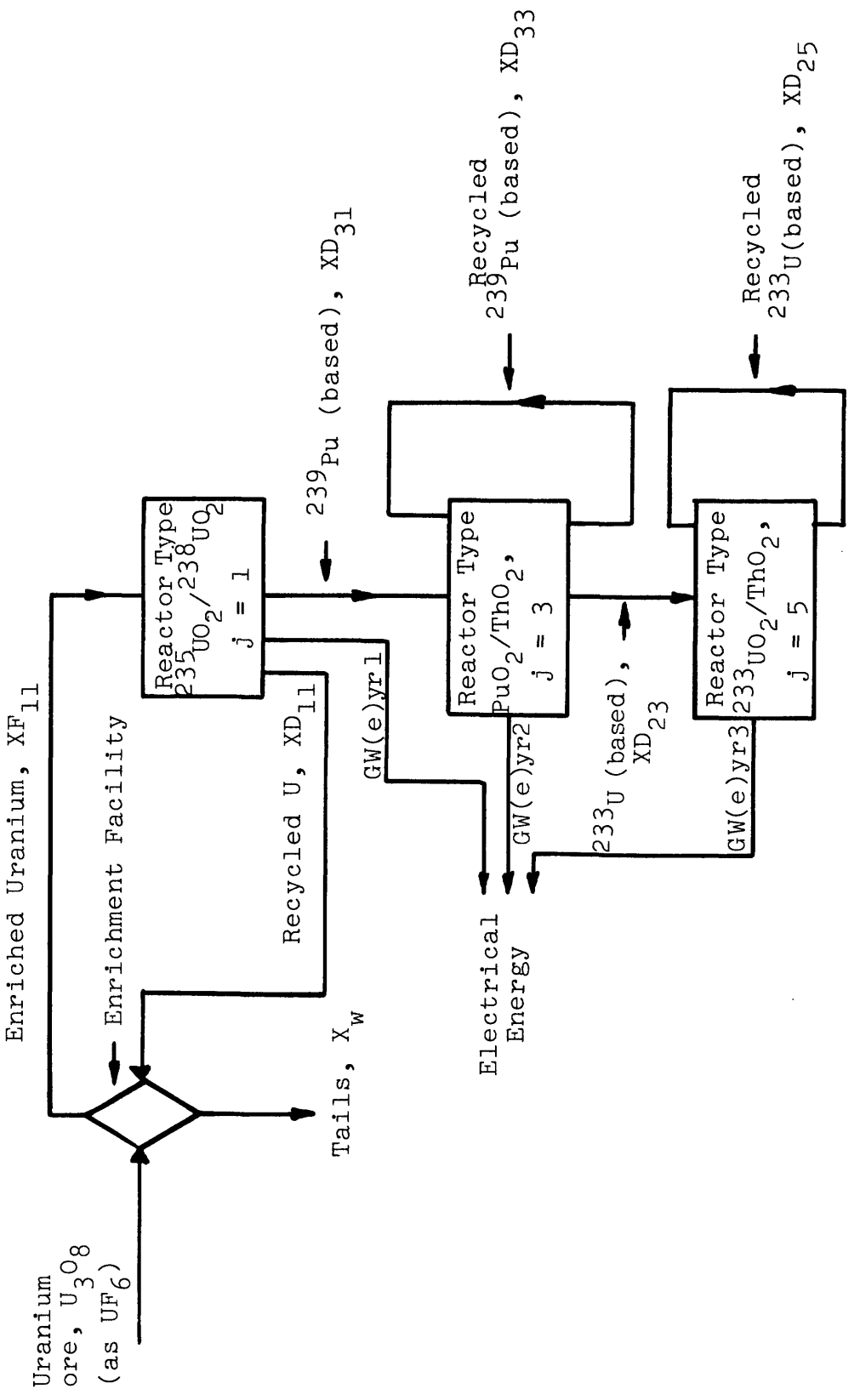


FIG. 3.3. COUPLING SCHEMATIC FOR A THREE TIER SYSTEM

In the general case the reactors in this system can have different thermal efficiencies, heavy metal loadings, capacity factors and the like. Thus the balancing of fissile mass flows between different reactor types in this system is best done on a basis of actual energy delivered in GW(e)yr (delivered). However, since for present purposes it is assumed that all reactor types in the system have a capacity factor L, then for present purposes, an equivalent technique for balancing the mass flows between different reactor types in this system is on the basis of GW(e)yr (rated) at a capacity factor L.

One can show that in generating one GW(e)yr (rated) at a capacity factor L, reactor type j = 1 produced PuD metric tons of Pu-239 (based) fuel, where PuD is given by

$$PuD = \frac{XD_{31} P_1 SL}{E_1 T_1} \tag{3.18}$$

Using the same methodology as before, in order to generate one GW(e)yr (rated) at capacity factor L, and a growth rate r% per year, reactor type j = 3 needs PuC metric tons of Pu-239 (based) fuel where PuC is given by

$$PuC = \frac{P_3}{E_3 T_3} \left[(XF_{33} - XD_{33} SL) + N_3 \frac{r}{100} XF_{33} T_3 \right] \tag{3.19}$$

where E_3, the rated capacity of reactor j = 3, is given by

$$E_3 = \frac{B_3 P_3 \eta_3}{365250\ T_3 L} \quad GW(e)\ (rated) \qquad (3.20)$$

Hence using Eqs. (3.18) and (3.19), in producing one GW(e)yr (rated) at capacity factor L, reactor type j = 1 allows reactor type j = 3 to generate R_{31} GW(e)yr (rated) at capacity factor L, where R_{31} is given by:

$$R_{31} = \left(\frac{P_1}{P_3}\right)\left(\frac{E_3 T_3}{E_1 T_1}\right) \frac{XD_{31}\ SL}{[(XF_{33} - XD_{33}\ SL) + N_3 XF_{33} \frac{r}{100} T_3]} \qquad (3.21)$$

Using a similar approach a U-233 (based) fuel transfer balance can be set up by making a mass energy balance between reactor j = 3 and reactor type j = 5. Here R_{53}, the energy in GW(e)yr (rated at capacity factor L generated by reactor j = 3 is given by:

$$R_{53} = \left(\frac{P_3}{P_5}\right)\left(\frac{E_5 T_5}{E_3 T_3}\right) \cdot \frac{XD_{23}\ SL}{[(XF_{25} - XD_{25}\ SL) + N_5 XF_{25} \frac{r}{100} T_5]} \qquad (3.22)$$

where E_5 is defined similarly to E_3 in Eq. (3.20).

The system ore consumption per GW(e)yr (rated) at capacity factor L produced by the entire system, F_{S3}, is the important parameter here, it is given by:

$$F_{S3} = \frac{S_{ysc}}{[\,GW(e)yr1 + GW(e)yr3 + GW(e)yr5]} \quad \begin{matrix} STU_3O_8 \text{ per } GW(e)yr \\ \text{(rated) at capacity factor L.} \end{matrix} \qquad (3.23)$$

where S_{ysc} is the ore charged to the system in STU_3O_8, and GW(e)yr1 is the electrical energy produced by reactor type 1; GW(e)yr is the electrical energy produced by reactor type j = 3 permitted by the transfer of Pu-239 (based) fuel produced in the generation of GW(e)yr 1 of electrical energy in reactor type 1, and GW(e)yr5 is the electrical energy produced by reactor type j = 3, permitted by the transfer of U-233 (based) fuel produced in the generation of GW(e)yr3 of electrical energy in reactor type 3.

Eq. (3.23) can be written as:

$$F_{S3} = \frac{S_{ysc}}{GW(e)yr1 \;\left[1 + \frac{GW(e)yr3}{GW(e)yr1} + \frac{GW(e)yr5}{GW(e)yr1}\right]} \qquad (3.24)$$

or

$$F_{S3} = \frac{S_{ysc}}{GW(e)yr1 \;\left[\,1 + \frac{GW(e)yr3}{GW(e)yr1}\;\left(1 + \frac{GW(e)yr5}{GW(e)yr3}\right)\,\right]} \qquad (3.25)$$

However, from Eqs. (3.14), (3.21), (3.22):

$$F_{S1} = \frac{S_{ysc}}{GW(e)yr1} \qquad (3.26)$$

$$R_{31} = \frac{GW(e)yr3}{GW(e)yr1} \tag{3.27}$$

$$R_{53} = \frac{GW(e)yr5}{GW(e)yr3} \tag{3.28}$$

Therefore Eq. (3.25) becomes:

$$F_{S3} = \frac{F_{S1}}{[1 + R_{31}\ (1 + R_{53})]} \quad \frac{STU_3O_8}{\text{System GW(e)yr (rated) at capacity factor L}} \tag{3.29}$$

where F_{S1}, R_{31} and R_{53} have been previously evaluated (Eqs. 3.14, 3.21, 3.22). Equation 3.29 gives the ore requirements per system GW(e)yr (rated) at capacity factor L for a three tier system.

The system separative work requirements, SWU-3 MTSWU per system GW(e)yr (rated) at capacity factor L, of the above three-tier system is given by

$$\text{SWU-3} = \frac{\text{SWU-1}}{[1 + R_{31}(1+R_{53})]} \quad \frac{\text{MTSWU}}{\text{SYSTEM GW(e)yr (rated) at capacity factor L}} \tag{3.30}$$

where SWU-1 is the separative work requirement per GW(e)yr (rated) at capacity factor L, generated by reactor type j = 1, given by

$$\text{SWU-1} = \frac{P_1}{SL} \frac{SWF1}{E_1 T_1} \left[1 - (SL)^2\ RD1\ \frac{SWD1}{SWF1} + \frac{r}{100} SN_1\ T_1 \right] \tag{3.31}$$

where SWF1 and SWD1 are the separative work requirements per metric ton of fuel charged to and discharged from reactor type $j = 1$, respectively, and are given by the general expression:

$$S/P\ (X_p) = [2X_p - 1] \cdot \ln \frac{X_p}{1-X_p} + \left[\frac{X_p - X_{NAT}}{X_{NAT} - X_w}\right] \cdot [2X_w - 1] \cdot \ln \frac{X_w}{1-X_w} \left[\frac{X_p - X_W}{X_{NAT} - X_W}\right] \cdot [2X_{NAT} - 1] \ln \frac{X_{NAT}}{1-X_{NAT}} \quad (3.32)$$

where $SWF1 = S/P\ (0.01\ XF_{11})$ (3.33)

$SWD1 = S/P\ (0.01\ XD_{11})$ (3.34)

(Note that XD_{11} and XF_{11} are in wt% enrichment) and SN_1 is the number of equivalent reload batches in the initial inventory of reactor type $j = 3$, in terms of separative work units.

Note that the term $[1 + R_{31}(1 + R_{53})]$ appears in Eq. 3.30, again because of the balance coupling bred fissile material to the quantity of electrical energy generated.

For a system with identical values of refueling time, thermal efficiency, heavy metal loading per batch, and final fuel burnup, Eq. 3.14 reduces to

$$F_{S1} = \frac{1.3P}{SL.E.T} \left(\frac{XF_{11} - X_W}{X_{NAT} - X_N}\right) \left[1 - (SL)^2\ RD1 \frac{(XD_{11} - X_W)}{(XF_{11} - X_W)} + \frac{r}{100}\ N_1 T\right] \quad (3.35)$$

Eq. (3.21) becomes,

$$R_{31} = \frac{XD_{31} \quad SL}{[(XF_{33}-XD_{33} \; SL + N_3 \; XF_{33} \; \frac{r}{100} \; T]} \tag{3.36}$$

Eq. (3.22) becomes:

$$R_{53} = \frac{XD_{23} \quad SL}{[(XF_{25} - XD_{25} \; .SL) + N_5 \; . \; XF_{25} \; . \frac{r}{100} \; T]} \tag{3.37}$$

and the system ore consumption per system GW(e)yr (rated) at capacity factor L, is given by Eq. (3.29).

MASFLO-2 can be cast in a form applicable to cases in which mass parameters are given in terms of metric tons per GW(e)yr (rated) at a capacity factor L, ie. actual fissile inventories and flow rates. For the present three-tier system Eq. (3.14) becomes

$$F_{S1} = 1.1 \; (qa_{11} + \frac{r}{100} \; Q_{11}) \tag{3.38}$$

where qa_{11} is the equilibrium requirements with uranium recycle, in metric tons U_3O_8 per GW(e)yr (rated) at capacity factor L, and Q_{11} is the initial inventory, MTU_3O_8 per GW(e).

Eq. (3.21) becomes:

$$R_{31} = \frac{qd_{31} \; SL}{[qa_{33} + Q_{33} \; \frac{r}{100} \;]} \tag{3.39}$$

where qd_{31} is the net discharge of Pu-239 (based fuel from reactor type j=1 in metric tons per GW(e)yr (rated) at capacity factor L, qa_{33} is the net equilibrium requirement of Pu-239 (based) fuel of reactor type j=3, in metric tons per GW(e)yr (rated) at capacity factor L, and Q_{33} is the specific inventory of Pu-239 (based) fuels in reactor type j=3, in MT per GW(e).

A similar expression can also be written for R_{53} using Eq. (3.22):

$$R_{53} = \frac{qd_{23}\ SL}{[qa_{25} + Q_{25}\ \frac{r}{100}]} \qquad (3.40)$$

where qd_{23} is the net discharge of U-233 (based) fuel from reactor type j=3 in metric tons per GW(e)yr (rated) at capacity factor L, qa_{25} is the net equilibrium requirement of U-233 (based) fuel of reactor type j=5, in metric tons per GW(e)yr (rated) at capacity factor L, and Q_{25} is the specific inventory of U-233 (based) fuel in reactor type j=5 in MT per GW(e). The system ore consumption per system GW(e)yr (rated) at capacity factor L, F_{S3}, is given by Eq. (3.29).

Note that the equilibrium requirement, qaij, in metric tons of isotope i per GW(e)yr (rated) at capacity factor L of reactor type j, is a net requirement. Alternatively, this term could be replaced by the difference between charged and discharged masses qc_{ij} and qd_{ij}, respectively, per GW(e)yr (rated) at capacity factor L, taking into account the system loss terms. In this application Eq. (3.38) becomes for the ore requirements, F_{S1}, (STU_3O_8 per GW(e)yr (rated) at capacity factor L), for reactor

type j=1:

$$F_{S1} = 1.1[(qc_{11}-qd_{11} \cdot SL)+ \frac{r}{100} Q_{11}] \qquad (3.41)$$

A similar approach can be used for separative work requirements.

Before discussing the limitations of this model, it is appropriate to show its similarity to MASFLO-1. Equation (3.29) can be written out explicitly using Eq. (3.14) to get:

$$F_{S3} = \frac{1.3.P_1}{SL.E_1T_1} \left(\frac{XF_{11} - X_w}{X_{NAT} - X_w}\right) \frac{[1-(SL)^2 .RD1(\frac{XD_{11} - X_w}{XF_{11} - X_w}) + \frac{r}{100} N_1T_1]}{[\,1 + R_{31} \;(1 + R_{53})]} \qquad (3.42)$$

Multiplying out terms yields:

$$F_{S3} = \frac{1.3\ P_1}{SL\ E_1T_1} \left(\frac{XF_{11} - X_w}{X_{NAT} - X_w}\right) \frac{\frac{r}{100}\ N_1T_1}{[1 + R_{31}\ (1 + R_{53})]} + (1)$$

$$\frac{1.3P_1}{SLE_1T_1} \left(\frac{XF_{11} - X_w}{X_{NAT} - X_w}\right) \frac{[1-(SL)^2RD1(\frac{XD_{11}-X_w}{XF_{11}-X_w})]}{[1 + R_{31}\ (1 + R_{53})]} \quad (2) \qquad (3.43)$$

The expression corresponding to F_{S3} in MASFLO-1 is Eq. (3.5), where m_j STU_3O_8/GW(e)yr is given by

$$m_j = \frac{r}{100} SI_j + SC_j \qquad (3.44)$$

(3) (4)

By a careful inspection, one can see the similarity between terms 1 and 2, and terms 3 and 4 of these equations. Term 2 is the equilibrium ore requirements per GW(e)yr of the

three tier system in full 235-Uranium, 233-Uranium (based) and Pu-239 (based) recycle. The factor $[1 + R_{31} (1 + R_{53})]$ corrects for the power bonus due to Pu-239 (based) and U-233 (based) recycle. Thus if the factor SC_j in MASFLO-1 properly accounted for the effects of Pu-239 (based) and U-253 (based) recycle in a three tier system, it would be fully compatible with MASFLO-2. Similarly one can see that term 1 of Eq. (3.43) is composed of the product of an equivalent SI_j and the growth rate per year.

MASFLO-2 differs from MASFLO-1 in that (1) it is not limited to single type reactor systems, as MASFLO-1 is; and (2) it is more explicit and flexible in terms of reactor parameters, such as the heavy metal loading per batch, while MASFLO-1 considers only integral ore usage parameters; (3), in dealing with chains of reactor systems, it takes into account the initial inventory of consumer reactors (See expressions for R_{31} and R_{53}).

MASFLO-2 is a simple model for assessing the ore and separative work requirements of multi-tiered systems. The model simulates a more rigid system than real-life systems in that (1) all bred fissile materials are immediately recycled, and no stockpile is maintained (except for the out-of-core inventory associated with each new reactor, which will be discussed later). This is in some ways an attractive feature in that such a system of reactors is utilizing all the fissile material to the maximum, and the model therefore gives the ultimate relative ore utilization capability of a well-managed nuclear economy. Excessive stockpiling is not only economically unsound, but increases system vulnerability to diversion and weapons proliferation.

In fact other models (N-1), (Z-1) assidiously try to minimize these stockpiles either year-by-year or at the end of the study.

Another related assumption is that both the installed capacity of the consumer and producer reactor types grow at the same rate. If this were not so, either stockpiling or fissile purchases from outside the system would be required. Note that this growth rate may be positive, zero, or negative (See Chapter 4). The above derivation applies to full, i.e., repetitive to extinction, recycle; however, the cases of no uranium-235 recycle or single pass plutonium recycle can be easily accommodated. Furthermore, degradation of the uranium 235 due to U-236 buildup, plutonium due to Pu-240 and Pu-242 build-up, and U-233 (based) fuel due to U-234 buildup in multi-recycle are easily accounted for by introducing appropriate weighting factors on the mass flow terms in the expressions for recycle. Thus, plutonium degradation would be accounted for by applying a weighting term, W_3, in the expression for R_{31}, the GW(e)yr produced by reactor $j = 3$, due to the production of one GW(e)yr (rated) at capacity factor L by reactor $j = 1$.

$$R_{31} = \left(\frac{P_1}{P_3}\right)\left(\frac{E_3 T_3}{E_1 T_1}\right) \cdot \frac{SL \cdot XD_{31}}{\left[(XF_{33} - W_3 SL \cdot XD_{33}) + N_3 XF_{33} \frac{r}{100} T_3\right]} \tag{3.45}$$

For single pass plutonium recycle, W_3 is simply set equal to zero.

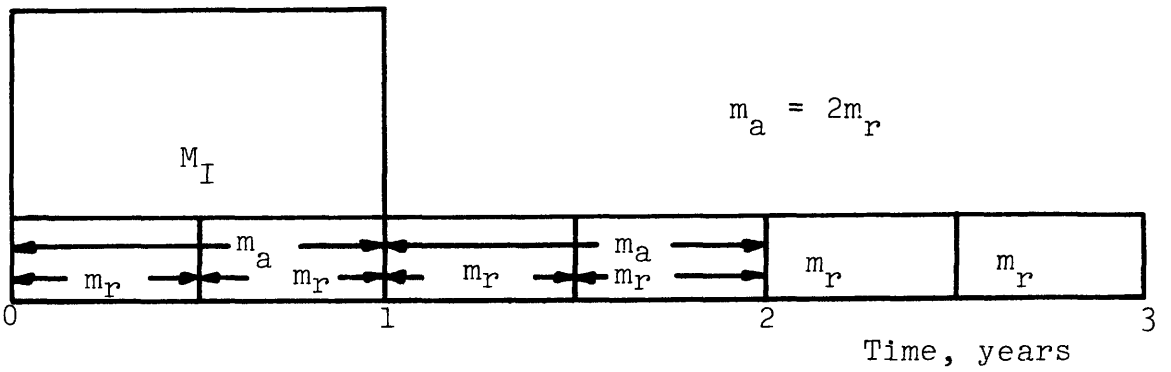
In contrast to MASFLO-1 where the variation of the re-

fueling interval T_j, was not taken into account explicitly, MASFLO-2 explicitly takes into account the refueling interval. Figure 3.4 demonstrates the effect of the refueling interval T_j on the treatment of MASFLO-2 of the first year requirements of a reactor. For a refueling interval of 0.5 years, the model calculates the first year's requirements as the sum of the initial inventory plus two equilibrium reload batches (i.e., one year's requirement), one batch of which is burned in the first yearly interval. For a 1.5 year reload interval MASFLO-2 calculates the first year's requirements again as the sum of the initial inventory plus a pro-rated annual reload (i.e., the equivalent annual reload is 2/3 of a batch reload). Finally, for a one year refueling interval, the model calculates the first year's requirement as the initial inventory plus the first annual requirement, i.e., one reload.

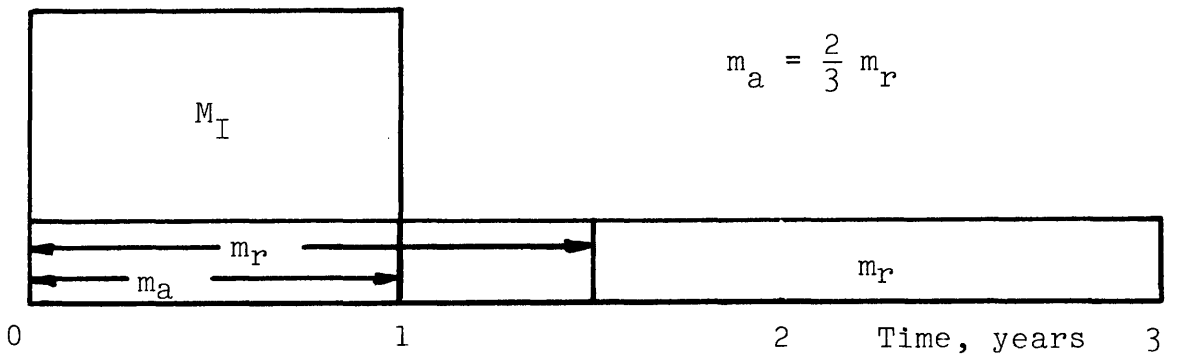
In general, therefore, the first year's requirements of a reactor are calculated as the initial inventory plus the equivalent of one year's requirements for the reactor. Note that the first yearly reload arbitrarily assumes the availability of recycled fissile material for its fabrication, in advance of actual reactor operation, when MASFLO-2 is utilized in a recycle study.

In effect, therefore, the model assumes that in starting up a reactor, the initial inventory is purchased, plus one year's worth of reload batches are purchased and debited to system usage in the first year of reactor operation.

1) Refueling Interval T_j = 0.5 years



2) Refueling Interval T_j = 1.5 years



3) Refueling Interval T_j = 1.0 year

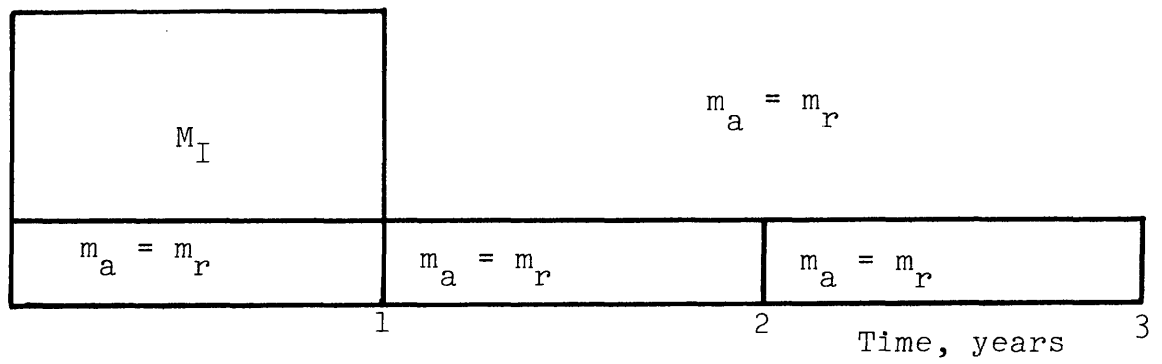


FIG. 3.4. THE REFUELING INTERVAL AND MASFLO-2'S TREATMENT OF THE FIRST YEAR REQUIREMENTS (1)

(1) M_I is the initial inventory, m_r the equilibrium reload requirement, m_a the annual requirement.

For a refueling period of one year, which is the value used in most fuel cycles, the model calculates the first year's requirement as the initial inventory plus one reload, and that extra reload (or its successor) always remains on site, awaiting insertion into the reactor during the following year, a practice which is not unlike actual practice.

It should be noted that MASFLO-2 is not limited to only converter reactors, but it can also handle breeder reactors. This will be discussed in more detail in Chapter 4. It is sufficient to note here that the breeder reactor is simply considered as a special type of consumer reactor, and an appropriate R_{ij} inserted for it in the above model.

Before going on to the next section, a few comments are in order on the terms N_j and SN_j, the equivalent ore and separative work requirements (measured in terms of the number of standard reload batches) contained in the initial core. The calculation of N_j and SN_j are based on the linear reactivity model of core burnup. These derivations are presented in detail in Appendix D.

The out-of-core off-site inventory, accounting for the material needed to fill the enrichment, reprocessing, etc., "pipelines", is accounted for very simply by modifying N_j according to the relation (H-7):

$$N_j' = N_j \left(1 + \frac{T_{lagj}}{T_{resj}}\right) = N_j + \Delta N_j \qquad (3.46)$$

where T_{lagj} is the out-of-core time spent by the fuel in processes such as mining, fabrication or reprocessing and refabrication, etc. for the j^{th} reactor type, and T_{resj} is the residence time of the fuel (years) in the core of reactor type j . Thus for reactor type j = 3, with plutonium as the fissile fuel, T_{lagj} would include reprocessing and refabrication periods for the plutonium fuel. Equation 3.46 can be more readily understood if one recognizes that the term $N_j \frac{T_{lagj}}{T_{resj}}$ is the number of reload batches in the ex-reactor phases of the fuel cycle per in-core batch.

3.5 Equivalent Weighted Mass Parameters

Reactor grade "plutonium" consists of four isotopes: Pu-239, Pu-240, Pu-241 and Pu-242; and reactor grade "U-233" consists of U-233, U-234, U-235 and small amounts of U-236. Isotopic separation processes are not currently applied to these materials, (although in the future this may become possible). Thus these fuels must be considered to be appropriate mixtures of these isotopes.

In most previous work in the area of reactor systems modeling (N-1),(W-1), (Z-1), when dealing with the isotopic mixtures of "plutonium" and "U-233" fuels, there is no distinction usually made between the various fissile isotopes. Thus they refer to "plutonium" fuels as Pu (fissile), a single species, and do not take into account the different fuel values of Pu-239 and Pu-241, with the same being done for the U-235 and

U-233 in "U-233" based fuels.

K.O. Ott et al. (O-4), have shown that for a fast breeder reactor employing plutonium fuel, the worth of the isotopes of plutonium (based on their breeding worth) are (w_{49} = 1.0, w_{40} = 0.5, w_{41} = 1.2, w_{48} = 0.11). Ott plans to examine thermal systems in future work (O-6).

In the present work, the criterion used was not the "breeding worth" of the isotope, but an "energy worth". For simplicity, the worth of an isotope was based on the amount of energy produced by the isotope per unit mass destroyed. This gives an energy per mass worth factor, but does not include the "extra" worth due to the production by capture of the other fissile isotopes. In fact, again for simplicity, only fissile isotopes were considered (fertile isotopes were given a weighting value of zero).

By using these "weighting" factors all discharged and charged isotopic compositions can be reduced to a limited number of equivalent single-isotope compositions.

In Appendix F, a detailed example of an adjustment in effective isotopic composition using the subject technique is presented, using the output of LEOPARD (B-1); also presented are the "weighting" factors used for the present work.

In this section, a general outline of the techniques used to calculate energy worths will be given. As an example, let us take the case of recycling plutonium fuel from a UO_2 (slightly enriched U-235) reactor j = 1, to a PuO_2/UO_2 reactor, j=4. Since the plutonium fuel is being burned in the PuO_2/UO_2 reactor

the energy worth of the fissile isotopes of recycled plutonium fuel must be characteristic of this reactor.

For plutonium fuel, Pu-239 was used as the base isotope, with an assigned "energy worth" factor of 1.0.

Thus, considering one metric ton of heavy metal charged to reactor type j = 4:

Let

E_k^j = total energy in MWD produced by isotope k per metric ton of heavy metal charged to reactor type j at the equilibrium discharge fuel burnup;

G_k^j = corresponding gross mass in metric tons of isotope k destroyed per metric ton of heavy metal charged to reactor type j, at the final equilibrium discharge fuel burnup;

and, $w_k^{\ell}(j)$ = the "energy" worth of isotope k relative to reference isotope ℓ (U-233 or Pu-239) in reactor type j.

Then in the present example the energy worth of Pu-241 relative to Pu-239 in reactor type j = 4 is given by

$$w_{41}^{49}(4) = \frac{E_{41}^4}{G_{41}^4} \cdot \frac{G_{49}^4}{E_{49}^4} \qquad (3.47)$$

Equation (3.47) is to be interpreted to say that one kg of Pu-241 is equivalent to $w_{41}^{49}(4)$ kg of Pu-239 in reactor type j = 4.

Having calculated the "energy" worth factors, the next task is to calculate the equivalent mass flows.

Let

Gd_k^j = the mass of isotope k in MT discharged per metric ton of heavy metal charged to reactor type j, at the equilibrium discharge fuel burnup.

$G_{deq}{}_j^{\ell}$ = the equivalent mass of reference isotope ℓ in MT discharged per metric ton of heavy metal charged to reactor type j, at the equilibrium discharge fuel burnup.

Then for the present example the equivalent mass of Pu-239, discharged from reactor type j = 1 in metric tons, per MT heavy metal charged at the final equilibrium fuel burnup, $G_{deq}{}_1^{49}$ is given by

$$G_{deq}{}_1^{49} = Gd_{49}^1 + w_{49}^{41}(1) \cdot Gd_{41}^1 \qquad (3.48)$$

One can readily see that the quantity $G_{deq}{}_1^{49}$ is identical to the quantity XD_{31}, used in MASFLO-2 previously; XD_{31} being the discharged weight fraction of Pu-239 (based) fuel from reactor type j = 1 with respect to heavy metal initially charged.

Using the same process, the same procedure was repeated for all reactor systems considered in the present study.

A few points should be made in reference to the analysis of the effect of Vf/Vm on ore and SWU requirements. For plutonium recycled from reactor j = 1, there were two possible "energy" weighting factors for Pu-241: (1) based on the data from reactor type j = 3 (PuO_2/ThO_2) and (2) based on the data from reactor

type j = 4. Rather than using two different sets of "energy" weighting factors for Pu-239 and therefore using two values of XD_{31} or $G_{deq}{}^{49}_{1}$ at each value of Vf/Vm of reactor type j = 1; a mean energy weighting factor for the two consumer reactors j = 3, j = 4 was used at each value of V_f/V_m. It was found that the individual XD_{31} values varied a maximum of 0.2% from the mean value, hence this simplification is readily justified.

For reactor type j = 3, UO_2 (93% enriched U-235) ThO_2 only U-233 was considered recycled to the U-233/ThO_2 reactor. This was due to the fact that the burnup code used, EPRI-LEOPARD, did not distinguish between feed U-235 and U-235 produced in the Th-232 chain. Furthermore, all fissile plutonium produced in the UO_2(93% enriched U-235) ThO_2 reactor was recycled as equivalent U-235 fuel using the same techniques as above, which is not a large approximation considering the relatively small amounts of plutonium produced.

