

INTERFUEL SUBSTITUTION AND THE INDUSTRIAL DEMAND
FOR ENERGY: AN INTERNATIONAL COMPARISON*

by

Robert S. Pindyck

Massachusetts Institute of Technology

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INTERFUEL SUBSTITUTION AND THE INDUSTRIAL DEMAND FOR ENERGY:

AN INTERNATIONAL COMPARISON

I. Introduction

Until recently most studies of production concentrated on the substitutability of capital and labor, assuming that in the underlying production function these factors were separable from energy and raw material inputs. Dramatic increases in the price of energy, together with the appearance of studies such as that of Berndt and Wood [9] which indicated that energy and capital may in fact be complements rather than substitutes, resulted in an increased interest in the substitutability of energy and other factors of production. In addition, rapid changes in the prices of individual fuels raised the issue of how rapidly and to what extent the industrial sector could substitute among different fuels. Finally, there has been an increased concern with the growth in energy demand brought about by growth in industrial activity.

In this paper we report on results of an econometric study of the industrial demand for energy in a number of industrialized countries. Our objectives are to determine the extent to which capital, labor, and energy can be substituted for one another, and vary across time. In addition, we will attempt to measure the extent to which alternative fuels can be substituted for one another both in the short-run and the long-run, and estimate the total elasticities of demand for individual fuels.

We model industrial energy demand using a two-stage approach. In the first stage total energy demand in the industrial sector is modelled as a derived factor demand base on a translog cost function.¹ Factor inputs include capital, labor, and energy; a lack of data makes it impossible to include materials as an explicit

¹Griffen and Gregory [34] have already fitted translog cost functions to pooled international data in order to measure elasticities of substitution for capital, labor, and energy, but that work does not explain inter-country differences in elasticities since only four observations were used for each country, and does not deal with interfuel substitution.

input, so we must assume separability of materials from the other factors in the underlying structure of production. In the second stage expenditures on energy are broken down into expenditures on oil, natural gas, coal and electricity. Here translog cost functions are again used, but we have also experimented with the use of a multinomial logit model to explain fuel shares. The use of this two-stage approach requires certain additional assumptions about the underlying structure of production. In particular, we must assume that the production function is homothetically separable in the capital, labor, and energy aggregates, i.e. first, that expenditure shares for fuels are independent of the expenditure shares for capital and labor, and second, that expenditure shares for fuels are independent of total energy expenditures.²

The use of the translog cost function has the advantage that it permits us to obtain relatively unrestricted estimates of elasticities of substitution and demand elasticities. We need not a priori assume that the cost function is homothetic (at least in modelling the demand for capital, labor, and energy inputs), and we need not assume that the elasticities of substitution between different inputs are all the same. Of course there are other "generalized" cost and production functions that could be used that also impose no a priori restrictions on elasticities of substitution, and we might find that these provide tighter estimates than does the translog function.³ For the time being, however, we use only the translog function.

As one might expect, in work like this one is continually bound by data limitations. For many countries there is no good data available for some or all of the variables of interest to us. For other countries data exists, but obtaining that

²The necessity for these assumptions is shown by Berndt and Christensen [8] and Denny and Fuss [26]. Note that this second assumption of homotheticity of fuel shares will not be violated in the use of a logit model if total energy expenditures is not an explanatory variable in that model.

³For example, the generalized Leontief function and the generalized Cobb-Douglas function, both of which were introduced by Diewert [27,28]. Lau and Tamura [44] used the generalized Leontief production function in estimating input substitution in petrochemical refining. Magnus [46] used the generalized Cobb-Douglas cost function in estimating substitution between capital, labor, and energy inputs in Dutch manufacturing.

data can be an extremely time consuming and laborious task. These data limitations were one of the factors that helped define and delimit the modelling approaches used here. In particular, it necessitated restricting this analysis of industrial demand to a set of only ten countries.⁴ Even for these countries, however, the quality of the data varies, and compromises had to occasionally be made. The data used in this study is described briefly in this report; a much more detailed description is provided in a separate report entitled "A User's Guide to the M.I.T. World Energy Demand Data Base."⁵

In the next section we outline the specifications of alternative models of industrial energy demand, and discuss the characteristics of each specification. Section 3 discusses some methodological issues in the estimation of industrial energy demand models using pooled data, and describes our estimation method. Section 4 describes some of the characteristics and limitations of our data, and Section 5 includes the statistical results.

2. Alternative Specifications for Models of Industrial Energy Demand

As explained before, our industrial demand models are based on a two-stage approach where energy demand is explained as a factor input share, and energy expenditures are then broken down into expenditures on fuels. We begin here by summarizing our assumptions about the structure of production. We then review the properties of the static translog cost function, and discuss its application to the industrial demand model. Next we describe some alternative dynamic specifications of the translog cost function; these specifications permit non-constant returns to scale in the short-run, with an adjustment to constant returns in the

⁴ A much less detailed model of the demand for petroleum products is being constructed for a number of countries for which only partial data is available; the results of this work will be described in a forthcoming paper.

⁵ Working Paper No. MIT EL 76-011WP, M.I.T. World Oil Project, May 1976. A revised version of this report will appear shortly.

long-run. Finally we discuss the use of the multinomial logit model as an alternative means of describing fuel shares.

2.1 The Structure of Production.

Our approach involves certain assumptions about the structure of production. First, we assume that capital, labor, and energy inputs are as a group weakly separable from the fourth input, materials.⁶ This assumption is made necessary by the fact that we have no data from which to construct price indices of materials inputs, and therefore we can only estimate unrestricted elasticities of substitution between capital, labor, and energy. Second, we assume that the production function is weakly separable in the major categories of capital, labor, and energy.⁷ This implies that the marginal rates of substitution between individual fuels is independent of the quantities of capital and labor. The assumption permits us to use aggregate price indices for capital, labor, and energy inputs; in particular to construct an energy price index that aggregates the price of the four fuels, and to construct a price index of capital services that aggregates different types of capital. Finally, we assume that the capital, labor, and energy aggregates are homothetic in their components - in particular, that the energy aggregate is homothetic in its oil, gas, coal, and electricity inputs.⁸ This last assumption provides a necessary and sufficient condition for an underlying two-stage optimization process, i.e. optimize the mix of fuels that make up the energy input, and

⁶Materials includes intermediate inputs as well as non-energy raw materials. Weak separability here means that the marginal rate of substitution between any two of the first three inputs is independent of the quantity of materials used as an input. This is a necessary and sufficient condition for the production function to be of the form $Q = F[f(K,L,E);M]$. For a proof and further discussion, see Berndt and Christensen [8].

⁷Halvorsen and Ford [37] recently used translog cost functions to test for separability of the energy aggregate for each of eight individual two-digit industries in the U.S. They found separability to hold for four of the eight countries.

⁸The second and third assumptions together are referred to as homothetic separability

then optimally choose quantities of capital, labor, and energy.⁹ Equivalently we can express these three assumptions by writing the production function as:

$$Q = [f(K, L, e(F_1, F_2, F_3, F_4)); M]. \quad (1)$$

where e is a homothetic function of the four fuels.

If the factor prices and output level are exogenously determined, the production structure described by (1) can alternatively be described by a cost function that is also weakly separable, i.e. a function of the form:¹⁰

$$C = G[g(P_K, P_L, P_E(P_{F1}, P_{F2}, P_{F3}, P_{F4})Q); P_M, Q]. \quad (2)$$

Here P_E is an aggregate price index of energy, i.e. a function that aggregates the fuel prices P_{F1} . This "aggregator function" is homothetic, and thus does not include the total quantity of energy as one of its arguments.

2.2 Use of the Translog Cost Function

Our approach here is similar to that used recently by Fuss and Waverman [31] in estimating the demand for energy in Canadian manufacturing. We first represent the price of energy (which is the unit cost of energy to a producer choosing fuel inputs) by a homothetic translog cost function with constant returns to scale. Estimation of the share equations implied by this cost function gives us the own and cross partial price elasticities for the four fuels. In addition, the cost function itself provides an instrumental variable for the price of energy. The second step is to represent the cost of industrial output by a non-homothetic translog cost function. Estimation of the share equations implied by this cost function gives us the elasticities of substitution and the own and cross price elasticities for capital, labor, and energy.

⁹See Denny and Fuss [26].

¹⁰This was shown by Shephard [54].

It is important to point out that we could have chosen to use translog production functions rather than cost functions in estimating elasticities. Since the translog cost and production functions are not self-dual, different elasticity estimates would result, and as Burgess [11] has recently shown, the difference could be significant. However, we choose to use the cost function since it is more appropriate to take prices as exogenous than quantities.

We begin by reviewing the properties of the translog cost function.¹¹ The translog cost function is a second-order approximation to an arbitrary cost function, and has the form

$$\log C = \alpha_0 + \alpha_Q \log Q + \sum_i \alpha_i \log P_i + \frac{1}{2} \gamma_{QQ} (\log Q)^2 + \frac{1}{2} \sum_{ij} \gamma_{ij} \log P_i \log P_j + \sum_i \gamma_{Qi} \log Q \log P_i \quad (3)$$

where C is total cost, Q is output, and P_i are the factor prices. From Shephard's Lemma [54], the derived demand functions are found by differentiating the cost function with respect to the prices, i.e. $X_i = \partial C / \partial P_i$. Thus the share equations are given by $S_i = \partial \log C / \partial \log P_i = (P_i X_i) / C$, or

$$S_i = \alpha_i + \gamma_{Qi} \log Q + \sum_j \gamma_{ij} \log P_j, \quad i=1, \dots, n. \quad (4)$$

Since the shares must add to 1, only $n-1$ of the share equations need be estimated.

Note, however, that the parameters α_0 , α_Q , and γ_{QQ} are not identified unless the cost function itself is estimated.¹²

¹¹The translog production function and cost function were introduced by Christensen, Jorgenson, and Lau [21]. Applications of the translog cost function can be found in the work of Berndt and Christensen [7], Berndt and Wood [9], Christensen and Greene [34], Fuss and Waverman [31], Fuss [30], Griffen and Gregory [34], Halvorsen [36], and Hudson and Jorgenson [38]. The translog production function has been used by Humphrey and Moroney [39] and Moroney and Toevs [47,48].

¹²Corbo and Meller [23] recently estimated the translog production function directly (instead of the derived share of equations) using capital and labor input data individual firms. This allowed them to test whether the underlying production function is really translog, and to test for competitive behavior.

The cost function must be homogenous of degree 1 in prices, and must satisfy the conditions corresponding to a well-behaved production function. This implies the following parameter restrictions that must be imposed:¹³

$$\sum_1 \alpha_1 = 1 \quad (5)$$

$$\sum_1 \gamma_{Q1} = 0 \quad (6)$$

$$\gamma_{ij} = \gamma_{ji}, i \neq j \quad (7)$$

$$\sum_1 \gamma_{ij} = \sum_j \gamma_{ij} = 0. \quad (8)$$

Note that the cost function as specified so far is non-homothetic, and may have non-constant returns to scale.¹⁴ The cost function would be homothetic if it could be written as a separable function of output and factor prices. Thus the following parameter restrictions can be added to impose homotheticity:

$$\gamma_{Q1} = 0. \quad (9)$$

The cost function is also homogenous if the elasticity of cost with respect to output ($\partial \log C / \partial \log Q$) is constant. This implies the additional restriction:

$$\gamma_{QQ} = 0. \quad (10)$$

Finally, we could impose the restriction that the elasticities of substitution between all factors are equal to 1 (so that the cost function corresponds to a Cobb-Douglas production function). This implies the additional parameter restrictions:

$$\gamma_{ij} = 0. \quad (11)$$

¹³See Christensen, Jorgenson, and Lau [21].

¹⁴Christensen and Greene [19] define the following index of scale economies: $SCE = 1 - \partial \log C / \partial \log Q = 1 - (\alpha_Q + \gamma_{QQ} \log Q + \sum_1 \gamma_{Q1} \log P_1)$. Note that if SCE is positive (negative), there is increasing (decreasing) returns to scale. This is a useful index, and has a natural interpretation in percentage terms. However, it can only be computed if α_Q and γ_{QQ} are known (which means estimating the cost function) or are assumed to be 1 and 0 respectively.

Rather than impose these restrictions a priori, we can test them using a simple chi-square test. The appropriate test statistic is

$$-2 \log \Lambda = N(\log |\hat{\Omega}_r| - \log |\hat{\Omega}_u|) \quad (12)$$

where $|\hat{\Omega}_r|$ and $|\hat{\Omega}_u|$ are the determinants of the estimated error covariance matrices for the restricted and unrestricted models respectively, and N is the number of observations. This statistic is distributed as chi-square with degrees of freedom equal to the number of parameter restrictions being tested.

Uzawa [56] showed that the Allen partial elasticities of substitution¹⁵ can be computed from $\sigma_{ij} = CC_{ij}/C_i C_j$, so that for the translog cost function these are given by

$$\begin{aligned} \sigma_{ij} &= (\gamma_{ij} + S_i S_j) / S_i S_j, \quad i \neq j \\ \sigma_{ii} &= [\gamma_{ii} + S_i (S_i - 1)] / S_i^2 \end{aligned} \quad (13)$$

It is easy to show that the own and cross price elasticities of demand are given by

$$\begin{aligned} \eta_{ii} &= \partial \log X_i / \partial \log P_i = \sigma_{ii} S_i \\ \eta_{ij} &= \partial \log X_i / \partial \log P_j = \sigma_{ij} S_j \end{aligned} \quad (14)$$

Note that these are partial price elasticities; when applied to fuels they account only for substitution between fuels, under the constraint that the total quantity of energy consumed remains constant.¹⁶ The total own price elasticity for each fuel $\eta_{ii}^* = d \log X_i / d \log P_i$ accounts for the effect of a change in the price of a fuel on total energy consumption, and is given by

$$\eta_{ii}^* = \frac{P_i}{X_i} \left[\frac{\partial X_i}{\partial P_i} \right]_{E \text{ const.}} + \frac{\partial X_i}{\partial E} \frac{\partial E}{\partial P_E} \frac{\partial P_E}{\partial P_i} \quad (15)$$

where E is the total quantity of energy consumed, and P_E is the price index for energy. However, since the price of energy is given by the homothetic translog cost function with constant returns to scale:

$$\log P_E = \alpha_0 + \sum_i \alpha_i \log P_i + \sum_{i,j} \gamma_{ij} \log P_i \log P_j, \quad (16)$$

which implies the fuel share equations $S_i = \gamma_i + \sum_j \gamma_{ij} \log P_j$, we have

¹⁵See Allen [3].

¹⁶Note that expenditures on energy will not remain constant.

$$\frac{\partial P_E}{\partial P_1} = \frac{P_E}{P_1} S_1 \quad (17)$$

$$\frac{\partial E}{\partial P_E} = \frac{E}{P_E} \eta_{EE} \quad (18)$$

where η_{EE} is the own price elasticity of energy, and, since the energy cost function is homothetic,

$$\frac{\partial X_1}{\partial E} = \frac{\partial X_1}{\partial M_E} \frac{M_E}{\partial E} = \frac{X_1}{M_E} P_E = \frac{X_1}{E} \quad (19)$$

where $M_E = P_E \cdot E$ is total expenditures on energy. Then, by substituting (17), (18), and (19) into (15), we have

$$\eta_{11}^* = \eta_{11} + \eta_{EE} S_1 \quad (20)$$

Similarly, we can compute the total cross price elasticity η_{ij}^* from

$$\eta_{ij}^* = \frac{P_j}{X_i} \left[\frac{\partial X_i}{\partial P_j} \Big|_{E \text{ const.}} + \frac{\partial X_i}{\partial E} \frac{\partial E}{\partial P_E} \frac{\partial P_E}{\partial P_j} \right] = \eta_{ij} + \eta_{EE} S_j \quad (21)$$

and the total output elasticity from¹⁷

$$\eta_{1Q}^* = \frac{d \log X_1}{d \log Q} = \frac{Q}{X_1} \frac{\partial X_1}{\partial M_E} \frac{\partial M_E}{\partial Q} \quad (22)$$

Since the energy cost function is homothetic, this reduces to

$$\eta_{1Q}^* = \eta_{EQ} \quad (23)$$

where η_{EQ} is the elasticity of energy with respect to output changes. This

¹⁷ This assumes that the value of output is equal to the value (cost) of inputs. This would be the case under perfect competition, or under oligopoly pricing based on a fixed percentage markup over cost. Otherwise this elasticity would be better referred to as a total cost elasticity, i.e. the percent change in the demand for fuel 1 corresponding to a 1 percent change in the total cost of production.

elasticity can in turn be computed as follows:

$$\begin{aligned}
 \eta_{EQ} &= \frac{d \log E}{d \log Q} = \frac{d \log E}{d \log C} \cdot \frac{\partial \log C}{\partial \log Q} \\
 &= \left[\frac{\partial \log S_E}{\partial \log Q} \frac{\partial \log Q}{\partial \log C} + 1 \right] \frac{\partial \log C}{\partial \log Q} \\
 &= \frac{\partial \log S_E}{\partial \log Q} + \frac{\partial \log C}{\partial \log Q}
 \end{aligned} \tag{24}$$

Obtaining these derivatives from equations (3) and (4), we have

$$\eta_{EQ} = \frac{\gamma_{QE}}{S_E} + \alpha_Q + \gamma_{QQ} \log Q + \sum_{i=K}^{L,E} \gamma_{Qi} \log P_i \tag{25}$$

Since the cost function is not estimated directly, we assume that α_Q and γ_{QQ} are 1 and 0 respectively.

