

Structural Steel Framing Options for Mid- and High Rise Buildings

by

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B.S., Civil and Environmental Engineering (2005)
Michigan Technological University

Submitted to the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for the Degree of
Master of Engineering in Civil and Environmental Engineering

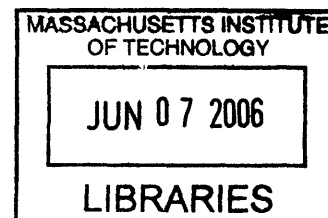
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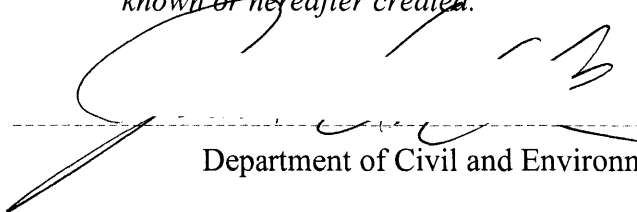
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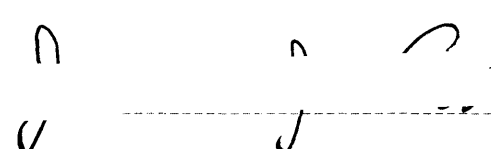


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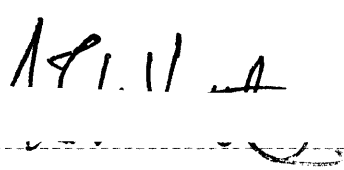
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ABSTRACT

Selecting a structural system for a building is a complex, multidisciplinary process. No design project is the same; however, there are certain criteria that are commonly true in the initial phase of evaluating different structural schemes. These criteria encompass all aspects of a full, functioning building, forcing the design team to be creative in their approach of satisfying all facets. An investigation was carried out for several structural steel framing options available to designers. The schemes describe how each successfully resist lateral loads explaining the advantages and disadvantages of each. Many of the structural design tools available for initial structural system evaluation are strength based. The demand for cheaper, more efficient and taller structures has paved the way for performance based design. A simple cantilever beam performance based analysis was utilized to evaluate three common structural framing schemes in order to gain a better understanding of the performance of each. Results give recommendations for efficient structural solutions for proposed buildings as a function of height.

Thesis Supervisor: Jerome J. Connor

Professor of Civil and Environmental Engineering

Acknowledgements

I would like to thank Sir Isaac Newton for his contribution to mathematics and science; without out your unwarranted devotion this thesis could not exist. Let it also be known that your Laws are the only ones I live by.

Like I always say, "When life gives you apples, invent calculus."

“... loads should be allowed to flow naturally and let the form of the building arise in its own way, even mathematically, without being subjected to arbitrariness.”

- Dr. Fazlur Rahman Khan (1929 – 1982)

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Chapter 1 Introduction

For a proposed building there is no right or wrong way for a structure to carry gravity loads to the foundation or resist lateral loads. There are, however, structurally efficient solutions, architectural solutions, economic solutions and other radical, unique, and challenging solutions. Most projects are probably some combinations of all of these solutions. So how does one select a structural system for a building? There is no short answer to this question. Building design is a complex, multidisciplinary process and no two buildings are alike.

This thesis will analyse six criteria that are commonly true for any building. Each will be evaluated for their level of influence and applicability to the structural system selection process. It will hopefully offer insight in the process of selection and the expectations of all parties involved in building design.

The bulk of this thesis is devoted to investigating the many structural systems available to designers. Some of these systems, such as rigid and braced frames, have been used for decades in a variety of projects. Others, such as the tube concept and outriggers, are a direct result of structural engineers seeking creative and efficient solutions to resist lateral loads and achieve new heights in engineering. All the systems will have descriptions of their lateral resistance mechanism and their performance will be loosely evaluated based on height of the structure. To develop a better understanding of the structural efficiency three common structural systems are analyzed using a performance based design. A simplified vertical cantilever model is used and individual member properties of the building will essentially define themselves based on predefined performance parameters. Evaluating these three schemes over a variety of heights a true relationship between the structural systems and building height can be determined.

Hopefully, this thesis will aid in developing a pragmatic way to generate efficient structural design solutions that offer economy, performance and elegance.

Chapter 2 Structural System Selection Criteria

Design of a building requires an intricate interaction between a team of creative professionals. The design team usually consists of the owner, the architects, the engineers, the contractor and occasionally a project manager. At times these members are from the same organization. The process of structural system selection begins with an initial meeting between the owner and the architect to determine the programmatic requirements that need to be satisfied within a distinct budget. The next step is the creation and convening of the total design team. At their early meetings the parameters which need to be examined are identified. Each and every member of the team, based on their experiences, is called upon to discuss the relevant issues based on their professional understanding of the problems involved. The goal of the team is to determine and analyze a number of options so that they can develop different design schemes for the project. The final choice will depend upon the best value for the budget.

In structural engineering there are essentially three building materials: structural steel, reinforced concrete, and a composite of the two. For each material there are a number of structural systems that one could choose. The design process is both a process of elimination as well as creation. Options must be eliminated so that the best value can be determined. At the same time, creative solutions to the specific project constraints must be considered.

The evaluation criteria for the choice of a structural system can vary between projects. However, there are usually six distinct criteria that are commonly true:

1. Economics
2. Construction time
3. Construction risk
4. Architectural desires and structural needs
5. Mechanical and structural needs
6. Local conditions

Each of these six criteria will be elaborated upon in the following sections.

2.1 Economics

Economics is the driving force for most projects. The developer or owner will typically have a budget for their project and an undeveloped vision. Typically they hire an architect and together they will collectively assemble the remainder of the design team. It is then the design team's responsibility to satisfy, to the best of their ability, the owner's desires and budget constraints.

If the owner already has property available then the site constraints naturally produce building geometry, or at the least establish an approximate floor area. Experience of the design team can then begin architecturally and structurally mapping out the building.

Structurally speaking, some framing schemes are more efficient than others in certain situations and although not always the case, efficiency translates into reduction of costs for a project.

Nearly all the criteria to follow have a direct link to the economics of a project.

2.2 Construction Time

Time is money. The longer a project takes from initial conception to its completion is time lost where the owner could be profiting from their property. Typically the developer wants to capitalize on the market while it is expanding or recovering, not while it is in recession. For the design team this means selecting a structure that can be constructed quickly and efficiently.

Historically materials governed the costs of construction. However, with the advent of labor unions and proper wage distributions, labor costs are now the governing costs associated with construction. Construction typically accounts for 60 to 70 percent of total

project costs for a building, with a substantial portion of these costs associated with labor. It is in the design team's best interest to control the construction time and labor cost.

Steel structures are known to be erected much quicker than concrete structures. Proper curing of concrete is lost time on a construction schedule, especially if it is a critical element of the structure. Many improvements have been made over the past century to decrease the curing time and increase early strength of concrete, but in many instances it still cannot compete with the ease and simplicity of bolted connection in structural steel.

Structural steel also has its pitfalls. Field welding structural steel is both time consuming and costly. Designs minimizing field welding either by specifying shop welded, prefabricated sections or bolted connections are favorable.

2.3 Construction Risk

Construction risk is coupled with many different aspects on a project. Site constraints, construction time, and difficulty factors associated with the location and structural system selection just to name a few.

For typically projects, the contractor assumes all construction risk. The system of ways and means, where a contractor is given the freedom to assemble the structure by nearly any method they so choose, is an example how the contractor assumes this risk.

Involving a contractor early on in a project is an easy way to reduce the construction risk of project. Contractors are much more knowledgeable in terms of difficulties faced on the construction site than other members of the design team. By being involved in the initial design phases the contractor can predict and foresee construction issues and work to eliminate or reduce their impact.

2.4 Architectural Desires and Structural Needs

Architects and structural engineers have many clashing ideas about framing a building. Architects have their desires for the building and there are certain instances where these desires prove to be structurally inefficient and would incur additional, unnecessary costs into a project. There are many tradeoffs between architects and structural engineers.

The ultimate goal of the design team is to select a framing scheme that integrates efficient use of structural material while integrating the structure into the architecture. Lateral bracing of a building, for example, tends to disrupt floor plans. However, integrating the lateral system into the service core or along preconceived walls will provide the stiffness necessary and satisfy architectural desires.

Irregular or asymmetrically shaped buildings can be an additional challenge for engineers. Dynamically a building is more efficient if the center of mass and center of stiffness coincide. Irregularities in the building can force the two to be different. In that case, additional measures must be enforced to redistribute stiffness or mass to produce a dynamically stable structure under heavy winds or seismic activity.

2.5 Mechanical and Structural Needs

Not only does the structural have to accommodate architectural features of the building but to be functional it must also integrate its mechanical needs. As mentioned previously, lateral resistance systems can be coupled with the service core. Integrating mechanical floors in high rise buildings with outrigger and belt trusses or other structural elements are efficient. Since the floors would not necessarily be considered rentable space they can at least be functional.

Castellated beams, Figure 2-1, and open web steel joists, Figure 2-2, are efficient at integrating mechanical duct work with the structural system by providing web openings. Both support systems reduce the weight of the structure while providing depths necessary for large shear loads or deflection control. Mechanical and electrical ducts as well as other systems are always necessary and the structural design team must accommodate for these in their design and calculations or be prepared for costly field work associated the retrofitting of them on-site.

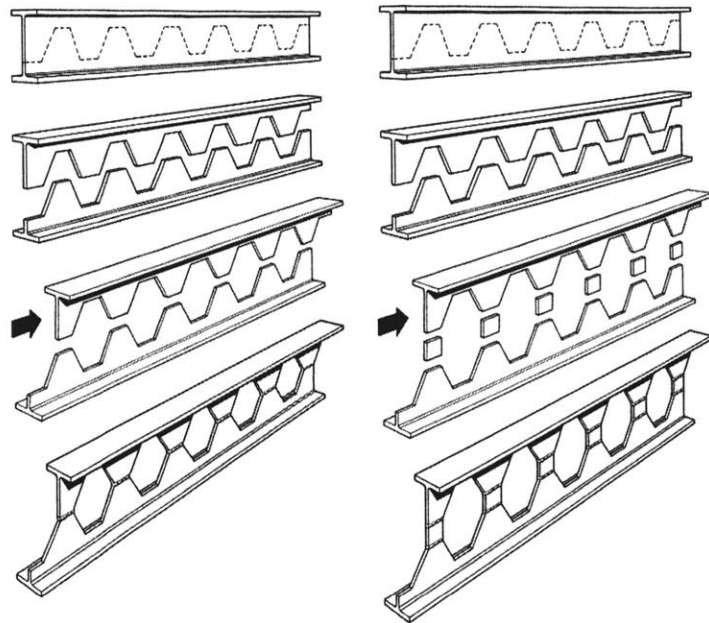


Figure 2-1: Castellated Beam
(Courtesy of Grünbauer BV)

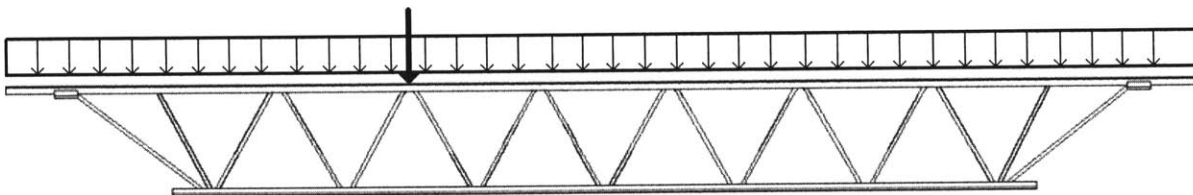


Figure 2-2: Open Web Steel Joist
(Courtesy of EPC Server)

2.6 Local Constraints

The property itself can influence the design team’s structural system. Site layout and soil conditions can pose certain issues with foundations and building geometry. The proposed building geometry can limit the available structural schemes due to the complexities that can result from acute angles or curvilinearity. Zoning laws, city ordinances, and other applicable codes govern in certain locations; height limitations are common.

Common practice and resource availability can force the design team into or away from some structural options. Contractors and construction crews operate quickly and efficiently if they are working with familiar materials, equipment, and practices. Ways and means is a tool which allows the contractor to efficiently erect structures in any acceptable matter they choose. It is always in the contractor's best interest to proceed with a project that will be profitable. Risks associated with foreign practices can either increase contractor's interest in a project because they will be justly compensated for the unique construction or deter them from the project altogether because the chance of profiting may seem slim.

Resource availability also plays an important role in the structural selection process. Proximity to concrete batch plants and steel mills must be considered as well as transporting the material to the construction site. Restricted access to the site can create many problems for the concrete batch trucks or truck trailers transporting large steel section. Sites must also have ample space for on-site storage of excess materials and unused equipment. Otherwise material must be transported to site on a demand basis which can prove costly and inefficient. For most high rise applications in urban settings this is generally the case.

2.7 References

- [1] Khan, F. R., "Influence of Design Criteria on Selection of Structural Systems for Tall Buildings," Canadian Structural Engineering Conference. Montreal, Canada, March 1972.
- [2] Suh, N. P., *The Principles of Design*, Oxford University Press Inc., New York, 1990.
- [3] Millais, M., *Building Structures*, E & FN Spon, an imprint of Chapman & Hall, London, England, 1997.

Chapter 3 Structural Steel Framing Options

Today there are innumerable structural steel systems that can be used for the lateral bracing of buildings. The different structural systems that are currently being used in the design for buildings are broadly divided into the following categories [1]:

1. Rigid frames
2. Semirigid frames
3. Braced frames
4. Rigid frames and braced frame interaction
5. Belt and outrigger truss systems
6. Tube structures

The structural systems all have a theoretical maximum height at which point they become inefficient at transferring lateral loads. The late Dr. Fazlur Khan and several other engineers have attempted to loosely define the maximum heights associated with each system, Figure 3-1.

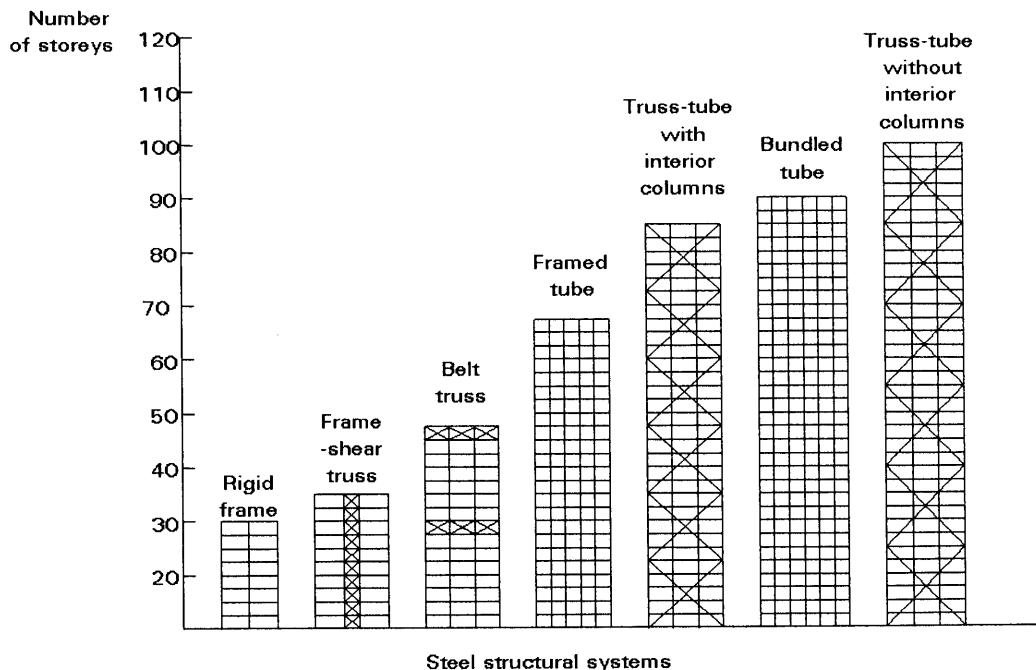


Figure 3-1: Steel Structural System Height

(Courtesy of Khan [10])

Descriptions of each system and its range of applicability are detailed in the following sections.

3.1 Rigid Frames

The use of portal frames, which consist of an assemblage of beams and columns, is one of the very popular types of bracing systems used in building design because of minimal obstruction to architectural layout created by this system. Rigid frames are most efficient for low rise to mid-rise buildings that are not excessively slender. To attain maximum frame action, the connections of beam to columns are required to be rigid.

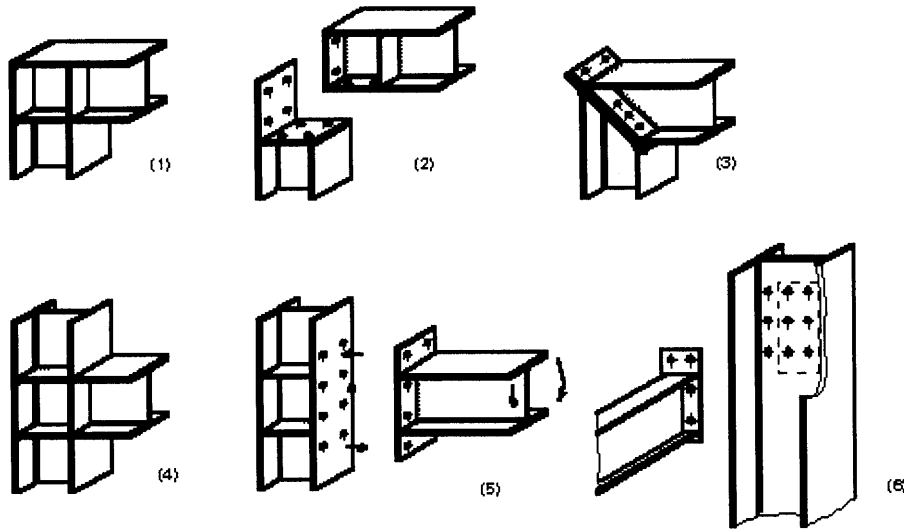


Figure 3-2: Rigid Connections

(1) Fully welded connection with stiffeners; (2) Bolted knee-connection; (3) Knee-connection with welded end plates; (4) Welded T-connection; (5) Bolted T-connection; (6) Bolted end plate connection

(Courtesy of ESDEP [3])

Rigid connections, Figure 3-2, are those with sufficient stiffness to hold the angles between members virtually unchanged under load. It gets strength and stiffness from the nondeformability of joints at the intersections of beams and columns, allowing the beam, in reality, to develop end moments which are about 90 to 95 percent of the fully fixed condition. Rigid frames generally consist of a rectangular grid of horizontal beams and vertical columns connected in the same plane by means of rigid connections. Because of

the continuity of members at the connections, the rigid frame resists lateral loads primarily through flexure of beams and columns, Figure 3-3.