As far as the treatment of the discharge masses of the primary fissile fuels of the consumer reactors (j = 3,4,5), i.e., Pu-239 (based) fuel in a PuO_2/UO_2 reactor, this is easily seen by again referring to the above system of the UO_2 (slightly enriched U-235) and the PuO_2/UO_2 reactors. Here the discharged masses of Pu-239 (based) fuels from the PuO_2/UO_2 reactor (j = 4) were calculated using Eqs. (3.47) and (3.48), however the parameters used in calculating the energy weighting factor $w_{49}^{41}(4)$ of Pu-241, and the equivalent mass of Pu-239 (based) fuels in metric tons per metric ton of heavy metal charged at the equi-

librium discharge fuel burnup of reactor type $j = 4$, $G_{deq}^{49}{}_4$ (or XD_{34}. See last section), were characteristic of reactor type $j = 4$.

Finally it should be noted that the question of the disposal of multiply-recycled fuel because of degradation due to poison buildup (U-236, Pu-242) would also require a further weighting factor to be put on these first recycle fuels. This will be discussed in Chapter 4.

3.6 Conclusions

In this chapter it has been pointed out that previous work in the area of energy modeling has concentrated mainly on large, complicated models. Due to their complexity, some are vulnerable to claims of hidden biases, which are difficult to verify, and they require large computer facilities and long computing times. Isolating the effect of individual variables is often a difficult task.

MASFLO-2, described in this chapter, was therefore developed to treat coupled reactor systems. The model has some limitations, caused by its very simplicity. However, as will be shown in Chapter 4, quite acceptable overall accuracy can be achieved. The model can handle any type of reactor of current interest, including breeders, as will be shown in Chapter 4.

Tables (3.3) and (3.4) list the key features and final set of equations, respectively, of MASFLO-2.

TABLE 3.3

SUMMARY OF THE KEY FEATURES OF MASFLO-2

Feature	Refer To
1. No variable system stockpile: inventories are proportional to installed capacity; also allows for out-of-core off-site inventory and on-site new fuel inventory.	Section 3.4
2. Allows for individual optimization of the reactor physics of each reactor type in a multi-tiered system: all reactors are dedicated to a single fissile-fertile fuel combination.	Section 3.4
3. Variation in isotopic composition treated by use of weighting factors: discharge streams are adjusted in composition to be equivalent to a limited number of charge streams.	Section 3.5
4. All reactors in the system grow at the same rate; rate can be zero, positive or negative, can vary on yearly basis.	Section 3.4
5. Applicable to both breeders and converters.	Section 3.4
6. Can be formulated in either mass flows or concentrations depending on the form in which fuel cycle data is available.	Section 3.4
7. Can explicitly handle reactor parameters such as final fuel burnup, thermal efficiency for each reactor type in a system.	Section 3.4
8. Calculates the system ore and separative work requirements per system GW(e)yr (rated) at a capacity factor L (assumed constant for present purposes (not an inherent limitation) for all reactor types in the system).	Section 3.4

TABLE 3.4

FINAL SET OF EQUATIONS OF MASFLO-2
FOR THREE-TIER SYSTEM OF UO_2 (SLIGHTLY ENRICHED U-235),
PuO_2/ThO_2 AND $^{233}UO_2/ThO_2$ REACTORS *

$$F_{S3} = \frac{F_{S1}}{[1 + R_{31}\ (1 + R_{53})]} \quad \frac{STU_3O_8}{\text{SYSTEM GW(e)yr (rated) at capacity factor L.}}$$

where

$$F_{S1} = \frac{1.3\ \ P_1}{SL\ E_1\ T_1} \left(\frac{XF_{11} - X_w}{X_{NAT} - X_w}\right) \left[1 - (SL)^2\ RDI\left(\frac{XD_{11} - X_w}{XF_{11} - X_w}\right) + \frac{rN_1T_1}{100} \right]$$

$$R_{31} = \left(\frac{P_1\ E_3\ T_3}{P_3\ E_1\ T_1}\right) \quad \frac{XD_{31}\ \ SL}{[(XF_{33} - W_3\ XD_{33}.SL + N_3XF_{33}\ \frac{r}{100}\ T_3]}$$

and

$$R_{53} = \left(\frac{P_3\ E_5\ T_5}{P_5\ E_3\ T_3}\right) \quad \frac{XD_{23}\ \ SL}{[XF_{25} - W_2\ XD_{25}\ .\ SL) + N_5\ XF_{25}\ \frac{r}{100}\ T_5]}$$

* See section 3.4 for definitions of terms.

CHAPTER 4

RESULTS

4.1 Introduction

In this chapter the main results of this study will be presented. In order to verify the calculational methodology of MASFLO-2, the results of benchmarking the model against the HEDL code ALPS will first be presented. Next results dealing with the effect of unit cell size (fuel pin diameter) on ore and separative work requirements will be documented to justify subsequent neglect of this parameter as an important variable. Then the MASFLO-2 results for various combinations of the 5 reactor types (four lattice pitches per type) considered in this investigation will be presented. Finally, a further application of MASFLO-2 to coupled breeder-converter systems will be dealt with briefly.

4.2 Benchmarking of MASFLO-2

In order to engender confidence in the rather simplistic MASFLO-2 model, it was decided to benchmark the model against results derived from the Hanford Engineering Laboratory (HEDL) model, ALPS (H-4),(H-5).

MASFLO-2 in its present form calculates system ore usage per GW(e) yr. In order to benchmark MASFLO-2 against ALPS, which gives cumulative ore and separative work usage, it was necessary to proceed in the manner outlined below.

Consider the simple case of the UO_2 (slightly enriched U-235) reactor. In Eq. (3.14) (Chapter 3) it was shown that the system ore consumption for a UO_2 (slightly enriched U-235) reactor per GW(e)yr (rated) at capacity factor L is given by:

$$F_{S1} = \frac{1.3\ P_1}{SL\ E_1\ T_1}\ \left[\frac{(XF_{11} - X_w)}{(X_{NAT} - X_w)}\right]\left[1 - (SL)^2.RD1.\left(\frac{XD_{11}-X_w}{XF_{11}-X_w}\right)\right.$$

$$\left. + \frac{r}{100}\ N_1T_1\right] \qquad (4.1)$$

where all parameters are as defined in Chapter 3.

In applying this result to cumulative usage, it will be assumed that all reactors introduced in year t are introduced at the beginning of year t (which is the same convention used in ALPS).

Given as input the year-by-year tabulation of newly installed capacity one can define the appropriate growth rate r(t) as:

$$r(t) = \frac{\Delta E_j(t)}{E_j(t-1)} \qquad (4.2)$$

where r(t) is now a time dependent growth rate; $\Delta E_j(t)$ is the newly-installed capacity (GW(e)) of reactor type j in year t and $E_j(t-1)$ is the total installed capacity of reactor type j in year (t-1).

Using Eqs. (4.1) and (4.2) one can now define an annual system ore usage (ASU(t), given by:

$$ASU(t) = \frac{1.3.P_1.}{SL.E_1T_1}\ \left[\frac{(XF_{11} - X_w)}{X_{NAT} - X_w)}\right]\left[1-(SL)^2.RD1.\left(\frac{XD_{11}-X_w}{XF_{11}-X_w}\right)\right.$$

$$\left. + \frac{r(t)}{100}\ N_1T_1\right].\ E_1(t-1) \qquad (4.3)$$

$$= \frac{1.3\ P_1}{SL\ E_1 T_1} \cdot \left[\frac{(XF_{11} - X_w)}{(X_{NAT} - X_w)}\right].\left[1 - (SL)^2.RD1.\ \left(\frac{XD_{11} - X_w}{X_{NAT} - X_w}\right)\right].E_1(t-1)$$

(1)

$$+ \frac{1.3\ P_1}{SL\ E_1\ T_1} \left[\frac{(XF_{11} - X_w)}{(X_{NAT} - X_w)}\right]\left[\frac{r(t)}{100}\ N_1 T_1\right]E_1(t-1) \qquad (4.4)$$

(2)

Term 1 in Eq. (4.4) is merely the ore required to sustain all reactors built before year t (employing uranium recycle). Term 2 of Eq. (4.4) is the ore required to start up the new reactors added at the beginning of year t, (this is easily shown by inserting the definition of r(t), Eq. (4.2) in term 2 of Eq. (4.4).)

The integrated ore requirements in year t, IOU(t), the cumulative ore usage from year t' = 0 to the end of year t'= t, is then given by

$$IOU(t) = \int_0^t ASU\ (t')\ dt' \qquad (4.5)$$

which in the present treatment involving finite yearly increments reduces to:

$$IOU(t) = \sum_{t'=0}^{t} ASU(t') \qquad (4.6)$$

Equation (4.6) gives the cumulative ore consumption from year t'=0 to the end of year t'=t, for a system of UO_2 (slightly enriched U-235) PWR's. A similar approach applies for separa-

tive work requirements. For two- and three-tier systems, the approach is very similar, i.e., calculating the yearly ore requirements per GW(e)yr using MASFLO-2 and then summing over the yearly system requirements.

From private communications with R.W. Hardie et al of HEDL (H-6) results were obtained from ALPS for two reactor cycles: (1) a UO_2 (slightly enriched U-235) PWR with uranium recycle only; and (2) the preceding case with recycle of plutonium to a PuO_2/UO_2 PWR, using as its fertile makeup uranium tails from the enrichment plant employed to feed the UO_2 (slightly enriched U-235) reactor.

As was mentioned in Chapter 3, the input to ALPS is very detailed, and therefore in benchmarking MASFLO-2 with this code some simplifications had to be made.

Table 4.1 presents the capacity factor curve used in the ALPS run of present interest (H-6). This represents the capacity factor curve for both types of reactors considered here (i.e. UO_2 (slightly enriched U-235) and the PuO_2/UO_2 reactor). Data is given on a year-by-year basis over the 30 year lifetime of the reactor. ALPS' uses this capacity factor history to calculate the yearly input and output data for each year of the 30 year lifetime of each reactor type. These numbers are then utilized by ALPS' to calculate the total ore and separative work requirements of a given growth scenario.

The major simplification used in the benchmarking of MASFLO-2 is the choice of an effective system capacity factor to be used in MASFLO-2. To do this a time-dependent capacity factor L(t)

TABLE 4.1

CAPACITY FACTOR HISTORY USED IN ALPS FOR PRESENT BENCHMARK

Year	Capacity Factor	Year	Capacity Factor
1	0.60	16	0.72
2	0.66	17	0.704
3	0.70	18	0.689
4	0.72	19	0.673
5	0.72	20	0.657
6	0.72	21	0.641
7	0.72	22	0.626
8	0.72	23	0.61
9	0.72	24	0.594
10	0.72	25	0.570
11	0.72	26	0.563
12	0.72	27	0.547
13	0.72	28	0.531
14	0.72	29	0.516
15	0.72	30	0.500

was defined such that:

$$L(t) = \frac{\sum_{k=1}^{t} \Delta E_{jk} \cdot C_{j(t-k+1)}}{\sum_{k=1}^{t} \Delta E_{jk}} \qquad (4.7)$$

where ΔE_{jk} is the net capacity of reactor type j installed in year k, and $C_j(t-k+1)$ is the capacity factor (see capacity factor table 4.1) of reactors built in year k, in year t. Thus L(t) is a system-weighted capacity factor. Note that since the present study extends beyond the 30 year lifetime of the present reactors for t-k+1 >30 years, the term $C_{j(t-k+1)}$ in Eq. (4.7) is replaced by $C_{j(t-k+29)}$.

Since MASFLO-2 allows for no bred fissile stockpiles in two-tier systems such as the second benchmark here (the UO_2 (slightly enriched U-235)-PuO_2/UO_2 ststem), the present calculations were terminated at the point at which ALPS' (H-6) calcula- a minimum value (0.01 kg) for the plutonium stockpile in the above two-tier system. This corresponded to the end of year 2002, or a time span of 34 years for the present benchmark.

Table 4.2 demonstrates the 34 year growth profiles for both scenarios considered in this study as used by ALPS' (H-6). The data from ALPS' (H-6) was given in 2 year stages. Thus reactors are installed every 2 years, at the beginning of the 2 year period. Hence for the last year of this benchmark, 1998, the newly installed capacity is zero. Note that since MASFLO-2

TABLE 4.2

GROWTH SCENARIO USED BY ALPS

(H-4),(H-5) IN PRESENT BENCHMARK (H-6)

Year	Capacity Built[1] in year (Gwe) =	UO_2 Capacity[2] Built (Gwe) +	PuO_2/UO_2[3] Capacity Built (Gwe)
1969	3.0	3.0	-
1971	9.0	9.0	-
1973	15.0	15.0	-
1975	16.3	16.3	-
1977	14.8	14.8	-
1979	18.7	18.7	-
1981	40.8	40.8	-
1983	50.0	50.0	-
1985	52.1	49.6	2.5
1987	62.7	57.7	5.0
1989	77.9	67.8	10.1
1991	87.5	67.3	20.2
1993	93.5	53.9	39.6
1995	101.5	48.8	52.7
1997	118.5	76.3	42.2
1999	140.5	106.7	33.7
2001	155.1	128.1	27.0

(1) Total newly installed capacity in year t for both scenarios.

(2) Individual newly installed capacity for both reactor types for scenario (2).

utilizes only net newly installed capacity, for $t > 30$ years the net newly installed capacity input into MASFLO-2 was the newly installed capacity in year t minus the newly installed capacity in year t-30.

The reactor parameters used in MASFLO-2, derived from the ALPS' data (H-6) are presented in Appendix E. Although this will be discussed more thoroughly in section 4.5, it should be noted here that in order to account for isotopic degradation, a weighting value of 0.8 (see Chapter 3) was used for multi-recycle plutonium ($\geq$ 2 recycles).

The results for both scenarios are now presented. Note that in compliance with the data obtained from HEDL, the ore and separative work requirements represent the net amounts consumed at the end of the two year stage, hence the values for year 2001, represent that consumed through the end of year 2002.

Table 4.3 compares cumulative ore requirements calculated by MASFLO-2 and by ALPS for the case of the UO_2 (slightly enriched U-235) reactor with uranium recycle. As one can see, the percentage deviation of MASFLO-2 converges to within approximately 1% at the end of the 34 year benchmark period, a quite acceptable result. The initial large deviations are to be expected because of the crude treatment of startup inventories in MASFLO-2 and its assumption of instantaneous recycle.

Very similar behavior is also demonstrated for separative work requirements, as shown in Table 4.4; here MASFLO-2 converges at the end of the 34 year study period to within 2.2% of ALPS'.

TABLE 4.3

COMPARISON OF ORE REQUIREMENTS:
MASFLO-2 VERSUS ALPS (H-6) FOR UO_2(U-RECYCLE)

Year	ST U_3O_8 (2) (ALPS)	ST U_3O_8 (2) (MASFLO-2)	$\Delta\%$ (1)
1969	4.44×10^3	2.021×10^3	54.4
1971	1.355×10^4	9.183×10^3	32.2
1973	2.861×10^4	2.350×10^4	17.9
1975	4.813×10^4	4.355×10^4	9.5
1977	7.237×10^4	6.764×10^4	6.5
1979	1.082×10^5	9.913×10^4	8.4
1981	1.634×10^5	1.522×10^5	6.9
1983	2.355×10^5	2.247×10^5	4.6
1985	3.252×10^5	3.142×10^5	3.4
1987	4.388×10^5	4.273×10^5	2.6
1989	5.804×10^5	5.704×10^5	1.7
1991	7.505×10^5	7.438×10^5	0.9
1993	9.501×10^5	9.477×10^5	0.3
1995	1.184×10^6	1.185×10^6	-0.1
1997	1.459×10^6	1.464×10^6	-0.3
1999	1.781×10^6	1.792×10^6	-0.6
2001	2.148×10^6	2.165×10^6	-0.8

$$(1)_{\Delta\%} = \frac{\text{ST } U_3O_8\text{(ALPS)} - \text{ST } U_3O_8\text{(MASFLO-2)}}{\text{ST } U_3O_8 \text{ (ALPS)}} \times 100$$

(2) Purchased at end of stage.

TABLE 4.4

COMPARISON OF SEPARATIVE WORK REQUIREMENTS:
MASFLO-2 VERSUS ALPS (H-6) FOR UO_2 (U-RECYCLE)

Year	MTSWU (ALPS)(2)	MTSWU (MASFLO-2)(2)	$\Delta\%$(1)
1969	2.640×10^3	3.649×10^2	86.2
1971	8.233×10^3	4.953×10^3	39.8
1973	1.780×10^4	1.434×10^4	19.4
1975	3.073×10^4	2.786×10^4	9.3
1977	4.720×10^4	4.453×10^4	5.7
1979	7.127×10^4	6.638×10^4	6.9
1981	1.079×10^5	1.024×10^5	5.1
1983	1.564×10^5	1.521×10^5	2.7
1985	2.178×10^5	2.142×10^5	1.7
1987	2.957×10^5	2.931×10^5	0.9
1989	3.933×10^5	3.931×10^5	0.1
1991	5.113×10^5	5.149×10^5	-0.7
1993	6.507×10^5	6.590×10^5	-1.3
1995	8.146×10^5	8.275×10^5	-1.6
1997	1.008×10^6	1.026×10^6	-1.8
1999	1.233×10^6	1.258×10^6	-2.0
2001	1.492×10^6	1.525×10^6	-2.2

(1) $\Delta\% = \dfrac{\text{ST } U_3O_8 \text{ (ALPS)} - \text{ST } U_3O_8 \text{(MASFLO-2)}}{\text{ST } U_3O_8 \text{ (ALPS)}} \times 100$

(2) Purchased at end of stage.

Tables 4.5 and 4.6 are comparisons of cumulative ore and separative work requirements calculated for the case of UO_2 (slightly enriched U-235) reactors coupled to PuO_2/UO_2 reactors with full recycle; as can be seen, MASFLO-2 converges to ALPS in both cases, with final deviations of 4.7% and 4.3%, respectively, for ore and SWU requirements at the end of the 34 year benchmark period; which is quite acceptable for present purposes. The large deviations in the major part of these two benchmark are due to , primarily, the growth scenario used by ALPS for this two-tier reactor system. As can be seen from Table 4.2, in the ALPS growth scenario, the installation of PuO_2/UO_2 reactors starts in 1985. However, MASFLO-2 starts up the PuO_2/UO_2 reactors at the beginning of the study. This contributes in the MASFLO-2 calculation, to a savings in the initial years of the study, resulting in much lower cumulative ore and separative work requirements. Note, however, that at the end of the study where as reported previously, ALPS calculated a zero plutonium stockpile (which is a built-in characteristic of MASFLO-2), the deviations of MASFLO-2 were small.

In conclusion, the above benchmarking has shown that over long periods of time, MASFLO-2 converges to results compatible with ALPS for both scenarios looked at here. Hence, MASFLO-2 is quite suitable for present purposes.

TABLE 4.5

COMPARISON OF ORE REQUIREMENTS:

MASFLO-2 VERSUS ALPS (H-6) FOR COUPLED UO_2 AND PuO_2 REACTORS WITH FULL RECYCLE

Year	ST U_3O_8 (2) (ALPS)	ST U_3O_8 (2) (MASFLO-2)	$\Delta\%$ (1)
1969	4.44×10^3	1.890×10^3	57.4
1971	1.355×10^4	8.402×10^3	38.0
1973	2.861×10^4	2.101×10^4	26.6
1975	4.813×10^4	3.794×10^4	21.2
1977	7.237×10^4	5.746×10^4	20.6
1979	1.082×10^5	8.285×10^5	23.4
1981	1.634×10^5	1.273×10^5	22.1
1983	2.349×10^5	1.870×10^5	20.4
1985	3.221×10^5	2.589×10^5	19.6
1987	4.299×10^5	3.493×10^5	18.7
1989	5.595×10^5	4.634×10^5	17.2
1991	7.048×10^5	6.004×10^5	14.8
1993	8.606×10^5	7.596×10^5	11.7
1995	1.035×10^6	9.434×10^5	8.9
1997	1.244×10^6	1.159×10^6	6.8
1999	1.493×10^6	1.412×10^6	5.4
2001	1.783×10^6	1.699×10^6	4.7

(1) $$\Delta\% = \frac{\text{ST } U_3O_8 \text{ (ALPS)} - \text{ST } U_3O_8 \text{ (MASFLO-2)}}{\text{ST } U_3O_8 \text{(ALPS)}} \times 100$$

(2) Purchased at end of stage.

TABLE 4.6

COMPARISON OF SEPARATIVE WORK REQUIREMENTS: MASFLO-2 VERSUS ALPS (H-6) FOR COUPLED UO_2 AND PuO_2 REACTORS WITH FULL RECYCLE

Year	MTSWU(2) (ALPS)	MTSWU(2) (MASFLO-2)	Δ%(1)
1969	2.640×10^3	2.666×10^2	89.9
1971	8.233×10^3	4.383×10^3	46.8
1973	1.780×10^4	1.255×10^4	29.5
1975	3.072×10^4	2.386×10^4	22.3
1977	4.720×10^4	3.727×10^4	21.0
1979	7.127×10^4	5.477×10^4	23.2
1981	1.078×10^5	8.464×10^4	21.5
1983	1.560×10^5	1.252×10^5	19.7
1985	2.159×10^5	1.749×10^5	19.0
1987	2.904×10^5	2.375×10^5	18.2
1989	3.804×10^5	3.167×10^5	16.7
1991	4.827×10^5	4.124×10^5	14.6
1993	5.924×10^5	5.243×10^5	11.8
1995	7.190×10^5	6.543×10^5	9.0
1997	8.664×10^5	8.071×10^5	6.8
1999	1.040×10^6	9.861×10^5	5.2
2001	1.243×10^6	1.190×10^6	4.3

(1) $$\Delta\% = \frac{ST\ U_3O_8\ (ALPS) - ST\ U_3O_8\ (MASFLO\text{-}2)}{ST\ U_3O_8\ (ALPS)} \times 100$$

(2) Purchased at end of stage.

4.3 Application of MASFLO-2 to Finite-Life Systems of LWR's

In this study, the primary emphasis has been on growing systems, i.e., positive system growth rates, r(t). However, MASFLO-2 is also applicable to systems which have negative growth rates. Figure 4.1 shows an example of a possible scenario for growth in a PWR economy, in which the installed capacity reaches some maximum value, $\hat{E}$ and then due to ore shortages (in the absence of a breeder), the PWR's are displaced by some other more economical technology and the installed capacity declines to zero after T years.

The model development so far has implicitly dealt with the yearly and cumulative characteristics of the rising part of the curve in Figure 4.1. Here we address the negative growth portion.

One of the assumptions built into MASFLO-2(see Chapter 3) is that in terms of ore and separative work requirements, the initial inventory required to start up a reactor is equivalent to the inventory remaining at the end of the reactor's life. This assumption is also made in at least one other model described in Chapter 3 (N-1). The linear reactivity model applied to obtain the equivalent number of startup batches, N, in the present work, will give the same value of N when applied to the end-of-life batches. Similarly, additional batches or fractions of a batch alloted to "fill the pipeline" will be recovered when their in-pile counterparts are no longer needed.

Following the above argument, if one chooses two times τ_1, τ_2, see Fig. 4.1, such that the installed capacity of the

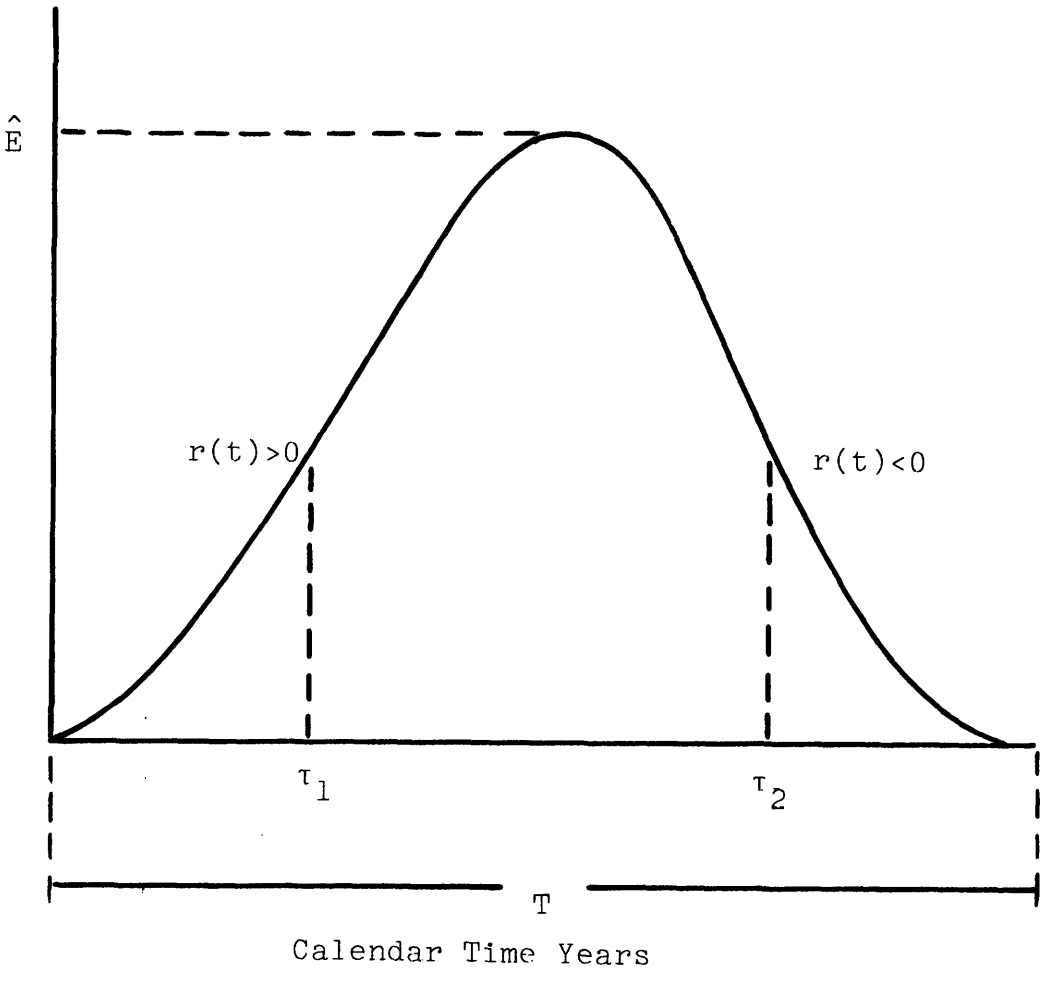


Fig. 4.1. A SCHEMATIC SCENARIO FOR A FINITE-LIFE SYSTEM OF LWR'S

system is the same, i.e., $E(\tau_1) = E(\tau_2)$ then for the simple case of a UO_2 (slightly enriched U-235) system with uranium recycle, the cumulative consumption, $IOU(\tau_2-\tau_1)$ between years T_2 and T_1 is given by

$$IOU(\tau_2 - \tau_1) = F_{S1}^{o} \int_{T_1}^{T_2} E(t)\, dt \qquad (4.8)$$

where F_{s1}^{o}, is the steady-state (r=0) consumption rate given by Eq. (4.1) with r=0:

$$F_{s1}^{o} = \frac{1.3P_1}{SL\ E_1\ T} \left[\frac{(XF_{11}-X_w)}{(X_{NAT}-X_W)}\right].\left[\ 1 - (SL)^2.RD1.\frac{(XD_{11}-X_w)}{(XF_{11}-X_w)}\right] \qquad (4.9)$$

In fact, for the period 0 to T years (i.e., the full period of the system) the cumulative ore consumption is given by:

$$IOU(T) = F_{S1}^{o} \int_{o}^{T} E(t)\, dt \qquad (4.10)$$

For finite incremental growth scenarios Eq. (4.10) reduces to

$$IOU(T) = F_{S1} \sum_{t=o}^{T} E(t).\ \Delta t \qquad (4.11)$$

The above results show that the zero-growth rate (r=o) output of MASFLO-2, namely F_{S1}^{o}, determines the overall cumulative ore usage of a system having a finite lifetime. The above also holds for SWU requirements.

Thus for a finite-life system of UO_2 (slightly enriched U-235) reactors, with a sinusoidal growth rate, E(t), given by:

$$E(t) = \hat{E}_o \, \text{Sin} \, \frac{\pi t}{T} \quad t \geq T \tag{4.12}$$

where $\hat{E}_o$ is the maximum installed capacity, the overall cumulative ore usage of the finite-life system IOU(T), is given by

$$IOU(T) = F_{S1}^{o} \int_0^T \hat{E}_o \, \text{Sin} \, \frac{\pi t}{T} \tag{4.13}$$

or, integrating

$$IOU(T) = \left(\frac{2}{\pi}\right)\hat{E}_o \cdot F_{S1}^{o} \cdot T \tag{4.14}$$

where F_{S1}^{o} is given by Eq. (4.9). Equation (4.14) gives the overall cumulative ore usage of a finite-life system having a sine-shaped capacity history - which is a reasonable approximation to expected scenarios (other histories would merely change the magnitude of the constant $2/\pi$ in Eq. (4.14)).

In conclusion, this section demonstrates that MASFLO-2 is applicable to any growth scenario, whether a growing or dying system, or indeed over the entire life of a finite-life system. In the latter case the zero growth rate (r=0) output of MASFLO-2 determines the overall cumulative ore and separative work usage.

4.4 The Effect of Unit Cell Size on Ore and SWU Utilization

As was mentioned in Chapter 2, a study of the effect of

unit cell size (fuel pin diameter on ore and SWU utilization was performed. In this investigation the fuel-to-coolant volume ratio was maintained constant(at the Maine Yankee base-case value). All cell dimensions were shrunk or expanded to obtain cell volumes of 0.5 V_o,V_o and 1.5 V_o; V_o being the volume of the standard Maine Yankee cell. Two reactor types were considered: UO_2 (slightly enriched U-235) and PuO_2/UO_2.

The calculational procedure followed to obtain the mass balance parameters was the same as discussed in Chapter 2 for the fuel-to-coolant volume ratio studies.

Having obtained these parameters four scenarios were considered (1) UO_2 (slightly enriched U-235) no uranium recycle, (2) the same as before but with uranium recycle (3) UO_2 (slightly enriched U-235 reactors coupled to a PuO_2/UO_2 reactor with full uranium and plutonium recycle (4) the same as scenario 3 but with single pass plutonium recycle. With reference to the full recycle of plutonium in scenario 3 above, although it will be discussed more thoroughly in the next section, in order to account for isotopic degradation a weighting value of 0.8 (see Chapter 3) was used for multi-recycle plutonium.

Scenarios (3) and (4) were also considered with a mixed coupling of the two reactor types, i.e., one cell volume, say V_o, for the UO_2 reactor coupled to a PuO_2/UO_2 reactor having a different cell volume, for example 1.5 V_o.

MASFLO-2 was used to generate the ore and SWU requirements per GW(e)yr for three growth rates, 0,5 and 10% per year. The parameters used are documented in Appendix E.

Table 4.7 shows the ore requirements per GW(e)yr for the four scenarios listed above. This table demonstrates that the variation from the standard case for all growth rates considered is at most 1.5% for the no recycle case, less than 1% for the uranium recycle case, less than 4% for full recycle, and less than 1% for single pass plutonium recycle.

The slight advantage gained at the smallest cell volume in the case of full recycle is due to the smaller spatial self-shielding, and hence slightly higher conversion ratio achieved at the smaller fuel pin diameter involved.

However, in the single pass plutonium case this advantage drops to less than 1%. Since, as will be discussed later, the real-life situation lies between these two extreme cases due to retirement of the plutonium after several recycles (due to degradation of the plutonium isotopic composition with multi-recycle) the conclusion that can be derived from the above results is that varying the unit cell size has little effect on the ore requirements of a system.

Table 4.8 demonstrates the separative work requirements per GW(e)yr for the same cases, and shows the small effect of unit cell size on separative work requirements. In fact, calculations have shown that even when reactors are coupled in a mixed-cell-volume system, in an attempt to separately optimize the producer and consumer reactors, the results are but very little better.