We would like to calculate standard errors for our estimates of elasticities, but since the elasticities are nonlinear functions of the estimated parameters (since the shares are themselves functions of the parameters), there is no straightforward way to do this without reverting to Monte Carlo simulation. However, we can obtain approximate estimates of the standard errors. To do this we follow Christensen and Greene [19] and calculate standard errors under the assumption that the shares S_i are constant and equal to the means (over the estimation time bounds) of their estimated values. Under this assumption we have, asymptotically,

$$\begin{aligned}
 \text{Var}(\hat{\sigma}_{ij}) &= \text{Var}(\hat{\gamma}_{ij}) / \hat{S}_i^2 \hat{S}_j^2 \\
 \text{Var}(\hat{\sigma}_{ii}) &= \text{Var}(\hat{\gamma}_{ii}) / \hat{S}_i^4 \\
 \text{Var}(\hat{\eta}_{ij}) &= \text{Var}(\hat{\gamma}_{ij}) / \hat{S}_i^2 \\
 \text{Var}(\hat{\eta}_{ii}) &= \text{Var}(\hat{\gamma}_{ii}) / \hat{S}_i^2
 \end{aligned} \tag{26}$$

Finally, it is useful to calculate the elasticity of the average cost of production with respect to the price of energy, i.e. $\eta_{CE} = \partial \log(AC) / \partial \log P_E$, and the elasticities of the average cost of production with respect to the prices of each fuel, i.e. $\eta_{Ci} = \partial \log(AC) / \partial \log P_i$. This will enable us to calculate the effect of a 1 percent change in the price of energy, or a 1 percent change in the price of a single fuel, on the cost of industrial output. We follow Fuss[30] in calculating point elasticities for η_{CE} and η_{Ci} . From equation (3) we have

$$\eta_{CE} = \alpha_E + \gamma_{EE} \log P_E + \gamma_{EK} \log P_K + \gamma_{EL} \log P_L + \gamma_{QE} \log Q \quad (27)$$

We obtain η_{Ci} from

$$\eta_{Ci} = \frac{\partial \log(AC)}{\partial \log P_E} \frac{\partial \log P_E}{\partial \log P_i} = \eta_{CE} S_i \quad (28)$$

Let us now review the steps involved in estimating a translog model of energy demand. First the fuel share equations

$$S_i = \alpha_i + \sum_j \gamma_{ij} \log P_j \quad (29)$$

are estimated, and the estimated parameters are used to calculate partial price elasticities. These equations are estimated subject to the parameter restrictions $\sum_i \alpha_i = 1$, $\gamma_{ij} = \gamma_{ji}$, and $\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$. We also test the additional restrictions $\gamma_{ij} = 0$. Next the estimated values of the α_i and γ_{ij} are used in equation (16) to obtain an aggregate price index for energy. To do this the parameter α_0 in equation (16) is determined so that the price of energy is equal to 1 in the U.S. in 1970. An energy price index is then calculated for each country.

Next we estimate the factor share equations (4), with i and j equal to capital, labor, and energy. In estimating these equations, we use our estimated aggregate price index for energy as an instrumental variable. We estimate these equations in stages, imposing additional parameter restrictions at each stage, and testing each set of restrictions. In the first stage we impose only the

restrictions implied by neoclassical production theory, i.e. $\sum_i \alpha_i = 1$, $\sum_i \gamma_{Qi} = 0$, $\gamma_{ij} = \gamma_{ji}$, and $\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$. Next we add the homotheticity restrictions $\gamma_{Q1} = 0$. Finally we test the restrictions that $\gamma_{ij} = 0$, i.e. that the elasticities of substitution between all three factors are equal to 1.¹⁸

2.3 Dynamic Versions of the Translog Cost Function

A problem with the translog cost functions described above is that they do not describe differences between short-run and long-run elasticities, or how the adjustment to the long-run takes place. It is reasonable, for example, to expect that in the long-run the aggregate production function exhibits constant returns to scale, but that it exhibits non-constant returns in the short-run. Our objective here is to specify a dynamic translog cost function that permits adjustment to constant returns over time.

Constant returns requires that $\alpha_Q = 1$, $\gamma_{QQ} = 0$, and $\gamma_{Q1} = 0$ in equation (3). One way, then, to build in an adjustment to constant returns in the long-run is to make these parameters functions of changes in output or prices. For example, the parameters α_Q and γ_{QQ} could be specified as:

$$\alpha_Q = \exp\left[\theta_1 \sum_{k=1}^K \lambda_1^k (\Delta Q_{t-k})^2\right] \quad (30)$$

$$\text{and } \gamma_{QQ} = \theta_2 \sum_{k=1}^K \lambda_2^k (\Delta Q_{t-k})^2 + \theta_3 \quad (31)$$

Here the parameters θ_1 , θ_2 and θ_3 are estimated. If θ_3 is not equal to zero, then non-constant returns can exist even in the long-run. Thus if long-run constant returns is taken as given a priori, the estimate of θ_3 (i.e. whether it is significantly different from 0) provides a test of the correctness of the specification of the lag distribution. The lag distribution parameters λ_1^k and λ_2^k could be estimated if the data permitted or they could be specified a priori

¹⁸ We will also test restrictions pertaining to the characteristics of inter-country differences in the cost function. This involves the use of dummy variables that permit some of the parameters of the translog cost function to vary across countries. This is discussed later.

(perhaps declining linearly). In either case K might be 3 to 5 years. Of course estimation of the parameters in (30) and (31) requires that the cost function (3) be estimated simultaneously with the share equations (4).

The parameters of γ_{Q1} can also be made to adjust to zero in long-run equilibrium by making them functions of a distributed lag in changes in prices:

$$\gamma_{Q1} = \theta_{Q1} \sum_{k=1}^K \lambda_{Q1}^k (\Delta \log P_{t-k})^2 \quad (32)$$

Thus if the data do not permit the simultaneous estimation of the cost function with the share equations, α_Q and γ_{QQ} could be assumed equal to 1 and 0 respectively, and adjustments could occur through the γ_{Q1} . In this case the production structure would be homothetic in the long-run.

An alternative approach is to introduce the dynamic adjustment directly into the share equations. This can be done by assuming that the shares adjust to a set of desired shares as follows:¹⁹

$$S_{i,t} = S_{i,t-1} + \sum_j \delta_{ij} (S_{j,t}^* - S_{j,t-1}) \quad (33)$$

where $S_{j,t}^*$ is given by equation (4). Adding up requires that the sum of all changes in shares be zero:

$$\sum_i (S_{i,t} - S_{i,t-1}) = 0, \quad (34)$$

so that

$$\sum_{ij} \delta_{ij} (S_{j,t}^* - S_{j,t-1}) = \sum_j (S_{j,t}^* \sum_i \delta_{ij} - S_{j,t-1} \sum_i \delta_{ij}) = 0. \quad (35)$$

Since the $S_{j,t}^*$ and $S_{j,t-1}$ sum to one, this equation implies the necessary condition that all of the columns of the matrix (δ_{ij}) sum to the same arbitrary constant, i.e.

$$\ell' \delta = c \ell' \quad (36)$$

¹⁹ This approach was suggested by L. Waverman and M. Fuss.

where \mathbf{l} is a vector of 1's (ones), δ is a matrix (δ_{ij}) , and c is an arbitrary constant.²⁰ Note that if the number of shares is greater than two, there are alternative constraints on the δ_{ij} that can be imposed to satisfy (36).

2.4 Multinomial Logit Models for Fuel Choice

Multinomial logit models have already been used to study the breakdown of energy consumption into demands for fuels in the residential sectors of the United States and other countries²¹ and in the industrial sector of the United States.²² Although the logit model is not based on assumptions of cost minimization, it has properties that make it appealing for this work. The model is consistent in terms of shares adding to one, and shares respond to price changes in a way that is intuitively appealing; as the share of, say, natural gas becomes small, it requires increasingly large price changes to make it still smaller. Finally the logit model is easy to estimate and permits us to easily introduce alternative dynamic specifications.

We follow our recent work in applying the logit model to the estimation of residential fuel demands [53]. The logit model for four fuels can be written as

$$\frac{Q_1}{Q_T} = \frac{e^{f_1(x\beta)}}{\sum_{j=1}^4 e^{f_j(x\beta)}} \quad (37)$$

where Q_1 is the quantity (in tcals) of fuel 1, $Q_T = \sum Q_1$, and the f_1 are functions of a vector of attributes x and vector of parameters β . Given this model, the relative shares of any two fuels can be represented as

$$\log(Q_1/Q_j) = \log(S_1/S_j) = f_1(x\beta) - f_j(x\beta). \quad (38)$$

Only three equations are estimated, since the parameters of the fourth equation are determined from the adding up constraint.

²⁰ For a discussion of adding up conditions for more general lag structures, see Wall [] and Berndt and Savin [].

²¹ See Baughman and Joskow [], Fuss and Waverman [31], and Pindyck [53].

²² See Joskow and Baughman [42].

In estimating fuel shares we include as attributes the relative price of each fuel. The relative oil price, for example, is the ratio of the real price of oil to the real price of energy, the latter being measured by the energy price index described earlier. We do not include total energy expenditures of industrial output as attributes since we a priori impose homotheticity on the fuel share model. Other attributes, however, can include lagged quantity or share variables that allow shares to adjust dynamically to changes in price. Functional forms for the f_i are somewhat arbitrary, but in the simplest model they might be linear functions of the relative fuel prices $\tilde{P}_i = P_i/P_E$, where P_E is the aggregate price of energy, and output Q :

$$f_i(x\beta) = a_i + b_i \tilde{P}_i + c_i Q \quad (39)$$

This yields the three estimating equations

$$\log(S_i/S_4) = (a_i - a_4) + b_i \tilde{P}_i - b_4 \tilde{P}_4 + (c_i - c_4)Q, \quad i = 1, 2, 3. \quad (40)$$

Note that these three equations must be estimated simultaneously, with b_4 constrained to be the same in each equation.

The simplest means by which the preference functions can be made dynamic (e.g. to account for stock adjustments) is to include the lagged share:

$$f_i(x\beta) = a_i + b_i \tilde{P}_i + c_i S_{i,t-1}. \quad (41)$$

The three estimating equations are then

$$\log(S_i/S_4) = (a_i - a_4) + b_i \tilde{P}_i - b_4 \tilde{P}_4 + c_i S_{i,t-1} - c_4 S_{4,t-1}, \quad i = 1, 2, 3. \quad (42)$$

Note that two lagged shares appear in each equation. The three equations must again be estimated simultaneously, with both b_4 and c_4 constrained to be the same in each equation.

3. Some Methodological Issues in Model Estimation.

There are a number of issues that must be resolved before the model presented in the last section can be estimated. Some of these issues have already been dealt with in some detail in an earlier paper by this author [53]. For example, purchasing power parity indices, rather than official exchange rates, are used to convert data measured in local currency units into U.S. dollars.²³ Secondly, we followed the approach used in our residential energy demand study in measuring energy consumption in "gross" rather than "net" terms.²⁴

There are two other issues, however, that must be treated. The first has to do with the identification of inter-country differences in the structure of production. The second important issue is the choice of estimation method. We deal with these in turn.

3.1 Identifying Inter-country Differences in Production Structure

One of our objectives in estimating energy demand models is to determine the extent to which elasticities vary across countries, and the possible reasons for such variations. To identify regional variations in elasticities, we must specify alternative ways of allowing for regional parameter variation when our models are estimated with pooled data.

At the one extreme, we could assume that the parameters of our models are the same for all countries (the resulting elasticities could still vary across countries since relative prices and total output levels are different in different countries).

At the other extreme we could estimate our models for each country separately;

²³ As before we use a Fisher "ideal" index (a geometric mean of a Laspeyre and Paasche index numbers) as a single index of relative purchasing power. Our purchasing power parities are binary index numbers with the U.S. as base country.

²⁴ That is, we do not adjust fuel quantities or prices by thermal efficiencies of utilization. There are two reasons for this; first, there are no good estimates of thermal efficiencies available; and second, there are other "economic" efficiency measures that could be equally important in affecting consumer demand. For a further discussion of this issue, see Pindyck [], pages 25 to 28.

this would be infeasible, however, due to insufficient data. Instead we use a compromise approach that we followed in our earlier work in residential energy demand [53]. The countries are pooled, but regional dummy variables are introduced that allow a subset of a model's parameters to vary across countries. In the translog models this could be done by assuming that the coefficients α_i of the first-order terms in the Taylor series approximation can vary across countries, while the coefficients γ_{Q1} and γ_{ij} are the same for each country. This would mean estimating the following share equations:

$$S_i = \sum_k \alpha_{ik} D_k + \gamma_{Q1} \log Q + \sum_j \gamma_{ij} \log P_j \quad (43)$$

where D_k are country dummy variables ($D_k = 1$ for country k and 0 otherwise).

Note that the usual restrictions on the γ_{ij} and the γ_{Q1} apply, but $\sum_i \alpha_{ik} = 1$ for each country k . Note that an advantage of this method is that it partially deals with heteroscedasticity of the error terms within each equation; it is essentially the covariance method for estimation with pooled data.

Alternatively, we could assume that the coefficients γ_{ij} or γ_{Q1} of the second-order terms can vary across countries, while the α_i 's are the same for each country. For example, we could estimate share equations of the form

$$S_i = \alpha_i + \gamma_{Q1} \log Q + \sum_{kj} \gamma_{ijk} D_k \log P_j \quad (44)$$

Note that the restrictions on the γ_{ijk} 's are now that $\gamma_{ijk} = \gamma_{jik}$ for each country k , and that $\sum_i \gamma_{ijk} = \sum_j \gamma_{ijk} = 0$ for every country k .

There is no a priori reason for preferring either specification (other than the econometric convenience of the first specification). However, for either specification, the null hypothesis that the corresponding coefficients are the same across countries can be tested using the straightforward chi-square test.

3.2 Estimation Methods.

The choice of estimation methods involves a trade-off between the richness of the stochastic specification (and hopefully a resulting gain in efficiency) and computational expense. This trade-off is particularly severe given that all of our models involve systems with equations. Ideally one would like to estimate a stochastic specification for which the error terms are heteroscedastic and autocorrelated both across time and across countries within each equation, and are correlated across equations in the system. Estimating such a specification (which amounts to full generalized least squares) however, would be unreasonably costly given the computer software available to us. We must therefore settle for a more restrictive stochastic specification that would still capture the more important characteristics of the error terms.

When estimating our models we ignore error term autocorrelation within equations, but account for error correlations across equations. In particular, we use iterative Zellner estimation which (under the assumption of no heteroscedasticity or autocorrelation within equations) is equivalent to full-information maximum-likelihood estimation. However, we limit the number of iterations on the error covariance matrix to five; this reduces computational expense while still capturing at least 90% of the added efficiency that results from accounting for cross-equation error correlations.

We will attempt to account for within-equation heteroscedasticity (at least as far as we can given the constraints on our computer budget). This

is done using the following procedure. First each equation in the system is estimated using ordinary least squares. The resulting regression residuals, which we can label u_{kt} , are then used to obtain consistent estimates of the regional (country) error variances σ_k^2 :

$$\hat{\sigma}_k^2 = \frac{1}{T-m-1} \sum_t (u_{kt})^2 \quad (45)$$

where T is the number of annual observations for country k and m is the number of independent variables in the equation. Different estimates of these error variances will of course be obtained for each equation in the system. We then transform the data by dividing each observation by the appropriate estimated error term standard deviation $\hat{\sigma}_k$, and then re-estimate the entire system of equations using iterative Zellner estimation. At this point, new estimates of the regional error variances can be computed, again using equation 43. Iterative Zellner estimation can then be repeated. Ideally this process should be iterated until convergence occurs; because of computational expense, however, we limit the process to one iteration.

Our estimation work has been carried out at the computer research center of the National Bureau of Economic Research, using the GREMLIN experimental non-linear estimation package on the TROLL Econometric software system. Some work was also done using the new version of TSP at M.I.T.'s Information Processing Center.

4. Characteristics of the Data

Estimation of the models described in Section 2 requires data for capital, labor, and energy price indices and expenditure shares of manufacturing output, and for the prices and quantities of petroleum, natural gas, coal and electricity used in the industrial sector. In some cases data was available from standard sources such as the UN Statistical Office or OECD publications, in other cases

we relied on the data collection efforts of the International Studies Division of the Federal Energy Administration. Finally, in some cases it was necessary to turn to the national statistical yearbook of individual countries.

