INTERNATIONAL COMPARISONS OF THE RESIDENTIAL DEMAND FOR ENERGY*

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1. Introduction

This paper reports on some results from an econometric study of the world demand for energy, the objectives of which have been to estimate the determinants of total energy demand and interfuel substitution in the residential and industrial sectors of several industrialized countries. Here we concentrate on the residential sector, and examine some very preliminary estimates of inter-country differences in the structure of demand.[1]

Our approach here is to assume that consumers make two decisions in purchasing fuels. First, with the amount of money to be spent on energy taken as given, consumers decide which fuels to purchase, i.e. the fractions of energy expenditures allotted to oil, natural gas, coal, and electricity. Next, they decide which fraction of their total budget will be spent on energy, as opposed to such other consumption categories as food, clothing, etc.[2] Thus we assume (and we will empirically test this assumption) that consumers' utility functions are separable between energy and other commodities, i.e. that expenditure shares on fuels may depend on total energy expenditures, but are independent of the expenditure shares for other consumptions categories.[3]

Estimating the demand for energy requires a model for the breakdown of total consumption expenditures. A number of such models have been constructed by others, some of them additively consistent (i.e. shares add to one) and some

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[1]Our work on the industrial sector is reported in Pindcyk [71].

[2]We do not treat energy as a derived demand determined by the stock of energy-using appliances (in fact durable goods are a separate consumption category). In the dynamic models to be discussed later the effects of changes in appliance stocks will be included implicitly.

[3]We thus have a "utlity tree" along the lines described by Strotz [60], and the marginal rate of substitution between any two variables in the class of energy expenditures is independent of the expenditure on any other consumption category.

inconsistent.[4] Typical model choices have included the additive logarithmic model, the linear expenditure system, and the additive quadratic model.[5] Usually these models have been estimated using time series data for single countries, but in some cases cross-country comparisons have been made using pooled time series-cross section data for a number of countries.[6]

We will extend this work by estimating versions of the indirect translog utility function with pooled data. The advantage of this functional form is that it is a general approximation to any indirect utility function, and therefore its use does not a priori impose contraints of homotheticity or additivity. We will also estimate alternative models for the breakdown of fuel and consumption expenditures, including static and dynamic logit models of consumer choice.

In estimating models of consumption expenditures we wish to explore the extent to which higher energy prices might reduce the total consumption of energy. Although energy prices did not increase substantially during the period covered by our data (through 1974), there is enough cross-sectional variation in prices to allow us to obtain long-run price and income elasticity estimates. In addition, by estimating models with country dummy variables and/or alternative groupings of countries, we can determine the extent to which elasticities vary across countries.

In estimating the demand for individual fuels we will also test a number of alternative model structures. We will again use our pooled data to estimate both static and dynamic versions of the indirect translog utility function. In doing so we can obtain unrestricted estimates of own-price, cross-price, and total expenditure elasticities, and test for homotheticity, additivity, and stationarity. Also, by estimating a translog model that includes both non-

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[4]For an overview of models of consumer behavior see Brown and Deaton [11] and Philips [52].

[5]See Houthakker [30], Pollak and Wales [55], Houthakker and Taylor [32], Philips [52], and Theil [63].

[6]See Houthakker [31], and Goldberger and Gameletsos [23].

energy consumption expenditures and fuel expenditures we can test for energy separability.

We also use the multinominal logit model to break energy expenditures down into fuel shares. One advantage of this model is that it is relatively easy to estimate; as long as the share data represents aggregated samples of individuals decisions (i.e. average shares for a large number of consumers) rather than individual decisions, ordinary or generalized least squares can be used. Another advantage of the logit model is that it allows considerable flexibility for working a dynamic structure into the specification. The logit model also has disadvantages, however. Estimates become inefficient when there are zeros in the share data. In addition, all cross-elasticities for a given price are equal; as Hausman and Wood have shown [26], they are the sum of the price elasticity for total expenditure minus the own price elasticity weighted by the share. This fact that cross elasticities are determined by total and own elasticities is restrictive, but must less so than the restrictions inherent in the linear expenditure system, additive quadratic model, and other "consistency" models.

In estimating demand models for individual fuels we will explore the extent to which fuel shares shift in response to price changes and changes in total energy expenditure over both the short- and long-run. We have found that there is enough variation in prices through the combined use of time-series and cross-section data to obtain low-variance elasticity estimates, and determine the extent to which elasticities vary across countries.

As one might expect, in work like this we are continually bound by data limitations. For many countries there is no good data available for some or all of the variables of interest to us. For other countries data exists, but obtaining that data can be an extremely time consuming and laborious task, so that choices had to be made as to which data were to be collected. These data limitations were one of the factors that helped define and delimit the modelling approaches used here. In particular, it necessitated restricting our detailed analysis of demand to a small set of countries.[7] Even for these countries, however, the quality of the data varies, and compromises had to occasionally be made.[8] The data used in this study is described briefly in this report; a much more detailed description is provided in a separate report entitled "A User's Guide to the MIT World Energy Demand Data Base."[9]

In the next section we outline alternative specifications of alternative models of residential energy demand, and discuss the characteristics of each specification. Section 3 discusses some methodological issues in the estimation of energy demand models using pooled data. These issues include the use of purchasing power parities to make international comparisons, the question of accounting for thermal efficiencies in the use of energy consumption data, the formulation of an aggregate price index for energy, and the use of alternative estimation methods. Section 4 describes some of the characteristics and limitations of our data, Section 5 includes the statistical results, and Sections 6 and 7 provide a comparison of our results with those of other studies, and a summary.

2. Alternative Specifications for Models of Residential Energy Demand

As explained above, all of our models of residential energy demand involve a two-stage approach where first consumption expenditures are broken down into energy and other consumption categories, and second energy expenditures are broken down into expenditures on fuels. We begin here by reviewing the pro-

[7] A much less datailed model of the demand for petroleum products is being constructed for a number of "secondary" countries for which only partial data is available; the results of this work will be described in a forthcoming paper.

[8] For example, one of our consumption expenditure categories is "food, alcohol, and tobacco." For some countries a price index is available only for food, and this index was used since food is by far the largest component of the category.

[9] Working Paper #MIT EL 76-011WP, MIT World Oil Project, May 1976. Revised and updated version to appear shortly.

perties of the indirect translog utility function with a time trend and discuss its application to both stages of the residential model. Next we describe some alternative dynamic specifications of the indirect translog utility function. As we will see, these specifications will permit us to explicitly include stock adjustment or habit formation effects. We then discuss the multinominal logit model and its application to the estimation of fuel shares. Finally we discuss alternative model specifications, including some simple logarithmic models that will be used for countries where our data is more limited.

2.1 Use of the Indirect Translog Utility Function

The indirect translog utility function is a second-order approximation to any indirect utility function. The indirect translog function with time-varying preferences, introduced by Jorgenson and Lau [37], has the form:[10]

$$\begin{aligned}\log V = \alpha_0 + \sum_i \alpha_i \log(P_i/M) + \alpha_t t + \tfrac{1}{2}\sum_{ij}\sum \beta_{ij} \log(P_i/M)\log(P_j/M) \\ + \sum_i \beta_{it} \log(P_i/M)\cdot t + \tfrac{1}{2}\beta_{tt}\cdot t^2\end{aligned} \tag{1}$$

When the indirect translog function is used to model expenditure shares for energy and non-energy consumption categories, P_i is the price index for consumption category i and M is total consumption expenditures. When this function is used to model fuel shares, P_i is the price of fuel i, and M is total expenditures on energy. The indirect translog function implies the budget share equations:

$$S_j = \frac{P_j X_j}{M} = \frac{\alpha_j + \sum_i \beta_{ji} \log(P_i/M) + \beta_{jt}\cdot t}{\alpha_M + \sum_i \beta_{Mi} \log(P_i/M) + \beta_{Mt}\cdot t}, \quad j = 1, \ldots, n \tag{2}$$

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[10] The indirect translog utility function without time was introduced by Christensen, Jorgenson, and Lau [15]. The homothetic form of the indirect translog function was used by Christensen and Manser [14] to study consumer preferences for food, and the non-homothetic form was used by Jorgenson [35] to study a three-category breakdown of consumer expenditures in the United States. Berndt, Darrough, and Diewert [70] demonstrated empirically that the translog specification is more robust than other generalized functional forms such as the generalized Leontief or generalized Cobb-Douglas utility functions.

where X_j is the quantity consumed of category i (or fuel i), t is a time trend (equal to zero at the beginning of the estimation period), and

$$\alpha_M = \sum_k \alpha_k , \quad \beta_{Mi} = \sum_k \beta_{ki} , \quad \beta_{Mt} = \sum_k \beta_{kt} .$$

Note that the parameters α_0, α_t, and β_{tt} in equation (1) do not affect the utility-maximizing quantities consumed, and therefore cannot be identified. In addition note that the budget constraint implies that $\Sigma S_j = 1$, so that only (n-1) of the share equations need be estimated to determine all of the parameters.

The budget share equations are homogeneous of degree zero in the parameters, and therefore a parameter normalization is required for estimation. We use the normalization $\alpha_M = \Sigma\alpha_k = -1$. A number of parameter restrictions are also required if the share equations are indeed based on utility maximization. In particular, the parameters β_{Mi} and β_{Mt} must be the same in each of the n share equations. Since there are (n+1) parameters involved, and (n-1) equations are estimated, this implies a total of $\frac{1}{2}$(n+1)(n-1) restrictions. Also, we assume that that log V is twice differentiable in its arguments, so the Hessian of log V must be symmetric. This implies the following $\frac{1}{2}$(n-1)(n-2) symmetry restrictions:

$$\beta_{ij} = \beta_{ji}, \quad i \neq j, \; i,j = 1, \ldots, n \tag{3}$$

There are an additional (n-1) restrictions resulting from the fact that the parameters of the n^{th} equation are determined from the parameters of the first (n-1) equations and the definitions of β_{Mi} and β_{Mt}. Thus, the total number of parameter restrictions is $\frac{1}{2}n(n-1)$.

There are other restrictions that might be imposed on the indirect translog function, and tests can be performed to determine whether such restrictions are supported by the data. We will test some of these restrictions in this work, so we list them here.

The indirect translog function is stationary if preferences do not change with time.[11] Stationarity implies that the parameters β_{jt} are all equal to zero, j=1, . . ., n.[12]

In estimating the consumption breakdown model we can test for groupwise separability between energy and the other consumption categories. Letting P_1 and S_1 be the price index and expenditure share for energy, and $P_2, P_3, \ldots, P_n$ and $S_2, S_3, \ldots, S_n$ be prices and shares for the other categories, separability would imply that the underlying indirect utility function can be written as

$$\log V = F(\log V^1(P_2/M, P_3/M, \ldots, t), P_1/M, t) \quad (4)$$

If the underlying indirect utility function is groupwise separable, then the following restrictions must hold:[13]

$$\beta_{12} = \rho_1\alpha_2, \quad \beta_{13} = \rho_1\alpha_3, \ldots, \quad \beta_{1n} = \rho_1\alpha_n \quad (5)$$

where ρ_1 is a constant. Even if the underlying indirect utility function is groupwise separable, the translog approximation need not be. Explicit groupwise separability ensures that the translog approximation is also groupwise separable. This requires the additional restriction that $\rho_1 = 0$.

We will also estimate share equations based on homothetic indirect utility functions.[14] Under homotheticity the budget shares S_j are independent of total

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[11] A dynamic translog function, in which long-run elasticities differ from short-run elasticities, may still be stationary as long as the elasticities themselves do not depend on the particular time in which prices or income change. This is discussed further later.

[12] Note that stationarity is equivalent to explicit neutrality. An indirect utility function is explicitly neutral if it can be written as

$$\log V = \log V^1(P_1/M, P_2/M, \ldots, P_n/M) + F(t)$$

[13] See Jorgenson and Lau [37] for a derivation of these restrictions.

[14] V is homothetic if it can be written as $\log V = F(\log H(P_1/M, \ldots, P_n/M, t), t)$ where H is homogeneous of degree 1.

expenditures M, which implies that the income elasticities of demand for every commodity are the same and equal to unity. We can test for homotheticity in our model of total consumption expenditures, but the model of fuel expenditures must be assumed to be homothetic to be consistent with a two-stage model of consumer spending. The underlying indirect utility function is homothetic if

$$\beta_{Mj} = \sigma\alpha_j, \quad j = 1, \ldots, n \tag{6}$$

where σ is a constant. Explicit homotheticity will ensure that the translog approximation is also homothetic, and this requires the additional assumption that $\sigma = 0$. If the indirect utility function is explicitly homothetic and $\beta_{Mt} = 0$, then it is also homogeneous.

Finally, it is straightforward to test for explicit additivity, since a necessary and sufficient condition for explicit additivity in the commodities is that the indirect translog function is explicitly groupwise separable in any pair of commodities from the remaining commodity.[15] Thus the parameter constraints are that $\beta_{ij} = 0$, $i \neq j$.

We can test these parameter restrictions using a simple chi-square test. The appropriate test statistic is

$$- 2 \log \Lambda = n(\log|\hat{\Omega}_r| - \log|\hat{\Omega}_u|) \tag{7}$$

where $|\hat{\Omega}_r|$ and $|\hat{\Omega}_u|$ are the determinants of the estimated error covariance matrices for the restricted and unrestricted models respectively. This statistic is distributed as chi-square with degrees of freedom equal to the number of parameter restrictions being tested.

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[15] An indirect translog utility function is explicitly additive if it can be written in the form

$$\log V = \log V^1(P_1/M,t) + \ldots + \log V^n(P_n/M,t).$$

It is important to remember that there are only certain ranges of inputs over which the indirect translog utility function is a meaningful approximation to the underlying utility function. Consider, for example, the marginal utility of income (or of total expenditure), $\lambda = \partial V/\partial M$:

$$\lambda = \frac{V}{M}\frac{\partial \log V}{\partial \log M} = -\frac{V}{M}\sum_i (\alpha_i + \sum_j \beta_{ij}\log\frac{P_j}{M} + \beta_{ij}\cdot t) \tag{8}$$

The α_i sum to -1 by the normalization, while the β_{ij} can be positive or negative. If some β_{ij} are positive, then as M becomes zero λ can become negative, and if some β_{ij} are negative, then as M becomes increasingly large λ can become negative. Thus, there are ranges of input space for which the translog approximation may not be meaningful. It is important therefore to check estimated translog models by determining whether the marginal utility of income is positive over the range of historical (and forecasted) input data.

After estimating a model for the breakdown of consumption expenditures, it is useful to compute Frisch's welfare indicator. This indicator is simply the income elasticity of the marginal utility of income, i.e. $\eta_{\lambda M} = \partial\log\lambda/\partial\log M$. For a utility function that is well-behaved over the <u>entire</u> input space (which the translog is not) $\eta_{\lambda M}$ would range from a large negative number (when M is zero) to zero (as M approaches infinity). Taking the log of equation (8) and differentiating, we have for the indirect translog function:[16]

$$\eta_{\lambda M} = -1 - \frac{\sum_i\sum_j \beta_{ij}}{\sum_i \alpha_i + \sum_{ij}\sum \beta_{ij}\log\frac{P_j}{M} + \sum_i \beta_{it}\cdot t} \tag{9}$$

[16] The same equation also applies to the <u>direct</u> translog utility function.

As $M \to 0$, we see that $\eta_{\lambda M} \to -1$, so that a very small level of income is clearly out of the "meaningful" range. The same is true for $M \to \infty$. If the utility function is homothetic, however, this indicator is independent of income (but time-varying if the utility function is non-stationary). Since for most of the models that we will estimates, the β_{ij} and β_{it} parameters will be held constant across countries, $\eta_{\lambda M}$ would also be constant across countries.

We also need formulas for the calculation of income and price elasticities. The income elasticity of demand for good j, $\eta_{jM} = \partial \log X_j / \partial \log M$, is found by multiplying the share equation (2) by M/P_j and differentiating:

$$\eta_{jM} = 1 + \frac{\sum_i \beta_{Mi} - \sum_i \beta_{ji}/S_j}{\alpha_M + \sum_i \beta_{Mi} \log \frac{P_i}{M} + \beta_{Mt} \cdot t} \quad (10)$$

Note that this is also the formula for the expenditure elasticity $\partial \log(P_j X_j)/\partial \log M$. The own price elasticity $\eta_{jj} = \partial \log X_j / \partial \log P_j$ is

$$\eta_{jj} = -1 + \frac{\beta_{jj}/S_j - \beta_{Mj}}{\alpha_M + \sum_i \beta_{Mi} \log \frac{P_i}{M} + \beta_{Mt} \cdot t} \quad (11)$$

and the cross price elasticities $\eta_{ji} = \partial \log X_j / \partial \log P_i$ are

$$\eta_{ji} = \frac{\beta_{ji}/S_j - \beta_{Mi}}{\alpha_M + \sum_i \beta_{Mi} \log \frac{P_i}{M} + \beta_{Mt} \cdot t} \quad (12)$$

These elasticity formulas are based on the assumption that total expenditure stays constant. They can be applied to each stage of our two-stage demand model, but they cannot be used to determine the total effect of a change in price or income on the demand for a particular fuel. If the price of oil changes, there will be a change in total expenditures on energy, and this will also effect the demand for oil. The <u>total</u> own price elasticity $\eta^*_{jj} = d\log X_j/d\log P_j$ is given by

$$\eta^*_{jj} = \frac{P_j}{X_j}\left[\frac{\partial X_j}{\partial P_j} + \frac{\partial X_j}{\partial M_E}\frac{\partial M_E}{\partial P_E}\frac{\partial P_E}{\partial P_j}\right] \tag{13}$$

where P_j and X_j are the price and quantity of fuel j, M_E is expenditures on energy, and P_E is the price index for energy. To determine the total elasticity we therefore need an expression for the price of energy in terms of the prices of individual fuels. Since fuels are not perfect substitutes we cannot determine P_E as a simple weighted average of the fuel prices. Instead we view P_E as the cost of producing heat from fuel inputs, and use a translog cost function with constant returns to model this "production" process:

$$\log P_E = \gamma_o + \sum_i \gamma_i \log P_i + \sum_i \sum_j \gamma_{ij} \log P_i \log P_j \tag{14}$$

This is an energy price aggregator, and can be determined up to a scalar γ_o by estimating the share equations[17] $S_i = \gamma_i + \sum_j \gamma_{ij} \log P_j$. Given equation (14) for the price of energy, we have

$$\frac{\partial P_E}{\partial P_j} = \frac{P_E}{P_j} S_j \tag{15}$$

We can thus compute η^*_{jj} from the fact that

$$\frac{\partial X_j}{\partial P_j} = \frac{X_j}{P_j}\eta_{jj} \tag{16}$$

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[17] We will discuss the energy price aggregator in more detail later.

where η_{jj} is the partial own price elasticity for fuel j given by equation (11),

$$\frac{\partial X_j}{\partial M_E} = \frac{X_j}{M_E} \eta_{jM_E} \qquad (17)$$

where η_{jM_E} is the expenditure elasticity for fuel j given by equation(10), and

$$\frac{\partial M_E}{\partial P_E} = X_E(1 + \eta_{EE}) \qquad (18)$$

where X_E is the total quantity of energy consumed and η_{EE} is the partial own price elasticity of energy consumption. Now substituting (15), (16), (17), and (18) into (13), we have

$$\eta^*_{jj} = \eta_{jj} + \eta_{jM_E}(1+\eta_{EE}) \; . \qquad (19)$$

We can similarly compute the <u>total</u> cross price elasticity η^*_{ji} from

$$\eta^*_{ji} = \frac{P_i}{X_j}\left[\frac{\partial X_j}{\partial P_i} + \frac{\partial X_j}{\partial M_E}\frac{\partial M_E}{\partial P_E}\frac{\partial P_E}{\partial P_i}\right] = \eta_{ji} + S_i\eta_{jM_E}(1+\eta_{EE}) \qquad (20)$$

and the total income elasticity η^*_{jM} from

$$\eta^*_{jM} = \frac{M}{X_j}\frac{\partial X_j}{\partial M_E}\frac{\partial M_E}{\partial M} \qquad (21)$$

Note that since $\partial M_E/\partial M = (M_E/M)\eta_{EM}$, where η_{EM} is the income elasticity of energy expenditures, we obtain

$$\eta^*_{jM} = \eta_{jM_E} \cdot \eta_{EM} \qquad (22)$$

Since the indirect utility function for fuels is assumed to be homothetic, this reduces to $\eta^*_{jM} = \eta_{EM}$.

2.2 Dynamic Versions of the Indirect Translog Utility Function

A problem with the static translog model is that they do not explain differences between short-run and long-run elasticities. Even when the time trend is included in the indirect utility function, the model is not dynamic - tastes can change slowly over time, but there is no dynamic (lagged) response in demand to a sudden change in price. Thus adjustments (while possibly non-stationary) are assumed to occur instantaneously.

There are two basic approaches that can be used to introduce dynamic adjustments into the translog utility function. The first involves specifying the translog approximation to the utility function (direct or indirect) to include lagged quantities, prices, or shares. The advantage of this approach is that "adding up" is always preserved in the resulting share equations without the introduction of additional parameter constraints. The disadvantage is that the translog approximation makes the dynamic specification somewhat arbitrary.

As an example of this approach, we could write the indirect translog utility function as:

$$\log V = \alpha_0 + \sum_i \alpha_i \log(P_i/M) + \tfrac{1}{2}\sum_{ij}\sum \beta_{ij} \log(P_i/M)\log(P_j/M) + \sum_i d_i \log(P_i/M) D_{i,t-1} \tag{23}$$

where $D_{i,t-1}$ is a lagged term in price, quantity, or share that is considered an exogenous input to the determination of current share.[18] Logical choices for

[18] We are assuming that consumers determine their budget shares via static utility maximization, i.e. they maximize utility at each instant of time ignoring the future, rather than maximizing the sum over time of discounted utilities. The $D_{i,t-1}$ (together with current prices and income) simply represent the current state of the world. As shown by Hoel [28], even in a static model dynamic utility maximization can result in a different marginal utility of income.

$D_{i,t-1}$ would be the quantity $X_{i,t-1}$ or the share $S_{i,t-1}$.[19] Applying Roy's identity[20] to equation (23) yields the share equations:

$$S_j = \frac{P_jX_j}{M} = \frac{\alpha_j + d_jD_{j,t-1} + \sum_i \beta_{ji}\log(P_i/M)}{\alpha_M + \sum_i d_iD_{i,t-1} + \sum_i \beta_{Mi}\log(P_i/M)}, j=1,\ldots,n \tag{24}$$

where α_M and β_{Mi} are defined as before. Note that unless all of the d_i are zero, the homothetic form of equation (24) - for which the β_{Mi} are zero - is nonlinear in the parameters. As a result, estimation of (24) can be costly, even under the assumption of homotheticity. The shares in (24) will always add to one, however, even if lagged shares are used as the $D_{i,t-1}$, and - assuming that the errors are not serially correlated - the parameter estimates will be invariant to the choice of share that is dropped.

A second approach that can be used is to introduce the dynamic adjustment directly into the share equations. This has the advantage of facilitating the use of simple and intuitively pleasing adjustment mechanisms. It has the disadvantage that "adding up" will not be preserved unless additional (and possibly highly restrictive) parameter restrictions are introduced.

We consider two examples of this approach. In the first, each quantity is assumed to adjust to a desired level:

$$X_{i,t} = X_{i,t-1} + \delta_i(X^*_{i,t} - X_{i,t-1}) \tag{25}$$

where $X^*_{i,t}$ is the desired quantity of commodity i as determined from static utility maximization, and δ_i is an adjustment parameter. This yields the share equations

$$S_{j,t} = \delta_j S^*_{j,t} + (1-\delta_j)S_{j,t-1}\left[\frac{P_{j,t}/M_t}{P_{j,t-1}/M_{t-1}}\right] \tag{26}$$

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[19] The form of equation (23) using lagged quantity $X_{i,t-1}$ was suggested by Manser [44,45], who applied it to the estimation of food demand.

[20] See [57]. The identity is:

$$\frac{P_iX_i}{M} = S_i = \frac{\partial \log V/\partial \log P_i}{\partial \log V/\partial \log M}$$

or, using the indirect translog function for $S^*_{j,t}$,

$$S_{j,t} = \frac{\alpha'_j + \sum_i \beta'_{ji}\log(P_i/M)}{\alpha_M + \sum_i \beta_{Mi}\log(P_i/M)} + (1-\delta_j)S_{j,t-1}\left[\frac{P_{j,t}/M_t}{P_{j,t-1}/M_{t-1}}\right] \tag{27}$$

The parameters of the share equations (27) are estimated subject to the constraints $\beta'_{ij} = \beta'_{ji}$, $\beta_{Mj} = \sum_k \beta'_{kj}/\delta_k$ are the same in each equation, and $\alpha_M = \sum_k \alpha'_k/\delta_k = -1$.

Note that the shares $S_{j,t}$ need not add to one. Adding up can be imposed by estimating only n-1 of the share equations, and determining the parameters of the n^{th} equation from $\Sigma S_{j,t} = 1$, but the estimated parameters will depend on the particular equation that is not estimated. Despite this deficiency, however, the specification of equation (27) permits the introduction of dynamic adjustments in a simple and appealing manner.

Alternatively, we can assume that the shares adjust to the desired shares as follows:[21]

$$S_{i,t} = S_{i,t-1} + \sum_j \delta_{ij}(S^*_{j,t} - S_{j,t-1}) \tag{28}$$

Adding up requires that the sum of all changes in shares be zero:

$$\sum_i (S_{i,t} - S_{i,t-1}) = 0, \tag{29}$$

so that

$$\sum_i\sum_j \delta_{ij}(S^*_{jt} - S_{j,t-1}) = \sum_j (S^*_{j,t}\sum_i \delta_{ij} - S_{j,t-1}\sum_i \delta_{ij}) = 0 \tag{30}$$

Since the $S^*_{j,t}$ and $S_{j,t-1}$ sum to one, this equation implies the necessary condition that <u>all of the columns of the matrix</u> (δ_{ij}) <u>sum to the same arbitrary constant</u>, i.e.

[21] This approach was suggested by Leonard Waverman in the context of dynamic adjustments in the translog production function.

$$\ell'\delta = c\ell' \tag{31}$$

where ℓ is a vector of 1's (ones), δ is the matrix (δ_{ij}), and c is an arbitrary constant.[22] Note that if the number of shares is greater than two, there are alternative constraints on the δ_{ij} that can be imposed to satisfy (31). Note also that (31) implies that δ cannot be diagonal unless the adjustment coefficients for every share are the same, so that the adjustment of the i^{th} commodity share would generally not depend only on that share, but would depend on other shares as well.