Thus, varying the unit cell size (fuel pin diameter) at constant fuel-to-coolant volume ratio has a negligible effect

TABLE 4.7

ORE REQUIREMENTS, STU_3O_8/GW(e)yr,[3]
AS A FUNCTION OF UNIT CELL VOLUME

Fuel Cycle	Cell Volume U/U	Cell Volume Pu/U	System Growth Rate 0%/yr	5%/yr	10%/yr
1) U-235/U-238	Vo/2	-	184.6	205.2	225.7
(No Recycle)	Vo	-	182.0	202.1	222.2
	1.5Vo	-	181.4	200.8	220.3
2) U-235/U-238	Vo/2	-	154.8	175.8	195.9
(Uranium Recycle)	Vo	-	155.6	175.7	195.8
	1.5Vo	-	156.5	176.0	195.5
3) U-235/U-238[1]	Vo/2	Vo/2	107.1	127.8	148.4
Pu/U (Fuel U+Pu	V_o	Vo	111.2	131.2	151.2
Recycle)	1.5Vo	1.5Vo	114.2	133.4	152.7
4) U-235/U-238[2]	Vo/2	Vo/2	127.1	146.2	165.6
Pu/U (U + Pu	Vo	Vo	128.1	146.9	165.9
recycle/single pass Pu	1.5Vo	1.5Vo	129.5	147.7	166.1

(1) Repetitive plutonium recycle to extinction.

(2) Single pass plutonium recycle

(3) Per GW(e)yr (rated) at 75% capacity, 0.2% tails.

TABLE 4.8

SEPARATIVE WORK REQUIREMENTS, MTSWU/GW(e)yr[3]
AS A FUNCTION OF UNIT CELL VOLUME

Fuel Cycle	Cell Volume U/U	Cell Volume Pu/U	System Growth Rate 0%/yr	System Growth Rate 5%/yr	System Growth Rate 10%/yr
1) U-235/U-238	Vo/2	-	111.7	124.2	137.2
(no recycle)	Vo	-	109.4	121.7	134.1
	1.5Vo	-	108.8	121.0	133.2
2) U-235/U-238	Vo/2	-	112.3	125.0	137.7
(Uranium Recycle)	Vo	-	111.3	123.7	136.0
	1.5Vo	-	111.3	123.5	135.7
3) U-235/U-238,	Vo/2	-	77.6	91.1	104.4
Pu/U (full U+Pu	Vo	Vo	79.6	92.3	105.0
recycle)	1.5Vo	1.5Vo	81.2	93.6	106.0
4) U-235/U-238[2]	Vo/2	Vo/2	92.1	104.2	116.4
Pu/U (u+Pu	Vo	Vo	91.7	103.4	115.2
recycle, single pass Pu)	1.5Vo	1.5Vo	92.0	103.6	115.3

(1) Repetitive plutonium recycle to extinction.

(2) Single pass plutonium recycle.

(3) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

on ore and SWU requirements.

Hence as asserted in Chapter 2, we are justified in excluding fuel pin diameter as an important variable in subsequent neutronic studies. Conversely, we may leave the determination of pin diameter to other criteria, such as those on fuel temperature, stored energy, heat transfer and fluid flow (pressure drop).

4.5 The Effect of Fuel-to-Coolant Volume Ratio on Ore and Separative Work Utilization

In this section, the main results of this work will be presented, namely a study of the effect of fuel-to-coolant volume ratio on ore and separative work requirements of PWR reactor systems. The results which will be presented here are limited to aspects of primary interest.

Five different fuel cycles are considered here:

(1) UO_2 (slightly enriched U-235) reactors with no recycle (here designated U-235/U-238),

(2) the same as (1) but with uranium recycle (here designated U-235/U-238 U Recycle),

(3) UO_2 (slightly enriched U-235) reactor with uranium recycle, recycling plutonium to a PuO_2/UO_2 reactor (here designated U-235/U-238, U Recycle, Pu/U-238),

(4) UO_2 (slightly enriched U-235) reactor with uranium recycling its bred plutonium to a Pu/ThO_2 reactor, which in turn recycles bred U-233 to a $^{233}UO_2/ThO_2$ reactor (here designated U-235/U-238, Pu/Th, U-233/Th), and

(5) UO_2 (93% enriched U-235)/ThO_2 reactor recycling uran-

ium-235 (and small amounts of bred plutonium) to itself, and recycling bred U-233 to a $^{233}UO_2/ThO_2$ reactor (here designated U-235/Th, U-233/Th).

For systems (3), (4) and (5) above two limiting cases were considered: repetitive recycle of the bred U-233 and plutonium to extinction; and single pass plutonium and U-233 recycle.

The first option is the most optimistic scenario, in that it allows for the complete utilization of bred fissile material. This does not correspond to a real life situation, since the repetitive recycle of U-233 and plutonium builds up fertile and poison isotopes (U-234, U-236 and Pu-242 respectively), resulting in the isotopic degradation of the fuel, which in turn creates a situation where economics dictates the retirement of the multi-recycled fuel. It should be noted, however, that in this option an allowance was made for the effect of isotopic degradation. Based on previous work (H-3), and the results of Chapter 2, best estimate values of the weighting factor, W,(Chapter 3) were made for multi-recycle plutonium and U-233. These were taken as 0.9 for U-233 and 0.8 for plutonium. Since these weighting factors are expected to vary with Vf/Vm and the fuel cycle, these approximations are a limitation of the present study which will warrant future investigation.

The second option, that of single pass recycle, represents the overly pessimistic case, in which a single recycle serves to degrade isotopic content to an extent precluding further use. Although reality will be somewhere in between, being determined primarily by economics (H-3), in view of recent increases in

yellowcake prices, the full recycle to extinction case will probably prove to be closer to the real-life situation in a LWR-only economy.

Although both ore and separative work utilization will be evaluated, the emphasis here will be on ore utilization, because ore costs are the single largest contributor to fuel cycle costs, because ore is subject to scarcity-related escalation, and because separative work may become cheaper in the future due to technological improvements.

It must be noted here that the results presented for ore and separative work requirements are on the basis of per GW(e)yr (rated) at 75% capacity factor.

As a final note before the presentation of the data, the important system growth rates of those considered here (0,5, and 10% per year) are the zero and ten percent per year growth rates.

The former is important for reasons discussed in Section 4.3, in that the resource utilization at this growth rate determines the overall cumulative resource requirements of a finite-life system, which will be the case in a non-breeder nuclear economy. The latter growth rate of 10% per year is important in that it represents a lower bound of recently predicted nuclear growth rates for the world of 11-14% per year over the period 1975-2000, (W-3),(E-5) (world electric growth rates in this period are expected to be 5.3% per year). Hence the results presented at this growth rate will dictate a possible short term strategy.

In the first group of comparisons full plutonium and U-233 recycle is assumed (i.e., recycle to extinction) and the fuel-

to-coolant volume ratio of both producer and consumer reactors are the same.

Table 4.9 presents the calculated system ore requirements per GW(e)yr at three system growth rates (0,5 and 10% per year) for the above cycles, for four fuel-to-coolant volume ratios. The results from this table for zero and ten percent per year growth rates are presented in Figs. 4.2 and 4.3, respectively.

Before searching for an overall optimum, it is appropriate to first discuss each cycle in more detail. Examining Figs. 4.2 and 4.3 and Table 4.9, it is observed that for the U-235/ U-238 reactor system utilizing either once-through,(no recycle) or uranium recycle (i.e., fuel cycles one and two), the optimum Vf/Vm value is close to that of current PWR's for all system growth rates.

For the third fuel cycle considered, the U-235/U-238, U Recycle, Pu/U-238 fuel cycle, the optimum Vf/Vm value moves from a value of 0.6 at 0%/yr growth rate to a value very close to current PWR's (~0.54) at 10% growth. In contrast to the above two cycles (cycles 1 and 2), this cycle exhibits behavior at a zero growth rate (see Fig. 4.2) that indicates a potential further decrease in steady-state ore consumption at Vf/Vm values exceeding the range of the present study which calls to mind Edlund's proposal of a super dry all plutonium light water breeder (Vf/Vm ~ 2-3) (E-3).

At zero growth rates, the two last cycles considered, the UO_2/ PuO_2/ThO_2, $^{233}UO_2/ThO_2$ and the $^{235}UO_2/ThO_2/ThO_2$ fuel cycles, which are also the only cycles here using the thorium fuel cycle,

TABLE 4.9

ORE USAGE (ST U_3O_8/GW(e)yr)[3] AS A FUNCTION OF FUEL-TO-COOLANT VOLUME RATIO (FULL RECYCLE)

	Fuel-to-Coolant Volume Ratio	System Growth Rate		
		0%/yr	5%/yr	10%/yr
1) One Reactor	A = 0.338	191.2	211.4	231.6
U-235/U-238	B = 0.4816[2]	182.0	202.1	222.2
No Recycle	C = 0.9161	256.5	285.4	314.3
	D = 1.497	403.6	450.8	498.0
2) One Reactor	A	166.8	187.0	207.1
U-235/U-238	B	155.6	175.7	195.8
U-Recycle only	C	175.3	204.1	233.0
	D	218.0	265.2	312.4
3) Two Reactors[1]	A	131.2	151.8	172.2
U-235/U-238	B	111.2	132.1	152.8
U-Recycle,	C	118.0	154.7	188.2
Pu/U-238	D	128.9	182.1	233.0
4) Three Reactors[1]	A	130.4	153.3	175.3
U-235/U-238	B	109.9	133.7	156.4
Pu/Th	C	106.4	145.4	181.1
U-233/Th	D	103.4	160.5	215.5
5) Two Reactors[1]	A	107.2	138.8	170.7
U-235/Th	B	91.9	122.9	154.3
U-233/Th	C	81.6	115.6	150.5
	D	81.3	128.4	177.8

[1] Plutonium and U-233 recycled to extinction

[2] Typical of current PWR lattices

[3] Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

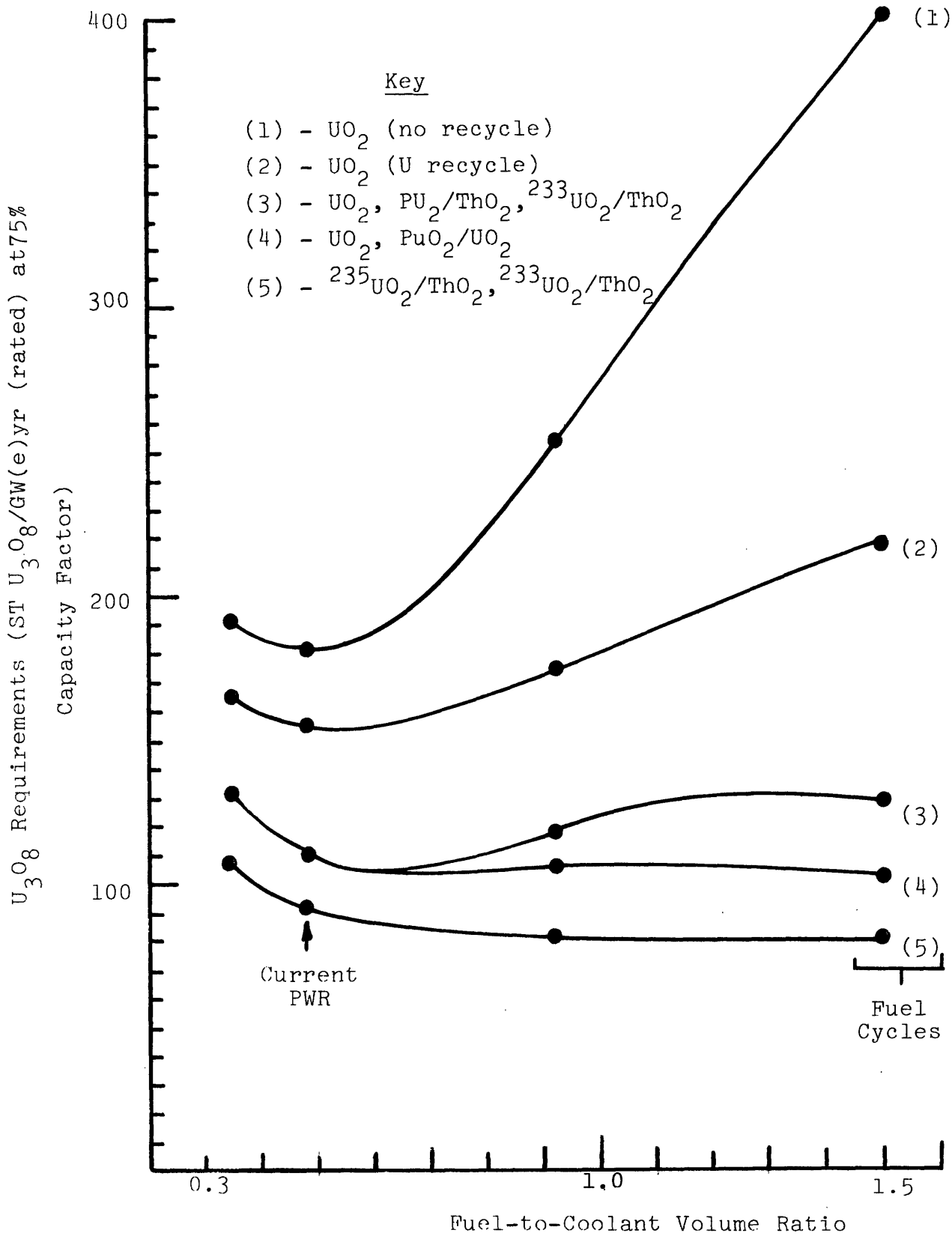


FIG. 4.2. ORE USAGE AS A FUNCTION OF FUEL-TO-COOLANT VOLUME RATIO FOR 0%/YR SYSTEM GROWTH RATE (FULL RECYCLE)

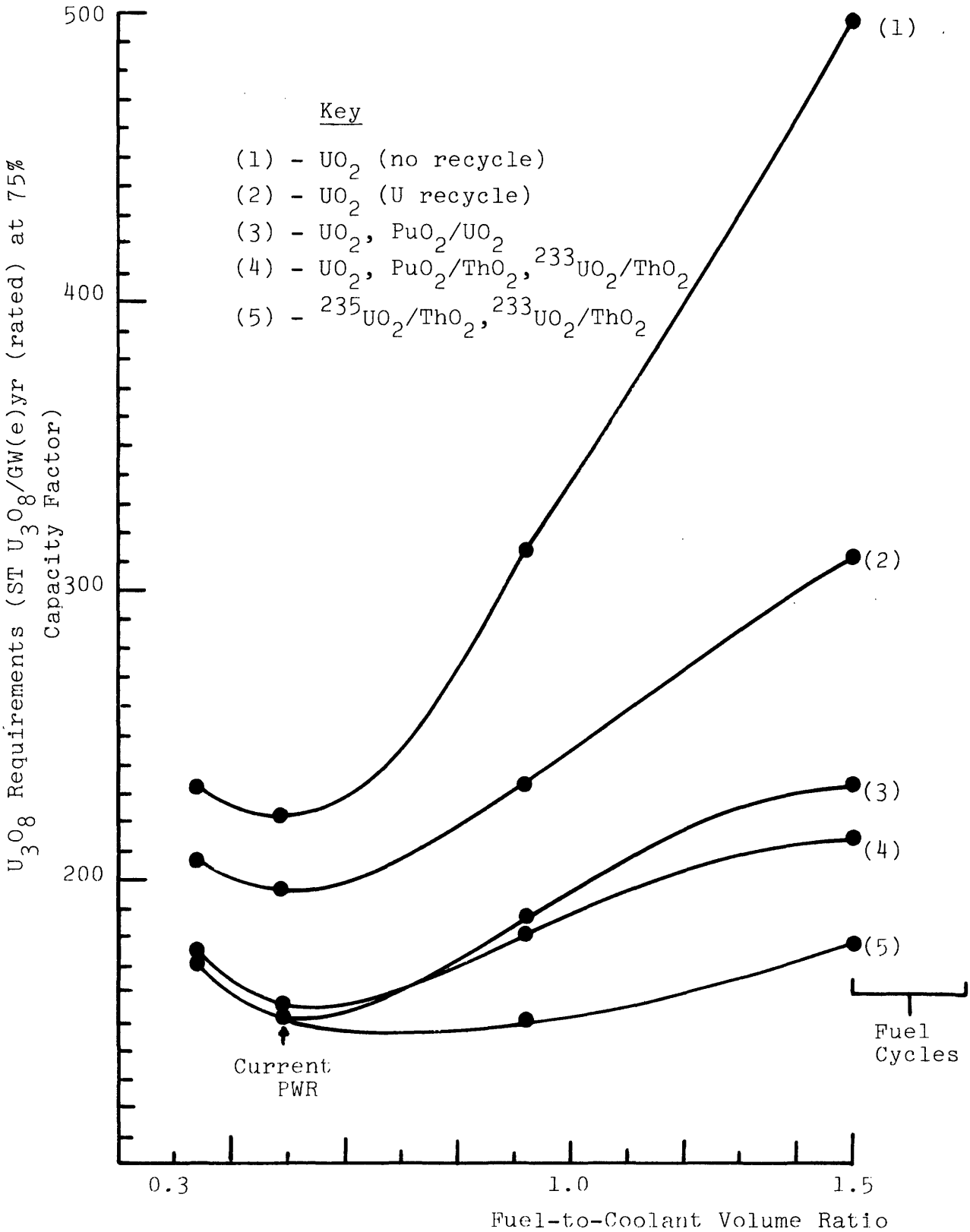


FIG. 4.3. ORE USAGE AS A FUNCTION OF FUEL-TO-COOLANT VOLUME RATIO FOR 10% PER YEAR SYSTEM GROWTH RATE (FULL RECYCLE)

exhibit very similar behavior, i.e., both have very flat optimum Vf/Vm values at the tightest pitches investigated here. This would seem to indicate that in considering the cumulative ore requirements of a finite life reactor system, for these fuel cycles, there is little incentive to go to very tight lattices.

As regards short term ore usage at 10%/yr growth rate the optimum Vf/Vm for these fuel cycles moves towards the Vf/Vm value of today's lattices with the UO_2, PuO_2/ThO_2, $^{233}UO_2/ThO_2$ fuel cycle exhibiting an optimum around a Vf/Vm of value of 0.55 and that of the $^{235}UO_2/ThO_2$, $^{233}UO_2/ThO_2$ fuel cycle exhibiting a slighter, tighter optimum at a Vf/Vm value of around 0.7.

In the consideration of cumulative ore requirements of a finite life reactor system, at zero system growth rate the overall system optimum in terms of minimum ore utilization, for the range of fuel-to-coolant volume ratios, (Vf/Vm) considered here, occurs in the tightest lattice of the fifth fuel cycle investigated (U-235/Th, U-233/Th at a value of 81.3 STU_3O_8/Gw(e)yr a factor of two less than for current once through PWR's (Case 1B), and a savings of 21% over the next best system's value of 103.4 STU_3O_8: the three reactor system, utilizing thorium again (Case 4D). This is of considerable interest since the Light Water Breeder Reactor, LWBR (E-4), advocates using cycle 5 as one of its options, with a very tight lattice (i.e., a metal-to-water ratio of 1.58 for the fully enriched U-235/Th prebreeder, and a metal-to-water ratio of 1.72 for the U-233/Th breeder). Furthermore, the second best fuel cycle, also corresponds to another option of the light water prebreeder, i.e., the use of

a tight lattice Pu/Th prebreeder (metal-to-water ratio of 1.31), and a tight lattice U-233/Th) breeder (metal-to-water ratio of 1.72).

Similarly, when considering short-term cumulative system ore requirements, at 10%/yr (and for that matter 5% per year) system growth rate, the overall optimum cycle is again cycle 5 (U-235/Th, U-233/Th), with an optimum Vf/Vm value at a 10%/yr growth rate of about 0.7, showing a significantly smaller savings of 5% in ore requirements (145 STU_3O_8/GW(e)yr versus 152 STU_3O_8/GW(e)yr over the next best cycle, cycle 3 UO_2, PuO_2/UO_2). This behavior is expected from the discussion in Chapter 1, since the penalty of higher initial core inventories in thorium based fuel cycles is the dominant factor at non-zero system growth rates, and therefore their steady state (zero growth rate) advantage is eroded at higher system growth rates, i.e., in the consideration of short term ore requirements.

A further point must be noted in that the penalty due to isotopic weighting of multi-recycled fuel increases with Vf/Vm, due to the increasing conversion ratio, and therefore higher recycled mass fraction (see Appendix B); in spite of this, however, for the fuel recycle cases (3,4 and 5), steady state or single reactor (0%/yr growth) ore utilization is attractive at higher Vf/Vm values.

One final observation is pertinent. If one examines the ore usage values at Vf/Vm = 0.4816 (conventional PWR) in Table 4.9, cycle 5 (U-235/Th, U-233/Th) is superior except at 10% growth, where cycle 3 (U-235/U-238) has a slight advantage. The single

reactor case (r-0%), which determines the long-term or finite-life system cumulative ore requirements favors cycle 5; this latter observation agrees with findings by CE (S-1). In comparing ore savings at this fuel-to-moderator volume ratio, CE concluded that the long-term, single reactor savings of cycle 5 over the full plutonium recycle fuel cycle (U-235/U-238, Pu/U-238), cycle 3, was 16%; here the savings are 17% (111.2 STU_3O_8/GW(e)yr versus 91.9STU_3O_8/Gw(e)yr). Similarly in a comparison of cycle 5 to the current once through PWR cycle, (Cycle 1), CE found a savings of 42% in ore requirements; here the savings are slightly higher, reaching 50% (182.0STU_3O_8/Gw(e)yr versus 91.9 STU_3O_8/GW(e)yr).

Additional variations were next examined for the full-recycle option to determine whether improvements could be obtained using systems comprised of reactors having different fuel-to-coolant volume ratios, i.e., with producer and consumer reactors at different fuel-to-coolant volume ratios.

Due to the large number of combinations possible, the results presented here are only those near the optimum values for each cycle.

Table 4.10 presents the results for ore utilization, together with the corresponding Vf/Vm values for the producer and consumer reactors, for the case of full recycle of bred material to extinction. These results can be evaluated by comparing them to the corresponding results in Table 4.9.

The first general observation is again the trend of the optimum Vf/Vm values of the system to go towards present PWR Vf/Vm values with increasing growth rates.

TABLE 4.10

ORE UTILIZATION IN MIXED Vf/Vm SYSTEMS
(STU_3O_8/GW(e)yr)[1], FUEL RECYCLE TO EXTINCTION, ALL CASES

Fuel Cycle	J, Reactor Type	Optimum Vf/Vm of J at r%/yr Growth Rate			Optimum System Ore Consumption at r%/yr Growth Rate		
		0	5	10	0	5	10
3) U-235/U-238 U Recycle	U-235/U-238	0.9161	0.9161	0.4816	100.8	126.7	152.8
Pu/U-238	Pu/U-238	0.4816	0.4816	0.4816			
4) U-235/U-238	U-235/U-238	0.9161	0.9161	0.4816			
Pu/Th	Pu/Th	0.4816	0.4816	0.4816	93.6	126.6	155.3
U-233/Th	U-233/Th	1.497	1.497	0.9161			
5) U-235/Th	U-235/Th	0.9161	0.9161	0.4816			
					77.5	115.2	148.6
	U-233/Th	1.497	1.497	0.9161			

(1) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

Another notable observation is that in both systems using the thorium fuel cycle, the optimum occurs with a looser producer reactor and a tighter consumer (U-233/Th) reactor for all growth rates, which agrees with the strategy being used for the Light Water Breeder Reactor and its prebreeder (or producer reactor).

From the perspective of cumulative ore requirements for a finite life system (or long term ore utilization), at zero system growth rate the overall mixed system optimum again occurs in cycle 5 (U-235/Th, U-233/Th) with up to 17% savings in ore requirements over the next best cycle (also optimized for a mixed system) cycle 4. A comparison of Tables 4.9 and 4.10 shows a 5% savings in ore consumption, in going from a system with the tightest lattice (Vf/Vm=1.497 (Table 4.9)) to a system with a looser producer (U-235/Th) lattice (Vf/Vm = 0.9161) and the tightest consumer (U-233/Th) lattice (Vf/Vm = 1.497). Likewise, for the other cycles (cycles 3 and 4) savings in ore requirements of mixed Vf/Vm systems (over the optimum values for uniform fuel-to-coolant systems) are 5 and 9% respectively. Thus it appears that based on long term considerations, separate optimization of consumer and producer reactors is attractive.

Similarly, in considering short term requirements, at ten percent per year system growth rate, the overall mixed system optimum occurs in cycle 5(U-235/Th, U-233/Th). For this cycle a comparison of Table 4.9 and 4.10 shows a savings of only about 1% in ore requirements in going from a system with a Vf/Vm value of 0.9161 to a system with a looser producer lattice (Vf/Vm = 0.4816) and a tighter consumer lattice (Vf/Vm = 0.9161). Similar savings in ore requirements are observed for the other two cycles (3 and

4 which suggests that individual optimization of consumer and producer lattices is not very attractive in the short-term.

As a final note of importance, for short term system ore requirements, at ten percent per year growth rate, the current uranium cycle with plutonium recycle optimizes in terms of ore requirements at a fuel-to-coolant ratio typical of present day reactors, a result which was also characteristic of results presented in Table 4.9 and Fig. 4.3.

Again in the discussion of ore utilization, we now deal with the other limiting case in this study, in which single-pass recycle of plutonium or U-233 is employed.

Table 4.11 presents the calculated system ore requirements per GW(e)yr at three system growth rates for the fuel cycles considered here under single pass recycle, i.e., cycles 3, 4 and 5. The results of this table are presented in Fig. 4.4 for zero and ten percent per year system growth rates.

One of the first pertinent observations is that the optimum Vf/Vm value (see Fig. 4.4) for each cycle now occurs nearer that of present producer reactor PWR's (Vf/Vm = 0.4816) for all system growth rates. This is a natural consequence of the reduced importance of the consumer reactors to the overall system economy, since they now become considerably less efficient at fissile utilization.

Examining Fig. 4.4 and Table 4.11, it is observed that in the consideration of long-term ore requirements, at zero system growth rate the optimum fuel cycle is a trade-off between cycles 3 and 4, (U-235/U-238,Pu/U-238 and U-235/U-238,Pu/Th,U-233/Th), which differ in optimum ore consumption by less than 1%, at a

TABLE 4.11

SYSTEM ORE USAGE (STU_3O_8/GW(e)yr)[1] FOR

SINGLE PASS RECYCLE

Fuel Cycle	Fuel-to-Coolant Volume ratio	System Growth Rate		
		0%/yr	5%/yr	10%yr
3) Two Reactors	A=0.338	141.3	160.9	180.6
U-235/U-238	B=0.4816[2]	128.1	147.3	166.7
U Recycle	C=0.9161	154.6	182.7	211.1
Pu/U-238	D=1.497	183.6	227.6	272.5
4) Three Reactors	A	142.5	162.8	183.2
U-235/U-238	B	127.5	147.7	167.9
Pu/Th	C	149.9	178.7	207.6
U-233/Th	D	173.6	218.4	263.9
5) Two Reactors	A	143.1	172.1	201.7
U-235/Th	B	133.0	161.4	190.4
U-233/Th	C	140.0	165.5	198.0
	D	152.7	198.2	245.3

(1) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

(2) Typical of current PWR lattices.

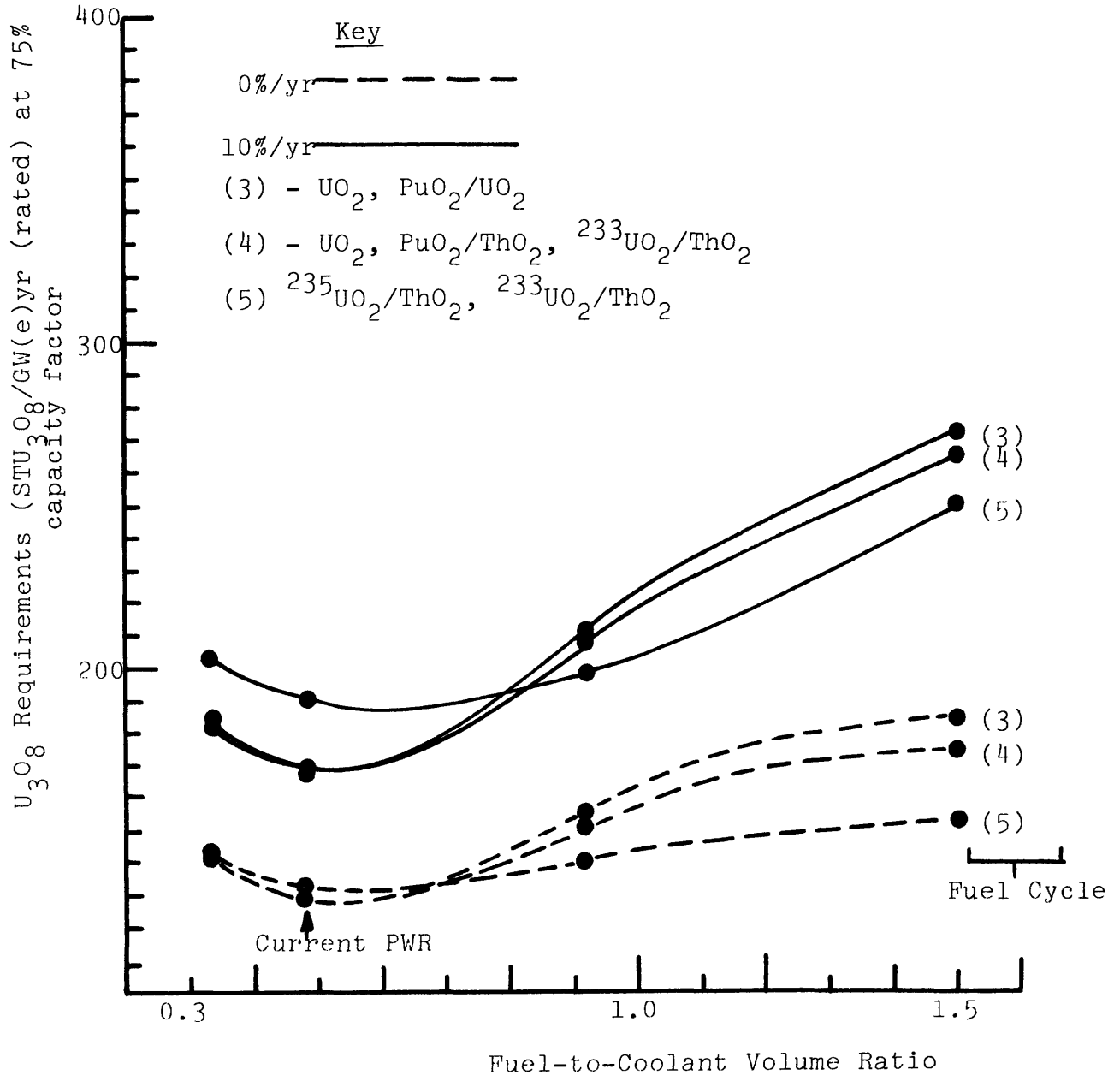


FIG. 4.4. ORE USAGE AS A FUNCTION OF FUEL-TO-COOLANT VOLUME RATIO FOR 0 AND 10%/YR SYSTEM GROWTH RATE (SINGLE PASS RECYCLE).

Vf/Vm value slightly higher (~0.5) than current PWR's. Furthermore, their advantage over cycle 5(U-235/Th, U-233/Th) is at most 5%. Note that this significant turn around in relative ore consumption between cycle 5 and the other cycles, when single pass recycle is considered (versus full recycle to extinction) shows the high importance of an efficient consumer reactor to cycle 5, the efficiency of which is severly degraded by allowing only single pass recycle.