Ten countries are included in our sample: Canada, France, Italy, Japan, The Netherlands, Norway, Sweden, U.K., U.S.A. and West Germany. The data collected for these countries are described briefly below.²⁵

Expenditures on Labor: Expenditures on labor include wages and salaries plus supplements paid to the manufacturing sector. For some countries, Canada, Italy, The Netherlands, Norway and West Germany, this was available from the United Nations' Growth of World Industry. For other countries, where the UN publication lacked data on supplements for all years, it was necessary to extend supplements by using the national percentages of supplements indicated by GWI, UN National Accounts or the International Labor Organization's Statistical Yearbook. For Sweden and the United Kingdom it was necessary to determine what percentage of total national compensation went to manufacturing, using data from UN National Accounts and ILO Statistics. Finally, for the U.S., Japan and France, national statistical yearbooks were used. Data is in local currency units and is converted to U.S. dollars using the purchasing power parity numbers for GDP.

Price of Labor: The price of labor was determined implicitly by dividing labor expenditures by total manhours of employees. Manhours of employees was calculated for 1967 by multiplying manhours of operatives by the ratio

²⁵The data used here are part of a larger international energy data base assembled for use in this and several related studies. For a more detailed description of that data base, see "A User's Guide to the M.I.T. World Energy Demand Data Base" (M.I.T. Energy Laboratory Working Paper). Other researchers wishing to replicate or extend this study or perform studies of their own can access the data directly through the TROLL computer system of the NBER.

of numbers of employees to number of operatives, for Canada, Italy, Japan, Norway, Sweden, U.S.A. and W. Germany. Data is from the UN Growth of World Industry. Where GWI did not have information, manhours were calculated from UN data on number of employees and ILO data on average working hours. Then, for every country except Norway, a wage index (1967 = 100) from the U.S. Bureau of Labor Statistics, which includes wages and supplements, was used to convert our price/hour for 1967 to a time series, 1955 to 1974. The time series for Norway was available directly from Growth of World Industry. (Note that the resulting index is not quality adjusted.)

Price of Capital Services: We compute a capital service price index separately for non-residential structures (P_{NR}) and producers durables (P_D), and aggregate these two series into a final price of capital services using a Divisia index, where the investment shares of non-residential structures and durables serve as the Divisia weights. The computation of the price of capital services of each component is based on Christensen and Jorgenson [20], i.e. we assume that the investment price of an asset q is equal to the present value of its future services evaluated at the service price P (which is the price we wish to ascertain).²⁶ We also assume that the service from an asset declines geometrically over time. Then, disregarding taxes, the asset price is related to the service price by

$$q_t = \sum_{j=t}^{\infty} [(1-d)^{j-t} P_{j+1} \prod_{s=t+1}^{j+1} \frac{1}{1+r_s}] \quad (46)$$

where d is the depreciation rate and r is the appropriate interest rate. From this we can obtain the equations that relate the price index for each type of capital service to the corresponding asset price index:

²⁶See also Hall and Jorgenson [35] and Coen [22].

$$P_{NR}(t) = R(t)q_{NR}(t-1) + d_{NR}q_{NR}(t) - (q_{NR}(t) - q_{NR}(t-1)) \quad (47)$$

$$P_D(t) = R(t)q_D(t-1) + d_Dq_D(t) - (q_D(t) - q_D(t-1)) \quad (48)$$

Here R is a long-term government bond interest rate (source: International Finance Statistics of the IMF), and q_{NR} and q_D are the asset price indices for non-residential structures and durables.²⁷

For some countries (Canada, France, Italy, The Netherlands, U.K. and U.S.) asset price indices and depreciation rates were obtained from Christensen et al. [12,13,14,15,16,17,18]. For the remaining countries it was necessary to compute implicit asset price indices from gross fixed capital formation in current and constant units using national statistical yearbooks, the U.N. or OECD National Accounts. Remaining depreciation rates were obtained from life of capital figures in Denison [25], or implicit rates from OECD National Accounts were used. Asset price indices were deflated and then converted into indices relative to the U.S. using the appropriate purchasing parity indices. Data on the investment shares (gross fixed capital formation for producer durables and non-residential structures) used to compute the Divisia index were obtained from national statistical yearbooks, or UN or OECD National Accounts. Note that this method of computing the price of capital does not take into account differences in corporate tax structures across countries; we simply did not have access to the data needed to take taxes into account. This means, however, that our price index for capital services must be viewed as approximate.

Expenditures on Capital Services: Expenditures on capital services were determined by subtracting labor expenditures from value added. Data on value added at factor cost was obtained from the United Nations' Growth of World Industry or Annual Yearbook. Value added for France and Germany was

²⁷For West Germany the discount rate was used as the interest rate, since the government bond yield was unavailable.

available only at producer costs, and value added tax data obtained from the EEC Tax Yearbook was used to arrive at a factor cost figure. All of this data is measured in local currency, was deflated using the local GDP price deflator, and converted to U.S. dollars using the purchasing power parity for GDP. Note that this does not include depreciation. Since the concept of depreciation varies between countries and comparable data is not available, the gross figures are used.

Fuel Quantities: Quantities of fuels used in the industrial sector (excluding energy conversion) are all obtained from OECD energy publications. Two different publications were used, Energy Balances of OECD Countries: 1960-1974, Paris, 1976, and Energy Statistics of OECD Countries. The 1976 publication is used for 1960-1974 since it contains the most recent and revised data and clearly excludes chemical feedstocks. These data series are related to those in the earlier OECD publications via simple linear regressions, together with the earlier data, are then used to extrapolate our 1960-1974 series back to 1955. The U.S. was treated differently from other countries in that the 1976 publication showed a large amount of "crude and NGL" consumed by industry. Investigations into other publications and consultations with the Paris office of the OECD and the International Studies Division of the FEA have led us to conclude that this category probably erroneously contains some petroleum products used for petrochemical feedstocks, non-petroleum hydrocarbons and other refinery gas. To keep our accounting consistent with other countries, this category was not included in our petroleum total.

Fuel Prices: Industrial price of heavy fuel oil, natural gas, coal and electricity were obtained from EEC publications and the OECD statistical office. These data are measured in local currency units, and converted to U.S. dollars

using the appropriate purchasing power parities. Final units are U.S. dollars/tcal.

Purchasing Power Parities: Purchasing power parities for gross domestic product, producers durables, and non-residential structures were obtained from Gilbert and Kravis [58], Gilbert et al. [59], and Kravis et al. [60], and are all bilateral indices with the U.S. base country.

Our basic models require cost shares and price indices for capital, labor and energy, expenditure shares and prices for the four fuels, and the value of output. The available range of our data is shown in Table 1. Note that for France and the U.K., factor share data is not available for some of the early years. It is useful to examine some of the share and price data before turning to the estimation results. Data for 1962 and 1970 are shown in Table 2. (Note that the energy price index is computed from a "preferred" translog fuel share model; this model is presented and discussed in the next section.) We see from this table that there is considerable variation in fuel expenditure shares across countries, and through time in any one country. Fuel prices also vary considerably across countries, and have generally decreased over time. Factor shares and prices show much more variation across countries than across time, so that our capital, labor and energy elasticity estimates should probably be viewed as long-term.

5. Statistical Results

In this section we present the results of the estimation of the models set forth in Section 2. We have estimated static translog models of fuel shares and factor shares, and static and dynamic logit models of fuel shares. At this point we have not yet estimated dynamic versions of the translog model. We begin with the two stages of the translog model - first the fuel share model, which in turn is used to generate a price index for energy, and then the factor share model.

Table 1 - Range of Data

PRICE	CANADA	FRANCE	ITALY	JAPAN	NETH.	NORWAY	SWEDEN	U.K.	U.S.A.	W. GER
Fuel Price	Sol. Fuel	55-74	55-74	55-75	55-74	55-74	55-74	55-74	54-7, 59-74	55-74
	Liq. Fuel	55-74	55-75	55-74	55-74	55-75	55-74	55-74	54-7, 59-74	55-74
	Gas	55-74	55-74	55-75	55-75	55-74	55-75	55-74	54-7, 59-74	55-75
	Elec.	55-74	55-74	55-74	55-74	55-75	55-74	55-74	54-7, 59-74	55-74
Fuel Consumption	Sol. Fuel	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
	Liq. Fuel	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
	Gas	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
	Elec.	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
Exp. of Fuels	Sol. Fuel	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
	Liq. Fuel	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
	Gas	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
	Elec.	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74	55-74
Other	Capital Price	55-73	55-73	55-73	55-73	56-73	58-73	56-73	55-73	58-73
	Labor Expen.	58-74	60-74	58-74	58-74	58-74	60-74	60-74	58-73	60-73
	Labor Price	60-74	60-73	60-73	60-73	58-73	60-74	60-74	60-74	60-74
	Value Added	58-73	58-73	58-73	58-72	53, 58-73	53, 58-73	58, 63-73	58-73	58-73

Table 2 - Share and Price Data

Fuel Expenditure Shares					Fuel Price - 1970, US \$ per ton				Price Index of Energy 1970 US=1	
	Solid Fuel	Liq. Fuel	Gas	Elec.	Solid Fuel	Liq. Fuel	Gas	Elec.		
Canada	1962	.14	.23	.07	.57	1950.77	3693.84	1658.82	9472	1.1
	1970	.09	.26	.12	.52	1919.33	3187.48	1498.22	8058	.94
France	1962	.34	.14	.10	.42	3986.67	3738.77	11107.4	21816.3	1.75
	1970	.23	.19	.06	.52	3953.2	1835.1	3982.76	20218.9	1.73
Italy	1962	.13	.22	.08	.56	6123.38	4476.98	3638.33	33163.1	2.9
	1970	.12	.23	.09	.56	5846.13	2711.22	2898.58	23147.9	2.04
Japan	1962	.19	.15	.03	.57	4317.05	4102.77	2980.97	30592.1	2.72
	1970	.13	.20	.03	.65	3065.6	2696.5	1832.62	24639.3	2.21
Netherlands	1962	.14	.18	.001	.67	4268.71	3281.06	8593.59	66391.4	5.49
	1970	.06	.10	.15	.69	3384.83	2194.8	2572.89	37697.1	3.27
Norway	1962	.09	.19	0	.74	2533.29	2922.16	10250.3	6006.92	.95
	1970	.10	.30	0	.61	3479.78	3305.39	16817.3	5318.52	.93
Sweden	1962	.21	.14	.01	.66	4037.07	3364.28	10933.4	26752.4	2.68
	1970	.15	.24	.01	.60	3015.79	2826.3	6540.04	15715.9	1.66
U.K.	1962	.37	.15	.07	.46	3543.11	3774.79	11949.7	29415.	2.16
	1970	.16	.21	.06	.56	2512.06	2581.53	5724.2	24269.2	1.81
USA	1962	.1	.08	.19	.64	1120.63	1749.	1376.74	14254.8	1.29
	1970	.06	.07	.21	.65	1011.43	1541.05	1111.11	11059.3	1.0
W. Germany	1962	.31	.11	.003	.58	4079.97	3360.8	4252.76	33188.8	2.91
	1970	.15	.15	.05	.64	4053.35	2415.6	3201.81	26605.8	2.45

Table 2 - Share and Price Data (Cont.)

		<u>Capital</u>		<u>Labor</u>		<u>Energy</u>		Net Value Output
		Exp. Share	Price	Exp. Share	USA 70=1 Price	Exp. Share	USA 70=1 Price	
Canada	1962	.422	.77	.523	.61	.0545	1.10	166.6
	1970	.39	.999	.56	.75	.05	0.94	226.65
France	1962	na	1.17	na	.3	na	1.75	na
	1970	.45	1.22	.50	.4	.05	1.73	499.69
Italy	1962	.47	1.39	.45	.33	.08	2.9	234.96
	1970	.363	1.15	.564	.53	.072	2.04	333.24
Japan	1962	.54	1.47	.38	.21	.08	2.72	524.19
	1970	.57	1.49	.37	.41	.06	2.21	1232.52
Netherlands	1962	.37	.78	.56	.36	.07	5.5	92.23
	1970	.404	.82	.533	.59	.063	3.27	134.03
Norway	1962	.45	.78	.5	.45	.05	.95	28.39
	1970	.41	1.07	.52	.62	.064	.93	37.64
Sweden	1962	.33	1.35	.57	.57	.1	2.68	78.45
	1970	.35	1.45	.584	.87	.064	1.66	119.01
U.K.	1962	na	1.1	na	.33	na	2.16	na
	1970	.35	1.06	.6	.42	.05	1.81	622.02
USA	1962	.434	1.1	.533	.86	.03	1.29	2550.72
	1970	.46	1.0	.51	1.0	.03	1.0	3291.5
W. Germany	1962	.62	1.26	.33	.33	.05	2.91	673.61
	1970	.553	1.06	.403	.5	.044	2.45	921.7

From these we determine partial and total elasticities for fuel demand, and elasticities of substitution and demand for capital, labor and energy. Finally, we describe the results (which were less successful) of estimating the logit models.

5.1 Fuel Share Model

In estimating the fuel share equations (29) a number of choices must be made regarding the pooling of data and the choice of time bounds. First, regional dummy variables can be used to allow the intercept terms of the share equations to vary across countries as in equation (43), or to allow the γ_{ij} parameters to vary across countries as in equation (44). We found the latter alternative to involve a considerable reduction in degrees of freedom, and the resulting elasticities had large standard errors, so in our "preferred" model we use regional dummy variables for the intercept terms. We also consider using no dummy variables at all, and we use the standard chi-square test to determine the need for intercept dummies. Second, since fuel prices in Canada and the U.S. have been significantly lower than in the other eight countries in our sample, this might have resulted in a production structure different enough to suggest pooling these countries separately. We therefore estimate models in which all ten countries are pooled together, and in which Canada and the U.S. are pooled separately. Third, although our data span the period 1959-1974, there is a question as to whether the 1974 data should be considered to have come from the same population as the 1959-1973 data, i.e. whether the 1974 data point lies on the same long-run cost function. We therefore estimate models both including and excluding the 1974 data. Finally, it is useful to test whether we are indeed estimating a long-run cost function. To do this we estimate one of the models using data at three-year intervals, and compare it to the same model estimated with annual data. If the resulting estimates are nearly the same, we can conclude that we have estimated a long-run cost function.

Estimated parameters for the various versions of the model are shown in Table 3. (Standard errors are in parentheses.) In the first version all ten countries are pooled and the 1959-1973 data are used, but the model is restricted in that no intercept dummy variables are included. We can test this restriction by comparing the model to the equivalent unrestricted model of column 6. The value of the chi-square statistic is 628, and given that there are nine parameter restrictions, this is well above the critical 1% level of 27.8. We therefore include intercept dummy variables in all other versions of the model. (A model is also estimated using regional dummy variables for the second-order terms, but we do not report the estimated parameters here. The resulting own price elasticities, however, are shown in Table 6, and as can be seen from that table, many of the elasticities are statistically insignificant.)

In columns 2, 3, 4 and 5 Canada and the U.S. are pooled separately, and in columns 6 and 7 they are pooled together with the European countries and Japan. Also, the effects of including the 1974 data can be seen by comparing columns 2 and 3, columns 4 and 5, and columns 6 and 7. Note that in all cases the β_{ij} estimates change considerably when the additional data is added, leading us to believe that it should not be included with the 1959-1973 sample. Also note by comparing columns 2, 4 and 6 that the β_{ij} parameter estimates for Canada and the U.S. are quite different from those for Europe when these countries are pooled separately. In addition, the parameter estimates for Canada and the U.S. are still statistically significant when the pooling is separate. This is an indication that it is probably preferable to pool Canada and the U.S. separately.

By comparing columns 8 to column 4 we see that using data at three-year intervals results in little change in the estimated parameters. The resulting price elasticities also do not change much (compare Tables 4 and 7), so that we conclude that we are indeed estimating long-run elasticities.