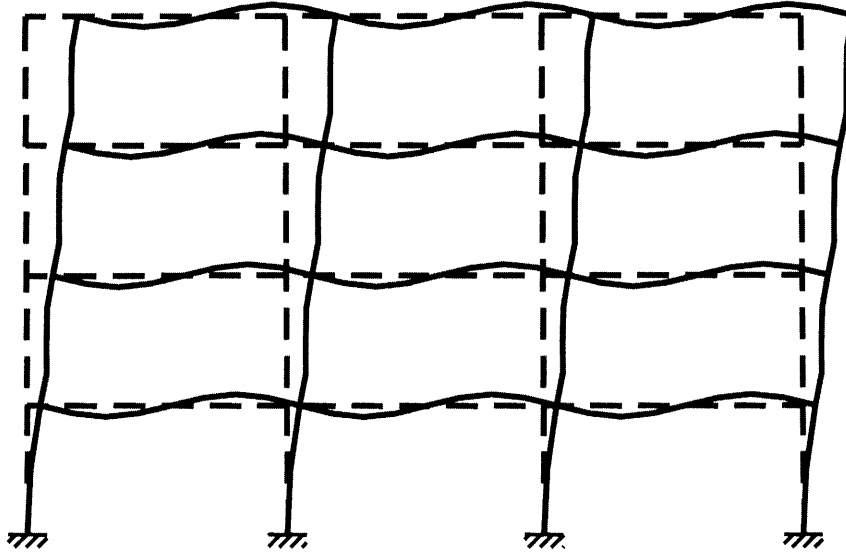


Figure 3-3: Rigid Frame Lateral Load Resistance Mechanism

(Courtesy of Buyukozturk [4])

Rigid frames can have good ductile characteristics if detailed properly. Performance is very sensitive to detailing and craftsmanship of the connections. Designers have numerous options available for plastic design of rigid frames including elastic-perfectly plastic and elasto-plastic analyses. Plastic hinges (or equivalent elasto-plastic zones) form at the base of columns, beam-column connections, and in beam spans, Figure 3-4. Failure occurs if a mechanism of fully plasticized zones has developed. The number of fully plasticized zones depends on

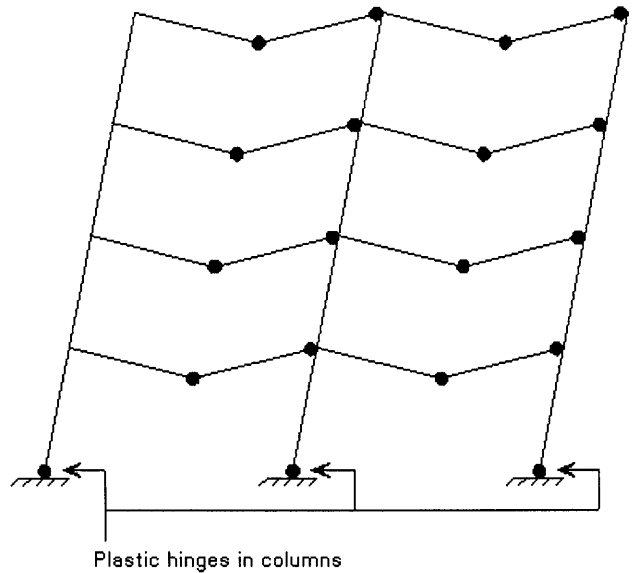


Figure 3-4: Plastic Analysis of Rigid Frame

(Courtesy of ESDEP [3])

the redundancy of the frame [1]. Ductility provides large capacities in the system, but typically their low system stiffness results in large deflections which can lead to high non-structural damage in heavy winds or seismic activity.

The rigid frame can prove to be quite expensive. Resisting the lateral loads through bending of the columns exhibits inefficiency in the system and requires more material than would another structural system. Rigid frames also require labor-intensive moment resisting connections. Limited field welding is desirable by using bolted connections where possible; however, achieving full rigidity of a connection with bolts only is nearly impossible.

3.2 Semirigid Frames

Rigid frames require certain boundary conditions to develop its full frame lateral resistance potential. In such frames rigid connections are specified to assure the stiffest building frame. However, in other situations rigid framing may not yield the optimum solution. This is because heavier connection elements, along with fully developed welds or large connections, are needed to obtain the desired fixity. In addition the gravity moment induced in external or unsymmetrical loaded interior columns may offset the advantages of reduced beam bending requirements and their attendant economic reduction in beam weight [1]. At the other end of the spectrum is the simple framing with very little resistance to bending (usually referred to as a pin connection). This framing requires some other provision for carrying lateral loads in buildings; shear walls, braced frames, or some other lateral bracing system is required in the planning and design of the building.

Semirigid connections can be defined as those connections whose behavior is intermediate between fully rigid and simple connections. Such connection offer substantial restraint to the end moment and can affect sufficient reduction in the midspan moment of a gravity loaded beam. However, they are not rigid enough to restrain all

rotation at the end of the beam. Although the actual behavior of the connection is complex, in practice simplified approaches are used in the design of such connections.

3.3 Braced Frames

Rigid or semirigid frames are not efficient for buildings higher than about 30 stories because the shear racking component of deflection produced by the bending of columns and girders causes the building drift to be too large [1]. A braced frame attempts to improve upon the efficiency of pure rigid frame action by providing a balance between shear racking and bending, Figure 3-5. This is achieved by adding truss members, such as diagonals, between the floor systems. The shear is now primarily absorbed by the diagonals and not by the girders. The diagonals carry the lateral forces directly in predominantly axial action, providing for nearly pure cantilever behavior. All members are subject to axial loads only, thereby creating an efficient structural system [1]. Efficient and economical braced frames use less material and have simpler connections than moment frames and compact braced frames can lead to lower floor-to-floor heights, which can be an important economic factor in tall buildings, or in a region where there are height limits.

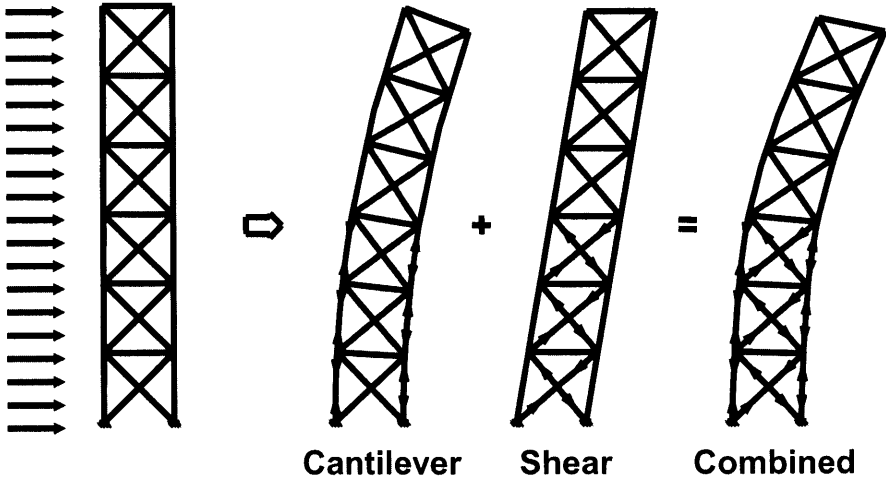


Figure 3-5: Braced Frame Lateral Load Resistance Mechanism
(Courtesy of Buyukozturk [4])

Any rational configuration of bracing can be used for bracing systems. Bracing types, Figure 3-6, available for incorporation into the structural system range from a concentric K or X brace between two columns to knee bracing and eccentric bracing with complicated geometry requiring computer solutions.

3.3.1 Concentric Bracing

The selection of bracing type is a function of the required stiffness, but most often it is influenced by the size of wall opening required for circulation. Because of architectural requirements, sometimes only certain bays around elevator and stairs are braced. On occasion it may be possible to brace portions of the building without compromising the architecture, and in very rare cases it may even be possible to truss across the full width of the building, resulting in sloping interior columns.

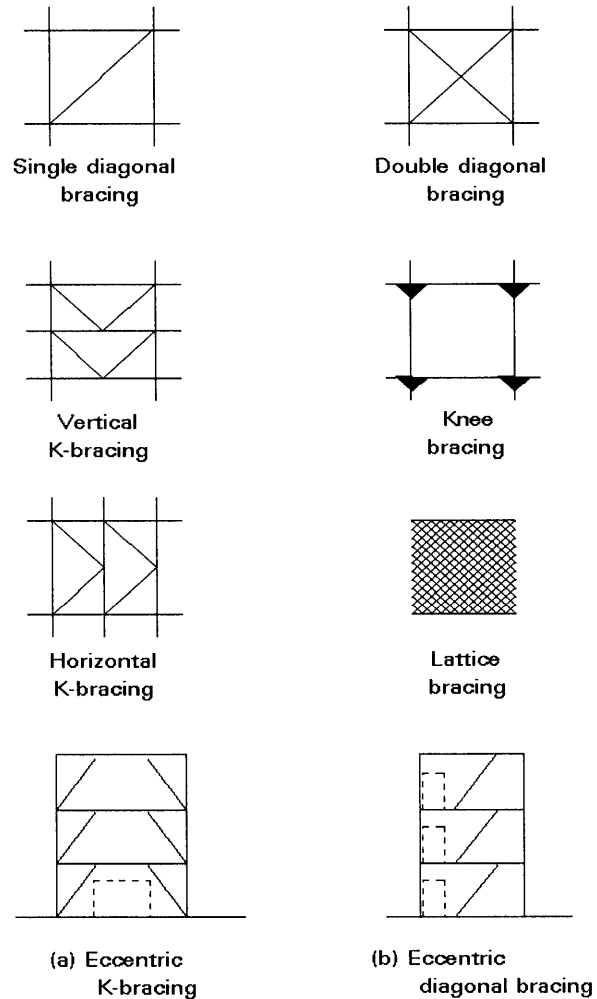


Figure 13 Bracing types

Figure 3-6: Types of Bracing

(Courtesy of ESDEP [3])

Common types of interior bracing are bracing across single bays in one-story increments. Typically these are single diagonals or double diagonals in an X or K bracing scheme. K bracing is increasing popularity because it still allows for circulations through the middle of the bay whereas X bracing or single diagonals virtually eliminate circulations possibilities. Diagonal braced bays, whether single or double, are sometimes paired with intermittent vierendeel bays allowing for unrestricted circulations. The braced bays can then be disguised within walls.

Any reasonable pattern of braces with singly or multiply braced bays can be designed, provided that shear is resisted at every story. Finding an efficient and economical bracing system for buildings of any types presents the structural engineer with an excellent opportunity to use innovative design concepts. However, availability of proper depth for bracing trusses is often an overriding consideration. Truss stiffness varies as the square of the depth and as a preliminary guide, a height-to-width ratio of 8 to 10 is considered proper for a reasonable efficient bracing system. Finding space for multi-story depth and optimum height-to-width truss bracing without disrupting architectural planning may not always be possible, forcing the structural engineer to use less efficient bracing systems.

3.3.2 *Eccentric Bracing*

In an eccentric bracing system the connection of the diagonal brace is deliberately offset from the connection between the beam and the column. By keeping the beam-to-brace connections close to the columns, the stiffness of the system can be made very close to that of concentric bracing.

Concentric braced frames are excellent from a strength and stiffness considerations and are therefore used widely either by themselves or in conjunction with moment frames when lateral loads are caused by the wind. However, they are of questionable value when used alone in seismic regions because of their poor inelastic behavior. Moment resistant frames possess considerable energy dissipation characteristics but are relatively flexible when sized from strength considerations alone. Eccentric bracing is a unique structural system that attempts to combine the strength and stiffness of a braced

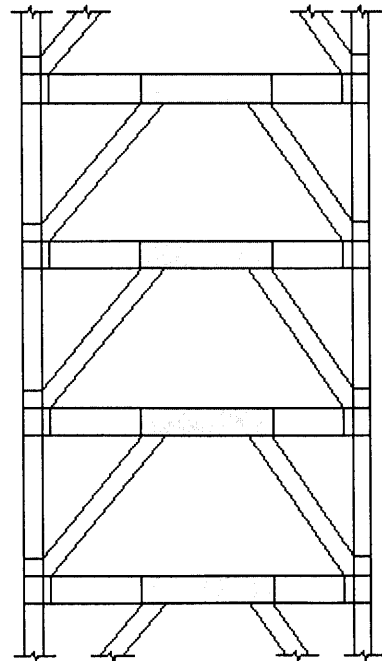


Figure 3-7: Eccentric Bracing Energy Absorbing Hinge
(Courtesy of ESDEP [3])

frame with the inelastic behavior and energy dissipation characteristics of a moment frame. The system is called eccentric bracing because deliberate eccentricities are employed between the beam-to-column and beam-to-brace connections. This offset or eccentricity promotes formation of energy absorbing hinges in the portion of the beam between the two connections, Figure 3-7. This element functions as a fuse by undergoing flexural and shear yielding prior to formation of any additional plastic hinges in the bending members and well before buckling of any compression members [1]. The system maintains stability even under large inelastic deformations. Required stiffness during wind or minor earthquakes is maintained because plastic hinges are not formed under these loads and all behavior is elastic.

3.4 Rigid Frame and Braced Frame Interaction

Unreasonably heavy columns can result if wind bracing is confined to the building's braced service core because the available depth for bracing is usually limited. In addition, high uplift forces occurring at the bottom of core columns can present foundation problems [1]. In such instances an economical structural solution can be arrived at by creating rigid frames to act in conjunction with the core bracing system. Although deep girders and moment connections are required for frame action, rigid frames are often preferred because they are the least objectionable from an architectural planning perspective. Although each building has its own set of criteria, many times architecturally it may be tolerable to use deep spandrels and additional columns on the building's exterior because additional columns will not interfere with interior planning or circulation and the depth of spandrels may not present any problems for HVAC and additional mechanical and electrical ducts.

As an alternative to perimeter frames, a set of interior frames can be made to act in conjunction with the core bracing. Yet another option would be to simply provide rigid girder connections between the braced core and the perimeter columns.

For slender buildings it becomes less practical to use an interaction system of moment frames and braces whose depths are limited by the depths of building cores. An economical structural solution is to increase the bracing system the full width of the building. If done properly it would not compromise the architecture of the building.

3.5 Outrigger and Belt Truss Systems

Traditional approaches to wind bracing for mid-height structures is to provide trussed bracing at the core or around stair wells and to supplement the lateral resistance by providing additional moment connected frames at other convenient plan locations. However, as building height increases, the core, if kept consistent with the elevator, stair well, and other mechanical requirements does not have sufficient stiffness to keep wind drift at acceptable levels.

One way of limiting drifts is a technique of using a cap truss on a braced core combined with perimeter columns. In this system columns are tied to the cap truss through a system of outrigger and belt trusses. In addition to traditional function of supporting gravity loads, the columns restrain the lateral movement of the building, Figure 3-8. When the building is subjected to lateral forces, the cap truss restrains the bending of the core by introducing a point of inflection in its deflection curve. This reversal in curvature reduces the lateral movement at the top [1]. The belt trusses acts as horizontal stiffeners engaging the exterior columns which are not directly connected to the outrigger trusses. This system can improve stiffness up to 25 to 30 percent over the same system without outrigger trusses [1].

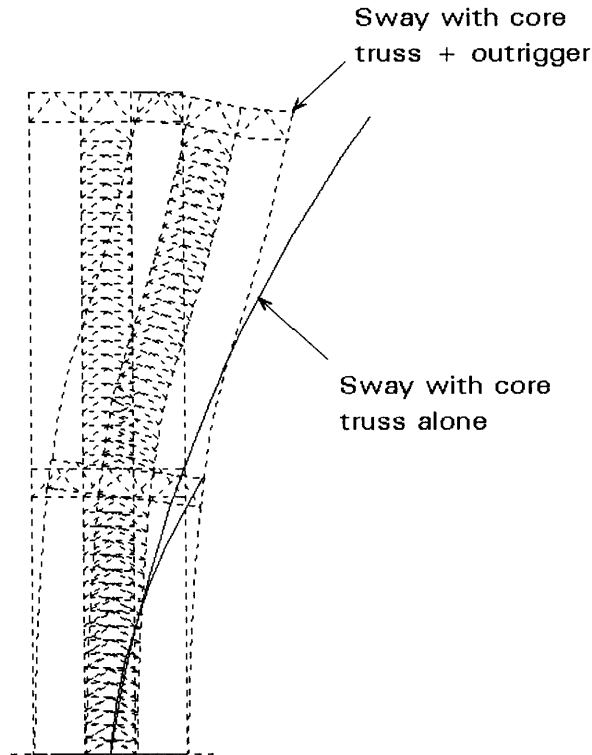


Figure 5 Improvement in overall stiffeners due to outrigger-bolt trusses

Figure 3-8: Outrigger Lateral Load Resistance Mechanism
(Courtesy of ESDEP [3])

3.6 Tube Structures

Tube design can be defined simply as a structural system that prompts the building to behave as an equivalent hollow tube. In past decades, tubular structures were several of the world's tallest buildings. The tubular concept is credited to Dr. Fazlur Khan. Tubular systems are so efficient that in most cases the amount of structural material used is comparable to that used in conventionally framed buildings half the size. Their development is the result of the continuing quest for structural engineers to design the most economical yet safe and serviceable system. Until the development of the tubular structures, buildings were designed as an arrangement of vertical column and horizontal beams and girders spanning between the columns. Lateral loads were resisted by various connections, rigid or semirigid, supplemented where necessary by bracing and truss

elements. Further improvement in the structural economy was achieved by engaging the exterior frame with the braced service core by tying the two systems together with outrigger and belt trusses. This was perhaps the beginning of tubular behavior since the engagement of the exterior columns is similar to that of the tube structures.

The revolutionary design of the tubular system essentially strives to create a rigid wall around the structure's exterior. Since all lateral loads are resisted by the perimeter frame, the interior floor plan is kept relatively free of bracing and columns thus increasing rentable value. A trade-off for structural efficiency is the reduction in window wall space with the presence of larger and closely spaced exterior columns. Maximum efficiency for lateral strength and stiffness using the exterior wall alone is achieved by making the entire building act as a hollow tube cantilevering out of the ground. The tubular action produces uniform axial stress on the flanges and triangular distribution on the webs, an efficient configuration for a cantilever structure such as building.