2.3 Multinominal Logit Models for Fuel Choice

Multinominal logit models have already been used to study the breakdown of energy consumption into demands for fuels in the United States[23] and Canada.[24] Although the logit model is not based on assumptions of utility maximization, it has properties that make it appealing for this work. The model is consistent in terms of shares adding to one, and shares respond to price changes in a way that is intuitively appealing; as the share of, say, natural gas becomes small, it requires increasingly large price changes to make it still smaller. Finally the logit model is easy to estimate and permits us to easily introduce a variety of alternative dynamic specifications.

We can write the logit model for the four fuel breakdown (oil, gas, coal and electricity) as follows:[25]

$$\frac{Q_i}{Q_T} = \frac{e^{f_i(x\beta)}}{\sum_{j=1}^{4} e^{f_j(x\beta)}} \tag{31}$$

[22] For a discussion of adding up conditions for more general lag structures, see Wall [56], and Berndt and Savin [9].

[23] See Baughman and Joskow [6].

[24] See Fuss and Waverman [68].

where Q_i is the quantity (in tcals) of fuel i, $Q_T = \Sigma Q_i$, and the f_i are functions of a vector of attributes X and vector of parameters β. Given this model, the relative shares of any two fuels can be represented as

$$\log(Q_i/Q_j) = \log(S_i/S_j) = f_i(x\beta) - f_j(x\beta). \tag{33}$$

Note that only three equations are estimated, since the parameters of the fourth equation are determined from the adding up constraint.

In estimating fuel shares we include as attributes the relative price of each fuel. The relative oil price, for example, is the ratio of the real price of oil to the real price of energy, the latter being measured by the translog price aggregator described earlier. Other attributes may include per capita income, average temperature, and lagged quantity variables that allow shares to adjust dynamically to changes in price. Functional forms for the f_i are somewhat arbitrary, but in the simplest model they might be linear functions of the relative fuel prices $\tilde{P}_i = P_i/P_E$, where P_E is the aggregate price of energy, as well as income Y and temperature T:

$$f_i(x\beta) = a_i + b_i\tilde{P}_i + c_iY + d_iT . \tag{34}$$

This yields the three estimating equations

$$\log(S_i/S_4) = (a_i-a_4) + b_i\tilde{P}_i - b_4\tilde{P}_4 + (c_i-c_4)Y + (d_i-d_4)T, \quad i=1,2,3 \tag{35}$$

Note that these three equations must be estimated simultaneously, with b_4 constrained to be the same in each equation.

---

[25] In effect we are assuming that consumer preferences are represented by a choice index which for the i^{th} fuel, has the form $f_i(x\beta)+\varepsilon_i(x)$, where ε_i is an error term. Then the probability that a consumer would choose fuel i is

$$P_i = \text{Prob}[f_i(x\beta)+ \varepsilon_i(x) > f_j(x\beta)+ \varepsilon_j(x)] \text{ for } i \neq j.$$

If the error terms $\varepsilon_i(x)$ are independently and identically distributed with the Weibull distribution

$$\text{Prob}[\varepsilon_i(x) < \varepsilon] = e^{-e^{-\varepsilon}},$$

then the probability that fuel i will be chosen is given by equation (32). For further discussion, see McFadden [46], Domencich and McFadden [19], Cox [16], Theil [62], and Chapter 8 of Pindyck and Rubinfeld [53]. For an interesting application to aggregate demand analysis, see Park [51].

The simplest means by which the preference functions can be made dynamic (e.g. to account for habit formation or stock adjustments) is to include the lagged share:

$$f_i(x\beta) = a_i + b_i\tilde{P}_i + c_i S_{i,t-1}. \tag{36}$$

The three estimating equations are then

$$\log(S_i/S_4) = (a_i - a_4) + b_i\tilde{P}_i - b_4\tilde{P}_4 + c_i S_{i,t-1} - c_4 S_{4,t-1}, \quad i=1,2,3. \tag{37}$$

Note that two lagged shares appear in each equation. The three equations must again be estimated simultaneously, with both b_4 and c_4 constrained to be the same in each equation.[26]

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[26] It is straightforward to calculate income and price elasticities of shares for the static logit model. From equation (35) we can obtain the income elasticity for a model with n shares as follows:

$$dS_k/S_k - dS_n/S_n = (c_k - c_n)dY$$

Since $\sum dS_k = 0$, we have that

$$\sum_{k=1}^{n-1} dS_k = \sum_{k=1}^{n-1} \left\{ \frac{S_k}{S_n} dS_n + S_k(c_k - c_n)dY \right\} = -dS_n \quad \text{, or,}$$

$$dS_n/S_n = c_n dY(1-S_n) - dY \sum_{k=1}^{n-1} c_k S_k \quad .$$

After some manipulation, this reduces to

$$\eta_{iY}^S = \frac{dS_i/S_i}{dY/Y} = (c_i - c_n)Y - \sum_{k=1}^{n-1} (c_k - c_n)S_k Y \quad .$$

To obtain the own price elasticities of shares, note that

$$dS_k/S_k - dS_n/S_n = b_k dP_k \quad .$$

Then using the fact that $\sum dS_k = 0$ and $\sum S_k = 1$, we can obtain:

$$\eta_{ii}^S = \frac{dS_i/S_i}{dP_i/P_i} = b_i(1-S_i)P_i \text{ , } i=1, \ldots, n$$

We do not derive analytical expressions for long-term and short-term dynamic elasticities. Instead, these elasticities can be calculated by simulating the dynamic model.

2.4 Other Models of Residential Energy Demand

When working with pooled cross-section time series it is often difficult to separately identify short-term and long-term effects, and determine the relative contributions to each from the cross-section versus time series variation in the data. This can be particularly true in the case of the translog function, where nonlinear estimation is involved. It is therefore useful to estimate logarithmic demand equations for expenditures on each consumption category and each fuel. Houthakker in his 1965 study [31] separated short-run and long-run elasticities by running separate regressions across countries and across time. We can apply Houthakker's approach to our own consumption data, and also to the estimation of fuel demands.[27] In addition, we will specify a dynamic version of Houthakker's basic model that should enable us to isolate the cross-section versus time series contributions to lag adjustments in demand.

The basic demand equation for commodity i is

$$\log q_{ijt} = \alpha_i + \beta_i \log y_{jt} + \gamma_i \log p_{ijt} + \delta_i t + \varepsilon_{ijt} \tag{38}$$

where j is the country index and t the time index, q_i is per capita expenditure on commodity i at constant prices, y is total consumer expenditure at constant prices per capita, and p_i is the relative price of commodity i. These equations can initially be estimated using simple weighted least squares, e.g. dividing each observation by the population of the country in the year concerned.

Short-run and long-run effects can be identified by estimating "within country" and "between country" regressions. The "within country" (short-run) regressions is

[27] Houthakker's consumption breakdown had only five categories: food, clothing, rent, durables, and "other." We use our own consumption breakdown for which energy is a separate category. Goldberger and Gamaletsos [23] also reestimated Houthakker's log-log demand functions, but using pre- 1961 data and the same categories that Houthakker used. What is more interesting is their estimation of a linear expenditure system using the same data, and the comparison of the two demand systems. The approach was also used by Gregory and Griffen [24] to identify international differences and intertemporal change in industry structure.

$$\log q_{ijt} - \overline{\log q_{ij}} = \beta_i(\log y_{jt} - \overline{\log y_j}) + \gamma_i(\log p_{ijt} - \overline{\log p_{ij}}) \quad (39)$$

and the "between country" (long-run) regression is

$$\overline{\log q_{ij}} = \beta_i\overline{\log y_j} + \gamma_i\overline{\log p_{ij}} + \delta_i \quad (40)$$

Here the bar represents averaging over time. Note that the "within country" regression is pooled, while the "between country" is purely cross sectional, and that deviations from means (over time) are used in the "within country" regressions in order to eliminate long-run effects. Equation (39) can also be run for each country separately, in order to determine how elasticities vary across countries. Alternatively, a pooled regression can be performed, with a multiplicative district dummy variable introduced to one coefficient at a time, e.g.:

$$\log q_{ijt} - \overline{\log q_{ij}} = \sum_j \beta_{ij} D_j(\log y_{jt} - \overline{\log y_j}) + \gamma_i(\log p_{ijt} - \overline{\log p_{ij}}) \quad (41)$$

Equations (39) and (40) can also be estimated in first-differenced form. This crudely reduces trend effects, and also eliminates problems associated with the arbitrary choice of purchasing power parities. The "within country" version is:

$$\Delta\log q_{ijt} - \overline{\Delta\log q_{ij}} = \beta_i(\Delta\log y_{jt} - \overline{\Delta\log y_j}) + \gamma_i(\Delta\log p_{ijt} - \overline{\Delta\log p_{ij}}) \quad (42)$$

and the "between country" version is:

$$\overline{\Delta\log q_{ij}} = \beta_i\overline{\Delta\log y_j} + \gamma_i\overline{\Delta\log p_{ij}} \,. \quad (43)$$

We can also specify a dynamic version of (39). We assume that demand q_{it} depends not only on price and income in period t, but also on a "state variable" s_{it}:

$$\log q_{it} = a_i + b_i\log y_t + c_i\log p_{it} + d_i\log s_{it}. \quad (44)$$

If the demand is for durables, then s_{it} will represent a stock, and d_i should be negative. If the demand is for a non-durable commodity, then s_{it} will represent a "habit" level, and d_i should be positive. The dynamics of s_{it} can be expressed as

$$\Delta \log s_{it} = \log q_{it} - w_i \log s_{i,t-1} \qquad (45)$$

where w_i is effectively a depreciation rate. To obtain the demand equation rewrite equation (44) as

$$s_{it} = \frac{1}{d_i}[\log q_{it} - a_i - b_i \log y_t - c_i \log p_{it}] \qquad (46)$$

which can be substituted into equation (45):

$$\Delta \log s_{it} = \log q_{it} - \frac{w_i}{d_i}[\log q_{i,t-1} - a_i - b_i \log y_{t-1} - c_i \log p_{i,t-1}] \qquad (47)$$

Now first-difference equation (46):

$$\Delta \log s_{it} = \frac{1}{d_i}[\Delta \log q_{it} - b_i \Delta \log y_t - c_i \Delta \log p_{it}]. \qquad (48)$$

Substituting this into (47) and rewriting, we have the estimating equation

$$\log q_{it} = \alpha_0 + \alpha_1 \log q_{i,t-1} - \alpha_2 \Delta \log y_t + \alpha_3 \log y_{t-1} - \alpha_4 \Delta \log p_{it} + \alpha_5 \log p_{i,t-1}. \qquad (49)$$

This equation can also be estimated within countries and between countries. If this results in differences in the estimated value of α_1, it would indicate that adjustment response is not constant over time.

3. Methodological Issues in the Estimation of Residential Demand Models

There are a number of problems that must be considered in estimating the models described in the previous section. These result from the fact that pooled international data are being used to obtain estimates, and from the nature of the models themselves. First, the comparison of expenditures or prices in different countries requires valuing different currencies in terms of a common unit. Although using purchasing power parities for this purpose is probably more desirable than using official exchange rates, the choice of a particular index is not always clear. Second, a choice must be made whether to value energy quantities in gross or net terms, i.e. whether to adjust for the thermal efficiencies of different fuels. Next, an energy price index must be obtained that accounts for fuel choice differences across countries. Finally, there are a number of econometric issues associated with the estimation of our models. We examine these problems in this section.

3.1 Use of Purchasing Power Parities

Since all of the price and expenditure data for each country in our sample are measured in terms of the local currency of that country, a method is needed to convert these numbers into common units. One method which has been used by a number of researchers is to simply use official exchange rates.[28] This can be misleading, however, since official rates can differ considerably from equilibrium exchange rates, and tariffs, quotas, subsidies, and other controls can result in price structures that differ considerably from relative international prices. Alternatively, one could attempt to identify "free market" exchange rates between individual countries over time periods thought to reflect equilibrium conditions, e.g. during which trade balances were near zero. Even under free trade, however, equilibrium exchange rates only reflect the price equalization of internationally traded goods, which for most countries represent a small subset of all market goods.[29]

A better approach is to use purchasing power parities (PPP's) to convert national currencies to some base currency. Purchasing power parities can be obtained explicitly by making binary comparisons between a base country (e.g.

[28] This approach has been used by Adams and Griffen [1], and Goldberger and Gameletsos [23].

[29] As Chenery and Syriquin [12] point out, the relative price of non-traded goods can be expected to increase with real per capita income, so that the use of official (or "free market") exchange rates leads to an underestimate of the purchasing power of the currencies of lower income countries.

the U.S.) and various other countries, using a fixed set of quantity weights.[30] The problem, of course, is that two sets of price index numbers (Laspeyre and Paasche) can be obtained depending on whether base country or other country weights are used. In this work we use a Fisher "ideal" index (a geometric mean of these two index numbers) as a single index of relative purchasing power.[31]

We use purchasing power parities by consumption category calculated by the German Statistical Office (Statistiches Bundesamt [59]) by means of detailed price comparisons.[32] These are binary index numbers for which Germany is the base country, and we therefore use Germany as a "bridge" to convert to the U.S. as base country.[33] The resulting parities apply to a base year, but we must construct intertemporal indices to deflate our time series. We do this using implicit price indices for each consumption category in each country, thus constructing an implicit ratio of relative intertemporal purchasing power in terms of a base year numeraire (normalized so that 1970 is our base year). The resulting base year purchasing power parities for each consumption category are shown in Table 1.[34]

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[30] Purchasing power parities can also be obtained implicitly by dividing a nominal national currency estimate of national product (or one of its components) by a base currency estimate of the same national product. This procedure was used recently by Lluch and Powell [42]. For a general discussion of explicit purchasing power parities, see Balassa [4] and Allen [3].

[31] The use of a Fisher "ideal" index is suggested on theoretical grounds by Samuelson [58] and on empirical grounds by Kloek and Theil [38].

[32] Binary purchasing power parities were more recently calculated by Kravis et al. [39], but for only a subset of the countries in our sample.

[33] Note that although binary PPP's permit us to make a transitive international ordering of purchasing powers, this ordering is not invariant with respect to the choice of "bridge" country. Kravis et. al. [39] also calculated multilateral PPP's by means of a regression model that estimates the purchasing power parity for a single category of expenditure as a function of all other international price ratios. Again, some of the countries in our sample are not included in the Kravis study.

[34] For those countries also covered by Kravis et al., our numbers are at all times within 10 percent of the 1970 Kravis numbers.

Table 1 - Base Year (1970) Purchasing Power Parities

Consumption Category / Country	Total Consumption	Apparel	Food	* Durables	** Energy	Transportation and Communication	*** Other
Belgium	43.50	38.60	48.18	43.44	43.50	46.03	43.50
Canada	1.08	0.88	1.13	0.85	1.08	1.08	1.08
France	4.64	4.23	5.49	4.00	5.13	5.99	4.64
Italy	493.00	385.03	616.65	423.47	465.80	558.85	493.00
Netherl.	2.75	2.34	3.03	2.30	1.47	3.32	2.75
Norway	6.36	5.06	8.01	4.34	4.30	6.12	6.36
U.K.	0.31	0.24	0.33	0.22	0.34	0.31	0.31
U.S.A.	1.00	1.00	1.00	1.00	1.00	3.40	1.00
W.Germany	3.32	2.40	3.88	2.61	2.84	3.68	3.32

*For Belgium, Canada, Netherlands, and Norway, no "durables" PPP exists. A PPP for "other household" was used in going from these countries to Germany, and the "durables" PPP was used in bridging from Germany to the U.S.

**For Belgium and the Netherlands, the PPP relative to Germany refers to "electricity, gas, and water."

***Other set equal to total consumption.

3.2 "Gross" versus "Net" Energy Consumption

As pointed out by Adams and Miovic [2], alternative fuels are not equivalent on a calorific basis as a result of the differing thermal efficiencies of energy consuming equipment. Since more efficient fuels are substituted for less efficient fuels over time, the measurement of an overall "energy elasticity" (i.e. the percent change in energy use associated with a 1 percent change in GNP) will yield a larger number if thermal efficiencies are taken into account.[35] This has led some individuals to suggest the use of "net" energy consumption (adjusted for thermal efficiencies) rather than gross energy consumption in the estimation of demand models. In recent studies of energy demand, for example, Nordhaus [48], Adams and Griffen [1], and Fuss and Waverman [68], made the assumption that within each sector fuels are perfect substitutes, so that (given equal levels of non-fuel cost) interfuel competition is determined by relative net prices of fuels. Net consumption and net price are given by

$$QN_{ij} = \eta_{ij} Q_{ij} \tag{50}$$

and

$$PN_{ij} = P_{ij}/\eta_{ij} \tag{51}$$

where η_{ij} is the efficiency of fuel i in sector j.

We see two problems with this approach. First, it is difficult to obtain reliable estimates of thermal efficiencies. Identification problems make econometric estimates infeasible unless unduly restrictive structural assumptions are imposed, and engineering estimates differ considerably from

[35] Adams and Miovic estimate an overall energy elasticity for the U.S. and several European countries of about 0.8 when gross energy quantities are used, and about 1.0 when energy quantities are adjusted for thermal efficiencies.

source to source.[36] As an example of this problem, we show in Table 2 engineering estimates of thermal efficiencies cited or used in four different studies. Note that these estimates differ considerably from study to study.

A second and more fundamental problem is that fuels are not perfect substitutes (particularly in the short run), and there are non-thermal efficiencies (which we could label "economic") that also affect consumer demand. Fuel choice is also based on convenience, controllability, cleanliness, capital costs, etc., and the effects of these "efficiencies" (as well as thermal ones) will hopefully be manifested in the estimated parameters of our demand models.

Table 2 - Alternative Engineering Estimates of Thermal Efficiencies

Citation or Use of Estimate	Adams and Miovic		Adams and Griffen		Nordhaus		Fuss and Waverman	
Fuel	Residential	Industrial	Residential	Industrial	Residential	Industrial	Residential	Industrial
Gas	.65-.72	.39	.60	.65	.70	.85	.75	.85
Solid	.05-.60	.33	.50	.45	.20	.70	.50	.87
Liquid	.65	.40	.65	.59	.60	.80	.65	.87
Electricity	.80	.80	--	.80	.95	.99	1.00	1.00

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[36] Adams and Miovic [2] attempted to measure thermal efficiences by assuming that fuel inputs are a constant proportion of aggregate economic output, and that there is no substitution between fuel inputs and labor and capital. Their production function was thus

$$Y = \min(\alpha F, f(L,K))$$

where fuel input F is given by $F = \sum_i \eta_i h_i F_i$, where h_i is the calorific content of fuel i (F_i). Since the h_i's are known, they can estimate the η_i's up to a scalar multiple. The assumptions are extremely restrictive, however, and their results differ considerably from engineering estimates that they cite. For an engineering discussion of thermal efficiencies, as well as a set of estimates, see Hottel and Howard [29].

It thus does not seem particularly relevant to measure fuel consumption in efficiency-adjusted thermal units, any more than it would be to measure food consumption in net calorific terms.[37]

We therefore choose to measure all of our energy quantities in "gross" rather than "net" terms. We assume that both thermal and non-thermal efficiencies have effects on interfuel competition, and that these effects will be picked up in the way that estimated fuel expenditure shares change as relative prices and income change.

3.3 A Price Index for Energy

Estimation of our consumption breakdown models requires a price index for energy, and since price series for individual fuels are available, it would be preferable to use this data rather than an implicit index constructed from nominal and real energy expenditure series. Since fuels are not perfect substitutes, however, a price index that truly reflects the unit cost of energy will not equal a simple weighted average of fuel prices. A typical approach is to construct an approximate Divisia index as a means of aggregation.[38] An alternative approach is to specify (and estimate) an aggregator function that relates the aggregate price index to the component prices. Any unit cost function could be used to represent the aggregate price of energy, but a logical choice is the translog cost function.[39] As an incidental advantage, the translog cost function (or "aggregator") provides us with an instrumental variable for estimation purposes.

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[37] This is discussed further by Turvey and Nobay [64].

[38] See Jorgenson and Griliches [36] and Hulten [34].

[39] This is appealing as an unrestrictive representation of unit cost. Also, as Diewert [8] has shown, the Divisia index is "exact" for the translog cost aggregator function, i.e. it retrieves the actual values of the function.

The translog cost function (which is equivalent to a homothetic and stationary indirect utility function with unit total expenditure) is given by

$$\log P_E = \gamma_o + \sum_i \gamma_i \log P_i + \sum_{ij}\sum \gamma_{ij} \log P_i \log P_j \tag{52}$$

Assuming cost-minimizing behavior, the fuel share equations are then

$$S_i = \gamma_i + \sum_j \gamma_{ij} \log P_j \quad , \; i=1,\ldots,n \tag{53}$$

The first (n - 1) share equations are estimated subject to the restrictions $\Sigma\gamma_i = 1$, $\gamma_{ij} = \gamma_{ji}$, and $\sum_i \gamma_{ij} = 0$. The estimated parameters γ_i and γ_{ij} are then substituted in equation (52). Using data for the fuel prices, P_i, the price index $\hat{P}_E$ can be computed. Note that the energy price index $\hat{P}_E$ is determined only up to an unknown scalar multiple γ_o. The procedure is to pick one country (say the U.S.) as a base country, and then solve equation (52) for γ_o so that the price of energy in the base country is equal to 1 in some base year (say 1970). Relative price indices are thereby determined for all of the other countries.

A problem remains regarding the number of fuels to be included in equations (52) and (53). Although four fuels are included in our demand model, almost no coal is consumed in the residential sectors of the U.S. and Canada. This suggests that equations (52) and (53) should apply to a three-fuel aggregation (oil, gas, and electricity) for the U.S. and Canada, and a four-fuel aggregation for the remaining countries. Should this approach be used--as opposed to a four-fuel aggregation for all countries--a method is needed to "bridge" the U.S.-Canadian aggregator with the aggregator for the remaining countries. We propose the following bridging method:

(1) Equation (53) is estimated for four fuels for all countries except the U.S. and Canada. The unidentified parameter γ_o in equation (52) is chosen so that the price of energy is equal to 1 in Belgium in 1970. This permits the calculation of the price of energy for all countries except the U.S. and Canada relative to Belgium in 1970.

(2) Equation (53) is estimated for four fuels for all nine countries. The parameter γ_o is chosen so that the price of energy is equal to 1 in the U.S. in 1970. The resulting index gives us the price of energy in Belgium in 1970 relative to that in the U.S. in 1970.

(3) Equation (53) is then estimated for three fuels (oil, gas, and electricity) for the U.S. and Canada only. The parameter γ_o is chosen so that the price of energy is equal to 1 in the U.S. in 1970. Now, using the Belgium-to-U.S. conversion ratio determined in step (2), the price indices calculated in step (1) are converted to a U.S. 1970 = 1 base.

Because the quantities of coal consumed in the residential sectors of the U.S. and Canada are small but not zero, it is not clear whether the bridging approach described above or a simple four-fuel aggregation across all countries should be used to construct the energy price index. We therefore estimate energy price indices using both methods. If the resulting indices are nearly the same, this would indicate that the relative size of the coal shares in the U.S. and Canada do not distort the fit of a fuel choice model that includes four fuels for all countries. In this case coal could be included for all countries in the fuel demand models. If the results are significantly different, then coal should not be included in the U.S. and Canada demand models, and the bridging method should be used to calculate the energy price index.

3.4 Identifying Inter-Country Differences in Elasticities

One of our objectives in estimating energy demand models is to determine the extent to which elasticities vary across countries, and the possible reasons for such variation. To identify regional variations in elasticities, we must

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specify alternative ways of allowing for regional parameter variation when our models are estimated with pooled data.

At the one extreme, we might assume that the parameters of our models are the same for all countries. Estimating the translog share equations (2) by simply pooling all of the data would restrict the parameters α_j, β_{ji}, and β_{jt}, to be the same in each country. The resulting elasticities would still vary across countries, but only because relative prices and total expenditures are different in different countries. At the other extreme we could estimate our models for each country separately; in the translog case the α_j's, β_{ji}'s, and β_{jt}'s could be different for every country. While this specification is least restrictive, it is likely to be infeasible due to insufficient data.

There are two compromise approaches that could be followed. One is to estimate models by pooling subsets of countries, so that parameters can differ across subsets but are the same within each subset. This might involve, for example, pooling the U. S. and Canada, pooling France, U.K., Italy and Germany, and pooling the Netherlands, Belgium and Norway. Re-estimation using alternative groupings could then be used to determine the validity of constraining parameters to be the same across countries in a subset.

A second approach is to pool all of the countries, but to introduce regional dummy variables that would allow a subset of a model's parameters to vary across countries. In the translog case, we might assume that the coefficients α_j of the first-order terms in the Taylor series approximation can vary across countries, while the coefficients β_{ji} and β_{jt} are the same for each country. This would involve estimating the following share equations:

$$S_j = \frac{\sum_k \alpha_{jk} D_k + \sum_i \beta_{ji} \log(P_i/M) + \beta_{jt} \cdot t}{\sum_k \alpha_{Mk} D_k + \sum_i \beta_{Mi} \log(P_i/M) + \beta_{Mt} \cdot t}, \quad j = 1, \ldots, (n-1) \qquad (54)$$

where D_k are country dummy variables (D_k = 1 for country k, and 0 otherwise). Note that the usual restrictions on the β_{ji} and β_{jt} apply, but $\alpha_{Mk} = \sum_j \alpha_{jk} = -1$ for each country k.

Alternatively we might assume that the coefficients β_{ji} of the second-order terms can vary across countries, while the α_j's and β_{jt}'s are the same for each country. The share equations are then

$$S_j = \frac{\alpha_j + \sum_{k}\sum_{i} \beta_{jik} D_k \log(P_i/M) + \beta_{jt} \cdot t}{\alpha_M + \sum_{k}\sum_{i} \beta_{Mik} D_k \log(P_i/M) + \beta_{Mt} \cdot t}, \quad j = 1, \ldots, (n-1) \tag{55}$$

Note that the restrictions on the β_{jik}'s are now that $\beta_{jik} = \beta_{ijk}$ for each country k, and β_{Mik} is the same in each share equation for every country k.