Table 3 - Parameter Estimates: Fuel Choice Models

(1 = solid, 2 = liquid, 3 = gas, 4 = electricity)

	1 10 countries, 1959-1973, country dummy variables.	2 U.S. & Canada 1959-1973, country dummy variables.	3 U.S. & Canada 1959-1974, country dummy variables.	4 Europe & Japan, 1959-1973, country dummy variables.	5 Europe & Japan 1959-1974, country dummy variables.	6 10 countries, 1959-1973, country dummy variables.	7 10 countries 1959-1974 country dummy variables.	8 Europe & Ja 1961-73 with yr. interv country dummies variables
α_1	0.1038 (0.0242)							
α_1^D		0.2335 (0.0218)	0.2092 (0.0186)	0.2438 (0.0268)	0.2560 (0.0243)	0.1836 (0.0262)	0.2028 (0.0240)	0.2472 (0.0332)
α_1^D		0.2555 (0.0382)	0.1950 (0.0331)	0.2379 (0.0231)	0.2503 (0.0219)	0.2763 (0.0242)	0.2890 (0.0220)	0.2524 (0.0277)
α_1^D				0.2571 (0.0314)	0.2728 (0.0287)	0.2780 (0.0220)	0.2950 (0.0204)	0.2907 (0.0376)
α_1^D				0.1486 (0.0368)	0.1686 (0.0330)	0.2959 (0.0280)	0.3167 (0.0261)	0.1702 (0.0417)
α_1^D				-0.0613 (0.0213)	-0.0504 (0.0197)	0.1991 (0.0332)	0.2226 (0.0303)	-0.0640 (0.0293)
α_1^D				0.1221 (0.0280)	0.1418 (0.0251)	-0.0575 (0.0188)	-0.0518 (0.0174)	0.1291 (0.0348)
α_1^D				0.1768 (0.0346)	0.1961 (0.0316)	0.1504 (0.0249)	0.1688 (0.0223)	0.1879 (0.0413)
α_1^D				0.2654 (0.0286)	0.2963 (0.0259)	0.2088 (0.0303)	0.2282 (0.0278)	0.2381 (0.0331)
α_1^D						0.1531 (0.0358)	0.1815 (0.0330)	
α_2	0.3045 (0.0136)					0.3289 (0.0263)	0.3422 (0.0242)	
α_2^D		0.2099 (0.0162)	0.2385 (0.0132)	0.3807 (0.0228)	0.4107 (0.0204)	0.3082 (0.0209)	0.3143 (0.0188)	0.4046 (0.0337)
α_2^D		0.0501 (0.0305)	0.1265 (0.0253)	0.4052 (0.0212)	0.4371 (0.0188)	0.3477 (0.0213)	0.3761 (0.0186)	0.4408 (0.0278)
α_2^D				0.3573 (0.0255)	0.3843 (0.0228)	0.3636 (0.0207)	0.3895 (0.0180)	0.3900 (0.0341)
α_2^D				0.4137 (0.0301)	0.4456 (0.0262)	0.3240 (0.0234)	0.3433 (0.0205)	0.4388 (0.0375)
α_2^D				0.3317 (0.0178)	0.3422 (0.0166)	0.3657 (0.0284)	0.3918 (0.0241)	0.3287 (0.0308)
α_2^D				0.3839 (0.0226)	0.4113 (0.0199)	0.3277 (0.0158)	0.3438 (0.0146)	0.4000 (0.0319)
α_2^D				0.0037 (0.0000)	0.4311 (0.0000)	0.3567 (0.0000)	0.3443 (0.0000)	0.4012 (0.0000)

Table 3 (Cont.) - Parameter Estimates: Fuel Choice Models

Para.	1	2	3	4	5	6	7	8
$\alpha_2 D_8$				0.3513 (0.0244)	0.3858 (0.0214)	0.3757 (0.0238)	0.4013 (0.0208)	0.3891 (0.0312)
$\alpha_2 D_9$						0.2292 (0.0274)	0.2425 (0.0240)	
$\alpha_2 D_{10}$						0.3096 (0.0233)	0.3390 (0.0200)	
α_3	0.0544 (0.0122)							
$\alpha_3 D_1$		0.2997 (0.0220)	0.2944 (0.0210)	0.1934 (0.0147)	0.1901 (0.0141)	0.2066 (0.0132)	0.2069 (0.0126)	0.2237 (0.0247)
$\alpha_3 D_2$		0.4705 (0.0325)	0.4680 (0.0298)	0.1466 (0.0125)	0.1432 (0.0120)	0.1991 (0.0134)	0.1933 (0.0132)	0.1554 (0.0186)
$\alpha_3 D_3$				0.1638 (0.0153)	0.1640 (0.0146)	0.1555 (0.0129)	0.1484 (0.0126)	0.1733 (0.0223)
$\alpha_3 D_4$				0.2311 (0.0179)	0.2366 (0.0167)	0.1771 (0.0151)	0.1758 (0.0148)	0.2749 (0.0256)
$\alpha_3 D_5$				0.0888 (0.0140)	0.0843 (0.0133)	0.2438 (0.0176)	0.2462 (0.0169)	0.1056 (0.0257)
$\alpha_3 D_6$				0.1362 (0.0151)	0.1301 (0.0141)	0.0872 (0.0128)	0.0826 (0.0121)	0.1611 (0.0243)
$\alpha_3 D_7$				0.2362 (0.0171)	0.2336 (0.0161)	0.1418 (0.0140)	0.1341 (0.0131)	0.2684 (0.0272)
$\alpha_3 D_8$				0.1356 (0.0145)	0.1371 (0.0138)	0.2445 (0.0160)	0.2411 (0.0151)	0.1585 (0.0211)
$\alpha_3 D_9$						0.3756 (0.0172)	0.3732 (0.0164)	
$\alpha_3 D_{10}$						0.1458 (0.0144)	0.1443 (0.0140)	
α_4	0.5374 (0.0222)							
$\alpha_4 D_1$		0.2569 (0.0233)	0.2579 (0.0191)	0.1821 (0.0223)	0.1432 (0.0183)	0.3015 (0.0196)	0.2760 (0.0174)	0.1245 (0.0254)
$\alpha_4 D_2$		0.2238 (0.0386)	0.2105 (0.0300)	0.2104 (0.0226)	0.1694 (0.0184)	0.1769 (0.0220)	0.1416 (0.0180)	0.1514 (0.0258)
$\alpha_4 D_3$				0.2218 (0.0271)	0.1789 (0.0229)	0.2029 (0.0229)	0.1671 (0.0190)	0.1460 (0.0319)
$\alpha_4 D_4$				0.2067 (0.0334)	0.1492 (0.0273)	0.2030 (0.0270)	0.1642 (0.0233)	0.1161 (0.0374)
$\alpha_4 D_5$				0.6408 (0.0116)	0.6239 (0.0101)	0.1915 (0.0332)	0.1395 (0.0275)	0.6297 (0.0136)
$\alpha_4 D_6$				0.3577 (0.0116)	0.3168 (0.0176)	0.6426 (0.0110)	0.6254 (0.0097)	0.3093 (0.0244)
$\alpha_4 D_7$				0.1833	0.1392	0.3511	0.3123	0.1225

Table 3 (Cont.) - Parameter Estimates: Fuel Choice Models

Para.	1	2	3	4	5	6	7	8
$\alpha_4 D_8$	-0.0723 (0.0196)			0.2268 (0.0266)	0.1808 (0.0219)	0.1710 (0.0258)	0.1294 (0.0216)	0.1643 (0.0302)
$\alpha_4 D_9$	0.0195 (0.0109)					0.2421 (0.0301)	0.2027 (0.0260)	
$\alpha_4 D_{10}$	0.0252 (0.0324)					0.2158 (0.0267)	0.1745 (0.0221)	
β_{11}	0.0276 (0.0124)	-0.1017 (0.0406)	-0.0842 (0.0352)	-0.1039 (0.0293)	-0.0826 (0.0272)	-0.1153 (0.0243)	-0.0910 (0.0224)	-0.0792 (0.0384)
β_{12}	0.0195 (0.0109)	0.0739 (0.0256)	0.0215 (0.0211)	0.0093 (0.0210)	-0.0001 (0.0196)	0.0239 (0.0176)	0.0134 (0.0160)	-0.0138 (0.0302)
β_{13}	0.0252 (0.0324)	0.1195 (0.0201)	0.1203 (0.0183)	0.1184 (0.0112)	0.1150 (0.0103)	0.1238 (0.0100)	0.1243 (0.0092)	0.1320 (0.0172)
β_{14}	0.0276 (0.0124)	-0.0917 (0.0164)	-0.0576 (0.0142)	-0.0237 (0.0142)	-0.0323 (0.0128)	-0.0115 (0.0126)	-0.0517 (0.0115)	-0.0391 (0.0173)
β_{21}	0.0195 (0.0109)	0.0739 (0.0256)	0.0215 (0.0211)	0.0093 (0.0210)	-0.0001 (0.0196)	0.0239 (0.0176)	0.0134 (0.0160)	-0.0138 (0.0302)
β_{22}	0.0445 (0.0099)	-0.0152 (0.0233)	0.0801 (0.0216)	0.1063 (0.0210)	0.1282 (0.0194)	0.0762 (0.0182)	0.1090 (0.0161)	0.1321 (0.0336)
β_{23}	0.0024 (0.0057)	-0.0754 (0.0128)	-0.0686 (0.0100)	-0.0240 (0.0105)	-0.0234 (0.0098)	-0.0279 (0.0092)	-0.0318 (0.0084)	-0.0171 (0.0192)
β_{24}	-0.0664 (0.0069)	0.0167 (0.0151)	-0.0330 (0.0135)	-0.0126 (0.0110)	-0.1047 (0.0095)	-0.0763 (0.0101)	-0.0665 (0.0085)	-0.1012 (0.0132)
β_{31}	0.0252 (0.0384)	0.1195 (0.0201)	0.1203 (0.0183)	0.1184 (0.0112)	0.1150 (0.0103)	0.1238 (0.0100)	0.1243 (0.0092)	0.1320 (0.0172)
β_{32}	0.0024 (0.0057)	-0.0754 (0.0128)	-0.0686 (0.0100)	-0.0240 (0.0105)	-0.0234 (0.0098)	-0.0279 (0.0092)	-0.0318 (0.0084)	-0.0171 (0.0192)
β_{33}	-0.0429 (0.0064)	0.0557 (0.0177)	0.0477 (0.0171)	-0.0340 (0.0087)	-0.0316 (0.0081)	-0.0299 (0.0080)	-0.0275 (0.0075)	-0.0451 (0.0153)
β_{34}	0.0153 (0.0061)	-0.0998 (0.0141)	-0.0995 (0.0127)	-0.0604 (0.0063)	-0.0998 (0.0141)	-0.0561 (0.0063)	-0.0650 (0.0060)	-0.0697 (0.0084)
β_{41}	0.0276 (0.0124)	-0.0917 (0.0164)	-0.0576 (0.0142)	-0.0237 (0.0142)	-0.0323 (0.0128)	-0.0415 (0.0126)	-0.0517 (0.0115)	-0.0391 (0.0173)
β_{42}	-0.0664 (0.0069)	0.0167 (0.0151)	-0.0330 (0.0134)	-0.0110 (0.0110)	-0.1047 (0.0095)	-0.0763 (0.0101)	-0.0865 (0.0035)	-0.1012 (0.0132)
β_{43}	0.0153 (0.0061)	-0.0998 (0.0141)	-0.0995 (0.0127)	-0.0604 (0.0063)	-0.0998 (0.0141)	-0.0561 (0.0063)	-0.0650 (0.0060)	-0.0697 (0.0084)
β_{44}	0.0234 (0.0116)	0.1748 (0.0176)	0.1902 (0.0139)	0.1758 (0.0126)	0.1969 (0.0105)	0.1839 (0.0124)	0.2031 (0.0105)	0.2100 (0.0143)
Rsq.								
Eq. 1	.045	.260	.316	.746	.733	.777	.769	.799
Eq. 2	.400	.959	.928	.471	.538	.685	.795	

Estimated elasticities are shown in Tables 4, 5, 6 and 7, (again, with standard errors in parentheses). Table 4 shows partial own and cross price elasticities for Canada and the U.S. pooled separately, for both the 1959-1973 and 1959-1974 time bounds. The same elasticities are shown in Table 5 for all ten countries pooled together. Note that while elasticities for solid fuel and electricity are more or less the same for the four different versions of the model, elasticities for liquid fuel (largely residual fuel oil) and natural gas vary across the four versions. In Table 5 we see that pooling all ten countries results in own price elasticities for liquid fuel (η_{22}) and natural gas (η_{33}) that vary little across countries. We see in Table 4 that pooling Canada and the U.S. separately results in the oil elasticity becoming much larger for these two countries and smaller for the other countries, while the opposite is true for the natural gas elasticity. In addition the own and cross price elasticities for oil are statistically significant for Canada and the U.S. in Table 4, whereas many of them are insignificant in Table 5. This is a further indication that Canada and the U.S. should be pooled separately.

Note in Table 4 that including the 1974 data results in a large change in the price elasticities for oil, and in particular these elasticities become smaller and in many cases insignificant. This is not surprising. Oil prices rose considerably in that year, but demand can adjust to these higher prices only slowly. The 1974 data thus lies on the short-run cost function, and including this data gives us demand elasticities for oil that are somewhere between the short- and long-run. This, however, is of little value given that we are estimating a static cost function. By leaving out the 1974 data we can safely assume that our estimates represent long-run elasticities. (As mentioned before, this assumption is supported by the elasticity estimates of Table 7.) We therefore choose as our "preferred" model version (a) in Table 4 - i.e. Canada and the U.S. pooled separately, and estimation over the 1959-1973 time bounds.

TABLE 4 - PARTIAL FUEL PRICE ELASTICITIES

(US & Canada Estimated Separately, Country Dummy Variables)*

(a) 1959 - 1973

(b) 1959 - 1974

Elast	Version	CAN	FRAN	ITAL	JAP	NETH	NOR	SWED	UK	USA	WGER
η_{11}	(a)	-1.80 (0.36)	-1.04 (0.10)	-1.49 (0.18)	-1.32 (0.15)	-1.67 (0.22)	-2.08 (0.33)	-1.26 (0.13)	-1.12 (0.11)	-2.17 (0.50)	-1.09 (0.10)
	(b)	-1.64 (0.32)	-0.97 (0.09)	-1.36 (0.17)	-1.21 (0.13)	-1.50 (0.21)	-1.84 (0.31)	-1.15 (0.12)	-1.04 (0.10)	-1.95 (0.43)	-0.99 (0.09)
η_{12}	(a)	0.91 (0.23)	0.20 (0.07)	0.27 (0.13)	0.21 (0.11)	0.21 (0.16)	0.37 (0.24)	0.24 (0.10)	0.21 (0.08)	0.99 (0.31)	0.15 (0.07)
	(b)	0.44 (0.19)	0.17 (0.06)	0.21 (0.12)	0.17 (0.10)	0.14 (0.15)	0.27 (0.22)	0.20 (0.09)	0.18 (0.07)	0.34 (0.26)	0.12 (0.07)
η_{13}	(a)	1.17 (0.18)	0.46 (0.04)	0.83 (0.07)	0.65 (0.06)	0.98 (0.09)	1.34 (0.13)	0.55 (0.05)	0.52 (0.04)	1.66 (0.25)	0.43 (0.04)
	(b)	1.17 (0.16)	0.44 (0.03)	0.81 (0.06)	0.64 (0.05)	0.95 (0.08)	1.30 (0.12)	0.53 (0.05)	0.50 (0.04)	1.66 (0.22)	0.42 (0.04)
η_{14}	(a)	-0.28 (0.15)	0.39 (0.05)	0.39 (0.09)	0.45 (0.07)	0.48 (0.11)	0.37 (0.16)	0.46 (0.07)	0.39 (0.05)	-0.48 (0.20)	0.49 (0.05)
	(b)	0.03 (0.13)	0.36 (0.04)	0.34 (0.08)	0.41 (0.06)	0.41 (0.10)	0.28 (0.14)	0.42 (0.06)	0.36 (0.05)	-0.06 (0.17)	0.46 (0.04)
η_{21}	(a)	0.41 (0.10)	0.36 (0.12)	0.20 (0.10)	0.26 (0.13)	0.20 (0.15)	0.12 (0.08)	0.27 (0.10)	0.32 (0.12)	0.97 (0.31)	0.37 (0.18)
	(b)	0.20 (0.08)	0.30 (0.12)	0.16 (0.10)	0.20 (0.12)	0.13 (0.14)	0.09 (0.07)	0.22 (0.10)	0.27 (0.11)	0.34 (0.25)	0.29 (0.17)
η_{22}	(a)	-0.81 (0.09)	-0.20 (0.12)	-0.29 (0.10)	-0.20 (0.13)	-0.11 (0.15)	-0.34 (0.08)	-0.27 (0.10)	-0.22 (0.12)	-1.10 (0.28)	+0.03 (0.18)
	(b)	-0.42 (0.09)	-0.07 (0.11)	-0.18 (0.10)	-0.06 (0.12)	0.04 (0.14)	-0.25 (0.07)	-0.16 (0.10)	-0.09 (0.11)	0.05 (0.25)	0.22 (0.17)
η_{23}	(a)	-0.21 (0.05)	-0.08 (0.06)	-0.03 (0.05)	-0.08 (0.06)	-0.10 (0.07)	-0.09 (0.04)	-0.11 (0.05)	-0.06 (0.06)	-0.72 (0.15)	-0.18 (0.09)
	(b)	-0.18 (0.04)	-0.07 (0.06)	-0.02 (0.05)	-0.03 (0.06)	-0.10 (0.07)	-0.09 (0.03)	-0.11 (0.05)	-0.06 (0.06)	-0.64 (0.12)	-0.18 (0.08)
η_{24}	(a)	0.61 (0.06)	-0.08 (0.07)	0.11 (0.05)	0.02 (0.07)	0.01 (0.08)	0.30 (0.04)	0.12 (0.05)	-0.04 (0.06)	0.85 (0.18)	-0.22 (0.10)
	(b)	0.41 (0.05)	-0.15 (0.06)	0.04 (0.05)	-0.05 (0.06)	-0.08 (0.07)	0.25 (0.04)	0.05 (0.05)	-0.11 (0.05)	0.24 (0.16)	-0.34 (0.03)

* - 1 = solid, 2 = liquid, 3 = gas, 4 = electricity.

