

*Archive*

Non-Aggressive Bidding Behavior  
and the "Winner's Curse"

James L. Smith\*

April 1980  
Energy Laboratory Working Paper No. MIT-EL 80-013WP

\*MIT Energy Laboratory and the University of Illinois, Urbana

This analysis was sponsored by the Center for Energy Policy Research of the MIT Energy Laboratory, and that support is gratefully acknowledged.



# NON-AGGRESSIVE BIDDING BEHAVIOR AND THE "WINNER'S CURSE"

James L. Smith

Center for Energy Policy Research  
Massachusetts Institute of Technology

## Abstract

Previous authors have noted a curious result that arises in the context of sealed-bid auctions: in certain situations it is in the bidder's interest to respond non-aggressively to increased competition. We consider a decision-theoretic formulation of the bidder's problem, and derive necessary conditions for the choice of a non-aggressive bidding strategy. The resulting conditions relate closely to a phenomenon that has been described rather loosely by bidding practitioners as the "winner's curse". In the course of this paper we develop a specific definition of the winner's curse, and demonstrate how it affects the firm's competitive behavior.



## NON-AGGRESSIVE BIDDING BEHAVIOR AND THE "WINNER'S CURSE"

James L. Smith\*

A notion that is central to the study of industrial structure and market performance is that the entrance of additional competitors to a market is likely to enhance the level of competition which prevails there. This effect may be reflected in various aspects of the industry's conduct, but perhaps most prominently in the individual pricing behavior adopted by its members. The common presumption is that additional rivals will engender more aggressive pricing behavior on the part of all participants. The apparent generality of this conclusion has been established by studies of the competitive pricing response in diverse market situations, including markets for both differentiated and undifferentiated products, and markets characterized by varying degrees of cooperative interaction among firms.<sup>1</sup> In each case, the equilibrium price offered by all firms can be shown to decrease and ultimately converge to the level of marginal cost as the number of rivals increases.

A possible exception to this paradigm arises in the context of sealed-bid auction markets, where independent firms compete either for the right

---

\*I am grateful to Keith Brown, Michael Fuerst, Edward Rice, Michael Rothkopf and an anonymous referee for comments on an earlier draft.

This research was supported in part by the U.S. Geological Survey, grant no. 14-08-0001-G-419. However, the views and conclusions reported here are those of the author, and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Government.

<sup>1</sup>See, for example: Phillips [1962, p. 29]; Shubik [1970]; Silberston [1970]; and Friedman [1977, p. 30].

to provide some service ("buyer's auction"), or to acquire an item of potential value ("seller's auction"). Several previous studies provide specific examples in which it is in the firm's interest to bid less aggressively as the number of rivals increases. The potential optimality of such behavior was first suggested by Rothkopf [1969], who found that non-aggressive strategies constitute a Nash equilibrium for a particular set of  $n$ -person bidding games. Further cases in which equilibrium behavior consists of non-aggressive bidding strategies have subsequently been reported by Zinn, Lesso, and Givens [1975], and by Reece [1977]. Similarly, Capen, Clapp, and Campbell [1971], and Dougherty and Nozaki [1975] have investigated the performance of alternative bidding strategies in a partial equilibrium framework (where competitors' actions are regarded as exogenous), and provide additional examples where expected profit is maximized by a non-aggressive response to competition.

Non-aggressive bidding strategies are not appropriate in all bidding environments. The previous studies provide only isolated examples of the phenomenon, and many counterexamples may be cited where it is in the firm's interest to respond aggressively to additional competition (e.g., Vickrey [1961]). The objective of the present paper is to characterize more generally the firm's choice between aggressive and non-aggressive bidding strategies. We consider a decision-theoretic formulation of the firm's bidding problem, and derive necessary conditions for the optimality of a non-aggressive bidding strategy. The resulting conditions relate closely to a phenomenon that has been described rather loosely by bidding practitioners as the "winner's curse." In the course of this paper we develop a specific definition of the winner's curse, and demonstrate how it affects the firm's competitive behavior.

### The Bidder's Problem

We formulate the firm's decision problem in fairly general terms. The firm must compete against  $n$  opposing bidders to acquire an item of uncertain value,  $v$ . The firm will select a bid,  $b$ , that maximizes the expected profit obtained in the auction. The nature of our results would be essentially unchanged under the convention of maximizing utility.