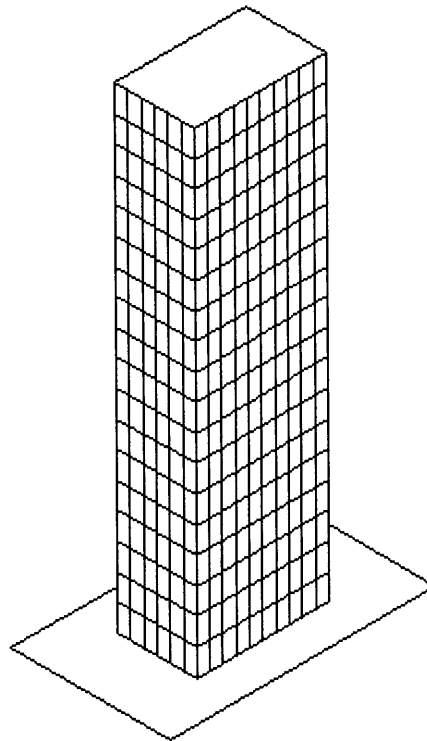


Figure 3-9: Framed Tube Structure
(Courtesy of ESDEP [3])

3.6.1 Framed Tube

The method of achieving the tubular behavior by using closely spaced exterior columns connected by deep spandrel beams is the most used system because rectangular windows can be accommodated in this design. The framed tube requires large columns and deep beams with welded connections for rigidity. When such frames are provided on all four faces of a building, one obtains a hollow tubular configuration.

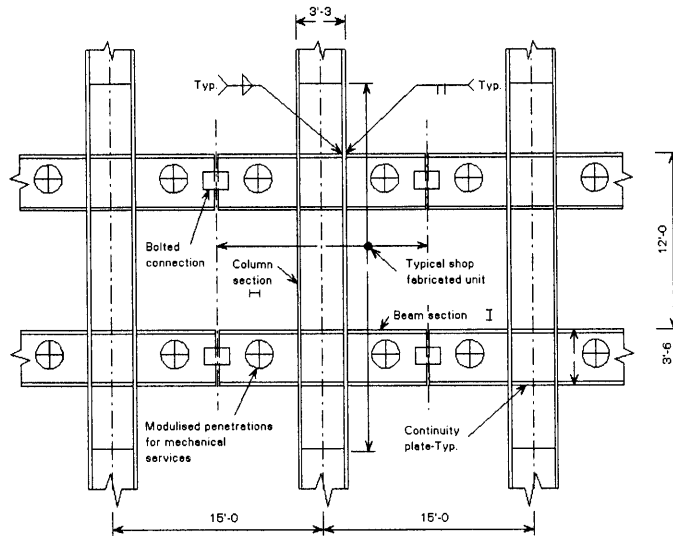


Figure 3-10: Prefabricated Framed Tube Element

(Courtesy of ESDEP [3])

The use of prefabricated framed tube elements, Figure 3-10, where all welding can be performed in a controlled environment has made the framed tube more practical and efficient. The prefabricated elements are then erected by bolting at mid-span of the beams.

One negative aspect of the framed tube design is a phenomenon commonly referred to as shear lag. Shear lag is essentially a nonlinear stress distribution across the flange and web sections of a beam, Figure 3-11. Design of the tube structure assume a linear distribution and shear lag results in corner column experiencing greater stresses than central perimeter columns. This shear lag is a result of local deformation of beams which leads to a reduction of axial stress near the center of the flange [2]. The redistribution of these axial loads result in the corner perimeter columns becoming overstressed. Designer should be prepared for this increased stress in the corner columns and adjust the design as necessary.

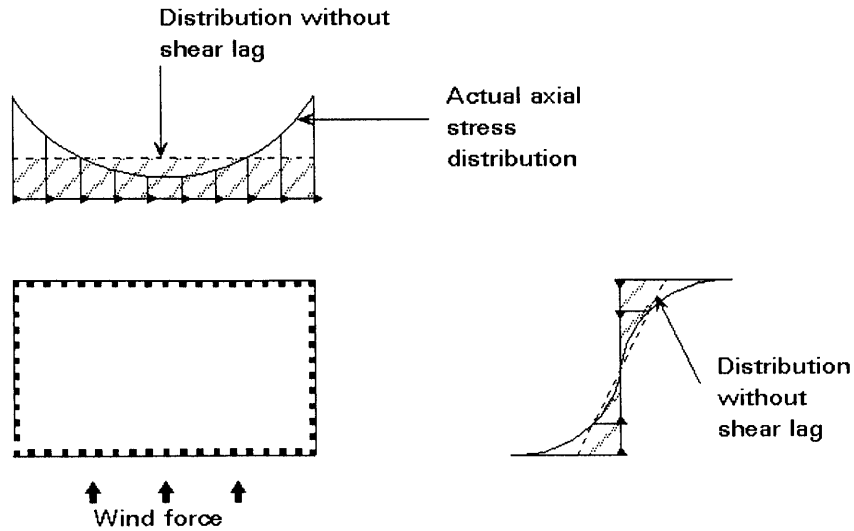


Figure 6b Framed tube : Axial load distribution

Figure 3-11: Framed Tube Shear Lag

(Courtesy of ESDEP [3])

3.6.2 Truss Tube

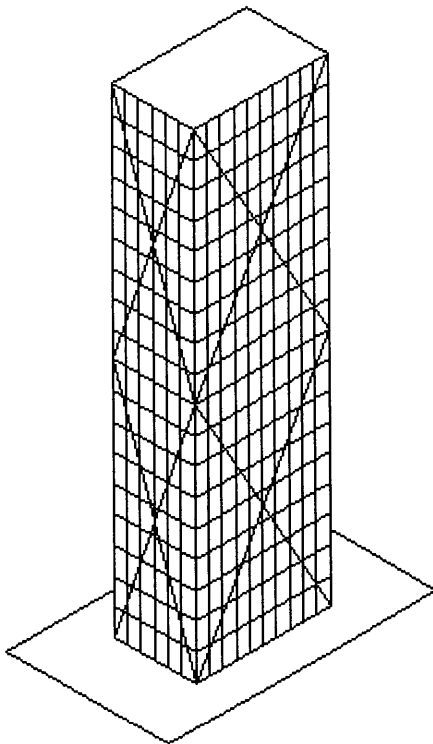


Figure 3-12: Trussed Tube Structure

(Courtesy of ESDEP [3])

The stiffness and strength of the framed tube reside in the rigidity of the connections between closely spaced columns and spandrels that require welded connections at the joints. Even with its rigid connections the framed tube is still somewhat flexible. The frames parallel to the wind essentially act as multi-bay rigid frames with bending moments in the columns and beams becoming controlling factors in the design. As much as 75 percent of the total lateral sway results from racking of the frame as a direct consequent of shear lag [1]. Because of the racking of the frame, the columns at the corners of the building take more than their share of the load, while columns in between do less work than an ideal

tube. One method of overcoming the problems resulting from the framed tube is to stiffen the exterior frames with diagonals or trusses.

The resulting system is commonly known as a trussed tube, Figure 3-12. This system creates problems in terms of window wall details because of the large number of joints between the diagonals resulting in odd shape windows and façade elements.

3.6.3 Bundled Tube

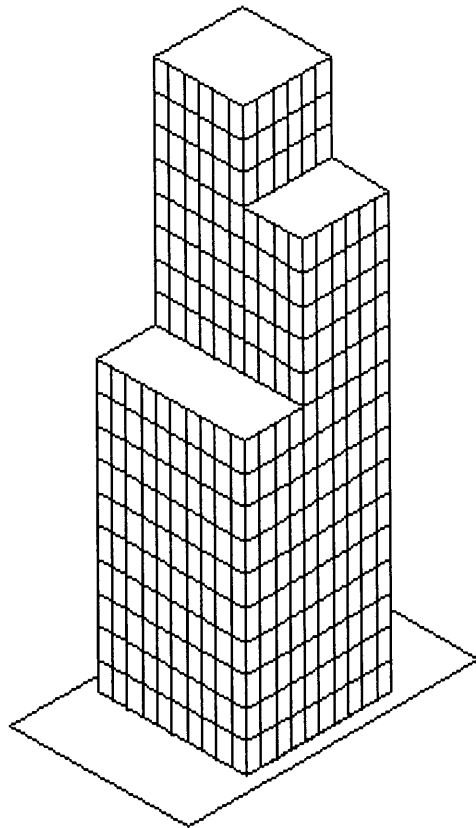


Figure 3-13: Bundled Tube Structure

(Courtesy of ESDEP [3])

A bundled tube structure is essentially a structure resulting from combining two or more independent tube structures designed to act as one, Figure 3-13. The idea of a bundled tube is that individual tube can be terminated at any desired level, creating a variety of floor plans for a building. This becomes a distinct advantage because the tubes can be assembled in nearly any configuration and terminated at any level without loss of structural integrity. This allows the architect to create multiple floor plans within the same building. Of course the obvious disadvantage to the bundled tube concept is each individual tube needs to be completely framed as a tube, resulting in columns that invade the floor plans.

The structural concept behind the bundled tube is that the interior columns from the individual tube act as internal webs of the cantilever structure. This results in a substantial increase in shear stiffness over the other tubular designs with no lateral

resisting interior frames or columns. Increased shear resistance results in a reduction in the shear lag effect. The decrease in shear lag improves torsion and bending behavior of the structure. Interior frames of the individual tube provide this additional bending resistance. Because more frame elements are resisting lateral loads in both shear and bending, the column spacing of the bundled tubular building can be increased considerably over the perimeter column-to-column spacing of the framed tube.

3.7 References

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Chapter 4 Performance Based Design

Performance based design is an alternative to conventional strength based design where a structural designer employs a structural system, determines the structural elements necessary to achieve this system, and then checks this design against serviceability requirements. Serviceability includes cracking (with concrete structures), gravity loaded deflections, drift associated with wind, seismic excitations, and probably most importantly, human perception of accelerations due to heavy winds or seismic activity. A performance based approach specifies the serviceability parameters that the structure shall attain in the initial design phase, prior to a selection a structural system, and views strength as a constraint, not as a primary requirement [1]

When first proposing tubular structures and first classifying structural systems based on height, Figure 4-1, Dr. Fazlur Khan was not familiar with performance based design. He had practical knowledge of mathematics and solid mechanics with aid of early computing power, but his design were still strength based.

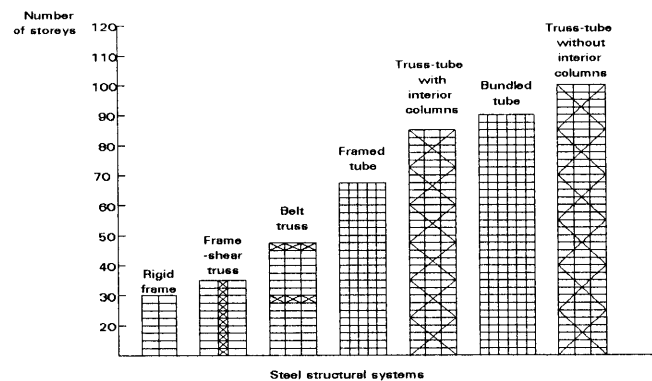


Figure 7 Steel structural systems and the number of storeys

Figure 4-1: Steel Structural System Height
(Courtesy of Khan [6])

Controlling motion of a building is accomplish by adjusting and tweaking three parameters: stiffness, mass, and damping. The mass of a building, generally governed by dead weight of the structure rather than live loads, is fixed without much variability. Stiffness and damping are the two topics that are considered in modern structural design.

4.1 Optimum Stiffness Distribution

Optimal design from a motion perspective corresponds to a state of uniform shear and bending deformations under the design loading. Uniform deformations are only possible for statically determinate structures [1]. This may well be a gross approximation for typical buildings, however, for simplification, building structures are modeled as cantilever beams, Figure 4-2, and therefore static determinacy holds.

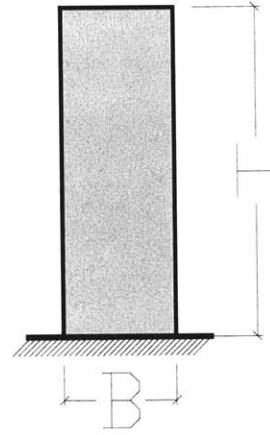


Figure 4-2: Building Structure Cantilever Beam Model

The vertical cantilever beam is defined by two parameters: u , deflection and β , the cross-sectional rotation. The deformation relation is defined by

$$\gamma := \frac{du}{dx} - \beta \quad (4.1)$$

$$\chi := \frac{d\beta}{dx} \quad (4.2)$$

where γ denotes the transverse shearing strain and χ denotes the bending deformation.

Considering the uniform deformation idealization for the building structures, the vertical cantilever beam can be denoted as

$$\beta(x) := \chi \cdot x \quad (4.3)$$

$$u(x) := \gamma \cdot x + \frac{\chi \cdot x^2}{2} \quad (4.4)$$

where γ and χ are both taken as constants.

The deflection at the end of the beam, at height H , is given by

$$u(H) := \gamma \cdot H + \frac{\chi \cdot H^2}{2} \quad (4.5)$$

where γH is the contribution from shear deformation and $\chi H^2 / 2$ is the contribution from bending deformation. For low rise buildings the majority of the deformation is attributed to shear racking; these building are sometimes referred to as shear beam models. On the other hand, high rise buildings tend to display bending beam behavior since the bulk of the deformation is contributed from bending.

Values for γ and χ are established using predefined performance constraints. Introducing a dimensionless factor s , which is equal to the ratio of the displacement due to bending and the displacement due to shear at $x = H$ [1]

$$s := \frac{H}{2 \cdot f \cdot B} \quad (4.6)$$

A pure shear beam is defined by $s = 0$ and tall buildings $s = O(1)$ and defined by another dimensionless parameter, f , which is the ratio of the diagonal strain to the chord strain for lateral loading [1]

$$f := \frac{\varepsilon_d}{\varepsilon_c} \quad (4.7)$$

where ε_d is the diagonal strain and ε_c is the chord, or column, strain. An estimate for initial design is to take $f = 3$ for elastic behavior to $f = 6$ for inelastic behavior for building structures models as truss beam, or in other words a braced frame building. For all other structural models $f = 1$. In these cases f is not applicable to the situation and since it is merely a scale factor a value of unity does not affect results.

As was noted before, γ and χ are functions of these two dimensionless parameters and are defined as

$$\gamma := \frac{1}{(1 + s) \cdot \alpha} \quad (4.8)$$

$$\chi := \frac{2 \cdot \gamma \cdot s}{H} \quad (4.9)$$

where α is a drift constraint, typically $\alpha = 400$ for normal construction to $\alpha = 500$ for drift critical structures that cannot risk damaged walls, façade pieces, or windows.

Force-deformation relations depend on the characteristics of the materials that make up the element. For static loading and linear elastic behavior, the expression relating the shear force, $V(x)$, and bending moment, $M(x)$, to the shear deformations and bending deformations are given by [1]

$$V(x) := D_T(x) \cdot \gamma(x) \quad (4.10)$$

$$M(x) := D_B(x) \cdot \chi(x) \quad (4.11)$$

where D_T and D_B are defined as the transverse shear and bending rigidities along the continuous beam element. Equations (4.10) and (4.11) can be rearranged to yield the shear and bending rigidities as a function of the prescribed shear and bending deformations and known loading characteristics

$$D_T(x) := \frac{V(x)}{\gamma} \quad (4.12)$$

$$D_B(x) := \frac{M(x)}{\chi} \quad (4.13)$$

4.2 Structural System Evaluation

In order to sufficiently compare the structural systems there needs to be geometric consistence, equivalent performance constraints, and lateral load criteria for the vertical cantilever beam models. For the sake of simplicity the proposed structure is located in a seismically inactive zone where the only performance constraint is that of drift associated with wind lateral loading.

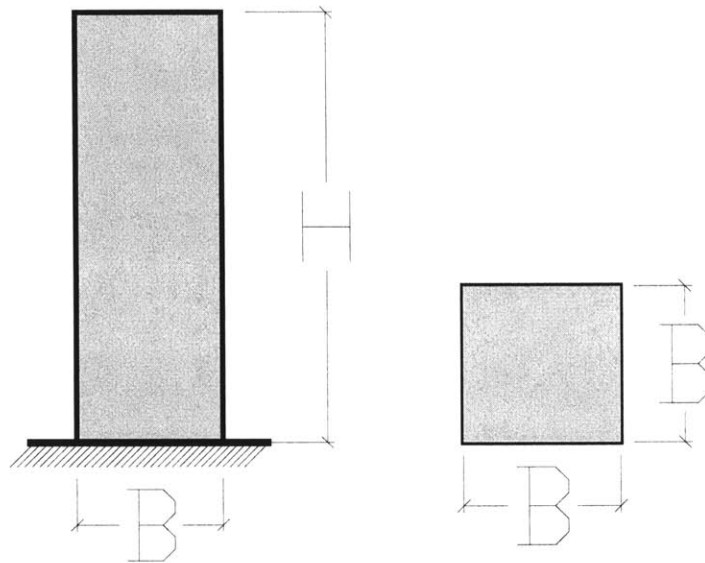


Figure 4-3: Proposed Cantilever Beam Model

The proposed model, Figure 4-3, will have a constant base width, $B = 120 \text{ ft}$ and floor-to-floor height, $h = 12 \text{ ft}$, Figure 4-4. The number of stories, n_{story} , and consequently the building height, H , will vary to establish a relationship for the height of the building structure and approximate member sizes. This relationship will inevitably be compared with the different structural systems evaluated here.

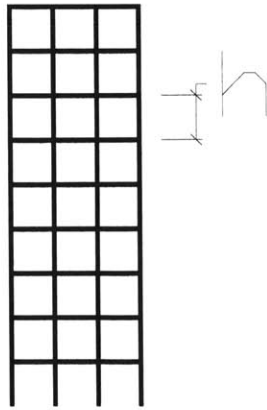


Figure 4-4: Proposed Structural Frame Floor-to-Floor Height

$$H := n_{\text{story}} \cdot h \quad (4.14)$$

The governing performance parameter associated with drift will be constant for all structural systems, $\alpha = 400$. The other dimensionless parameters, s and f , will depend on the system being evaluated.

Lateral wind loading pressure, w , is determined from the Municipal Code of Chicago, Illinois [3]. It is simplified to a uniform load, b , along the vertical cantilever beam, Figure 4-5, as a function of the building height, H .

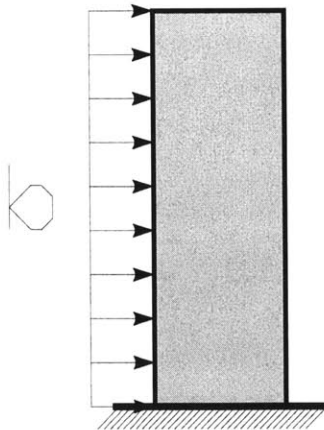


Figure 4-5: Proposed Structure Uniform Lateral Load

$$w := \begin{cases} 20\text{psf} & \text{if } H \leq 200\text{ft} \\ 21\text{psf} & \text{if } 200\text{ft} < H \leq 300\text{ft} \\ 25\text{psf} & \text{if } 300\text{ft} < H \leq 400\text{ft} \\ 28\text{psf} & \text{if } 400\text{ft} < H \leq 500\text{ft} \\ 31\text{psf} & \text{if } 500\text{ft} < H \leq 600\text{ft} \\ 33\text{psf} & \text{if } 600\text{ft} < H \leq 700\text{ft} \\ 36\text{psf} & \text{if } 700\text{ft} < H \leq 800\text{ft} \\ 39\text{psf} & \text{if } 800\text{ft} < H \leq 900\text{ft} \\ 42\text{psf} & \text{otherwise} \end{cases} \quad (4.15)$$

$$b := \text{bay} \cdot w \quad (4.16)$$

where *bay* is the bay width, Figure 4-6, essentially the column-to-column spacing, for the proposed building.