** - Almost no natural gas is consumed in the industrial sectors of Norway & Sweden, so that these

TABLE 4 - PARTIAL FUEL PRICE ELASTICITIES (CONT.)
(US & Canada Estimated Separately, Country Dummy Variables)*

(a) 1959 - 1973

(b) 1959 - 1974

Elast	Version	CAN	FRAN	ITAL	JAP	NETH	NOR	SWED	UK	USA	WGER
η_{31}	(a)	1.35 (0.21)	2.21 (0.18)	1.52 (0.13)	2.12 (0.18)	1.84 (0.16)	**	**	1.86 (0.15)	0.72 (0.11)	4.98 (0.44)
	(b)	1.36 (0.19)	2.16 (0.17)	1.48 (0.12)	2.06 (0.17)	1.80 (0.15)	**	**	1.82 (0.14)	0.72 (0.10)	4.84 (0.41)
η_{32}	(a)	-0.53 (0.13)	-0.22 (0.17)	-0.06 (0.12)	-0.22 (0.17)	-0.21 (0.15)	**	**	-0.15 (0.14)	-0.32 (0.07)	-0.83 (0.42)
	(b)	-0.45 (0.10)	-0.21 (0.16)	-0.06 (0.11)	-0.21 (0.16)	-0.20 (0.14)	**	**	-0.14 (0.13)	-0.28 (0.05)	-0.81 (0.39)
η_{33}	(a)	-0.33 (0.18)	-1.49 (0.14)	-1.30 (0.10)	-1.49 (0.14)	-1.42 (0.13)	**	**	-1.38 (0.12)	-0.52 (0.09)	-2.31 (0.34)
	(b)	-0.41 (0.18)	-1.45 (0.13)	-1.28 (0.09)	-1.45 (0.13)	-1.39 (0.12)	**	**	-1.35 (0.11)	-0.55 (0.09)	-2.23 (0.32)
η_{34}	(a)	-0.49 (0.15)	-0.51 (0.10)	-0.15 (0.07)	-0.41 (0.10)	-0.22 (0.09)	**	**	-0.33 (0.09)	0.12 (0.08)	-1.82 (0.25)
	(b)	-0.49 (0.13)	-0.50 (0.09)	-0.15 (0.07)	-0.40 (0.09)	-0.21 (0.08)	**	**	-0.33 (0.08)	0.12 (0.07)	-1.81 (0.23)
η_{41}	(a)	-0.06 (0.03)	0.25 (0.03)	0.12 (0.03)	0.16 (0.02)	0.09 (0.02)	0.05 (0.02)	0.18 (0.02)	0.22 (0.03)	-0.06 (0.03)	0.25 (0.03)
	(b)	0.01 (0.03)	0.23 (0.03)	0.10 (0.02)	0.14 (0.02)	0.08 (0.02)	0.03 (0.02)	0.16 (0.02)	0.20 (0.03)	-0.01 (0.02)	0.23 (0.02)
η_{42}	(a)	0.28 (0.03)	-0.03 (0.02)	0.04 (0.02)	0.01 (0.02)	0.00 (0.02)	0.12 (0.02)	0.04 (0.02)	-0.01 (0.02)	0.11 (0.02)	-0.05 (0.02)
	(b)	0.19 (0.02)	-0.05 (0.02)	0.02 (0.02)	-0.02 (0.02)	-0.02 (0.01)	0.10 (0.01)	0.02 (0.01)	-0.04 (0.02)	0.03 (0.02)	-0.07 (0.02)
η_{43}	(a)	-0.09 (0.03)	-0.07 (0.01)	-0.02 (0.01)	-0.04 (0.01)	-0.02 (0.01)	-0.09 (0.01)	-0.10 (0.01)	-0.05 (0.01)	0.03 (0.02)	-0.08 (0.01)
	(b)	-0.09 (0.02)	-0.07 (0.01)	-0.02 (0.01)	-0.04 (0.01)	-0.02 (0.01)	-0.09 (0.01)	-0.10 (0.01)	-0.05 (0.01)	0.03 (0.02)	-0.08 (0.01)
η_{44}	(a)	-0.14 (0.03)	-0.16 (0.03)	-0.13 (0.02)	-0.12 (0.02)	-0.07 (0.02)	-0.08 (0.02)	-0.12 (0.02)	-0.15 (0.03)	-0.08 (0.03)	-0.12 (0.02)
	(b)	-0.11 (0.03)	-0.11 (0.02)	-0.09 (0.02)	-0.08 (0.02)	-0.04 (0.02)	-0.05 (0.02)	-0.08 (0.02)	-0.11 (0.02)	-0.06 (0.02)	-0.08 (0.02)

* - 1 = solid, 2 = liquid, 3 = gas, 4 = electricity.

** - Almost no natural gas is consumed in the industrial sectors of Norway & Sweden, so that these elasticities are meaningless.

TABLE 5 - PARTIAL FUEL PRICE ELASTICITIES
(10 Countries Pooled, Country Dummy Variables)*

(a) 1959 - 1973

(b) 1959 - 1974

Elas	Version	CAN	FRAN	ITAL	JAP	NETH	NOR	SWED	UK	USA	WGER
η_{11}	(a)	-1.87 (0.22)	-1.07 (0.08)	-1.53 (0.15)	-1.35 (0.12)	-1.71 (0.19)	-2.15 (0.27)	-1.29 (0.11)	-1.14 (0.09)	-2.27 (0.30)	-1.10 (0.08)
	(b)	-1.70 (0.20)	-1.00 (0.07)	-1.41 (0.14)	-1.25 (0.11)	-1.56 (0.17)	-1.93 (0.25)	-1.20 (0.10)	-1.07 (0.08)	-2.04 (0.28)	-1.02 (0.08)
η_{12}	(a)	0.50 (0.16)	0.26 (0.06)	0.39 (0.11)	0.31 (0.09)	0.36 (0.13)	0.58 (0.20)	0.32 (0.08)	0.28 (0.07)	0.43 (0.22)	0.21 (0.06)
	(b)	0.41 (0.14)	0.23 (0.05)	0.33 (0.10)	0.26 (0.08)	0.28 (0.12)	0.47 (0.18)	0.29 (0.07)	0.24 (0.06)	0.31 (0.20)	0.18 (0.06)
η_{13}	(a)	1.20 (0.09)	0.47 (0.03)	0.86 (0.06)	0.68 (0.05)	1.02 (0.08)	1.40 (0.11)	0.58 (0.05)	0.54 (0.04)	1.71 (0.12)	0.45 (0.03)
	(b)	1.21 (0.08)	0.47 (0.03)	0.87 (0.06)	0.68 (0.05)	1.02 (0.07)	1.40 (0.10)	0.58 (0.04)	0.54 (0.03)	1.71 (0.11)	0.45 (0.03)
η_{14}	(a)	0.17 (0.11)	0.33 (0.04)	0.28 (0.08)	0.36 (0.06)	0.34 (0.10)	0.17 (0.14)	0.38 (0.06)	0.33 (0.05)	0.14 (0.15)	0.43 (0.04)
	(b)	0.08 (0.10)	0.30 (0.04)	0.22 (0.07)	0.31 (0.06)	0.26 (0.09)	0.06 (0.13)	0.34 (0.05)	0.29 (0.04)	0.01 (0.14)	0.40 (0.04)
η_{21}	(a)	0.22 (0.07)	0.47 (0.10)	0.29 (0.08)	0.37 (0.11)	0.33 (0.12)	0.19 (0.07)	0.36 (0.09)	0.43 (0.10)	0.42 (0.21)	0.53 (0.15)
	(b)	0.18 (0.06)	0.41 (0.09)	0.25 (0.08)	0.31 (0.10)	0.26 (0.11)	0.15 (0.06)	0.31 (0.08)	0.37 (0.09)	0.30 (0.19)	0.45 (0.14)
η_{22}	(a)	-0.44 (0.07)	-0.38 (0.10)	-0.43 (0.09)	-0.38 (0.11)	-0.32 (0.13)	-0.45 (0.07)	-0.42 (0.09)	-0.39 (0.10)	0.00 (0.21)	-0.23 (0.16)
	(b)	-0.35 (0.06)	-0.24 (0.10)	-0.31 (0.08)	-0.23 (0.10)	-0.15 (0.11)	-0.36 (0.06)	-0.30 (0.08)	-0.26 (0.09)	0.29 (0.19)	-0.02 (0.14)
η_{23}	(a)	-0.02 (0.04)	-0.10 (0.05)	-0.05 (0.04)	-0.11 (0.06)	-0.13 (0.07)	-0.10 (0.03)	-0.13 (0.05)	-0.08 (0.05)	-0.15 (0.11)	-0.22 (0.08)
	(b)	-0.03 (0.03)	-0.13 (0.05)	-0.06 (0.04)	-0.13 (0.05)	-0.15 (0.06)	-0.12 (0.03)	-0.15 (0.04)	-0.11 (0.05)	-0.20 (0.10)	-0.25 (0.07)
η_{24}	(a)	0.24 (0.04)	0.02 (0.06)	0.18 (0.05)	0.11 (0.06)	0.12 (0.07)	0.36 (0.04)	0.19 (0.05)	0.05 (0.06)	-0.27 (0.12)	-0.09 (0.09)
	(b)	0.19 (0.03)	-0.04 (0.05)	0.13 (0.04)	0.05 (0.05)	0.05 (0.06)	0.32 (0.03)	0.14 (0.04)	-0.01 (0.05)	-0.39 (0.10)	-0.18 (0.07)

* - 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

** - see note on Table 4

TABLE 5 - PARTIAL FUEL PRICE ELASTICITIES (CONT.)

(10 Countries Pooled, Country Dummy Variables)*

(a) 1959 - 1973

(b) 1959 - 1974

Elas	Version	CAN	FRAN	ITAL	JAP	NETH	NOR	SWED	UK	USA	WGER
η_{31}	(a)	1.39 (0.10)	2.30 (0.16)	1.58 (0.11)	2.21 (0.16)	1.93 (0.14)	**	**	1.94 (0.13)	0.74 (0.05)	5.19 (0.40)
	(b)	1.40 (0.09)	2.31 (0.15)	1.59 (0.11)	2.22 (0.15)	1.93 (0.13)	**	**	1.94 (0.12)	0.74 (0.05)	5.12 (0.36)
η_{32}	(a)	-0.04 (0.10)	-0.28 (0.15)	-0.11 (0.11)	-0.29 (0.15)	-0.26 (0.13)	**	**	-0.20 (0.12)	-0.07 (0.05)	-0.99 (0.36)
	(b)	0.08 (0.09)	-0.35 (0.13)	-0.15 (0.10)	-0.35 (0.14)	-0.32 (0.12)	**	**	-0.25 (0.11)	-0.09 (0.04)	-1.14 (0.33)
η_{33}	(a)	-1.21 (0.08)	-1.42 (0.13)	-1.26 (0.09)	-1.42 (0.13)	-1.36 (0.12)	**	**	-1.33 (0.11)	-0.97 (0.04)	-2.16 (0.32)
	(b)	-1.19 (0.08)	-1.38 (0.12)	-1.23 (0.09)	-1.38 (0.12)	-1.33 (0.11)	**	**	-1.30 (0.10)	-0.96 (0.04)	-2.06 (0.30)
η_{34}	(a)	-0.14 (0.06)	-0.60 (0.10)	-0.22 (0.07)	-0.50 (0.10)	-0.30 (0.09)	**	**	-0.41 (0.08)	0.30 (0.03)	-2.04 (0.24)
	(b)	-0.13 (0.06)	-0.58 (0.10)	-0.20 (0.07)	-0.48 (0.10)	-0.28 (0.09)	**	**	-0.39 (0.08)	0.30 (0.03)	-2.00 (0.24)
η_{41}	(a)	0.04 (0.02)	0.21 (0.03)	0.08 (0.02)	0.13 (0.02)	0.07 (0.02)	0.02 (0.02)	0.14 (0.02)	0.18 (0.03)	0.02 (0.02)	0.22 (0.02)
	(b)	0.02 (0.02)	0.19 (0.02)	0.06 (0.02)	0.11 (0.02)	0.05 (0.02)	0.01 (0.02)	0.13 (0.02)	0.16 (0.02)	0.00 (0.02)	0.20 (0.02)
η_{42}	(a)	0.11 (0.02)	0.01 (0.02)	0.07 (0.02)	0.03 (0.02)	0.03 (0.02)	0.15 (0.02)	0.07 (0.02)	0.02 (0.02)	-0.03 (0.02)	-0.02 (0.02)
	(b)	0.09 (0.02)	-0.02 (0.02)	0.05 (0.02)	0.02 (0.01)	0.01 (0.01)	0.13 (0.01)	0.05 (0.01)	0.00 (0.02)	-0.05 (0.01)	-0.04 (0.01)
η_{43}	(a)	-0.02 (0.01)	-0.07 (0.01)	-0.03 (0.01)	-0.05 (0.01)	-0.03 (0.01)	-0.10 (0.01)	-0.11 (0.01)	-0.06 (0.01)	0.09 (0.01)	-0.09 (0.01)
	(b)	-0.02 (0.01)	-0.08 (0.01)	-0.03 (0.01)	-0.05 (0.01)	-0.03 (0.01)	-0.10 (0.01)	-0.11 (0.01)	-0.06 (0.01)	0.09 (0.01)	-0.09 (0.01)
η_{44}	(a)	-0.12 (0.02)	-0.14 (0.03)	-0.12 (0.02)	-0.11 (0.02)	-0.06 (0.02)	-0.07 (0.02)	-0.11 (0.02)	-0.14 (0.03)	-0.07 (0.02)	-0.11 (0.02)
	(b)	-0.03 (0.02)	-0.10 (0.02)	-0.08 (0.02)	-0.07 (0.02)	-0.03 (0.02)	-0.04 (0.02)	-0.07 (0.02)	-0.10 (0.02)	-0.04 (0.02)	-0.07 (0.02)

* - 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

** - see note on Table 4

Table 6: Own Price Elasticities - 10 Countries, Dummy Variables
on Second-Order Terms, 1959-1974*

	CANADA	FRANCE	ITALY	JAPAN	NETHERLANDS	NORWAY	SWEDEN	UK	U.S.A.	W. GERMANY
η_{11}	-2.91 (0.45)	-1.44 (0.19)	-0.97 (0.27)	-0.08 (0.98)	0.30 (0.48)	1.68 (0.66)	0.03 (0.56)	-0.93 (0.21)	0.41 (1.58)	-2.25 (0.23)
η_{22}	-0.39 (0.16)	-0.68 (0.28)	-0.76 (0.18)	0.26 (0.25)	0.36 (0.36)	-0.09 (0.13)	0.77 (0.32)	-0.06 (0.24)	-0.30 (0.31)	-0.90 (0.44)
η_{33}	-1.18 (0.35)	-0.45 (0.23)	-0.29 (0.44)	1.11 (1.53)	-2.77 (0.19)	**	**	-1.34 (0.16)	-0.27 (0.52)	-3.89 (1.64)
η_{44}^*	-0.15	-0.32	-0.25	-0.22	-0.15	0.11	-0.04	-0.33	-0.14	-0.23

* Standard errors could not easily be calculated for this elasticity.

Table 7: Own Price Elasticities - Europe & Japan, Country Dummy
Variables, Data at 3-Year Intervals

	FRANCE	ITALY	JAPAN	NETHERLANDS	NORWAY	SWEDEN	UK	W. GERMANY
η_{11}	-0.96 (0.13)	-1.34 (0.24)	-1.20 (0.19)	-1.48 (0.29)	-1.80 (0.43)	-1.14 (0.18)	-1.03 (0.14)	-0.98 (0.13)
η_{22}	-0.05 (0.20)	-0.17 (0.16)	-0.04 (0.20)	-0.08 (0.24)	-0.24 (0.13)	-0.14 (0.17)	-0.07 (0.19)	+0.25 (0.29)
η_{33}	-1.67 (0.26)	-1.43 (0.19)	-1.67 (0.26)	-1.59 (0.24)	**	**	-1.53 (0.22)	-2.76 (0.65)
η_{44}	-0.08 (0.03)	-0.07 (0.03)	-0.06 (0.03)	-0.02 (0.02)	-0.03 (0.03)	-0.06 (0.03)	-0.08 (0.03)	-0.06 (0.03)

** Almost no natural gas is consumed in the industrial sectors of Norway & Sweden, so that these

Note that, except for electricity, the elasticities of our "preferred" model are large in magnitude. (Remember that these are partial price elasticities, i.e. based on constant energy consumption, so that the total own price elasticities will be even larger in magnitude.) Own price elasticities for coal range from -1 in France and West Germany to about -2 in Norway, Canada, and the U.S. (where coal has had a smaller share of industrial energy consumption). Own price elasticities for natural gas are less than -1 in the European countries and Japan, but -.33 in Canada and -.52 in the U.S. This is reasonable given that natural gas prices have been much lower in Canada and the U.S. than in the rest of the world. On the other hand, Canada and the U.S. have the largest own price elasticities for oil, even though they had relatively low prices. We can only explain this on the basis of a greater availability of alternative fuels at low prices (notably natural gas), so that producers chose technologies allowing for greater interfuel substitution possibility. Finally, note that the own price elasticities for electricity are all quite small in magnitude. This is not surprising; since electricity is a much more expensive fuel on a total basis, we expect it to be used only where there is no possibility of using an alternative fuel.