In considering short term ore requirements, at 10 percent per year growth rate (as for zero growth rate) the optimum fuel cycle is a trade-off between cycles 3 and 4, which differ in ore utilizaiton by less than 1%, at an optimum Vf/Vm value near that of current PWR's (see Fig. 4.4). From Fig (4.4), one can observe by a comparison of optimum ore requirements of the three cycles, that cycles 3 and 4 exhibit a savings of about 11% over cycle 5(~168STU_3O_8/GW(e)yr versus~188STU_3O_8/GW(e)yr). This is expected due to the relatively higher initial inventory of the U-235/Th reactor in cycle 5, which uses 93% enriched U-235.

Finally, in spite of the penalty of single-pass recycle on the systems employing full recycle, at present day Vf/Vm values they exhibit considerable savings over a once-through cycle. Examining Tables 4.9 and 4.11 for the zero growth rate case, by a comparison of either cycle 3 or 4 of Table 4.11 to cycle 1 of Table 4.9, one observes savings of 30% in ore requirements; at five percent per year growth rate this value is 27%, and is 24% at ten percent per year growth rate.

As before, mixed Vf/Vm scenarios were studied for the case of multi-tiered systems (fuel cycles 3,4 and 5). Again as before,

by a comparison of the results of Table 4.12 to Table 4.10, we see that the optimum Vf/Vm values move to wetter lattices as the option of single-pass recycle is employed in lieu of fully repetitive recycle. As in Table 4.11, cycles 3 and 4 exhibit optimum ore utilization at all growth rates.

As a final note here, it must be said that for single-pass recycle, cycle 4(U-235/U-238,Pu/Th,U-233/Th), which utilizes the thorium fuel cycle, is at least comparable to the full-recycle uranium fuel cycle, cycle 4(U-235/U-238, U recycle, Pu/U-238).

The results of the calculations of separative work requirements (MTSWU/GW(e)yr) will now be presented for the above cycles. Here the discussion will be less detailed than that pertaining to ore requirements, due to the much lesser emphasis placed on separative work requirements, for reasons discussed in an earlier part of this section.

Table 4.13 presents the calculated system separative work requirements (MTSWU/GW(e)yr) for the case of full recycle to extinction. The results here demonstrate behavior parallel to the corresponding results for ore usage (Table 4.9) with two important exceptions. First, the advantage of cycle 5 in ore usage is not seen, in that the optimum at all growth rates is shared by cycles 3 and 4, whose values differ by less than 2 percent at all growth rates. Thus while demonstrating superior ore utilization, cycle 5 has the penalty of increased separative work requirements over the other cycles (3 and 4). In considering the optimum separative work requirements for cycle 5 versus

TABLE 4.12

ORE UTILIZATION IN MIXED Vf/Vm SYSTEMS

(ST U_3O_8/GW(e)yr)[1] FOR SINGLE PASS RECYCLE

Fuel Cycle	J, Reactor	Optimum Vf/Vm of J at r%/yr Growth Rate			Optimum System Consumption at r%/yr Growth Rate		
		0	5	10	0	5	10
3) U-235/U-238	U-235/U-238	0.9161	0.4816	0.4816			
U Recycle					122.9	145.6	164.8
Pu/U-238	Pu/U-238	0.338	0.338	0.338			
4) U-235/U-238	U-235/U-238	0.9161	0.4816	0.4816			
Pu/Th	Pu/Th	0.338	0.338	0.338	124.4	147.5	167.6
U-233/Th	U-233/Th	0.4816	0.4816	0.4816			
5) U-235/Th	U-235/Th	0.4816	0.4816	0.4816			
U-233/Th					133.0	161.4	190.4
	U-233/Th	0.4816	0.4816	0.4816			

[1] Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

TABLE 4.13

SEPARATIVE WORK REQUIREMENTS (MTSWU/GW(e)yr)[3]
AS A FUNCTION OF FUEL-TO-COOLANT VOLUME RATIO (FULL RECYCLE)[1]

Fuel Cycle	Fuel-to-Coolant Volume Ratio	System Growth Rate 0%/yr	5%/yr	10%/yr
1)One Reactor	A=0.338	117.4	127.8	138.3
U-235/U-238	B=0.4816[2]	109.4	119.8	130.2
No Recycle	C=0.9161	175.0	192.7	210.5
	D=1.497	310.2	344.4	378.6
2) One Reactor	A	120.0	130.4	130.8
U-235/U-238	B	111.3	121.7	132.1
U Recycle	C	144.0	161.8	179.6
	D	193.7	227.9	262.1
3) Two Reactors	A	94.4	105.9	117.1
U-235/U-238	B	79.6	91.5	103.1
U-Recycle	C	97.0	122.7	145.1
Pu/U-238	D	114.6	156.5	195.4
4) Three Reactors	A	93.8	106.9	119.2
U-235/U-238	B	78.6	92.6	105.5
Pu/Th	C	87.5	115.3	139.5
U-233/Th	D	91.9	137.9	180.8
5) Two Reactors	A	107.8	139.5	171.4
U-235/Th	B	92.5	123.5	154.9
U-233/Th	C	82.2	116.3	151.2
	D	82.1	129.4	178.9

(1) Plutonium and U-233 recycled to extinction.
(2) Typical of current PWR lattices.
(3) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

cycles 3 or 4, at zero growth rate the penalty for cycle 5 is only 4% (cycles 5D versus 4B), but increases to 47%/cycles 3B/ cersus 5B) for the short term situation represented by the 10%/yr growth rate. The second exception is that, for all growth rates all cycles, except cycle 5, optimize at close to the current PWR Vf/Vm value.

As before, mixed Vf/Vm scenarios were considered for cycles 3,4 and 5. Table 4.14 demonstrates the results of this work. The important observations to be extracted from this table are that it more explicitly points out the two important observations from the previous table (Table 4.13): (1) the superiority of cycles 3 and 4 over cycle 5, and (2) the observation that the optimum Vf/Vm for the cycles is close to current PWR's with the exception of cycle 5.

Table 4.15 demonstrates the separative work requirements of the fuel recycle systems (3,4 and 5) for the option of single-pass recycle. Again as in the previous two tables (Table 4.13, 4.14), the same two major observations are demonstrated, however, in this option, they are even more absolute. First, all cycles optimize at a Vf/Vm value close to current PWR's for all system growth rates. Second, the advantage of cycles (3 and 4), which are again very similar in performance, over cycle 5 is even more dominant here than in the previous option of full recycle (here at zero growth rate cycle 5 exhibits a 46% higher SWU consumption rate, and at ten percent per year a 70% higher SWU consumption rate).

Finally, as before, mixed Vf/Vm scenarios were considered

TABLE 4.14

SEPARATIVE WORK REQUIREMENTS IN MIXED Vf/Vm SYSTEMS (MTSWU/GW(e)yr)[1], FULL RECYCLE TO EXTINCTION, ALL CASES

Fuel Cycle	J, Reactor Type	Optimum Vf/Vm of J at r%/yr Growth Rate			Optimum System Consumption at r% Growth Rate		
		0	5	10	0	5	10
3) U-235/U-238	U-235/U-238	0.4861	0.4816	0.4816			
U-Recycle					79.6	91.5	103.1
Pu/U-238	Pu/U-238	0.4816	0.4816	0.4816			
4) U-235/U-238	U-235/U-238	0.4816	0.4816	0.4816			
Pu/Th	Pu/Th	0.4816	0.4816	0.4816	75.7	91.4	104.7
U-233/Th	U-233/Th	1.497	1.497	0.9161			
5) U-235/Th	U-235/Th	0.9161	0.9161	0.4816			
U-233/Th					78.1	115.9	149.2
	U-233/Th	1.497	1.497	0.9161			

(1) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

TABLE 4.15

SEPARATIVE WORK REQUIREMENTS (MTSWU/GW(e)yr)[1]
FOR SINGLE PASS RECYCLE

Fuel Cycle	Fuel-to-Coolant Volume Ratio	System Growth Rate 0%/yr	5%/yr	10%/yr
3) Two Reactors	A=0.338	101.7	112.2	122.8
U-235/U-238	B=0.4816[2]	91.7	102.2	112.4
U-Recycle	C=0.9161	127.1	144.9	162.6
	D=1.497	163.1	195.7	228.6
4) Three Reactors	A	102.5	113.6	124.6
Pu/Th	B	91.2	102.3	113.3
U-233/Th	C	123.3	141.7	160.0
	D	154.3	187.7	221.3
5) Two Reactors				
U-235/Th	A	144.0	173.0	202.6
U-233/Th	B	133.9	162.0	191.2
	C	135.0	166.5	199.0
	D	154.3	199.7	246.8

(1) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

(2) Typical of current PWR lattices.

for cycles 3,4 and 5. Table 4.16 demonstrates these results. Again the same observations as made with respect to Table 4.15 previously are demonstrated, and even more explicitly.

The conclusions of this section are now reviewed.

Before discussing those conclusions which are dependent on the particular scenario, a few general observations are pertinent.

1) For all system growth rates, (i.e., both short-term and long term considerations), for both full recycle and single pass recycle, and for both ore and separative work requirements, the optimum fuel-to-coolant volume ratio for both the present day once-through uranium fuel cycle and the uranium fuel cycle with uranium recycle, is very close to the value of current PWR's. Hence, if these cycles are considered no major lattice re-design is warranted.

2) The identification of the best strategy for resource utilization in PWR fuel cycles will depend on whether short-term optimization or long-term optimization of resource utilization is preferred, since these optima depend strongly on system growth rates, favoring tighter lattices at lower growth rates and wetter lattices at higher growth rates.

In the consideration of long term system ore requirements, i.e., zero growth rate under the option of full recycle of bred material to extinction, the following conclusions are most pertinent.

1) In terms of ore utilization, the most attractive fuel cycle is the U-235/Th,U-233/Th fuel cycle, with a mixed Vf/Vm

TABLE 4.16

SEPARATIVE WORK REQUIREMENTS IN MIXED Vf/Vm SYSTEMS (MTSWU/GW(e)yr)[1]

FOR SINGLE PASS RECYCLE

Fuel Cycle	J, Reactor Type	Optimum Vf/Vm of J at r%/yr growth rate			Optimum System Consumption at r%/yr Growth Rate		
		0	5	10	0	5	10
3) U-235/U-238	U-235/U-238	0.4816	0.4816	0.4816			
U-Recycle					90.5	100.8	111.2
Pu/U-238	Pu/U-238	0.338	0.338	0.338			
4) U-235/U-238	U-235/U-238	0.4816	0.4816	0.4816			
Pu/Th	Pu/Th	0.338	0.338	0.338	91.2	102.1	113.0
U-233/Th	U-233/Th	0.4816	0.4816	0.4816			
4) U-235/Th	U-235/Th	0.4816	0.4816	0.4816			
U-233/Th					133.9	162.2	191.2
	U-233/Th	0.4816	0.4816	0.4816			

(1) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails

system (Vf/Vm of U-235/Th ≈ 0.9161 and Vf/Vm of U-233/Th ≈ 1.497) a result which agrees with the strategy of one of the options suggested for the LWBR (E-4). The savings in ore requirements of this system over the present once-through conventional PWR's in long term ore utilization is up to 57%. Results also show that the savings of the U-235/Th, U-233/Th fuel cycle at the Vf/Vm value of todays reactors over the present day conventional PWR once through-cycle is up to 50%. This latter finding agrees quite well with results found by CE (S-1). Finally, at this growth rate, the second best fuel cycle is the (U-235/U-238, Pu/Th, U-233/Th) fuel cycle using a mixed system Vf/Vm of 0.9161, 0.4816, 1.497 respectively. This also has significant implications in that it is another fuel cycle option considered for the LWBR, (E-4) however, contrary to the use of a tight pitch Pu/Th prebreeder in the LWBR program, the results here advocate the use of a looser lattice in the Pu/Th prebreeder. However, this contradiction should be tempered by the following considerations:

(A) the Pu/Th prebreeder of the LWBR, program and the U-233/Th breeder are quite intricately designed, utilizing inner and outer blankets, whereas the present result applies to a uniform lattice.

(B) the present option of full recycle only approximates the real situation.

2) The separate optimization of producer/consumer reactors is attractive, with savings of up to 9% in ore requirements over uniform Vf/Vm systems. The optimum here involves a looser con-

sumer and a tighter producer lattice (e.g. for the U-235/U-238, U recycle, Pu/U-238 cycle the optimum producer and consumer Vf/Vm values were 0.9161 and 0.4816, respectively).

In the consideration of short term ore utilization, at 10% per year system growth rate, which represents a lower bound of recently predicted nuclear growth rates for the world in the period 1975-2000, (E-5), (W-3), the following conclusions are pertinent.

1) The overall optimum fuel cycle at this growth rate is again the U-235/Th,U-233/Th fuel cycle using a mixed Vf/Vm system of around 0.4816 and 0.9161 respectively, which shows a shift towards wetter lattices at higher system growth rates. However, the savings in ore requirements over the Uranium cycle with full recycle (U-235/U-238, U recycle, Pu/U-238) is only about 1%, a result which severly penalizes this cycle, if the strategy adopted by the nuclear industry is to optimize short term ore usage; since a 1% savings would not justify the development of a thorium cycle, (note that the uranium cycle with full recycle shows better ore utilization than the other thorium cycle (U-235/U-238, Pu/Th, U-233/Th) at this growth rate).

Before reviewing the conclusions for the single pass LWR recycle option the following observation is pertinent. Single-pass LWR recycle is best contemplated in a mixed LWR fast breeder economy, where the single-pass fissile mass discharged from the LWR is recycled to the fast breeders. The buildup of higher isotopes is of no substantial disadvantage in the fast spectrum of FBR's. The use of this option in this study is primarily as

a pessimistic version to be contrasted with the optimistic version of full recycle to extinction. The economics of the particular cycle will dictate at which point bred fissile material is to be retired.

For the option of single-pass recycle, and with regard to long term strategies, for ore utilization at zero growth rate the following conclusions are pertinent:

1) In terms of ore utilization, the optimum system is the present day uranium cycle with fuel recycle (using a tight producer (U-235/U-238) Vf/Vm $\approx$ 0.9161, and a wetter (Vf/Vm ~ 0.338) consumer (U-235/U-238). Although its advantage over the (U-235/U-238, Pu/Th, U-233/Th) system and the U-235/Th, U-233/Th system is small (1% and 7.6%, respectively), this result dictates that if the interim strategy is to use single pass recycle (anticipatory reuse of the once-recycled bred materials in future fast breeders) the incentive for introducing the thorium cycle is essentially eliminated, particularly in view of the development costs of this cycle.

In the consideration of short term strategies for ore utilization, at 10% growth rate the same conclusions apply as for long term strategies.

With respect to separative work requirements, the conclusions here will be brief, since the emphasis is on ore utilization, because ore costs are the the single largest contributor to fuel cycle costs, because ore is subject to scarcity related escalation and because separative work may become cheaper in the future due to technological improvement.

The conclusions regarding separative work utilization are:

(1) For single pass-recycle, for both long and short term considerations, wetter lattices are favored for all cycles (close to the Vf/Vm value of today's PWR's) with the U-235/U-238, U Recycle, Pu/U-238 and U-235/U-238, Pu/Th, U-233/Th fuel cycles demonstrating similar performance, and significantly superior to the U-235/Th, U-233/Th fuel cycle. A notable exception is in full recycle, for long term (zero-growth rate): here the advantages of the Pu-burning cycles above are only at most 3%; however the U-235/Th, U-233/Th cycle shows a savings of up to 17% over these cycles, the optimum for the cycle (cycle 5 see Fig. 4.20) occurring at the same Vf/Vm mix of producer/consumer reactors as for minimum ore requirements.

This further establishes the attractiveness of this cycle.

Finally, in determination of weighting factors needed to account for isotopic degradation in multiply-recycled bred fuel, further investigations are recommended: in particular the effect of Vf/Vm in the various fuel cycles. Related work of this nature is presently being carried out by K.O. Ott (O-6).

4.6 Results of MASFLO-2 Applied to PWR-Breeder Systems

The application of MASFLO-2 to a breeder LWR economy is done quite simply provided that a correct interpretation of parameters is made. In Chapter 3, MASFLO-2 was cast in a form applicable to cases in which mass parameters are given in terms of metric tons per GW(e)yr (rated) at a capacity factor L.

For a two tier system, it can be similarly shown that

the system ore consumption was given by:

$$F_{S2} = \frac{1.1\ (qa_{ij} + \frac{r}{100}\ Q_{ij})}{(\ 1 + R_{j'j})} \tag{4.15}$$

where

$$R_{j'j} = \frac{qd_{ij}}{(\frac{r}{100}\ Q_{i'j'} + qa_{i'j'})} \tag{4.16}$$

F_{S2} is the system ore requirements in STU_3O_8 per GW(e)yr (rated) at a given capacity factor L; Q_{ij} is the initial inventory of the j^{th} reactor per GW(e); qa_{ij} is the requirements per GW(e)yr (rated) at a given capacity factor L of isotope i in reactor j in metric tons; qd_{ij} is the discharge fissile mass of bred isotope i from producer reactor j in metric tons per GW(e)yr (rated) at capacity factor L. (Note that isotope i' is the isotope traded to consumer reactor j' by producer reactor j). Finally, i=1 refers to U_3O_8 rather than U-235, which was the convention used in the prior applications of MASFLO-2.

In the application of MASFLO-2 to breeder-LWR systems, the breeder reactor becomes the consumer reactor and the LWR reactor the producer reactor. Note however, that due to the net production of fissile isotopes by the breeder reactor the term $qa_{ij'}$ in Eq. (4.16) is negative.

In this application, it was decided to look at three scenarios (1) a UO_2 (slightly enriched U-235) reactor coupled to a LMFBR utilizing plutonium, (2) a UO_2 (93% enriched U-235)/ ThO_2 reactor coupled to a LWBR on the U-233/Th cycle and (3) a

UO_2 (93% enriched U-235)/ThO_2 reactor coupled to a LMFBR utilizing the U-233/Th cycle.

The effect of varying the fuel-to-coolant volume ratio, Vf/Vm, of the producer reactors was also examined.

Table 4.17 lists the reactor mass flow parameters of the three breeder reactors considered. In view of the fact that the references presented these mass flow parameters such that all fissile isotopes were weighted equally, the relative worth of the various fuel isotopes could not be allowed for; as reported before, K. Ott (0-4) has shown that for plutonium, the proper weighting, W, of the isotopes in a typical LMFBR should be (W_{49}, W_{40}, W_{41}, W_{42}) = (1.0, 0.5, 1.2, 0.11).

Table 4.18 summarizes the calculated ore requirements, STU_3O_8 per GW(e)yr (rated) at 75% capacity factor, of the systems investigated. Note that for all fuel cycles, the zero percent growth rate case is not considered.

This is because at this system growth rate the breeder reactors can more than sustain themselves without the need for coupling to a LWR producer reactor. Referring to Eq. (4.16), the reactors become coupled only when the breeders need fissile mass from the LWR producer reactors, i.e.,

$$(qa_{i'j'} + \frac{r}{100} Q_{i'j'}) > 0, \qquad (4.17)$$

so that the initial inventory, $Q_{i'j'}$, and equilibrium production, $qa_{i'j'}$, determine the growth rate at which the reactors become coupled. Thus for the LMFBR (Pu) with its high breeding ratio, this occurs at greater than 5%/yr (~6%/yr) growth rate,

TABLE 4.17

BREEDER REACTOR PARAMETERS

Reactor Type	Fissile Material	Initial Inventory MT(fissile) /GW(e)	Consumption MT(fissile) /GW(e)yr[1]	Reference
1) LWBR[2]	U-233	4.0	0.0	(E-4)
2) LMFBR[3]	Pu(LWR grade)	3.424	- 0.242	(C-8)
3) LMFBR[2]	U-233	3.822	- 0.034	(C-8)

(1) 75% capacity factor.

(2) Fertile material is thorium.

(3) Fertile material is depleted uranium.

TABLE 4.18

ORE REQUIREMENTS, STU_3O_8/GW(e)yr, FOR COUPLED PWR-BREEDER SYSTEMS[1]

Fuel Cycle	Vf/Vm (Producer)[2]	System Growth Rate		
		5%/yr	10%/yr	15%/yr
1) UO_2/ThO_2	A=0.338	90.9	148.6	195.3
(93% U-235	B=0.4816	84.1	139.9	185.7
	C=0.9161	82.0	141.0	191.3
LWBR	D=1.497	89.7	163.0	229.5
2) UO_2(slightly enriched U-235)	A	--	95.6	157.7
	B	--	78.6	138.1
	C	--	62.0	128.2
LMFBR(Pu)	D	--	62.5	143.2
3)UO_2/ThO_2	A	78.2	139.7	188.2
(93%U-235)	B	72.1	131.3	178.7
	C	69.9	131.7	183.5
LMFBR(U-233)	D	75.9	151.5	219.4

(1) 75% capacity factor, 0.2% enrichment tails.

(2) Vf/Vm of PWR.

and hence no entries are reported for this cycle at the 5% per year system growth rate.

Referring to Table 4.18, the first observation is that the two LWR-breeder systems utilizing U-233/thorium fuels, (1) UO_2/ThO_2, LWBR, (2) UO_2/ThO_2, LMFBR (U-233) are very comparable. This is due to the very similar fissile inventories and breeding ratios of the breeder component of these systems.

As is expected, the high breeding ratio of the LMFBR (Pu), 1.277, results in the UO_2, LMFBR (Pu) fuel cycle being the most attractive at all system growth rates, with savings of up to 53% at 10% growth rate and 28% at 15% growth rate over the best breeder LWR fuel cycle using U-233/Th, the UO_2/ThO_2, LMFBR (U-233). In fact, a quick glance at the results of the previous section, Tables 4.11 and 4.12, demonstrates the overall superiority of this system to <u>any</u> other system looked at in this study in terms of ore and separative work utilization.

Similar to the behavior seen in the previous section, Table 4.18 demonstrates that for these LWR-breeder systems, the optimum Vf/Vm value of the producer (LWR) reactor decreases with increasing system growth rate, from a tight lattice optimum at low growth rates, to a value close to that of current PWR's at higher growth rates. Note, however, that for the LMFBR (Pu) system the optimum at all growth rates investigated here indicates an incentive to go to cores tighter than today's lattices, a conclusion that has been drawn in previous studies (U-1).

Table 4.19 shows comparable results for the separative work requirements. Again the UO_2 (slightly enriched U-235),

TABLE 4.19

SEPARATIVE WORK REQUIREMENTS, MTSWU/GW(e)yr,

FOR COUPLED PWR-BREEDER SYSTEMS[1]

Fuel Cycle	Vf/Vm Producer[2]	System Growth Rate		
		5%/yr	10%/yr	15%/yr
1) UO_2/ThO_2	A=0.338	91.4	149.3	196.0
(93% U-235)	B=0.4816	84.6	140.5	186.4
	C=0.9161	82.5	141.7	192.0
LWBR	D=1.497	89.7	164.0	230.7
2) UO_2 (slightly enriched U-235)	A	--	65.0	105.0
	B	--	53.0	91.1
	C	--	47.8	96.6
LMFBR(Pu)	D	--	52.4	118.0
3) UO_2/ThO_2	A	78.6	140.4	188.9
(93% U-235)	B	72.5	131.8	179.3
	C	70.3	132.4	184.3
LMFBR(U-233)	D	76.4	152.5	220.5

(1) 75% capacity factor; 0.2% enrichment tails.

(2) Vf/Vm of PWR.

LMFBR (Pu) reactor system is by far the most attractive in terms of separative work requirements, for reasons already discussed. The same pattern of performance as a function of Vf/Vm again applies.

In conclusion the work presented in this section shows the following:

(1) The applicability of MASFLO-2 to LWR breeder reactor systems has been demonstrated.

(2) In terms of both ore and separative work utilization, the UO_2 (slightly enriched U-235), LMFBR (Pu) reactor system has been shown to be the optimum fuel cycle, (LWR-LWR, and LWR-Breeder systems included) for both short and long-term strategies with savings of up to 53% in ore and up to 64% in separative work requirements at the lower bound (10% per year) of recently predicted nuclear-growth rates of 11-14% per year for the world (1975-2000)(E-5)(W-3) over the second best system, the UO_2/ThO_2, LMFBR (U-233): see tables 4.10, 4.14, 4.18 and 4.19.

(3) In agreement with previous work, (U-1), it has been shown that an incentive exists to use a producer reactor with a Vf/Vm higher than today's reactors for the UO_2 (slightly enriched U-235), LMFBR (Pu) reactor system.

(4) It has been demonstrated that the LWBR and LMFBR U-233/thorium LWR-breeder cycles are very comparable in terms of resource requirements at all system growth rates, a result which raises questions as to whether both systems should be developed.

(5) Finally, similar to results demonstrated in the previous section for LWR-breeder systems, as the system growth rate increases, the optimum Vf/Vm of the producer (PWR) reactor decreases from a tight lattice to pitches typical of today's lattices.

4.7 Conclusions

In this chapter several aspects of ore and separative work utilization have been looked at. First, MASFLO-2, the model developed in the present study to determine ore and separative work requirements of reactor systems, was benchmarked against ALPS (H-4),(H-5), one of the premier state-of-the art codes in this area, developed by HEDL. Over a thirty four year period, for given nuclear growth scenarios (H-6), MASFLO-2 was observed to converge to within quite acceptable limits to values of ore and SWU calculated by ALPS for two fuel cycles (UO_2 U, Recycle), and (UO_2, U Recycle, PuO_2/UO_2).

Second, it was shown that MASFLO-2 was applicable to a variety of growth scenarios, (with positive, zero or negative growth rates). Furthermore, it was shown that cumulative ore and separative work usage of a finite-life reactor system are determined by the zero growth rate (r=0) output of MASFLO-2; non zero growth

therefore indicate short-term performance in finite-life systems. Furthermore, it should be noted that the common practice of quoting ore savings over the life of a single reactor is equivalent to basing one's evaluation on a zero growth system.

Third, it was shown that the effect of unit cell size at constant fuel-to-coolant volume ratio, and therefore fuel pin size, on ore and SWU requirements was small (at most >4%); and thus one of the main assumptions of this thesis, that fuel-to-coolant volume ratio is a more important criterion for optimizing ore and SWU requirements, was validated.

Fourth, the results of the main work of this thesis, the effect of fuel-to-coolant volume ratio on ore and SWU utilization of PWR fuel cycles, (see Table 4.20 for a description of the cycles,) concluded the following:

(1) For both the present day once-through uranium fuel cycle and the uranium fuel cycle with uranium recycle, the optimum lattice design points were found to be close to the present day PWR design for all growth rates and hence for either short-term (>0%/yr growth rate) or long-term (0%/yr growth rate) considerations.

(2) The decision of an optimum strategy for resource utilization in PWR fuel cycles will depend on whether short-term or long-term optimization is perferred, since these optimums (in particular for ore utilization) are very dependent on system growth rates, going from tighter lattices at lower growth rates to looser lattices at higher growth rates.

(3) In accounting for isotopic degradation in multiply-recycled fuel, further investigations are recommended, in particular to investigate the effect of Vf/Vm on these factors in the various fuel cycles. Related work of this nature is presently being carried out by K.O. Ott (O-6).

(4) For cycles 3, 4 and 5 (see Table 4.20) which employ recycle of bred plutonium and/or U-233, two limiting cases were considered, that of full recycle to extinction of bred plutonium and U-233, with best-estimate weighting factors of 0.8 and 0.9, respectively (See section 4.5), and single-pass recycle of these same bred fuels. The former represents an optimistic and the latter a pessimistic view of resource utilization in a LWR economy. The real-life situation lies somewhere between, with the point of retirement of these multiply-recycled fuels being subject to economic considerations. However, in view of the recent price escalation history of yellowcake (U_3O_8), the optimistic full-recycle-to-extinction option may more closely parallel the real-life situation in the future.

(5) For the case of full recycle to extinction, the optimum fuel cycle at all growth rates considered here, in the range of Vf/Vm values considered here, is the U-235/Th, U-233/Th fuel cycle (see cycle 5, Table 4.20). With regard to long term considerations (zero growth rate), its optimum design corresponds to a mixed Vf/Vm system (Vf/Vm of U-235/Th $\approx$0.9161, and Vf/Vm of U-233/Th $\approx$1.497), demonstrating savings in ore requirements of up to 17% over the next best cycle, the U-235/U-238, Pu/Th, U-233/Th cycle, and up to 57% over the present day once through

TABLE 4.20

PWR FUEL CYCLES CONSIDERED IN THE FUEL-TO-COOLANT VOLUME RATIO STUDY

1. UO_2 (slightly enriched U-235) reactor with no recycle (here designated U-235/U-238).
2. The same as (1) but with uranium recycle (here designated U-235/U-238 U Recycle).
3. UO_2 (slightly enriched U-235) reactor with uranium recycle, recycling plutonium to a PuO_2/UO_2 reactor (here designated U-235/U-238, U, Recycle, Pu/U-238).
4. UO_2 (slightly enriched U-235) reactor with uranium recycle, recycling its bred plutonium to a PuO_2/ThO_2 reactor, which in turn recycles bred U-233 to a $^{233}UO_2/ThO_2$ reactor (here designated U-235/U-238, Pu/Th, U-233/Th)
5. UO_2 (93% enriched U-235)/ThO_2 reactor recycling uranium-235 (and small amounts of bred plutonium) to itself, and recycling bred U-233 to a $^{233}UO_2/ThO_2$ reactor (here designated U-235/Th, U-233/Th).

conventional PWR's.

(6) A comparison of long-term, zero growth rate, ore requirements (at present Vf/Vm values) of the U-235/Th, U-233/Th fuel cycle to present day PWR's using a once-through fuel cycle, shows a savings in ore requirements achieved by the U-235, U-233/Th fuel cycle of up to 50%.

(7) In the consideration of short term requirements, at 10% per year growth rate (the lower bound of recently predicted nuclear growth rates for the world (E-5), (W-3) (1975-2000)), although the U-235/Th, U-233/Th fuel cycle exhibits the optimum results, its advantage over the uranium cycle with full recycle (U-235/U-238, U-recycle, Pu/U-238) is only about 1%. Thus if the strategy adopted by the nuclear industry is to optimize short term ore usage, the development of a thorium cycle would not be attractive (note the uranium cycle with full recycle shows better ore utilization than the other thorium cycle (U-235/U-238, Pu/Th, U-233/Th) at this growth rate)

(8) If the strategy of utilizing single pass recycle is adopted the development a thorium cycle for PWR's (cycles 4 or 5, Table 4.20) is not warranted at either zero, 5 or ten percent growth rates, i.e., whether short-term or long-term considerations are viewed. This is a result of the superior performance of the uranium cycle with full recycle (U-235/U-238, U-recycle, Pu-U-238), which exhibits an optimum value in a mixed Vf/Vm system with the producer lattice U-235/U-238) being tighter (Vf/Vm ≈0.9161, 0.4816, 0.4816 at 0, 5 and 10% per year growth rates, respectively) than the (Pu/U-238) consumer lattice

optimum Vf/Vm$\simeq$0.338 at all system growth rates)

(9) Separative work requirements were examined with much less emphasis, because ore costs are the largest contributor to fuel cycle costs, because ore is subject to scarcity-related escalation and because separative work may become cheaper in the future due to technological improvements.

The results of the calculations showed that for single pass recycle, for all growth rates, wetter lattices are favored for all cycles (close to the Vf/Vm value of today's PWR's), with the U-235/U-238, U-recycle, Pu/U-238 and U-235/U-238, Pu/Th, U-233/Th cases demonstrating similar performance, and significantly superior to the U-235/Th, U-233/Th fuel cycle. However, the optimum Vf/Vm mix for the U-235/Th, U-233/Th case is very nearly the same for both SWU and ore requirements.

Furthermore its disadvantage in SWU requirements is at most 3% over cycles 3 and 4,whereas savings in ore requirements of 17% are achieved over cycles 3 and 4. This further demonstrates the attractiveness of this cycle based on long term considerations. However it is not clear that engineering constraints would allow one to contemplate actual use of such tight lattices.