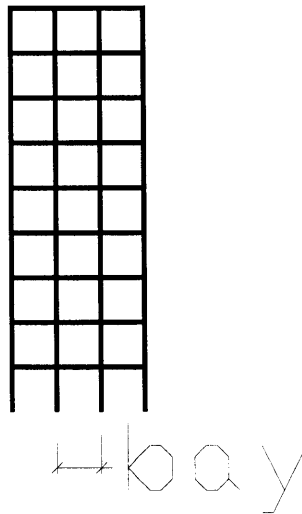


Figure 4-6: Proposed Structural Frame Bay Width

From the given uniform lateral wind loading the shear force, V , and bending moment, M , can be determined

$$V(x) := b \cdot (H - x) \quad (4.17)$$

$$M(x) := \frac{b \cdot (H - x)^2}{2} \quad (4.18)$$

All the necessary information is available to develop a deflection profile of the vertical cantilever beam, Figure 4-7.

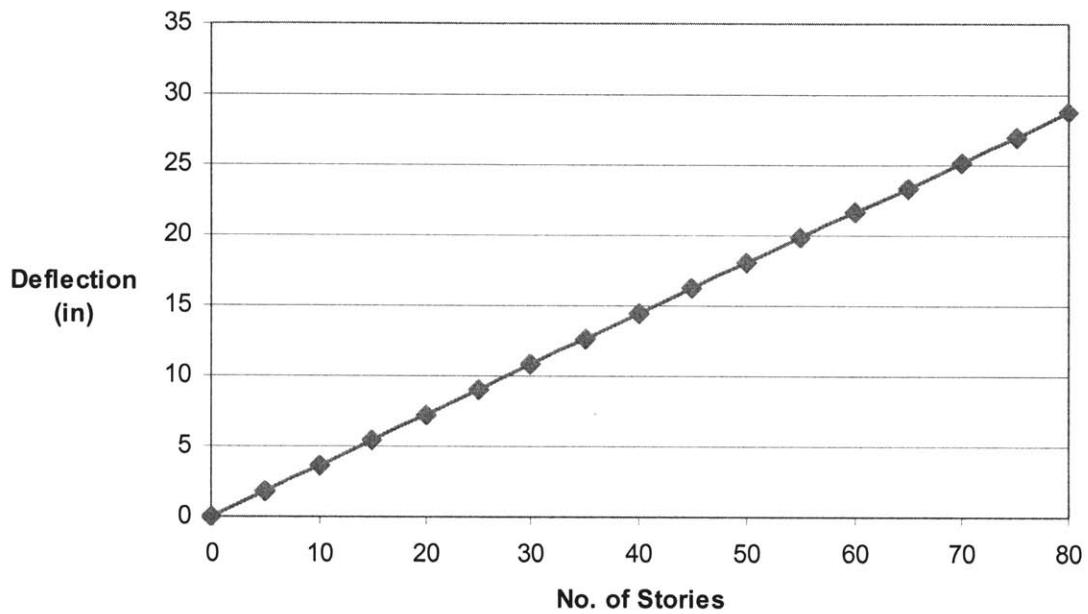


Figure 4-7: Proposed Structure Deflection Profile

With this available information each of the structural systems can be evaluated using performance based design. Optimal shear and bending stiffness can be determined for the vertical cantilever beam model and distributed to beams, columns, and diagonal bracing of the real structure.

4.2.1 Rigid Frame

To satisfy the geometric constraint of $B = 120 \text{ ft}$ the proposed rigid frame building will have five, 24 ft bays, $n_{bay} = 5$ and $bay = 24 \text{ ft}$. The dimensionless parameter $f = 1$ for rigid frame design.

The shear and bending rigidities are calculated according to the aforementioned manner and evaluated at h to find the required rigidities for the first story.

$$D_{Tframe} := D_T(h) \quad (4.19)$$

$$D_{Bframe} := D_B(h) \quad (4.20)$$

A good approximation for the column shear stiffness in a rigid frame can be obtained by assuming the location of the inflection points in the columns and beams. Taking these points at the midpoints, yields the following estimates for interior and exterior columns [1]

$$k_{interior} := \frac{12E \cdot I_c}{h^3 \cdot (1 + r)} \quad (4.21)$$

$$k_{exterior} := \frac{12E \cdot I_c}{h^3 \cdot (1 + 2r)} \quad (4.22)$$

where E is the modulus of elasticity for steel, I_c is the moment of inertia for the column members and r is a dimensionless parameter,

$$r := \frac{I_c}{h} \cdot \frac{L_b}{I_b} \quad (4.23)$$

where L_b is the length of the beams framing out the bays, essentially the bay width, bay , and I_c is the moment of inertia for the beam members. A typical frame has $r = O(1)$ and the shear stiffness required for one frame of the structure is calculated

$$K_{Tframe} := \frac{D_{Tframe}}{h} \quad (4.24)$$

Knowing the shear stiffness relationships for interior and exterior columns, assuming each to have the same properties, an equivalent required moment of inertia for the columns can be determined

$$I_c := \frac{K_{Tframe}}{2 \cdot \left(\frac{4E}{h^3} \right) + (n_{col} - 2) \cdot \left(\frac{6E}{h^3} \right)} \quad (4.25)$$

where n_{col} is the number of columns in the frame, signifying two exterior columns and the remainder as interior columns.

$$n_{col} := n_{bay} + 1 \quad (4.26)$$

Having calculated the required bending rigidity beforehand, a relationship can be constructed relating the column properties to the bending rigidity of the structure.

$$D_{Bframe} := E \cdot A_c \cdot n_{col} \sum_{n_{col}} \left(l_{dist}^2 \right) \quad (4.27)$$

where A_c is the required column area and l_{dist} is the distance from the neutral axis of the building to the columns; the neutral axis is taken as $B/2$. Given this relationship the required column area for this proposed structures with $n_{bay} = 5$ is as follows

$$A_c := \frac{D_{Bframe}}{E \cdot n_{col} \left[2 \cdot \left[\left(\frac{B}{2} \right)^2 + \left(\frac{bay}{2} \right)^2 + \left(\frac{3 \cdot bay}{2} \right)^2 \right] \right]} \quad (4.28)$$

Calculating the required moment of inertia, I_c , and area, A_c , for the columns for various story heights one can evaluate the system's premium for height, Figure 4-8 and Figure 4-9. Sample calculations and additional results can be found in Appendix A.

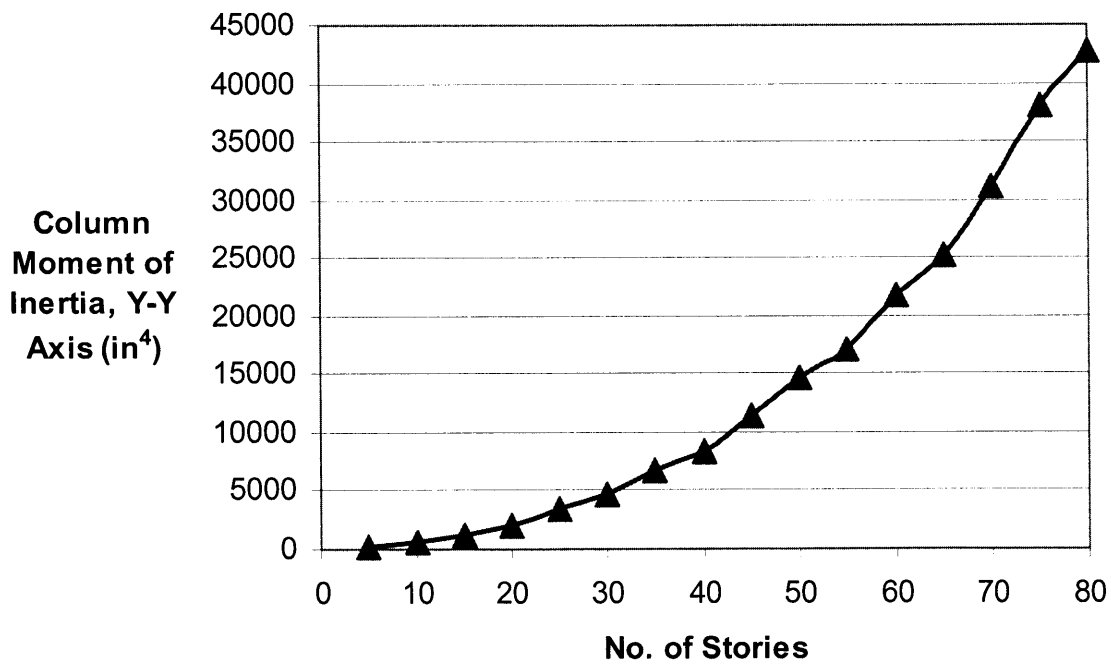


Figure 4-8: Rigid Frame Column Moment of Inertia for Story Height

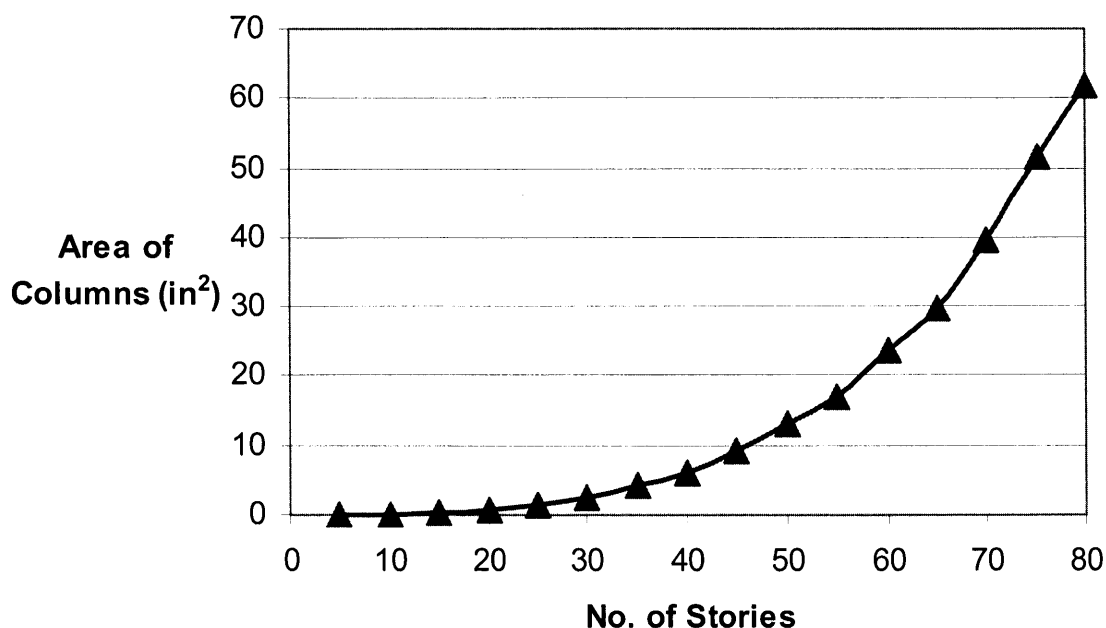


Figure 4-9: Rigid Frame Column Area for Story Height

As the number of stories approaches 80 the area requirements for the column are still well within reason for typical W-Shape steel section. However, the least moment of inertia, the weak axis Y-Y, for the larger W-Shape steel sections becomes unreasonable around 2000 in⁴. Examining this location

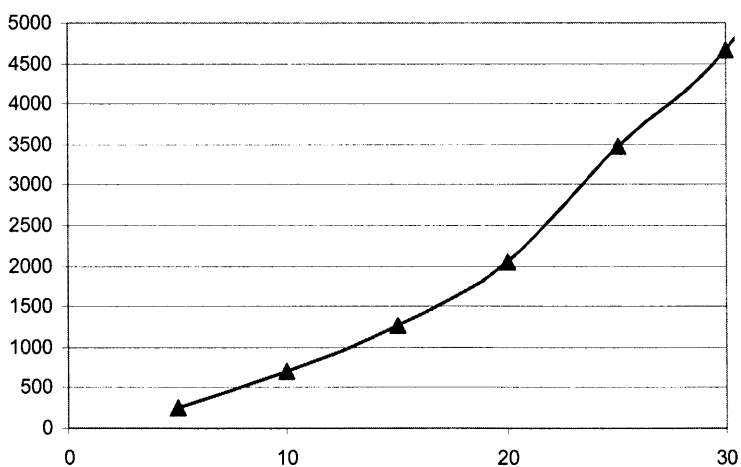


Figure 4-10: Close Examination of Rigid Frame Column Moment of Inertia

on the plot closer, Figure 4-10, it appears this occurs at 20 stories. It is important to note that the required moment of inertia for these columns is a direct result of bending in the columns due to shear racking and shear stiffness requirements for the frame. If there was

some way of reducing the shear racking and shear stiffness requirements rigid frame construction could be feasible to 50, upwards of 60 stories since the strong moment of inertia for typical W-Shapes can be as large as 20,000 in⁴. In the next section will utilize diagonal bracing to control the shear racking of the rigid frames.

4.2.2 Braced Frame

As was alluded to earlier, rigid frame construction suffers from large shear deformations, shear racking. One way to limit these shear deformations is to increase the stiffness of the structure by adding diagonal bracing.

Four bracing elements, $n_{brace} = 4$, in a concentric pattern spanning the entire width of a bay, will be used for evaluation purposes. These can be arranged in any fashion to be determined. A popular and efficient scheme is two diagonals for one bay arranged in an X-brace. The angle, θ , formed by the bracing, Figure 4-11, is easily determined based on the known geometry of the proposed structure.

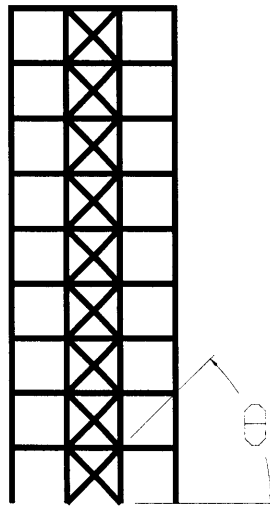


Figure 4-11: Proposed Structural Diagonal Bracing Angle

$$\theta := \text{atan}\left(\frac{h}{\text{bay}}\right) \quad (4.29)$$

For the braced frame the ratio of diagonal strain to chord strain is taken as $f = 3$. This is a typical value for mid- to high rise buildings experiencing only elastic deformations. The designer also defines distribution of shear stiffness to the bracing and the frame elements. Here, 80 percent of the shear stiffness is allocated to the bracing, $v_{brace} = 80\%$, where the remaining 20 percent is left to the framing elements.

The shear and bending rigidities for the vertical cantilever beam are similar in nature to those of the rigid frame, however, in this case there is both shear rigidity for the brace and the frame elements and will be distinguished as such

$$D_{Tbrace}(x) := v_{brace} \cdot D_T(x) \quad (4.30)$$

$$D_{Tframe}(x) := D_T(x) - D_{Tbrace}(x) \quad (4.31)$$

The shear and bending rigidities can then be evaluated at h for calculating the required shear and bending rigidities for the first floor columns and diagonal bracing

$$D_{Tbrace1} := D_{Tbrace}(h) \quad (4.32)$$

$$D_{Tframe1} := D_{Tframe}(h) \quad (4.33)$$

$$D_{B1} := D_B(h) \quad (4.34)$$

Column shear stiffness is calculated in the same manner as the rigid frame case, Eq. (4.21) to (4.24), assuming column and beam inflection points at the midpoint yielding essentially the same required shear stiffness for the frame reduced because of the allocation of stiffness to the bracing elements

$$K_{Tframe} := \frac{D_{Tframe1}}{h} \quad (4.35)$$

Again, knowing the shear stiffness relationships for interior and exterior columns the required moment of inertia for the columns can be determined in the same manner as Eq. (4.25) previously

$$I_c := \frac{K_{Tframe}}{2 \cdot \left(\frac{4E}{h^3} \right) + (n_{col} - 2) \cdot \left(\frac{6E}{h^3} \right)} \quad (4.36)$$

Derivation of the shear stiffness of the bracing element is more involved. Neglecting the extensional strain in the diagonals due to the bending moment rotations leads to the equivalent continuous beam properties for the shear rigidity required for the bracing [1]

$$D_{Tbrace1} := A_d \cdot E \cdot \cos(\theta)^2 \cdot \sin(\theta) \quad (4.37)$$

where A_d is the cross sectional area for the diagonal bracing elements. This relationship can then be rearranged since the required shear rigidity for the bracing elements is known yielding the area required for an individual diagonal bracing element

$$A_d := \frac{D_{Tbrace1}}{E \cdot \sin(\theta) \cdot \cos(\theta)^2 \cdot n_{brace}} \quad (4.38)$$

In a similar manner beforehand for the rigid frame case, Eq. (4.27) the required column area can be calculated for the given bending rigidity

$$A_c := \frac{D_{B1}}{E \cdot \left[2 \cdot \left[\left(\frac{B}{2} \right)^2 + \left(\frac{bay}{2} \right)^2 + \left(\frac{3 \cdot bay}{2} \right)^2 \right] \right]} \quad (4.39)$$

Results again were compiled for a variety of heights, Figure 4-12 and Figure 4-13. Sample calculations and additional results can be found in Appendix B.