Our "preferred" model can now be used to generate the aggregate price index for energy. This is done by applying the estimated parameters to equation (16) (note that the first-order parameters α_1 will vary across countries), and choosing α_0 , the unobservable parameter of the equation, so that the price of energy P_E is equal to 1.0 in the U.S. in 1970. Then using the data on fuel prices, we can generate the relative energy price index over time for each country. The resulting energy price indices are shown in Table 8. These indices will in turn serve as the instrument variable for the price of energy in the estimation of our factor share model.

(U.S. = 1 in 1970)

	CANADA	FRANCE	ITALY	JAPAN	NETHERLANDS
1955	1.0707	2.1349	3.6862	3.4706	7.5825
1956	1.2583	2.0541	3.5704	3.3103	6.9176
1957	1.2738	2.0432	3.5807	3.1635	6.9216
1958	1.2187	1.9325	3.4128	3.1860	6.9786
1959	1.1550	1.9805	3.3691	3.1010	6.6800
1960	1.1574	1.8590	3.3315	2.9548	6.4169
1961	1.1452	1.8086	3.0781	2.8187	6.1131
1962	1.1040	1.7484	2.9146	2.7179	5.4860
1963	1.1039	1.6921	2.6765	2.5977	5.0274
1964	1.0730	1.6991	2.4554	2.4869	4.5576
1965	1.0633	1.6738	2.3863	2.7431	4.3033
1966	0.9826	1.6495	2.3320	2.6305	4.3704
1967	0.9578	1.6738	2.2923	2.5159	4.0833
1968	0.9648	1.7420	2.2749	2.4010	3.9185
1969	0.9338	1.7254	2.1749	2.3594	3.5203
1970	0.9357	1.7286	2.0375	2.2061	3.2744
1971	0.9248	1.7568	1.9222	2.1358	3.0467
1972	0.8438	1.7432	1.8195	2.0930	2.7999
1973	0.8525	1.6517	1.7615	2.0201	2.4709
1974	1.0646	1.6831	1.6346	NA	2.6157

	NORWAY	SWEDEN	U.K.	U.S.A.	W. GERMANY
1955	1.1147	2.7962	2.1636	NA	3.3351
1956	1.1020	2.7825	2.1752	1.4599	3.1851
1957	1.1235	2.8569	2.2623	1.4532	3.2232
1958	1.0727	2.6824	2.2512	NA	3.2704
1959	1.0135	2.5866	2.2051	1.3698	3.1876
1960	0.9468	2.6763	2.1629	1.3466	3.2221
1961	0.9238	2.6258	2.2206	1.3276	3.0536
1962	0.9515	2.6346	2.1628	1.2956	2.9051
1963	0.9437	2.4719	2.1808	1.2372	2.8344
1964	0.9217	2.3112	2.1125	1.1897	2.7768
1965	0.8801	2.1428	2.0970	1.1579	2.8191
1966	0.8990	2.0422	2.0547	1.1111	2.7484
1967	0.9428	1.9952	2.0402	1.0887	2.6623
1968	0.8938	1.8700	1.9372	1.0495	2.8022
1969	0.9025	1.7619	1.8422	1.0088	2.5817
1970	0.9264	1.6633	1.8091	1.0000	2.4469
1971	0.9512	1.6143	1.9275	1.0603	2.3168
1972	1.0002	1.5811	1.8696	1.0900	2.3266
1973	0.9404	1.4522	1.7409	1.1197	2.2439
1974	1.0938	1.6794	2.0924	1.7096	2.2744

5.2 Factor Share Model

We turn now to the model of capital, labor, and energy shares. Once again, choices must be made regarding the use of dummy variables and the pooling of data.²⁸ In addition, we cannot assume homotheticity, but must test this as a possible restriction on the model.

Parameter estimates for several forms of the model are given in Table 9, although other forms were estimated as well. In columns 1 and 2 the share equations have been estimated without regional dummy variables. In column 1 the cost function is homothetic ($\gamma_{Q1} = 0$, $i = K, L, E$), and in column 2 it is non-homothetic. These two models can be used as a first test for homotheticity; the value of the test statistic is 68.4, and this is significant at the 1% level, indicating that homotheticity cannot be accepted. In column 3 the cost function is non-homothetic, and intercept dummy variables are added to the share equations. A comparison of this model with that in column 2 allows us to test for constancy of the first-order parameters across countries. The test statistic is 359.8, which is significant at the 1% level (9 degrees of freedom), so that intercept dummy variables are retained in the share equations.

In columns 4 and 5 a non-homothetic cost function is again estimated with intercept dummy variables in the share equations, but now Canada and the U.S. are pooled separately. Note that the γ_{1j} parameter values change considerably, with the values in column 3 generally midway between those in columns 4 and 5. In addition, the parameter estimates generally become more significant for Europe and Japan, but less significant for Canada and the U.S. This provides us with no firm indication of whether Canada and the U.S. should be pooled separately.

²⁸Inclusion of 1974 data is not a question here; our derived capital price data extends only through 1973.

Table 9 - Parameter Estimates: Factor Share Model
(1= capital, 2 = labor, 3 = energy)

Parameters	1 Homothetic 10 countries, 1963-73	2 Non-Homothetic, 10 countries, 1963-73	3 Non-homothetic country dummy variables, 10 countries 1963-1973	4 Non-homothetic, U.S. & Canada, country dummy variables, 1963-1973	5 Non-homothetic, Europe & Japan, country dummy variables, 1963-1973	6 Non-homothetic, countries, country dummy variables, data at 3 year intervals
α_1	0.3761 (0.0171)	0.2137 (0.0399)				
$\alpha_1 D_1$			0.3138 (0.1881)	0.8375 (0.3930)	0.0649 (0.2584)	0.8517 (0.3252)
$\alpha_1 D_2$			0.2887 (0.2254)	1.0104 (0.5869)	0.0432 (0.2356)	0.9604 (0.3908)
$\alpha_1 D_3$			0.2451 (0.2056)		0.0969 (0.2890)	0.8540 (0.3576)
$\alpha_1 D_4$			0.3468 (0.2520)		0.0919 (0.1899)	1.1027 (0.4370)
$\alpha_1 D_5$			0.2492 (0.1658)		0.1748 (0.1465)	0.7229 (0.2867)
$\alpha_1 D_6$			0.3018 (0.1281)		0.0829 (0.1898)	0.6762 (0.2190)
$\alpha_1 D_7$			0.2452 (0.1658)		-0.1353 (0.2575)	0.7191 (0.2850)
$\alpha_1 D_8$			0.0852 (0.2247)		0.1438 (0.2792)	0.7403 (0.3916)
$\alpha_1 D_9$			0.2947 (0.2718)			1.0511 (0.4689)
$\alpha_1 D_{10}$			0.3841 (0.2436)			1.1119 (0.4268)
α_2	0.5754 (0.0160)	0.7021 (0.0383)				
$\alpha_2 D_1$			0.5749 (0.1780)	0.1920 (0.3614)	0.7559 (0.2479)	0.0624 (0.3090)
$\alpha_2 D_2$			0.5950 (0.2137)	0.0710 (0.5393)	0.7677 (0.2261)	-0.0456 (0.3721)
$\alpha_2 D_3$			0.6232 (0.1950)		0.6934 (0.2773)	0.0429 (0.3407)
$\alpha_2 D_4$			0.5139 (0.2389)		0.7542 (0.1828)	-0.2055 (0.4163)
$\alpha_2 D_5$			0.6438 (0.1577)		0.6880 (0.1405)	0.1982 (0.2745)
$\alpha_2 D_6$			0.5961 (0.1213)		0.7521 (0.1820)	0.2400 (0.2082)
$\alpha_2 D_7$			0.6358 (0.1571)		0.9448 (0.2471)	0.1839 (0.2713)
$\alpha_2 D_8$			0.7867 (0.2131)		0.6701 (0.2679)	0.1620 (0.3731)
$\alpha_2 D_9$			0.5842 (0.2572)			-0.1368 (0.4455)
$\alpha_2 D_{10}$			0.4978 (0.2310)			-0.1961 (0.4065)

Parameters	1 Homothetic 10 countries 1963-73	2 Non-Homothetic, 10 countries 1963-73	3 Non-homothetic country dummy vari- ables, 10 countries 1963-1973	4 Non-homothetic, U.S. & Canada, country dummy variables, 1963-73	5 Non-homothetic, Europe & Japan country dummy variables, 1963-73	6 Non-homothetic, 10 countries, country dummy variables, data at 3 year in- tervals
α_3	0.0485 (0.0022)	0.0841 (0.0040)				
$\alpha_3 D_1$			0.1112 (0.0332)	-0.0295 (0.9794)	0.1791 (0.0447)	0.0859 (0.0608)
$\alpha_3 D_2$			0.1163 (0.0412)	-0.0813 (0.9003)	0.1891 (0.0400)	0.0852 (0.0748)
$\alpha_3 D_3$			0.1317 (0.0387)		0.2097 (0.0500)	0.1031 (0.0707)
$\alpha_3 D_4$			0.1393 (0.0447)		0.1539 (0.0361)	0.1028 (0.0843)
$\alpha_3 D_5$			0.1070 (0.0316)		0.1375 (0.0245)	0.0790 (0.0600)
$\alpha_3 D_6$			0.1021 (0.0223)		0.1650 (0.0332)	0.0838 (0.0424)
$\alpha_3 D_7$			0.1190 (0.0283)		0.1908 (0.0448)	0.0969 (0.0548)
$\alpha_3 D_8$			0.1280 (0.0412)		0.1862 (0.0480)	0.0977 (0.0762)
$\alpha_3 D_9$			0.1211 (0.0480)			0.0857 (0.0883)
$\alpha_3 D_{10}$			0.1182 (0.0424)			0.0843 (0.0825)
γ_{11}	0.0685 (0.0211)	0.0525 (0.0190)	0.0467 (0.0244)	-0.1068 (0.0795)	0.0732 (0.0270)	-0.0387 (0.0434)
γ_{12}	-0.0697 (0.0193)	-0.0569 (0.0179)	-0.0411 (0.0228)	0.0957 (0.0728)	-0.0628 (0.0254)	0.0446 (0.0403)
γ_{13}	0.0011 (0.0036)	0.0044 (0.0027)	-0.0056 (0.0030)	0.0111 (0.0083)	-0.0104 (0.0032)	-0.0059 (0.0054)
γ_{21}	-0.0697 (0.0193)	-0.0569 (0.0179)	-0.0411 (0.0228)	0.0957 (0.0728)	-0.0628 (0.0254)	0.0446 (0.0403)
γ_{22}	0.0775 (0.0180)	0.0675 (0.0172)	0.0422 (0.0215)	-0.0808 (0.0673)	0.0585 (0.0243)	-0.0406 (0.0382)
γ_{23}	-0.0078 (0.0024)	-0.0106 (0.0018)	-0.0011 (0.0036)	-0.0149 (0.0087)	0.0043 (0.0039)	-0.0041 (0.0070)
γ_{31}	0.0011 (0.0036)	0.0044 (0.0027)	-0.0056 (0.0030)	0.0111 (0.0083)	-0.0104 (0.0032)	-0.0059 (0.0054)
γ_{32}	-0.0078 (0.0024)	-0.0106 (0.0018)	-0.0011 (0.0036)	-0.0149 (0.0087)	0.0043 (0.0039)	-0.0041 (0.0070)
γ_{33}	0.0067 (0.0021)	0.0062 (0.0016)	0.0067 (0.0028)	0.0038 (0.0042)	0.0061 (0.0031)	0.0100 (0.0047)
γ_{Q1}		0.0293 (0.0066)	0.0216 (0.0333)	-0.0648 (0.0727)	0.0539 (0.0380)	-0.0701 (0.0574)
γ_{Q2}		-0.0229 (0.0063)	-0.0104 (0.0315)	0.0513 (0.0667)	-0.0335 (0.0364)	0.0771 (0.0545)
γ_{Q3}		-0.0065 (0.0007)	-0.0113 (0.0059)	0.0135 (0.0091)	-0.0203 (0.0065)	-0.0069 (0.0108)
Eqn. 1	RSQ .228	RSQ .095	RSQ .859	RSQ .259	RSQ .880	RSQ .871
2	.214	.126	.864	.165	.835	.877

A second test for homotheticity is performed by estimating homothetic versions of the cost functions in columns 4 and 5 (the resulting parameter estimates are not reported here). These models can then be compared to the corresponding non-homothetic model. For Europe and Japan the test statistic is 8.58, and this is significant at the 2.5% level, so that homotheticity cannot be accepted here. For Canada and the U.S. the test statistic is 1.25, which is not significant at the 10% level, so that homotheticity could be accepted. However, if the regional dummy variables are eliminated, the same test for homotheticity gives a test statistic of 32.4, which is significant at the 1% level. In addition, testing constancy of the first-order terms in the non-homothetic model gives a test statistic of 2.35 which is not significant at the 10% level. We therefore consider the test for homotheticity to be inconclusive, and retain a non-homothetic model.²⁹

In column 6 the non-homothetic cost function is estimated, with all ten countries pooled, using data at 3-year intervals. Comparing columns 3 and 6 we see that many of the estimated parameter values do change considerably. We are thus less confident that we have estimated a long-run cost function than we are for the fuel share model.

The resulting elasticities of substitution and price elasticities of demand are shown in Table 10a and 10b for the model in which all ten countries are pooled together, and the model in which Canada and the U.S. are pooled separately. Note that the choice of pooling method has little effect on the elasticities of the European countries and Japan, but almost all of the elasticities for Canada and the U.S. are larger in magnitude when these countries are pooled separately.

²⁹ We also tested the hypotheses that the γ_{ij} are all zero, i.e. that the production function is Cobb-Douglas. The test statistics are significant at the 1% level, so that this hypotheses can be rejected.

Table 10a - Elasticities of Substitution for Capital, Labor, and Energy

(a) Ten countries pooled										
(b) U. S. and Canada pooled separately from Europe and Japan										
	CANADA	FRANCE	ITALY	JAPAN	NETH	NOR	SWED	UK	USA	NGER
σ_{KK}										
(a)	-0.99 (0.12)	-0.91 (0.11)	-1.14 (0.14)	-0.68 (0.08)	-1.35 (0.18)	-1.12 (0.14)	-1.39 (0.18)	-2.18 (0.37)	-0.95 (0.11)	-0.57 (0.07)
(b)	-1.75 (0.39)	-0.79 (0.13)	-0.98 (0.16)	-0.59 (0.09)	-1.16 (0.19)	-0.97 (0.15)	-1.20 (0.20)	-1.78 (0.41)	-1.66 (0.37)	-0.49 (0.08)
σ_{KL}										
(a)	0.82 (0.10)	0.82 (0.10)	0.81 (0.11)	0.81 (0.11)	0.81 (0.11)	0.82 (0.10)	0.80 (0.11)	0.77 (0.13)	0.82 (0.10)	0.81 (0.11)
(b)	1.43 (0.32)	0.72 (0.11)	0.70 (0.12)	0.70 (0.12)	0.70 (0.12)	0.71 (0.12)	0.69 (0.12)	0.64 (0.15)	1.41 (0.31)	0.71 (0.12)
σ_{KE}										
(a)	0.76 (0.13)	0.76 (0.13)	0.82 (0.09)	0.86 (0.07)	0.77 (0.12)	0.78 (0.12)	0.80 (0.11)	0.66 (0.18)	0.61 (0.21)	0.82 (0.10)
(b)	1.48 (0.36)	0.56 (0.13)	0.67 (0.10)	0.74 (0.08)	0.59 (0.13)	0.59 (0.13)	0.63 (0.11)	0.36 (0.19)	1.77 (0.58)	0.66 (0.10)
σ_{LL}										
(a)	-0.84 (0.09)	-0.90 (0.09)	-0.79 (0.09)	-1.32 (0.15)	-0.65 (0.08)	-0.76 (0.08)	-0.65 (0.07)	-0.38 (0.05)	-0.81 (0.08)	-1.42 (0.16)
(b)	-1.33 (0.27)	-0.83 (0.11)	-0.73 (0.09)	-1.22 (0.17)	-0.60 (0.08)	-0.70 (0.09)	-0.60 (0.08)	-0.34 (0.05)	-1.29 (0.26)	-1.30 (0.18)
σ_{LE}										
(a)	0.96 (0.14)	0.95 (0.15)	0.97 (0.09)	0.96 (0.13)	0.97 (0.09)	0.96 (0.11)	0.96 (0.08)	0.97 (0.08)	0.93 (0.23)	1.94 (0.19)
(b)	0.42 (0.35)	1.17 (0.16)	1.11 (0.10)	1.15 (0.14)	1.11 (0.10)	1.14 (0.12)	1.10 (0.09)	1.10 (0.09)	0.05 (0.56)	1.23 (0.21)
σ_{EE}										
(a)	-15.86 (1.07)	-16.20 (1.12)	-10.96 (0.50)	-11.17 (0.51)	-12.25 (0.61)	-13.75 (0.78)	-10.77 (0.47)	-13.10 (0.70)	-24.21 (2.93)	-15.82 (1.06)
(b)	-16.96 (1.58)	-16.45 (1.22)	-11.06 (0.54)	-11.28 (0.56)	-12.39 (0.67)	-13.92 (0.86)	-10.88 (0.52)	-13.25 (0.78)	-27.21 (4.35)	-16.05 (1.16)