The bid tendered by the  $i^{\text{th}}$  competitor, denoted  $z_i$ , is assumed to follow a conditional distribution function,  $G(\cdot|v)$ , which depends on the item's true underlying value. The only restriction placed on this distribution is that items of higher value are presumed more likely to draw high bids:

$$(1) \quad G(z|v_1) < G(z|v_2); \quad \text{for all } z, \text{ with } v_1 > v_2.$$

This implies that rivals' bids, once revealed, are informative regarding the item's true value. This aspect of the bidding environment is critical, as we demonstrate below, to the choice between aggressive and non-aggressive bidding strategies. For comparison, the bidding models of Vickrey [1961], Rothkopf [1969], and Reece [1977] are also structured such that rivals' bids are informative, but in each case the specification is more restrictive. For example, Rothkopf assumes that each competitor enters a bid that reflects a "sample" or indicator of the item's true value, drawn from an unbiased Weibull distribution. The models of Vickrey and Reece are similar, but respectively substitute rectangular and lognormal sampling distributions for the Weibull. The present analysis does not restrict the form of the sampling distribution, nor require that it be unbiased.

The highest competing bid, denoted  $x$ , is defined simply as  $x = \max \{z_1, \dots, z_n\}$ . This random variable follows the extreme value distribution,  $H(\cdot|v,n)$ ;

$$(2) \quad H(x|v,n) = [G(x|v)]^n.$$

The density of the highest competing bid may then be written as  $h(\cdot|v,n)$ :

$$(3) \quad h(x|v,n) = n \cdot g(x|v) \cdot [G(x|v)]^{n-1},$$

... where  $g(\cdot|v)$  is the density of each firm's bid.

If we represent the firm's own beliefs regarding the item's true value by the prior density,  $p(\cdot)$ , we may then write explicitly the joint density of highest competing bid and underlying value,  $f(v,x|n)$ :

$$(4) \quad f(v,x|n) = h(x|v,n) \cdot p(v) = n \cdot p(v) \cdot g(x|v) \cdot [G(x|v)]^{n-1}.$$

As Equation (4) demonstrates, the random variables  $v$  and  $x$  cannot generally be regarded as statistically independent. This is essentially what it means for the competitors' bids to be informative. One implication is that if the amount of the highest competing bid were revealed, our firm would then revise its appraisal of the item's value to incorporate this additional information. The two variables would truly be independent only if the firm were prevented from relating the magnitude of competing bids to the underlying value of the contested item. This would occur, for example, if the separate bidders were to enter the auction with a consensus of opinion regarding the item's value; because then the range of observed bids could only be interpreted as the manifestation of divergent bidding strategies, not as new information regarding the item's value. A special case of consensus beliefs is, of course, that in which the item's value is known with certainty.



The firm's objective is to maximize expected profits by appropriate choice of bid:<sup>1</sup>

$$(5) \quad \max_{(b)} E[\pi(b)] = \int_0^{\infty} \int_0^{b^*} (v-b) \cdot f(v,x|n) \, dx \, dv.$$

The optimal bid,  $b^*$ , must satisfy the first-order condition obtained upon differentiation of Equation (5), which appears after simplification:

$$(6) \quad \phi(b^*|n) \equiv \tilde{v}(b^*|n) - b^*;$$

$$\dots \text{ where: } \phi(b^*|n) = \frac{\int_0^{\infty} \int_0^{b^*} f(v,x|n) \, dv \, dx}{\int_0^{\infty} f(v,b^*|n) \, dv}$$

$$\text{and: } \tilde{v}(b^*|n) = E[v|x=b^*,n].$$

Each component of Equation (6) has an intuitive interpretation. The function  $\phi(b^*|n)$  is the inverse of the "failure rate" function, which represents the probability that the bid,  $b^*$ , will be defeated, given that any larger bid would have been successful. The expression  $\tilde{v}(b^*|n)$  represents the posterior expectation of the item's value, conditional on the event that the highest of  $n$  competing bids equals precisely the value  $b^*$ .

### The Winner's Curse

One implication of Equation (6) is that the optimal strategy requires the firm to underbid its expectation of the item's value. This follows directly since the degree of underbidding ( $\tilde{v}-b^*$ ) is equated to a probability

---

<sup>1</sup>We abstract from possible costs of preparing the bid, so profit is zero if the auction is not won. Because the competing bid distribution is assumed continuous throughout, the probability of a tie is zero.

density,  $\phi(b^*|n)$ , which is necessarily positive. However, the nature of the underbidding phenomenon is less straightforward than the simple equation might suggest. It is not sufficient for the firm to bid less than its a priori expectation of value. Rather, Equation (6) requires the firm to enter a bid which, if successful, will not exceed its posterior expectation which incorporates the knowledge that none of its rivals were prepared to bid any higher. In other words, the underbidding strategy provides a course of action that sustains the winner's confidence even in the equivocal but inevitable event that no other bidder shows a comparable interest in the contested item.