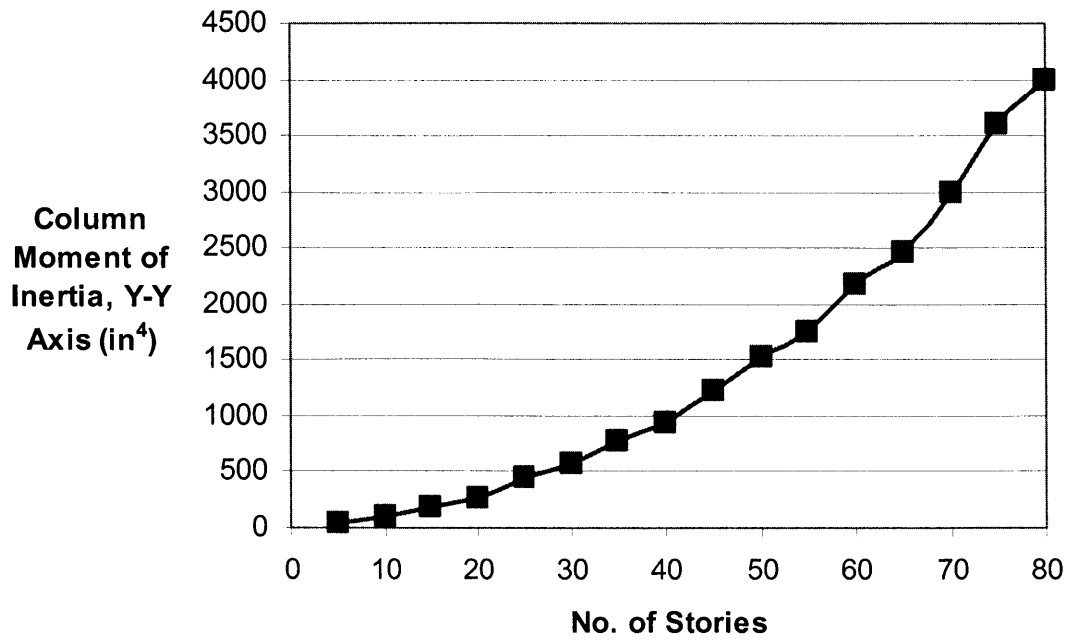


Figure 4-12: Braced Frame Column Moment of Inertia for Story Height

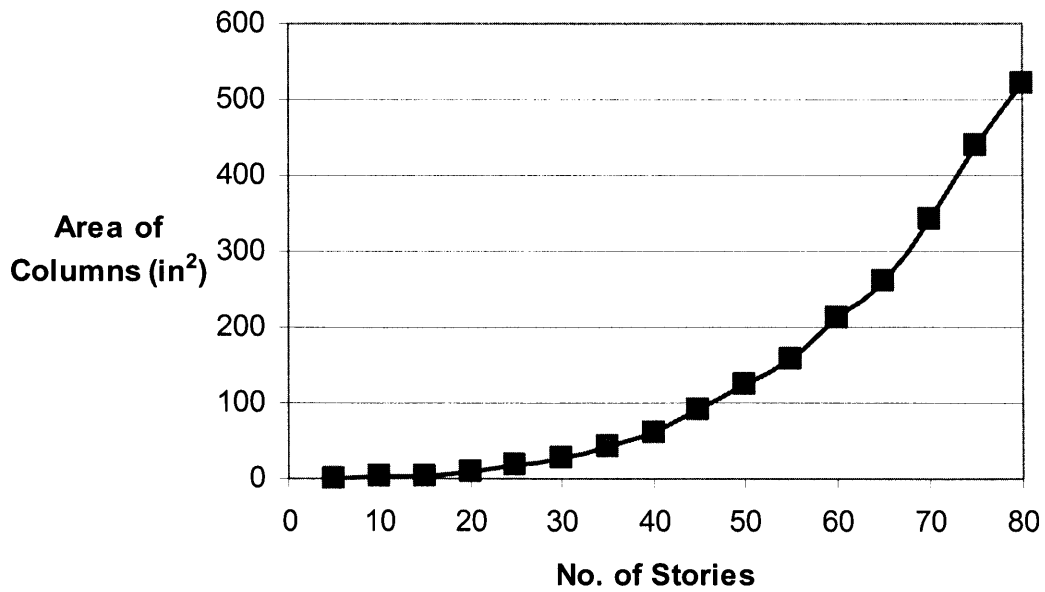


Figure 4-13: Braced Frame Column Area for Story Height

Near 60 stories the weak axis bending moment of inertia becomes unreasonable, there are not many available W-Shape steel section with the Y-Y axis moment of inertia greater than 2000 in⁴. However, at approximately 50 stories the required cross section area to satisfy bending rigidity becomes larger than most available steel sections. The largest sections are approximately 150 in² and even then are not usually considered an economical structural solution. So for the braced frame case the design is actually controlled by the column bending stiffness rather than the shear stiffness. This is due to the additional shear stiffness of the diagonals, Figure 4-14.

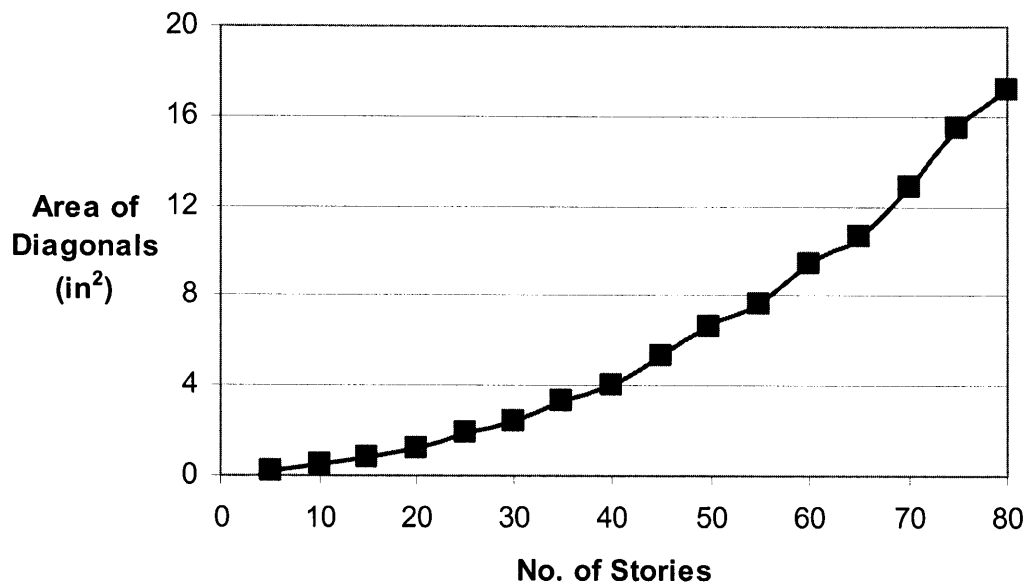


Figure 4-14: Braced Frame Diagonal Bracing Area for Story Height

Even for some of the tallest building structures evaluated, the area of the diagonals do not become completely unreasonable. There are plenty of structural elements available with cross sectional areas in excess of 20 in². It is relevant to note that the larger diagonal bracing elements may be feasible from a structural standpoint, but architecturally they may be unpractical. Increasing the bracing elements' size reduces the overall window wall area in the case of an exterior wall, or decreases circulation patterns and available door and pass-through openings for interior walls.

4.2.3 Tube Structure

Braced frames taller than about 50 stories were controlled by the bending stiffness of the columns. One way of increasing the bending stiffness and rigidity of the entire structure is to allow it to act as a tubular structure in which only perimeter columns resist the lateral loads. It is a well know fact that the most optimum shape to resist bending is a tube. Tube elements do not have weak axis bending and their moment of inertia is quite large compared to its overall cross sectional area. In order to achieve tube-like behavior in the proposed structural model there shall be no interior columns and the exterior perimeter columns must be closely spaced, $col_{space} = 3.75 \text{ ft}$, to simulate the tubular action without completely eliminating potential window wall space.

Again, for this model, as way the case with the rigid frame model, the ratio of diagonal strains to chord strains is nonexistent and not applicable, $f = 1$.

The lateral loading of this structure is not divided among the frames of the structure but rather treated as an entire uniform wind loading on the structure since tubular action can only be achieved with the whole system. Required shear and bending rigidities are calculated in the same manner as the rigid and braced frames

$$D_{Ttube} := D_T(h) \quad (4.40)$$

$$D_{Btube} := D_B(h) \quad (4.41)$$

The required shear stiffness is also found in a similar manner.

$$K_{Ttube} := \frac{D_{Ttube}}{h} \quad (4.42)$$

Column design for the tubular structure is slightly different than the rigid or braced frame design. Rigid and braced frame design used individual frame elements for calculations,

however to tube does not have frame elements, it must be treated in its entirety. The required column moment of inertia was calculated with a similar approach. The exterior column line parallel to the lateral wind load was treated as interior column as previously defined in Eq. (4.21) and likewise the exterior column line perpendicular to the lateral wind load was treated as exterior columns as previously defined in Eq. (4.22). Already knowing the number of column in each column line, n_{col} , an equation for moment of inertia for the columns satisfying the shear stiffness is defined as

$$I_c := \frac{K_{Ttube}}{2n_{col} \left(\frac{4E}{h^3} \right) + 2 \cdot n_{col} \left(\frac{6E}{h^3} \right)} \quad (4.43)$$

Since the proposed structure is emulating a bending tube it is a fair assumption to calculate the moment of inertia, I_{tube} , to satisfy the bending rigidity for the vertical cantilever model and then further simplify to a column area required to achieve this. Bending rigidity for the proposed structure is defined by

$$D_{Btube} := E \cdot I_{tube} \quad (4.44)$$

A simple rearrangement gives the moment of inertia of the tube in terms of its bending rigidity

$$I_{tube} := \frac{D_{Btube}}{E} \quad (4.45)$$

The geometry of the proposed structure is essentially know, therefore the only unknown factor is an equivalent thickness of the tube wall. A general equation for calculating the moment of inertia for a rectangular tube is as follows

$$I_{tube} := \frac{1}{12} \left(b_{exterior} \cdot h_{exterior}^3 - b_{interior} \cdot h_{interior}^3 \right) \quad (4.46)$$

where b and h are the base width and height for the respective rectangular section. Using Eq. (4.46) as a reference, the moment of inertia for the proposed tubular structure is known as well as the exterior geometry, $b_{exterior} = B$ and $h_{exterior} = B$, leaving the only unknown piece of information the interior geometry which can be defined as

$$b_{interior} := B - t \quad (4.47)$$

$$h_{interior} := B - t \quad (4.48)$$

where B is the proposed structure base width and t is a tube wall thickness. Eq. (4.46) can then be rearranged and evaluated for t , the tube wall thickness

$$t := B - \left(B^4 - 12 \cdot I_{tube} \right)^{\frac{1}{4}} \quad (4.49)$$

This equivalent tube wall thickness can then be lumped to the column locations to give a column cross sectional area required

$$A_c := t \cdot col_{space} \quad (4.50)$$

Results again were compiled for a variety of heights, Figure 4-15 and Figure 4-16. Sample calculations and additional results can be found in Appendix C.

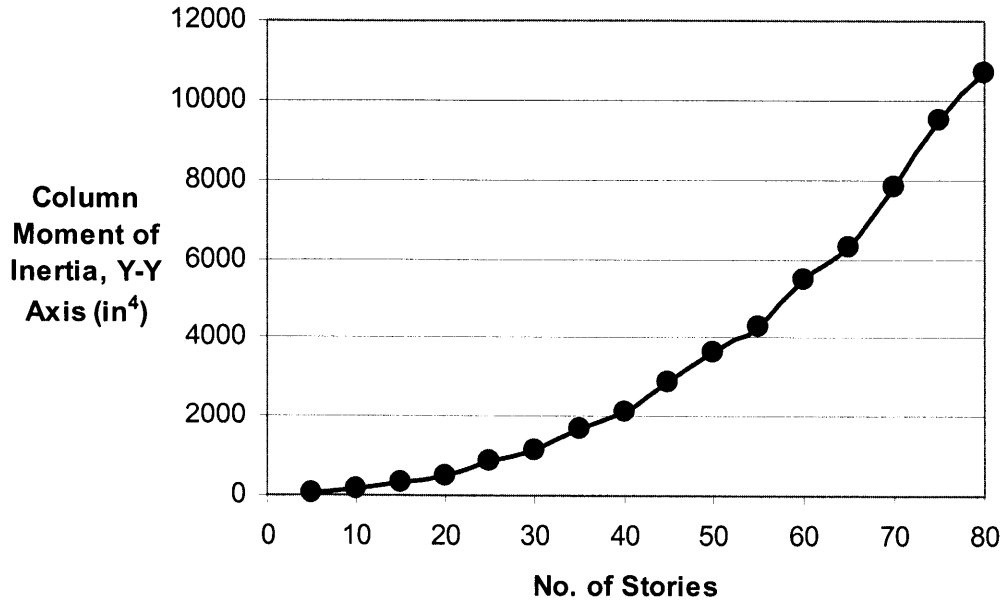


Figure 4-15: Tube Structure Column Moment of Inertia for Story Height

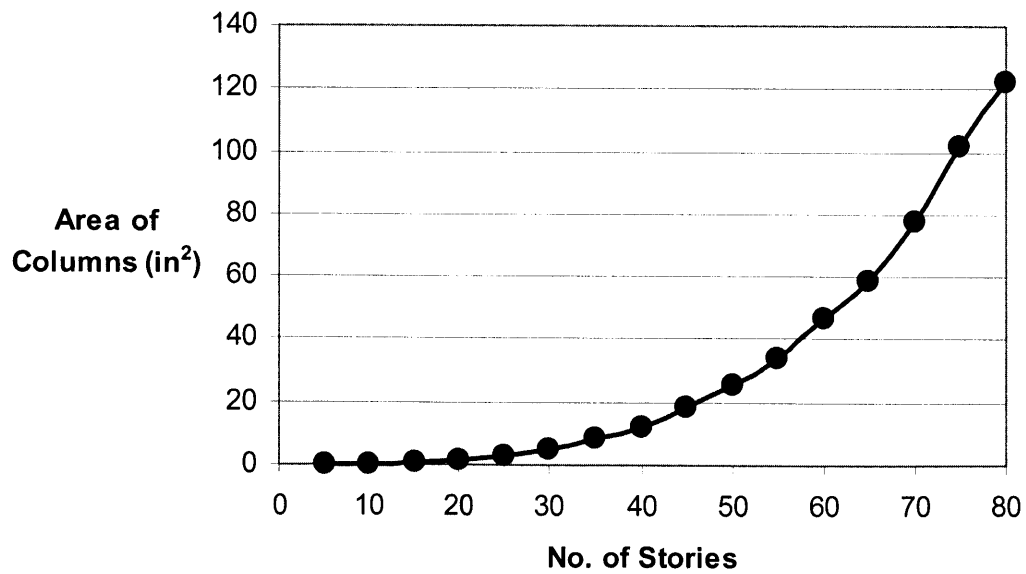


Figure 4-16: Tube Structure Column Area for Story Height

For the tube structure the use of traditional W-Shape steel sections will not suffice. It is even apparent in the results. As was previously noted, Y-Y axis moment of inertia for some of the largest W-Shape steel sections only approach 2000 in⁴ and this occurs at approximately 40 stories. This is a direct result of shear racking of the column. One way to possibly increase the shear stiffness of this structure would be to consider a truss or bundled tube which try to capitalize on the strengths of the different systems.

4.3 References

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- [5] Taranath, B. S., *Structural Analysis & Design of Tall Buildings*, McGraw-Hill Inc., New York, 1988.
- [6] Khan, F. R., and Rankine, J., "Structural Systems," Tall Building Systems and Concepts, Council on Tall Buildings and Urban Habitat (CTBUH)/American Society of Civil Engineers (ASCE), Vol. SC, 1980.
- [7] Chopra, A. K., *Dynamics of Structures*, Theory and Applications to Earthquake Engineering, Prentice Hall, New Jersey, 2001.
- [8] Anderson, D., "Design of Multi-Storey Steel Frames to Sway Deflection Limitations," Ed. Narayanan, R., *Steel Frame Structures, Stability and Strength*, Elsevier Applied Science Publishers, New York, 1985.

Chapter 5 Conclusion

Collectively, Figure 5-1 and Figure 5-2, one can truly see the results from the analysis. Additional analysis and comparisons can be found in Appendix D.

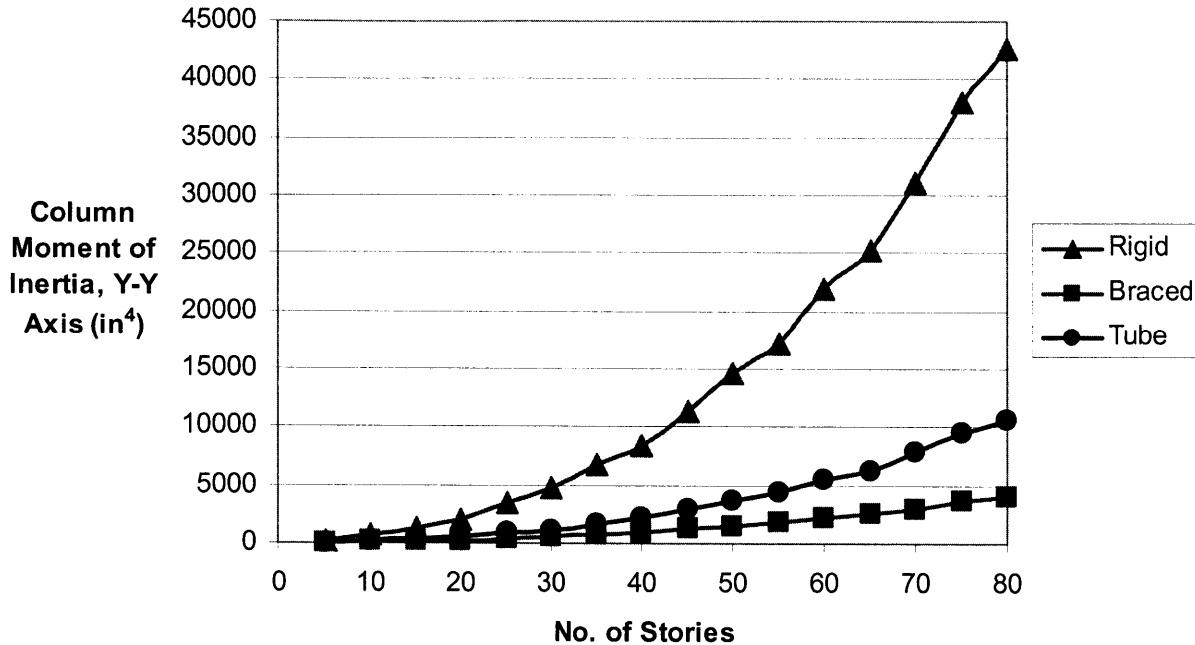


Figure 5-1: Structural Systems Column Moment of Inertia for Story Height

Although this analysis was a gross approximation for the modeling the three structural systems, rigid frame, braced frame and tube structure, the results are generally what was expected based on previous knowledge. The column moment of inertia is governed by the shear stiffness of the structures and most obviously the rigid frame has the lowest stiffness, requiring the largest weak axis moment of inertia. Probably most surprisingly the tube structure has the second lowest shear stiffness. Upon further analysis, one would conclude that the column stiffness estimations for the tube structure, Eq. (4.21) and (4.22), would not hold true. The deep perimeter girders would, in reality, provide more bending resistance and stiffen the structure. In any case, the tube structure does still lack the shear resistance of the braced frame.