Table 10b - Price Elasticities of Demand for Capital, Labor, and Energy

(a) Ten countries pooled (b) U.S. & Canada pooled separately from Europe & Japan

	CANADA	FRANCE	ITALY	JAPAN	NETHERLANDS	NORWAY	SWEDEN	U.K.	U.S.A.	W. GERMANY
η_{KK} (a)	-0.45 (0.05)	-0.43 (0.05)	-0.47 (0.06)	-0.37 (0.05)	-0.50 (0.07)	-0.47 (0.06)	-0.51 (0.07)	-0.56 (0.10)	-0.44 (0.05)	-0.33 (0.04)
(b)	-0.78 (0.18)	-0.37 (0.06)	-0.41 (0.06)	-0.32 (0.05)	-0.43 (0.07)	-0.41 (0.06)	-0.43 (0.07)	-0.46 (0.10)	-0.71 (0.17)	-0.29 (0.05)
η_{KL} (a)	0.41 (0.05)	0.39 (0.05)	0.41 (0.05)	0.31 (0.04)	0.45 (0.06)	0.42 (0.05)	0.44 (0.06)	0.52 (0.09)	0.42 (0.05)	0.30 (0.04)
(b)	0.71 (0.16)	0.35 (0.05)	0.36 (0.06)	0.27 (0.05)	0.39 (0.07)	0.37 (0.06)	0.38 (0.06)	0.44 (0.10)	0.71 (0.16)	0.26 (0.04)
η_{KE} (a)	0.04 (0.01)	0.04 (0.01)	0.06 (0.01)	0.06 (0.01)	0.05 (0.01)	0.05 (0.01)	0.06 (0.01)	0.04 (0.01)	0.02 (0.01)	0.04 (0.01)
(b)	0.08 (0.02)	0.03 (0.01)	0.05 (0.01)	0.06 (0.01)	0.04 (0.01)	0.04 (0.01)	0.05 (0.01)	0.02 (0.01)	0.06 (0.02)	0.03 (0.01)
η_{LK} (a)	0.37 (0.05)	0.38 (0.05)	0.33 (0.04)	0.44 (0.06)	0.30 (0.04)	0.34 (0.04)	0.29 (0.04)	0.20 (0.03)	0.38 (0.04)	0.47 (0.06)
(b)	0.64 (0.15)	0.84 (0.05)	0.29 (0.05)	0.38 (0.07)	0.26 (0.05)	0.30 (0.05)	0.26 (0.05)	0.17 (0.04)	0.65 (0.14)	0.42 (0.07)
η_{LL} (a)	-0.42 (0.04)	-0.43 (0.04)	-0.41 (0.04)	-0.51 (0.06)	-0.36 (0.04)	-0.40 (0.04)	-0.36 (0.04)	-0.26 (0.03)	-0.41 (0.04)	-0.52 (0.06)
(b)	-0.66 (0.14)	-0.40 (0.05)	-0.37 (0.05)	-0.46 (0.06)	-0.34 (0.04)	-0.37 (0.05)	-0.33 (0.04)	-0.23 (0.04)	-0.65 (0.13)	-0.47 (0.07)
η_{LE} (a)	0.05 (0.01)	0.05 (0.01)	0.07 (0.01)	0.07 (0.01)	0.06 (0.01)	0.06 (0.01)	0.08 (0.01)	0.06 (0.01)	0.03 (0.01)	0.05 (0.01)
(b)	0.02 (0.02)	0.06 (0.01)	0.08 (0.01)	0.09 (0.01)	0.08 (0.01)	0.07 (0.01)	0.09 (0.01)	0.07 (0.01)	0.00 (0.02)	0.06 (0.01)

Table 10b (Cont.) - Price Elasticities of Demand for Capital, Labor and Energy

(a) Ten countries pooled (b) U.S. & Canada pooled separately from Europe & Japan

	CANADA	FRANCE	ITALY	JAPAN	NETH.	NORWAY	SWEDEN	U.K.	U.S.A.	W. GERM.
η_{EK} (a)	0.34 (0.06)	0.36 (0.06)	0.34 (0.04)	0.47 (0.04)	0.29 (0.04)	0.33 (0.05)	0.29 (0.04)	0.17 (0.05)	0.28 (0.10)	0.48 (0.06)
(b)	0.66 (0.16)	0.26 (0.06)	0.28 (0.04)	0.40 (0.04)	0.22 (0.05)	0.25 (0.05)	0.23 (0.04)	0.09 (0.05)	0.82 (0.27)	0.38 (0.06)
η_{EL} (a)	0.48 (0.07)	0.46 (0.07)	0.50 (0.05)	0.37 (0.05)	0.54 (0.05)	0.50 (0.06)	0.55 (0.05)	0.66 (0.06)	0.47 (0.12)	0.34 (0.07)
(b)	0.21 (0.17)	0.56 (0.08)	0.57 (0.05)	0.44 (0.05)	0.62 (0.06)	0.59 (0.06)	0.62 (0.05)	0.75 (0.06)	0.03 (0.29)	0.45 (0.08)
η_{EE} (a)	-0.82 (0.05)	-0.82 (0.06)	-0.84 (0.04)	-0.84 (0.04)	-0.84 (0.04)	-0.83 (0.05)	-0.84 (0.04)	-0.84 (0.04)	-0.75 (0.09)	-0.82 (0.05)
(b)	-0.87 (0.08)	-0.83 (0.06)	-0.84 (0.04)	-0.84 (0.04)	-0.84 (0.05)	-0.84 (0.05)	-0.84 (0.04)	-0.84 (0.05)	-0.85 (0.14)	-0.85 (0.06)

Pooling Canada and the U.S. separately, however, also results in elasticities with larger standard errors, so we are inclined to choose as a "preferred" model that for which all of the countries are pooled.

Let us now consider the implications of these elasticities for energy demand and the substitutability of energy with other factors of production. First we see that the elasticity of substitution for energy and capital (σ_{KE}) is positive, although small (0.61 to 0.86). We thus find that energy and capital are substitutes, and not complements as earlier studies for the U.S. had indicated.³⁰ We find that labor and energy are also substitutes (with elasticities of substitution close to 1); this is not surprising, and is supported by most other work. Similarly, we find that capital and labor are substitutes, as expected. The own price elasticities of demand for capital and labor are around -0.3 to -0.5, which is in agreement with most earlier work. The own price elasticities for energy, however, are larger in magnitude than most of those obtained earlier by others. We find this elasticity to be about -0.8, whereas most earlier estimates were around -0.4 to -0.5. Most earlier work, however, was based on data for a single country, so that it is more likely that we have estimated the long-term elasticity.³¹ Finally, note from the cross-price elasticities of energy and capital that over the long-run a doubling in the price of energy should result in a 5% increase in the demand for capital and a 6% or 7% increase in the demand for labor as substitution away from energy takes place.

Since our model is non-homothetic, it is also interesting to note that what our results imply about economies of scale in aggregate production, and about

³⁰ Berndt and Wood [9], for example, found strong complementarity between energy and capital. Griffen and Gregory [34], using pooled international data at four-year intervals, obtain results very similar to ours.

³¹ Our estimate is close to that found by Griffen and Gregory [34], who also used international data.

the elasticity of energy demand with respect to output changes (η_{EQ}). In Table 11 we show the index of scale (SCE) introduced by Christensen and Greene [19] (see footnote 14), and the elasticity of energy demand with respect to output changes, as given by equation (25). (The indices and elasticities for each country are calculated at the point of means.) Note that the index of scale economies is insignificantly different from zero for each country, so that the aggregate cost functions exhibit nearly constant returns to scale. The output elasticity of energy demand, however, is significantly less than unity. Thus, even if energy prices remain constant relative to other prices, as output increases there will be substitution away from energy.³²

We can now examine the total price elasticities for the individual fuel demands. These elasticities are computed using equations (20) and (21), and are shown in Table 12. The own price elasticities of energy are obtained from Table 10b based on all ten countries pooled together. Note that these total elasticities are larger than the partial elasticities of Tables 4 and 5, since they account for decreased use of energy as well as interfuel substitution. We find that coal has the largest own price elasticities, ranging from -1.29 to -2.24. For Europe and Japan, own price elasticities for natural gas are large (-1.37 to -2.34), while those for oil are small (-0.6 to -.56). We attribute this to the fact that for two countries (Netherlands and W. Germany), as oil and gas prices fell, there was a large increase in the share of natural gas (from almost zero) as supplies became available for the first time. This might have tended to bias the natural gas elasticities upwards. It is more difficult to explain the low oil price elasticities; oil prices on a total basis were generally the lowest of any fuel, but oil did not gain a dominant share in Europe. For Canada and the U.S. (which was pooled separately in the fuel choice model) the situation is reversed - the price elasticities for oil are larger than those for natural gas. Here natural gas prices were lower than oil prices, and the share of oil was small (in the U.S.) and roughly constant over time.

³²This result is not surprising given the data. Note from Table 2 that for large-output countries like to U.S., the share of energy is smaller than for low-output countries.

Table 11 - Index of Scale Economies and
Output Elasticity of Energy Demand *
(all ten countries pooled)

Country	SCE	η_{EQ}
Canada	.0015 (.0080)	0.785 (0.108)
France	.0086 (.0383)	0.783 (0.113)
Italy	.0032 (.0335)	0.855 (0.078)
Japan	.0105 (.0487)	0.849 (0.087)
Netherlands	-.0124 (.0170)	0.818 (0.093)
Norway	.0056 (.0172)	0.807 (0.097)
Sweden	.0019 (.0205)	0.864 (0.070)
UK	.0041 (.0327)	0.778 (0.113)
U.S.A.	.0003 (.0037)	0.624 (0.188)
W. Germany	.0012 (.0316)	0.761 (0.122)

* Standard errors are computed based on constancy of shares and prices at their mean values. The standard error of SCE is thus computed from:

$$\text{Var}(\text{SCE}) = \sum_{i=K,}^{L,M} (\log P_i)^2 \text{Var}(\gamma_{Q1}) + \sum_{i \neq j} \log P_i \log P_j \text{Covar}(\gamma_{Q1} \gamma_{Qj})$$

and the standard error of η_{EQ} is computed from:

$$\begin{aligned} \text{Var}(\eta_{EQ}) = & \text{Var}(\text{SCE}) + [2\log P_E / S_E + 1/S_E^2] \text{Var}(\gamma_{EQ}) \\ & + (2/S_E) \log P_K \text{Covar}(\gamma_{QE} \gamma_{QK}) + (2/S_E) \log P_L \text{Covar}(\gamma_{QE} \gamma_{QL}) \end{aligned}$$

Table 12 - Total Fuel Price Elasticities*

(Using preferred fuel share and factor share models)

ELAS.	CANADA	FRANCE	ITALY	JAPAN	NETHERLANDS	NORWAY	SWEDEN	UK	U.S.A.	W. GERMANY
η_{11}	-1.89	-1.29	-1.63	-1.49	-1.78	-2.15	-1.44	-1.35	-2.24	-1.31
η_{12}	0.69	0.06	0.09	0.07	0.09	0.15	0.07	0.06	0.92	0.05
η_{13}	1.08	0.40	0.76	0.60	0.92	1.33	0.54	0.45	1.50	0.41
η_{14}	-0.75	0.0	-0.06	-0.03	-0.08	-0.16	-0.02	-0.02	-1.03	0.01
η_{21}	0.31	0.11	0.07	0.09	0.09	0.05	0.08	0.10	0.90	0.13
η_{22}	-1.03	-0.34	-0.46	-0.35	-0.22	-0.56	-0.44	-0.37	-1.17	-0.06
η_{23}	-0.29	-0.13	-0.10	-0.13	-0.16	-0.09	-0.12	-0.12	-0.88	-0.20
η_{24}	0.14	-0.46	-0.35	-0.46	-0.54	-0.24	-0.37	-0.44	0.30	-0.70
η_{31}	1.25	1.96	1.39	1.95	1.73	**	**	1.64	0.65	4.73
η_{32}	-0.75	-0.36	-0.24	-0.36	-0.33	**	**	-0.30	-0.38	-0.93
η_{33}	-0.41	-1.54	-1.37	-1.54	-1.48	**	**	-1.44	-0.67	-2.34
η_{34}	-0.96	-0.89	-0.61	-0.89	-0.77	**	**	-0.74	-0.43	-2.29
η_{41}	-0.15	0.0	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01	-0.13	0.01
η_{42}	0.06	-0.17	-0.14	-0.13	-0.11	-0.10	-0.13	-0.16	0.04	-0.14
η_{43}	-0.17	-0.12	-0.10	-0.09	-0.08	-0.09	-0.10	-0.11	-0.13	-0.10
η_{44}	-0.61	-0.54	-0.59	-0.60	-0.63	-0.62	-0.60	-0.56	-0.63	-0.59

* - 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

** - 1992: no natural gas is consumed in the industrial sectors of Norway & Sweden, so that these

In Table 13 we show the elasticities of average cost of output with respect to the price of energy and the prices of the individual fuels. These elasticities are based on equations (27) and (28), and are shown for two versions of the factor share model - Canada and the U.S. pooled separately, and all ten countries pooled together.³³ These elasticities give us the effects of increases in energy prices on the cost of output, assuming the level of output stays fixed. They thus provide information about the inflationary impact of an energy price rise, but they do not provide information about the effect on the level of GNP. Note that in the United States a 10% increase in the cost of energy would result in about a 0.3% increase in the cost of output, whereas for Italy, Japan, and Sweden the cost of output would rise about 0.7%.

5.3 Logit Models of Fuel Shares

We estimate a number of static and dynamic logit models to describe the dependence of fuel shares on prices and the level of output. The "decision functions" in these models are linear or logarithmic functions of relative fuel prices \tilde{P}_i (the price of fuel i divided by the price of energy), the level of output, and in the case of the dynamic models, lagged shares. Recall from equations (40) and (42) that this leads to a set of three equations that must be estimated simultaneously since certain coefficients are constrained to be the same across equations.³⁴ We therefore use iterative Zellner estimation to estimate all of the models.

Our static logit models are of the general form

$$\log(S_i/S_4) = \sum_{k=1}^{10} a_{14k} D_k + b_{11} \tilde{P}_1 - b_{44} \tilde{P}_4 + c_{14} Q, \quad i=1,2,3 \quad (49)$$

³³ In comparing these elasticities across countries, remember that they are dependent on the shares of energy, and the fuel shares. Thus Italy, Japan and Sweden have the largest values of η_{CE} in part because they have the largest shares of energy in the cost of output.

³⁴ Even without cross-equation coefficient constraints, simultaneous equation estimation is desirable in that insofar as errors are correlated across equations, it yields more efficient parameter estimates.

Table 13 - Elasticity of Average Cost of Output with Respect to Price of Energy & Fuels*

A. U.S. & Canada Pooled Separately from Europe & Japan

	CANADA	FRANCE	ITALY	JAPAN	NETH.	NORWAY	SWEDEN	UK	U.S.A.	W. GERMANY	
η_{CE}	1963 1972	.045 .050	.053 .046	.076 .067	.073 .063	.069 .060	.062 .062	.078 .066	.065 .059	.029 .032	.051 .043
η_{C1}	1963 1972	.006 .004	.017 .008	.012 .007	.010 .004	.004 .006	.015 .010	.019 .008	.003 .002	.003 .002	.014 .005
η_{C2}	1963 1972	.011 .013	.008 .012	.018 .014	.012 .014	.016 .016	.012 .015	.010 .016	.002 .003	.002 .003	.007 .007
η_{C3}	1963 1972	.004 .007	.005 .004	.006 .007	.006 .002	.00 .00	.001 .00	.005 .005	.005 .006	.005 .006	.001 .003
η_{C4}	1963 1972	.028 .026	.023 .023	.043 .037	.044 .040	.045 .041	.050 .041	.030 .030	.019 .020	.019 .020	.030 .028

B. All Ten Countries Pooled

	CANADA	FRANCE	ITALY	JAPAN	NETH.	NORWAY	SWEDEN	UK	U.S.A.	W. GERMANY
η_{CE}	1963 1972	.053 .047	.076 .068	.072 .063	.069 .060	.064 .062	.075 .066	.065 .060	.033 .028	.051 .044
η_{C1}	1963 1972	.007 .004	.009 .010	.012 .007	.009 .004	.004 .006	.014 .010	.019 .008	.003 .002	.014 .005
η_{C2}	1963 1972	.011 .012	.018 .014	.012 .014	.013 .004	.016 .016	.012 .015	.010 .017	.002 .003	.007 .007
η_{C3}	1963 1972	.005 .006	.006 .007	.006 .002	.000 .011	.00 .00	.001 .00	.005 .005	.006 .006	.00 .003
η_{C4}	1963 1972	.031 .024	.043 .037	.042 .040	.046 .041	.044 .039	.048 .041	.030 .030	.022 .018	.030 .028

* E = energy, 1 = solid fuel, 2 = liquid fuel, 3 = gas, 4 = electricity

where $a_{14k} = a_{1k} - a_{4k}$, and $c_{i4} = c_i - c_4$, as in equation (40). The D_k are country dummy variables (countries are ordered alphabetically), and the fuels are ordered (1) solid, (2) liquid, (3) gas, and (4) electricity.