(10) The separate optimization of consumer/producer reactor lattices for ore utilization has been shown to be significant only for the long-term (zero growth rate) for full recycle. Here, significant savings of up to 9% are observed over uniform Vf/Vm systems with the producer lattice being wetter (lower Vf/Vm) than the consumer lattice. (e.g., savings of 5% in

ore requirements are observed for the uranium system with fuel recycle (cycle 3) when a mixed system (Vf/Vm producer = 0.9161, Vf/Vm consumer = 0.4816) is compared to the uniform Vf/Vm optimum system (Vf/Vm ~ 0.6).

Finally, in the application of MASFLO-2 to LWR-Breeder systems the following conclusions are pertinent:

(1) MASFLO-2 can be applied to either LWR-LWR systems or LWR-Breeder systems.

(2) In terms of both ore and separative work utilization, the UO_2 (slightly enriched U-235), LMFBR(Pu) reactor system is the optimum fuel cycle, (LWR-LWR, and LWR-Breeder systems included) considering both short and long-term strategies, with savings of up to 53% in ore (and up to 64% in separative work units) at the lower bound (10%/year) of recently predicted world nuclear power growth rates over the second best system, the UO_2/ThO_2, LMFBR (U-233).

(3) Significant gains in ore utilization can be achieved by utilizing a tighter lattice producer reactor for the UO_2 (slightly enriched U-235), LMFBR (Pu) reactor system.

(4) It has been demonstrated that the LWBR and LMFBR U-233/thorium LWR-breeder cycles are very comparable in terms of resource requirements, a result which raises questions as to whether both systems should be developed. The LMFBR U-233/Thorium option might be favored since it could be readily converted over to the superior LMFBR(Pu) system if the ban on plutonium recycle is lifted.

In summary, it appears that in addressing the question

of optimum use of fuel resources the determining factors are the growth rate to be sustained over the time frame to be considered, and the limitations imposed upon recycle of fissile material.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The high ore (and separative work) consumption rates of light water reactors, the mainstay of the current nuclear industry, have not been of particular concern in the past because such reactors were viewed as precursors of fast breeder reactors capable of extracting thirty times the amount of energy from a given mass of U_3O_8. However, recent developments in the U.S. have not only locked the LWR into its most inefficient mode of operation -- the "once-through" fuel cycle, but have also led to a hiatus of indefinite duration in the fast breeder reactor development program. The resulting shortages and price escalation have led ERDA to devote greater attention to improvements in the LWR fuel cycle, with a special interest in thorium. One outcome of these circumstances was the establishment of a thorium assessment program in 1976, of which the present work is a part -- and is being carried out within the Nuclear Engineering Department at MIT under block-grant authorization funded by ERDA through the MIT Energy Laboratory.

The work reported here focuses on one important aspect of the overall assessment: determination of the optimum fuel-to-moderator volume ratio in terms of ore (and separative work)

requirements of PWR's fueled by various combinations of fissile and fertile isotopes.

The primary emphasis in this study is on ore utilization, because ore utilization is subject to scarcity-related price increases, because ore costs are the largest contributor to fuel cycle costs and because separative work (the second largest cost component) may become cheaper (relatively) in the future due to technological improvements.

5.1.1 Background

The lifetime ore (and separative work) requirements of a reactor consists of two components (1) ore (and SWU) to supply the initial loading of the reactor, and (2) ore (and SWU) needed to provide the net yearly makeup requirements.

An examination of basic neutronic considerations shows that, in general, high conversion ratio cores have low annual makeup but high initial fissile mass requirements. For a given reactor the annual requirements dominate the lifetime ore requirements, hence high conversion ratio cores are attractive; for growing systems, on the other hand, the high initial inventory strongly affects the ore requirements and lower inventory cores are favored. On the balance, however, a high conversion ratio is to be preferred. Since the conversion ratio is proportional to $\eta - 1$, , (η being the number of neutrons produced by fission per neutron absorbed) fissile isotopes having large spectrum-averaged η values (such as U-233 in thermal and epithermal spectra) are preferred.

Increasing the fuel-to-moderator ratio in LWR lattices also increases the conversion ratio due to improved neutron economy (eg. less neutron capture in light water coolant, fission products, control poison); although, as mentioned before, simultaneously requiring higher initial fissile loadings.

Motivated by these considerations, the main objective of this study has been to investigate the overall benefits, in terms of improved resource utilization, of fuel-to-moderator variations for individual reactors and for systems of reactors.

5.1.2 Previous Work

Combustion Engineering (S-1) has recently done a major study of thorium utilization in PWR's. The bulk of the study was concerned with a comparison of various fuel cycle options at present-day fuel-to-moderator ratios. Their major conclusion was that in the long term thorium cycles employing highly enriched uranium (UO_2 (93% U-235)/ThO_2) could increase the energy generated per mined ton of uranium ore on the order of 18 to 34%. A brief study of the effect of increased fuel-to-moderator ratio showed that for the UO_2 (93% U-235)/ThO_2 fuel cycle, increasing the fuel-to-moderator ratio to about twice that of today's reactors resulted in an additional savings in long term ore requirements of 15% with respect to the same fuel cycle using current lattice designs.

Other studies, by Brazilian workers (O-1)(C-2), followed a similar format, and found similar results.

Finally, extensive work on more radical design alternatives has been done under the Light Water Breeder Reactor (LWBR) Program (E-4).

It was concluded that previous work in the area of this study warranted extension since:

(a) all fissile and fertile combinations of interest had not been examined;

(b) there appeared to be insufficient emphasis on fuel-to-moderator ratio optimization;

(c) changes in fuel-to-moderator ratio were usually obscured due to concurrent variations in fuel pin diameter;

(d) previous studies generally focused on a single reactor, not the more realistic picture of a growing system.

5.2. Determination of Reactor Fuel Cycle Parameters

In the present work, as in prior studies of the present type (S-1),(O-1) a step-by-step procedure is in order. First a representative standard reactor design was chosen as a point of reference, to which all modifications are to be compared. Then, due to the large number of fuel cycles investigated, a relatively simple depletion code was chosen, to reduce computational time and expense.

Next the depletion code was benchmarked against experimental results. Finally, a method was prescribed to permit use of the burnup code in developing a fissile input/output mass balance for critical multi-batch cores.

The reference reactor design chosen here was the operation-

al Maine Yankee PWR. Table 5.1, (M-3),(Y-1), summarizes the reactors' key core parameters.

In this study, four different fuel-to-coolant volume ratios were considered (0.338, 0.4816 (standard), 0.9161 and 1.497) for each of five reactor types (Tables 5.2 and 5.3 list the reactor types and their respective fuel composition). It should be noted here that in contrast to previous work (S-1), all reactors chosen for this study are of the "dedicated" type i.e., each reactor accepts as feed, fuel containing only one of the major fissile species: U-235, U-233 (with U-234 and U-235) and fissile plutonium Pu-239 and Pu-241 (with their accompanying isotopes Pu-240 and Pu-242).

The burnup code chosen for this study was an EPRI version of the LEOPARD code (B-1) containing an up-to-date ENDF/B-IV cross-section library.

Although some criticisms can be made of LEOPARD, particularly of its treatment of plutonium isotopes, the benchmarking of this version of the code and its ENDF/B-1V cross-section library against light water critical and exponential experiments at varying fuel-to-moderator ratios, showed that the code was satisfactory for the accuracy needed in the present scoping studies. Benchmark calculations were made against 63 criticals and exponentials of slightly-enriched uranium (U-235/U-238) light water lattices, 42 plutonium-enriched, uranium oxide light water criticals and exponentials, and five ^{233}U- enriched thorium oxide exponential experiments. The mean calculated effective multiplication factors, K_{eff}, were 1.00257 for the uranium cases, 1.01783 for the plutonium-fueled cores and 1.0103 for the thorium

TABLE 5.1

MAINE YANKEE CORE PARAMETERS [1]

Parameters	Value	Units
Core thermal power	2440	MWTh
Nominal Electric Output	790	MW(e)
Nominal Thermal Efficiency	32.5%	
Core Heavy Metal Loading	87	MTU
Fuel Management	3 batch, mixed central zone	
Equilibrium Discharge burnup	33,000	MWD/MTHM
Power Density	75.016	KW/Liter
Nominal System Pressure	2100.0	psia
Nominal Fuel Temperature (Average)	1209.5	^{o}F
Nominal Coolant Bulk Temperature	562.5	^{o}F
Number of Fuel Assemblies	217	
Fuel Rod Array	14x14	
Fuel Rod Pitch	0.580	inches
Fuel Pellet Diameter	0.390	inches
Clad Material	Zircaloy 2	
Clad thickness, nominal	0.031	inches
Diametrical Gap, nominal	0.008	inches
UO_2/H_2O Volume Ratio, Vf/Vm:		
Supercell	0.4816	
Unit Cell	0.621	
Fuel Density, % theoretical	92%	

[1] Data applicable as of June, 1976 (subject to design changes).

TABLE 5.2

REACTOR LATTICES AND CORRESPONDING FUEL TYPES INVESTIGATED IN THE PRESENT STUDY

	Lattice Type	Fuel Type Charged
1.	$^{235}UO_2/^{238}UO_2$	Slightly enriched uranium (on the order of 3%).
2.	$^{235}UO_2/ThO_2$	93% enriched uranium oxide mixed with ThO_2.
3.	PuO_2/ThO_2	Plutonium-isotopic mix characteristic of LWR (base case) discharge mixed with ThO_2.
4.	PuO_2/UO_2	Same Pu mix as in #3, Uranium is tails at 0.2 w% U-235.
5.	$^{233}UO_2/ThO_2$	Uranium isotopic mix characteristic of discharge from PuO_2/ThO_2 at standard Vf/Vm.

TABLE 5.3

ISOTOPIC COMPOSITION OF FUEL CHARGED TO CONSUMER REACTORS[1]

Fuel Type	Isotope	Weight Fraction
1. Plutonium	Pu-239	0.542196
	Pu-240	0.259582
	Pu-241	0.139364
	Pu-242	0.058858
2. ^{233}Uranium[2]	U-233	0.907158
	U-234	0.0803653
	U-235	0.0124766

[1] Consumer reactors - those whose fissile makeup is bred by other reactor types

[2] U-236 is neglected due to its small concentration (<0.06%)

containing lattices. The lattices examined covered the range of fuel-to-moderator ratios up to 1.282.

In the interest of consistency, and because burnup and fuel management optimization is the subject of other studies (F-2), both the discharge burnup and the number of in-core batches were maintained constant, and consistent with that of Maine Yankee (Table 5.1), at values of 33,000 MWD/MT and 3 batches respectively.

In simulating a critical reactor (and, in particular, determining the equivalent number of reload batches in a startup core) a linear reactivity model was employed:

$$B_n = \left\lfloor \frac{2n}{n+1} \right\rfloor \cdot B_o, \text{ and}$$

$$B_o = A(X - X_o) \qquad (5.1)$$

where n is the number of batches per core, B_o the fuel burnup at which a one-batch core would just be critical without poison (here 22,000 MWD/MT) and B_n the fuel discharge burnup allowed in a n batch core, and finally, X is the beginning of life enrichment, and A and X_o are constants (determined by fitting LEOPARD results).

The calculational methodology followed was to first adjust the fissile concentration in the heavy metal oxide until the computed effective multiplication constant was unity at 22,000 MWD/MT for a control-poison-free core with leakage. This determined the composition of the fresh fuel. Natural

boron was then added homogeneously to the coolant until the "average" (i.e. midcycle - 16,500 MWD/MT) effective multiplication constant was unity; the lattice was then burned to 33,000 MWD/MT to find the composition of the discharged fuel.

The fuel-to-coolant volume ratio, Vf/Vm, was changed by varying the lattice pitch at constant fuel pin diameter. All temperatures were maintained constant, and while the thermal/hydraulic consequences of this requirement are of obvious concern, they were considered to be beyond the scope of the present work, being the subject of other related research at MIT.

5.2.1 Results of Unit Cell Calculations

For the most part the output from the LEOPARD calculations are important only as intermediate results, which are to be used in computing the parameters of real interest. However, some of the trends observed are of intrinsic interest, and hence worthy of note.

Previous work (H-2), (E-2),(A-3) (A-4), (G-2) has indicated that fuel which is high in Pu-240 content (here 26% by weight) should experience a flatter reactivity swing during fuel depletion than comparable uranium fuels, as a consequence of the large Pu-240 resonance at 1.05 ev, which enables the Pu-240 to behave as a "burnable poison", which not only adds reactivity by its removal, but also produces the high-worth fissile isotope Pu-241. In a comparison of the reactivity behavior with burnup for plutonium lattices fueled with discharge PWR plutonium

(containing Pu-240) to plutonium lattices fueled without Pu-240 but with all the other plutonium isotopes, this effect was clearly seen.

As expected, the neutron spectrum was observed to harden with increasing fuel-to-coolant volume ratio (for the UO_2 (slightly enriched U-235) reactor, the ratio of fast ($\geq$ 0.625 ev) to thermal fluxes was observed to rise from a value of 5.3 at the standard fuel-to-coolant volume ratio, Vf/Vm of Maine Yankee (0.4816) to a value of 60.91 at Vf/Vm = 1.497).

It is well known (H-3), that the multiple-recycle of plutonium, U-233 or U-235 results in isotopic degradation due to the buildup of higher isotopes. The results of the unit cell calculations showed this to be particularly true for plutonium cores. Furthermore, it was observed that there are two phenomenon of fuel degradation which are greatly enhanced in the plutonium-fueled cores (1) a lower fuel-to-coolant volume ratio lattice suffers severe isotopic degradation but large fissile mass destruction, (2) a high fuel-to-coolant volume ratio lattice has large residual fissile masses but smaller isotopic degradation. This suggested the possibility of defining a single representative weighting factor, W, between 0 and 1 to account for fuel degradation due to multiple recycle. The implications of this will be discussed in the next section.

5.2.2. Conclusions

In common with previous studies of this genre, an

appropriate compromise between rigor and simplification (hence computation time and cost) has been established; here in the form of the LEOPARD code, whose appropriateness was verified using 110 benchmark lattice calculations.

It should be noted that in contrast to previous work, where the fuel pin diameter was varied to effect changes in the fuel-to-moderator ratio, in the present study the lattice pitch was varied, a difference which should be kept in mind when comparing both the absolute results and the trends to other work.

Finally, a note of caution should be raised in regard to Vf/Vm values beyond a value of roughly 1.0, where the reactor becomes highly epithermal and thermal-reactor oriented methods, codes and cross section sets are pushed to their limits of proven applicability. The paucity of experimental benchmarks in this region is also to be noted.

5.3 Reactor Systems Model

The parameters of importance to a nuclear economy are the total requirements for ore (and separative work) per unit of electrical energy produced by the system. Thus a "systems model" is required to convert reactor mass flow data into energy demand scenario dependent ore and SWU needs.

Previously developed models such as NEEDS (W-1) and ALPS (H-4) were reviewed, but for the most part were found to be very involved with many degrees of freedom, requiring large computer facilities and long computing time. Hence,

it was decided that a simpler model was more appropriate for present purposes. Since presently existing simple models (A-7), (F-4) do not lend themselves well to the present approach, i.e., use of coupled multi-tiered dedicated reactor systems, an in-house model, designated MASFLO-2, incorporating the most attractive features of these previous models, was tailored to serve our needs.

The structure of MASFLO-2 can be best described by considering its application to a three-tier system such as a UO_2 (slightly enriched U-235) reactor, which recycles uranium to itself, while feeding bred plutonium to a PuO_2/ThO_2 reactor, which in turn supplies U-233 to a $^{233}UO_2/ThO_2$ reactor recycling bred U-233 to itself.

Table 5.4 summarizes the classification of reactor types and fissile species considered in the present work. Although the isotopic composition of the fuel charged or discharged from the various reactors differed, it was convenient to adjust the fissile compositions to make all streams equivalent to a small set of reference compositions by using fissile isotopic weighting factors. These weighting factors were based on ratios of computed energy-delivered-per-mass-destroyed. Note that several models (NEEDS (W-1), GAECON (N-1)), merely lump fissile isotopes together (i.e., Pu-239 and Pu-241 are considered equivalent: all fissile weighting factors are 1.0). A detailed treatment of this process is presented in Appendix F.

Table 5.5 summarizes the definitions of parameters used in the model. Here it should be noted that a capacity factor, L,

TABLE 5.4

CLASSIFICATION OF REACTOR TYPES AND FISSILE SPECIES OF MASFLO-2

Reactor Type - Subscript j	
j	Reactor type
1	UO_2 (slightly enriched U-235/U-238
2	UO_2(93% enriched U-235)/ThO_2
3	PuO_2/ThO_2
4	PuO_2/UO_2
5	$^{233}UO_2/ThO_2$

Fissile Species - Subscript i	
i	Fissile Species
1	U-235
2	U-233 (based)*
3	Pu-239 (based)*

* "based" indicates that this is the reference (dominant) fissile isotope in the fuel; in practice a representative isotopic mixture is employed.

TABLE 5.5

PARAMETERS USED IN MASFLO-2

1. XF_{ij}: (1) i = 1, and j = 1,2 - wt% U-235 in charged uranium heavy metal (2) i ≠ 1, j ≠ 1, 2 weight fraction of isotope i in heavy metal charged to reactor type j.

2. XD_{ij} (1) i = 1, j = 1, 2 - wt% U-235 in discharged uranium heavy metal (2) i ≠ 1, j ≠ 1,2 discharge weight fraction of isotope i from reactor type j with respect to heavy metal charged.

3. SL: process loss factor: 1.0 minus fraction of process stream lost.

4. r: system growth rate in percent per year.

5. E_j: installed capacity of reactor type j in GW(e).

6. T_j: refueling interval, in years, of reactor type j; time between refuelings.

7. B_j: final discharge burnup of the fuel of reactor type j. MW(th)/MT, in its equibrium cycle.

8. η_j: thermal efficiency of reactor type j.

9. N_j: (1) i = 1, j = 1,2; - equivalent number of batch reloads in the initial startup core for reactor type j in terms of ore usage (ST U_3O_8).

   (2) i ≠ 1, j ≠ 1,2 - ratio of fissile loading of isotope i needed to startup reactor type j to the fissile loading of isotope i in an equilibrium reload batch of reactor type j.

10. P_j: heavy metal loading per fuel batch of reactor type. j (MT).

11. L: time-averaged system capacity factor (assumed constant for all reactor types in the system).

12. RD1: Ratio of discharged heavy metal uranium to the charged heavy metal uranium for reactor type j = 1.

is specified and that all ore (and separative work) usage is computed on a per GW(e)yr rated basis.

A major point here concerns the term N_j (see Table 5.5), the number of batch reloads (without recycle) in the initial core. This parameter was obtained using the linear reactivity model (see Appendix D), and thus enabled the calculation of initial core startup requirements from equilibrium core reload results. Its use in this model is demonstrated in Figure 5.1. Figure 5.1 also demonstrates one of the key assumptions in the derivation of this model - the ore requirements of a reactor can be considered as (1) an initial inventory composed of N_j batch reloads (with no recycle) and (2) equilibrium batch reloads, the (partially-burned) last two of which (for a three batch core) are used in the startup of a replacement core: i.e., the model considers only net installed new capacity when debiting the system for startup inventory. This assumption is also basic to some other models (N-1).

Table 5.6 presents other key assumptions and features of MASFLO-2. In this regard three other points should be made: (1) MASFLO-2 is applicable to any growth rate (zero, positive or negative), (2) MASFLO-2 can be formulated in either mass flows or concentrations depending on the form in which fuel cycle data is available (3) MASFLO-2 is applicable to breeder-converter systems.

Table 5.7 presents the final set of equations of MASFLO-2 for the ore consumption by a three-tier system of reactors. (The equations for separative work consumption are quite similar;

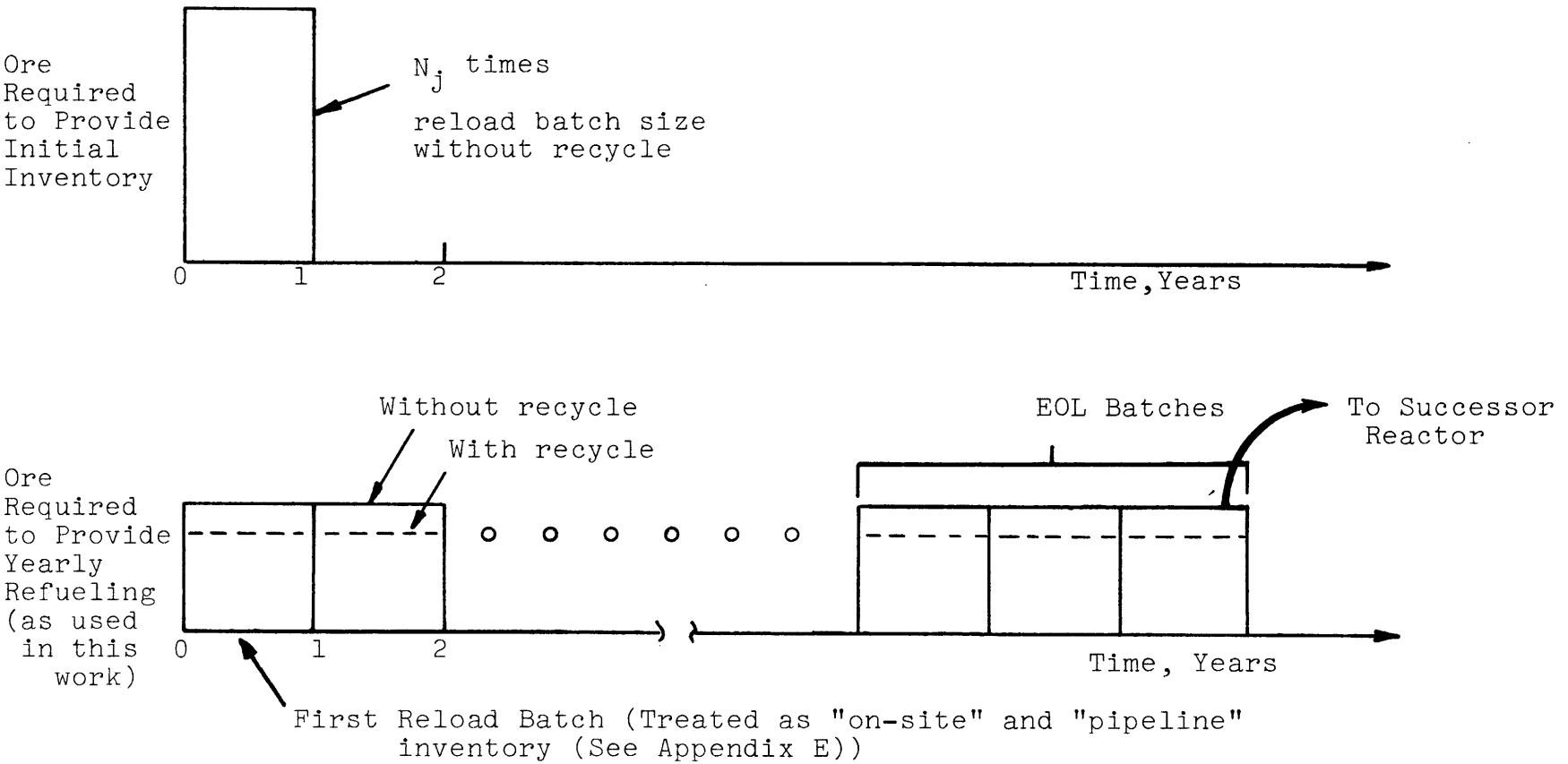


FIG. 5.1. MASS FLOWS VERSUS TIME - AS USED IN MASFLO-2

TABLE 5.6

KEY FEATURES AND ASSUMPTIONS OF MASFLO-2

1. No variable system stockpile: all bred fissile materials are immediately recycled (similar to NEEDS (W-1)); inventories are proportional to installed capacity; also allows for out-of-core, off site inventory and on-site new fuel inventory.

2. Allows for individual optimization of the reactor physics of each reactor type in a multi-tiered system: all reactors are dedicated to a single fissile-fertile fuel combination.

3. Variation in isotopic composition treated by use of weighting factors: discharge streams are adjusted in composition to be equivalent to a limited number of charge streams.

4. All reactors in the system grow at the same rate; rate can be zero, positive or negative, rate can be varied on a yearly basis.

5. Applicable to both breeders and converters.

6. Can be formulated in either mass flows or concentrations depending on the form in which fuel cycle data is available.

7. Can explicitly handle reactor parameters such as final fuel burnup, thermal efficiency, refueling interval, for each reactor type in a system.

8. Calculates the system ore and separative work requirements per system GW(e) (rated) at a capacity factor L (assumed to be constant for present purposes (not an inherent limitation) for all reactor types in the system).

TABLE 5.7

FINAL SET OF EQUATIONS OF MASFLO-2 FOR THREE-TIER SYSTEM OF UO_2 (SLIGHTLY ENRICHED U-235), PuO_2/ThO_2 AND $^{233}UO_2/ThO_2$ REACTORS FOR SYSTEM ORE CONSUMPTION.

$$F_{S3} = \frac{F_{S1}}{[\,1 + R_{31}(1 + R_{53})\,]} \quad \frac{ST\ U_3O_8}{\text{System GW(e)yr (rated) at capacity factor L}} \qquad (A)$$

where

$$F_{S1} = \underbrace{\frac{1.3\ P_1}{SL\ E_1\ T_1}\ \left(\frac{X_{F11} - X_W}{X_{NAT} - X_W}\right)}_{1}\ [\,1 - (SL)^2 RD1\ \underbrace{\left(\frac{XD_{11} - X_W}{XF_{11} - X_W}\right)}_{2} + \underbrace{\frac{r}{100}\ N_1 T_1}_{3}\,] \qquad (B)$$

$$R_{31} = \left(\frac{P_1\ E_3\ T_3}{P_3\ E_1\ T_1}\right)\ \frac{XD_{31} \cdot SL}{[(XF_{33} - W_3 \cdot XD_{33} \cdot SL) + N_3\ XF_{33}\ \frac{r}{100}\ T_3]} \qquad (C)$$

$$R_{53} = \left(\frac{P_3\ E_5\ T_5}{P_5\ E_3\ T_3}\right)\ \frac{XD_{23} \cdot SL}{[XF_{25} - W_2 \cdot XD_{25} \cdot SL) + N_5\ XF_{25}\ \frac{r}{100}\ T_5]} \qquad (D)$$

and the equations are readily reduced to the two-tier case). With reference to Table 5.7, the term F_{S3} in Eq. (A) is the ore consumption per system GW(e)yr (rated) at capacity factor L, of the three tier system. The term F_{S1} represents (see Eq. (B)) the ore consumption for the UO_2 (slightly enriched U-235) reactor, with uranium recycle. With reference to Eq. (B): term (1) represents the equilibrium ore consumption of the UO_2 (slightly enriched U-235) reactor without recycle, term (2) accounts for uranium recycle and term (3) accounts for net system growth of this reactor type (i.e. ore needed to start up new UO_2 (slightly enriched U-235) reactors).

The term R_{31} in Eq. (A) is the amount of energy (GW(e)yr (rated) at capacity factor L) produced by the PuO_2/ThO_2 reactor, growing at r% per year and recycling plutonium, due to the transfer from the UO_2 (slightly enriched U-235) reactor of a certain amount of plutonium produced by this reactor in the process of delivering one GW(e)yr (rated) at a capacity factor L. Term R_{53} represents a similar mass-energy balance between the PuO_2/ThO_2 reactor and the $^{233}UO_2/ThO_2$ reactor, accounting for the transfer of bred U-233. Note that (see Eqs. (C) and (D)) the startup requirements of the PuO_2/ThO_2 and $^{233}UO_2/ThO_2$ reactors growing at r% per year are accounted for in R_{31} and R_{53}, respectively.

Finally, the terms W_3 and W_2 in Eqs. (C) and (D) are weighting factors for plutonium fuel and U-233 fuel to account for isotopic degradation due to the buildup of poisons (Pu-242, U-236) in these fuels (in full recycle: single pass recycle

implies W_2, W_3 = 0). These weighting factors should be distinguished from the fissile isotope weighting factor which is designed to account for differences in fissile isotopic values (e.g. Pu-239 and Pu-241 in fissile plutonium). Taking into account previous work (H-3) and the results of our own cell calculations, the best estimate for W_3 (plutonium recycle) was taken to be 0.8 and for W_2 (U-233 recycle) 0.9. The utilization of a single, case-independent, weighting factor in these calculations for all fuel-to-coolant volume ratios has certain ramifications, which become more important as lattice pitch is tightened. For example, for the case of twice recycled plutonium, for the PuO_2/UO_2 lattice the degradation of the fuel, measured by Pu-242 "poison" buildup is far lower at Vf/Vm = 1.497 than that at Vf/Vm = 0.4816 (current PWR's) - (6.6 wt% Pu-242 versus 14.5 wt% Pu-242). This implies that the weighting factors at these very tight pitches should be somewhere between 0.8 (the value used) and unity (which would apply to a hard-spectrum fast breeder). Changing W_3 from 0.8 to 0.9 would affect ore utilization at Vf/Vm = 0.4816 (current PWR) by only about 3%, but at Vf/Vm = 1.497 (tightest pitch studied) improvements as large as 17% could be realized in some cases. Hence, in the event that engineering constraints can be satisfied for these very tight lattices, future work will have to be done in the area of isotopic-degradation weighting factors.

5.3.1 Conclusions

With the development of MASFLO-2 we have the ability to calculate yearly (and cumulative) ore (and SWU) usage of a coupled system of reactors for nuclear-powered energy systems under any given growth scenario (growing, static, declining). The model has been formulated to require as input only information readily derivable from the output of the cell burnup code, LEOPARD. We are now prepared to consider, with the above reservations, the main results of this investigation.

5.4 Results of System Studies

In this section, a summary of the main results of the present work will be presented.

5.4.1 Application of MASFLO-2 to Growth Scenarios

Here the MASFLO-2 model was cast in a form applicable to the calculation of cumulative ore (and separative work) requirements of a growing system of reactors.

In order to engender confidence in the rather simplistic MASFLO-2 model, it was benchmarked against results derived from the Hanford Engineering Development Laboratory (HEDL) model ALPS (H-4), (H-5), (H-6) which were in the form of cumulative ore (and separative work) requirements for a given history of nuclear-generated electric energy over a period of 34 years. The systems benchmarked were (1) UO_2 (slightly enriched U-235) with uranium (but not plutonium) recycle (2) UO_2 (slightly enriched U-235) with both uranium and plutonium

recycle. (At the end of the 34 year period, deviations between ALPS-calculated and MASFLO-2-calculated values were 0.8% for ore and 2.2% for SWU requirements in the uranium recycle only case and 4.7% and 4.3% for ore and separative work respectively in the case of full recycle. These deviations were judged quite acceptable for present purposes.

MASFLO-2 was shown to be applicable to positive, zero, and negative growth rates; and, in particular, it can be used to predict cumulative ore (and separative work) requirements of a finite-life reactor economy (one which grows, peaks, then dies out). In this regard, it was shown that the zero growth rate (r=0%/yr) output of MASFLO-2 determines the overall cumulative ore (and separative work) requirements of a finite-life system.

5.4.2. The Effect of Unit Cell Size on Ore and SWU Utilization

In order to investigate the effect of unit cell size on ore and SWU utilization, the cell dimensions of the standard Maine Yankee core were shrunk and expanded to obtain cell volumes half of and 50% larger than, the standard cell volume while maintaining the fuel-to-moderator ratio constant. Application of MASFLO-2 to LEOPARD results for the cases of (1) a UO_2 (slightly enriched U-235) reactor recycling uranium and (2) the same reactor recycling plutonium to a PuO_2/UO_2 reactor, showed that ore and separative work utilization varied by at most 4% with cell size. From this result it was concluded that the fuel pin diameter is not an important parameter in determining ore and separative work requirements. This is

important in that it gives the thermal/hydraulic designer an important degree of freedom in designing new lattices at higher fuel-to-coolant volume ratios.