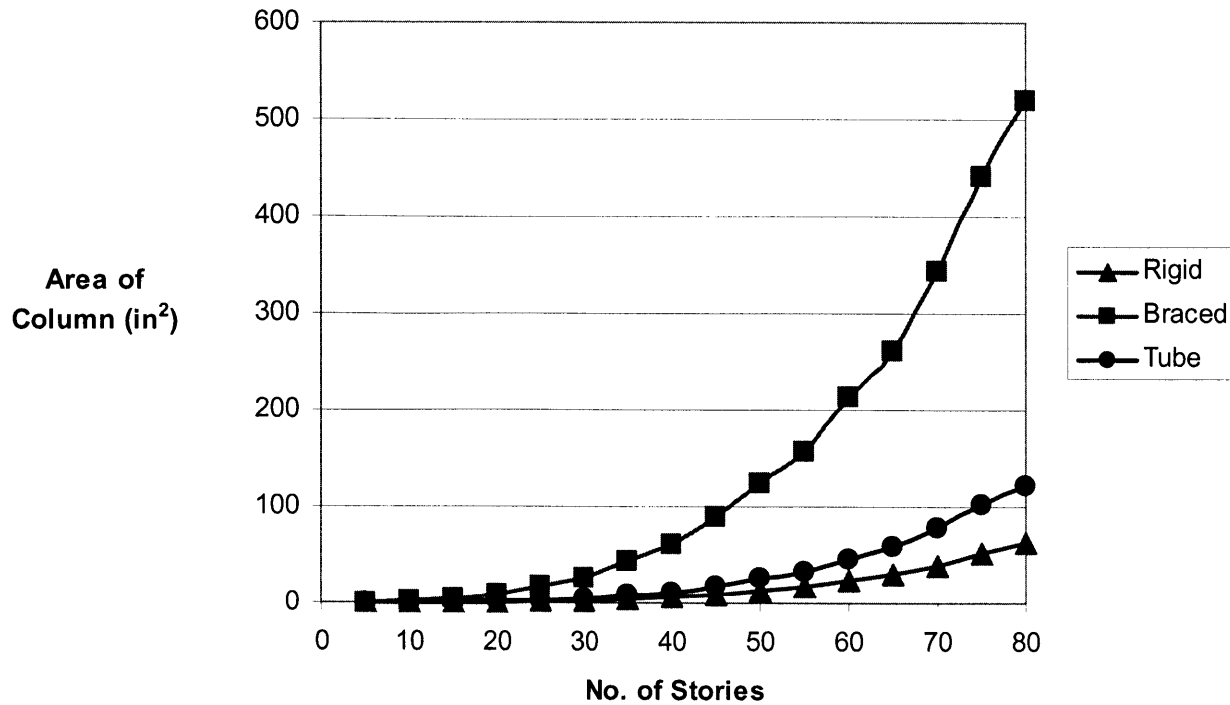


Figure 5-2: Structural Systems Column Area for Story Height

Bending rigidities of the tube structure is, as one would expect, quite high requiring smaller column cross sectional areas to achieve the required rigidity. One would probably not expect the rigid frame to be as efficient in bending as the results predict.

Efficient structural solutions to 40 or 50 stories can be obtained by combining one or more of the three options. The rigid frame proves to be extremely efficient if the columns were always oriented in the strong axis. Since this is not the case, due to the equal probability that the lateral wind loads will come N-S direction as W-E direction. In the weak axis of the rigid frame shear stiffness can be provided with the aid of bracing. This would create a rigid and braced frame interaction. Another highly efficient structure would be to brace the tube structure to develop a truss tube design that is equally as efficient in shear and in bending creating a super structure for tall buildings.

With the advent of the composite super-tall structural systems, the structural steel tall buildings are beginning to be phased out of construction. Concrete has many admirable

qualities for high rise building design including better damping, cheaper labor than steel and extremely high compression capacity so it is quickly becoming the material of choice for the super-tall building structural systems. At a performance level structural steel can be an efficient material on its own by combining structural systems and reaping the benefits of the individual systems without accumulating negative effects. It will continue to be a favorable building material for low- and mid-rise structures, appreciated by engineers, architects and contractors alike.

Appendices

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Appendix A: Rigid Frame Design

Parameters Governing Frame Design

Height

$n_{\text{story}} := 80$	Number of Stories in Building
$h := 12 \text{ ft}$	Typical Story Height of Building
$H := n_{\text{story}} \cdot h$	Height of Building
$H = 960 \text{ ft}$	

Width

$n_{\text{bay}} := 5$	Number of Bays in Building
$\text{bay} := 24 \text{ ft}$	Typical Bay Width of Building
$B := \text{bay} \cdot n_{\text{bay}}$	Width of Building
$B = 120 \text{ ft}$	
$\frac{B}{H} = 0.13$	Aspect Ratio

Performance Parameters

$E := 29000 \text{ ksi}$	Steel Modulus of Elasticity
$\alpha := 400$	Design Deflection Parameter
$f := 1$	Ratio of Diagonal Strain to Chord Strain (Not Applicable in this case)

Summary of Proposed Building

The building under design is $n_{\text{story}} = 80$ stories tall and has a height of, $H = 960 \text{ ft}$, and a width of, $B = 120 \text{ ft}$, giving an aspect ratio of

$$\frac{B}{H} = 0.125 \text{ .}$$

Frame Design

Lateral Loading

$$w := \begin{cases} 20\text{psf} & \text{if } H \leq 200\text{ft} \\ 21\text{psf} & \text{if } 200\text{ft} < H \leq 300\text{ft} \\ 25\text{psf} & \text{if } 300\text{ft} < H \leq 400\text{ft} \\ 28\text{psf} & \text{if } 400\text{ft} < H \leq 500\text{ft} \\ 31\text{psf} & \text{if } 500\text{ft} < H \leq 600\text{ft} \\ 33\text{psf} & \text{if } 600\text{ft} < H \leq 700\text{ft} \\ 36\text{psf} & \text{if } 700\text{ft} < H \leq 800\text{ft} \\ 39\text{psf} & \text{if } 800\text{ft} < H \leq 900\text{ft} \\ 42\text{psf} & \text{otherwise} \end{cases}$$

Simplified Minimum Wind
Design Pressure
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Municipal Code of Chicago, IL

$$b := \text{bay} \cdot w$$

Minimum Uniform Wind Loading on
Frame Element

$$b = 1.008 \frac{\text{kip}}{\text{ft}}$$

$$V(x) := b \cdot (H - x)$$

Shear Force

$$M(x) := \frac{b \cdot (H - x)^2}{2}$$

Bending Moment

Deflection Characteristics

$$s := \frac{H}{2 \cdot f \cdot B}$$

Dimensionless Parameter

$$\gamma := \frac{1}{(1 + s) \cdot \alpha}$$

Design Shear Deformation

$$\chi := \frac{2 \cdot \gamma \cdot s}{H}$$

Design Bending Deformation

$$u(x) := \gamma \cdot x + \frac{\chi \cdot x^2}{2}$$

Deflection of Building

$$u(H) = 28.8 \text{ in}$$

Maximum Deflection of Building

Design Shear and Bending Rigidity

$$D_T(x) := \frac{V(x)}{\gamma}$$

Shear Rigidity

$$D_B(x) := \frac{M(x)}{\chi}$$

Bending Rigidity

$$D_{Tframe} := D_T(h)$$

Shear Rigidity Required

$$D_{Bframe} := D_B(h)$$

Bending Rigidity Required

$$K_{Tframe} := \frac{D_{Tframe}}{h}$$

Shear Stiffness Required

$$n_{col} := n_{bay} + 1$$

Number of Columns per Frame Element

Column Design

$$I_c := \frac{K_{Tframe}}{2 \cdot \left(\frac{4E}{h^3}\right) + (n_{col} - 2) \cdot \left(\frac{6E}{h^3}\right)}$$

Least Moment of Inertial for a First Floor Column to Satisfy Shear Rigidity

$$I_c = 42705 \text{ in}^4$$

$$A_c := \frac{D_{Bframe}}{E \cdot n_{col} \left[2 \cdot \left[\left(\frac{B}{2}\right)^2 + \left(\frac{\text{bay}}{2}\right)^2 + \left(\frac{3 \cdot \text{bay}}{2}\right)^2 \right] \right]}$$

Area of Columns to Satisfy Bending Rigidity

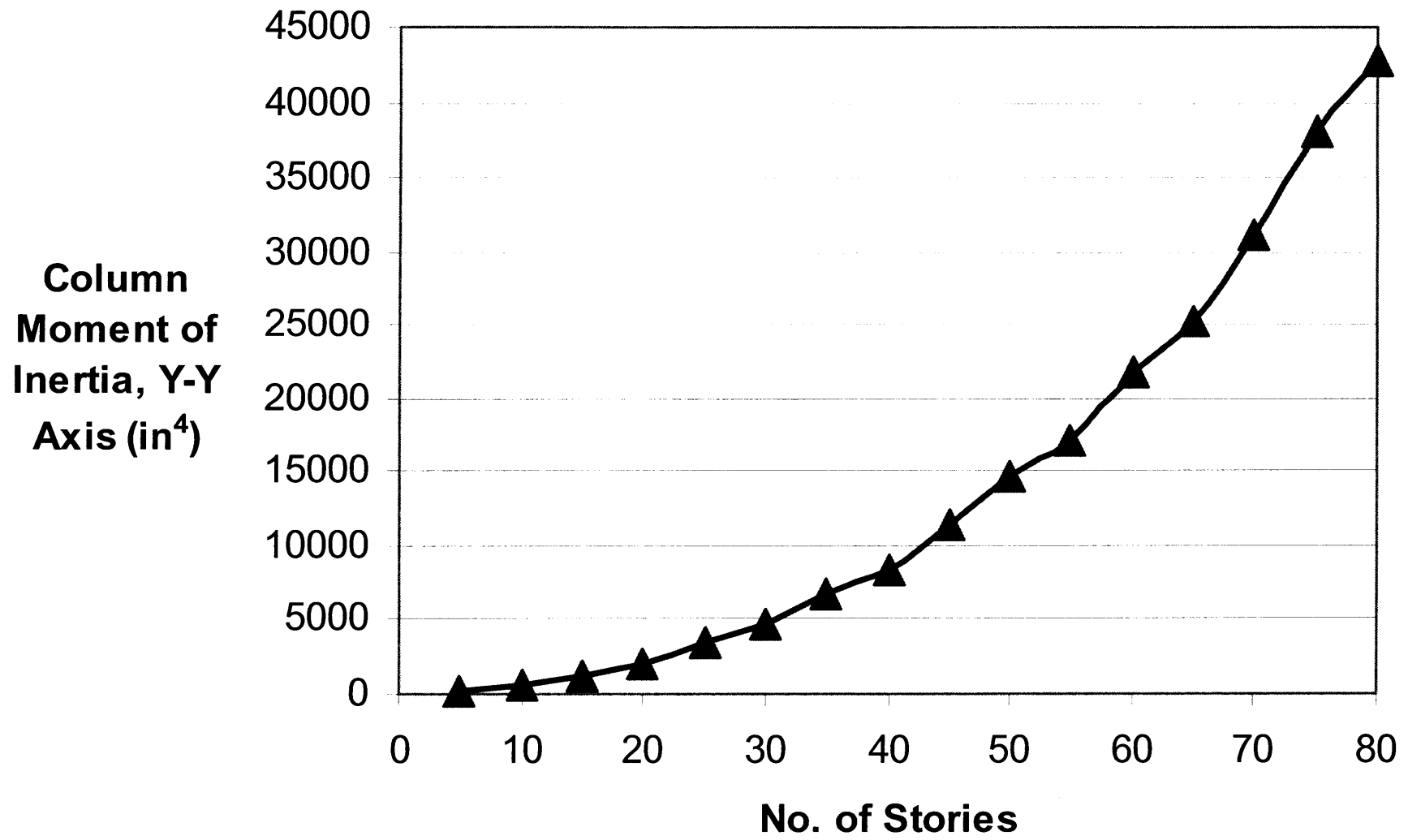
$$A_c = 62 \text{ in}^2$$

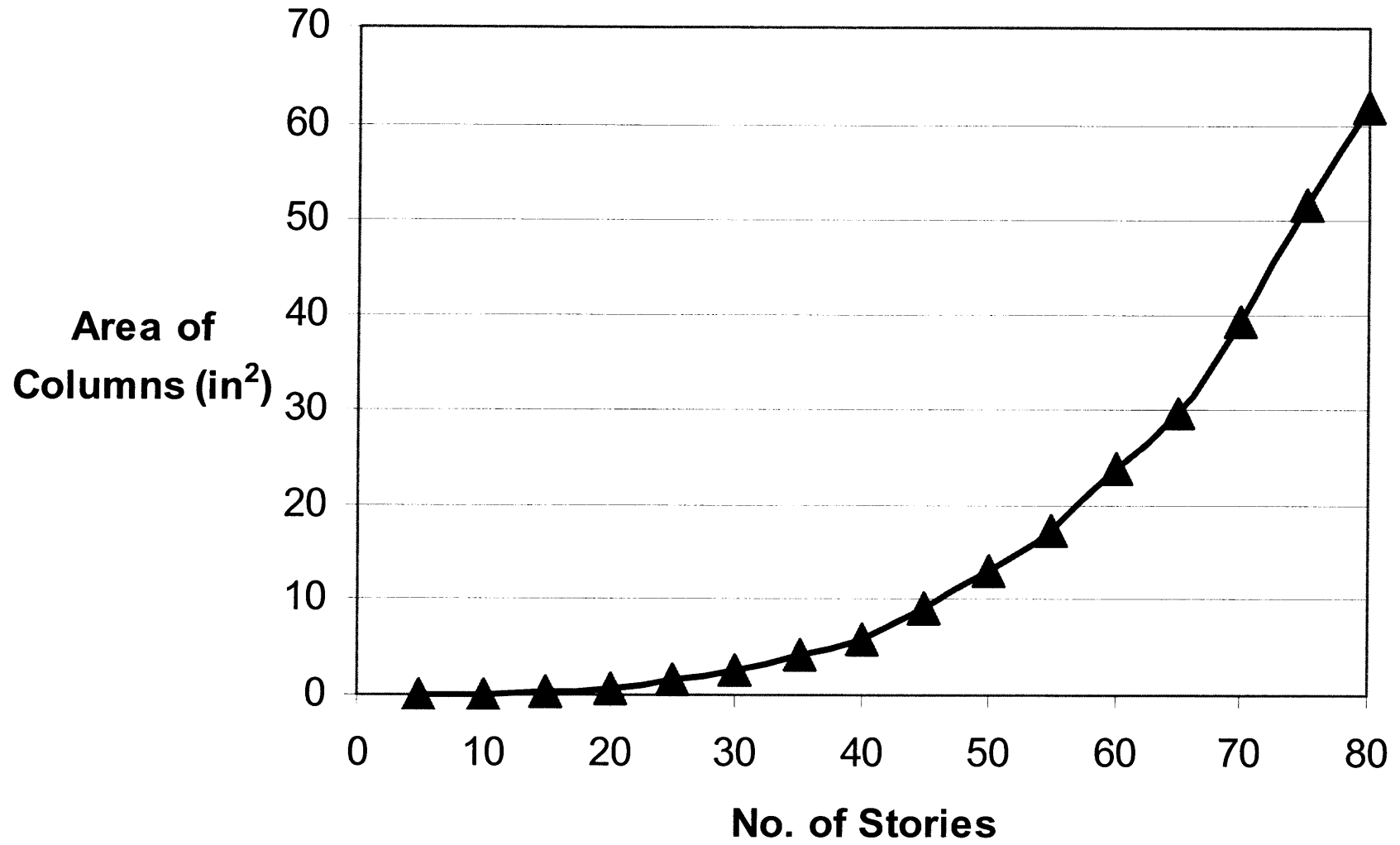
Design Summary

Select structural steel columns that satisfy: area, $A_c = 62 \text{ in}^2$

Y-Y axis Moment of Inertia, $I_c = 42705 \text{ in}^4$

No. of Stories	Story Height (ft)	Building Height (ft)	No. of Bays	Bay Width (ft)	Building Width (ft)	Aspect Ratio	Max. Deflection (in)		Shear Rigidity	Bending Rigidity
									Y-Y axis Moment of Inertia (in ⁴)	Area of Column (in ²)
5	12	60	5	24	120	2.00	1.8		257	0
10	12	120	5	24	120	1.00	3.6		695	0.1
15	12	180	5	24	120	0.67	5.4		1261	0.3
20	12	240	5	24	120	0.50	7.2		2054	0.7
25	12	300	5	24	120	0.40	9		3475	1.5
30	12	360	5	24	120	0.33	10.8		4666	2.5
35	12	420	5	24	120	0.29	12.6		6739	4.2
40	12	480	5	24	120	0.25	14.4		8433	6
45	12	540	5	24	120	0.22	16.2		11411	9.2
50	12	600	5	24	120	0.20	18		14568	13.1
55	12	660	5	24	120	0.18	19.8		17202	17.1
60	12	720	5	24	120	0.17	21.6		21870	23.7
65	12	780	5	24	120	0.15	23.4		25206	29.6
70	12	840	5	24	120	0.14	25.2		31171	39.5
75	12	900	5	24	120	0.13	27		38002	51.7
80	12	960	5	24	120	0.13	28.8		42705	62





Appendix B: Braced Frame Design

Parameters Governing Frame Design

Height

$$n_{\text{story}} := 80$$

Number of Stories in Building

$$h := 12 \text{ ft}$$

Typical Story Height of Building

$$H := n_{\text{story}} \cdot h$$

Height of Building

$$H = 960 \text{ ft}$$

Width

$$n_{\text{bay}} := 5$$

Number of Bays in Frame

$$\text{bay} := 24 \text{ ft}$$

Typical Bay Width of Frame

$$B := \text{bay} \cdot n_{\text{bay}}$$

Width of Building

$$B = 120 \text{ ft}$$

$$\frac{B}{H} = 0.13$$

Aspect Ratio

$$\theta := \text{atan}\left(\frac{h}{\text{bay}}\right)$$

Angle Formed by Concentric Bracing within Each Bay

$$\theta = 26.6 \text{ deg}$$

Performance Parameters

$E := 29000$ ksi	Steel Modulus of Elasticity
$\alpha := 400$	Design Deflection Parameter
$f := 3$	Ratio of Diagonal Strain to Chord Strain
$v_{\text{brace}} := 80\%$	Allocation of Shear Stiffness to Bracing
$n_{\text{brace}} := 4$	Number of Bracing Elements in Frame

Summary of Proposed Building

The building under design is $n_{\text{story}} = 80$ stories tall and has a height of, $H = 960$ ft, and a width of, $B = 120$ ft, giving an aspect ratio of $\frac{B}{H} = 0.125$. It will have $n_{\text{brace}} = 4$ bracing elements in any given frame. The detailing of the bracing scheme can be left to the designer, using one diagonal element per bay or two per bay in an X-brace format.