Finally, note that variables whose variation is largely regional (as opposed to time-wise) can be introduced in additional to the regional dummy variables. In the translog case, for example, we might assume that α_j is a function of temperature T (which has both regional and time-wise variation),

$$\alpha_j = a_j + b_j T \tag{56}$$

with a_j varying across countries, so that the share equations are

$$S_j = \frac{\sum_k a_{jk} D_k + b_j T + \sum_i \beta_{ji} \log(P_i/M) + \beta_{jt} \cdot t}{\sum_k a_{Mk} D_k + \sum_i b_i T + \sum_i \beta_{Mi} \log(P_i/M) + \beta_{Mt} \cdot t} \tag{57}$$

with $a_{MK} = \sum_i a_{ik} = -1$ for each country k.

We will estimate demand models by pooling data for alternative subsets of country, and by using regional dummy variables as described above. This should enable us to identify sources of inter-country elasticity variation.

3.5 Estimation Methods

The choice of estimation methods involves a trade-off between the richness of the stochastic specification (and hopefully a resulting gain in efficiency) and computational expense. This trade-off is particularly severe given that all of our models involve systems of equations (even though for some models, e.g. the log-log models, the systems are not consistent, i.e. "adding up" does not hold). Ideally one would like to estimate a stochastic specification for which the error terms are heteroscedastic and autocorrelated both across time and across countries within each equation, and are correlated across equations in the system. Estimating such a specification (which amounts to full generalized least squares), however, would be unreasonably costly even if the individual equations were linear in the parameters. If individual equations are nonlinear in the parameters (as is the case with our non-homothetic translog model), the estimation might be computationally impossible. We must therefore settle for a more restrictive specification that would hopefully capture the more important characteristics of the error terms.

When estimating translog models (which can be nonlinear in the parameters and/or have cross-equation parameter constraints that are nonlinear), we ignore error term heteroscedasticity and autocorrelation within equations, and account only for error correlations across equations.[40] In particular, we use iterative nonlinear Zellner estimation, which (under the assumption of no heteroscedasticity or autocorrelation within equations) is equivalent to full-information maximum-

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40 Accounting for within-equation heteroscedasticity and autocorrelation is certainly possible even if the equation is nonlinear in the parameters. One might use an algorithm that repeatedly linearized each equation and iteratively computed an error covariance matrix and estimated the linear equation for each linearization (see, for example, Eisner and Pindyck [20]). There is no guarantee, however, that final convergence would ever occur, and if it did the process would be extremely expensive.

likelihood estimation.[41] However we limit the number of iterations on the error covariance matrix to five; this reduces computational expense while still capturing at least 90% of the added efficiency that results from accounting for cross-equation error correlations.

Our logit models and logarithmic models are all linear in the parameters, and therefore our stochastic specification can be somewhat richer here. When estimating the equations of these models, we also account for within-equation heteroscedasticity. This is done using the following procedure. First each equation in the system is estimated using ordinary least squares. The resulting regression residuals, which we can label u_{kt}, are then used to obtain consistent estimates of the regional (country) error variances σ_k^2:

$$\hat{\sigma}_k^2 = \frac{1}{T-m-1} \sum_t (u_{kt})^2 \qquad (58)$$

where T is the number of annual observations for country k and m is the number of independent variables in the equation. Different estimates of the $\hat{\sigma}_k^2$'s will be of course obtained for each equation in the system. We then transform the data by dividing each observation by the appropriate estimated error term standard deviation $\hat{\sigma}_k$, and then re-estimate the entire system of equations using iterative Zellner estimation.[42]

All of our estimation work has been carried out at the Computer Research Center of the National Bureau of Economic Research, using the GREMLIN experimental nonlinear estimation package on the TROLL econometric software system.[43] This package permits one to perform iterative nonlinear Zellner estimation conveniently and with reasonable computational expense.

[41] See Zellner [67] and Gallant [22]. Oberhofer and Kmenta [49] prove that iterative Zellner estimation (iterating to convergence on the cross-equation error covariances) is equivalent to full-information maximum likelihood.

[42] MacAvoy and Pindyck [43] used a similar approach to single-equation estimation that also accounted for time-wise autocorrelation.

[43] For details on the estimation algorithm and its use, see Belsley [7]. For a discussion of alternative nonlinear estimation algorithms, see Berndt, Hall, Hall, and Hausman [8], Chow [13], and Gallant [22].

4. Characteristics of the Data

Although much of the data needed for this study could be obtained from such standard sources as the OECD or the U.N. Statistical Office, it was necessary to look elsewhere for some of the data. In some cases the need data (such as retail fuel prices) are not collected by the U.N. or the OECD, and in other cases the data have been collected, but have been aggregated or categorized in ways limiting their usefulness for this study. As a result it was often necessary or desirable to go to the national statistical yearbooks of individual countries to obtain data.

Nine countries are included in our sample: Belgium, Canada, France, Italy, the Netherlands, Norway, U.K., U.S., and West Germany. The data collected for these countries are described briefly below.[44]

> Consumption Expenditures. These are broken down into six categories: food (including alcohol and tobacco), clothing, durable goods, transportation and communication, energy, and "other." This last category includes housing expenditures (actual and imputed rental payments), expenditures on health services, and any other consumption expenditures. Data were obtained from the OECD's *National Accounts*, the national accounts publications of the EEC Statistical Office, the U.N. *Yearbook of National Accounts*, and national statistical yearbooks. The data are measured in current local currency units.

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[44] The data used here are part of a larger international energy data base assembled for use in this and several related studies. For a more detailed description of that data base, see [66]. Other researchers wishing to replicate or extend this study, or perform studies of their own, can access the data directly through the TROLL computer system of the NBER.

Price Indices for Consumption Expenditures. A retail price index (1970=100) was collected for each of the categories of consumption expenditures listed above. Although for some countries retail price indices were available directly, we constructed implicit price indices for all countries from consumption expenditure series in current and constant monetary units. Although price indices for energy are available, we use the energy price aggregator function described earlier in estimating our consumption breakdown model.[45] Data were obtained from the OECD's National Accounts, the U.N. National Accounts, the EEC's National Accounts publications, and the national statistical yearbooks of individual countries.

Fuel Expenditures. Data were collected for total residential consumption expenditures on petroleum products (largely light fuel oil), natural gas, coal, and electricity. The data through 1970 were generally obtained from the Statistical Office of the EEC's National Accounts publications, or from national statistical yearbooks. In a few cases for data before 1971, and for all data covering 1971-1974, figures were obtained by multiplying the retail price of the fuel by the physical quantity of the fuel consumed, with physical quantity data obtained from the OECD's Energy Statistics tape. The data are measured in current local currency units.

[45] The energy price aggregator requires data on fuel prices. For some countries our data on fuel prices does not go back as far as our consumption expenditure data. In these cases the estimated energy price aggregator was regressed against the implicit energy price index, so that data for the index could be used to extend the aggregator backwards.

Fuel Prices. For each fuel, the data represent countrywide averages of the average retail price. In the cases of natural gas and electricity, for countries with tariffs the price level chosen was the average price facing an average size household.[46] When the price of natural gas differs from that of manufactured gas, an average of the prices weighted by the relative amounts consumed was calculated. Data were obtained from the Statistical Office of the EEC's Energy Statistics and Studien und Erhabungen, publications and documents of the International Energy Agency, the Petroleum Times, and from national statistical yearbooks of individual countries. All prices were originally measured in local currency units per tcal of energy, but have been converted to 1970 U.S. dollers per tcal for estimation purposes.

Fuel Quantities. Some of our models use physical quantity data for fuels. All quantities are implicitly derived from data on fuel expenditures and fuel prices. Units are tcals.

Other Variables. Data were also collected for net disposable income, population, and temperature. The income data represent total net disposable income of all households, although for some countries only total private income data (personal income plus income going to non-profit institutions) were available. (These data are put into per capita terms for estimation of our demand models.) Income data were obtained from the OECD's National Accounts, and have been converted to 1970 U.S. dollars. Data for the total population of

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[46] Some researchers, e.g. Halvorsen [25], have used the marginal price of electricity as a measure of price. The marginal price alone is inappropriate, however, as has been demonstrated by Taylor [61]. The correct procedure is to use the average price at a normalized and constant rate of consumption, or to incorporate both average and marginal prices. We use only the average price, since that is the only data that is available.

each country came from the U.N. Demographic Yearbook, and are measured in millions of people. Finally, our temperature data represents the average temperature over the five winter months (November - March) averaged over the principle city or cities of each country. The source is the U.S. Weather Bureau's Monthly Climatic Data for the World, and the units of measurement are degrees Fahrenheit.

In Table 3 we show the range of the data that have been collected for all of the variables described above. This range does not necessarily represent the time bounds used in model estimation, however. In order to have roughly overlapping bounds, only a subset of the data is used for estimation work.

Our static translog models of consumption expenditures require consumption expenditure shares and price indices, and the models of fuel expenditures require data on fuel shares and fuel prices. It is useful to examine some of the share and price data before turning to the estimation results. Data for 1962 and 1972 are shown in Table 4. The price index for energy is not shown, since it is computed from the translog price aggregator, using the fuel prices.

Observe from Table 4 that the shares of energy in consumption expenditures remained fairly constant over time for most countries, but varies by as much as 100% across countries (from .03 in Canada and Italy to .06 in Belgium and the Netherlands). This same pattern is true for the clothing, durables, and transportation shares; only the food and "other" shares change significantly over time (decreasing and increasing respectively in every country). Since most of the variation in shares is across countries, we would expect our models of consumption expenditures to capture long-run elasticities. Fuel shares, on the other hand, show considerable variation both across time and across countries,

Table 3 - Range of Data

		BELG	CANADA	FRANCE	ITALY	NETH	NORWAY	UK	USA	W. GERMANY
CONSUMPTION EXPENDITURES	Food	55-74	50-74	58-74	55-74	55-74	55-74	55-74	50-74	50-74
	Clothing	55-74	50-74	58-74	51-74	55-74	50-74	57-74	50-74	55-74
	Durables	55-74	55-74	58-74	55-74	55-74	55-74	54-74	50-74	55-74
	Trans & Comm.	55-74	50-74	58-74	51-74	53-74	53-74	55-74	50-74	55-74
	Energy	60-74	61-74	60-74	60-74	50-74	50-74	57-74	50-74	59-74
	"Other"	60-74	61-74	60-74	60-74	53-74	53-74	57-74	50-74	59-74
CONSUMPTION EXPENDITURE PRICE SERIES	Food	55-74	50-74	58-74	55-74	55-74	55-74	57-74	50-74	55-74
	Clothing	55-74	50-74	58-74	55-74	55-74	53,55-74	57-74	50-74	55-74
	Durables	55-74	50-74	58-74	55-74	55-74	53,55-74	57-74	50-74	55-74
	Trans. & Comm.	55-74	53-74	58-74	55-74	55-74	53,55-74	57-74	50-74	55-74
	Energy	60-74	61-74	60-74	60-74	60-74	64-74	60-74	60-74	60-63 65-74
	"Other"	55-74	50-74	58-74	55-74	55-74	53,5-74	57-74	50-74	55-74
FUEL EXPENDITURES	Elec	60-74	50-74	60-74	60-74	60-74	50-74	57-74	50-74	55-74
	Liquid	60-74	58-74	60-74	60-74	60-74	50-74	57-74	50-74	56-74
	Solid	60-74	61-74	60-74	59-74	60-74	50-74	57-74	50-74	55-74
	Gas	60-74	50-74	60-74	60-74	59-74	50-74	57-74	50-74	59-74
FUEL PRICES	Elec	55-74	58-74	58-74	60-74	55-74	64-74	57-74	52-74	55-74
	Liquid	55-74	58-74	58-74	57-74	55-74	62-74	60-74	56-74	56-74
	Solid	58-74	55-74	58-74	55-74	55-74	53,55-74	57-74	50-74	55-74
	Gas	55-74	58-74	58-74	55-74	55-74	55-74	57-74	59-74	55-74
MISC.	Per. Disp. Income	55-74	50-74	50-74	51-74	50-74	55-74	53-74	50-74	50-74
	Population	50-74	50-74	50-74	50-74	50-74	50-74	51-74	50-74	50-74
	Temperature	55-74	54-74	56-74	55-74	55-74	55-74	55-74	55-74	55-74

and it is therefore more difficult to know a priori whether our estimated partial fuel price elasticities will be short- or long-term. One way to help determine this is to estimate a model using pooled annual data, and then repeat the estimation using only data at three or four year intervals. If the resulting estimates do not differ much, we can be more certain that the estimated elasticities are long-term.

5. Statistical Results

In this section we present the results of the estimation of the models set forth in Section 2. We begin by discussing the estimation of the translog energy price aggregator, and we present our estimates of relative energy price indices for the nine countries. Next we discuss the estimation results - and the implied demand elasticities - for the static translog models. We have not estimated the dynamic versions of the translog model; they are highly nonlinear, and we found the computational expense involved in estimating them to be inordinate. In addition, as we will see, we can expect that all of our elasticity estimates are long-term, so that intermediate-term elasticities can be found by applying a simple dynamic adjustment based on engineering estimates of appliance depreciation rates. Finally, we do not present estimates of the logarithmic models here; these models are applied to countries where limited data is available, and will be described in a forthcoming working paper.

We estimate a price index for energy using both of the methods described in Section 3.3. First, a translog price aggregator is estimated assuming a choice of four fuels in all nine countries. This version of the aggregator is in turn estimated in two ways. First the share equations (53) are estimated directly assuming that all of the parameters are the same across all countries, and second, the equations are estimated with regional intercept dummy variables that allow

Table 4 - Share and Price Data

		Expenditure Shares						Expenditure Price Indices					Fuel Shares				Fuel Prices			
		Clothing	Food	Durables	T&C	Energy	Other	Food	Clothing	Durables	T&C	Other	Solid Fuel	Liq. Fuel	Gas	Elec.	Solid Fuel	Liq. Fuel	Gas	Elec.
Belgium	1962	.10	.32	.11	.09	.06	.32	1.09	.82	1.11	.92	1.03	.56	.10	.15	.19	11069.70	6447.2	26241.30	86837.30
	1972	.08	.29	.15	.11	.06	.31	1.12	.88	1.01	1.05	1.06	.23	.30	.20	.27	11334.70	5204.8	22485.2	60981.50
Canada	1962	.09	.25	.08	.14	.03	.41	1.02	.79	.81	.90	.94	.03	.38	.16	.43	5623.65	4598.48	4589.02	19177.60
	1972	.09	.23	.09	.15	.03	.42	1.03	.83	.83	1.04	.97	.01	.36	.17	.46	6197.83	4656.45	3452.15	16247.50
France	1962	.10	.38	.09	.09	.04	.30	1.13	.79	.86	1.23	1.19	.44	.08	.24	.24	13555.70	6138.03	28675.80	70642.0
	1972	.09	.27	.08	.11	.04	.41	1.17	.91	.91	1.32	1.00	.09	.41	.20	.30	11772.40	6058.19	25380.90	55661.70
Italy	1962	.093	.44	.08	.08	.03	.28	1.22	.75	.88	1.05	1.01	.16	.09	.28	.47	8940.68	5311.31	26918.90	76763.9
	1972	.10	.32	.06	.11	.031	.38	1.26	.78	.93	1.18	.99	.03	.22	.26	.49	9784.66	3902.12	19798.00	46565.10
Neth	1962	.12	.34	.15	.04	.05	.30	1.05	.82	.81	1.10	1.17	.35	.25	.15	.25	10857.10	5184.33	23915.0	61273.1
	1972	.10	.27	.11	.08	.06	.38	1.17	.83	.88	1.21	1.01	.02	.20	.60	.18	11228.20	4482.82	11803.10	32805.20
Norway	1962	.15	.37	.11	.10	.03	.24	1.18	.72	.72	.81	.94	.09	.14	.01	.76	6831.32	6529.05	13293.80	ng
	1972	.1	.30	.08	.12	.04	.36	1.27	.79	.71	.96	.99	.07	.20	.01	.72	8500.93	4822.79	19986.80	12839.30
UK	1962	.09	.37	.08	.10	.05	.31	1.04	.65	.79	.91	.94	.39	.07	.18	.36	6096.33	8084.18	19472.60	35356.40
	1972	.08	.31	.08	.14	.05	.34	1.1	.75	.73	1.04	1.01	.18	.06	.28	.48	7607.69	7489.38	12215.0	30656.60
U.S.A.	1962	.08	.26	.14	.03	.04	.45	.99	.97	1.14	.95	.94	.01	.32	.26	.41	8506.91	5527.97	3535.53	34725.20
	1972	.08	.21	.15	.04	.04	.48	1.01	.99	1.03	1.03	1.01	.0	.30	.23	.47	7907.33	6077.64	3760.37	24909.2
West Germany	1962	.13	.32	.14	.10	.04	.27	1.16	.65	.83	1.01	1.03	.11	.20	.11	.58	10063.40	5271.72	24685.10	66068.60
	1972	.114	.284	.14	.12	.06	.282	1.18	.71	.82	1.13	.96	.03	.22	.21	.54	11116.60	3661.70	31729.10	44369.7

U.S. 1970 = 1.00

Units: 1970 U.S. $/tcal

the parameters γ_i to vary across countries. The resulting estimated parameters are shown in columns (1) and (2) respectively in Table 5. The chi-square test (equation (7)) is used to test the restrictive hypothesis of "homogeneity," i.e. γ_i parameters constant across countries. The test statistic is 336.0, which, with eight degrees of freedom, is significant at the 1% level, leading us to reject "homogeneity" and include dummy variables in the estimation of the price aggregator.

Next a price index is estimated using the bridging method described in Section 3.3, so that there is a choice of three fuels in Canada and the U.S., and four fuels in the other countries. First, a four-fuel aggregator is estimated for the seven European countries (including country dummy variables), and the parameter γ_0 is chosen so that the price of energy is equal to 1 in Belgium in 1970. The resulting parameter estimates are shown in column (3) of Table 5. Next, the four-fuel model estimated previously for all nine countries is used to compute the price of energy in Belgium in 1970 relative to that in the U.S.; this price is 1.9315. Next, the price aggregator is estimated for three fuels for the U.S. and Canada, and the parameter γ_0 is chosen so that the price of energy is equal to 1 in the U.S. in 1970. The estimation results are shown in column (4) of Table 5. Finally, using the Belgium-to-U.S. ratio of 1.9315, the four-fuel price indices calculated for the European countries are converted to a U.S. 1970 = 1 base.

The resulting energy price indices computed under the two methods are shown in Table 6 for all nine countries. Note that for all countries except the U.S. the two indices are quite close to each other. For the U.S. the bridging method yields an unrealistically low price of energy in the early years. This is due in large part to a heavy positive weight attached to natural gas, based on a coefficient (α_{33}) that is not statistically significant. We therefore use the four-fuel price index for all countries in estimating our consumption breakdown models.

Table 5 - Parameter Estimates: Translog Energy Price Aggregator

Parameters	1. Four Fuels Nine Countries	2. Four Fuels, Nine Countries, Dummies	3. Four Fuels, Seven Countries, Dummies	4. Three Fuels, U.S. & Canada, Dummies
α_1	-0.0438 (0.0380)			
$\alpha_1 D_1$		0.0657 (0.0488)	0.0450 (0.0536)	
$\alpha_1 D_2$		-0.0454 (0.0360)		
$\alpha_1 D_3$		-0.0051 (0.0457)	-0.0193 (0.0501)	
$\alpha_1 D_4$		-0.2445 (0.0493)	-0.2634 (0.0545)	
$\alpha_1 D_5$		-0.0191 (0.0405)	-0.0327 (0.0430)	
$\alpha_1 D_6$		-0.0872 (0.0308)	-0.0873 (0.0349)	
$\alpha_1 D_7$		-0.0104 (0.0450)	-0.0338 (0.0517)	
$\alpha_1 D_8$		-0.0186 (0.0452)		
$\alpha_1 D_9$		-0.2516 (0.0478)	-0.2653 (0.0532)	
α_2	0.2517 (0.0229)			
$\alpha_2 D_1$		0.2922 (0.0399)	0.3359 (0.0426)	
$\alpha_2 D_2$		0.5236 (0.0282)		0.2902 (0.0226)
$\alpha_2 D_3$		0.2673 (0.0404)	0.2991 (0.0434)	
$\alpha_2 D_4$		0.2498 (0.0408)	0.2867 (0.0439)	
$\alpha_2 D_5$		0.3546 (0.0352)	0.3921 (0.0372)	
$\alpha_2 D_6$		0.1469 (0.0260)	0.1339 (0.0290)	
$\alpha_2 D_7$		0.1547 (0.0302)	0.1906 (0.0336)	
$\alpha_2 D_8$		0.5336 (0.0356)		0.1949 (0.0279)
$\alpha_2 D_9$		0.2923 (0.0416)	0.3166 (0.0453)	
α_3	0.0824 (0.0299)			
$\alpha_3 1$		0.5983 (0.0411)	0.7015 (0.0433)	
$\alpha_3 D_2$		0.2865 (0.0363)		0.0665 (0.0234)
$\alpha_3 D_3$		0.6416 (0.0393)	0.7421 (0.0420)	
$\alpha_3 D_4$		0.7065 (0.0414)	0.8091 (0.0442)	

Table 5 (Cont.) - Parameter Estimates: Translog Energy Price Aggregator

Parameters	1.	2.	3.	4.
$\alpha_3 D_5$		0.6367 (0.0346)	0.7269 (0.0361)	
$\alpha_3 D_6$		0.1947 (0.0275)	0.2251 (0.0294)	
$\alpha_3 D_7$		0.5380 (0.0327)	0.5957 (0.0345)	
$\alpha_3 D_8$		0.3492 (0.0494)		0.1335 (0.0327)
$\alpha_3 D_9$		0.5767 (0.0410)	0.6753 (0.0444)	
α_4	0.7097 (0.0403)			
$\alpha_4 D_1$		0.0439 (0.0634)	-0.0820 (0.0429)	
$\alpha_4 D_2$		0.2353 (0.0548)		0.6433 (0.0291)
$\alpha_4 D_3$		0.0963 (0.0544)	-0.0220 (0.0389)	
$\alpha_4 D_4$		0.2881 (0.0426)	0.1680 (0.0419)	
$\alpha_4 D_5$		0.0279 (0.0365)	-0.0860 (0.0356)	
$\alpha_4 D_6$		0.7456 (0.0199)	0.7282 (0.0203)	
$\alpha_4 D_7$		0.3177 (0.0348)	0.2734 (0.0347)	
$\alpha_4 D_8$		0.1352 (0.0485)		0.6715 (0.0377)
$\alpha_4 D_9$		0.3829 (0.0365)	-0.7266 (0.0389)	
α_0	9.066	9.115	9.773	9.161
α_{11}	-0.3701 (0.0424)	-0.2596 (0.0450)	-0.2797 (0.0529)	
α_{12}	0.1257 (0.0239)	0.0007 (0.0292)	0.0105 (0.0342)	
α_{13}	0.0915 (0.0169)	0.1537 (0.0271)	0.1469 (0.0292)	
α_{14}	0.1529 (0.0265)	0.1051 (0.0244)	0.1222 (0.0264)	
α_{21}	0.1257 (0.0239)	0.0007 (0.0292)	0.0105 (0.0342)	
α_{22}	-0.0292 (0.0179)	-0.0032 (0.0278)	-0.0118 (0.0320)	0.0555 (0.0180)
α_{23}	-0.0933 (0.0108)	0.1032 (0.0212)	0.1459 (0.0397)	-0.1062 (0.0117)
α_{24}	-0.0032 (0.0150)	-0.1007 (0.0178)	-0.1445 (0.0192)	0.0507 (0.0163)
α_{31}	0.0915 (0.0169)	0.1537 (0.0271)	0.1469 (0.0292)	

Table 5 (Cont.) - Parameter Estimates: Translog Energy Price Aggregator

Parameter	1.	2.	3.	4.
α_{32}	-0.0933 (0.0108)	0.1031 (0.0212)	0.1459 (0.0237)	-0.1062 (0.0117)
α_{33}	-0.0747 (0.0152)	-0.1241 (0.0316)	-0.1181 (0.0325)	0.0303 (0.0194)
α_{34}	0.0764 (0.0160)	-0.1328 (0.0229)	-0.1747 (0.0227)	0.0759 (0.0157)
α_{41}	0.1529 (0.0265)	0.1051 (0.0244)	0.1222 (0.0264)	
α_{42}	-0.0032 (0.0150)	-0.1007 (0.0178)	-0.1445 (0.0192)	0.0507 (0.0163)
α_{43}	0.0764 (0.0160)	-0.1328 (0.0229)	-0.1747 (0.0227)	0.0759 (0.0157)
α_{44}	-0.2260 (0.0283)	0.1283 (0.0262)	0.1970 (0.0254)	-0.1266 (0.0203)
RSQ				
Eqn. 1	0.374	0.795	0.729	
Eqn. 2	0.435	0.711	0.518	0.674
Eqn. 3	0.142	0.746	0.782	0.944

Table 6 - Energy Price Indices

(1) Four fuels for all countries (2) "Bridging" method - three fuels for U.S. and Canada

	Belgium (1)	Belgium (2)	Canada (1)	Canada (2)	France (1)	France (2)	Italy (1)	Italy (2)	Netherlands (1)	Netherlands (2)
1955	-	-	-	-	-	-	-	-	1.9248	1.9121
1956	-	-	-	-	-	-	-	-	1.9585	1.9391
1957	-	-	-	-	-	-	-	-	1.9853	1.9725
1958	2.2344	2.2902	0.9979	0.9730	2.6068	2.6279	-	-	1.9747	1.9810
1959	2.1719	2.1979	1.0060	0.9863	2.6541	2.6594	-	-	1.9404	1.9519
1960	2.2227	2.2473	0.9867	0.9601	2.6727	2.6864	3.4913	3.5180	1.8876	1.9000
1961	2.1314	2.1422	0.9750	0.9466	2.6431	2.6561	3.3705	3.4002	1.7850	1.7970
1962	2.1230	2.1377	0.9580	0.9341	2.5045	2.5042	3.2250	3.2484	1.7264	1.7228
1963	2.0643	2.0681	0.9410	0.9275	2.4536	2.4566	3.0037	3.0198	1.7030	1.6922
1964	2.1289	2.1411	0.9243	0.9057	2.4551	2.4657	2.9525	2.9746	1.6783	1.6723
1965	2.1164	2.1247	0.9044	0.8858	2.3630	2.3741	2.9405	2.9636	1.5520	1.5507
1966	1.9856	2.0003	0.8615	0.8492	2.3431	2.3700	2.8788	2.9075	1.5588	1.5855
1967	1.9263	1.9349	0.8754	0.8409	2.3381	2.3800	2.7040	2.6913	1.5365	1.5535
1968	1.8922	1.9073	0.8905	0.8499	2.2264	2.2341	2.6092	2.5928	1.5238	1.5334
1969	1.8523	1.8649	0.8763	0.8371	2.1808	2.1852	2.4359	2.4048	1.4067	1.4080
1970	1.9315	1.9315	0.8685	0.8244	2.1790	2.2074	2.3128	2.2801	1.4082	1.4047
1971	2.0218	2.0170	0.8771	0.8432	2.0735	2.0861	2.5631	2.5457	1.3974	1.3920
1972	1.8737	1.8520	0.8544	0.8325	2.1048	2.0719	2.4661	2.4439	1.3054	1.3013
1973	1.8229	1.7725	0.8493	0.8483	1.9570	1.9174	2.4280	2.3978	1.2894	1.2578
1974	1.7878	1.7490	1.0105	1.0324	1.8190	1.7645	2.2953	2.2734	1.3352	1.2812

	Norway (1)	Norway (2)	UK (1)	UK (2)	USA (1)	USA (2)	W. Germany (1)	W. Germany (2)
1955	-	-	-	-	-	-	-	-
1956	-	-	-	-	-	-	3.7413	3.8100
1957	-	-	-	-	-	-	3.7579	3.7949
1958	-	-	-	-	-	-	3.7123	3.8358
1959	-	-	-	-	1.3976	1.0934	3.6608	3.7903
1960	-	-	1.5776	1.5579	1.3657	1.0960	3.3493	3.4658
1961	-	-	1.5561	1.5320	1.3430	1.1060	3.2496	3.3752
1962	-	-	1.6193	1.5972	1.3149	1.0879	3.2551	3.3444
1963	-	-	1.6501	1.6299	1.2849	1.0847	3.1409	3.2135
1964	1.1937	1.2013	1.6712	1.6584	1.2604	1.1141	2.9953	3.0864
1965	1.1455	1.1505	1.7199	1.7190	1.2173	1.0996	2.8751	2.9644
1966	1.1287	1.1349	1.7157	1.7203	1.1764	1.0912	2.7529	2.8463
1967	1.1579	1.1672	1.7820	1.7897	1.1387	1.0728	2.6015	2.6198
1968	1.1515	1.1597	1.8508	1.8680	1.0869	1.0495	2.7237	2.7226
1969	1.1308	1.1300	1.7315	1.7466	1.0365	1.0183	2.5262	2.5289
1970	0.9742	0.9548	1.6798	1.6958	1.0000	1.0000	2.4103	2.3912
1971	1.1702	1.1611	1.7092	1.7320	1.0097	1.0215	2.3652	2.3108
1972	1.1134	1.1006	1.7616	1.7780	1.0427	1.0901	2.2262	2.1516
1973	1.1105	1.1033	1.7050	1.6973	1.1011	1.3109	2.5113	2.4245
1974	1.2660	1.2772	1.6272	1.6281	1.1557	1.3329	2.6600	2.5699

5.2 Translog Model of Consumption Expenditures

We now turn to the model of aggregate consumption expenditures. In order to test various restrictions on the structure of the model, a number of alternative versions were estimated. Parameter estimates for several of these versions are shown in Table 7. All of the versions estimated are homothetic (so that the income elasticity of aggregate energy demand is equal to 1); the computational expense of estimating a non-homothetic translog system with six categories of expenditures proved to be inordinate.