This discussion indirectly raises the question of the winner's curse. In the auction of any item whose value is uncertain, there is a possibility that the winning bidder is led to that position because he has most over-estimated the item's true value.<sup>1</sup> In retrospect, the winner will revalue the item in accordance with the revealed bids of his rivals, all of whom bid lower than himself. Consequently, the winner's appraisal may be diminished by the very act of winning. If the firm does not respect the underbidding rule described by Equation (6), it may enter a bid that exceeds its final appraisal of the item's value, even though the bid started out well below the initial (pre-auction) estimate. In this unfortunate event, the firm would regret its action immediately upon winning the auction, and could be said to have experienced the "winner's curse."

---

<sup>1</sup>Brown [1974] has discussed this tendency in the context of capital budgeting decisions, where "acceptable" investment projects may be those whose returns have been most over-estimated.

The firm's desire to avoid the winner's curse not only motivates the practice of underbidding, but may in certain cases induce the firm to bid less aggressively as the degree of competition increases. An intuitive explanation of this paradoxical result is that the posterior expectation,  $\tilde{v}$ , which imposes a natural ceiling on the bid, is itself pushed downward as the number of bidders increases. Greater competition diminishes the perceived value of winning because the act of outbidding a relatively large number of rivals strongly suggests that the firm has been unduly optimistic. Because greater competition diminishes the perceived value of winning, it may also reduce the firm's willingness to pay and ultimately cause the bid to be lowered. The formal elements which enter this decision problem are discussed below.

Equation (6) characterizes the optimal bid as a function of the number of competitors. Upon total differentiation this relation also describes the firm's response to additional competitors:

$$(7) \quad \frac{db^*}{dn} = \frac{\tilde{v}_n - \phi_n}{1 - \tilde{v}_b + \phi_b} ;$$

$$\dots \text{ where } \tilde{v}_n = \frac{\partial}{\partial n} \tilde{v}(b^*|n), \text{ etc.}$$

If the sign of  $db^*/dn$  were positive, the firm could be said to display an aggressive competitive response, in accordance with conventional theory. Conversely, if the sign were negative, the firm would display a non-aggressive response.

Determination of the sign of  $db^*/dn$  is simplified by two observations:

- (1) The denominator,  $1 - \tilde{v}_b - \phi_b$ , must, by the second-order optimality conditions, be positive. Consequently, the sign of  $db^*/dn$  is that of the numerator,  $\tilde{v}_n - \phi_n$ .

- (2) The sign of  $\phi_n$  must be negative under fairly broad assumptions regarding the distribution of competing bids.<sup>1</sup>

Several conclusions follow. First, a sufficient condition for the firm to display an aggressive response to competition is that its appraisal of the item's value be independent of rivals' behavior (which implies  $\tilde{v}_n = 0$ ). This condition would be satisfied, for example, if the item's value were known with certainty, or if all firms were to share common indications (samples) of its unknown value.

Conversely, necessary conditions for a non-aggressive response to competition are that the item's value be uncertain, and that rivals' bids reflect independent indications of the true underlying value. These necessary conditions are satisfied by the models of Vickrey [1961], Rothkopf [1969], and Reece [1977]. They are satisfied also in the present analysis by the assumption that rival bids are drawn independently from the conditional distribution,  $G(\cdot|v)$ . This assumption is sufficient to prove that  $\tilde{v}_n < 0$ , as demonstrated in the appendix. That is, under the conditions of uncertainty described here, rivals' bids do convey some information regarding the item's value, and the winner's posterior valuation is necessarily a decreasing function of the number of opposing bidders. Consequently, there is a potential for the firm to adopt a non-aggressive competitive response.

The sufficient condition for a non-aggressive response requires not only that  $\tilde{v}_n$  be negative, but also be of greater absolute magnitude than

---

<sup>1</sup>A sufficient (not necessary) condition for this result is that the probability of a competing bid falling in any interval above  $b^*$  be increased by the advent of additional competition. That is,  $f_n(v, x|n) > 0$ ; for all  $v$ , and  $x \geq b^*$ .