5.4.3 The Effect of Fuel-to-Coolant Volume Ratio on Ore and Separative Work Utilization

In regard to this topic, which embodies the main results of this study, only the most significant output will be summarized due to the extensive amount of data involved, much of which relates to systems which proved to be of marginal ultimate interest.

Table 5.8 presents an overall summary of the fuel cycles considered in this study. In the multiple-recycle cases (cases 3,4,5, of Table 5.8), two options were considered (1) single pass-recycle (2) full recycle to extinction, with isotopic degradation taken into account using weighting factors. Single-pass recycle is best contemplated in a mixed LWR-fast breeder economy, where the single-pass fissile mass discharged from the LWR is recycled to the fast breeders, in which the buildup of higher isotopes is of no substantial disadvantage. Since the second option - that of full recycle, is of greatest present interest, the discussions here will be limited to this option. Furthermore, since, as mentioned previously, ore utilization is the main point of emphasis, the results for separative work utilization will only be very briefly mentioned. Note that all values discussed here are per GW(e)yr (rated) at 75% capacity factor and 0.2% diffusion plant tails.

TABLE 5.8

PWR FUEL CYCLES CONSIDERED IN THE FUEL-TO-COOLANT VOLUME RATIO STUDY

1. UO_2 (slightly enriched U-235) reactor with no recycle (here designated U-235/U-238).
2. The same as (1) but with uranium recycle (here designated U-235/U-238 U Recycle).
3. UO_2 (slightly enriched U-235) reactor with uranium recycle, recycling plutonium to a PuO_2/UO_2 reactor (here designated U-235/U-238, U, Recycle, Pu/U-238).
4. UO_2 (slightly enriched U-235) reactor with uranium recycle, recycling its bred plutonium to a PuO_2/ThO_2 reactor, which in turn recycles bred U-233 to a $^{233}UO_2/ThO_2$ reactor (here designated U-235/U-238, Pu/Th, U-233/Th),
5. UO_2 (93% enriched U-235)/ThO_2 reactor recycling uranium-235 (and small amounts of bred plutonium) to itself, and recycling bred U-233 to a $^{233}UO_2/ThO_2$ reactor (here designated U-235/Th, U-233/Th).

Figure 5.2 shows the results of calculations for the case of full recycle. In the interest of brevity only the three most interesting cycles, ((1) UO_2 (no recycle), (2) UO_2 U recycle, PuO_2/UO_2 and (3) $^{235}UO_2/ThO_2$, $^{233}UO_2/ThO_2$) are presented for zero and ten percent per year system growth rates.

The first observation is that for both growth rates considered, the optimum Vf/Vm of the once through uranium cycle is very near the value used in present PWR's. (The same behavior was also observed for the uranium fuel cycle with uranium recycle).

Another observation is that the optimum Vf/Vm value for those cycles employing full recycle (cycle 3 and 5) decreases as the growth rate increases (again approaching that of current lattices), which demonstrates the consequences of the initial startup penalty inherent in high Vf/Vm systems discussed previously.

At zero system growth rate (and at 10% per year system growth rate which is a representative lower bound of recently predicted world nuclear growth rates for the period 1975-2000 (W-3), (E-5)), the best fuel cycle is the U-235/Th, U-233/Th fuel cycle.

At zero growth rate, the lowest ore usage occurs in the tightest lattice, showing a savings of 55% in ore consumption the present once-thru uranium cycle. Furthermore, when mixed Vf/Vm systems were considered a further savings of 5% (as much as 9% for other cycles) was observed with a producer Vf/Vm (U-235/Th) of about 0.9161 and a consumer lattice Vf/Vm (U-233/Th)

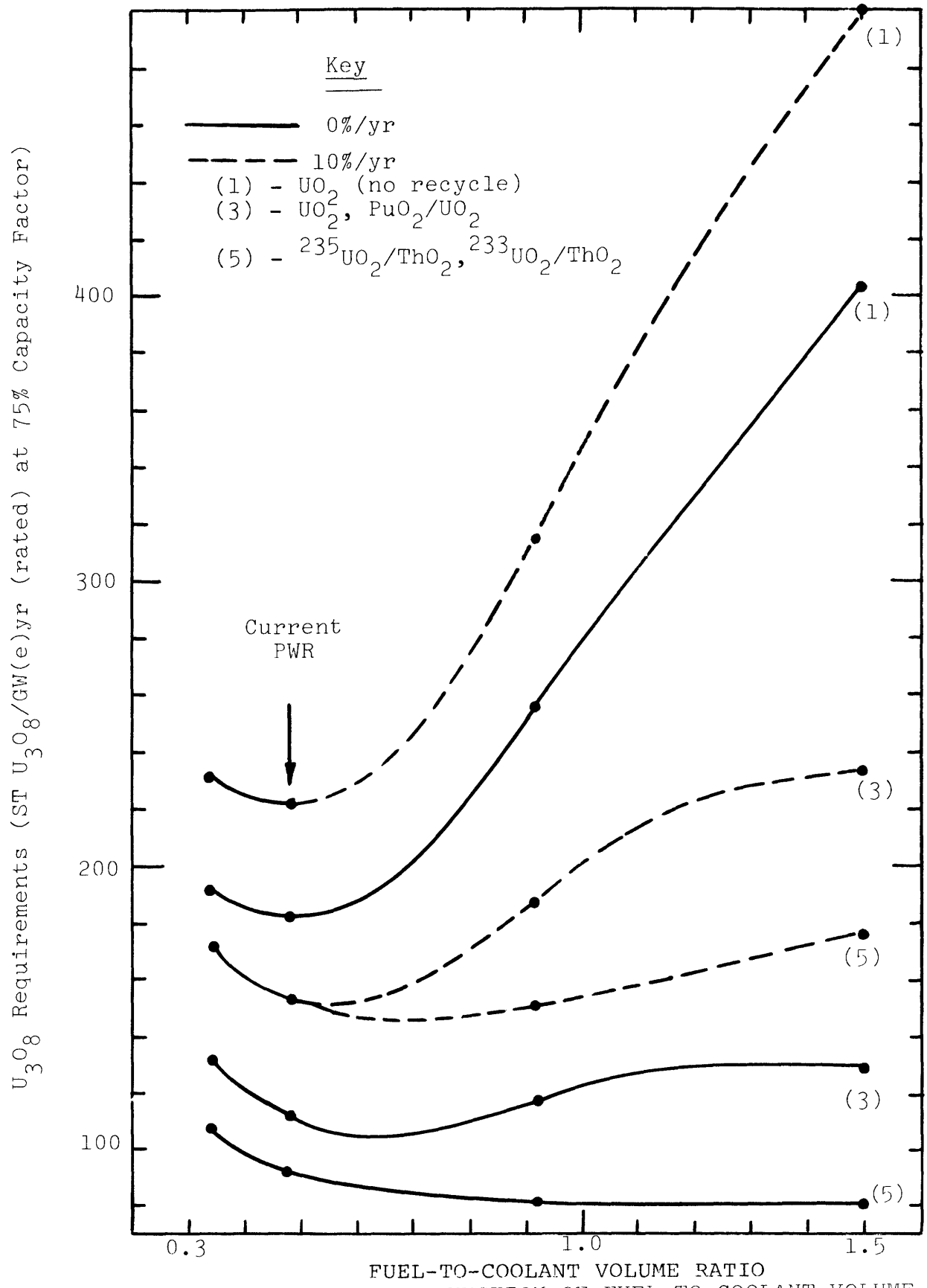


FIG. 5.2. ORE USAGE AS A FUNCTION OF FUEL-TO-COOLANT VOLUME RATIO FOR 0 AND 10% PER YEAR SYSTEM GROWTH RATES (FOR FULL RECYCLE)

of about 1.497, which represents a savings of 23% over the best combination of full recycle uranium cycle lattices (U-235/U-238, U recycle, Pu/U-238). This has two implications (1) separate optimization of consumer and producer lattices is attractive (2) For zero growth, or in the very long term, the U-235/Th, U-233/Th fuel cycle is very attractive. The second point is of further interest in that this cycle, (and the U-235/U-238, Pu/Th, U-233/Th cycle (not shown here)), which was the second most attractive cycle) are two of the options being considered for the LWBR program (E-4), which uses tight prebreeder and breeder reactor lattices.

In confirmation of previous work done by CE (S-1) using present day lattice designs, the single reactor lifetime ore savings of cycle 5 (Fig. 5.2) over the current once-through PWR cycle approaches 50%; its margin over the full-recycle uranium cycle (U-235/U-238, U recycle, Pu/U-238) is on the order of 17%. Hence even if it is not possible to construct tight-lattice PWR's because of heat transfer and safety limitations, considerable savings can still be realized by going to the U-235/Th, U-233/Th fuel cycle.

Thus for zero growth or in the very long term the U-235/Th, U-233/Th fuel cycle is attractive in terms of ore utilization.

For the short term (or in a rapidly growing system): see the results in Fig. 5.2 at 10% per year system growth rate, the savings of this cycle (and cycle 4 (not shown here)) over the present uranium cycle with full recycle are on the order of 1% and thus do not warrant the imminent deployment of the thorium

fuel cycle.

Single-pass recycle results showed that based on either short or long term considerations, the performance of the thorium fuel cycles with respect to the U-235/U-238, Pu/U-238 fuel cycle did not warrant development of the thorium fuel cycle. In other words, interim use of thorium in LWR's is not attractive if FBR deployment is in view.

As regards separative work requirements, and as previous work has shown (S-1), the U-235/Th, U-233/Th system is penalized: its SWU requirements are always higher than those of the uranium cycle (with full recycle). The three tier thorium cycle (cycle 4) was at best comparable (at optimum Vf/Vm values) to the full recycle uranium cycle case.

5.4.4 Application of MASFLO-2 to Systems Containing Breeder Reactors

In this area three systems were considered (1) a UO_2 (slightly enriched U-235) PWR reactor feeding a Pu/U fueled LMFBR, (2) a $^{235}UO_2/ThO_2$ PWR reactor feeding a U-233/Th fueled LMFBR and (3) a $^{235}UO_2/ThO_2$ PWR reactor feeding a U-233/Th fueled LWBR, all with full recycle.

The main results were:

1) in terms of ore (and separative work) utilization, the UO_2 (slightly enriched U-235), FBR (Pu) reactor system is the optimum fuel cycle of <u>all</u> cycles considered in this study considering both short and long-term strategies, with savings of up to 53% in ore (and 64 % in separative work requirements) for systems growing at 10% per year compared to the

second best system, the $^{235}UO_2/ThO_2$, LMFBR (U-233) system (2) significant gains in ore utilization (up to 21%) were obtained by utilizing a tighter lattice producer reactor for the UO_2 (slightly enriched U-235), LMFBR (Pu) reactor system, over present designs; a strategy which has also been suggested in previous work (U-1); (3) it is observed that at a growth rate of 10% per year the $^{235}UO_2/ThO_2$, LMFBR (U-233) fuel cycle is essentially comparable to the $^{235}UO_2/ThO_2$, LWBR fuel cycle raising doubts as to whether both systems should be developed.

With the above in mind, and other results not summarized here, the final conclusions of this research can be presented.

5.5 Conclusions

Before discussing the conclusions, the results of Table 5.9, a comparative summary of results for zero and ten percent per year system growth rates for full recycle, should be noted.

The following conclusions are drawn from the results of this study.

1. Thorium is only attractive in light water reactors if fuel reprocessing and recycle are permitted.

2. The degree to which thorium appears attractive is very scenario dependent. For periods of high system growth rate (eg. 10% per year), for full recycle, the performance of thorium is comparable to uranium with respect to ore utilization, and may not warrant the penalty of development costs and increased SWU consumption. When the study encompasses a period of low growth, zero growth or decline, thorium becomes the preferred fertile species over uranium. Since nuclear must grow faster than fossil in the near term, if it is to become a significant contributor, it is clear that thorium benefits will not be significant for some time to come in terms of its impact on national and international resource conservation.

TABLE 5.9

IMPORTANT RESULTS FOR 0 AND 10%/YEAR GROWTH RATES - FULL RECYCLE

A. Zero Percent Per Year Growth Rate (i.e., Single Reactor Case)

Fuel Cycle	Fuel-to-Coolant Volume ratio[1]	System Ore Usage ST U_3O_8/GW(e)yr[2]	% Savings Over Today's Once Through PWR
1. Uranium (Once through)	Present day lattices	182.0	--
2. Fully enriched Uranium/Thorium (full recycle)	Present day lattices	91.9	49.5(42)[3,4]
3. Uranium Cycle (full recycle)	Present day lattices	111.2	38.9
4. Fully enriched Uranium/Thorium (full recycle)	Tight Lattice (producer) very tight lattice (consumer)	77.5	57.4
5. Uranium Cycle (full recycle)	Tight pitch (producer) Present day lattice (consumer)	100.8	44.6

B. Ten Percent Per Year Growth Rate

1. Uranium (once through)	Present day lattices	222.2	--
2. Fully enriched Uranium/Th (full recycle)	Present day lattices	154.3	30.6
3. Uranium Cycle (full cycle)	Present day lattices	152.8	31.2
4. Fully enriched Uranium/Thorium (full recycle)	Present day lattices (producer) tight lattice (consumer)	148.6	33.1

TABLE 5.9 (Continued)
IMPORTANT RESULTS FOR 0 AND 10%/YEAR GROWTH RATES - FULL RECYCLE

Fuel Cycle	Fuel-to-Coolant Volume Ratio(1)	System Ore Usage ST U_3O_8/GW(e)yr(2)	% Savings Over Today's Once Through PWR
5. Uranium Cycle (full recycle)	Present day lattices	152.8	31.2

(1) Cycles 4 and 5 are optimized mixed Vf/Vm systems - tight lattice refers to Vf/Vm = 0.9161. Very tight lattice refers to Vf/Vm = 1.497.

(2) Per GW(e)yr (rated) at 75% capacity factor, 0.2% tails.

(3) Corresponding CE value (S-1).

(4) Savings here are 17.3% over cycle 3: CE value is 16%

3. One should also note that the common practice of quoting ore savings over the life of a single reactor is equivalent to basing one's evaluation on a zero growth system.

4. In low or zero growth systems, thorium becomes progressively more attractive as lattice pitch is tightened (i.e., fuel-to-moderator ratio is increased). We have not yet established a practical limit to such modifications, which will be set by engineering constraints. However, in confirmation of previous work done by CE (S-1), results have shown that the $^{235}UO_2/ThO_2$, $^{233}UO_2/ThO_2$ fuel cycle exhibits ore savings of up to 17% over the uranium cycle with full recycle when both fuel cycles are considered at present day lattice designs and zero growth (single reactor) conditions. Hence even if it is not possible to construct tight lattice PWR's because of thermal/hydraulic limitations, for low or zero growth systems thorium is moderately attractive.

5. It is important to note that we have not independently studied other reactor options such as the HTGR, CANDU or the spectral shift PWR. Thus, the remarks here should not be construed as denigrating their capabilities, which are generally superior to the PWR with respect to ore utilization. We have, however, examined several cases in which U-233 produced in a PWR is consumed in either a LWBR or a FBR operating on the U-233/Th-232 fuel cycle. Interestingly enough, the performance of these latter two reactors, while superior to LWR systems, are comparable (comparable fissile inventory needs and (low) breeding gains). This implies that the rationale of developing both thorium breeder concepts is somewhat redundant. In view of the fact that the FBR operating on a plutonium fuel cycle showed far superior resource utilization, it would appear to be preferable to develop the FBR concept utilizing a U-233/Th-232 fuel cycle (if need be) in lieu of the LWBR, thus allowing for the subsequent introduction of a FBR utilizing plutonium with only minor core design modifications, in the event that plutonium recycle is ultimately allowed.

6. For both the once-through and uranium only recycle fuel cycles, the cores of present uranium cycle PWR's were found to be very near the optimum lattice pitch design at all growth rates.

7. Restricting recycle of plutonium or U-233 to certain reactors is advantageous in that one can separately optimize all lattices for a particular fissile-fertile combination, whereas self-generated recycle uses the same subassembly throughout. This finding is of interest since reducing the risk of proliferation and diversion of fissile materials also favors confinement of recycle plutonium and U-233 to a minimum number of reactors. Our work indicates that this would also reduce system ore requirements by a modest amount, perhaps as much as 9%.

8. If a gradual transition to a LWR thorium economy is contemplated, then U-235/thorium lattices having tighter pitches than used in current PWR's are preferable; perhaps similar to one of the prebreeder options proposed in the LWBR program.

9. If the strategy of single-pass recycle of bred fissile materials is adopted in a LWR, results show that the uranium cycle and present day lattices designs are favored.

10. Fuel pin diameter is not an important parameter in terms of resource utilization, hence it is a free variable, available for use in the engineering design of tighter lattices.

5.6 Recommendations for Future Work

The recommendations for future work are the following:

1. Further work is needed should the extension of the present work to even tighter lattices prove desirable. This would imply development of the tools (depletion codes and cross sections) necessary for treating highly epithermal systems, since the present tools, designed for thermal reactors have been pushed to their (verified) limits. Such work should also prove useful in assessing the neutronic advantages advanced for ultra-tight pitch lattices by other investigators (E-3). Additional work in this area is underway at MIT (C-4), (A-5).

2. The degree of priority assigned to the above work is contingent on the satisfaction of engineering constraints by these tighter lattices. Future work is recommended in this area and is also in progress at MIT (G-1). It should be noted, however, that the tightest core lattices in widespread use today are those of the LMFBR, which have Vf/Vm $\approx$0.75, indicating that practical constraints may have already been exceeded in the tightest lattices of the present study.

3. In the event that future work shows the engineering feasibility of the tight lattices investigated here, future work on the relative worths (in terms of fuel cycle performance) of the various fuel isotopes in LWR's is recommended to enable more accurate determinations of ore usage in these tight lattices. Related research in this area is in progress at Purdue (O-6).

4. Future work is recommended on the fuel cycle economics of the tighter lattices and fissile/fertile combinations dealt with in this study, to determine the economic incentives for adopting a particular fuel cycle. Since ore costs are the largest component of fuel cycle costs in a PWR, these results should not shift the optimum far from those determined for minimum ore usage. However, commercial attractiveness is more closely tied to fuel cycle cost than it is to resource conservation per se. This too is the subject of current work at MIT (A-6).

5. Future work is required to investigate other means of fuel cycle optimization (radial and axial power flattening, burnup optimization, fuel zoning and end-of-cycle power/temperature coastdown); work is currently being done on these topics at MIT (F-1).

To recapitulate: the present study has found that the resource conservation benefits of using thorium in PWR's are modest to moderate but not negligible, that they are likely to be realized only in the very long term, after an initial period in which ore (and SWU) usage may actually be increased. Should the benefits prove sufficiently attractive, a gradual transition to tighter pitch lattices is favored.

APPENDIX A

BENCHMARKING OF EPRI-LEOPARD AND ITS ENDF/B-IV CROSS SECTION LIBRARY AGAINST EXPERIMENTAL DATA

Table A.1 presents the lattice parameters, and the calculated k_{eff} values for 63 UO_2 light water criticals and exponentials.

Table A.2 presents similar data for 42 PuO_2/UO_2 light water criticals Table A.2.1 gives the isotopic composition of the plutonium fuel for each case considered.

Finally, Table A.3 presents the lattice parameters and calculated k_{eff} values for 5 light water $U^{233}O_2/ThO_2$ exponentials. Table A.3.1 gives the isotopic composition of the fuel.

References to the literature documenting the lattice experiments are included.

TABLE A.1

LATTICE PARAMETERS AND CALCULATED k_{eff} FOR UO_2 CRITICALS AND EXPONENTIALS

Case #	Lattice Type (1)	Reference (REF)	Enrichment at %	Fuel / H_2O Volume Ratio	Fuel Density g/cc	Pellet Diameter cm	Clad Material	Clad OD cm	Clad Thickness cm	Lattice Pitch cm	Critical Buckling m-2	Calculated Multiplication Factor
1	H	S-3	1.328	0.3311	7.53	1.5265	AL	1.6916	0.0711	2.205	28.37(3)	1.006602
2	H	S-3	1.328	0.2532	7.53	1.5265	AL	1.6916	0.0711	2.359	30.17(3)	1.008985
3	H	S-3	1.328	0.2020	7.53	1.5265	AL	1.1506	0.0711	2.512	29.06(3)	1.006405
4	H	S-3	1.328	0.2545	7.52	0.9855	AL	1.1506	0.0711	1.558	25.28(3)	1.005505
5	H	S-3	1.328	0.2045	7.52	0.9855	AL	1.1506	0.0711	1.652	25.21(3)	1.005712
6	H	S-3	1.328	0.3472	10.53	0.9728	AL	1.1506	0.0711	1.558	32.59(3)	1.008380
7	H	S-3	1.328	0.2793	10.53	0.9728	AL	1.1506	0.0711	1.652	35.47(3)	1.008512
8	H	S-3	1.328	0.2070	10.53	0.9728	AL	1.1506	0.0711	1.806	34.22(3)	1.007700
9	S	S-3	2.734	0.4587	10.18	0.7620	SS-304	0.8594	0.04085	1.0287	40.75	1.004089
10	S	S-3	2.734	0.3413	10.18	0.7620	SS-304	0.8594	0.04085	1.1049	53.23	1.008508
11	S	S-3	2.734	0.2591	10.18	0.7620	SS-304	0.8594	0.04085	1.1938	63.26	1.008490
12	S	S-3	2.734	0.1425	10.18	0.7620	SS-304	0.8594	0.04085	1.4554	65.64	1.013980
13	S	S-3	2.734	0.1178	10.18	0.7620	SS-304	0.8594	0.04085	1.5621	60.07	1.015264
14	S	S-3	2.734	0.0963	10.18	0.7620	SS-304	0.8594	0.04085	1.6891	52.92	1.009573
15	S	S-3	2.734	0.400	10.18	0.7620	SS-304	0.8594	0.04085	1.0617	47.50	1.003896
16	S	S-3	2.734	0.2217	10.18	0.7620	SS-304	0.8594	0.04085	1.2522	68.8	1.003420
17	S	S-3	3.745	0.400	10.37	0.7544	SS-304	0.8600	0.0406	1.0617	68.3	1.006816
18	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2522	95.1	1.010013
19	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2522	95.68	1.008455
20	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2522	74.64	1.004568
21	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2252	63.66	1.003112
22	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2252	40.99	1.002160
23	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2252	38.39	1.001574
24	S	S-3	3.745	0.2217	10.37	0.7544	SS-304	0.8600	0.0406	1.2252	33.38	1.000282

TABLE A.1 (continued)

LATTICE PARAMETERS AND CALCULATED k_{eff} FOR UO_2 CRITICALS AND EXPONENTIALS

Case #	Lattice Type (1)	Reference (REF)	Enrichment at %	$\frac{\text{Fuel}}{H_2O}$ Volume Ratio	Fuel Density g/cc	Pellet Diameter cm	Clad Material	Clad OD cm	Clad Thickness cm	Lattice Pitch cm	Critical Buckling m-2	Calculated Multiplication Factor
25	S	S-3	4.069	0.3922	9.46	1.1278	SS-304	1.2090	0.0406	1.5113	88.0	0.9995884
26	S	S-3	4.069	0.3922	9.46	1.1278	SS-304	1.2090	0.0406	1.5113	17.2	1.009368
27	S	S-3	4.069	0.4673	9.46	1.1278	SS-304	1.2090	0.0406	1.450	79.0	0.9937482
28	S	S-3	2.490	0.3521	10.24	1.0297	AL	1.2060	0.0813	1.5113	70.10	1.025139
29	S	S-3	3.037	0.3788	9.28	1.1268	SS-304	1.2701	0.07163	1.555	50.75	0.9977289
30	S	S-3	3.037	0.1225	9.28	1.1268	SS-304	1.2701	0.07163	2.198	68.81	0.9851319
31	S	S-3	4.069	0.3861	9.45	1.1268	SS-304	1.2701	0.07163	1.555	69.25	1.005637
32	S	S-3	4.069	0.2833	9.45	1.1268	SS-304	1.2701	0.07163	1.684	85.52	1.002795
33	S	S-3	4.069	0.1247	9.45	1.1268	SS-304	1.2701	0.07163	2.198	92.84	1.011119
34	S	S-3	4.069	0.1010	9.45	1.1268	SS-304	1.2701	0.07163	2.381	91.79	0.9938607
35	S	S-3	2.096	0.4854	10.38	1.524	AL	1.6916	0.07112	2.1737	58.0	1.063637
36	S	S-3	2.096	0.3236	10.38	1.524	AL	1.6916	0.07112	2.4052	80.6	1.013274
37	S	S-3	2.096	0.2427	10.38	1.524	AL	1.6916	0.07112	2.6162	85.7	0.9940549
38	S	S-3	2.096	0.1629	10.38	1.524	AL	1.6916	0.07112	2.9891	77.0	0.9743959
39	S	S-3	2.096	0.1220	10.38	1.524	AL	1.6916	0.07112	3.3255	61.6	0.9643520
40(4)	H	P-3(2)	3.006	0.7582	9.299	1.1280	SS-304	1.2675	0.0697	1.7188	56.6	0.9993485
41	H	P-3	3.006	0.7582	9.299	1.1280	SS-304	1.2675	0.0697	1.7188	36.71	0.9986718
42	H	P-3	3.006	0.7582	9.299	1.1280	SS-304	1.2675	0.0697	1.7188	18.26	0.9985092
43	H	P-3	3.006	0.6127	9.299	1.1280	SS-304	1.2675	0.0697	1.8194	65.81	0.9953153
44	H	P-3	3.006	0.6127	9.299	1.1280	SS-304	1.2675	0.0697	1.8194	45.00	1.00018
45	H	P-3	3.006	0.6127	9.299	1.1280	SS-304	1.2675	0.0697	1.8194	26.20	0.9991273
46	H	P-3	3.006	0.6127	9.299	1.1280	SS-304	1.2675	0.0697	1.8194	14.62	0.9985029
47	H	P-3	3.006	0.4782	9.299	1.1280	SS-304	1.2675	0.0697	1.9573	70.49	0.9989164
48	H	P-3	3.006	0.4782	9.299	1.1280	SS-304	1.2675	0.0697	1.9573	46.34	0.9989534

TABLE A.1 (continued)

LATTICE PARAMETERS AND CALCULATED k_{eff} FOR UO_2 CRITICALS AND EXPONENTIALS

Case #	Lattice Type (1)	Reference (REF)	Enrichment at %	$\frac{\text{Fuel}}{H_2O}$ Volume Ratio	Fuel Density g/cc	Pellet Diameter cm	Clad Material	Clad OD cm	Clad Thickness cm	Lattice Pitch cm	Critical Buckling m-2	Calculated Multiplication Factor
49	H	P-3	3.006	0.4782	9.299	1.1280	SS-304	1.2675	0.0697	1.9573	27.70	0.9987571
50	H	P-3	3.006	0.4782	9.299	1.1280	SS-304	1.2675	0.0697	1.9573	12.94	0.9970450
51	H	P-3	3.006	0.3493	9.299	1.1280	SS-304	1.2675	0.0697	2.1697	70.22	1.001411
52	H	P-3	3.006	0.3493	9.299	1.1280	SS-304	1.2675	0.0697	2.1697	47.61	0.9986315
53	H	P-3	3.006	0.3493	9.299	1.1280	SS-304	1.2675	0.0697	2.1697	25.22	0.9969191
54	H	P-3	3.006	0.3493	9.299	1.1280	SS-304	1.2675	0.0697	2.1697	15.05	0.9938596
55	H	P-3	3.006	0.2456	9.299	1.1280	SS-304	1.2675	0.0697	2.4656	61.73	0.9960579
56	H	P-3	3.006	0.2500	9.299	1.1280	SS-304	1.2675	0.0697	2.4656	41.18	0.9956138
57	H	P-3	3.006	0.2456	9.299	1.1280	SS-304	1.2675	0.0697	2.4656	32.41	0.9934591
58	H	P-3	3.006	0.2500	9.299	1.1280	SS-304	1.2675	0.0697	2.4656	20.51	0.9885852
59	H	P-3	3.006	0.2500	9.299	1.1280	SS-304	1.2675	0.0697	2.4656	60.4	0.9884218
60	S	F-5	3.003	1.000	10.44	1.0119	SS-348	1.0925	0.0267	1.320	66.0	1.008246
61	S	F-5	3.003	1.000	10.44	1.0119	SS-348	1.0925	0.0267	1.320	62.58	1.004642
62	S	F-5	3.003	0.3165	10.44	1.0119	SS-348	1.0925	0.0267	1.866	100.44	1.009664
63	S	F-5	3.003	1.2821	10.44	1.0119	SS-348	1.0925	0.0267	1.2056	50.96	1.007158

(1) Lattice Type H - Hexagonal (triangular)

S - Square

(2) Weight percent

(3) These bucklings were not measured directly but were inferred from critical loadings.

(4) Cases 40 to 59 are exponential experiments.