Frame Design

Lateral Loading

$$w := \begin{cases} 20\text{psf} & \text{if } H \leq 200\text{ft} \\ 21\text{psf} & \text{if } 200\text{ft} < H \leq 300\text{ft} \\ 25\text{psf} & \text{if } 300\text{ft} < H \leq 400\text{ft} \\ 28\text{psf} & \text{if } 400\text{ft} < H \leq 500\text{ft} \\ 31\text{psf} & \text{if } 500\text{ft} < H \leq 600\text{ft} \\ 33\text{psf} & \text{if } 600\text{ft} < H \leq 700\text{ft} \\ 36\text{psf} & \text{if } 700\text{ft} < H \leq 800\text{ft} \\ 39\text{psf} & \text{if } 800\text{ft} < H \leq 900\text{ft} \\ 42\text{psf} & \text{otherwise} \end{cases}$$

Simplified Minimum Wind
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Municipal Code of Chicago,
IL

$$b := \text{bay} \cdot w$$

Uniform Wind Loading on
Frame Element

$$b = 1.008 \frac{\text{kip}}{\text{ft}}$$

$$V(x) := b \cdot (H - x)$$

Shear Force

$$M(x) := \frac{b \cdot (H - x)^2}{2}$$

Bending Moment

Deflection Characteristics

$s := \frac{H}{2 \cdot f \cdot B}$	Dimensionless Parameter
$\gamma := \frac{1}{(1 + s) \cdot \alpha}$	Design Shear Deformation
$\chi := \frac{2 \cdot \gamma \cdot s}{H}$	Design Bending Deformation
$u(x) := \gamma \cdot x + \frac{\chi \cdot x^2}{2}$	Deflection of Building
$u(H) = 28.8 \text{ ir}$	Maximum Deflection of Building

Design Shear and Bending Rigidity

$D_T(x) := \frac{V(x)}{\gamma}$	Total Shear Rigidity
$D_{Tbrace}(x) := v_{brace} \cdot D_T(x)$	Brace Shear Rigidity
$D_{Tframe}(x) := D_T(x) - D_{Tbrace}(x)$	Frame Shear Rigidity
$D_B(x) := \frac{M(x)}{\chi}$	Bending Rigidity
$D_{Tbrace1} := D_{Tbrace}(h)$	Brace Shear Rigidity Required
$D_{Tframe1} := D_{Tframe}(h)$	Frame Shear Rigidity Required
$D_{B1} := D_B(h)$	Bending Rigidity Required
$K_{Tframe} := \frac{D_{Tframe1}}{h}$	Frame Shear Stiffness Required
$n_{col} := n_{bay} + 1$	Number of Columns per Frame Element

Brace and Column Design

$$A_d := \frac{D_{Tbrace1}}{E \cdot \sin(\theta) \cdot \cos(\theta)^2 \cdot n_{brace}}$$

Area of Diagonal Bracing

$$A_d = 17.2 \text{ in}^2$$

$$I_c := \frac{K_{Tframe}}{2 \cdot \left(\frac{4E}{h^3}\right) + (n_{col} - 2) \cdot \left(\frac{6E}{h^3}\right)}$$

Least Moment of Inertial for a Column to Satisfy Shear Rigidity

$$I_c = 3986 \text{ in}^4$$

$$A_c := \frac{D_{B1}}{E \cdot \left[2 \cdot \left[\left(\frac{B}{2}\right)^2 + \left(\frac{\text{bay}}{2}\right)^2 + \left(\frac{3 \cdot \text{bay}}{2}\right)^2 \right] \right]}$$

Area of a Column to Satisfy Bending Rigidity

$$A_c = 520.6 \text{ in}^2$$

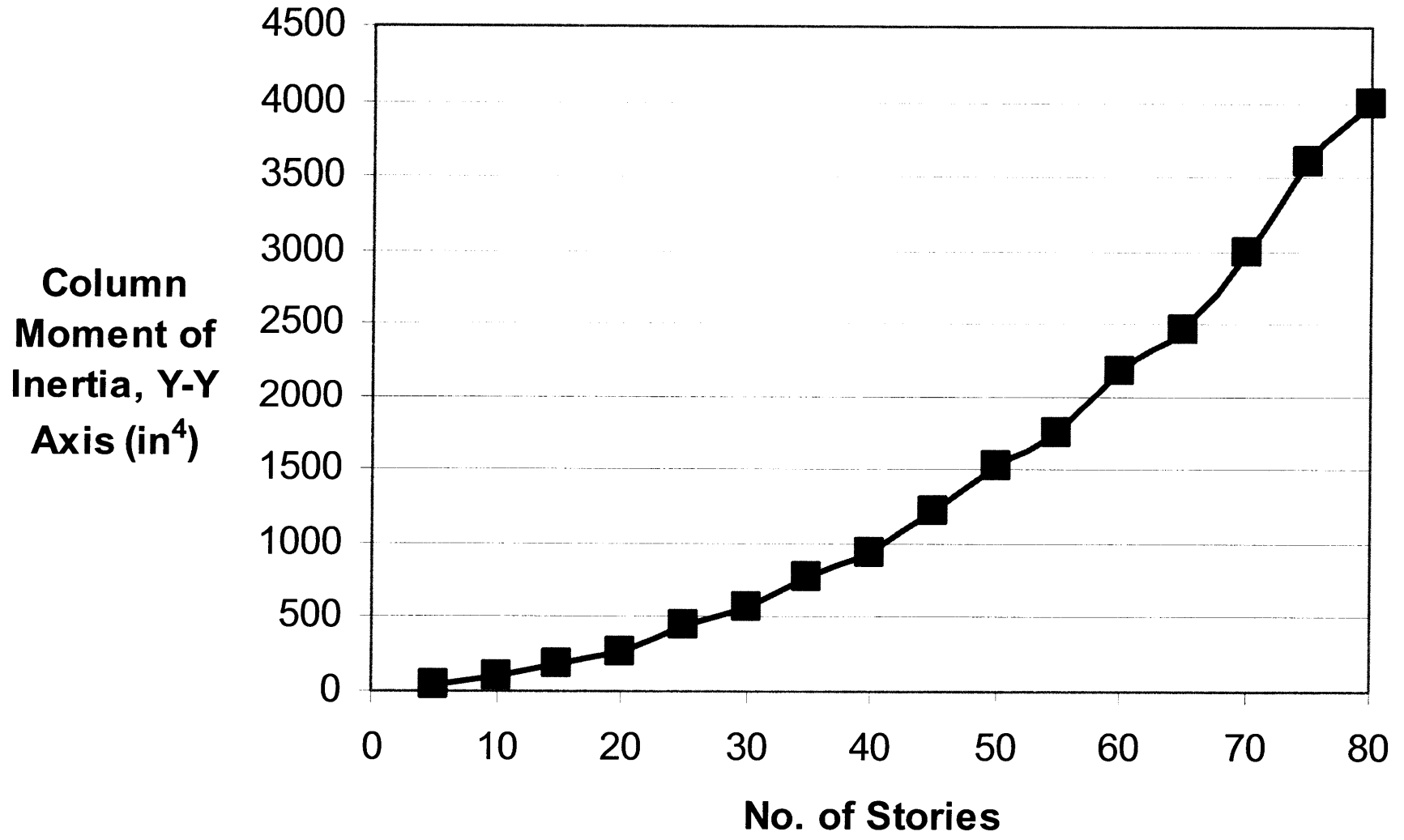
Design Summary

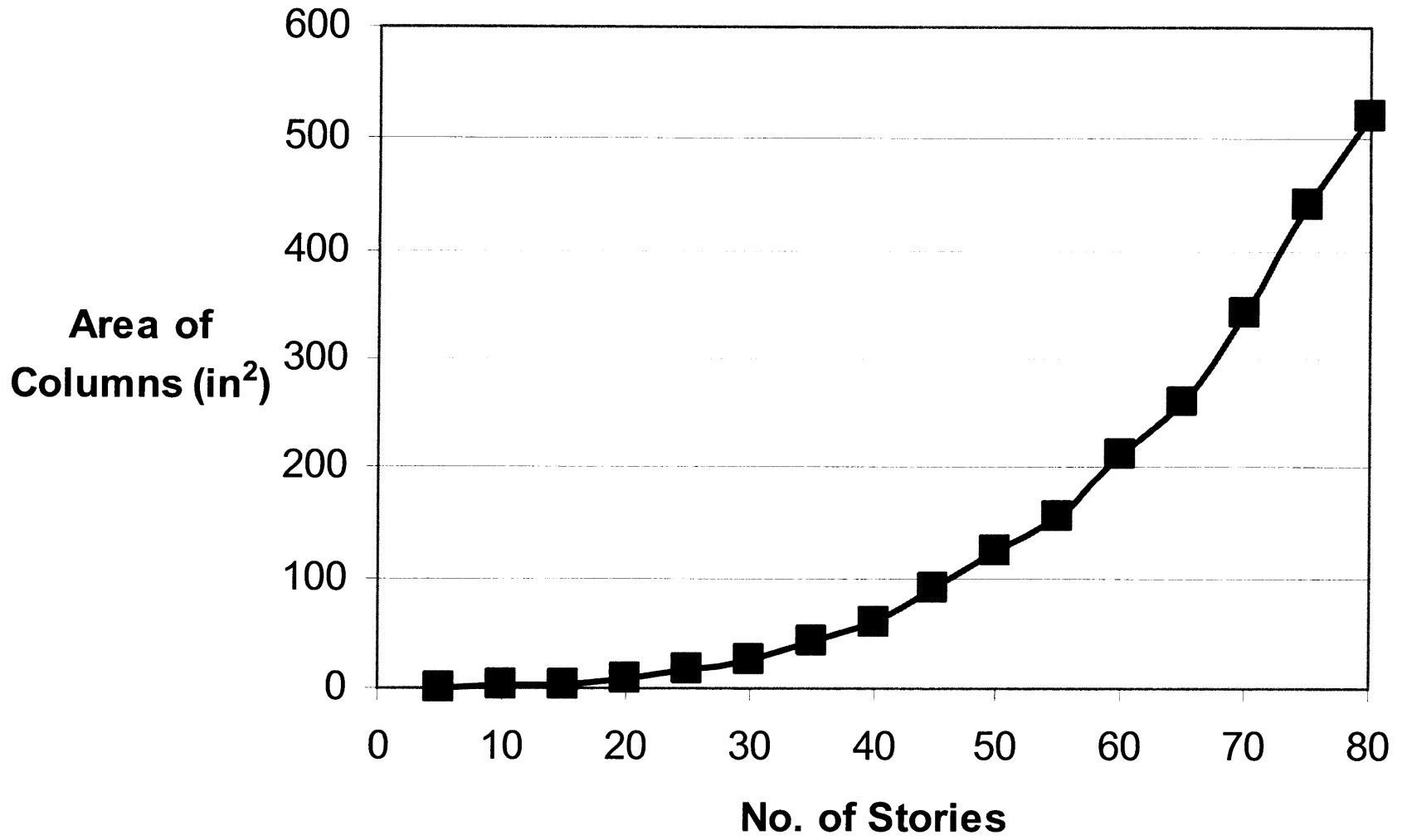
Select structure steel columns that satisfy: Area, $A_c = 520.6 \text{ in}^2$

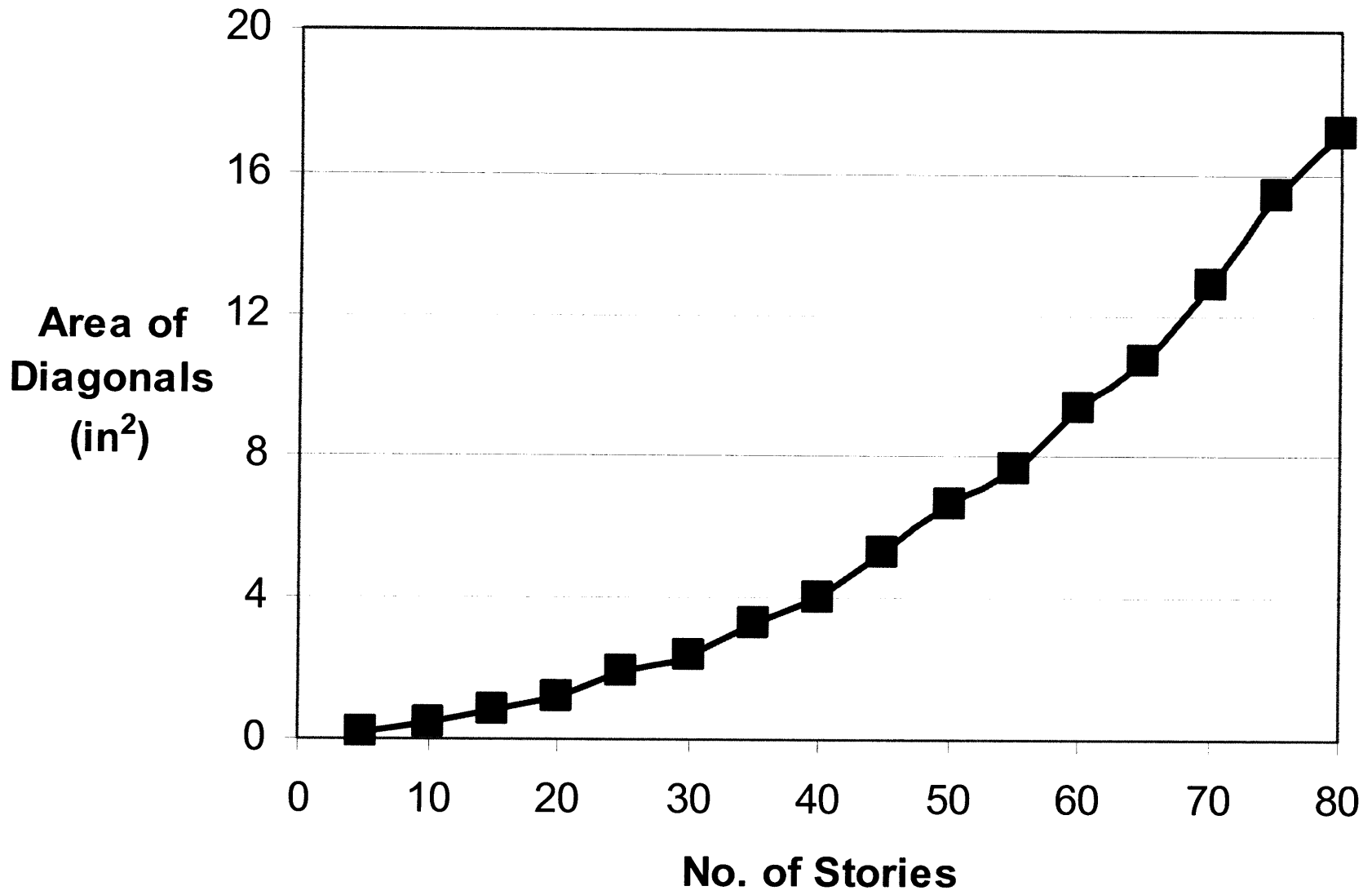
Y-Y axis Moment of Inertia, $I_c = 3986 \text{ in}^4$

Select diagonal bracing elements that satisfy: Area, $A_d = 17.2 \text{ in}^2$

No. of Stories	Story Height (ft)	Building Height (ft)	No. of Bays	Bay Width (ft)	Building Width (ft)	Aspect Ratio	Max. Deflection (in)	Shear Rigidity		Bending Rigidity
								Y-Y axis Moment of Inertia (in ⁴)	Area of Diagonals (in ²)	Area of Columns (in ²)
5	12	60	5	24	120	2.00	1.8	45	0.2	0.3
10	12	120	5	24	120	1.00	3.6	108	0.5	1.6
15	12	180	5	24	120	0.67	5.4	180	0.8	4.2
20	12	240	5	24	120	0.50	7.2	274	1.2	8.6
25	12	300	5	24	120	0.40	9	438	1.9	17.4
30	12	360	5	24	120	0.33	10.8	560	2.4	26.8
35	12	420	5	24	120	0.29	12.6	776	3.3	43.6
40	12	480	5	24	120	0.25	14.4	937	4	60.4
45	12	540	5	24	120	0.22	16.2	1229	5.3	89.4
50	12	600	5	24	120	0.20	18	1526	6.6	123.7
55	12	660	5	24	120	0.18	19.8	1758	7.6	157
60	12	720	5	24	120	0.17	21.6	2187	9.4	213.3
65	12	780	5	24	120	0.15	23.4	2471	10.7	261.5
70	12	840	5	24	120	0.14	25.2	3002	12.9	342.5
75	12	900	5	24	120	0.13	27	3600	15.5	440.5
80	12	960	5	24	120	0.13	28.8	3986	17.2	520.6







Appendix C: Tubular Structure Design

Parameters Governing Frame Design

Height

$n_{\text{story}} := 80$	Number of Stories in Building
$h := 12 \text{ ft}$	Typical Story Height of Building
$H := n_{\text{story}} \cdot h$	Height of Building
$H = 960 \text{ ft}$	

Width

$\text{col}_{\text{space}} := 3.75 \text{ ft}$	Center-to-Center Perimeter Column Spacing
$n_{\text{col}} := 32$	Number of Perimeter Columns
$B := \text{col}_{\text{space}} \cdot n_{\text{col}}$	Width of Building
$B = 120 \text{ ft}$	
$\frac{B}{H} = 0.13$	Aspect Ratio

Performance Parameters

$E := 29000 \text{ ksi}$	Steel Modulus of Elasticity
$\alpha := 400$	Design Deflection Parameter
$f := 1$	Ratio of Diagonal Strain to Chord Strain (Not Applicable in this case)

Summary of Proposed Building

The building under design is $n_{\text{story}} = 80$ stories tall and has a height of,

$H = 960 \text{ ft}$, and a width of, $B = 120 \text{ ft}$, giving an aspect ratio of $\frac{B}{H} = 0.13$.

The Framed Tube design will only have perimeter columns resisting lateral loads.