We would like to test restrictions of homogeneity (parameters the same across all countries), stationarity, energy separability, and addivity, as well as the assumption that the expenditure shares are based on utility-maximizing behavior. All of these tests are, of course, conditional on the restriction of homotheticity.

We take heterogeneity to imply variation of the first-order parameters (the intercept terms of the share equations) across countries, and we use regional dummy variables to estimate the variation of these parameters. We do not test variation of the second-order parameters, since the inclusion of dummy variables for these parameters would leave too few degrees of freedom for estimation.

In the first two columns of Table 7 the models are both homogeneous, but the first is non-stationary while the second is restricted to be stationary. A comparison of these two models gives us a first test of the stationarity restriction (conditional on homogeneity). The value of the test statistic is 106.1. This is well above the critical 1% level of 15.09, so that we reject stationarity, conditional on homogeneity.

Table 7a - Parameter Estimates: Consumption*

Parameter	1. Homothetic, non-stationary		2. Homothetic, stationary	
α_1	-.0792	(.0058)	-.0787	(.0023)
α_2	-.0672	(.0109)	-.1198	(.0046)
α_3	-.4390	(.0165)	-.2825	(.0081)
α_4	-.0579	(.0130)	-.0714	(.0047)
α_5	-.0383	(.0049)	-.0370	(.0018)
α_6	-.3190	(.0177)	-.4105	(.0068)
β_{11}	.0749	(.0094)	.0950	(.0092)
β_{12}	.0202	(.0076)	.0102	(.0075)
β_{13}	.0102	(.0079)	.0134	(.0083)
β_{14}	.0014	(.0079)	.0021	(.0081)
β_{15}	.0015	(.0079)	.0045	(.0028)
β_{16}	-.1075	(.0099)	-.1253	(.0088)
β_{21}	.0202	(.0076)	.0102	(.0075)
β_{22}	-.0908	(.0133)	-.0680	(.0135)
β_{23}	.1200	(.0147)	.0622	(.0169)
β_{24}	.0922	(.0129)	.0917	(.0125)
β_{25}	-.0119	(.0044)	-.0126	(.0044)
β_{26}	-.1296	(.0155)	-.0835	(.0157)
β_{31}	.0102	(.0079)	.0134	(.0083)
β_{32}	.1200	(.0147)	.0622	(.0169)
β_{33}	-.1823	(.0339)	.0334	(.0393)
β_{34}	-.1089	(.0205)	-.1179	(.0207)
β_{35}	-.0246	(.0067)	-.0268	(.0069)
β_{36}	.1856	(.0028)	.0356	(.0301)
β_{41}	.0014	(.0079)	.0021	(.0081)
β_{42}	.0922	(.0129)	.0917	(.0125)
β_{43}	-.1089	(.0205)	-.1179	(.0207)
β_{44}	.0152	(.0241)	-.0013	(.0230)
β_{45}	.0040	(.0054)	.0079	(.0051)
β_{46}	-.0040	(.0226)	.0174	(.0198)
β_{51}	.0015	(.0027)	.0045	(.0028)
β_{52}	-.0119	(.0044)	-.0126	(.0044)
β_{53}	-.0246	(.0067)	-.0268	(.0069)
β_{54}	.0040	(.0054)	.0079	(.0051)
β_{55}	-.0014	(.0026)	-.0038	(.0024)
β_{56}	.0324	(.0073)	.0308	(.0067)
β_{61}	-.1075	(.0099)	-.1253	(.0088)
β_{62}	-.1296	(.0155)	-.0835	(.0157)
β_{63}	.1856	(.0028)	.0356	(.0301)
β_{64}	-.0040	(.0226)	.0174	(.0198)
β_{65}	.0324	(.0073)	.0308	(.0067)
β_{66}	.0232	(.0372)	.1250	(.0345)
β_{1T}	-3.45×10^{-5}	(.0003)	0	
β_{2T}	-.0033	(.0006)	0	
β_{3T}	.0104	(.0009)	0	
β_{4T}	-.0007	(.0006)	0	
β_{5T}	-3.56×10^{-5}	(.0002)	0	
β_{6T}	-.0062	(9.62×10^{-4})	0	
Eqn	RSQ		RSQ	
1	0.4528		.4027	
2	0.3208		.1805	
3	0.5942		.2043	
4	0.3342		.3321	
5	-0.0833		-.0681	

*Consumption categories are: 1 - apparel, 2 - durables, 3 = food, 4 = transportation and communication, 5 = energy, 6 = all other.

Table 7b - Parameter Estimates: Consumption

Parameter	3. Homothetic, stationary w/ country dummies, 1960-74	4. Homothetic, non-stationary with country dummies, 1960-74	5. Homothetic, non-stationary with country dummies, energy separability, 1960-74	6. Homothetic, non-stationary with country dummies, additivity, 1960-74	7. Homothetic, non-stationary with country dummies, 1962-73
$\alpha_1 D_1$	-.0755 (.0038)	-.0828 (.0081)	-.0875 (.0051)	-.1079 (.0027)	-.1019 (.0084)
$\alpha_1 D_2$	-.0845 (.0028)	-.0894 (.0049)	-.0911 (.0045)	-.1110 (.0028)	-.0986 (.0049)
$\alpha_1 D_3$	-.0848 (.0055)	-.0932 (.0082)	-.0989 (.0050)	-.1231 (.0027)	-.1160 (.0090)
$\alpha_1 D_4$	-.0849 (.0064)	-.0917 (.0095)	-.0979 (.0055)	-.1187 (.0027)	-.1206 (.0107)
$\alpha_1 D_5$	-.1010 (.0041)	-.1080 (.0067)	-.1121 (.0051)	-.1376 (.0027)	-.1239 (.0070)
$\alpha_1 D_6$	-.1231 (.0047)	-.1247 (.0063)	-.1268 (.0056)	-.1422 (.0031)	-.1439 (.0072)
$\alpha_1 D_7$	-.0797 (.0050)	-.0855 (.0079)	-.0901 (.0056)	-.1143 (.0027)	-.1064 (.0086)
$\alpha_1 D_8$	-.0840 (.0016)	-.0908 (.0053)	-.0934 (.0041)	-.1082 (.0027)	-.0988 (.0051)
$\alpha_1 D_9$	-.1034 (.0068)	-.1107 (.0103)	-.1173 (.0063)	-.1438 (.0028)	-.1402 (.0114)
$\alpha_2 D_1$	-.1453 (.0050)	-.1388 (.0096)	-.1422 (.0064)	-.1371 (.0047)	-.1338 (.0095)
$\alpha_2 D_2$	-.0700 (.0041)	-.0764 (.0064)	-.0776 (.0060)	-.0845 (.0049)	-.0768 (.0062)
$\alpha_2 D_3$	-.0811 (.0075)	-.0764 (.0100)	-.0811 (.0064)	-.0926 (.0047)	-.0743 (.0105)
$\alpha_2 D_4$	-.0776 (.0083)	-.0708 (.0111)	-.0761 (.0061)	-.0748 (.0047)	-.0705 (.0119)
$\alpha_2 D_5$	-.1238 (.0060)	-.1255 (.0084)	-.1287 (.0068)	-.1426 (.0047)	-.1249 (.0085)
$\alpha_2 D_6$	-.0816 (.0072)	-.0941 (.0082)	-.0969 (.0075)	-.1033 (.0053)	-.0976 (.0088)
$\alpha_2 D_7$	-.0681 (.0070)	-.0696 (.0095)	-.0737 (.0070)	-.0826 (.0047)	-.0692 (.0098)
$\alpha_2 D_8$	-.1567 (.0028)	-.1540 (.0072)	-.1553 (.0062)	-.1529 (.0047)	-.1531 (.0069)
$\alpha_2 D_9$	-.1355 (.0089)	-.1300 (.0119)	-.1357 (.0071)	-.1392 (.0048)	-.1285 (.0126)
$\alpha_3 D_1$	-.2866 (.0073)	-.3825 (.0104)	-.4013 (.0082)	-.4185 (.0072)	-.3874 (.0123)
$\alpha_3 D_2$	-.2395 (.0063)	-.3226 (.0086)	-.3259 (.0087)	-.3516 (.0074)	-.3167 (.0092)
$\alpha_3 D_3$	-.2934 (.0085)	-.3906 (.0112)	-.4133 (.0088)	-.4423 (.0072)	-.3905 (.0137)
$\alpha_3 D_4$	-.3741 (.0102)	-.4554 (.0119)	-.4835 (.0081)	-.5074 (.0072)	-.4582 (.0156)
$\alpha_3 D_5$	-.2728 (.0072)	-.3707 (.0102)	-.3837 (.0095)	-.4172 (.0072)	-.3661 (.0117)
$\alpha_3 D_6$	-.3462 (.0095)	-.4125 (.0100)	-.4201 (.0099)	-.4548 (.0081)	-.4066 (.0127)
$\alpha_3 D_7$	-.3178 (.0082)	-.4073 (.0107)	-.4246 (.0093)	-.4596 (.0072)	-.4034 (.0131)
$\alpha_3 D_8$	-.2416 (.0056)	-.3358 (.0088)	-.3433 (.0083)	-.3522 (.0072)	-.3432 (.0094)
$\alpha_3 D_9$	-.2622 (.0103)	-.3540 (.0126)	-.3820 (.0091)	-.4151 (.0073)	-.3501 (.0162)
$\alpha_4 D_1$	-.1179 (.0045)	-.0791 (.0084)	-.0660 (.0054)	-.0623 (.0030)	-.0717 (.0083)
$\alpha_4 D_2$	-.1512 (.0032)	-.1224 (.0049)	-.1184 (.0045)	-.1085 (.0031)	-.1198 (.0050)
$\alpha_4 D_3$	-.1187 (.0070)	-.0886 (.0083)	-.0743 (.0048)	-.0600 (.0030)	-.0789 (.0091)
$\alpha_4 D_4$	-.1203 (.0077)	-.0871 (.0096)	-.0703 (.0049)	-.0591 (.0030)	-.0734 (.0106)
$\alpha_4 D_5$	-.0847 (.0050)	-.0534 (.0066)	-.0438 (.0049)	-.0289 (.0030)	-.0469 (.0070)
$\alpha_4 D_6$	-.1240 (.0055)	-.0920 (.0065)	-.0858 (.0058)	-.0712 (.0034)	-.0843 (.0074)

Table 7b (Cont.) - Parameter Estimates: Consumption

Parameter	3.		4.		5.		6.		7.	
$\alpha_4 D_7$	-.1415	(.0058)	-.1069	(.0078)	-.0949	(.0053)	-.0797	(.0030)	-.0973	(.0084)
$\alpha_4 D_8$	-.0329	(.0022)	-.0001	(.0060)	-.0066	(.0050)	.0057	(.0030)	.0023	(.0057)
$\alpha_4 D_9$	-.1408	(.0080)	-.1039	(.0102)	-.0865	(.0056)	-.0712	(.0031)	-.0909	(.0111)
$\alpha_5 D_1$	-.0561	(.0037)	-.0500	(.0069)	-.0446	(.0022)	-.0446	(.0022)	-.0482	(.0073)
$\alpha_5 D_2$	-.0273	(.0020)	-.0271	(.0036)	-.0227	(.0023)	-.0227	(.0023)	-.0240	(.0035)
$\alpha_5 D_3$	-.0411	(.0051)	-.0352	(.0073)	-.0255	(.0022)	-.0255	(.0022)	-.0327	(.0080)
$\alpha_5 D_4$	-.0339	(.0061)	-.0283	(.0085)	-.0216	(.0022)	-.0216	(.0022)	-.0272	(.0096)
$\alpha_5 D_5$	-.0539	(.0033)	-.0509	(.0054)	-.0424	(.0022)	-.0424	(.0022)	-.0475	(.0057)
$\alpha_5 D_6$	-.0225	(.0036)	-.0265	(.0048)	-.0243	(.0025)	-.0243	(.0025)	-.0252	(.0054)
$\alpha_5 D_7$	-.0482	(.0043)	-.0460	(.0065)	-.0389	(.0022)	-.0389	(.0022)	-.0435	(.0071)
$\alpha_5 D_8$	-.0379	(.0015)	-.0331	(.0044)	-.0290	(.0022)	-.0290	(.0022)	-.0317	(.0043)
$\alpha_5 D_9$	-.0569	(.0063)	-.0515	(.0089)	-.0433	(.0022)	-.0433	(.0022)	-.0463	(.0099)
$\alpha_6 D_1$	-.3182	(.0087)	-.2666	(.0128)	-.2580	(.0117)	-.2292	(.0106)	-.2567	(.0127)
$\alpha_6 D_2$	-.4272	(.0075)	-.3618	(.0125)	-.3638	(.0126)	-.3214	(.0109)	-.3638	(.0119)
$\alpha_6 D_3$	-.3807	(.0097)	-.3156	(.0141)	-.3064	(.0128)	-.2562	(.0106)	-.3074	(.0145)
$\alpha_6 D_4$	-.3089	(.0117)	-.2664	(.0143)	-.2502	(.0114)	-.2181	(.0106)	-.2498	(.0152)
$\alpha_6 D_5$	-.3636	(.0083)	-.2912	(.0140)	-.2891	(.0139)	-.2311	(.0106)	-.2904	(.0140)
$\alpha_6 D_6$	-.3022	(.0102)	-.2499	(.0139)	-.2458	(.0138)	-.2040	(.0210)	-.2421	(.0137)
$\alpha_6 D_7$	-.3443	(.0091)	-.2844	(.0141)	-.2774	(.0134)	-.2246	(.0106)	-.2800	(.0141)
$\alpha_6 D_8$	-.4467	(.0069)	-.3860	(.0114)	-.3855	(.0114)	-.3633	(.0106)	-.3752	(.0110)
$\alpha_6 D_9$	-.3009	(.0114)	-.2495	(.0153)	-.2349	(.0129)	-.1871	(.0108)	-.0243	(.0160)
β_{11}	-.0542	(.0099)	.0501	(.0113)	.0481	(.0110)	0		.0330	(.0111)
β_{12}	-.0185	(.0074)	-.0123	(.0093)	-.0123	(.0090)	0		-.0154	(.0094)
β_{13}	-.0570	(.0078)	.0386	(.0084)	.0336	(.0078)	0		-.0610	(.0115)
β_{14}	-.0312	(.0082)	-.0217	(.0085)	-.0195	(.0084)	0		-.0189	(.0083)
β_{15}	-.0075	(.0046)	-.0044	(.0056)	0		0		.0090	(.0060)
β_{16}	-.0539	(.0082)	-.0502	(.0091)	-.0498	(.0090)	0		-.0686	(.0094)
β_{21}	-.0185	(.0074)	-.0123	(.0093)	-.0123	(.0090)	0		-.0154	(.0094)
β_{22}	.0796	(.0137)	.0538	(.0147)	.0502	(.0146)	0		.0523	(.0154)
β_{23}	.0192	(.0110)	.0426	(.0119)	.0423	(.0117)	0		.0611	(.0161)
β_{24}	-.0151	(.0096)	-.0272	(.0105)	-.0271	(.0104)	0		-.0220	(.0109)
β_{25}	.0084	(.0059)	-.0041	(.0068)	0		0		-.0082	(.0070)
β_{26}	-.0737	(.0131)	-.0527	(.0140)	-.0531	(.0138)	0		-.0677	(.0151)
β_{31}	.0570	(.0078)	.0386	(.0084)	.0336	(.0078)	0		-.0610	(.0115)
β_{32}	.0192	(.0110)	.0426	(.0119)	.0423	(.0117)	0		.0611	(.0161)

Table 7b (Cont.) - Parameter Estimates: Consumption

Parameter	3.	4.	5.	6.	7.
β_{33}	.1718 (.0210)	.0142 (.0184)	.0105 (.0183)	0	.0013 (.0277)
β_{34}	-.0242 (.0108)	-.0081 (.0098)	-.0009 (.0094)	0	-.0217 (.0133)
β_{35}	-.0416 (.0062)	-.0224 (.0071)	0	0	-.0149 (.0088)
β_{36}	-.1823 (.0221)	-.0649 (.0207)	-.0855 (.0202)	0	-.0867 (.0256)
β_{41}	-.0312 (.0082)	-.0217 (.0085)	-.0195 (.0084)	0	-.0189 (.0083)
β_{42}	-.0151 (.0096)	-.0272 (.0105)	-.0271 (.0104)	0	-.0220 (.0109)
β_{43}	-.0219 (.0108)	-.0081 (.0098)	-.0009 (.0094)	0	-.0217 (.0133)
β_{44}	-.0203 (.0131)	.0161 (.0134)	.0168 (.0134)	0	.0135 (.0135)
β_{45}	.0219 (.0058)	.0128 (.0064)	0	0	.0052 (.0065)
β_{46}	.0690 (.0105)	.0281 (.0103)	.0307 (.0101)	0	.0440 (.0121)
β_{51}	-.0075 (.0046)	-.0044 (.0056)	0	0	.0090 (.0060)
β_{52}	.0084 (.0059)	-.0041 (.0068)	0	0	.0082 (.0070)
β_{53}	-.0416 (.0062)	-.0224 (.0071)	0	0	-.0149 (.0088)
β_{54}	.0219 (.0058)	.0128 (.0064)	0	0	.0052 (.0065)
β_{55}	.0102 (.0051)	.0041 (.0060)	0	0	.0066 (.0063)
β_{56}	.0085 (.0074)	.0139 (.0076)	0	0	.0023 (.0078)
β_{61}	-.0539 (.0082)	-.0502 (.0091)	-.0498 (.0090)	0	-.0686 (.0094)
β_{62}	-.0737 (.0131)	-.0527 (.0140)	-.0531 (.0138)	0	-.0677 (.0151)
β_{63}	-.1823 (.0221)	-.0649 (.0207)	-.0855 (.0202)	0	-.0867 (.0256)
β_{64}	.0690 (.0105)	.0281 (.0103)	.0307 (.0101)	0	.0440 (.0121)
β_{65}	.0085 (.0074)	.0139 (.0076)	0	0	.0023 (.0078)
β_{66}	.2324 (.0292)	.1259 (.0317)	.1577 (.0297)	0	.1767 (.0360)
β_{1T}	0	.0003 (.0002)	.0004 (.0002)	.0013 (.0001)	.0006 (.0002)
β_{2T}	0	3.82×10^{-5} (.0003)	8.53×10^{-5} (.0003)	.0002 (.0002)	-3.22×10^{-5} (.0003)
β_{3T}	0	.0053 (.0004)	.0055 (.0004)	.0062 (.0003)	.0056 (.0004)
β_{4T}	0	-.0017 (.0002)	-.0019 (.0002)	-.0020 (.0001)	-.0018 (.0002)
β_{5T}	0	-.0001 (.0001)	-.0004 (.0001)	-.0004 (.0001)	-.0003 (.0001)
β_{6T}	0	-.0038 (5.87×10^{-4})	-.0036 (5.91×10^{-4})	-.0053 (4.88×10^{-4})	-.0042 (.0005)
Eqn	RSQ	RSQ	RSQ	RSQ	RSQ
1	.8788	.8858	.8848	.8503	.9114
2	.9053	.9101	.9092	.8864	.9252
3	.8660	.9330	.9298	.9224	.9391
4	.9491	.9637	.9632	.9581	.9646
5	.7833	.7903	.7775	.7775	.8447

All of the remaining models in Table 7 are heterogeneous, i.e. have regional dummy variables for the first-order parameters. Note that the R^2's for these models are all much higher than those for the homogeneous models, so that we would expect statistical tests to lead to rejection of homogeneity restrictions. This is indeed the case. The models in columns 3 and 4 are heterogeneous, but the first is stationary and the second is non-stationary. Comparisons of models (3) and (2) and of models (4) and (1) provide two tests of homogeneity, the first of which is conditional on stationarity. The test statistics for these two comparisons are 1082.8 and 1111.5, both well above the critical 1% level, and indicating rejection of homogeneity.[47]

A comparison of models (3) and (4) provides a second test of stationarity. The test statistic is 134.8, again indicating rejection of stationarity. We now test further restrictions conditional on a heterogeneous, non-stationary indirect utility function.

The model in column (5) has the restriction that energy is explicitly groupwise separable from the other categories of consumption, i.e. that the parameters β_{Ej}, $j = 1, \ldots, 6$, are all zero. The test statistic is 14.8, and with 5 degrees of freedom this is above the critical 2.5% level, but below the 1% level. We reject the restriction of energy separability, but recognize that since several of the β_{Ej} parameters are statistically insignificant, the own price elasticity of total energy consumption will be close to -1.

The model in column (6) is restricted to be explicitly additive, and since it is also homothetic, this means that all of the β_{ij} parameters are zero. Note that additivity implies, therefore, that all of the own-price elasticities are -1 and all of the cross-price elasticities are 0. The test statistic for this

[47] We also tested the restriction of homogeneity conditional on the restriction of additivity. The test statistic was well above the 1% level, again supporting the retention of the country dummy variables.

model, compared to the unrestricted model in column (4), is 77.1. With a total of 30 degrees of freedom, this is above the critical 1% level of 50.1. We thus reject additivity.

Finally, we test the basic assumption of utility maximization. Utility maximization implies the symmetry restrictions $\beta_{ij} = \beta_{ji}$, and so far we have imposed these restrictions on our models. We can test these restrictions by estimating a model in which all of the β_{ij} parameters are estimated freely (although we still impose the homotheticity restrictions that $\beta_{mj} = \sum_i \beta_{ij} = 0$). Comparing this model (the parameters for which are not shown here) to that of column (4), we obtain a test statistic of 11.0. With 10 degrees of freedom, this is below the critical 10% level, allowing us to reject the restrictions. Nonetheless, we will maintain these restrictions, and calculate elasticities based on the assumption of utility maximization.[48]

The results of these tests indicate that a non-stationary model based on a non-additive indirect utility function with first-order coefficients that vary across countries is needed to estimate price elasticities of consumption expenditures.[49] We therefore retain the model of column 4 as our preferred model.

As we mentioned earlier, it would also be desirable to estimate our preferred model using data at three or four year intervals, as a means of verifying that the implied elasticities are long-run. However, because the number of parameters involved in the consumption model, estimation with only one-third of the data is

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[48] In applications of the direct and indirect translog utility functions to consumption behavior in the U.S., Christensen, Jorgenson and Lau [15] and Jorgenson and Lau [37] also found that the assumption of utility maximization could be rejected. However, they went on, as we do, to estimate models based on this assumption. This is in fact reasonable, given the nature of the test. It is a "weak" test, so that given our test statistic above, we may but need not, reject the assumption of utility maximization.

[49] We must stress, however, that these tests are all conditional on the assumption of homotheticity, and we have not tested the homotheticity restriction since we have been unable to estimate the non-homothetic model.

not possible given the reduction in degrees of freedom.[50] It would still be useful to re-estimate the model eliminating some of the data in order to test the stability of the coefficients and implied elasticities to changes in the data set. The model in column (7) of Table 7 is the same as that in column (4), except that it is estimated using data over the period 1962-1973, instead of 1960-1974. Comparing columns (4) and (7), we see that most of the estimated parameters are the same. We found that elasticities computed for the two models are also very close to each other. This is encouraging, and indicates that the model is fairly robust.

Own and cross price elasticities for our "preferred" consumption model are shown in Table 8. Since these elasticities depend on the particular prices and shares, they are calculated for each of two years, 1965 and 1973. The numbers in parentheses below each elasticity are the standard errors.