TABLE A-2

LATTICE PARAMETERS AND CALCULATED k_{eff} FOR PuO_2/UO_2 CRITICALS AND EXPONENTIALS

Case #	Lattice Type (1)	Reference (REF)	Enrichment PuO_2 w %	$\frac{\text{Fuel}}{H_2O}$ Volume Ratio	Fuel Density cm	Pellet Diameter cm	Clad Material	Clad OD cm	Clad Thickness cm	Lattice Pitch cm	Critical Buckling m-2	Calculated Multiplication Factor
1 (2)	H	U-2	1.5	1.1930	9.59	0.9448	ZY-2	1.082	0.06858	1.397	48.00	1.004190
2	H	U-2	1.5	0.8425	9.59	0.9448	ZY-2	1.082	0.06858	1.524	65.1	1.014366
3	H	U-2	1.5	0.4847	9.59	0.9448	ZY-2	1.082	0.06858	1.8034	78.5	1.022292
4	H	U-2	1.5	0.3461	9.59	0.9448	ZY-2	1.082	0.06858	2.0320	74.9	1.018835
5	H	U-2	1.5	0.2550	9.59	0.9488	ZY-2	1.082	0.06858	2.2860	60.9	1.013853
6	H	U-2	1.5	0.2350	9.59	0.9478	ZY-2	1.082	0.06858	2.3622	55.2	1.012787
7	H	U-2	2.0	0.8258	9.54	1.2828	ZY-2	1.4352	0.0762	2.032	93.7	1.995937
8	H	U-2	2.0	0.5033	9.54	1.2828	ZY-2	1.4352	0.0762	2.3622	103.3	1.035449
9	H	U-2	2.0	0.3561	9.54	1.2828	ZY-2	1.4352	0.0762	2.667	101.3	1.041947
10	H	U-2	2.0	0.2847	9.54	1.2828	ZY-2	1.4352	0.0762	2.9032	97.0	1.032756
11	H	U-2	2.0	0.1992	9.54	1.2828	ZY-2	1.4352	0.0762	3.3528	75.6	1.019633
12	H	U-2	2.0	0.1775	9.54	1.2828	ZY-2	1.4332	0.0762	3.5204	68.9	1.005136
13	H	U-2	2.0	0.5033	9.54	1.2828	ZY-2	1.4352	0.0762	2.3622	86.3	1.03434
14	H	U-2	2.0	0.3561	9.54	1.2828	ZY-2	1.4352	0.0762	2.667	85.4	1.038365
15	H	U-2	2.0	0.2847	9.54	1.2828	ZY-2	1.4352	0.0762	2.9032	81.5	1.028393
16	H	U-2	2.0	0.1992	9.54	1.2828	ZY-2	1.4352	0.0762	3.3528	61.6	1.012907
17	H	U-2	2.0	0.1775	9.54	1.2828	ZY-2	1.4352	0.0762	3.5204	55.6	1.9970399
18	H	U-2	2.0	0.8258	9.54	1.2828	ZY-2	1.4352	0.0762	2.0320	63.1	1.003424
19	H	U-2	2.0	0.5033	9.54	1.2828	ZY-2	1.4352	0.0762	2.3622	79.4	1.019001
20	H	U-2	2.0	0.3561	9.54	1.2828	ZY-2	1.4352	0.0762	2.667	77.6	1.025420
21	H	U-2	2.0	0.2847	9.54	1.2828	ZY-2	1.4352	0.0762	2.9032	72.2	1.019647
22	H	U-2	2.0	0.1992	9.54	1.2828	ZY-2	1.4352	0.0762	3.3528	53.7	1.000644
23	H	U-2	2.0	0.1775	9.54	1.2828	ZY-2	1.4352	0.0762	3.5204	44.3	1.9941918

TABLE A-2 (continued)

LATTICE PARAMETERS AND CALCULATED k_{eff} FOR PuO_2/UO_2 CRITICALS AND EXPONENTIALS

Case #	Lattice Type (1)	Reference (REF)	Enrichment PuO_2 w %	Fuel / H_2O Volume Ratio	Fuel Density cm	Pellet Diameter cm	Clad Material	Clad OD cm	Clad Thickness cm	Lattice Pitch cm	Critical Buckling m-2	Calculated Multiplication Factor
24	S	L-1	2.0	1.1127	9.54	1.2828	ZR-2	1.4352	0.0762	1.7526	69.1	1.012611
25	S	L-1	2.0	0.8044	9.54	1.2828	ZR-2	1.4352	0.0762	1.905	90.0	1.016239
26	S	L-1	2.0	0.3575	9.54	1.2828	ZR-2	1.4352	0.0762	2.47853	105.9	1.030011
27	S	L-1	2.0	0.2868	9.54	1.2828	ZR-2	1.4352	0.0762	2.69418	98.4	1.029979
28	S	L-1	2.0	0.1433	9.54	1.2828	ZR-2	1.4352	0.0762	3.5052	50.3	1.003411
29	S	L-1	2.0	0.3145	9.54	1.2828	ZR-2	1.4352	0.0762	2.47853	79.5	1.020430
30	S	L-1	2.0	0.2868	9.54	1.2828	ZR-2	1.4352	0.0762	2.69418	73.3	1.017623
31	H	U-2	4.0	0.6667	9.46	1.26366	ZR-2	1.4351	0.085598	2.159	94.7	1.009754
32	H	U-2	4.0	0.5018	9.46	1.26366	ZR-2	1.4351	0.085598	2.3622	107.9	1.018895
33	H	U-2	4.0	0.3552	9.46	1.26366	ZR-2	1.4351	0.085598	2.667	108.7	1.041125
34	H	U-2	4.0	0.2840	9.46	1.26366	ZR-2	1.4351	0.085598	2.90322	107.9	1.038308
35	H	U-2	4.0	0.1771	9.46	1.26366	ZR-2	1.4351	0.085598	3.52044	88.4	1.019348
36	H	U-2	4.0	0.1272	9.46	1.26366	ZR-2	1.4351	0.085598	4.06400	59.4	0.9977180
37	H	U-2	4.0	0.1111	9.46	1.26366	ZR-2	1.4351	0.085598	4.31800	41.1	0.9952763
38	S	T-1	6.6	0.7983	10.3334	0.857	ZR-4	0.993	0.05840	1.3208	108.8	0.9943199
39	S	T-1	6.6	0.6202	10.3334	0.857	ZR-4	0.993	0.05840	1.4224	121.5	1.018560
40	S	T-1	6.6	0.2857	10.3334	0.857	ZR-4	0.993	0.05840	1.8669	159.6	1.028610
41	S	T-1	6.6	0.2367	10.3334	0.857	ZR-4	0.993	0.05840	2.0117	159.3	1.032102
42	S	T-1	6.6	0.1248	10.3334	0.857	ZR-4	0.993	0.05840	2.6416	128.4	1.030224

[1]Lattice Type H - triangular (hexagonal)

S - square

[2]Cases 1 - 23, 31 - 37 are approach-to-critical experiments (see Reference U-2)

TABLE A.2.1

ISOTOPIC COMPOSITION OF Pu FUEL USED IN EXPERIMENTS ON PuO_2/UO_2 LATTICES

	Isotope	AT %(1)
1. Cases 1 - 6	Pu-239	91.41
	Pu-240	7.83
	Pu-241	0.73
	Pu-242	0.03
2. Cases 7 - 12	Pu-239	91.62
	Pu-240	7.65
	Pu-241	0.70
	Pu-242	0.03
3. Cases 13 - 17	Pu-239	81.11
	Pu-240	16.54
	Pu-241	2.15
	Pu-242	0.20
4. Cases 18 - 23	Pu-239	71.76
	Pu-240	23.50
	Pu-241	4.08
	Pu-242	0.66
	Isotope	W %(2)
5. Cases 24 - 28	Pu-239	91.615
	Pu-240	7.654
	Pu-241	0.701
	Pu-242	0.031

TABLE A.2.1 (continued)

ISOTOPIC COMPOSITION OF Pu FUEL USED

IN EXPERIMENTS ON PuO_2/UO_2 LATTICES

	Isotope	W %
6. Cases 28 - 30	Pu-239	71.762
	Pu-240	23.503
	Pu-241	4.08
	Pu-242	0.656
	Isotope	**AT %**
7. Cases 31 - 37	Pu-238	0.28
	Pu-239	75.38
	Pu-240	18.10
	Pu-241	5.08
	Pu-242	1.15
	Isotope	**W %**
8. Cases 38 - 42	Pu-239	90.49
	Pu-240	8.57
	Pu-241	0.89
	Pu-242	0.04

(1) AT % = atom percent

(2) W % = weight percent

TABLE A-3

LATTICE PARAMETERS AND CALCULATED k_{eff} FOR ^{233}U O_2/ThO_2 EXPONENTIALS

Case #	Lattice Type[1]	Reference	Enrichment W% U-233	Fuel H_2O Volume Ratio	Fuel Density g/cc	Pellet Diameter cm	Clad Material	Clad OD cm	Clad Thickness cm	Lattice Pitch cm	Critical Buckling m-2	Calculated Multiplication Factor
1	H	(W-2)	2.6343	1.0030	8.96	1.0922	ZY-2	1.2675	0.089	1.592	75.88	1.0017
2	H	(W-2)	2.6343	0.7225	8.96	1.0922	ZY-2	1.2675	0.089	1.719	86.06	1.0066
3	H	(W-2)	2.6343	0.5838	8.96	1.0922	ZY-2	1.2675	0.089	1.819	89.34	1.0117
4	H	(W-2)	2.6343	0.4558	8.96	1.0922	ZY-2	1.2675	0.089	1.957	90.35	1.0151
5	H	(W-2)	2.6343	0.3329	8.96	1.0922	ZY-2	1.2675	0.089	2.170	85.54	1.0166

[1] H - Hexagonal (triangular)

TABLE A-3.1

COMPOSITION OF FUEL USED IN $^{233}UO_2/ThO_2$ EXPERIMENTS

Isotope	Weight Fraction
^{16}O	0.121197
Th-232	0.851764
U-233	0.026343
U-234	0.000394
U-235	0.000012
U-238	0.000289
B	0.0000044

APPENDIX B

RESULTS OF EPRI-LEOPARD CALCULATIONS

In this appendix the results of the calculations using EPRI-LEOPARD for the reactor types considered, and a study on the effect of varying unit cell size (fuel pin diameter) are presented. All masses are presented on the basis of kg per metric ton heavy metal charged, and the discharged fuel burnup is 33,000 MWD/MT in all cases.

Note that the Maine Yankee PWR, the base case for the present work, has a fuel-to-coolant volume ratio of 0.482.

The results tabulated are all directly available in the LEOPARD output printout, as described in Reference (B-1).

TABLE B.1

CHARGE AND DISCHARGE MASSES FOR THE UO_2
(SLIGHTLY ENRICHED U-235) REACTOR

Case	A	B	C	D
Fuel-to-Coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
Boron [1] (PPM)	500	545	1220	3800
Initial Fissile Enrichment	3.10	2.96	4.09	6.32
INITIAL INVENTORIES (kg/Initial MT HM)				
U-235	31.0	29.6	40.9	63.2
U-238	969.0	970.4	959.1	936.8
DISCHARGED INVENTORIES (kg/Initial MT HM)				
U-235	5.737	6.038	14.589	30.899
U-236	3.973	3.807	5.122	7.588
U-238	948.583	946.479	930.385	906.442
Pu-239	3.788	4.800	9.408	14.701
Pu-240	2.164	2.221	2.096	1.874
Pu-241	0.950	1.249	1.938	1.763
Pu-242	0.449	0.511	0.351	0.151

[1] Natural Boron

TABLE B.2

CHARGE AND DISCHARGE MASSES FOR THE
UO_2 (93% ENRICHED U-235)/ThO_2 REACTOR

Case	A	B	C	D
Fuel-to-Coolant Volume Ratio	0.338	0.4816	0.916	1.497
Boron[1] (PPM)	305	342	580	1180
INITIAL INVENTORIES (kg/INITIAL MT HM)				
Fissile Enrichment	3.95	3.82	4.47	6.47
U-235	39.538	38.208	44.729	64.689
U-238	2.976	2.876	3.367	4.869
Th-232	957.486	958.916	951.904	930.442
DISCHARGED INVENTORIES (kg/INITIAL MT HM)				
U-233	10.984	11.818	13.545	15.687
U-234	1.225	1.447	1.654	1.553
U-235	10.063	9.853	15.320	31.342
U-236	4.739	4.651	5.330	7.090
U-238	2.627	2.446	2.676	3.815
Th-232	934.616	933.618	924.315	902.390
Pu-239	0.073	0.089	0.177	0.424
Pu-240	0.035	0.034	0.039	0.056
Pu-241	0.021	0.029	0.060	0.101087
Pu-242	0.009	0.012	0.016	0.013
Pa-233	1.027	1.084	1.095	1.071

[1] Natural Boron

TABLE B.3

CHARGE AND DISCHARGE MASSES FOR THE PuO_2/ThO_2 REACTOR

Case	A	B	C	D
Fuel-to-Coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
Boron[1] (PPM)	440	350	3400	2400
INITIAL INVENTORIES (kg/INITIAL MT HM)				
Fissile Enrichment	3.57	3.67	9.39	9.21
Pu-239	28.420	29.220	74.668	73.300
Pu-240	13.606	13.990	35.748	35.093
Pu-241	7.305	7.511	19.192	18.841
Pu-242	3.085	3.172	8.106	7.957
Th-232	947.583	946.107	862.287	864.809
DISCHARGED INVENTORIES (kg/INITIAL MT HM)[2]				
U-233	10.014	11.123	13.327	16.588
U-234	0.815	0.974	0.871	1.063
U-235	0.097	0.142	0.117	0.134
Pu-239	3.405	4.569	46.902	47.014
Pu-240	8.217	7.372	24.742	27.077
Pu-241	5.642	6.841	22.086	19.108
Pu-242	5.040	4.813	8.034	7.778
Th-232	930.407	926.976	842.588	840.153
Pa-233	0.765	0.778	0.690	0.865

(1) Natural Boron

(2) U-236 is ignored due to its small concentration.

TABLE B.4

CHARGE AND DISCHARGE MASSES FOR THE PuO_2/UO_2 REACTOR

Case	A	B	C	C
Fuel-to-Coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
Boron [1] (PPM)	470	450	530	6000
INITIAL INVENTORIES (kg/INITIAL MT HM)				
Fissile Enrichment	2.78	2.97	8.51	8.80
U-235	1.924	1.918	1.755	1.747
U-238	960.063	957.315	875.967	871.673
Pu-239	20.610	22.103	66.298	68.631
Pu-240	9.867	0.582	31.741	32.858
Pu-241	5.298	5.681	17.041	17.641
Pu-242	2.237	2.399	7.197	7.450
DISCHARGED INVENTORIES (kg/INITIAL MT HM)				
U-235	0.694	0.805	1.087	1.008
U-236	0.219	0.220	0.193	0.217
U-238	941.578	935.472	851.924	844.313
Pu-239	6.578	10.188	55.568	60.954
Pu-240	7.346	5.889	22.650	27.233
Pu-241	4.254	5.889	20.834	18.746
Pu-242	4.138	3.971	7.349	7.513

[1] Natural Boron

TABLE B.5

CHARGE AND DISCHARGED MASSES FOR THE $^{233}UO_2/ThO_2$ REACTOR

Case	A	B	C	D
Fuel-to-Coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
Boron[(1)] (PPM)	400	535	1115	2280
INITIAL INVENTORIES (kg/INITIAL MT HM)				
Fissile Enrichment	3.23	3.08	3.15	3.61
U-233	31.865	30.378	31.104	35.598
U-234	2.823	2.691	2.755	3.154
U-235	0.438	0.418	0.428	0.490
Th-232	964.870	966.513	965.713	960.759
DISCHARGED INVENTORIES (kg/INITIAL MT HM)				
U-233	18.079	18.345	21.174	26.982
U-234	5.052	4.956	5.088	5.498
U-235	1.145	1.662	1.672	2.040
U-236	0.205	0.234	0.260	0.283
Th-232	940.222	939.673	935.820	928.835
Pa-233	1.086	1.140	1.199	1.238

[(1)] Natural Boron

TABLE B.6

CHARGE AND DISCHARGE MASSES FOR THE
CELL SIZE STUDY ON A UO_2 (SLIGHTLY ENRICHED) REACTOR

Case			
Relative Cell Volume[1][2]	0.5	1.0	1.5
Fuel Pellet Dia. (IN)	0.2616	0.3900	0.4532
Boron[3] (PPM)	535	545	540
INITIAL INVENTORIES(kg/INITIAL MT HM)			
Fissile Enrichment	3.00	2.96	2.95
U-235	30.00	29.6	29.5
U-238	970.0	970.4	970.5
DISCHARGED INVENTORIES (kg/INITIAL MT HM)			
U-235	6.578	6.038	5.794
U-236	3.819	3.807	3.813
U-238	945.230	946.479	947.132
Pu-239	5.262	4.800	4.538
Pu-240	2.235	2.221	2.204
Pu-241	1.352	1.249	1.188
Pu-242	0.518	0.511	0.504

(1) Compared to Standard Maine Yankee Cell Size (see B-7),

(2) Fuel-to-Coolant volume ratio maintained at 0.4816 (Maine Yankee)

(3) Natural Boron

TABLE B.7

CHARGE AND DISCHARGE MASSES FOR CELL SIZE STUDY ON A PuO_2/UO_2 REACTOR

Case	1	2	3
Relative Cell Volume[1][2]	0.5	1.0	1.5
Fuel Pellet Dia. (in)	0.2616	0.3900	0.4532
Boron[3] (PPM)	405	450	475
INITIAL INVENTORIES (kg/INITIAL MT HM)			
Fissile Enrichment	3.18	2.97	2.90
U-235	1.912	1.918	1.921
U-238	954.309	957.315	958.420
Pu-239	23.737	22.103	21.503
Pu-240	11.364	10.582	10.295
Pu-241	6.101	5.681	5.527
Pu-242	2.577	2.399	2.334
DISCHARGED INVENTORIES (kg/INITIAL MT HM)			
U-235	0.871	0.805	0.773
U-236	0.213	0.220	0.224
U-238	931.639	935.472	937.133
Pu-239	12.206	10.188	9.294
Pu-240	7.631	7.287	7.173
Pu-241	6.857	5.889	5.466
Pu-242	4.116	3.971	3.915

[1] Compared to standard Maine Yankee cell size. Fuel pin diameter varies as square root of cell size ratio

[2] Fuel-to coolant volume ratio maintained at a value of 0.4816 (MAINE YANKEE)

[3] Natural Boron

APPENDIX C

VARIATION OF k_∞ WITH ENRICHMENT

In this appendix we will show that the following approximate relation holds:

$$\left(\frac{\Delta X}{X}\right) \simeq 2 \left(\frac{\Delta k_\infty}{k_\infty}\right) \tag{C.1}$$

where X is the fissile enrichment of the fuel, and k is the infinite medium multiplication constant.

In this treatment, a one group description will be used, and a UO_2 (slightly enriched U-235) reactor will be used as a numerical example.

In a one group model k_∞ is given by:

$$k_\infty = \frac{\nu\Sigma_f}{\Sigma_a} \tag{C.2}$$

where ν is the average numer of neutrons per fission, Σ_f is the macroscopic fission cross section and Σ_a, the macroscopic absorption cross-section.

The numerator of Equation C.2 can be written:

$$\nu\Sigma_f = \nu\Sigma_f^{25}(1 + \delta^{28}) = N^{25}\nu\sigma_f^{25}(1 + \delta^{28}) \tag{C.3}$$

where δ^{28} is the fertile-to-fissile fission ratio, $\frac{\text{fissions in U-238}}{\text{fissions in U-235}}$; σ_f^{25} is the microscopic fission cross-section of U-235, and N^{25} is the number density of U-235.

The denominator of Equation C.2 can be written as:

$$\Sigma_a = N^{28}\sigma_a^{28} + N^{25}\sigma_a^{25} + \Sigma_p \tag{C.4}$$

where N^{28}, N^{25} are the number densities of U-238 and U-235, respectively, σ_a^{25} and σ_a^{28} are their microscopic absorption cross sections and Σ_p is the non-fuel macroscopic absorption cross-section.

The variation of k_∞ with N^{25} is desired. Therefore, using Equations (C.3) and (C.4) in Equation (C.2) one gets:

$$\frac{\partial k_\infty}{\partial N^{25}} = \frac{\partial}{\partial N^{25}}\left[\frac{\nu N^{25}\sigma_f^{25}(1 + \delta^{28})}{(N^{28}\sigma_a^{28} + N^{25}\sigma_a^{25} + \Sigma_p)}\right] \tag{C.5}$$

$$= \frac{\nu\sigma_f^{25}(1 + \delta^{28})}{\Sigma_p} - \frac{N^{25}\nu\sigma_f^{25}(1 + \delta^{28})\sigma_a^{25}}{(\Sigma_a)^2} \tag{C.6}$$

or

$$\frac{\partial k_\infty}{\partial N^{25}} = \frac{[\Sigma_a\nu\sigma_f^{25}(1 + \delta^{28}) - N^{25}\nu\sigma_f^{25}(1 + \delta^{28})\sigma_a^{25}]}{(\Sigma_a)^2} \tag{C.7}$$

However,

$$\Delta k_\infty \approx \frac{\partial k_\infty}{\partial N^{25}} \cdot \Delta N^{25} \approx \frac{[\Sigma_a\nu\sigma_f^{25}(1 + \delta^{28}) - N^{25}\nu\sigma_f^{25}(1 + \delta^{28})\sigma_a^{25}]}{(\Sigma_a)^2} \cdot \Delta N^{25} \tag{C.8}$$

Using Equations (C.2) and (C.3):

$$\frac{\Delta k_\infty}{k_\infty} \approx [\Sigma_a\nu\sigma_f^{25}(1 + \delta^{28}) - N^{25}\nu\sigma_f^{25}(1 + \delta^{28})\sigma_a^{28}]\Delta N^{25} \cdot \left[\frac{\Sigma_a}{N^{25}\nu\sigma_f^{25}(1 + \delta^{28})}\right] \tag{C.9}$$

Therefore:

$$\frac{\Delta k_\infty}{k} \approx \left[1 - \frac{\Sigma_a^{25}}{\Sigma_a}\right] \frac{\Delta N^{25}}{N^{25}} \tag{C.10}$$

However $k_\infty \approx 1$, since the reactor is close to critical.

Therefore:

$$\Sigma_a \approx \nu\Sigma_f = N^{25}\nu\sigma_f^{25}(1 + \delta^{28}) \tag{C.11}$$

Substituting Equation (C.11) into Equation (C.10):

$$\frac{\Delta k_\infty}{k} \approx \left[1 - \frac{\sigma_a^{25}}{\nu\sigma_f^{25}} (1 + \delta^{28}) \quad \frac{\Delta N^{25}}{N^{25}}\right] \tag{C.12}$$

However,

$$\eta^{25} = \frac{\nu\sigma_f^{25}}{\sigma_a^{25}} , \tag{C.13}$$

where η^{25} is the average number of neutrons produced per neutron absorbed in U-235.

Using Equation (C.13) in Equation (C.12) one gets:

$$\frac{\Delta k_\infty}{k_\infty} \approx \left[1 - \frac{1}{\eta^{25}(1 + \delta^{28})}\right] \frac{\Delta N^{25}}{N^{25}} \tag{C.14}$$

$$= \left[\frac{\eta^{25} + \eta^{25}\delta^{28} - 1}{(\eta^{25})(1 + \delta^{28})}\right] \frac{\Delta N^{25}}{N^{25}} \tag{C.15}$$

However, η^{25} is approximately 2, and δ^{28} is approximately 0.1. Therefore, one can ignore terms containing $\eta^{25}\delta^{28}$ and δ^{28}, in which case Equation (C.15) becomes:

$$\frac{\Delta k_\infty}{k_\infty} \approx \frac{\eta^{25} - 1}{\eta^{25}} \frac{\Delta N^{25}}{N^{25}} \tag{C.16}$$

Since $\eta^{25} \approx 2.0$:

$$\frac{\Delta N^{25}}{N^{25}} \approx \frac{2\Delta k_\infty}{k_\infty} \tag{C.17}$$

But,

$$\frac{\Delta N^{25}}{N^{25}} = \frac{\Delta M^{25}}{M^{25}} = \left(\frac{\Delta X}{X}\right) \cdot \left(\frac{Mu}{Mu}\right) \tag{C.18}$$

where Mu is the total mass of Uranium heavy metal in the fuel, and M^{25} is the mass of U-235.

Therefore Equation (C.17) becomes:

$$\frac{\Delta M^{25}}{M^{25}} = \left(\frac{\Delta X}{X}\right) \approx 2 \left(\frac{\Delta k_\infty}{k_\infty}\right), \tag{C.19}$$

as was to be shown.

Equation (C.19) is useful for adjusting fissile concentration estimates in trial-and-error computer calculations to achieve a given $k_\infty(t)$ behavior. It is also useful for estimating errors in fissile mass flows resulting from errors or uncertainties in burnup history calculations. It should be noted here also, that the same results hold for $k_{effective}$ i.e. where leakage is taken into account, since

Σ_p, the non-fuel macroscopic absorption cross section (Equation 3.4) could include a DB^2 leakage contribution.

In order to verify the relation (Equation C.19), two separate runs of LEOPARD for a UO_2 (slightly enriched U-235) reactor at the Maine Yankee V_f/V_m were compared.

Here the constant R defined by:

$$R = \left(\frac{\Delta X}{X}\right) / \left(\frac{\Delta k}{k}\right) \tag{C.20}$$

was evaluated for both k_{eff} and k_∞ . For the variation of k_{eff} with enrichment R was found to be 2.68, and for k_∞, 2.72. These numbers verify the approximate validity of Equation (C.19). In general the accuracy would be expected to improve as k_∞ approaches 1.0: the above values were determined in the range $0.968 \leq k_\infty \leq 0.946$.

APPENDIX D

APPLICATION OF THE LINEAR REACTIVITY MODEL TO THE INITIAL CORE LOADING

D.1 Ore Requirements for the Initial Core

To a very good approximation the reactivity limited burnup of fuel discharged from a PWR is a linear function of the beginning-of-life fissile enrichment, X, (F-2). In this model it is assumed that each core batch is irradiated in a medium composed of other batches which are collectively maintained at k = 1, and hence a given batch is responsible only for its own reactivity; so that:

$$B = A(X - X_o) \tag{D.1}$$

where A and X_o are constants.

For an n batch core, if B_0 is the equilibrium burnup (33,000 MWD/MT for a typical PWR); then the burnup B_1 reached by the first batch in the n-batch initial core is

$$B_1 = \frac{1}{n} B_o \tag{D.2}$$

More generally, the final burnup, B_j, of the j^{th} startup batch in an n- batch initial core is:

$$B_j = \frac{j}{n} B_o \tag{D.3}$$

$$j \leq n$$

Summing over B_j:

$$\sum_{j=1,n} B_j = \frac{n(n+1)}{2n} B_o \tag{D.4}$$

The average burnup, $\bar{B}$, to be sustained by the initial core batches, assuming equal heavy metal loading per batch is:

$$\bar{B} = \sum_j \frac{B_j}{n} = \frac{n+1}{2n} B_o \tag{D.5}$$

Referring back to Equation (D.1), for an equilibrium batch the equilibrium discharge burnup of the fuel, B_o, is given by:

$$B_o = A(X_p - X_o) \tag{D.6}$$

where X_p is the equilibrium batch's initial enrichment.

If $\bar{B}$ is the average final burnup of the n batches in the initial core, then:

$$\bar{B} = A(\bar{X} - X_o) \tag{D.7}$$

where $\bar{X}$ is the average enrichment of the initial core batches.

Using Equations (D.4), (D.6) and (D.7), one gets:

$$\frac{\bar{X} - X_o}{X_p - X_o} = \frac{n+1}{2n} \tag{D.8}$$

Therefore,

$$\bar{X} = X_o + \frac{n+1}{2n} (X_p - X_o) \tag{D.9}$$

or,

$$\bar{X} = \frac{(n+1)}{2n} X_p + \frac{(n-1)}{2n} X_o \qquad (D.10)$$

Equation (D.10) applies to all types of reactors considered in this study.

For a UO_2 (slightly enriched U-235) core; if Peq represents the equilibrium loading of a reactor batch in metric tons of heavy metal, then the feed of natural U_3O_8, Feq per batch is given by, (for a once through fuel cycle):

$$Feq = 1.3\ Peq \left(\frac{X_p - X_w}{X_f - X_w}\right) \frac{STU_3O_8}{batch} \qquad (D.11)$$

where X_w and X_f are the tails and natural uranium enrichments respectively.

The heavy metal loading of the initial core batches is the same as that of the equilibrium batches (only the enrichment differs). Furthermore Feq is a linear function of the enrichment. Therefore, for the initial core the natural U_3O_8 requirement, Fi, is given by

$$F_i = 1.3\ n\ Peq \frac{\bar{X} - X_w}{X_f - X_w} \qquad (D.12)$$

Using Equations (D.11) and(D.12), one obtains:

$$\frac{F_i}{Feq} = n \left(\frac{\bar{X} - X_w}{X_p - X_w}\right) = N \qquad (D.13)$$

where N is the equivalent number of equilibrium batches in the initial core in terms of U_3O_8 requirements.

But,

$$\bar{X} = \frac{(n+1)X_p}{2n} + \frac{(n-1)X_o}{2n} \tag{D.10}$$

Therefore, Equation (D.13) becomes

$$N = \left[\frac{\left(\frac{n+1}{2}\right)X_p + \left(\frac{n-1}{2}\right)X_o - nX_w}{X_p - X_w}\right] \tag{D.14}$$

For the typical core in which n = 3:

$$N = \left[\frac{2X_p + X_o - 3X_w}{X_p - X_w}\right] \tag{D.14a}$$

It can be shown that Equation (D.14) also holds for Pu and U-233 fueled cores, but with $X_w = 0$.

Table D.1 shows the parameters of the first four fuel batches of CE System 80^{TM}, which is a three batch core. The difference in N between Equation (D.14) and the actual masses is approximately 7.0% i.e. N = 2.18 using Equation (D.14), while N = 2.04 for the actual mass flows of Table D.1. Checks on other cores (eg. Westinghouses ZION PWR) have also confirmed the validity of Equation (D.14). It should also be noted that initial core batches usually contain burnable poison for additional reactivity control, which introduces another degree of freedom not considered here. Nevertheless, the agreement is more than adequate for present purposes. This approach to obtaining initial core loadings is particularly convenient since all information needed for our ore usage model can be obtained from a single equilibrium-cycle batch burnup computation.

Equation (D.14) is applicable to all cores except for the UO_2 (93% U-235)/ThO_2 core, where the Uranium is 93% by weight U-235. Here, the following modifications must be made.

If the enrichments $\bar{X}$ and $\bar{X}_p$ are taken as weight percent of U-235 in total heavy metal (i.e. both thorium and uranium) for this core type, then Equation (D.10) applies. Thus, rewriting (D.10):

$$\bar{X} = \left(\frac{n+1}{2n}\right) X_p + \left(\frac{n-1}{2n}\right) X_o \tag{D.15}$$

Where X_p is now the weight percent of U-235 in heavy metal in the equilibrium batch of the UO_2/ThO_2 core and $\bar{X}$ is the average weight percent of U-235 in heavy metal of the initial UO_2/ThO_2 core batches.

Using the same reasoning as before, Feq, the equilibrium requirement for natural uranium oxide in ST U_3O_8 per batch is:

$$Feq = 1.3\ Peq \left(\frac{X_p}{0.93}\right)\frac{(93.0 - 0.2)}{(0.711 - 0.2)} \tag{D.16}$$

Similarly, F_i, the initial core requirement of U_3O_8 in ST U_3O_8 is:

$$F_i = 1.3\ n\ Peq \left(\frac{\bar{X}}{0.93}\right)\frac{(93.0 - 0.2)}{(0.711 - 0.2)} \tag{D.17}$$

Therefore N, the number of equivalent equilibrium batches in the initial core using Equations (D.15), (D.16), (D.17) is given by

$$N = \frac{(n+1)}{2} + \frac{(n-1)}{2}\left(\frac{X_o}{X_p}\right) \tag{D.18}$$

Note that Equation (D.18) is the same as Equation (D.14) with $X_w = 0$.

D.2 SWU Requirements for the Initial Core

We are also interested in the separative work unit requirements of the initial cores for the UO_2 (slightly enriched U-235) and the UO_2 (93% enriched U-235)/ThO_2 reactors.

For the UO_2 (93% enriched U-235)/ThO_2 reactor, the metric tons of separative work units per metric ton of uranium fuel is constant i.e. the batches are all at 93% enrichment.

Hence, if S/P (0.93) is the metric tons of separative work units per metric ton of 93% enriched uranium, then the equilibrium S.W.U. requirement, S.W.Feq, of a reload batch is given by

$$SWFeq = Peq \cdot \frac{X_p}{0.93} \cdot S/P(0.93) \qquad (D.19)$$

where, again, X_p is the enrichment of U-235 in total heavy metal (uranium plus thorium).

Likewise if $\bar{X}$ is the average initial core enrichment of U-235 in heavy metal, then, SWF_i, the initial SWU requirement of the initial core is given by:

$$SWF_i = n\ Peq \frac{\bar{X}}{0.93} \cdot S/P(0.93) \qquad (D.20)$$

Thus, using Equations (D.19) and (D.20), the number of equivalent equilibrium reloads contained in the initial core in terms of separative work units, SN, is given by:

$$SN = \frac{SWF_i}{SWFeg} = \frac{n\ \bar{X}}{X_p} \qquad (D.20)$$

Using Equation (D.15) for $\bar{X}$:

$$SN = \left(\frac{n+1}{2}\right) + \left(\frac{n-1}{2}\right)\left(\frac{X_o}{X_p}\right) = N \text{ (for } UO_2\text{(93\% U-235)/Th}O_2\text{)} \qquad (D.21)$$

Hence for this case the number of equivalent equilibrium reloads contained in the initial inventory, in terms of both ore requirements and separative work units, is the same.

For the UO_2 (slightly enriched U-235) case, the problem is different.

Here, SWFeq, the equilibrium SWU requirement of a reload batch is given by:

$$SWFeq = Peq \cdot S/P(X_p) \qquad (D.22)$$

where X_p is the enrichment of U-235 in uranium heavy metal, and Peq and $S/P(X_p)$ have been previously defined.

Likewise, SWF_i, the initial SWU requirement of the initial core is given by:

$$SWF_i = \sum_{j=1}^{n} Peqj \, \frac{S}{P}(X_j) \qquad (D.23)$$

where X_j is the enrichment of U-235 in uranium heavy metal of the j^{th} batch in the initial core, and Peqj the heavy metal loading of the j^{th} batch in the initial core.

Since Peqj is assumed constant, SN, the number of equivalent equilibrium reloads per initial core for the UO_2 (slightly enriched U-235) reactor is given by:

$$SN = \frac{SWP_i}{SWFeq} = \frac{\sum_{j=1}^{n} S/P(X_j)}{S/P(X_p)} \qquad (D.24)$$

At this point the approximation is made that the sum of the separative work requirements for the n individual startup core batches is equivalent to n times the separative work requirements of one batch of the same heavy metal mass loading at an average enrichment $\bar{X}$,(where $\bar{X}$ is given by Equation (D.10)). This approximation is equivalent to saying that the separative work requirement per metric ton is a linear function of enrichment in the range of interest. For the type of reactor considered in this study, in which X_p is approximately 3%, this turns out to be a very good approximation.