$\phi_n$  (i.e., the sensitivity of the hazard rate to the degree of competition). Consequently, if the non-aggressive response is to be observed, the firm must not only be impressionable, but its posterior appraisal must be relatively volatile, as when the firm places little confidence in its own information. Not surprisingly, this sufficient condition for non-aggressive behavior describes the bidding environment in which the firm is most vulnerable to the winner's curse.

### Summary

We have described the conditions under which an individual bidder would adopt a non-aggressive competitive bidding strategy. The conditions presuppose uncertainty regarding both the magnitude of the highest competing bid and the value of the contested item. The interaction of these risks propagates a phenomenon known as the winner's curse. It is the firm's motivation to avoid the "curse" that leads to the adoption of non-aggressive competitive strategies.

The present paper constitutes a partial equilibrium analysis because we have focused on an individual firm's response to attributes of the market that are taken as exogenous. However, dynamic interaction among firms should be expected to influence these parameters and the resulting configuration of bids as the market proceeds from one equilibrium to another. Therefore, the short-term comparative static response of the individual--be it aggressive or not--is not necessarily a reliable guide to the behavioral patterns that would emerge if all firms were permitted to react simultaneously. The question arises whether the non-aggressive bidding strategies explored here would indeed constitute viable equilibrium strategies if adopted by all firms. The answer seems clearly to

be in the affirmative. Imagine that each individual bidder were to reduce the amount of its bid upon the arrival of a new competitor, in the manner described above. These mutual adjustments, if anticipated, would cause each bidder to reduce its expectation of highest competing bid, and induce a second round of downward bid revisions that reinforces the initial adjustment attributed to the winner's curse. The bidding simulation studies cited above provide examples of this phenomenon, where the Nash strategy for each firm is to reduce the level of its bid as additional firms enter the auction. By focussing on the individual firm's decision calculus we have been able to gain some insight regarding the economic rationale for this type of behavior.

## APPENDIX: THE POSTERIOR VALUATION

We present a theorem which demonstrates under general conditions that the winning bidder's posterior valuation is necessarily a decreasing function of the number of rivals:  $\tilde{v}_n < 0$ . The notation of the text is maintained here: the number of opposing bidders is represented by  $n$ , and  $x$  denotes the value of the highest competing bid. Individual competing bids are assumed to be generated independently by the conditional distribution function,  $G(\cdot|v)$ , where  $v$  denotes the true value of the contested item.

The theorem simply states: if it is possible to order the bid distributions,  $G(\cdot|v)$ , in terms of stochastic dominance on the basis of parameter  $v$ ; then it must also be possible to order the posterior value distributions,  $H(\cdot|x,n)$ , in terms of stochastic dominance on the basis of parameter  $n$ . The restriction to stochastically ordered bid distributions corresponds to our previous assumption [Equation (1) of the text] that items of higher value are more likely to draw high bids.<sup>1</sup>

### Theorem

Let  $(z_1, \dots, z_n)$  represent an independently and identically distributed sample drawn from parent distribution  $G(\cdot|v)$ ; this being one of a family of distribution functions indexed by parameter  $v$ , and having continuous density  $g(\cdot|v)$ . Let  $x$  represent the maximum value obtained in the sample. Finally, let  $F(\cdot|x,n)$  represent the posterior distribution of  $v$  conditional on the sample information  $(x,n)$ , and itself having continuous density  $f(\cdot|x,n)$ . Then, if  $G(z|v_1) \geq G(z|v_2)$  for all  $z$ , with  $v_1 < v_2$ ; it follows that  $F(v|x,n_1) \geq F(v|x,n_2)$  for all  $v$ , with  $n_1 > n_2$ .

---

<sup>1</sup>Technically, the value of the item must belong to the parameter class used to define Lehmann's second category of well-ordered distributions [1955, p. 400].

Proof

The theorem is established by proving the somewhat stronger proposition that, for  $n_1 > n_2$ , there exists some value,  $c$ , for which:

$$(A1) \quad f(v|x, n_1) \gtrless f(v|x, n_2) \quad \text{if and only if} \quad v \lesseqgtr c;$$

that is, the posterior densities are "simply intertwined" in the sense of Hammond [1974, p. 1052], with exactly one intersection at the point  $c$ .

The theorem follows immediately from this property of the posterior densities.