Note that none of the own price elasticities for energy are significantly different from -1, which is what would be implied by energy separability. This is actually quite reasonable, given that we presume to estimate a long-run elasticity, and, as we will see, is within the bounds of some alternative estimates obtained by others.

For the other categories of consumption, own price elasticities vary between -1 and about -1.7. As expected, the smallest own price elasticity (about -1.04) is for food, but even this may not reflect a true price response, but instead the result of food prices rising slightly with food shares dropping considerably as consumers spend larger incomes on other goods. In other words, this may be the result of an income effect that we cannot capture because of the imposed restriction

[50] On the other hand, because most of the variation in the consumption share data is cross-sectional rather than time-wise, we can be fairly sure that the elasticities are long-run.

Table 8 - Price Elasticities for "Preferred" Consumption Model

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{AA}	1965	-1.57	-1.59	-1.47	-1.54	-1.42	-1.36	-1.54	-1.60	-1.40
		(.12)	(.13)	(.10)	(.12)	(.09)	(.08)	(.12)	(.13)	(.09)
	1973	-1.67	-1.57	-1.6	-1.54	-1.51	-1.52	-1.57	-1.62	-1.45
		(.15)	(.12)	(.13)	(.12)	(.11)	(.11)	(.13)	(.14)	(.10)
η_{DD}	1965	-1.43	-1.70	-1.61	-1.71	-1.34	-1.49	-1.68	-1.35	-1.39
		(.12)	(.19)	(.16)	(.19)	(.09)	(.13)	(.18)	(.09)	(.10)
	1973	-1.35	-1.60	-1.63	-1.88	-1.48	-1.65	-1.70	-1.35	-1.40
		(.09)	(.16)	(.17)	(.24)	(.13)	(.17)	(.19)	(.09)	(.11)
η_{FF}	1965	-1.04	-1.05	-1.03	-1.03	-1.04	-1.03	-1.04	-1.05	-1.04
		(.06)	(.07)	(.05)	(.04)	(.05)	(.04)	(.05)	(.07)	(.06)
	1973	-1.04	-1.06	-1.05	-1.04	-1.05	-1.04	-1.04	-1.06	-1.05
		(.06)	(.08)	(.06)	(.05)	(.06)	(.06)	(.05)	(.08)	(.06)
η_{TT}	1965	-1.16	-1.10	-1.17	-1.18	-1.30	-1.15	-1.14	-1.55	-1.15
		(.13)	(.08)	(.14)	(.15)	(.25)	(.12)	(.12)	(.46)	(.12)
	1973	-1.14	-1.10	-1.14	-1.14	-1.18	-1.13	-1.11	-1.46	-1.13
		(.12)	(.09)	(.12)	(.12)	(.15)	(.11)	(.09)	(.38)	(.11)
η_{EE}	1965	-1.08	-1.13	-1.12	-1.13	-1.09	-1.13	-1.08	-1.11	-1.09
		(.11)	(.18)	(.18)	(.20)	(.13)	(.20)	(.12)	(.16)	(.14)
	1973	-1.06	-1.15	-1.11	-1.12	-1.07	-1.11	-1.09	-1.10	-1.05
		(.10)	(.22)	(.16)	(.18)	(.10)	(.17)	(.13)	(.15)	(.08)
η_{RR}	1965	-1.37	-1.30	-1.39	-1.45	-1.40	-1.51	-1.39	-1.27	-1.44
		(.09)	(.07)	(.09)	(.11)	(.10)	(.12)	(.09)	(.07)	(.11)
	1973	-1.40	-1.30	-1.30	-1.32	-1.33	-1.34	-1.36	-1.26	-1.44
		(.10)	(.07)	(.07)	(.08)	(.08)	(.08)	(.09)	(.06)	(.11)
η_{EA}	1965	.08	.14	.13	.14	.09	.15	.09	.12	.10
		(.10)	(.17)	(.17)	(.18)	(.12)	(.18)	(.11)	(.15)	(.13)
	1973	.07	.16	.11	.13	.08	.12	.09	.11	.06
		(.09)	(.20)	(.14)	(.17)	(.10)	(.16)	(.12)	(.14)	(.07)
η_{ED}	1965	.08	.12	.12	.13	.09	.13	.08	.11	.09
		(.13)	(.21)	(.20)	(.22)	(.15)	(.23)	(.14)	(.18)	(.16)
	1973	.06	.15	.10	.12	.07	.11	.09	.10	.05
		(.11)	(.25)	(.18)	(.21)	(.12)	(.19)	(.15)	(.17)	(.09)

A = apparel D = durables F = food T = transport E = energy R = all other

Table 8 (Cont.)

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{EF}	1965	.43	.70	.68	.74	.49	.75	.46	.60	.53
		(.14)	(.22)	(.21)	(.23)	(.15)	(.24)	(.15)	(.19)	(.17)
	1973	.37	.82	.59	.68	.40	.64	.49	.53	.31
		(.12)	(.26)	(.19)	(.21)	(.13)	(.20)	(.16)	(.18)	(.09)
η_{ET}	1965	-.25	-.40	-.39	-.42	-.28	-.43	-.26	-.34	-.30
		(.12)	(.20)	(.19)	(.21)	(.14)	(.21)	(.13)	(.17)	(.15)
	1973	-.21	-.47	-.34	-.39	-.23	-.36	-.28	-.32	-.17
		(.10)	(.23)	(.17)	(.19)	(.11)	(.18)	(.14)	(.16)	(.08)
η_{ER}	1965	-.27	-.43	-.42	-.46	-.30	-.46	-.29	-.37	-.33
		(.14)	(.23)	(.23)	(.25)	(.16)	(.25)	(.15)	(.20)	(.18)
	1973	-.23	-.51	-.37	-.42	-.25	-.40	-.31	-.35	-.19
		(.12)	(.28)	(.20)	(.23)	(.13)	(.21)	(.17)	(.19)	(.10)
η_{AE}	1965	.05	.05	.04	.05	.04	.03	.05	.05	.04
		(.06)	(.07)	(.05)	(.06)	(.05)	(.04)	(.06)	(.07)	(.05)
	1973	.06	.05	.05	.05	.05	.05	.05	.06	.04
		(.08)	(.06)	(.07)	(.06)	(.06)	(.06)	(.06)	(.07)	(.05)
η_{DE}	1965	.03	.05	.05	.05	.03	.04	.05	.03	.03
		(.06)	(.09)	(.08)	(.09)	(.04)	(.06)	(.09)	(.04)	(.05)
	1973	.03	.05	.05	.07	.04	.05	.05	.03	.03
		(.04)	(.08)	(.08)	(.11)	(.06)	(.08)	(.09)	(.04)	(.05)
η_{FE}	1965	.08	.09	.06	.05	.07	.06	.06	.09	.07
		(.02)	(.03)	(.02)	(.02)	(.02)	(.02)	(.02)	(.03)	(.02)
	1973	.08	.10	.08	.07	.08	.07	.07	.11	.08
		(.02)	(.03)	(.03)	(.02)	(.03)	(.02)	(.02)	(.03)	(.03)
η_{TE}	1965	-.13	-.08	-.13	-.14	-.24	-.12	-.11	-.44	-.12
		(.06)	(.04)	(.06)	(.07)	(.12)	(.06)	(.05)	(.22)	(.06)
	1973	-.11	-.08	-.11	-.11	-.14	-.10	-.09	-.37	-.10
		(.05)	(.04)	(.05)	(.05)	(.07)	(.05)	(.04)	(.18)	(.05)
η_{RE}	1965	-.04	-.03	-.04	-.05	-.05	-.06	-.04	-.03	-.05
		(.02)	(.02)	(.02)	(.03)	(.02)	(.03)	(.02)	(.02)	(.03)
	1973	-.05	-.03	-.03	-.04	-.04	-.04	-.04	-.03	-.05
		(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.03)

A = apparel D = durables F = food T = transport E = energy R = all other of homotheticity. Cross-price elasticities are presented only for energy in order to save space. Most of these are near zero except for food and transportation. The large cross-price elasticity for food and energy is reasonable; we expect these goods to be substitutes, particularly if incomes are low. The negative cross-price elasticities for energy and transportation are surprising, but again may represent something other than a true price effect; as energy became cheaper during the 1960's, the infrastructure grew that made expansion of the transportation and communication shares possible.

5.3 Translog Model of Fuel Expenditures.

We turn next to models for the breakdown of energy expenditures into expenditures on individual fuels. All of the models we estimate here are homothetic (so that the elasticities of fuel expenditures with respect to total energy expenditures are all equal to 1), but we would like to test restrictions of homogeneity, stationarity, and additivity.

Parameter estimates for the various models are given in Table 9. Both of the models in the first two columns of that table are homogeneous, i.e. all of the parameters are the same across all countries. The first model, however, has the additional restriction of stationarity. Comparing these two models provides a first test of stationarity. The test statistic is 43.6, and with three degrees of freedom this is well above the critical 1% level, leading us (conditioned on homogeneity) to reject stationarity.

All of the remaining models are heterogeneous - regional dummy variables are added to the intercept parameters of the share equations. The models in columns (3) and (4) of Table 9 have no additional restrictions. Model (3) is estimated using data over the entire range of 1960-1974, whereas model (4) is estimated without the 1974 data. By comparing columns (3) and (4) we can determine whether it is appropriate to include the 1974 data; since energy prices rose considerably in that year, and since we are estimating what we believe are long-run elasticities, there is some question as to whether the 1974 data belong to the same sample as the 1960-1973 data, i.e. whether the 1974 data were generated by the same indirect utility function. Since the estimated parameters in columns (3) and (4) are quite close (as are the implied elasticities), we can conclude that the 1974 data belongs to the same sample, and estimate our models using the full 1960-1974 range.

Comparing model (3) with model (2) provides a test of the homogeneity restrictions. The test statistic is 626.0, which is well above the critical 1% level, so that we reject homogeneity and include regional dummy variables in our model. Now, given a heterogeneous model, we again test for stationarity. Parameter estimates for the stationary version of model (3) are shown in column (7). The appropriate test statistic is 195.4, which is well above the 1% level, so that we again reject stationarity. Finally, we test for additivity. In the model shown in column (9), all of the β_{ij} parameters are construed to be 0. We compare this model to model (3), and obtain a test statistic of 189.3. This is above the critical 1% level, so that we can reject additivity.

Fuel prices have been lower in the U.S. and Canada than in the European countries, and incomes have been higher, which suggests pooling the U.S. and Canada separately. The results of such a pooling are shown in columns (5) (Europe only) and (6) (U.S. and Canada only). The models in this case are heterogeneous and non-stationary. We can compare the resulting parameter estimates (in particular the parameters of the second-order terms in the indirect

Table 9

Parameter Estimates: Fuel Expenditure Models

	1 1960-74 Stationary	2 1960-74	3 1960-74 Country Dummies	4 1960-73 Country Dummies
α_1	0.0414 (0.0383)	-0.2151 (0.0761)		
$\alpha_1 D_1$			-0.8124 (0.0644)	-0.8188 (0.1010)
$\alpha_1 D_2$			-0.5234 (0.0475)	-0.5491 (0.0739)
$\alpha_1 D_3$			-0.6746 (0.0610)	-0.6813 (0.0919)
$\alpha_1 D_4$			-0.4964 (0.0647)	-0.4830 (0.1021)
$\alpha_1 D_5$			-0.6089 (0.0551)	-0.6181 (0.0843)
$\alpha_1 D_6$			-0.3892 (0.0442)	-0.3608 (0.0576)
$\alpha_1 D_7$			-0.6954 (0.0569)	-0.6902 (0.0904)
$\alpha_1 D_8$			-0.5752 (0.0544)	-0.6261 (0.0848)
$\alpha_1 D_9$			-0.4455 (0.0630)	-0.4337 (0.0949)
α_2	-0.2360 (0.0228)	-0.0894 (0.0449)		
$\alpha_2 D_1$			-0.1246 (0.0524)	-0.1554 (0.0681)
$\alpha_2 D_2$			-0.3949 (0.0363)	-0.4219 (0.0491)
$\alpha_2 D_3$			-0.1008 (0.0531)	-0.1293 (0.0665)
$\alpha_2 D_4$			-0.0791 (0.0536)	-0.1122 (0.0688)
$\alpha_2 D_5$			-0.1994 (0.0475)	-0.2383 (0.0610)
$\alpha_2 D_6$			0.0165 (0.0395)	-0.0030 (0.0430)
$\alpha_2 D_7$			0.0020 (0.0402)	-0.0359 (0.0530)
$\alpha_2 D_8$			-0.4154 (0.0406)	-0.4519 (0.0575)
$\alpha_2 D_9$			-0.1189 (0.0547)	-0.1569 (0.0680)
α_3	-0.0870 (0.0236)	0.0855 (0.0501)		
$\alpha_3 D_1$			-0.6495 (0.0565)	-0.6474 (0.0684)
$\alpha_3 D_2$			-0.3221 (0.0428)	-0.2619 (0.0560)
$\alpha_3 D_3$			-0.6950 (0.0557)	-0.6921 (0.0649)
$\alpha_3 D_4$			-0.7579 (0.0579)	-0.7777 (0.0693)
$\alpha_3 D_5$			-0.6868 (0.0494)	-0.6617 (0.0598)
$\alpha_3 D_6$			-0.2238 (0.0452)	-0.2434 (0.0463)
$\alpha_3 D_7$			-0.5659 (0.0474)	-0.5816 (0.0571)
$\alpha_3 D_8$			-0.3897 (0.0513)	-0.3029 (0.0692)
$\alpha_3 D_9$			-0.6287 (0.0586)	-0.6329 (0.0671)
α_4	-0.6984 (0.0414)	-0.7810 (0.0832)		
$\alpha_4 D_1$			0.5866 (0.0695)	0.6216 (0.1142)
$\alpha_4 D_2$			0.2403 (0.0550)	0.2328 (0.0874)
$\alpha_4 D_3$			0.4704 (0.0594)	0.5026 (0.1093)
$\alpha_4 D_4$			0.3334 (0.0683)	0.3729 (0.1135)
$\alpha_4 D_5$			0.4953 (0.0594)	0.5181 (0.0968)
$\alpha_4 D_6$			-0.4036 (0.0345)	-0.3929 (0.0649)
$\alpha_4 D_7$			0.2593 (0.0575)	0.3078 (0.0950)
$\alpha_4 D_8$			0.3803 (0.0647)	0.3808 (0.1030)
$\alpha_4 D_9$			0.1932 (0.0636)	0.2235 (0.1048)
β_{11}	0.3708 (0.0423)	0.2657 (0.0415)	-0.0012 (0.0352)	0.0304 (0.0516)
β_{12}	-0.1265 (0.0239)	-0.0807 (0.0223)	0.0203 (0.0227)	0.0159 (0.0302)
β_{13}	-0.0914 (0.0171)	-0.0803 (0.0158)	-0.1363 (0.0258)	-0.1826 (0.0305)
β_{14}	-0.1522 (0.0273)	-0.1046 (0.0293)	0.1171 (0.0227)	0.1362 (0.0397)
β_{21}	-0.1265 (0.0239)	-0.0807 (0.0223)	0.0203 (0.0227)	0.0159 (0.0302)
β_{22}	0.0286 (0.0178)	0.0150 (0.0164)	0.0162 (0.0252)	0.0108 (0.0315)
β_{23}	0.0917 (0.0109)	0.0933 (0.0105)	-0.1285 (0.0208)	-0.1298 (0.0245)
β_{24}	0.0063 (0.0151)	-0.0276 (0.0155)	0.0920 (0.0146)	0.1031 (0.0242)

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Table 9

(Continued)

	1	2	3	4
β_{31}	-0.0914 (0.0273)	-0.0803 (0.0158)	-0.1363 (0.0258)	-0.1826 (0.0305)
β_{32}	0.0917 (0.0109)	0.0933 (0.0105)	-0.1285 (0.0208)	-0.1298 (0.0245)
β_{33}	0.0747 (0.0159)	0.0836 (0.0147)	0.1207 (0.0319)	0.1853 (0.0359)
β_{34}	-0.0750 (0.0168)	-0.0966 (0.0169)	0.1440 (0.0193)	0.1270 (0.0285)
β_{41}	-0.1522 (0.0273)	-0.1046 (0.0293)	0.1171 (0.0227)	0.1362 (0.0397)
β_{42}	0.0063 (0.0151)	-0.0276 (0.0155)	0.0920 (0.0146)	0.1031 (0.0242)
β_{43}	-0.0750 (0.0168)	-0.0966 (0.0169)	0.1440 (0.0193)	0.1270 (0.0285)
β_{44}	0.2209 (0.0294)	0.2289 (0.0323)	-0.3531 (0.0296)	-0.3663 (0.0484)
β_{1T}	0	0.0112 (0.0027)	0.0181 (0.0014)	0.0179 (0.0018)
β_{2T}	0	-0.0068 (0.0017)	-0.0068 (0.0012)	-0.0060 (0.0013)
β_{3T}	0	-0.0079 (0.0021)	0.0009 (0.0013)	-0.0005 (0.0014)
β_{4T}	0	0.0034 (0.0029)	-0.0121 (0.0011)	-0.0114 (0.0017)
Rsq.				
Eqn.1	0.375	0.464	0.894	0.904
Eqn.2	0.437	0.455	0.772	0.785
Eqn.3	0.114	0.210	0.746	0.769

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Table 9

(Continued)

	5	6	7	8	9
	1960-74 Country Dummies (Europe only)	1960-74 Country Dummies (US & Canada only)	1960-74 Country Dummies Stationary	1962-74 No Dummies 4-yr. intervals	1960-74 Additivity Country Dummies
α_1				-.2478 (.1328)	
$\alpha_1 D_1$	-1.012 (0.1010)		-.0796 (.0529)		-0.6811 (0.0251)
$\alpha_1 D_2$		-0.1021 (0.0259)	.0404 (.0406)		-0.3160 (0.0258)
$\alpha_1 D_3$	-0.8505 (0.0575)		-.0094 (.0486)		-0.5733 (0.0251)
$\alpha_1 D_4$	-0.6958 (0.0615)		.2301 (.0528)		-0.3860 (0.0251)
$\alpha_1 D_5$	-0.7716 (0.0527)		.0068 (.0431)		-0.4853 (0.0251)
$\alpha_1 D_6$	-0.5580 (0.0389)		.0810 (.0310)		-0.4056 (0.0283)
$\alpha_1 D_7$	-0.9261 (0.0545)		-.0045 (.0481)		-0.5974 (0.0251)
$\alpha_1 D_8$		-0.1099 (0.0287)	.0131 (.0514)		-0.2952 (0.0251)
$\alpha_1 D_9$	-0.6327 (0.0587)		.2368 (.0502)		-0.3692 (0.0256)
α_2				-.0593 (.0875)	
$\alpha_2 D_1$	-0.0533 (0.0553)	-0.5274 (0.0496)	-.3251 (.0419)		-0.0497 (0.0245)
$\alpha_2 D_2$			-.5518 (.0312)		-0.2353 (0.0253)
$\alpha_2 D_3$	-0.0331 (0.0562)		-.2952 (.0415)		-0.0484 (0.0245)
$\alpha_2 D_4$	-0.0031 (0.0566)		-.2807 (.0423)		-0.0231 (0.0245)
$\alpha_2 D_5$	-0.1436 (0.0502)		-.3815 (.0367)		-0.1223 (0.0245)
$\alpha_2 D_6$	0.1033 (0.0422)		-.1498 (.0256)		-0.0247 (0.0277)
$\alpha_2 D_7$	0.0892 (0.0418)		-.1821 (.0317)		0.0535 (0.0245)
$\alpha_2 D_8$		-0.4467 (0.0559)	-.5703 (.0398)		-0.1903 (0.0245)
$\alpha_2 D_9$	-0.0407 (0.0580)		-.3184 (.0424)		-0.0911 (0.0250)
α_3				.0968 (.0941)	
$\alpha_3 D_1$	-0.6964 (0.0593)	-0.0414 (0.0313)	-.6066 (.0448)		-0.0485 (0.0299)
$\alpha_3 D_2$			-.2776 (.0422)		-0.0278 (0.0308)
$\alpha_3 D_3$	-0.7414 (0.0587)		-.6530 (.0420)		-0.1139 (0.0299)
$\alpha_3 D_4$	-0.8030 (0.0605)		-.7171 (.0445)		-0.1429 (0.0299)
$\alpha_3 D_5$	-0.7285 (0.0528)		-.6435 (.0380)		-0.1871 (0.0299)
$\alpha_3 D_6$	-0.2069 (0.0475)		-.2062 (.0285)		0.1427 (0.0337)
$\alpha_3 D_7$	-0.5698 (0.0484)		-.5536 (.0355)		-0.1099 (0.0299)
$\alpha_3 D_8$		-0.1053 (0.0372)	-.3347 (.0574)		-0.1092 (0.0299)
$\alpha_3 D_9$	-0.6704 (0.0612)		-.5905 (.0432)		-0.0197 (0.0305)
α_4				-.7896 (.1392)	
$\alpha_4 D_1$	0.7616 (0.0730)	-0.3291 (0.0304)	.0113 (.0524)		-0.2207 (0.0238)
$\alpha_4 D_2$			-.2109 (.0452)		-0.4209 (0.0245)
$\alpha_4 D_3$	0.6253 (0.0641)		-.0423 (.0450)		-0.2644 (0.0238)
$\alpha_4 D_4$	0.5019 (0.0714)		-.2321 (.0510)		-0.4481 (0.0238)
$\alpha_4 D_5$	0.6437 (0.0578)		.0183 (.0439)		-0.2052 (0.0238)
$\alpha_4 D_6$	-0.3384 (0.0362)		-.7250 (.0233)		-0.7124 (0.0180)
$\alpha_4 D_7$	0.4066 (0.0590)		-.2596 (.0419)		-0.3463 (0.0238)
$\alpha_4 D_8$		-0.3382 (0.0361)	-.1081 (.0591)		-0.4053 (0.0238)
$\alpha_4 D_9$	0.3439 (0.0663)		-.3275 (.0396)		-0.5199 (0.0243)
β_{11}	-0.1199 (0.0300)	0.0179 (0.0137)	.2620 (.0456)	.2153 (.0753)	0
β_{12}	0.0716 (0.0195)	-0.0161 (0.0067)	-.0089 (.0290)	-.0544 (.0409)	0
β_{13}	-0.1123 (0.0224)	-0.0353 (0.0064)	-.1515 (.0284)	-.0706 (.0292)	0
β_{14}	0.1606 (0.0236)	0.0336 (0.0160)	-.1016 (.0282)	-.0903 (.0497)	0
β_{21}	0.0716 (0.0195)	-0.0161 (0.0067)	-.0089 (.0290)	-.0544 (.0409)	0
β_{22}	0.0016 (0.0261)	-0.1665 (0.0236)	.0012 (.0273)	.0069 (.0311)	0
β_{23}	-0.1698 (0.0229)	0.1322 (0.0146)	-.1142 (.0221)	.0843 (.0203)	0
β_{24}	0.0965 (0.0157)	0.0504 (0.0240)	.1212 (.0203)	-.0368 (.0279)	0

- continued on following page . . .

Table 9

(Continued)

	5	6	7	8	9
β_{31}	-0.1123 (0.0224)	-0.0353 (0.0064)	0.1515 (0.0284)	-0.0706 (0.0292)	0.
β_{32}	-0.1698 (0.0229)	0.1322 (0.0146)	-0.1142 (0.0221)	0.0843 (0.0203)	0
β_{33}	0.1044 (0.0336)	-0.0243 (0.0167)	-0.1383 (0.0348)	0.0727 (0.0279)	0
β_{34}	0.1777 (0.0216)	-0.0726 (0.0158)	0.1274 (0.0271)	-0.0864 (0.0291)	0
β_{41}	0.1606 (0.0236)	0.0336 (0.0160)	-0.1016 (0.0282)	-0.0903 (0.0497)	0
β_{42}	0.0965 (0.0157)	0.0504 (0.0240)	0.1219 (0.0203)	-0.0368 (0.0279)	0
β_{43}	0.1777 (0.0216)	-0.0726 (0.0158)	0.1274 (0.0271)	-0.0864 (0.0291)	0
β_{44}	-0.4348 (0.0313)	-0.0114 (0.0334)	-0.1477 (0.0320)	0.2134 (0.0523)	0
β_{1T}	0.0254 (0.0013)	0.0019 (0.0004)	0	0.0121 (0.0047)	0.0164 (0.0011)
β_{2T}	-0.0100 (0.0013)	0.0060 (0.0011)	0	-0.0076 (0.0033)	-0.0075 (0.0011)
β_{3T}	-0.0009 (0.0015)	-0.0011 (0.0007)	0	-0.0092 (0.0039)	-0.0073 (0.0014)
β_{4T}	-0.0146 (0.0012)	-0.0067 (0.0013)	0	0.0046 (0.0049)	-0.0016 (0.0011)
Rsq.					
Eqn. 1	0.940	0.899	0.7911	0.4603	0.880
Eqn. 2	0.667	0.821	0.7185	0.4093	0.739
Eqn. 3	0.785	0.960	0.7340	0.1889	0.636

translog utility function) to those in column (3). We do find that the β_{ij} parameters change considerably, and that the parameter values for the U.S. and Canada differ considerably from those for the European (so that elasticity estimates will also differ considerably). Furthermore, a slightly larger fraction of the β_{ij} parameter estimates are statistically significant when Canada and the U.S. are pooled separately. This would argue strongly for a separate pooling of Canada and the U.S. On the other hand, in pooling Canada and the U.S. separately, we are relying mostly on time-wise variation of prices and shares to obtain parameter estimates. This in turn could explain why the parameter estimates in columns (5) and (6) differ by as much as they do; the estimates for Europe represent a long-run utility function, while those for the U.S. and Canada represent a shorter-run utility function. With this in mind we report elasticities for both versions of the model - the U.S. and Canada pooled separately (Table 11), and all nine countries pooled together (Table 12). Indeed, we see that pooling the U.S. and Canada separately results in much smaller price elasticities for these countries than for the European countries.

Finally, we wish to verify that by pooling all nine countries together, we will estimate <u>long-run</u> elasticities. To do this we estimate the <u>homogeneous</u> (but non-stationary) version of our model using data at four-year intervals (1962, 1966, 1970, 1974). We estimate the homogeneous version because there are not enough degrees of freedom to identify the 32 additional parameters of the heterogeneous version. The results are shown in column (8) and note that the parameter estimates are quite close to those in column (2). Own price elasticities for the models in columns (2) and (8) are shown in Table 10. These results are very similar, which indicates that by pooling data for nine countries, we are indeed estimating long-run elasticities.