Hence Equation (D.24) becomes:

$$SN = \frac{n\ S/P(\bar{X})}{S/P(X_p)} \qquad (D.25)$$

Using the values in Table D.1 for the System 80^{TM} core, the value of SN using Equation (D.25) was found to be 1.86, or about 7 percent higher than the value of 1.73 calculated using the exact SWU relations and the actual mass flows in Table (D.1). Thus we also have an acceptable simple model for determining startup core SWU requirements using only equilibrium batch data.

D.3 The Calculational Methodology for Obtaining A and X_o, the Constants in the Linear Reactivity Model, from a LEOPARD Calculation

In determining the enrichment of a given reactor lattice, as was described in Chapter 2, an iterative process is used to determine an enrichment, X_p, at which the multiplication constant has a value of

1.000 at a fuel burnup of 22,000 MWD per metric ton heavy metal charged (MTHM). As was also described in Chapter 2, this burnup of 22,000 MWD/MT was chosen on the basis of the linear reactivity model, which requires that an equilibrium core batch be just critical at a burnup which is $\frac{n+1}{2n}$ (here 2/3) of the discharge burnup, which in this study was taken to be 33,000 MWD/MT.

Using the above criterion, that in a three batch core, the final burnup of the fuel possible is 3/2 of the just critical value, it was possible using iterative LEOPARD runs to obtain an initial enrichment, X, which would give the burnup, B, at k = 1.00.

It was then possible to do a least squares fit of sets of (B,X) data to Equation (D.1),

$$B = A(X - X_o) \tag{D.1}$$

to obtain values of A and X_o.

Note that if the linear reactivity model was an exact representation, X_o would be the enrichment of a just critical hot clean (zero-burnup) lattice with equilibrium Xenon and Samarium. However, because the model is only approximate, values of X_o determined using end-of-life data, as above, are not equal to beginning-of-life X_o values.

For the consumer reactors j = 3, 4, 5 where mixed fissile isotopes are used as feed material, the value of X in each case was determined using the techniques developed in Chapter 3 and Appendix F, using energy weighting values to define equivalent single-isotope enrichments.

Table (D.2) presents the values of A and X_o obtained for all reactor types considered, at different V_f/V_m values. Note that for Cases C and D of the PuO_2/ThO_2 reactor, no values are shown. This is due to the flat

reactivity swing observed in these cases.

Finally Table (D.2) lists the comparable values for the fuel cell size study.

TABLE D.1

MASS AND BURNUP PARAMETERS FOR THE FIRST FOUR

BATCHES OF THE C.E. SYSTEM 80^{TM} REACTOR

Batch #	Initial Enrichment	HM Charge, MT	Discharge Burnup MWD/MT
1	1.66	34.119	12748
2	2.21	32.232	21811
3	2.81	32.962	28997
4(steady state)	3.07	34.19	30360

(1) Source: Private Communication with C.E. (Physics Division)

(2) 3-batch, UO_2, low enrichment U-235 Core

TABLE D.2

VALUES OF THE CONSTANTS, A, X_o OF THE LINEAR REACTIVITY MODEL FOR VARIOUS REACTOR TYPES AT VARYING V_f/V_m VALUES

Case	A	B	C	D
Fuel-to-Coolant Volume Ratio	0.388	0.4816	0.9161	1.497
Reactor Type				
1. U-238/U-235				
A	12,760	15,130	11,290	8,170
X_o	0.5142	0.7786	1.168	2.281
2. U-235/ThO_2				
A	17,130	21,800	17,070	13,870
X_o	2.028	2.307	2.540	4.090
3. PuO_2/UO_2				
A	16,840	21,010	--	--
X_o	1.680	2.174	9.443	9.288
4. PuO_2/UO_2				
A	16,540	15,622	30,520	17,488
X_o	0.6438	0.7200	7.303	6.810
5. U-233/ThO_2				
A	20,150	20,430	18,530	20,800
X_o	1.588	1.463	1.366	2.015

TABLE D.3

VALUES OF THE CONSTANTS A, X_o OF THE LINEAR REACTIVITY MODEL FOR VARIOUS REACTOR TYPES FOR VARIOUS UNIT CELL SIZES

Case			
Relative Cell Volume[1]	0.5	1.0	1.5
Reactor Type			
1. U-235/U-238			
A	15,220	15,130	14,080
X_o	0.8325	0.7786	0.6070
2. PuO_2/UO_2			
A	15,150	15,620	15,730
X_o	0.8579	0.7200	0.6571

(1) Relative to the fuel cell size of Maine Yankee taken as 1.0; V_f/V_m is maintained constant at the Maine Yankee value

APPENDIX E

PARAMETERS USED BY MASFLO-2

The function of this appendix is to document the numerical results developed using the methods described in the preceding chapters and appendices, and for the most part only shown in graphical form previously.

E.1 Fuel-to-Coolant Volume Ratio Studies

In this section the parameters used in MASFLO-2 for all studies involving the variation of the fuel-to-coolant volume ratio are presented. All terms employed here have been defined in Chapter 3, and the methods used for their determination have been documented there and in Appendix F.

The data for each reactor type are presented in Tables E.1.1 through E,1.5. The following ground rules apply to all cases: (a) all reactors were considered to operate at a capacity factor of 0.75, with a fuel discharge burnup of 33,000 MWD/MT, (b) all reactors were assumed to have the nominal thermal efficiency of Maine Yankee (0.325); (c) 1.5 weight percent heavy metal process stream losses were assumed in fabrication and mining combined, and reprocessing and refabrication combined, (c) the refueling interval was taken to be one calendar year and (d) the enrichment plant tails composition was taken to be 0.2 weight percent. Finally, no additional "pipeline" inventory was taken into account by modification of N_j in Equation 3.46, i.e. $T_{lag} = 0$ (see Chapter 3); however in view of the one year refueling interval an "on-site" out-of-core inventory equal to one equilibrium reload (see Figure 3.4) was allowed for - one can, if desired, consider that part of this inventory is in process and part on-site.

E.2 Fuel Cell Size Study

In this section are listed the parameters used in the study of the effect of fuel cell size on reactor resource utilization. The same common parameters, such as discharge burnup, specified in Section E.1, apply here.

Tables E.2.1 and E.2.2 list the parameters used in this study for both types of reactors involved: UO_2 (slightly enriched U-235) and PUO_2/UO_2. The same terminology is used as in the previous section.

E.3 Benchmarking of MASFLO-2 Against ALPS.

In this section the parameters used in the comparison of MASFLO-2 to ALPS (H-4),(H-5) are presented. Before going into details, a few points should be made: (1) As stated in Chapter 4, the capacity factor was taken as a system-averaged capacity factor. (2) The final burnup of both reactor types involved was 33,000 MWD/MT (H-6) (3) Finally, the lead times (see Chapter 3) were specified as one year for the fabrication, etc. of virgin Uranium, and one year for the fabrication and reprocessing of Plutonium; and annual refueling was specified. Thus the ex-reactor inventory allowance ΔN was equal to 0.33 N for both producer and consumer reactors (see Equation 3.46); in addition as discussed in section 3.4 "on-site" inventory of one reload is also maintained because of the manner in which the model is formulated.

TABLE E.1.1

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

UO_2 (SLIGHTLY ENRICHED U-235) REACTOR (j = 1)

Case	A	B	C	D
Fuel-to-coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
XF_{11}	3.10	2.96	4.09	6.32
XD_{11}	0.5986	0.6314	1.536	3.270
XD_{31}	0.004826	0.006167	0.01140	0.01653
RD_1	0.9583	0.9563	0.9510	0.9450
N_1	2.108	2.210	2.251	2.340
SN_1	1.780	1.898	2.027	2.204

TABLE E.1.2

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

UO_2/ThO_2 (93% ENRICHED U-235) REACTOR (j = 2)

Case	A	B	C	C
Fuel-to-coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
XF_{12}	93.00	93.00	93.00	93.00
XD_{12}	57.93	58.37	65.97	74.46
XD_{22}	0.01211	0.01290	0.01464	0.01676
Γ	0.4118	0.4150	0.4892	0.6139
α	0.04251	0.04108	0.04810	0.06956
N_2	2.513	2.604	2.568	2.632
SN_2	2.513	2.604	2.568	2.632

TABLE E.1.3

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

PuO_2/ThO_2 REACTOR (j = 3)

Case	A	B	C	D
Fuel-to-coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
XF_{33}	0.03640	0.03744	0.09443	0.09288
XD_{33}	0.009564	0.01206	0.06964	0.06688
XD_{23}	0.01013	0.01125	0.01343	0.01670
N_3	2.462	2.581	3.000	3.000

TABLE E.1.4

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

PuO_2/UO_2 REACTOR (j = 4)

Case	A	B	C	D
Fuel-to-coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
XF_{34}	0.02640	0.02832	0.08384	0.08697
XD_{34}	0.01122	0.01664	0.07702	0.08044
N_4	2.244	2.254	2.871	2.783

TABLE E.1.5

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

$^{233}UO_2/ThO_2$ REACTOR (j = 5)

Case	A	B	C	D
Fuel-to-coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
XF_{25}	0.03226	0.03075	0.03148	0.03601
XD_{25}	0.02021	0.02098	0.02383	0.02992
N_5	2.492	2.476	2.434	2.560

TABLE E.2.1

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

UO_2 (SLIGHTLY ENRICHED U-235) REACTOR (j = 1) IN

THE CELL SIZE EFFECT STUDY

Case	1	2	3
Relative Cell Volume(1)	0.5	1.0	1.5
Fuel Pin Diameter (in)	0.2616	0.3900	0.4532
XF_{11}	3.00	2.96	2.95
XD_{11}	0.6883	0.6314	0.6056
XD_{31}	0.006732	0.006163	0.005838
RD_1	0.9556	0.9563	0.9567
N_1	2.226	2.210	2.148
SN_1	1.925	1.898	1.815

(1) with respect to Maine Yankee cell size taken as 1.0

TABLE E.2.2

REACTOR PARAMETERS USED IN MASFLO-2 FOR THE

PuO_2/UO_2 REACTOR IN THE CELL SIZE EFFECT STUDY

Case	1	2	3
Relative Cell Volume(1)	0.5	1.0	1.5
Fuel Pin Diameter (in)	0.2616	0.3900	0.4532
XF_{34}	0.03037	0.02831	0.02755
XD_{34}	0.01966	0.01662	0.01527
N_4	2.283	2.254	2.239

(1) with respect to Maine Yankee cell size taken as 1.0.

TABLE E.3.1

REACTOR PARAMETERS USED IN THE COMPARISON

OF MASFLO-2 TO ALPS[1]

UO_2 REACTOR (j = 1)

VARIABLE/	XF_{11}	XD_{11}	Γ	XD_{31}	N_1	SN_1
VALUE	3.242	0.8198	0.9487	0.006460	2.562	2.434

PuO_2/UO_2 REACTOR (j = 4)

VARIABLE/	XF_{34}	XD_{34}	N_4
VALUE	0.02751	0.01708	2.571

(1) Reference (H-6)

APPENDIX F

A SAMPLE CALCULATION FOR OBTAINING MASFLO-2 PARAMETERS

In this appendix a detailed sample calculation for obtaining MASFLO-2 parameters from the output of the LEOPARD code (B-1) will be presented. For this purpose the charge and discharge MASFLO-2 parameters will be calculated for the $^{233}UO_2/ThO_2$ reactor (j = 5) at the V_f/V_m value of Maine Yankee (0.4816).

Before this is done, however, the following definitions, some of which have already been given in Chapter 3, will be presented.

Let

$w_k^{\ell}(j)$ = the "energy worth" of isotope k, relative to reference isotope ℓ (U-233 or Pu-239) in reactor type j

E_k^j = total energy in MWD produced by isotope k per metric ton of heavy metal charged to reactor type j, at the equilibrium discharge fuel burnup

G_k^j = corresponding gross mass in metric tons of isotope k destroyed per metric ton of heavy metal charged to reactor type j, at the final equilibrium discharge fuel burnup

P_k^j = power sharing; fraction of total energy produced by isotope k, in reactor type j

$G_{c\,k}^{\ j}$ = the mass of isotope k in metric tons discharged per metric ton of heavy metal charged to reactor type j, at the equilibrium discharge fuel burnup.

$G_{d\ k}^{j}$ = mass of isotope k in metric tons charged per metric ton of heavy metal charged to reactor type j in its equilibrium cycle

$G_{d\ eq}^{\ell}(j)$ = the equivalent mass of reference isotope ℓ, in MT, discharged per metric ton of heavy metal charged to reactor type j, at the equilibrium discharge fuel burnup

$G_{c\ eq}^{\ell}(j)$ the equivalent mass of reference isotope ℓ, in MT, charged per metric ton of heavy metal charged to reactor type j, in its equilibrium cycle.

Table F.1 presents those of the above parameters which are readily available from either the user input to or output from LEOPARD (B-1), for the $^{233}UO_2/ThO_2$ reactor at Maine Yankee's V_f/V_m value of 0.4816. As was stated in Chapter 3, only the fissile isotopes (here U-233 and U-235) are considered. As a convention, isotope k will be defined by its usual nomenclature i.e. U-233 for uranium-233. Finally, the discharged mass of U-233 also includes that of its parent isotope Pa-233 which has a relatively short half-life of 27 days.

As was mentioned in Chapter 3, the task is to define equivalent single-isotope mass parameters for use in MASFLO-2. To do this, the fissile isotopes utilized in a given consumer reactor were given "energy worths" based on the energy produced by the isotope in the consumer reactor per unit mass destroyed.

With the above in mind, the relevant equations will now be derived in which the values of Table F.1 will be utilized to calculate the

required parameters for MASFLO-2.

For simplicity let us consider one metric ton of fuel charged to reactor type j. Then by definition E_k^j is given by

$$E_k^j = P_k^j \cdot B \tag{F.1}$$

where B is the final burnup or total energy produced per metric ton of fuel charged.

Again by definition $w_k^{\ell}(j)$, the "energy worth" of isotope k, relative to reference isotope ℓ in reactor type j, is given by

$$w_k^{\ell}(j) = \frac{E_k^j}{G_k^j} \cdot \frac{G_{\ell}^j}{E_{\ell}^j} \tag{F.2}$$

or using Equation (F.1)

$$w_k^{\ell}(j) = \frac{P_k^j}{G_k^j} \cdot \frac{G_{\ell}^j}{P_{\ell}^j} \tag{F.3}$$

where the final fuel burnup per metric ton, B, cancels. The term $w_k^{\ell}(j)$ in Equation (F.3) is therefore the energy worth of isotope k with respect to reference isotope ℓ.

With this in mind, then the equivalent mass of reference isotope ℓ, in metric ton charged per metric ton of heavy metal charged to reactor type j, in its equilibrium cycle, $G_{ceq}^{\ell}(j)$ is given by

$$G_{ceq}^{\ell}(j) = \sum_k w_k^{\ell}(j) \cdot G_{c\,k}^{\ell\ j} \tag{F.4}$$

or using Equation (F.3)

$$G_{ceq}^{\ell}(j) = \sum_k \frac{P_k^j}{G_k^j} \cdot \frac{G_\ell^j}{F_\ell^j} \cdot G_{c\,k}^{\,j} \qquad \text{(F.5)}$$

Similarly $G_{deq}^{\ell}(j)$, the equivalent mass of reference isotope ℓ in MT discharged per metric ton of heavy metal charged to reactor type j, at the equilibrium fuel burnup is given by

$$G_{deq}^{\ell}(j) = \sum_k \frac{P_k^j}{G_k^j} \cdot \frac{G_\ell^j}{P_\ell^j} G_{d\,k}^{\,j} \qquad \text{(F.6)}$$

However, by definition (see Chapter 3), $G_{ceq}^{\ell}(j)$ is equivalent to XF_{ij}, where isotope i or ℓ are the reference isotopes (see Table 3.2). A similar situation holds for XD_{ij} and $G_{deq}^{\ell}(j)$.

Therefore,

$$Xf_{ij} = \sum_k \frac{P_k^j}{G_k^j} \cdot \frac{G_\ell^j}{P_\ell^j} \cdot G_{c\,k}^{\,j} \qquad (i = \ell) \qquad \text{(F.7)}$$

and

$$XD_{ij} = \sum_k \frac{P_k^j}{G_k^j} \cdot \frac{G_\ell^j}{P_\ell^j} \cdot G_{d\,k}^{\,j} \qquad (i = \ell) \qquad \text{(F.8)}$$

where XF_{ij} and XD_{ij} are the required single-isotope parameters for input into MASFLO-2.

For the present example of reactor type j = 5, the reference isotope i = 2 (see Table 3.2), is U-233. Thus Equation (F.7) and (F.8) become,

$$XF_{25} = G_{c\ell}^{5} + \frac{P_k^5}{G_k^5} \cdot \frac{G_\ell^5}{E_\ell^5} \cdot G_{c\ k}^{5} \tag{F.9}$$

and

$$XD_{25} = G_{d\ell}^{5} + \frac{P_k^5}{G_k^5} \cdot \frac{G_\ell^5}{E_\ell^5} \cdot G_{d\ k}^{5} \tag{F.10}$$

where ℓ refers to the reference isotope, U-233, and k refers to U-235.

Using parameter values from Table F.1 in Equations (F.9) and (F.10) one gets:

$$XF_{25} = 0.03038 + \left(\frac{3.041 \times 10^{-2}}{1.346 \times 10^{-3}}\right) \cdot \left(\frac{0.03683}{0.9228}\right) \cdot (4.18 \times 10^{-4})$$

or

$$XF_{25} = 0.03075 \tag{F.11}$$

and

$$XD_{25} = 0.01949 + \left(\frac{3.041 \times 10^{-2}}{1.346 \times 10^{-3}}\right)\left(\frac{0.03683}{0.9228}\right) \cdot (1.662 \times 10^{-3})$$

or

$$XD_{25} = 0.02098 \tag{F.12}$$

The above values of XF_{25} and XD_{25}, obtained in Equations (F.11) and (F.12), are listed for this case in Table E.1.5 of Appendix E.

In Chapter 3 a detailed derivation is developed for the trading of fissile isotopes between a consumer reactor and a producer reactor i.e. Plutonium between the UO_2 (slightly enriched U-235) reactor and the

PuO_2/UO_2 reactor. That example, and the one just presented, cover the entire technique required to handle single isotope equivalent mass flows.

Finally, Tables F.2 and F.3 present the various weighting factors used for (1) the V_f/V_m study and (2) the cell size study. Note that these refer only to the consumer reactors (j = 3, 4, 5) which utilize a mixed isotopic feed, and are used to calculate equivalent single-isotope values for both the charged and discharged masses of these consumer reactors.

TABLE F.1

VALUES OF PARAMETERS DIRECTLY AVAILABLE

FROM LEOPARD OUTPUT FOR THE $^{233}UO_2/ThO_2$ REACTOR

AT V_f/V_m VALUE OF 0.4816[1]

Isotope	$G_{c\kappa}^5$	Isotope	G_{dk}^5
k = U-233	0.03038	k = U-233[2]	0.01949
k = U-235	4.18×10^{-4}	k = U-235	1.662×10^{-3}
Isotope	P_k^5	**Isotope**	G_k^5
k = U-233	0.9228	k = U-233	0.03683
k = U-235	3.041×10^{-2}	k = U-235	1.346×10^{-3}

(1) value of Maine Yankee core

(2) includes Pa-233.

TABLE F.2

WEIGHTING FACTORS $w_k^{\ell}(j)$ USED IN DETERMINING

SINGLE-ISOTOPE MASS FLOWS FOR THE CONSUMER REACTORS

IN THE V_f/V_m STUDY

Case	A	B	C	D
Fuel-to-coolant Volume Ratio	0.3380	0.4816	0.9161	1.497
1. $PuO_2/UO_2(j=3)$ [1]				
$w^{Pu-239}_{Pu-241}(3)$	1.092	1.095	1.030	1.039
2. $PuO_2/ThO_2(j=4)$				
$w^{Pu-239}_{Pu-241}(4)$	1.092	1.095	1.030	1.039
3. $^{233}UO_2/ThO_2(j=5)$				
$w^{U-233}_{U-235}(5)$	0.9119	0.9017	0.8712	0.8314

(1) value used at each V_f/V_m value is mean of values for reactor types j = 3 and 4 (see Chapter 3)

TABLE F.3

WEIGHTING FACTORS $w_k^{\ell}(j)$ USED IN DETERMINING EQUIVALENT SINGLE-ISOTOPE MASS FLOWS FOR THE PuO_2/UO_2 REACTOR IN THE CELL SIZE STUDY

Case	1	2	3
Relative Cell-Size(1)	0.5	1.0	1.5
PuO_2/UO_2			
w^{Pu-239}_{Pu-241}(3)	1.087	1.092	1.094

(1) Relative to cell size of standard Main Yankee Core.

APPENDIX G

SUPERCELL AND FUEL-TO-MODERATOR RATIO DEFINITION

G.1 Supercell Description of the Maine Yankee PWR

In this section, the supercell description of the base case reactor of the present study, the Maine Yankee PWR, will be presented. The description here will be limited to the minimum needed to do a supercell burnup with LEOPARD. For a further discussion of the parameters listed here, the reader is referred to reference (B-1).

Table G.1 lists the volume fractions of the various Maine Yankee supercell constituents according to their respective supercell region i.e. fuel region, clad region, moderator region and extra region. These volume fractions together with the assigned density of each constituent are readily converted into nuclide number densities in each region. Then specification of the fuel-to-moderator ratio suffices to determine the homogenized number density over the entire unit cell volume.

Table G.2 lists various required dimensional, thermodynamic and reactor physics parameters. A final point is that the fission product cross section scale factor quoted is for ^{235}U; a description of its modification for plutonium cores is given in Chapter 2.

G.2 Definition of Fuel-to-Moderator Volume Ratio

In this section a consistent definition of the fuel-to-moderator volume ratio used in this study will be presented.

From a reactor physics point of view the important parameter is the fuel-to-moderator atom ratio which is the ratio of the number density of the fuel to that of the moderator, both homogenized over the fuel cell (zero-dimensional codes like LEOPARD homogenize all cell contituents over the entire cell (B-1)).

Then if N_f and N_m are the cell homogenized number densities of the fuel and moderator respectively, the "reactor physics" fuel to moderator ratio R is given by:

$$R = \frac{N_f}{N_m} = \frac{\dfrac{\rho_f \cdot V_f \cdot A_v}{A_f \cdot V_{cell}}}{\dfrac{\rho_m \cdot V_m \cdot A_v}{A_m \cdot V_{cell}}} \qquad (G.1)$$

where

ρ_f and ρ_m are the mass densities of the fuel and moderator, respectively; V_f, V_m are the volumes occupied in the cell by the fuel and moderator, respectively.

V_{cell} is the cell volume

and

A_f and A_m are the atomic masses of the fuel and moderator respectively, Equation (G.1) can be reduced to

$$R = \frac{\rho_f}{\rho_m} \cdot \frac{A_m}{A_f} \cdot \frac{V_f}{V_m} \qquad (G.2)$$

If ρ_{ftd} is the theoretical density of the fuel and F_{td} the fraction of theoretical density of the fuel, then Equation (G.2) becomes:

$$R = \frac{F_{td}\ \rho_{ftd} \cdot A_m \cdot V_f}{A_f \quad \rho_m \quad V_m} \tag{G.3}$$

In this study the fraction of theoretical density, F_{td}, of all fuels considered was a constant value of 0.92. Furthermore, both the moderator mass density ρ_m and atomic mass A_m were maintained constant (i.e. ρ_m = 1.0 g/cc, A_m = 16 for light water). In other words the water density is quoted at 68°F; although the lattice is calculated at hot full power (LEOPARD temperature corrects densities for thermal expansion) the fuel-to-moderator ratio is calculated on a cold basis. Furthermore, for the fuels considered in this study the ratio ρ_{ftd}/A_f does not vary significantly. (The highly predominant species in these fuels are $^{238}UO_2$ and $^{232}ThO_2$, with ρ_{ftd}/A_f values of 4.055×10^{-2} moles/cc and 3.799×10^{-2} moles/cc, respectively; for a six percent difference) ($\rho_{ftd}(ThO_2)$ = 10.039/cc, $\rho_{ftd}(UO_2)$ = 10.95 g/cc).

Hence Equation (G.3) can be approximated in this study by

$$R \approx C \frac{V_f}{V_m} \tag{G.4}$$

where C is a constant: $C \equiv 0.60$ in this work. Thus, for this study the "reactor physics" fuel-to-moderator ratio, R, is reduced to a simple ratio of the fuel-to-moderator volume ratio, which is frequently referred to in this study as the fuel-to-coolant volume ratio, since for the model reactor in this study, the coolant and moderator are the same (H_2O).

The next task here is to derive V_f/V_m for a LEOPARD supercell in terms of the supercell parameters. For a further discussion of the parameters used here the reader is referred to Ref. (B-1).

Let

D_f = fuel pellet outer diameter

D_c = clad outer diameter

P = lattice pitch

f_m = volume fraction of water in the moderator region

f_e = volume fraction of water in the extra region

NLF = non-lattice fraction of the supercell.

In a code such as LEOPARD, one need only consider two dimensions, i.e. cell length is of no consequence. Hence, for two dimensions:

$$V_f = \frac{\pi D_f^2}{4} \tag{G.5}$$

The next task is to calculate the volume of the moderator V_m.

The volume of the (square) unit cell, (i.e. excluding the extra region), V_{uc}, is given by

$$V_{uc} = P^2 = (1 - NLF) \cdot V_{super} \tag{G.6}$$

where V_{super} is the volume of the supercell.

However

$$V_{uc} + V_e = V_{super} \tag{G.7}$$

where V_e is the volume of the extra region.

Using Equations (G.6) and (G.7), V_e becomes

$$V_e = \frac{NLF}{(1 - NLF)} \; P^2 \tag{G.8}$$

Using the definition of the volume fraction of a constituent in a region (B-1), the volume of water in the extra region, V_{me}, is given by

$$V_{me} = \frac{NLF}{(1 - NLF)} \cdot P^2 \cdot f_e \tag{G.9}$$

The only other region containing water is the moderator region. The volume of water in the moderator region, V_{mm}, is given by the product of the volume fraction of water in the moderator region and the volume of the moderator region:

$$V_{mm} = (P^2 - \pi D_c^2/4) \cdot f_m \tag{G.10}$$

Thus, combining Equations (G.5), (G.9) and (G.10), the fuel-to-coolant volume ratio V_f/V_m is given by

$$\frac{V_f}{V_m} = \frac{V_f}{V_{me} + V_{mm}} = \frac{\frac{\pi D_f^2}{4}}{\left\{\left[\frac{NLF}{(1 - NLF)} \cdot P^2 \cdot f_e\right] + \left[(P^2 - \pi D_c^2/4) \cdot f_m\right]\right\}}$$

or

$$\frac{V_f}{V_m} = \frac{\pi D_f^2}{\left\{\left[\frac{4 \cdot NLF \cdot P^2 \cdot f_e}{(1 - NLF)}\right] + \left[(4P^2 - \pi D_c^2) \cdot f_m\right]\right\}} \tag{G.11}$$

In this study the volume of the structural materials (Zircaloy, etc.) in the moderator region was kept constant. To allow for this, the volume fractions of the water and the structural materials were corrected as V_f/V_m was varied, maintaining the volume of the structural materials equal to that in the base case V_f/V_m (Maine Yankee).

Let

f_{mb} = base case volume fraction of water in the moderator region

P_b = base case (Maine Yankee) lattice pitch.

The sum of the non-H_2O volume fractions, f_{zb}, in the moderator region in the base case is given by

$$f_{zb} = (1 - f_{mb}) \tag{G.12}$$

In order to maintain the volume of this material constant, the volume fraction of the structural material at a given V_f/V_m, f_z, can be shown to be:

$$f_z = (1 - f_{mb}) \left[\frac{(P_b^2 - \pi D_c^2/4)}{(P^2 - \pi D_c^2/4)} \right] \tag{G.13}$$

where the lattice pitch P corresponds to the new V_f/V_m.

Equation (G.13) shows that the volume fraction of each constituent of the structural material in the moderator region in the base case, is to be multiplied by the factor, ϕ, to get its new value at the new V_f/V_m, where ϕ is given by:

$$\phi = \left[\frac{(P_b^2 - \pi D_c^2/4)}{(P^2 \quad \pi D_c^2/4)}\right] \tag{G.14}$$

Hence at a given lattice pitch, P, corresponding to a given V_f/V_m, the volume fraction of the water in the moderator region, f_m, is given by

$$f_m = (1 - f_z) = 1 - \left\{(1 - f_{mb}) \cdot \frac{\left(P_b^2 - \frac{\pi D_c^2}{4}\right)}{\left(P^2 - \frac{\pi D_c^2}{4}\right)}\right\} \tag{G.15}$$

Thus for this study the fuel-to-coolant volume ratio is given by:

$$\frac{V_f}{V_m} = \frac{\pi D_f^2}{\left\{\left[\frac{4 \cdot NLF \cdot P^2 \cdot f_e}{(1 - NLF)}\right] + \left[(4 \quad P^2 - \pi D_c^2) \cdot f_m\right]\right\}} \tag{G.16}$$

where f_m is given by Equation (G.15).

Note that all the parameters for Equations (G.15) and (G.16) are given in Table (G.1). The lattice pitches used and the corresponding values of V_f/V_m investigated in this study are given in Table (G.3).

TABLE G.1

VOLUME FRACTIONS OF VARIOUS MAINE YANKEE SUPERCELL CONSTITUENTS

Region / Constituent	Fuel	Clad(1)	Moderator	Extra
UO_2	1.00000	0.00000	0.00000	0.00000
Zircaloy - 2	0.00000	0.909573	0.004410	0.00000
Light - Water	0.00000	0.00000	0.994410	0.00000
304 Stainless Steel	0.00000	0.00000	0.00000	0.912349
Chromium	0.00000	0.00000	0.000224	0.087651
Nickel	0.00000	0.00000	0.000619	0.00000
Carbon	0.00000	0.00000	0.000001	0.00000
Manganese	0.00000	0.00000	0.000004	0.00000
Aluminum	0.00000	0.00000	0.000006	0.00000
Iron	0.00000	0.00000	0.000325	0.00000

(1) Volume fractions do not add to unity due to presence of fuel-clad gap in this region.

TABLE G.2

REQUIRED DIMENSIONAL, THERMODYNAMIC AND REACTOR PHYSICS

PARAMETERS FOR THE MAINE YANKEE SUPERCELL

Parameter	Value	Parameter	Value
"Resonance" Temperature (°F)	1209.5	Non-Lattice Fraction	0.115166
Pellet Temperature (°F)	1209.5	H_2O Density (g/cc)	1.0
Clad Temperature (°F)	614.8	Fuel Theoretical Density Fraction	0.92
Moderator Temperature (°F)	562.5	(1) Volume in cm^3	1.0
Geometrical Buckling (cm^{-2})	7.319×10^{-4}	(1) Power in Watts	75.016
Non-Lattice Peaking Factor	1.16	Fission Product	
Pellet Outer Radius (in)	0.185	Cross-Section Scale	
Clad Outer Radius (in)	0.220	Factor	0.84
Clad Inner Radius (in)	0.189		
Pitch (in)	0.580		
Power (fraction of full power)	1.0		

(1) Used for calculating power density.

TABLE G.3

VALUES OF V_f/V_m AND CORRESPONDING LATTICE PITCHES

Case	A	B[1]	C	D
Fuel-to-Coolant Volume Ratio	0.338	0.4816	0.9161	1.497
Lattice Pitch (in)	0.649	0.580	0.492	0.448

[1] Base Case, Maine Yankee PWR

APPENDIX H

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