The dynamic models are based on the assumption that the choice of fuels this period depends on the relative shares last period, as well as this period's relative prices and output. The dependence on past shares is intended to incorporate partial adjustment of the capital stock. It leads to equations of the form

$$\begin{aligned} \log(S_i/S_4) = & \sum_{k=1}^{10} a_{i4k} D_k + b_i \tilde{P}_i - b_4 \tilde{P}_4 + c_{i4} Q \\ & + d_i S_{i,t-1} - d_4 S_{4,t-1}, \quad i=1,2,3 \end{aligned} \quad (50)$$

Note that this is not a Koyck adjustment model. The coefficients d_i can be greater than 1 (although we would expect them to be positive), and in general a change in price will not lead to geometrically declining changes in shares over time.

The results of estimating various versions of these models are shown in Table 14. Unfortunately, the results are disappointing. In all of the models that we have estimated, the own price elasticities are positive for both solid and liquid fuel (note that b_1 and b_2 are positive for all of the models). In addition the coefficients b_3 and b_4 are insignificant or positive for some of the models. The logit models provides essentially an ad hoc description of fuel choice by industrial consumers of energy, and that description is not consistent with the data and our a priori expectations regarding the characteristics of fuel demands.

6. Comparison of Results with Other Studies

In Table 15 we present a survey of recent estimates by others of industrial energy demand elasticities. We can compare these estimates with our results to get an idea of what "consensus" elasticities would be, and how and why our estimates might differ from these.

Table 14: Parameter Estimates for Logit Models of Fuel Choice*

Parameters	1. Static, linear in prices & output, no interaction dummies.	2. Dynamic, linear in prices & output, no interaction dummies.	3. Static, linear in prices & output.	4. Static, linear in prices & output, gas price dummies for Norway (6), Sweden (7), & W. Germ (10).	5. Dynamic, linear in prices & output, gas dummies for Norway, Sweden, & W. Germany.	6. Static, linear in logs of prices & output.	7. Dynamic, linear in logs of price & output, gas price dummies for Norway, Sweden, & W. German
a ₁₄	-3.4784 (-17.939)	-2.6053 (-12.23)	-2.5352 (-4.53)	-2.3553 (-4.17)	-4.1231 (-9.27)	-2.6618 (-3.93)	.7162 (1.48)
a ₁₄₁			.1980 (.96)	.1555 (.76)	-.3131 (-2.39)	.6319 (3.06)	-.3330 (-2.53)
a ₁₄₂			.5983 (5.5238)	.6122 (5.66)	.2284 (3.07)	.8489 (8.26)	.0112 (.17)
a ₁₄₃			.6844 (7.12)	.6854 (7.12)	.0715 (1.04)	.9789 (8.80)	-.0773 (-.95)
a ₁₄₄			.1901 (2.31)	.1906 (2.32)	.0170 (.35)	.2718 (3.32)	-.0919 (-1.97)
a ₁₄₅			.3665 (3.68)	.3733 (3.76)	-.1706 (-2.46)	.9335 (5.50)	-.0454 (-.40)
a ₁₄₆			.1687 (.60)	.0832 (.29)	-.4172 (-2.10)	.7525 (1.84)	-.3929 (-1.48)
a ₁₄₇			.5297 (4.13)	.5012 (3.91)	.0492 (.57)	.9130 (4.68)	-.1373 (-1.03)
a ₁₄₈			.4871 (3.29)	.5208 (3.51)	-.4034 (-2.70)	.8786 (6.81)	-.1092 (-1.33)
a ₁₄₉			-1.4535 (-6.56)	-1.4463 (-6.56)	-.0158 (-.15)	-1.1931 (-8.93)	-.3287 (-3.85)
a ₂₄	-3.2813 (-15.25)	-2.5873 (-11.66)	-.8207 (-1.43)	-.6457 (-1.12)	-3.9559 (-8.78)	8.2076 (11.03)	.6691 (1.30)
a ₂₄₁			-1.0316 (-5.97)	-1.0750 (-6.18)	.1811 (1.58)	-2.4468 (-12.97)	.0412 (.30)
a ₂₄₂			-.0574 (-.41)	-.0461 (-.32)	.2403 (3.05)	-.3349 (-2.74)	.0424 (.64)

* t-statistics in parentheses

Table 14 (Cont.) - Parameter Estimates for Logit Models of Fuel Choice *

Para- meters	1.	2.	3.	4.	5.	6.	7.
a ₂₄₃			-.9881 (-7.02)	-.9909 (-7.00)	-.0955 (-1.16)	-1.8622 (-12.92)	-.1677 (-1.56)
a ₂₄₄			-.2778 (-2.67)	-.2771 (-2.65)	.0805 (1.49)	-.3384 (-2.74)	.0086 (.20)
a ₂₄₅			-1.3914 (-10.24)	-1.3828 (-10.12)	-.3786 (-4.37)	-2.8783 (-15.92)	-.0519 (-.32)
a ₂₄₆			-1.7552 (-6.16)	-1.8464 (-6.41)	-.2249 (-1.15)	-5.8022 (-13.42)	-.2364 (-.75)
a ₂₄₇			-.7600 (-4.89)	-.7894 (-5.05)	.0907 (.97)	-2.8328 (-13.13)	-.0761 (-.47)
a ₂₄₈			-.1554 (-1.01)	-.1217 (-.79)	.1175 (1.18)	-.1405 (-1.16)	.0695 (1.00)
a ₂₄₉			.9433 (3.10)	.9538 (3.12)	.3281 (2.00)	.7697 (4.54)	.2243 (2.39)
a ₃₄	-2.9679 (-4.65)	-4.0277 (-5.54)	-6.2493 (-3.68)	17.0764 (1.32)	23.4225 (1.77)	-21.1975 (-1.29)	-28.0535 (-2.05)
a ₃₄₁			4.1272 (2.54)	-20.1252 (-1.56)	-30.1351 (-2.33)	10.4219 (.68)	25.5272 (1.99)
a ₃₄₂			3.9913 (2.30)	-21.7997 (-1.70)	-29.5761 (-2.29)	6.7407 (.43)	25.4743 (1.91)
a ₃₄₃			3.9266 (2.53)	-20.1276 (-1.57)	-30.0476 (-2.35)	10.4777 (.68)	25.1851 (1.91)
a ₃₄₄			2.7323 (2.01)	-20.9194 (-1.65)	-30.3671 (-2.35)	8.5579 (.55)	24.4663 (1.89)
a ₃₄₅			.5820 (.34)	-23.309 (-1.81)	-32.726 (-2.51)	9.8609 (.64)	24.8269 (1.91)
a ₃₄₆			-9.9658 (-2.72)	-26.6065 (-1.92)	-32.2177 (-2.30)	.8714 (.05)	22.6419 (1.67)
a ₃₄₇			1.9029 (1.01)	-32.1899 (-1.86)	-32.5799 (-1.87)	8.5278 (.55)	25.5077 (1.94)
a ₃₄₈			3.7195 (2.18)	-22.1578 (-1.72)	-29.0803 (-2.24)	6.2470 (.40)	25.3685 (1.90)

Table 14 (Cont.) - Parameter Estimates for Logit Models of Fuel Choice*

Para-Meters	1.	2.	3.	4.	5.	6.	7.
a ₃₄₉			1.3916 (.36)	-21.8905 (-1.75)	-35.0585 (-2.82)	6.0806 (.37)	24.5529 (1.77)
b ₁	2.2725 (12.34)	.5417 (5.21)	1.8141 (4.9713)	1.8156 (4.99)	1.6900 (7.60)	.3041 (2.56)	.4907 (7.56)
b ₂	.8944 (3.17)	.4771 (3.97)	.5944 (1.7412)	.6123 (1.79)	.7995 (4.58)	.4669 (3.64)	.3873 (5.44)
b ₃	-5.3071 (-9.86)	-4.4968 (-8.78)	-.4209 (-.28)	2.9883 (1.30)	-1.2887 (-.57)	2.6761 (1.95)	-.4017 (-.35)
b ₃₆				-6.0953 (-2.02)	-2.6134 (-.8789)	-7.9501 (-1.44)	-1.4466 (-.32)
b ₃₇				9.6689 (.69)	4.8105 (.34)	7.0841 (.61)	2.7512 (.29)
b ₃₁₀				-83.861 (-1.90)	-108.186 (-2.4118)	-3.6752 (-.30)	-18.9185 (-1.78)
b ₄	-.7499 (-11.28)	-.2943 (-6.22)	-.1179 (-.51)	-.0400 (-.17)	-.4433 (-2.58)	1.0990 (2.45)	-.4533 (-1.76)
c ₁₄	-.0003 (-11.46)	-.0001 (-5.28)	.0003 (3.48)	.0003 (3.44)	-6.2086 (-1.01)	.3622 (4.86)	-.0216 (-.43)
c ₂₄	-.0002 (-.4.02)	-5.5186 (-2.68)	-.0008 (-6.95)	-.0009 (-6.95)	-.0002 (-2.92)	-1.1744 (-13.76)	-.0556 (-.84)
c ₃₄	.0004 (.99)	-.0015 (-2.89)	.0011 (.72)	.0009 (.59)	.0002 (.11)	2.3579 (1.58)	.2892 (.25)
d ₁		5.4454 (18.83)			4.8088 (15.54)		.8485 (20.60)
d ₂		4.3915 (27.31)			4.5694 (21.04)		.9188 (19.21)
d ₃		47.6408 (5.96)			43.7486 (3.89)		.6385 (10.45)
d ₄		.6455 (3.21)			-1.0096 (-4.45)		.1156 (1.13)
Eqn 1 R ²	.603	.8744	.7901	.7921	.9325	.8091	.9468
Eqn 2 R ²	.3798	.8831	.7277	.7247	.9346	.8291	.9605
Eqn 3 R ²	.4655	.5330	.6177	.6160	.6573	.5872	.7763

Table 15 (Cont.)

Elas.	Country	Estimate	Source
Price Elas. (cont.)	Canada	$\eta_{KK} = -0.76$, $\eta_{LL} = -0.49$, $\eta_{EE} = -0.49$, $\eta_{KE} = -0.05$, $\eta_{LE} = 0.55$	(g)
	Netherlands	$\eta_{KK} = 0.05$, $\eta_{LL} = -0.26$, $\eta_{EE} = -0.90$	(e)
	9 industrialized countries	$\eta_{KK} = -0.18$ to -0.38 , $\eta_{LL} = -0.12$ to -0.27 , $\eta_{EE} = -0.79$ to -0.80 , $\eta_{KE} \approx 0.13$, $\eta_{LE} \approx 0.11$	(f)
	six-country composite	$\eta_{EE} = -0.30$	(h)
Fuels -			
<u>own price</u> <u>elastici-</u> <u>ties, partial</u>	U.S.	elec: -0.66 , oil: -2.75 gas: -1.30 , coal: -1.46	(i)
	U.S.	elec., S.R.: -0.14 elec., L.R.: -1.20	(j)
	U.S.	elec., S.R.: -0.06 elec., L.R.: -0.52	(k)
<u>own price</u> <u>elastici-</u> <u>ties, total</u>	U.S.	elec: -0.92 , oil: -2.82 , gas: -1.47 , coal: -1.52	(i)
	Canada	elec: -0.74 , oil: -1.30 , gas: -1.30 , coal: -0.48	(g)

Sources:

- (a) Berndt and Wood [9]
- (b) Halvorsen and Ford [37]
- (c) Fuss and Waverman [31], translog
- (d) Fuss and Waverman [31], generalized Leontief
- (e) Magnus [46]
- (f) Griffen and Gregory [34]
- (g) Fuss [30]
- (h) Nordhaus [50]
- (i) Halvorsen [36]
- (j) Mount, Chapman, and Tyrrell [49]
- (k) Griffen [33]

Table 15 - Alternative Estimates of Industrial Demand Elasticities

Elas.	Country	Estimate	Source
Factor Inputs - elasticities of substitution	U.S.	$\sigma_{KL} = 1.01, \sigma_{KE} = -3.25,$ $\sigma_{LE} = 0.64$	(a)
	U.S. (2-digit industries)	$\sigma_{KE} = -1.03$ to $2.02,$ $\sigma_{LE} = .48$ to 2.88 (prod. workers), $\sigma_{LE} = -2.02$ to 5.59 (non-prod. workers)	(b)
	Canada	$\sigma_{KL} = 0.72, \sigma_{KE} = 0.42,$ $\sigma_{LE} = 1.70$	(c)
	Canada	$\sigma_{KL} = 5.46, \sigma_{KE} = -11.91,$ $\sigma_{LE} = 4.89$	(d)
	Netherlands	$\sigma_{KL} = 0.30, \sigma_{KE} = -4.50,$ $\sigma_{LE} = 3.80$	(e)
	9 industrialized countries	$\sigma_{KL} = 0.06$ to $0.52,$ $\sigma_{KE} = 1.02$ to $1.07,$ $\sigma_{LE} = 0.72$ to 0.87	(f)
Factor Inputs - price elasticities	U.S.	$\eta_{KK} = -0.44, \eta_{LL} = -0.45,$ $\eta_{EE} = -0.49, \eta_{KE} = -0.15,$ $\eta_{LE} = 0.03$	(a)
	U.S. (2-digit industries)	$\eta_{KK} = -0.67$ to $-1.16,$ $\eta_{LL} = -0.28$ to $-1.55,$ $\eta_{EE} = -0.66$ to -2.56	(b)
	Canada	$\eta_{KK} = -0.79, \eta_{LL} = -0.45,$ $\eta_{EE} = -0.36$	(c)
	Canada	$\eta_{KK} = -0.31, \eta_{LL} = -0.77,$ $\eta_{EE} = -0.59$	

Note that there is mixed evidence on the substitutability of energy and capital. Berndt and Wood [9], Fuss [30], and Magnus [46] find energy and capital to be strong complements, but they worked with time series data for a single country, and might have estimated a short-run cost or production function. Halvorsen and Ford [37] and Fuss and Waverman [31] obtain mixed results on energy-capital substitutability, depending on the particular disaggregated industry or the particular form of the cost function. Only Griffen and Gregory [34] find strong evidence of capital-energy substitutability, and their estimate of the Allen elasticity of substitution is close to ours (1.01 compared to about 0.8). This is reassuring since both their study and ours use international data and presume to estimate long-run elasticities. As for elasticities of substitution between other factors, our results are close to Griffen and Gregory for labor and energy, but we find greater substitution of capital and labor.

The own price elasticity of aggregate energy use is an important parameter to any energy policy debate. Our estimate (about -0.8), together with those of Magnus (-0.9) and Griffen and Gregory (also -0.8) are larger than most other estimates, which fall in the range of -0.3 to -0.6. Again, most other estimates are based on time series data for a single country, and may be short-run.

It is more difficult to find a consensus on partial and total fuel price elasticities. Although most would agree that electricity demand is less elastic than the demands for other fuels, partial long-run elasticities for the U.S. range from -0.5 to -1.2. Our study finds electricity demand to be even less elastic; we found partial own price elasticities to range from -.08 to -.16. Our total own price elasticity estimates, however, are closer to the estimates of others (largely because of our higher estimate of the own price elasticity of energy). We find this elasticity to range from -0.54 to -0.63, where Halvorsen [36] obtained an estimate of -0.92 and Fuss [30] -0.74. Our own price elasticity estimate (total) for oil is also well below the estimates of others; -.22 to -1.17 as com-

pared to Halvorsen's estimate of -2.82 and Fuss's of -1.30. An explanation for this discrepancy will probably require further work. There is less disagreement over the elasticities for coal and natural gas. Our estimates of the total own price elasticities for coal (-1.29 to -2.24) and natural gas (-0.41 to -2.34) are generally in line with other estimates.

7. Summary

We have seen that the use of a "two-stage" weakly separable cost function provides means of estimating demand elasticities for aggregate energy use and for individual fuels. In addition, by pooling international time series-cross-section data we can obtain a sample large enough to provide low-variance estimates of essentially long-run elasticities.

We have found that the own-price elasticity of aggregate industrial energy demand is larger than had been thought previously, and that energy and capital appear to be substitutes, rather than complements. We attribute these results to the long-run nature of our estimates. We found the total own-price elasticities of coal and natural gas to be large, as expected, but we found the total own-price elasticities of oil and electricity to be below 1 in magnitude. While we expect the small elasticity for electricity (there is little flexibility in its use), it is harder to justify the elasticities for oil.

We also found that the aggregate cost functions are mildly, but significantly, non-homothetic, so that the elasticity of aggregate energy use with respect to output changes is below 1 (generally around .7 or .8). This is not due to economies of scale in the long-run (we found cost functions to exhibit constant returns to scale), but rather to substitution away from energy as output increases. Finally we found that further increases in the price of energy would have only a small impact on the total cost of production. This is due in part to energy's small share in production, and in part to substitution possibilities.

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