Price elasticities for the model in which the U.S. and Canada are pooled separately are shown in Table 11. Note that for liquid fuel and gas the elasticities for the U.S. and Canada are about half the size of those for the other countries. This could be because per capita incomes have been much higher in the U.S. and Canada (so that fuel expenditures for home heating are not viewed by consumers as a discretionary component of their consumption baskets), or because we are simply estimating shorter-run elasticities. On the other hand the own-price elasticities for the U.S. and Canada are larger negative numbers than for the European country (where for the most part they are insignificantly different from 0 or positive). This could be because of greater discretion in the choice of electricity for home heating in the U.S. and Canada, where electricity tends to be used predominantly to heat vacation homes or homes in warmer climates.

Own- and cross-price elasticities for the version of the model in which all nine countries are pooled are shown in Table 12. Note that own-price elasticities for solid fuel and liquid fuel are all close to -1, and show little statistically significant variation across countries. Own price elasticities for natural gas are larger (-1.45 to -1.99) and show more regional variation. The magnitudes of these elasticities are reasonable, particularly given that they are long-run. The own-price elasticity estimates for electricity are disturbing, however, in that three of them (Belgium, France, and the Netherlands) are positive. This occurs because of the large negative value that is estimated for β_{44}. This parameter, when divided by the negative of the small expenditure shares for electricity in these countries, becomes greater than 1. Finally, note that all of the cross-price elasticities associated with electricity are negative (and statistically significant).

Table 10a - Own Price Elasticities for Models 2 and 8 *

Model 2: Non-stationary, homogeneous, annual data 1960-1974.

ELAST.	YEAR	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-1.56 (.08)	**	-1.74 (.12)	-3.21 (.34)	-1.94 (.15)	-3.71 (.42)	-1.80 (.13)	**	-3.87 (.49)
	1973	-2.48 (.23)	**	-4.45 (.54)	-12.64 (1.81)	-15.07 (2.19)	-10.58 (1.50)	-1.83 (.21)	**	-13.67 (2.00)
η_{22}	1965	-1.12 (.13)	-1.04 (.04)	-1.13 (.15)	-1.11 (.12)	-1.05 (.06)	-1.12 (.13)	-1.23 (.26)	-1.04 (.04)	-1.07 (.08)
	1973	-1.04 (.05)	-1.04 (.04)	-1.04 (.05)	-1.06 (.06)	-1.08 (.08)	-1.08 (.08)	-1.27 (.29)	-1.04 (.04)	-1.04 (.05)
η_{33}	1965	-1.48 (.09)	-1.46 (.08)	-1.34 (.06)	-1.31 (.05)	-1.49 (.08)	**	-1.44 (.07)	-1.33 (.06)	-1.69 (.12)
	1973	-1.41 (.07)	-1.56 (.10)	-1.31 (.06)	-1.32 (.06)	-1.14 (.02)	**	-1.27 (.05)	-1.46 (.08)	-1.40 (.07)
η_{44}	1965	-1.98 (.13)	-1.53 (.08)	-1.81 (.11)	-1.48 (.07)	-1.86 (.12)	-1.30 (.04)	-1.54 (.07)	-1.56 (.08)	-1.40 (.06)
	1973	-1.89 (.13)	-1.49 (.07)	-1.74 (.11)	-1.49 (.07)	-2.21 (.17)	-1.30 (.04)	-1.52 (.07)	-1.52 (.07)	-1.51 (.07)

*1 = solid, 2 = liquid, 3 = gas, 4 = electricity

**Almost no solid fuel is consumed in the residential sectors of Canada and the United States, and almost no natural gas is consumed in Norway, so that these elasticities are meaningless.

Table 10b - Own Price Elasticities for Models 2 and 8*

Model 8 : Non-stationary, **homogeneous, data at 4 year intervals, 1962-1974.**

ELAST.	YEAR	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-1.45 (.16)	**	-1.60 (.21)	-2.78 (.63)	-1.76 (.27)	-3.19 (.76)	-1.66 (.23)	**	-3.33 (.81)
	1973	-2.20 (.42)	**	-3.79 (.97)	-10.40 (3.27)	-12.41 (3.94)	-8.76 (2.69)	-2.11 (.39)	**	-11.27 (3.55)
η_{22}	1965	-1.06 (2.5)	-1.02 (.08)	-1.06 (0.27)	-1.05 (.23)	-1.02 (.11)	-1.05 (.25)	-1.11 (.50)	-1.02 (.09)	-1.03 (.15)
	1973	-1.02 (.07)	-1.02 (.08)	-1.02 (.09)	-1.03 (.12)	-1.03 (.16)	-1.03 (.16)	-1.12 (.55)	-1.02 (.08)	-1.02 (.09)
η_{33}	1965	-1.43 (.16)	-1.40 (.15)	-1.29 (.11)	-1.27 (.10)	-1.43 (.16)	**	-1.38 (.15)	-1.28 (.10)	-1.60 (.23)
	1973	-1.36 (.14)	-1.49 (.17)	-1.27 (.10)	-1.28 (.11)	-1.12 (.05)	**	-1.23 (.09)	-1.40 (.15)	-1.35 (.13)
η_{44}	1965	-1.92 (.23)	-1.50 (.12)	-1.75 (.19)	-1.45 (.11)	-1.80 (.20)	-1.27 (.07)	-1.51 (.12)	-1.52 (.13)	-1.37 (.09)
	1973	-1.83 (.20)	-1.46 (.11)	-.170 (.17)	-1.45 (.11)	-2.12 (.28)	-1.28 (.07)	-1.49 (.12)	-1.48 (.12)	-1.48 (.12)

*1 = solid, 2 = liquid, 3 = gas, 4 = electricity

**Almost no solid fuel is consumed in the residential sectors of Canada and the United States, and almost no natural gas is consumed in Norway, so that these elasticities are meaningless.

Table 11 - Partial Price Elasticities for U.S. and Canada Estimated Separately from Europe* (Country Dummy Variables, 1960-74)

Elast	YEAR	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-0.75	**	-0.66	-0.00	-0.57	0.22	-0.63	**	0.32
		(0.06)		(0.08)	(0.25)	(0.11)	(0.30)	(0.09)		(0.32)
	1973	-0.33	**	0.56	4.26	5.35	3.32	-0.38	**	4.72
		(0.17)		(0.39)	(1.31)	(1.59)	(1.08)	(0.15)		(1.43)
η_{12}	1965	-0.15	**	-0.20	-0.59	-0.25	-0.73	-0.22	**	-0.78
		(0.04)		(0.05)	(0.16)	(0.07)	(0.20)	(0.06)		(0.21)
	1973	-0.40	**	-0.93	-3.14	-3.79	-2.58	-0.37	**	-3.42
η_{13}	1965	0.24	**	0.31	0.93	0.40	1.14	0.34	**	1.22
		(0.05)		(0.06)	(0.18)	(0.08)	(0.23)	(0.07)		(0.24)
	1973	0.63	**	1.46	4.92	5.95	4.05	0.58	**	5.36
		(0.12)		(0.29)	(0.98)	(1.18)	(0.81)	(0.11)		(1.07)
η_{14}	1965	-0.34	**	-0.45	-1.33	-0.57	-1.63	-0.49	**	-1.74
		(0.05)		(0.06)	(0.19)	(0.08)	(0.24)	(0.07)		(0.25)
	1973	-0.90	**	-2.08	-7.04	-8.51	-5.79	-0.83	**	-7.66
		(0.13)		(0.30)	(1.03)	(1.25)	(0.85)	(0.12)		(1.12)
η_{21}	1965	-0.58	0.04	-0.63	-0.53	-0.25	-0.57	-1.14	0.048	-0.33
		(0.16)	(0.02)	(0.17)	(0.14)	(0.07)	(0.15)	(0.31)	(0.02)	(0.09)
	1973	-0.20	0.04	-0.20	-0.28	-0.37	-0.36	-1.28	0.04	-0.22
		(0.05)	(0.02)	(0.05)	(0.08)	(0.10)	(0.10)	(0.35)	(0.021)	(0.06)
η_{22}	1965	-1.01	-0.55	-1.01	-1.01	-1.00	-1.01	-1.02	-0.50	-1.01
		(0.21)	(0.06)	(0.23)	(0.19)	(0.09)	(0.21)	(0.42)	(0.07)	(0.12)
	1973	-1.00	-0.56	-1.00	-1.01	-1.01	-1.01	-1.03	-0.56	-1.00
		(0.07)	(0.06)	(0.07)	(0.10)	(0.13)	(0.13)	(0.46)	(0.06)	(0.08)
η_{23}	1965	1.38	-0.35	1.50	1.25	0.60	1.36	2.71	-0.39	0.79
		(0.18)	(0.04)	(0.20)	(0.17)	(0.08)	(0.18)	(0.36)	(0.04)	(0.11)
	1973	0.47	-0.35	0.48	0.66	0.87	0.85	3.03	-0.35	0.52
		(0.06)	(0.04)	(0.06)	(0.09)	(0.12)	(0.11)	(0.41)	(0.04)	(0.07)
η_{24}	1965	-0.78	-0.13	-0.85	-0.71	-0.34	-0.77	-1.54	-0.15	-0.45
		(0.13)	(0.06)	(0.14)	(0.11)	(0.05)	(0.12)	(0.25)	(0.07)	(0.07)
	1973	-0.27	-0.13	-0.27	-0.38	-0.49	-0.48	-1.72	-0.13	-0.29
		(0.04)	(0.06)	(0.04)	(0.06)	(0.08)	(0.08)	(0.28)	(0.06)	(0.05)
η_{31}	1965	0.66	0.19	0.45	0.42	0.66	**	0.59	0.14	0.93
		(0.13)	(0.03)	(0.09)	(0.08)	(0.13)		(0.12)	(0.03)	(0.18)
	1973	0.55	0.24	0.42	0.44	0.19	**	0.36	0.19	0.54
		(0.11)	(0.04)	(0.08)	(0.09)	(0.04)		(0.07)	(0.03)	(0.11)

Table 11 - continued

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{32}	1965	0.99	-0.73	0.68	0.63	1.00	**	0.89	-0.52	1.40
		(0.13)	(0.08)	(0.09)	(0.08)	(0.13)	**	(0.12)	(0.06)	(0.19)
	1973	0.83	-0.88	0.64	0.66	0.28	**	0.54	-0.73	0.82
		(0.11)	(0.10)	(0.08)	(0.09)	(0.04)	**	(0.07)	(0.08)	(0.11)
η_{33}	1965	-1.61	-0.87	-1.42	-1.39	-1.61	**	-1.55	-0.90	-1.86
		(0.20)	(0.09)	(0.13)	(0.12)	(0.20)	**	(0.18)	(0.06)	(0.28)
	1973	-1.51	-0.84	-1.39	-1.41	-1.17	**	-1.33	-0.87	-1.50
		(0.16)	(0.11)	(0.13)	(0.13)	(0.06)	**	(0.11)	(0.09)	(0.16)
η_{34}	1965	-1.04	0.40	-0.71	-0.66	-1.05	**	-0.94	0.29	-1.47
		(0.13)	(0.09)	(0.09)	(0.08)	(0.13)	**	(0.11)	(0.06)	(0.18)
	1973	-0.87	0.48	-0.67	-0.69	-0.30	**	-0.57	0.40	-0.86
		(0.10)	(0.10)	(0.08)	(0.08)	(0.04)	**	(0.07)	(0.09)	(0.10)
η_{41}	1965	-0.69	-0.08	-0.57	-0.34	-0.60	-0.21	-0.38	-0.08	-0.28
		(0.10)	(0.04)	(0.08)	(0.05)	(0.09)	(0.03)	(0.06)	(0.04)	(0.04)
	1973	-0.63	-0.07	-0.52	-0.34	-0.85	-0.21	-0.37	-0.08	-0.36
		(0.09)	(0.03)	(0.08)	(0.05)	(0.12)	(0.03)	(0.05)	(0.04)	(0.05)
η_{42}	1965	-0.41	-0.12	-0.34	-0.20	-0.36	-0.12	-0.23	-0.12	-0.17
		(0.07)	(0.06)	(0.05)	(0.03)	(0.06)	(0.02)	(0.04)	(0.06)	(0.03)
	1973	-0.38	-0.11	-0.32	-0.21	-0.51	-0.12	-0.22	-0.11	-0.22
		(0.06)	(0.05)	(0.51)	(0.03)	(0.08)	(0.02)	(0.03)	(0.05)	(0.03)
η_{43}	1965	-0.76	0.17	-0.63	-0.37	-0.67	-0.23	-0.42	0.18	-0.31
		(0.09)	(0.04)	(0.08)	(0.04)	(0.08)	(0.028)	(0.05)	(0.04)	(0.04)
	1973	-0.69	0.15	-0.58	-0.38	-0.94	-0.23	-0.40	0.16	-0.40
		(0.08)	(0.03)	(0.07)	(0.05)	(0.11)	(0.028)	(0.05)	(0.03)	(0.05)
η_{44}	1965	0.87	-0.97	0.55	-0.09	0.63	-0.44	0.04	-0.97	-0.24
		(0.13)	(0.08)	(0.11)	(0.06)	(0.12)	(0.04)	(0.07)	(0.08)	(0.05)
	1973	0.70	-0.97	0.42	-0.06	1.29	-0.43	-0.01	-0.97	-0.02
		(0.12)	(0.07)	(0.10)	(0.07)	(0.16)	(0.04)	(0.07)	(0.07)	(0.07)

*1 = solid, 2 = liquid, 3 = gas, 4 = electricity.

**Almost no solid fuel is consumed in the residential sectors of Canada and the United States, and almost no natural gas is consumed in Norway, so that these elasticities are meaningless.

Table 12 - Partial Price Elasticities for Model 4*

(Country Dummy Variables, 1960-74, Non-stationary)

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	- .99	**	- .99	- .99	- .99	- .99	- .99	**	- .99
		(.07)		(.09)	(.29)	(.12)	(.36)	(.11)		(.38)
	1973	- .99	**	- .98	- .95	- .94	- .96	- .99	**	- .94
		(.19)		(.46)	(1.50)	(1.86)	(1.27)	(.18)		(1.68)
η_{12}	1965	- .04	**	- .06	- .17	- .07	- .21	- .06	**	- .22
		(.05)		(.06)	(.19)	(.08)	(.23)	(.07)		(.25)
	1973	- .11	**	- .26	- .89	-1.07	- .73	- .10	**	- .97
		(.13)		(.29)	(1.00)	(1.20)	(.82)	(.12)		(1.08)
η_{13}	1965	.29	**	.38	1.13	.48	1.39	.41	**	1.47
		(.05)		(.07)	(.21)	(.09)	(.26)	(.08)		(.28)
	1973	.76	**	1.77	5.97	7.22	4.91	.71	**	6.50
		(.14)		(.33)	(1.13)	(1.37)	(.93)	(.13)		(1.23)
η_{14}	1965	- .25	**	- .33	- .97	- .42	-1.19	- .36	**	-1.27
		(.05)		(.06)	(.19)	(.08)	(.23)	(.07)		(.25)
	1973	- .65	**	-1.52	-5.13	-6.20	-4.22	- .61	**	-5.59
		(.13)		(.29)	(1.0)	(1.2)	(.82)	(.12)		(1.08)
η_{21}	1965	- .16	- .05	- .18	- .15	- .07	- .16	- .32	- .06	- .09
		(.18)	(.06)	(.20)	(.17)	(.08)	(.18)	(.36)	(.07)	(.11)
	1973	- .06	- .05	- .06	- .08	- .10	- .10	- .36	- .05	- .06
		(.06)	(.06)	(.06)	(.09)	(.12)	(.11)	(.40)	(.06)	(.07)
η_{22}	1965	-1.13	-1.04	-1.14	-1.11	-1.06	-1.13	-1.26	-1.05	-1.07
		(.20)	(.07)	(.22)	(.09)	(.09)	(.20)	(.40)	(.07)	(.12)
	1973	-1.04	-1.04	-1.04	-1.06	-1.08	-1.08	-1.29	-1.04	-1.05
		(.07)	(.07)	(.07)	(.10)	(.13)	(.13)	(.45)	(.07)	(.08)
η_{23}	1965	1.04	.34	1.14	.95	.45	1.03	2.05	.38	.60
		(.17)	(.05)	(.18)	(.15)	(.07)	(.17)	(.33)	(.06)	(.10)
	1973	.36	.34	.37	.50	.66	.64	2.29	.34	.39
		(.06)	(.05)	(.06)	(.08)	(.11)	(.10)	(.37)	(.05)	(.06)
η_{24}	1965	- .75	- .24	- .81	- .68	- .32	- .07	-1.47	- .27	- .43
		(.11)	(.04)	(.13)	(.11)	(.05)	(.12)	(.23)	(.04)	(.07)
	1973	- .25	- .24	- .26	- .36	- .47	- .46	-1.64	- .24	-.28
		(.04)	(.04)	(.04)	(.06)	(.07)	(.07)	(.26)	(.04)	(.04)
η_{31}	1965	.80	.75	.55	.51	.80	**	.72	.54	1.13
		(.15)	(.14)	(.10)	(.10)	(.15)		(.14)	(.10)	(.21)
	1973	.67	.91	.51	.53	.23	**	.43	.75	.66
		(.13)	(.17)	(.10)	(.10)	(.04)		(.08)	(.14)	(.12)

Table 12 (Continued).

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{32}	1965	.75	.71	.52	.48	.76	**	.68	.51	1.06
		(.12)	(.11)	(.08)	(.08)	(.12)		(.11)	(.08)	(.17)
	1973	.63	.86	.48	.50	.21	**	.41	.71	.62
		(.10)	(.14)	(.08)	(.08)	(.03)		(.07)	(.11)	(.10)
η_{33}	1965	-1.70	-1.60	-1.48	-1.45	-1.71	**	-1.64	-1.48	-1.99
		(.19)	(.18)	(.13)	(.12)	(.19)		(.17)	(.12)	(.26)
	1973	-1.59	-1.81	-1.45	-1.47	-1.20	**	-1.39	-1.66	-1.58
		(.16)	(.21)	(.12)	(.12)	(.05)		(.10)	(.18)	(.15)
η_{34}	1965	- .84	- .79	- .58	- .54	- .85	**	- .76	- .57	-1.19
		(.11)	(.11)	(.08)	(.07)	(.11)		(.10)	(.08)	(.16)
	1973	- .71	- .96	- .54	- .56	- .24	**	- .46	- .79	- .70
		(.09)	(.13)	(.07)	(.07)	(.03)		(.06)	(.11)	(.09)
η_{41}	1965	- .50	- .27	- .42	- .24	- .44	- .15	- .28	- .28	- .20
		(.10)	(.05)	(.08)	(.05)	(.08)	(.03)	(.05)	(.05)	(.04)
	1973	- .46	- .25	- .38	- .25	- .62	- .15	- .27	- .26	- .26
		(.09)	(.05)	(.07)	(.05)	(.12)	(.03)	(.05)	(.05)	(.05)
η_{42}	1965	- .40	- .21	- .33	- .19	- .34	- .12	- .22	- .22	- .16
		(.06)	(.03)	(.05)	).03)	(.05)	(.02)	(.03)	(.03)	(.02)
	1973	- .36	- .19	- .30	- .20	- .48	- .12	- .21	- .21	- .21
		(-.06)	(.03)	(.05)	(.03)	(.08)	(.02)	(.03)	(.03)	(.03)
η_{43}	1965	- .60	- .34	- .51	- .30	- .54	- .19	- .34	- .35	- .25
		(.08)	(.04)	(.07)	(.04)	(.07)	(.02)	(.05)	(.05)	(.03)
	1973	- .56	- .31	- .47	- .31	- .76	- .19	- .33	- .32	- .32
		(.07)	(.04)	(.06)	(.04)	(.10)	(.02)	(.04)	(.04)	(.04)
η_{44}	1965	.52	- .17	.25	- .25	.32	- .54	- .15	- .14	- .38
		(.13)	(.07)	(.10)	(.06)	(.11)	(.04)	(.07)	(.07)	(.05)
	1973	.38	- .25	.15	- .24	.86	- .54	- .19	- .20	- .21
		(.11)	(.06)	(.10)	(.06)	(.16)	(.04)	(.07)	(.07)	(.07)

* 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

** Almost no solid fuel is consumed in the residential sectors of Canada and the United States, and almost no national gas is consumed in Norway, so that these elasticities are meaningless.

Although we have rejected the hypothesis of stationarity, it is useful to examine elasticities for the model in which this restriction was imposed. It may be (and we have no way of knowing if this is the case) that time trend variables were significant in estimates of the share equations not because of a change over time in the indirect utility function, but because prices and expenditure shares all monotonically increased or decreased. If this was the the case the non-stationary model may result in underestimates of some of the elasticities. Own and cross-price elasticities for the stationary version of the model are shown in Table 13. Note that own price elasticities for solid fuel are much larger, perhaps unreasonably large in later years. Own price elasticities for liquid fuel and natural gas are about the same as in the non-stationary model, but those for electricity are now all negative, and all larger in magnitude. Since the statistical significance of the time trend variables may have been spurious, we use both this model and its non-stationary counterpart to compute total fuel price elasticities. Total fuel price elasticities are computed using the estimates of the own price elasticities of aggregate energy use for our "preferred" consumption model (these elasticities range from -1.05 to -1.15). Total price elasticities corresponding to the non-stationary model of fuel expenditures (all nine countries pooled) are shown in Table 14a, those corresponding to the stationary model are in Table 14b, and those corresponding to the fuel choice model in which Canada and the U.S. are pooled separately are given in Table 14c. Since the estimates of the own price elasticity of energy do not differ very much from -1, the total fuel price elasticities are within 10% of the partial elasticities.

Table 13: Partial Price Elasticities for Model 7
(Country Dummies, 1960-74, Stationary)*

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-1.55	**	-1.73	-3.18	-1.93	-3.67	-1.79	**	-3.84
		(.10)		(.13)	(.38)	(.16)	(.46)	(.14)		(.49)
	1973	-2.46	**	-4.40	-12.49	-14.88	-10.45	-2.36	**	-13.5
		(.25)		(.59)	(1.99)	(2.42)	(1.64)	(.24)		(2.17)
η_{12}	1965	.02	**	.02	.07	.03	.09	.03	**	.10
		(.06)		(.08)	(.24)	(.10)	(.29)	(.09)		(.31)
	1973	.05	**	.12	.39	.47	.32	.05	**	.43
		(.16)		(.38)	(1.27)	(1.54)	(1.05)	(.15)		(1.38)
η_{13}	1965	.32	**	.42	1.26	.54	1.54	.46	**	1.64
		(.06)		(.08)	(.24)	(.10)	(.29)	(.09)		(.31)
	1973	.84	**	1.96	6.64	8.0	5.46	.78	**	7.22
		(.16)		(.37)	(1.25)	(1.5)	(1.02)	(.15)		(1.36)
η_{14}	1965	.21	**	.28	.84	.36	1.03	.31	**	1.1
		(.06)		(.08)	(.23)	(.10)	(.29)	(.08)		(.31)
	1973	.57	**	1.32	4.45	5.38	3.66	.53	**	4.84
		(.16)		(.37)	(1.24)	(1.50)	(1.02)	(.15)		(1.35)
η_{21}	1965	.07	.02	.08	.07	.03	.07	.14	.03	.04
		(.23)	(.08)	(.26)	(.21)	(.10)	(.23)	(.46)	(.09)	(.13)
	1973	.02	.02	.02	.03	.04	.04	.16	.02	.03
		(.08)	(.08)	(.08)	(.11)	(.15)	(.14)	(.52)	(.08)	(.09)
η_{22}	1965	-1.00	-1.00	-1.01	-1.01	-1.00	-1.01	-1.01	-1.00	-1.00
		(.22)	(.07)	(.24)	(.20)	(.10)	(.22)	(.44)	(.08)	(.13)
	1973	-1.00	-1.00	-1.00	-1.00	-1.01	-1.01	-1.02	-1.00	-1.00
		(.07)	(.07)	(.08)	(.11)	(.14)	(.14)	(.49)	(.07)	(.08)
η_{23}	1965	.93	.30	1.01	.84	.40	.91	1.82	.34	.53
		(.18)	(.06)	(.20)	(.16)	(.08)	(.18)	(.35)	(.07)	(.10)
	1973	.32	.30	.32	.45	.58	.57	2.04	.30	.35
		(.06)	(.06)	(.06)	(.09)	(.11)	(.11)	(.39)	(.06)	(.07)
η_{24}	1965	-.99	-.32	-1.08	-.90	-.43	-.97	-1.94	-.36	-.57
		(.16)	(.05)	(.18)	(.15)	(.07)	(.16)	(.32)	(.06)	(.09)
	1973	-.34	-.32	-.35	-.48	-.62	-.61	-2.17	-.32	-.37
		(.06)	(.05)	(.06)	(.08)	(.10)	(.10)	(.36)	(.05)	(.06)

* 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

** Almost no solid fuel is consumed in the residential sectors of Canada and the United States, and almost no natural gas is consumed in Norway, so that these elasticities are meaningless.