We begin by writing the likelihood of the extreme value, conditional on  $v$  and  $n$  [Equation (3) of the text]:

$$(A2) \quad h(x|v, n) = n \cdot g(x|v) \cdot [G(x|v)]^{n-1}$$

The posterior density of  $v$ , conditional on  $x$  and  $n$ , may then be written:

$$(A3) \quad f(v|x, n) = \frac{g(x|v)[G(x|v)]^{n-1}p(v)}{\int_{-\infty}^{\infty} g(x|v) \cdot [G(x|v)]^{n-1}p(v) dv} = \frac{A(v|x, n)}{B(x, n)};$$

where  $p(\cdot)$  represents the prior distribution of  $v$ ; and where we have replaced the numerator and denominator in the final expression by the terms  $A(v|x, n)$  and  $B(x, n)$ , respectively.

To establish (A1) we characterize the distortion in the posterior density  $f(\cdot|x, n)$  induced by variations in  $n$ . Specifically, we demonstrate the existence of a value  $c$  for which:

$$(A4) \quad \frac{\partial}{\partial n} f(v|x, n) \lesseqgtr 0 \quad \text{if and only if} \quad v \gtrless c.$$

The derivative in (A4) is evaluated using (A3):



$$(A5) \quad \frac{\partial}{\partial n} f(v|x,n) = \frac{A_n \cdot B - A \cdot B_n}{B^2};$$

where  $A_n = \frac{\partial}{\partial n} A(v|x,n)$ , etc. Evaluation of (A5) reveals:

$$\text{sgn} \left( \frac{\partial}{\partial n} f(v|x,n) \right) = \text{sgn} \left( \ln[G(x|v)] \cdot \int_{-\infty}^{\infty} f(v,x|n) dv - \int_{-\infty}^{\infty} f(v,x|n) \cdot \ln[G(x|v)] dv \right).$$

Two remarks are sufficient to determine the sign of the expression on the right.

Remark 1: The expression cannot be uniformly positive (negative) for all values of  $v$ , for then the derivative would be positive (negative) throughout, which is impossible for any proper density function.

Remark 2: The value of the expression depends on  $v$  only through the leading term:  $\ln G(x|v)$ , which has been assumed monotonically decreasing in  $v$  for fixed  $x$ .

From this it follows that a point  $c$  exists such that  $\frac{\partial}{\partial n} f(v|x,n) \leq 0$  as  $v \geq c$ . By the continuity of  $f(v|x,n)$  it follows that  $f(v|x,n_1) \geq f(v|x,n_2)$  as  $v \geq c$ . QED

## REFERENCES

- Brown, Keith C., "A Note on the Apparent Bias of Net Revenue Estimates for Capital Investment Projects," The Journal of Finance. 1974, 29(4):1215-16.
- Capen, E. C., Clapp, R. V., and Campbell, W. M., "Competitive Bidding in High Risk Situations," Journal of Petroleum Technology. June, 1971, pp. 641-53.
- Dougherty, Elmer L., and Nozaki, M., "Determining Optimum Bid Fraction," Journal of Petroleum Technology. March, 1975, pp. 349-56.
- Friedman, James W., Oligopoly and the Theory of Games. Amsterdam: North Holland Publishing Co., 1977.
- Hammond, John S. III, "Simplifying the Choice Between Uncertain Prospects Where Preference is Nonlinear," Management Science. 1974, 20(7):1047-72.
- Lehmann, E. L., "Ordered Families of Distributions," Annals of Mathematical Statistics. 1955, 26(3):399-419.
- Phillips, Almarin, Market Structure, Organization and Performance. Cambridge, Mass.: Harvard University Press, 1962.
- Reece, Douglas, K., "Leasing Offshore Oil: An Analysis of Alternative Information and Bidding Systems," unpublished Ph.D. dissertation, University of California, Berkeley, 1977.
- Rothkopf, Michael H., "A Model of Rational Competitive Bidding," Management Science. 1969, 15(7):362-73.
- Shubik, Martin, "Price Strategy Oligopoly: Limiting Behavior with Product Differentiation," Western Economic Journal. 1970, 8(3): 226-32.
- Silberston, Aubrey, "Price Behavior of Firms," Economic Journal. 1970, 80(319): 511-82.
- Vickrey, William, "Counterspeculation, Auctions, and Competitive Sealed Tenders," Journal of Finance. 1961, 16(1):8-37.
- Zinn, C. D., Lesso, W. G., and Givens, G. R., "OILSIM: A Simulation Model for Evaluation of Alternative Bidding Procedures," paper presented at the American Society of Mechanical Engineers Winter Annual Meeting, November 30, 1975.