Table 13: Partial Price Elasticities for Model 7 (Country Dummies, 1960-74, Stationary)* - Continued

Elast	Year	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{31}	1965	.89	.84	.61	.56	.89	**	.80	.60	1.25
		(.17)	(.16)	(.11)	(.11)	(.17)		(.15)	(.11)	(.23)
	1973	.74	1.01	.57	.59	.25	**	.48	.83	.73
		(.14)	(.19)	(.11)	(.11)	(.05)		(.09)	(.16)	(.14)
η_{32}	1965	.67	.63	.46	.43	.67	**	.60	.45	.94
		(.13)	(.12)	(.09)	(.08)	(.13)		(.12)	(.09)	(.18)
	1973	.56	.76	.43	.44	.19	**	.36	.63	.55
		(.11)	(.15)	(.08)	(.09)	(.04)		(.07)	(.12)	(.11)
η_{33}	1965	-1.81	-1.76	-1.56	-1.51	-1.81	**	-1.73	-1.54	-2.14
		(.20)	(.19)	(.14)	(.13)	(.20)		(.18)	(.14)	(.29)
	1973	-1.68	-1.92	-1.52	-1.54	-1.23	**	-1.44	-1.76	-1.67
		(.17)	(.23)	(.13)	(.13)	(.06)		(.11)	(.19)	(.17)
η_{34}	1965	-.75	-.70	-.51	-.47	-.75	**	-.67	-.50	-1.05
		(.16)	(.15)	(.11)	(.10)	(.16)		(.14)	(.11)	(.22)
	1973	-.62	-.85	-.48	-.49	-.21	**	-.41	-.70	-.62
		(.13)	(.18)	(.10)	(.10)	(.04)		(.09)	(.15)	(.13)
η_{41}	1965	.44	.24	.36	.21	.38	.13	.24	.25	.18
		(.12)	(.06)	(.10)	(.06)	(.10)	(.04)	(.07)	(.07)	(.05)
	1973	.39	.22	.33	.22	.53	.13	.23	.23	.23
		(.11)	(.06)	(.09)	(.06)	(.15)	(.04)	(.06)	(.06)	(.06)
η_{42}	1965	-.52	-.28	-.43	-.25	-.46	-.16	-.29	-.29	-.21
		(.09)	(.05)	(.07)	(.04)	(.08)	(.03)	(.05)	(.05)	(.03)
	1973	-.48	-.26	-.39	-.26	-.64	-.16	-.28	-.27	-.27
		(.08)	(.04)	(.07)	(.04)	(.11)	(.03)	(.05)	(.05)	(.04)
η_{43}	1965	-.55	-.30	-.45	-.27	-.48	-.16	-.30	-.31	-.22
		(.12)	(.06)	(.09)	(.06)	(.10)	(.03)	(.06)	(.07)	(.05)
	1973	-.50	-.27	-.42	-.27	-.67	-.17	-.29	-.29	-.29
		(.11)	(.06)	(.09)	(.06)	(.14)	(.03)	(.06)	(.06)	(.06)
η_{44}	1965	-.36	-.65	-.47	-.69	-.45	-.81	-.65	-.64	-.74
		(.14)	(.07)	(.11)	(.07)	(.12)	(.04)	(.08)	(.08)	(.05)
	1973	-.42	-.68	-.52	-.68	-.22	-.81	-.67	-.66	-.67
		(.12)	(.07)	(.10)	(.07)	(.17)	(.04)	(.07)	(.07)	(.07)

* 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

** Almost no solid fuel is consumed in the residential sectors of Canada and the United States, and almost no natural gas is consumed in Norway, so that these elasticities are meaningless.

5.4 Logit Models of Fuel Choice

As an alternative to the translog model, we have also estimated several versions of static and dynamic logit models to describe the dependence of fuel shares on prices, income, and temperature. The "decision functions" in these models are linear or logarithmic functions of relative fuel prices $\tilde{P}_i$ (the price of fuel i divided by the price of energy), per capita income Y, temperature T, and, in the dynamic models, lagged shares. Recall from equation (37) that this leads to a set of three equations that must be estimated simultaneously, since certain coefficients are constrained to be the same across equations.

Our static logit models are of the general form

$$\log(S_i/S_4) = \sum_{k=1}^{9} a_{i4k}D_k + b_i\tilde{P}_i - b_4\tilde{P}_4 + c_{i4}Y + d_{i4}T, \quad i = 1,2,3. \qquad (59)$$

where $a_{i4k} = a_{ik}-a_{4k}$, $c_{i4}=c_i-c_4$, and $d_{i4}-d_i-d_4$, as in equation (35). The K_k are country dummy variables (countries are ordered alphabetically), and the fuels are ordered (1) liquid, (2) solid, (3) gas, and (4) electricity. Parameter estimates for four alternative versions of the model are shown in Table 15 (t-statistics are in parentheses).

The results are not encouraging. Own price elasticities are determined by the coefficients b_i, and for three of the models two or more of these are positive. The version of the model that gives the most sensible results in that shown in column (3) in which the decision function is linear in relative prices and income, price dummy variables are included for the shares of solid fuel in Canada and the U.S. and for the shares of natural gas in Norway and W.Germany,[51] and the temperature variable is not included. Even here, however, b_4 (the coefficient determining the own price elasticity for electricity) is positive (but insignificant), and only b_3 is significant at the 5% level.

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[51] The price dummy variables for solid fuel in Canada and the U.S. are CNSD and USSD, and for gas in Norway and West Germany are NRGD and WGGD. There is virtually no solid fuel used in the residential sectors of Canada and the U.S., and little or no gas used in the residential sectors of Norway and West Germany. This is not because prices are too high, but because in Canada and the U.S. other fuels are readily available that are cleaner and more convenient, and in Norway and West Germany the extremely limited supplies of gas are not made available to residential consumers. Note that the price dummy variables are indeed highly significant.

Table 14(a) - Total Fuel Price Elasticities for Fuel Model 3 with Consumption Model 4

ELAST.	YEAR	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-1.07	**	-1.12	-1.12	-1.08	-1.12	-1.08	**	-1.08
	1973	-1.06	**	-1.09	-1.08	-1.01	-1.07	-1.08	**	-1.00
η_{12}	1965	-0.05	**	-0.07	-0.18	-0.09	-0.22	-0.06	**	-0.24
	1973	-0.13	**	-0.30	-0.92	-1.09	-0.75	-0.11	**	-0.98
η_{13}	1965	0.27	**	0.34	1.09	0.46	1.38	0.39	**	1.46
	1973	0.74	**	1.73	5.94	7.17	4.91	0.67	**	6.48
η_{14}	1965	-0.26	**	-0.36	-1.03	-0.44	-1.30	-0.39	**	-1.32
	1973	-0.67	**	-1.55	-5.19	-6.21	-4.31	-0.64	**	-5.61
η_{21}	1965	-0.20	-0.06	-0.22	-0.16	-0.09	-0.17	-0.35	-0.06	-0.10
	1973	-0.06	-0.05	-0.06	-0.08	-0.10	-0.10	-0.38	-0.05	-0.06
η_{22}	1965	-1.21	-1.17	-1.27	-1.25	-1.14	-1.26	-1.34	-1.16	-1.17
	1973	-1.11	-1.19	-1.15	-1.19	-1.15	-1.20	-1.38	-1.14	-1.10
η_{23}	1965	1.02	0.31	1.10	0.91	0.43	1.02	2.03	0.35	0.58
	1973	0.34	0.31	0.33	0.47	0.61	0.64	2.26	0.32	0.38
η_{24}	1965	-0.76	-0.30	-0.85	-0.74	-0.35	-0.84	-1.50	-0.32	-0.48
	1973	-0.27	-0.31	-0.29	-0.41	-0.48	-0.55	-1.68	-0.29	-0.30
η_{31}	1965	0.75	0.75	0.50	0.49	0.77	**	0.69	0.53	1.11
	1973	0.65	0.91	0.50	0.52	0.22	**	0.41	0.75	0.65
η_{32}	1965	0.74	0.66	0.50	0.46	0.73	**	0.67	0.46	1.04
	1973	0.60	0.80	0.44	0.46	0.20	**	0.40	0.66	0.60
η_{33}	1965	-1.78	-1.79	-1.61	-1.58	-1.80	**	-1.72	-1.58	-2.09
	1973	-1.66	-1.95	-1.56	-1.59	-1.28	**	-1.47	-1.77	-1.64
η_{34}	1965	-0.86	-0.85	-0.61	-0.60	-0.87	**	-0.76	-0.61	-1.24
	1973	-0.72	-1.03	-0.57	-0.61	-0.25	**	-0.50	-0.84	-0.72
η_{41}	1965	-0.54	-0.27	-0.46	-0.26	-0.46	-0.16	-0.30	-0.28	-0.21
	1973	-0.47	-0.25	-0.39	-0.25	-0.61	-0.15	-0.28	-0.26	-0.26
η_{42}	1965	-0.40	-0.26	-0.34	-0.21	-0.37	-0.13	-0.22	-0.26	-0.18
	1973	-0.38	-0.25	-0.34	-0.23	-0.49	-0.14	-0.21	-0.24	-0.22
η_{43}	1965	-0.63	-0.35	-0.54	-0.33	-0.55	-0.18	-0.35	-0.37	-0.26
	1973	-0.57	-0.33	-0.50	-0.34	-0.80	-0.18	-0.35	-0.34	-0.33
η_{44}	1965	0.43	-0.30	0.12	-0.39	0.23	-0.68	-0.24	-0.25	-0.48
	1973	0.31	-0.39	0.04	-0.36	0.78	-0.65	-0.28	-0.30	-0.26

Table 14(b) - Total Fuel Price Elasticities for Fuel Model 7 with Consumption Model 4

ELAST.	YEAR	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-1.63	**	-1.85	-3.31	-2.02	-3.80	-1.88	**	-3.93
	1973	-2.53	**	-4.51	-12.61	-14.05	-10.57	-2.45	**	-13.55
η_{12}	1965	0.01	**	0.01	0.06	0.01	0.07	0.02	**	0.08
	1973	0.02	**	0.08	0.36	0.46	0.29	0.04	**	0.40
η_{13}	1965	0.30	**	0.39	1.22	0.52	1.54	0.44	**	1.63
	1973	0.83	**	1.93	6.60	7.98	5.46	0.75	**	7.21
η_{14}	1965	0.19	**	0.24	0.77	0.33	0.92	0.27	**	1.04
	1973	0.54	**	1.28	4.39	5.36	3.57	0.48	**	4.82
η_{21}	1965	0.03	0.02	0.03	0.04	0.01	0.06	0.11	0.02	0.03
	1973	0.01	0.02	0.01	0.03	0.04	0.04	0.14	0.02	0.02
η_{22}	1965	-1.09	-1.13	-1.13	-1.14	-1.09	-1.14	-1.10	-1.11	-1.10
	1973	-1.07	-1.15	-1.11	-1.13	-1.08	-1.12	-1.11	-1.10	-1.06
η_{23}	1965	0.91	0.28	0.98	0.80	0.38	0.91	1.80	0.31	0.52
	1973	0.30	0.27	0.29	0.41	0.53	0.57	2.00	0.28	0.33
η_{24}	1965	-1.00	-0.38	-1.11	-0.96	-0.45	-1.08	-1.98	-0.40	-0.62
	1973	-0.35	-0.39	-0.38	-0.53	-0.63	-0.70	-2.21	-0.37	-0.39
η_{31}	1965	0.84	0.83	0.56	0.54	0.86	**	0.77	0.59	1.24
	1973	0.72	1.01	0.55	0.58	0.25	**	0.46	0.83	0.73
η_{32}	1965	0.65	0.58	0.44	0.40	0.64	**	0.59	0.41	0.92
	1973	0.53	0.70	0.38	0.41	0.17	**	0.36	0.58	0.53
η_{33}	1965	-1.89	-1.89	-1.68	-1.65	-1.90	**	-1.81	-1.65	-2.24
	1973	-1.74	-2.07	-1.62	-1.66	-1.30	**	-1.53	-1.86	-1.72
η_{34}	1965	-0.76	-0.75	-0.54	-0.54	-0.77	**	-0.70	-0.54	-1.11
	1973	-0.64	-0.92	-0.51	-0.55	-0.22	**	-0.44	-0.74	-0.64
η_{41}	1965	0.39	0.23	0.31	0.19	0.35	0.11	0.21	0.24	0.16
	1973	0.38	0.21	0.32	0.21	0.53	0.12	0.21	0.22	0.22
η_{42}	1965	-0.53	-0.33	-0.44	-0.27	-0.48	-0.17	-0.29	-0.33	-0.23
	1973	-0.50	-0.31	-0.43	-0.29	-0.65	-0.18	-0.28	-0.31	-0.29
η_{43}	1965	-0.56	-0.32	-0.48	-0.30	-0.49	-0.16	-0.32	-0.33	-0.23
	1973	-0.51	-0.29	-0.44	-0.30	-0.71	-0.16	-0.31	-0.30	-0.29
η_{44}	1965	-0.44	-0.78	-0.60	-0.82	-0.53	-0.94	-0.73	-0.75	-0.84
	1973	-0.49	-0.83	-0.62	-0.80	-0.29	-0.92	-0.75	-0.77	-0.72

Table 14(c) - Total Fuel Price Elasticities: U.S. and Canada Pooled Separately

ELAST.	YEAR	BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
η_{11}	1965	-0.83	**	-0.79	-0.14	-0.66	0.08	-0.72	**	0.20
	1973	-0.40	**	0.44	4.13	5.28	3.20	-0.47	**	4.66
η_{12}	1965	-0.16	**	-0.21	-0.61	-0.28	-0.75	-0.22	**	-0.80
	1973	-0.42	**	-0.97	-3.17	-3.81	-2.61	-0.38	**	-3.44
η_{13}	1965	0.22	**	0.28	0.90	0.38	1.14	0.32	**	1.20
	1973	0.61	**	1.43	4.89	5.90	4.05	0.55	**	5.34
η_{14}	1965	-0.36	**	-0.48	-1.40	-0.60	-1.74	-0.52	**	-1.80
	1973	-0.91	**	-2.12	-7.10	-8.52	-5.88	-0.87	**	-7.68
η_{21}	1965	-0.62	0.04	-0.68	-0.54	-0.28	-0.59	-1.17	0.05	-0.34
	1973	-0.21	0.04	-0.21	-0.28	-0.37	-0.36	-1.29	0.04	-0.22
η_{22}	1965	-1.09	-0.69	-1.14	-1.15	-1.10	-1.15	-1.11	-0.61	-1.11
	1973	-1.07	-0.72	-1.11	-1.13	-1.08	-1.13	-1.12	-0.66	-1.06
η_{23}	1965	1.36	-0.38	1.47	1.22	0.58	1.35	2.70	-0.42	0.78
	1973	0.46	-0.37	0.45	0.63	0.82	0.85	3.00	-0.37	0.51
η_{24}	1965	-0.80	-0.19	-0.89	-0.78	-0.37	-0.88	-1.58	-0.20	-0.51
	1973	-0.28	-0.20	-0.31	-0.44	-0.51	-0.58	-1.76	-0.18	-0.32
η_{31}	1965	0.62	0.19	0.41	0.40	0.63	**	0.56	0.14	0.92
	1973	0.54	0.24	0.41	0.43	0.19	**	0.34	0.19	0.54
η_{32}	1965	0.98	0.78	0.67	0.61	0.97	**	0.89	0.56	1.38
	1973	0.81	0.94	0.60	0.63	0.27	**	0.54	0.77	0.80
η_{33}	1965	-1.69	-0.10	-1.55	-1.53	-1.71	**	-1.64	-1.02	-1.96
	1973	-1.58	-0.99	-1.50	-1.53	-1.25	**	-1.43	-0.97	-1.56
η_{34}	1965	-1.06	0.34	-0.75	-0.73	-1.07	**	-0.97	0.24	-1.53
	1973	-0.89	0.41	-0.70	-0.75	-0.31	**	-0.61	0.35	-0.89
η_{41}	1965	-0.73	-0.08	-0.62	-0.35	-0.63	-0.22	-0.41	-0.08	-0.29
	1973	-0.64	-0.07	-0.53	-0.35	-0.85	-0.21	-0.38	-0.08	-0.36
η_{42}	1965	-0.42	-0.17	-0.36	-0.22	-0.39	-0.14	-0.23	-0.16	-0.19
	1973	-0.40	-0.16	-0.35	-0.24	-0.52	-0.15	-0.22	-0.15	-0.23
η_{43}	1965	-0.78	0.14	-0.66	-0.41	-0.68	-0.23	-0.44	0.15	-0.32
	1973	-0.71	0.13	-0.61	-0.41	-0.98	-0.23	-0.43	0.14	-0.41
η_{44}	1965	0.79	-1.10	0.42	-0.22	0.54	-0.58	-0.05	-1.08	-0.34
	1973	0.63	-1.13	0.31	-0.19	1.21	-0.55	-0.1-	-1.08	-0.08

Table 15 - Parameter Estimates for Static Logit Models

Parameter	1 Linear in Relative Price and Income		2 Linear in Relative Price, Income and Temperature		3 Linear in Relative Price and Income; Dummy Variables		4 Linear in Logs of Relative Price and Income; Dummy Variables	
a_{14}	6.7256	(10.10)	8.6567	(8.75)	4.6183	(6.34)	26.43	(4.55)
a_{24}	0.9400	(1.55)	0.7536	(0.75)	-1.1002	(-1.66)	-13.03	(-2.35)
a_{34}	1.1740	(2.15)	-0.5394	-0.6335	1.9594	(3.25)	8.68	(1.71)
CN_{14}	-4.8884	(-20.48)	-5.4918	(-16.35)	0.3183	(0.26)	40.3656	(4.08)
CN_{24}	-0.7109	(-2.69)	-0.6725	(-1.82)	0.1322	(0.48)	-0.0022	(-0.01)
CN_{34}	-1.2883	(-5.79)	-0.7377	(-2.40)	-2.04	(-8.22)	-1.8879	(-6.60)
FR_{14}	-0.8825	(-5.22)	-0.6833	(-3.63)	-0.4874	(-2.92)	-0.3739	(-2.35)
FR_{24}	-0.7151	(-4.04)	-0.7428	(-3.71)	-0.3500	(-1.95)	-0.1395	(-0.82)
FR_{34}	-0.2900	(-1.85)	-0.4983	(-2.89)	0.0022	(0.01)	0.1220	(0.08)
It_{14}	-3.5782	(-12.82)	-3.1205	(-9.09)	-2.8435	(-9.77)	(-2.4560)	(-8.16)
It_{24}	-1.4759	(-5.89)	-1.5400	(-4.60)	-0.7820	(-2.95)	-0.3213	(-1.14)
It_{34}	-0.8752	(-3.99)	-1.3700	(-4.80)	-0.9612	(-4.13)	-0.6138	(-2.54)
ND_{14}	-1.7246	(-9.47)	-1.7623	(-9.75)	-1.366	(-7.60)	-1.4059	(-8.19)
ND_{24}	0.1651	(0.17)	0.1580	(0.91)	0.5675	(3.27)	0.5661	(3.45)
ND_{34}	0.1263	(0.85)	0.1705	(1.16)	0.2095	(1.41)	0.3520	(2.41)
NR_{14}	-4.0201	(-11.71)	-4.5917	(-11.25)	-2.8467	(-7.73)	-2.5151	(-5.16)
NR_{24}	-2.3643	(-6.49)	-2.3383	(-5.36)	-1.0996	(-2.82)	-1.0664	(-2.15)
NR_{34}	-6.5012	(-18.11)	-5.9818	(-14.73)	-7.9598	(-15.69)	-23.82	(-5.81)
UK_{14}	-1.7562	(-6.32)	-1.6575	(-5.90)	-0.9859	(-3.39)	-0.56	(-1.87)
UK_{24}	-2.1243	(-7.96012)	-2.1729	(-7.90)	-1.2533	(-4.47)	-1.3266	(-4.68)
UK_{34}	-0.9826	(-4.27)	-1.1300	(-4.81)	-0.7736	(-3.18)	-0.4693	(-1.85)
US_{14}	-8.7050	(-49.24)	-8.5911	(-47.53)	-12.00	(-9.67)	-21.3229	(-2.03)
US_{24}	-0.7155	(-3.73)	-0.7454	(-3.73)	-0.3594	(-1.91)	-0.4493	(-2.55)
US_{34}	-0.4900	(-2.49)	-0.6236	(-3.14)	-1.7326	(-7.77)	-1.9778	(-6.23)
WG_{14}	-3.5738	(-12.64)	-3.8422	(-13.10)	-2.8073	(-9.51)	-2.3900	(-7.81)
WG_{24}	-1.3589	(-5.24)	-1.3400	(-4.83)	-0.6365	(-2.32)	0.1648	(-0.56)
WG_{34}	-1.7872	(-7.71)	-1.5866	(-6.53)	-4.3407	(-9.73)	-24.50	(-6.18)
b_1	5.4863×10^{-6}	(0.10)	-8.3737×10^{-6}	(-0.15)	-4.0938×10^{-6}	(-0.07)	0.4894	(1.79)
b_2	7.1110×10^{-6}	(0.15)	1.6490×10^{-5}	(0.35)	-4.5742×10^{-5}	(-1.07)	0.4152	(2.50)
b_3	3.4942×10^{-5}	(2.10)	3.3989×10^{-5}	(2.08)	-0.0002	(-6.70)	-1.2639	(-5.15)
b_4	4.8882×10^{-5}	(4.03)	4.9800×10^{-5}	(4.10)	4.7887×10^{-6}	(0.36)	-0.1843	(-0.47)
CNSD					-0.0007	(-3.88)	-5.060	(-4.49)
USSD					0.0005	(2.88)	1.4289	(1.20)
NRGD					0.0002	(6.56)	1.9984	(4.63)
WGGD					0.0003	(7.36)	2.5392	(5.87)
c_{14}	-0.0023	(-19.88)	-0.0023	(-19.67)	-0.0020	(-17.84)	-4.2573	(-15.97)
c_{24}	0.0002	(1.84)	0.0002	(1.67)	0.0005	(4.16)	0.9737	(3.89)
c_{34}	-4.9263×10^{-5}	(-.49)	-8.2384×10^{-5}	(-0.83)	-6.0993×10^{-5}	(-0.61)	0.1266	(0.56)
d_{14}			-0.0484	(2.08)				
d_{24}			.0059	(0.27)				
d_{34}			0.0465	(2.65)				
Eqn(1) R^2	.9886		.9891		.9909		.9901	
Eqn(2) R^2	.7178		.7164		.7420		.7549	
Eqn(3) R^2	.9543		.9568		.9592		.9574	

The dynamic version of this model is based on the assumption that the choice of fuels this period depends on the relative shares last period, as well as this period's prices and income. The dependence on past shares is intended to incorporate both habit formation and stock adjustment effects. It leads to equations of the form

$$\log (S_i/S_4) = \sum_{k=1}^{9} a_{i4k}D_k + b_i\tilde{P}_i - b_4\tilde{P}_4 + c_{i4}Y + \lambda_i S_{i,t-1} - \lambda_4 S_{4,t-1}, \quad i = 1,2,3 \tag{60}$$

Again, this is not a Koyck adjustment model; the coefficients λ_i can be greater than 1 (although we would expect them to be positive), and in general a change in price will not lead to geometrically declining changes in shares over time.

Estimation results for three versions of the dynamic model are shown in Table 16. The first model is identical to static model (3) in Table 15, except that it contains lagged share terms. The second model is identical to static model (4), but with lagged share terms. The third model is the same as the first, but uses actual fuel prices, rather than relative prices.

The results are again discouraging. None of the b_i coefficients are significant at the 5% level, and at least one is positive in each model. This inability of the model to pick up price effects may be due to its restrictive form. All cross price elasticities for a given own price are constrained to be equal, and parameter estimates become inefficient when shares become very small.

We have calculated price and income elasticities for version (3) of the static logit model, and these are shown in Tables 17 and 18. Only the price elasticities for natural gas are reasonable; the others are insignificantly different from 0. Income elasticities are large and negative for solid fuel, which is reasonable, since we would expect consumers to shift to clearer and more convenient fuels as their incomes rise. The regional variation in income elasticities for gas and electricity, however, does not make sense.

Table 16 - Parameter Estimates for Dynamic Logit Models

Parameters	1. Linear in relative price, income and lagged share		2. Linear in logs of relative price, income and lagged share		3. Linear in price, income and lagged share	
a_{141}	4.5706	(6.37)	9.1927	(2.15)	6.5965	(8.84)
a_{241}	-1.0893	(-2.02)	-1.7969	(-0.45)	-0.2495	(-0.61)
a_{341}	-0.1113	(-0.20)	3.0236	(0.86)	0.6676	(1.53)
CN_{14}	1.5298	(1.03)	17.0989	(1.98)	-4.6891	(-2.44)
CN_{24}	-0.2392	(-0.89)	-0.2440	(-1.13)	-0.52	(-2.12)
CN_{34}	-0.3963	(-1.80)	-0.4428	(-2.31)	-0.7723	(-3.25)
FR_{14}	-0.2921	(-1.84)	-0.1406	(-1.24)	-0.4646	(-3.08)
FR_{24}	-0.0429	(-0.31)	0.0021	(0.0182)	-0.1767	(-1.80)
FR_{34}	0.0220	(0.18)	-0.0609	(0.64)	-0.0398	(-0.40)
It_{14}	-2.1697	(-6.92)	-0.7096	(-2.82)	-2.7483	(-9.77)
It_{24}	0.0039	(0.02)	-0.1185	(-0.57)	-0.2704	(-1.70)
It_{34}	-0.1448	(-0.69)	-0.3020	(-1.69)	-0.2670	(-1.75)
ND_{14}	-1.1588	(-5.69)	-0.5176	(-3.58)	-1.7439	(-7.95)
ND_{24}	0.1236	(0.86)	0.0152	(0.12)	-0.0489	(-0.34)
ND_{34}	0.0517	(0.46)	0.1250	(1.38)	-0.2167	(-1.79)
NR_{14}	-1.4014	(-3.47)	-0.8337	(-2.23)	-2.4255	(-6.04)
NR_{24}	0.0343	(0.08)	-0.5085	(-1.37)	-0.4642	(-1.29)
NR_{34}	-4.2717	(-9.27)	-10.0931	(-4.71)	-5.3241	(-11.77)
UK_{14}	-0.5633	(-2.15)	-0.1730	(-0.81)	-1.0461	(-4.01)
UK_{24}	-0.6417	(-2.72)	-0.5212	(-2.60)	-0.9827	(-4.52)
UK_{34}	-0.0835	(-0.41)	-0.1901	(-1.06)	-0.4565	(-2.70)
US_{14}	-10.4923	(-7.55)	-6.4424	(-0.81)	-3.2583	(-1.95)
US_{24}	-0.3169	(-1.96)	-0.2822	(-2.14)	-0.45	(-2.85)
US_{34}	-0.3619	(-2.00)	-0.4752	(-2.72)	-0.4420	(-2.42)
WG_{14}	-1.9051	(-5.92)	-0.6222	(-2.44)	-2.4607	(-8.64)
WG_{24}	0.1751	(0.66)	-0.0895	(-0.39)	-0.1280	(-0.68)
WG_{34}	-1.3432	(-3.38)	-8.2090	(-3.76)	-2.0327	(-3.99)
b_1	-4.5454×10^{-5}	(-0.78)	0.2706	(1.28)	-4.9784×10^{-6}	(-0.15)
b_2	5.7585×10^{-5}	(1.67)	0.2053	(1.78)	2.4267×10^{-5}	(1.12)
b_3	-3.6124×10^{-5}	(-1.68)	-0.2239	(-1.67)	-1.0576×10^{-5}	(-0.90)
b_4	-5.0433×10^{-6}	(-0.45)	-0.0088	(-0.03)	6.4775×10^{-6}	(1.77)
CNSD	-0.0008	(-3.54)	-2.0847	(-2.05)	6.2298×10^{-5}	(.20)
USSD	0.0004	(1.82)	0.4873	(0.54)	-0.0006	(-3.02)
NRGD	7.1644×10^{-5}	(2.81)	0.9924	(4.56)	7.9932×10^{-5}	(4.21)
WGGD	0.0001	(3.72)	0.8642	(3.65)	6.2513×10^{-5}	(3.20)
c_{14}	-0.0019	(-12.64)	-1.5112	(-4.94)	-0.0025	(-12.00)
c_{24}	9.1226×10^{-5}	(0.83)	0.0384	0.1864	5.6738×10^{-6}	(0.05)
c_{34}	-2.4279×10^{-5}	(-0.32)	-0.0834	-0.5623	-0.0002	(-2.38)
λ_1	0.2897	(0.55)	0.7529	(11.85)	-0.7330	(-1.07)
λ_2	4.0610	(9.83)	0.8074	(13.40)	3.9227	(9.54)
λ_3	3.4846	(9.2933)	0.9301	(15.55)	3.4574	(9.83)
λ_4	2.1464	(5.22)	0.6438	(5.98)	1.9725	(5.03)
Eqn(1)R^2	0.9910		0.9953		0.9907	
Eqn(2)R^2	0.8825		0.9005		0.8844	
Eqn(3)R^2	0.9833		0.9881		0.9856	

Table 17 - Relative Price Elasticities for Fuel Shares:
Static Logit Model (3)*

FUEL	YEAR	BELG	CANADA	FRANCE	ITAL	NETH	NORWAY	UK	USA	W. GERMANY
SOLID	1962	-.01	**	-.01	-.01	-.02	-.02	-.01	**	-.01
	1966	-.01	**	-.02	-.01	-.02	-.02	-.01	**	-.01
	1970	-.02	**	-.02	-.01	-.03	-.03	-.01	**	-.02
	1974	-.02	**	-.02	-.03	-.03	-.03	-.01	**	-.02
LIQUID	1962	-.13	-.14	-.10	-.07	-.10	-.20	-.21	-.13	-.06
	1966	-.09	-.15	-.08	-.06	-.08	-.18	-.18	-.14	-.05
	1970	-.08	-.15	-.08	-.06	-.11	-.18	-.16	-.17	-.05
	1974	-.12	-.17	-.10	-.07	-.27	-.25	-.22	-.24	-.10
GAS	1962	-1.69	-.65	-1.41	-.97	-1.92	**	-1.60	-.32	.85
	1966	-1.69	-.64	-1.32	-.91	-1.35	**	-1.22	-.40	.89
	1970	-1.60	-.58	-1.21	-1.12	-.89	**	-.96	-.43	1.18
	1974	-1.43	-.49	-1.78	-.75	-.52	**	-.82	-.46	1.24
ELECTRICITY	1962	.16	.05	.10	.06	.13	.01	.07	.04	.15
	1966	.14	.05	.09	.05	.10	.01	.06	.07	.04
	1970	.12	.05	.09	.05	.10	.02	.05	.06	.04
	1974	.11	.05	.09	.05	.09	.01	.05	.05	.05

*Elasticities refer to percent changes in fuel shares, not quantities.

Table 18 - Per Capita Disposable Income Elasticities for Fuel Shares: Static Logit Model (3)

FUEL	YEAR	BELG	CANADA	FRANCE	ITAL	NETH	NOR	UK	USA	W. GERMANY
SOLID	1962	-1.46	**	-1.90	-2.44	-2.04	-3.33	-2.10	**	-3.08
	1966	-2.25	**	-2.79	-3.06	-2.96	-3.66	-2.61	**	-3.60
	1970	-3.33	**	-3.96	-4.13	-3.98	-3.85	-2.98	**	-4.54
	1974	-5.07	**	-5.69	-4.52	-4.96	-4.79	-3.77	**	-5.11
LIQUID	1962	2.52	.69	2.26	1.11	1.57	1.04	2.14	.92	1.03
	1966	2.37	.73	2.12	1.04	1.42	1.13	2.11	1.06	1.03
	1970	2.31	.79	1.89	1.03	1.44	1.37	1.89	1.22	1.07
	1974	1.88	.96	1.26	.95	1.09	.97	1.86	1.31	.95
GAS	1962	1.64	-.32	1.34	.32	.77	**	1.20	-.53	.12
	1966	1.35	-.41	1.03	.13	.45	**	1.06	-.66	.01
	1970	1.06	-.50	.59	-.11	.24	**	.81	-.67	-.18
	1974	.34	-.83	-.28	-.26	-.25	**	.61	-.79	-.39
ELECTRICITY	1962	1.73	-.21	1.44	.41	.86	.26	1.30	-.37	.22
	1966	1.46	-.29	1.15	.23	.55	.19	1.18	-.47	.12
	1970	1.19	-.36	.73	.01	.37	.34	.92	-.47	-.04
	1974	.51	-.64	-.11	-.13	-.11	-.17	.74	-.56	-.25

Table 19 - Predicted and Actual Fuel Share

(1) Predicted share (2)(Actual share)

Fuel	Year		BELG	CAN	FRAN	ITAL	NETH	NOR	UK	USA	WGER
SOLID	1962	(1)	.597	.026	.450	.141	.317	.111	.420	<.001	.117
		(2)	(.560)	(.030)	(.444)	(.157)	(.354)	(.102)	(.394)	(<.001)	(.114)
	1966	(1)	.468	.009	.299	.092	.164	.087	.302	<.001	.078
		(2)	(.423)	(.012)	(.315)	(.100)	(.228)	(.086)	(.319)	(<.001)	(.080)
	1970	(1)	.266	.004	.157	.044	.074	.061	.262	<.001	.034
		(2)	(.312)	(.005)	(.199)	(.043)	(.128)	(.117)	(.243)	(<.001)	(.048)
	1974	(1)	.103	<.001	.084	.029	.040	.042	.158	<.001	.022
		(2)	.168)	(<.001)	(.067)	(.023)	(.012)	(.026)	(.167)	(<.001)	(.017)
LIQUID	1962	(1)	.094	.327	.099	.119	.231	.149	.059	.276	.193
		(2)	(.091)	(.381)	(.079)	(.084)	(.251)	(.144)	(.071)	(.321)	(.203)
	1966	(1)	.144	.361	.143	.136	.264	.164	.072	.332	.220
		(2)	(.144)	(.357)	(.124)	(.160)	(.309)	(.154)	(.055)	(.339)	(.211)
	1970	(1)	.225	.388	.191	.187	.331	.180	.073	.368	.250
		(2)	(.221)	(.352)	(.205)	(.193)	(.229)	(.146)	(.058)	(.306)	(.252)
	1974	(1)	.306	.478	.289	.169	.280	.183	.089	.386	.241
		(2)	(.340)	(.420)	(.369)	(.251)	(.209)	(.257)	(.047)	(.320)	(.310)
GAS	1962	(1)	.129	.167	.211	.270	.189	.002	.143	.275	.114
		(2)	(.155)	(.156)	(.240)	(.278)	(.140)	(.004)	(.174)	(.265)	(.114)
	1966	(1)	.157	.160	.270	.300	.324	.002	.228	.235	.120
		(2)	(.177)	(.187)	(.258)	(.273)	(.180)	(.002)	(.204)	(.249)	(.117)
	1970	(1)	.215	.164	.331	.240	.341	.005	.281	.215	.162
		(2)	(.183)	(.161)	(.270)	(.282)	(.450)	(.002)	(.255)	(.242)	(.148)
	1974	(1)	.269	.147	.210	.349	.453	.003	.331	.210	.221
		(2)	(.213)	(.132)	(.242)	(.279)	(.570)	(.005)	(.324)	(.165)	(.297)
ELECTRICITY	1962	(1)	.179	.480	.239	.469	.262	.737	.377	.449	.576
		(2)	(.195)	(.433)	(.237)	(.475)	(.254)	(.756)	(.360)	(.413)	(.569)
	1966	(1)	.231	.469	.288	.471	.246	.746	.396	.433	.582
		(2)	(.254)	(.444)	(.303)	(.467)	(.282)	(.758)	(.422)	(.412)	(.593)
	1970	(1)	.292	.443	.321	.529	.253	.753	.383	.418	.554
		(2)	(.283)	(.481)	(.326)	(.482)	(.193)	(.735)	(.443)	(.452)	(.519)
	1974	(1)	.322	.373	.417	.452	.226	.772	.422	.404	.519
		(2)	(.278)	(.447)	(.325)	(.447)	(.208)	(.711)	(.462)	(.515)	(.376)

Predicted and actual fuel shares for version (3) of the static logit model are shown in Table 19. Since the model is static there is no accumulation of errors when it is used to simulate shares, and yet for several countries the predicted shares differ considerably from the actual. These results, together with the elasticity estimates discussed above, lead us to reject the logit model as a means of explaining the demand for fuels.

6. Comparison of Results with Other Studies

In Table 20 we present a survey of recent estimates by others of residential energy demand elasticities. We can examine these estimates and compare them on our own to determine, first, if there is any concensus on price elasticities, and second, how and why our estimates differ from those of others.

Looking first at estimates of the long-run own price elasticity of total energy use, we find a range extending from -0.28 to -1.70. However, only Nordhaus [48] obtained elasticities greater in magnitude than -1. An unweighted average of the seven studies other than that of Nordhaus gives an elasticity of -0.43. Our estimate of about -1.1 would thus seem high. On the other hand, most of these other studies are based on time series data for a single country (usually the U.S.), and thus are more likely to have captured short-run rather than long-run elasticities.

Most of the estimates of the long-run income elasticity of total energy use do not differ very much from 1, and this is supportive of our having imposed an elasticity of 1 on our model by assuming homotheticity. Joskow and Baughman [69] obtain an elasticity of -.6, and Nelson [47] obtains an elasticity of 0.27 (both for the U.S.), but these studies are based on time series data for a single country and, again, are more likely to have captured short-run elasticities.

Table 20 - Alternative Estimates of Residential Energy Demand Elasticities

Elasticity	Country	Estimate	Source
Aggregate energy use - own price elasticity	U.S.	S.R.: -.12, L.R.: -.50	(a)
	U.S.	S.R.: -.16, L.R.: -.63	(b)
	U.S.	-.28	(c)
	U.S.	-.40	(d)
	U.S.	S.R.: -.50, L.R.: -1.70	(e)
	Canada	-.33 to -.56	(f)
	Norway	-0.30	(g)
	W. Germany	S.R.: -.35, L.R.: -.78	
	Italy	S.R.: -.63, L.R.: -1.30	
	Netherlands	S.R.: -.42, L.R.: -1.30	(e)
	UK	S.R.: -.38, L.R.: -.42	
	6 countries pooled	-0.71	
	20 OECD countries pooled	-0.42	(h)
Aggregate energy use - income elasticity	U.S.	S.R.: 0.10, L.R.: 0.60	(a)
	U.S.	S.R.: 0.20, L.R.: 0.80	(b)
	U.S.	0.27	(c)
	Canada	0.83 to 1.26	(f)
	Norway	1.08	(g)
	6 countries pooled	1.09	(e)
	20 OECD countries pooled	1.51	(h)

Table 20 (Cont.)

Elasticity	Country	Estimate	Source
Fuel consumption, partial own price elasticities	U.S.	elec: -.06(S.R.), -.52(L.R.)	(m)
	U.S.	elec: -.14(S.R.), -1.22(L.R.)	(n)
	Canada	gas & oil: -0.96 -0.34	(f)
	Norway	elec: -.22 to -.60	(g)
	20 OECD countries pooled	gas: -1.05 oil: -0.33 coal: -0.81	(h)
Fuel consumption, total on price elasticities	U.S.	elec: -1.0 to -1.2	(i)
	U.S.	gas: -.15(S.R.), -1.01(L.R.) oil: -.18(S.R.), -1.10(L.R.) elec: -.19(S.R.), -1.00(L.R.)	(a)
	U.S.	gas: -1.34 oil: -1.89 elec: -1.13	(b)
	U.S.	gas: -1.28 to -1.77 elec: -0.40	(j)
	U.S.	gas: -0.91 oil: -0.91 elec: -0.84	(1)
	Canada	gas: -0.20(S.R.), -1.30(L.R.)	(k)
	20 OECD countries pooled	gas: -1.11 oil: -0.52 coal: -0.98	(h)

SOURCES:

(a) Joskow and Baughman [69]
(b) Baughman and Joskow [6]
(c) Nelson [47]
(d) Jorgenson [35]
(e) Nordhaus [48]
(f) Fuss and Waverman [68]
(g) Rødseth and Strøm [56]
(h) Adams and Griffen [1]
(i) Halvorsen [25]
(j) Liew [41]
(k) Berndt and Watkins [10]
(1) Hirst, Lin, and Cope [27]
(m) Griffen [72]
(n) Mount, Chapman, and Tyrrell [73]

There is much less agreement on own price elasticities of individual fuels. Elasticities for natural gas and fuel oil range from -0.33 to -1.89, although most are larger in magnitude than -1. Our elasticity estimates for oil (about -0.6 to -1.2) tend to be in the middle of this range, while our estimates for natural gas (about -0.9 to -1.8) are at the higher end. Elasticity estimates for electricity also vary considerably, ranging from -0.34 (Canada, Fuss and Waverman [68]) to -1.2 (U.S., Halvorsen [25] and Mount, Chapman and Tyrrell [73]). Notably, the one estimate for a European country (Rødseth and Strøm [56] is at the low end of the spectrum. This is consistent with our result that electricity demand is more elastic in Canada and the U.S. (given that these countries are pooled separately) than in Europe. This is reasonable since we would expect that in Canada and the U.S. there is a greater discretionary use of electricity for such purposes as heating vacation homes or heating homes in warmer climates.

7. Summary

We have applied the indirect translog utility function to a two-stage model of residential energy demand, in which consumers determine their energy expenditures as a share of total consumption expenditures, and determine their expenditures on individual fuels as shares of energy expenditures. We found this model to yield reasonably low-variance and robust estimates of long-run demand elasticities for the energy aggregate (as well as the other component categories of the consumption basket), and for individual fuels. Although we were forced by computational constraints to impose the restriction of homotheticity on our model, the use of the translog form allowed us to test other restrictions on the structure of demand, rather than impose them *a priori*. In estimating the model of aggregate consumption shares we found that the symmetry restrictions resulting from the hypothesis of utility maximization could be rejected. Accepting these restrictions nonetheless, we found that, conditional on utility maximization and homotheticity, the indirect utility function is heterogeneous and non-stationary, and that no other restrictions can be maintained. Similarly, in estimating the model of fuel shares, we found (again conditional on utility maximization and homotheticity) the indirect utility function to be heterogeneous and non-stationary, with no other restrictions.

We found the long-run own price elasticity of energy to be about -1.10, which is within the range of estimates found by others, although higher than the "concensus" range of estimates usually used for policy analysis and forecasting in the U.S.[52] We also found considerable variation in price elasticities for individual fuels. If all nine countries are pooled together in estimating the model of fuel shares, we find own price elasticities for solid and liquid fuel to be between -1 and -1.25, elasticities for natural gas to be about -1.7, and elasticities for electricity to be between 0 and -0.4. There is some evidence, however, for pooling Canada and the U.S. separately. If this is done, we find that the elasticities for liquid fuel and gas are still about -1 to -1.25 and about -1.7 respectively for the European countries, but only half as large for the U.S. and Canada. This is reasonable if one believes that price elasticities for necessities such as fuel become smaller as incomes rise (and there is certainly evidence that this is the case). Pooling the U.S. and Canada separately yields elasticities for electricity near -1 for these countries, but near zero for some of the European countries, and these latter estimates are outside the range of most other studies. Finally, pooling the U.S. and Canada separately gives solid fuel elasticities that vary widely across countries and across time, but this is due largely to the fact that the share of solid fuel has become very small in most countries in recent years.

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[52] In their study of the effects of alternative energy policies in the U.S., Hall and Pindyck [74] used a "concensus" estimate of the own price elasticity of energy demand equal to -0.25.

REFERENCES

1. Adams, F.G. and J.M. Griffin, "Energy and Fuel Substitution Elasticities: Results from an International Cross Section Study," unpublished, October, 1974.

2. Adams, F.G. and P. Miovic, "On Relative Fuel Efficiency and the Output Elasticity of Energy Consumption in Western Europe," Journal of Industrial Economics, November, 1968.

3. Allen, R.G.D., Index Numbers in Theory and Practice, Aldine Publishing Co., Chicago, 1975.

4. Balassa, B., "The Purchasing-Power Parity Doctrine: A Reappraisal," Journal of Political Economy, June, 1964.

5. Barten, A.P., "Maximum Likelihood Estimation of a Complete System of Demand Equations," European Economic Review, Fall, 1969, pp. 7-73.

6. Baughman, M.L. and P.L. Joskow, "Interfuel Substitution in the Consumption of Energy in the United States," M.I.T. Energy Laboratory Working Paper, May, 1974.

7. Belsley, D.A., "Estimation of Systems of Simultaneous Equations, and Computational Specifications of GREMLIN," Annals of Economic and Social Measurement, October, 1974.

8. Berndt, E.R., B.H. Hall, R.E. Hall, and J.A. Hausman, "Estimation and Inference in Nonlinear Structural Models," Annals of Economic and Social Measurement, October, 1974.

9. Berndt, E.R. and N.E. Savin, "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances," Econometrica, September, 1975.

10. Berndt, E.R. and G.C. Watkins, "A Residential and Commercial Natural Gas Demand Model for Ontario and British Columbia," in The Economics of Oil and Gas Self-Sufficiency in Canada, Conference Proceedings, Department of Economics, University of Calgary, October, 1975.

11. Brown, A. and H. Deaton, "Surveys in Applied Economics: Models of Consumer Behavior," Economic Journal, December, 1972.

12. Chenery, H., and M. Syrquin, Patterns of Development, 1950-1970, Oxford University Press, London, 1975.

13. Chow, G., "On the Computation of Full-Information Maximum Likelihood Estimates for Nonlinear Equation Systems," Review of Economics and Statistics, February, 1973.

14. Christensen, L.R. and M.E. Manser, "Estimating U.S. Consumer Preferences for Meat, 1947-71," unpublished, February, 1974.

15. Christensen, L.R., D.W. Jorgenson, and L.J. Lau, "Transcendental Logarithmic Utility Functions," American Economic Review, June, 1975.

16. Cox, D.R., Analysis of Binary Data, Methuen & Co., London, 1970.

17. Deaton, A.S., "The Measurement of Income and Price Elasticities," European Economic Review, Vol. 6, 1975, pp. 261-273.

18. Diewert, W.E., "Homogeneous Weak Separability and Exact Index Numbers," Technical Report No. 122, Institute for Mathematical Studies in the Social Sciences, Stanford University, January, 1974.

19. Domencich, T., and D. McFadden, Urban Travel Demand: A Behavioral Analysis, North-Holland, Amsterdam, 1975.

20. Eisner, M., and R.S. Pindyck, "A Generalized Approach to Estimation as Implemented in the TROLL/1 System," Annals of Economic and Social Measurement, January, 1973.

21. Frisch, R., "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors," Econometrica, April, 1959.

22. Gallant, W.R., "Seemingly Unrelated Nonlinear Regressions," Journal of Econometrics, February, 1975.

23. Goldberger, A.S. and T. Gamaletsos, "A Cross-Country Comparison of Consumer Expenditure Patterns," European Economic Review, Vol. 1, pp. 357-400.

24. Gregory, P., and J.M. Griffen, "Secular and Cross-Section Industrialization Patterns: Some Further Evidence on the Kuznets-Chenery Controversy," Review of Economics and Statistics, August, 1974.

25. Halvorsen, R., "Residential Demand for Electric Energy," Review of Economics and Statistics, February, 1975.

26. Hausman, J. and D. Wood, "Using the Share Ratio (Logit) Specification for Estimation of Price Elasticities in Demand Models," 1975.

27. Hirst, E., W. Lin, and J. Cope, "An Engineering-Economic Model of Residential Energy Use," Oak Ridge National Laboratory, Technical Report #TM-5470, July, 1976.

28. Hoel, M., "A Note on the Estimation of the Elasticity of the Marginal Utility of Consumption," European Economic Review, Vol. 6, 1975, pp. 411-415.

29. Hottel, H.C., and J.B. Howard, New Energy Technology: Some Facts and Assessments, MIT Press, Cambridge, Mass., 1971.

30. Houthakker, H.S., "Additive Preferences," Econometrica, Vol. 28, No. 2, 1960.

31. Houthakker, H.S., "New Evidence on Demand Elasticities," Econometrica, Vol. 33, No. 2, April, 1965.

32. Houthakker, H.S. and L.D. Taylor, Consumer Demand in the United States 1929-1970, Harvard University Press, Cambridge, Mass., 1970.

33. Hudson, E.A., and D.W. Jorgenson, "U.S. Energy Policy and Economic Growth, 1975-2000," The Bell Journal of Economics and Management Science, Autumn, 1974.

34. Hulten, "Divisia Index Numbers," Econometrica, November, 1973.

35. Jorgenson, D.W., "Consumer Demand for Energy," Harvard Institute of Economic Research, Discussion Paper 386, November, 1974.

36. Jorgenson, D.W. and Z. Griliches, "Issues In Growth Accounting: A Reply to Edward F. Denison," Survey of Current Business, May, 1972, pp. 65-94.

37. Jorgenson, D.W., and L.J. Lau, "The Structure of Consumer Preferences," Annals of Economic and Social Measurement, Vol. 4, Winter, 1975, pp. 49-101.

38. Kloek, T. and H. Theil, "International Comparisons of Prices and Quantities Consumed," Econometrica, July, 1965.

39. Kravis, I.B., Z. Kenessey, A. Heston, and R. Summers, A System of International Comparisons of Gross Product and Purchasing Power, Johns Hopkins University Press, Baltimore, 1975.

40. Lau, L.J., "Duality and the Structure of Utility Functions," Journal of Economic Theory, Vol. 1, December, 1969.

41. Liew, C.K., "Measuring the Substitutibility of Energy Consumption," unpublished, December, 1974.

42. Lluch, C., and H. Powell, "International Comparisons of Expenditure Patterns," European Economic Review, Vol. 5, 1975, pp. 275-303.

43. MacAvoy, P.W. and R.S. Pindyck, The Economics of the Natural Gas Shortage, 1960-1980, North-Holland Publishing Co., Amsterdam, 1975.

44. Manser, M.E., "The Translog Utility Function with Changing Tastes," Discussion Paper No. 333, S.U.N.Y. at Buffalo, Economic Research Group, December, 1974.

45. Manser, M.E. "Elasticities of Demand for Food: An Analysis Using Non-Additive Utility Functions Allowing for Habit Formation," Discussion paper #357, S.U.N.Y. at Buffalo, Economic Research Group, August, 1975.

46. McFadden, D., "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka (editor), Frontiers in Econometrics, Academic Press, New York, 1973.

47. Nelson, J.P., "The Demand for Space Heating Energy," Review of Economics and Statistics, November, 1975.

48. Nordhaus, W.D., "The Demand for Energy: An International Perspective," Unpublished, September, 1975.

49. Oberhofer, W., and J. Kmenta, "A General Procedure for Obtaining Maximum Likelihood Estimates in Generalized Regression Models," Econometrica, May, 1974.

50. Organization for Economic Cooperation and Development, Energy Prospects to 1985, Paris, 1974.

51. Park, R.E., "Prospects for Cable in the 100 Largest Television Markets," Bell Journal of Economics and Management Science, Spring, 1972.

52. Philips, L., Applied Consumption Analysis, North-Holland, Amsterdam, 1974.

53. Pindyck, R.S., and D.L. Rubinfeld, Econometric Models and Economic Forecasts, McGraw-Hill, New York, 1976.

54. Pollak, R.A., "Habit Formation and Dynamic Demand Functions," Journal of Political Economy, Vol. 78, 1970.

55. Pollak, R.A. and P.J. Wales, "Estimation of the Linear Expenditure System," Econometrica, Vol. 37, 1969.

56. Rødseth, A. and S. Strøm, "The Demand for Energy in Norwegian Households with Special Emphasis on the Demand for Electricity," University of Oslo, Institute of Economics, Research Memorandum, April, 1976.

57. Roy, R., De l'Utilité-Contribution à la Théorie des Choix, Hermann, Paris, 1942.

58. Samuelson, P.A., "Analytical Notes on International Real Income Measures," Economic Journal, September, 1974.

59. Statistiches Bundesamt, "Internationaler Vergleich der Preise für die Lebenshaltung," Preise Lohne Wirtschaftsrechsnungen, Reihe 10, various years.

60. Strotz, R.H., "The Empirical Implications of a Utility Tree," Econometrica, Vol. 25, 1957.

61. Taylor, L.D., "The Demand for Electricity: A Survey," The Bell Journal of Economics and Management, Spring, 1975.

62. Theil, H., "A Multinomial Extension of the Linear Logit Model," International Economic Review, Vol. 10, 1969, pp. 251-259.

63. Theil, H., Theory and Measurement of Consumer Demand, North-Holland Publishing Co., Amsterdam, 1975.

64. Turvey, R. and A.R. Nobay, "On Measuring Energy Consumption," Economic Journal, December, 1965.

65. Wall, K.D., "Inter-Equation Constraints and the Specification of Dynamic Structure," Working Paper No. 119, Computer Research Center, National Bureau of Economic Research, January, 1976.

66. World Oil Project, MIT, "A User's Guide to the MIT World Energy Demand Data Base," MIT Energy Laboratory Working Paper #76-011WP, May, 1976.

67. Zellner, A., "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of the American Statistical Association, June, 1962.

68. Fuss, M. and L. Waverman, "The Demand for Energy in Canada," Working Paper, Institute for Policy Analysis, University of Toronto, 1975.

69. Joskow, P.L., and M.L. Baughman, "The Future of the U.S. Nuclear Energy Industry," Bell Journal of Economics, Spring 1976.

70. Berndt, E.R., M.N. Darrough, and W.E. Diewert, "Flexible Functional Forms and Expenditure Distributions: An Application to Canadian Consumer Demand Functions," Discussion Paper 76-06, University of British Columbia, January 1976.

71. Pindyck, R.S., "Interfuel Substitution and the Industrial Demand for Energy: An International Comparison," MIT Energy Laboratory Working Paper #MIT EL 77-026WP, August 1977.

72. Griffen, J.M., "The Effects of Higher Prices on Electricity Consumption," The Bell Journal of Economics and Management Science, Autumn, 1974.

73. Mount, T.D., L.D. Chapman, and T.J. Tyrrell, "Electricity Demand in the United States: An Econometric Analysis," Oak Ridge National Laboratory, Technical Report, June 1973.

74. Hall, R.E. and R.S. Pindyck, "The Conflicting Goals of National Energy Policy," The Public Interest, Spring 1977.