Frame Design

Lateral Loading

$$w := \begin{cases} 20\text{psf} & \text{if } H \leq 200\text{ft} \\ 21\text{psf} & \text{if } 200\text{ft} < H \leq 300\text{ft} \\ 25\text{psf} & \text{if } 300\text{ft} < H \leq 400\text{ft} \\ 28\text{psf} & \text{if } 400\text{ft} < H \leq 500\text{ft} \\ 31\text{psf} & \text{if } 500\text{ft} < H \leq 600\text{ft} \\ 33\text{psf} & \text{if } 600\text{ft} < H \leq 700\text{ft} \\ 36\text{psf} & \text{if } 700\text{ft} < H \leq 800\text{ft} \\ 39\text{psf} & \text{if } 800\text{ft} < H \leq 900\text{ft} \\ 42\text{psf} & \text{otherwise} \end{cases}$$

Simplified Minimum Wind
Design Pressure
Section 13-52-310
Municipal Code of Chicago, IL

$$b := B \cdot w$$

Uniform Wind Loading on Tube

$$b = 5 \frac{\text{kip}}{\text{ft}}$$

$$V(x) := b \cdot (H - x)$$

Shear Force

$$M(x) := \frac{b \cdot (H - x)^2}{2}$$

Bending Moment

Deflection Characteristics

$$s := \frac{H}{2 \cdot f \cdot B}$$

Dimensionless Parameter

$$\gamma := \frac{1}{(1 + s) \cdot \alpha}$$

Design Shear Deformation

$$\chi := \frac{2 \cdot \gamma \cdot s}{H}$$

Design Bending Deformation

$$u(x) := \gamma \cdot x + \frac{\chi \cdot x^2}{2}$$

Deflection of Building

$$u(H) = 28.8 \text{ in}$$

Maximum Deflection of Building

Design Shear and Bending Rigidity

$$D_T(x) := \frac{V(x)}{\gamma}$$

Total Shear Rigidity

$$D_B(x) := \frac{M(x)}{\chi}$$

Bending Rigidity

$$D_{T\text{tube}} := D_T(h)$$

Shear Rigidity Required

$$D_{B\text{tube}} := D_B(h)$$

Bending Rigidity Required

$$K_{T\text{tube}} := \frac{D_{T\text{tube}}}{h}$$

Shear Stiffness Required

Column Design

$$I_c := \frac{K_{Ttube}}{2n_{col} \left(\frac{4E}{h^3} \right) + 2 \cdot n_{col} \left(\frac{6E}{h^3} \right)}$$

Least Moment of Inertial for a Column to Satisfy Shear Rigidity

$$I_c = 10676 \text{ in}^4$$

$$I_{tube} := \frac{D_{Btube}}{E}$$

Moment of Inertia of the Tube

$$I_{tube} = 2.7 \times 10^9 \text{ in}^4.$$

$$t := B - \left(B^4 - 12 \cdot I_{tube} \right)^{.25}$$

Thickness of Equivalent Tube

$$t = 2.7 \text{ in}$$

$$A_c := t \cdot col_{space}$$

Area of Column to Satisfy Bending Rigidity

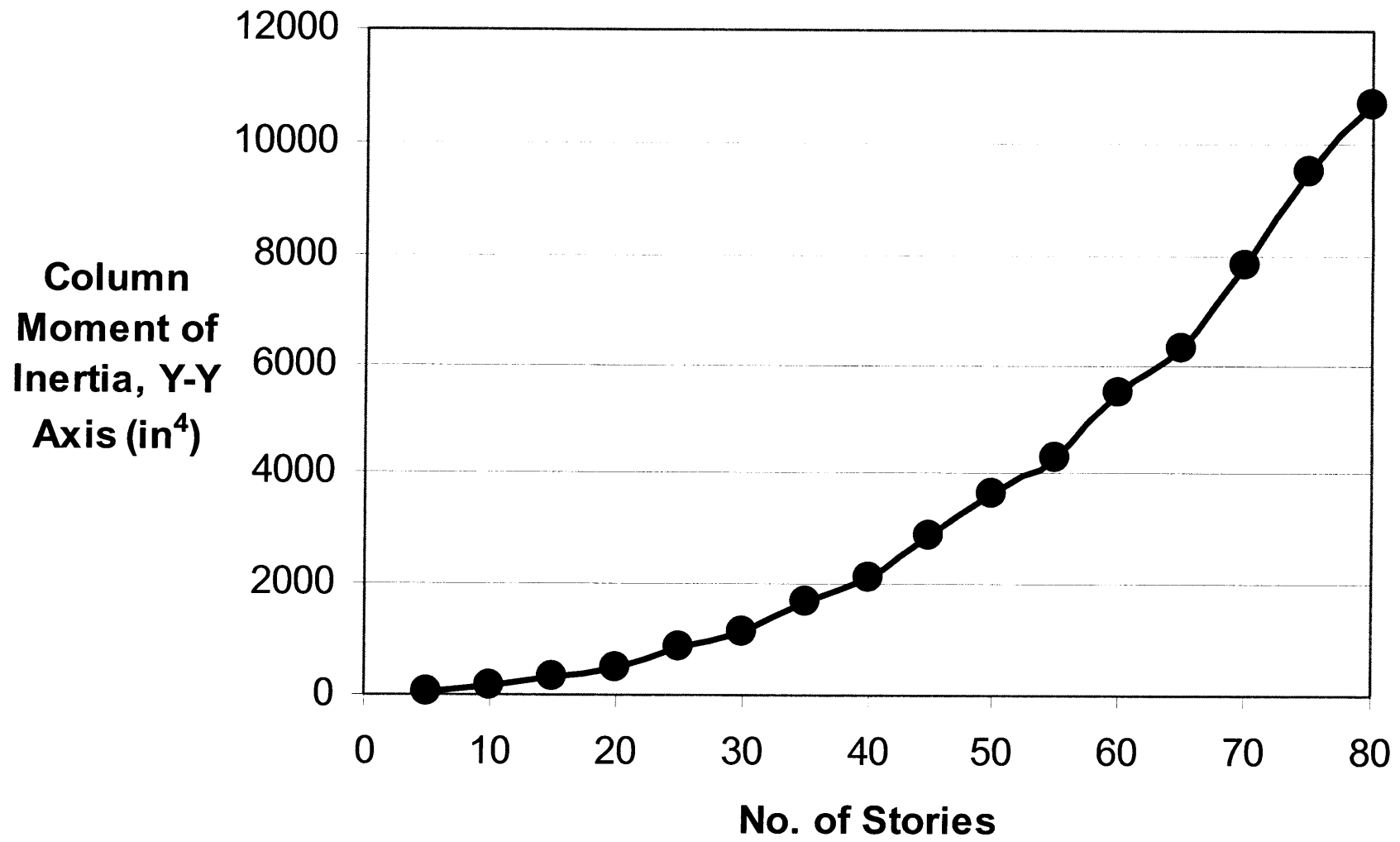
$$A_c = 122.4 \text{ in}^2$$

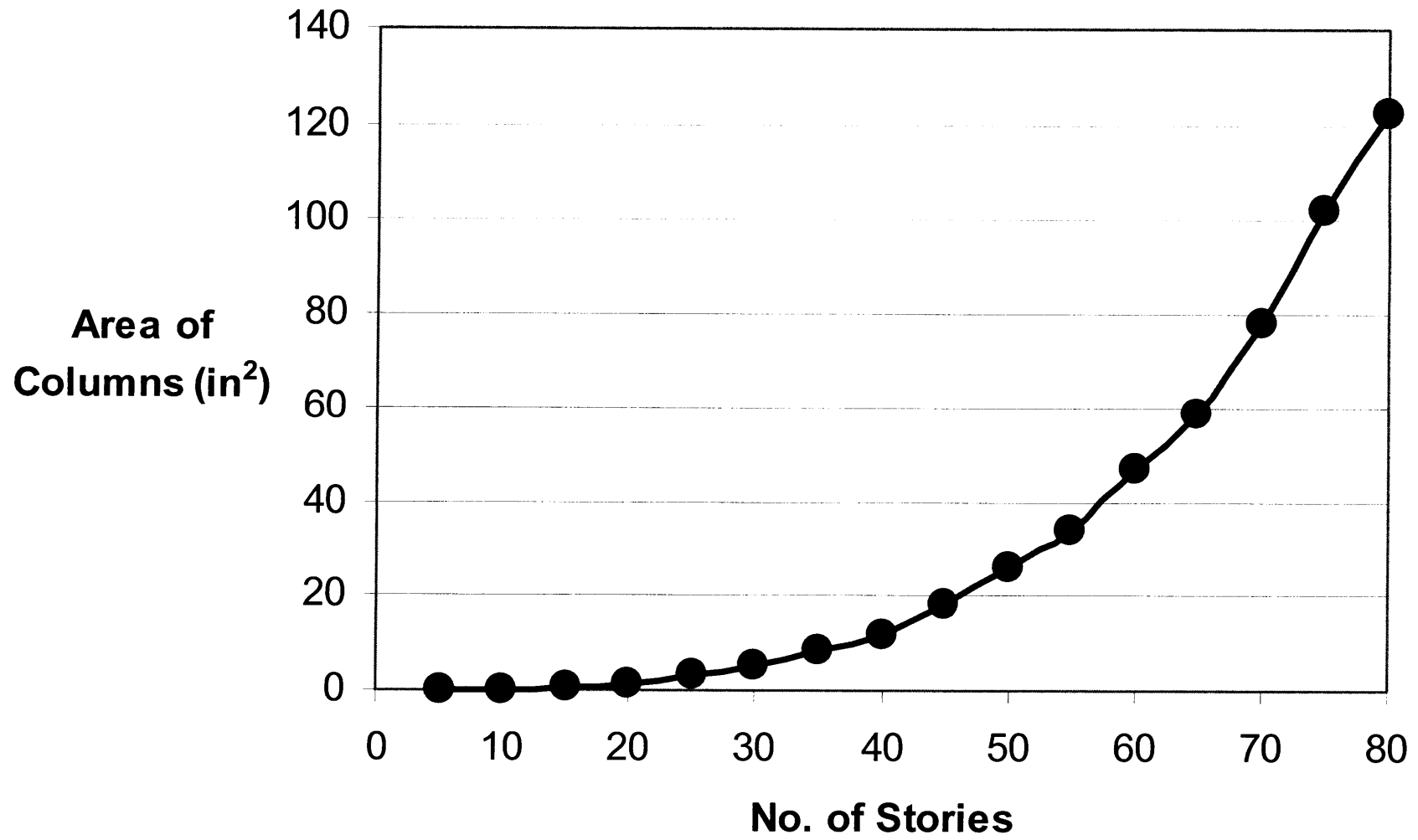
Design Summary

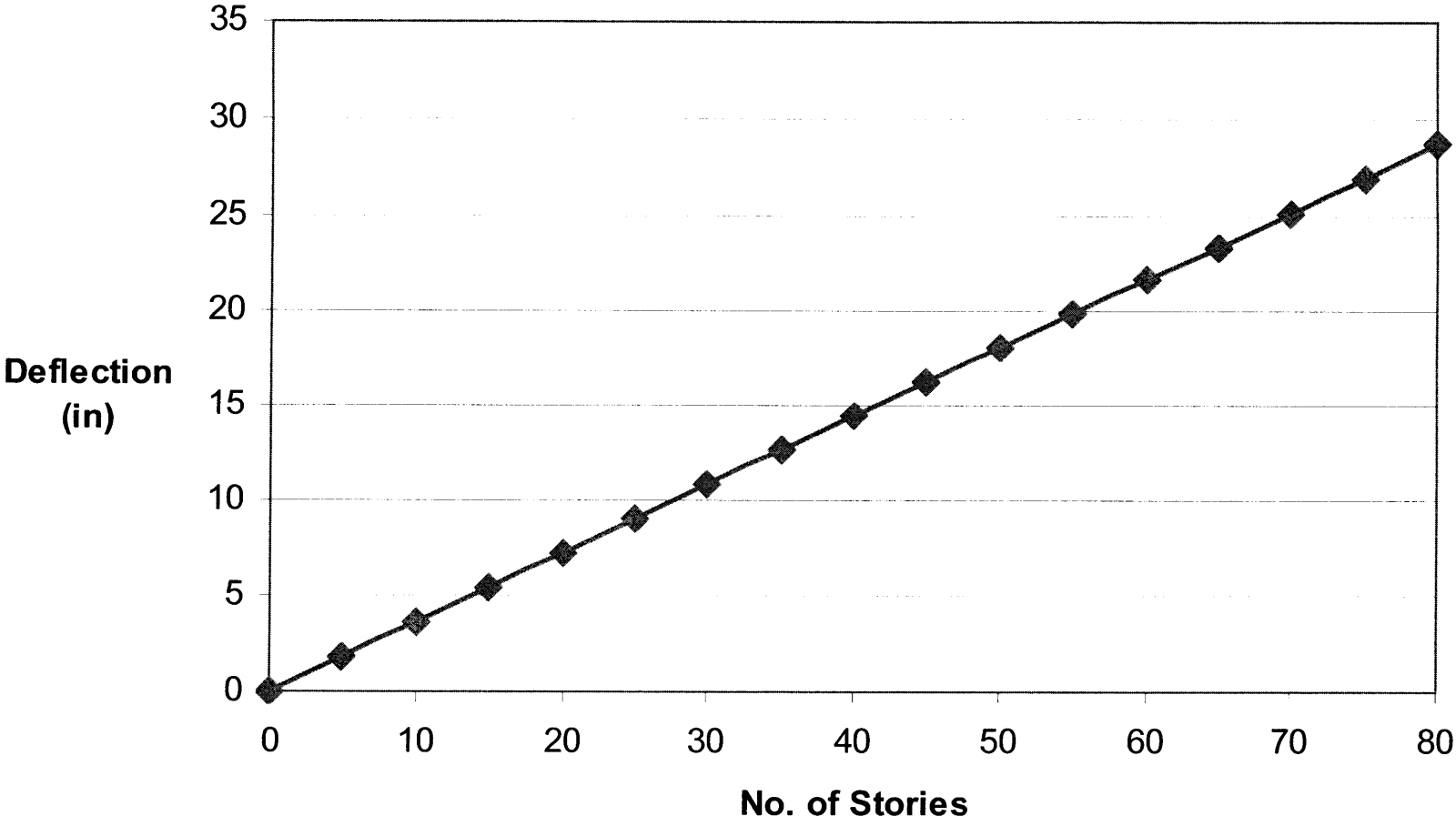
Select structural steel columns that satisfy: Area, $A_c = 122.4 \text{ in}^2$

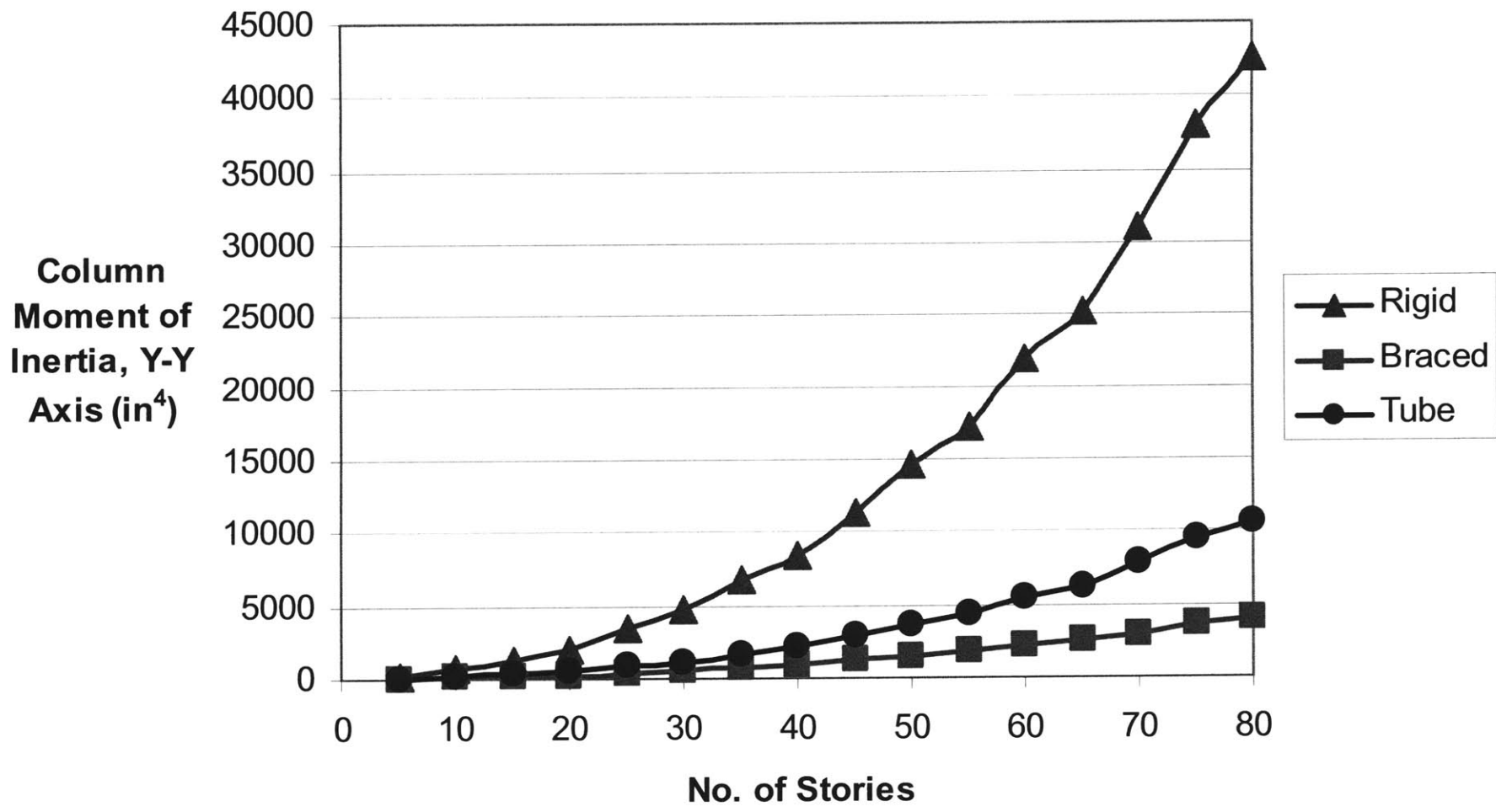
Y-Y axis Moment of Inertia, $I_c = 10676 \text{ in}^4$

No. of Stories	Story Height (ft)	Building Height (ft)	No. of Bays	Bay Width (ft)	Building Width (ft)	Aspect Ratio	Max. Deflection (in)		Shear Rigidity	Bending Rigidity
									Y-Y axis Moment of Inertia (in ⁴)	Area of Column (in ²)
5	12	60	5	24	120	2.00	1.8		64	0
10	12	120	5	24	120	1.00	3.6		174	0.2
15	12	180	5	24	120	0.67	5.4		315	0.6
20	12	240	5	24	120	0.50	7.2		514	1.4
25	12	300	5	24	120	0.40	9		869	3
30	12	360	5	24	120	0.33	10.8		1166	4.9
35	12	420	5	24	120	0.29	12.6		1685	8.3
40	12	480	5	24	120	0.25	14.4		2108	11.9
45	12	540	5	24	120	0.22	16.2		2853	18.2
50	12	600	5	24	120	0.20	18		3642	25.8
55	12	660	5	24	120	0.18	19.8		4300	33.6
60	12	720	5	24	120	0.17	21.6		5467	46.7
65	12	780	5	24	120	0.15	23.4		6301	58.4
70	12	840	5	24	120	0.14	25.2		7793	77.9
75	12	900	5	24	120	0.13	27		9500	102
80	12	960	5	24	120	0.13	28.8		10676	122.4

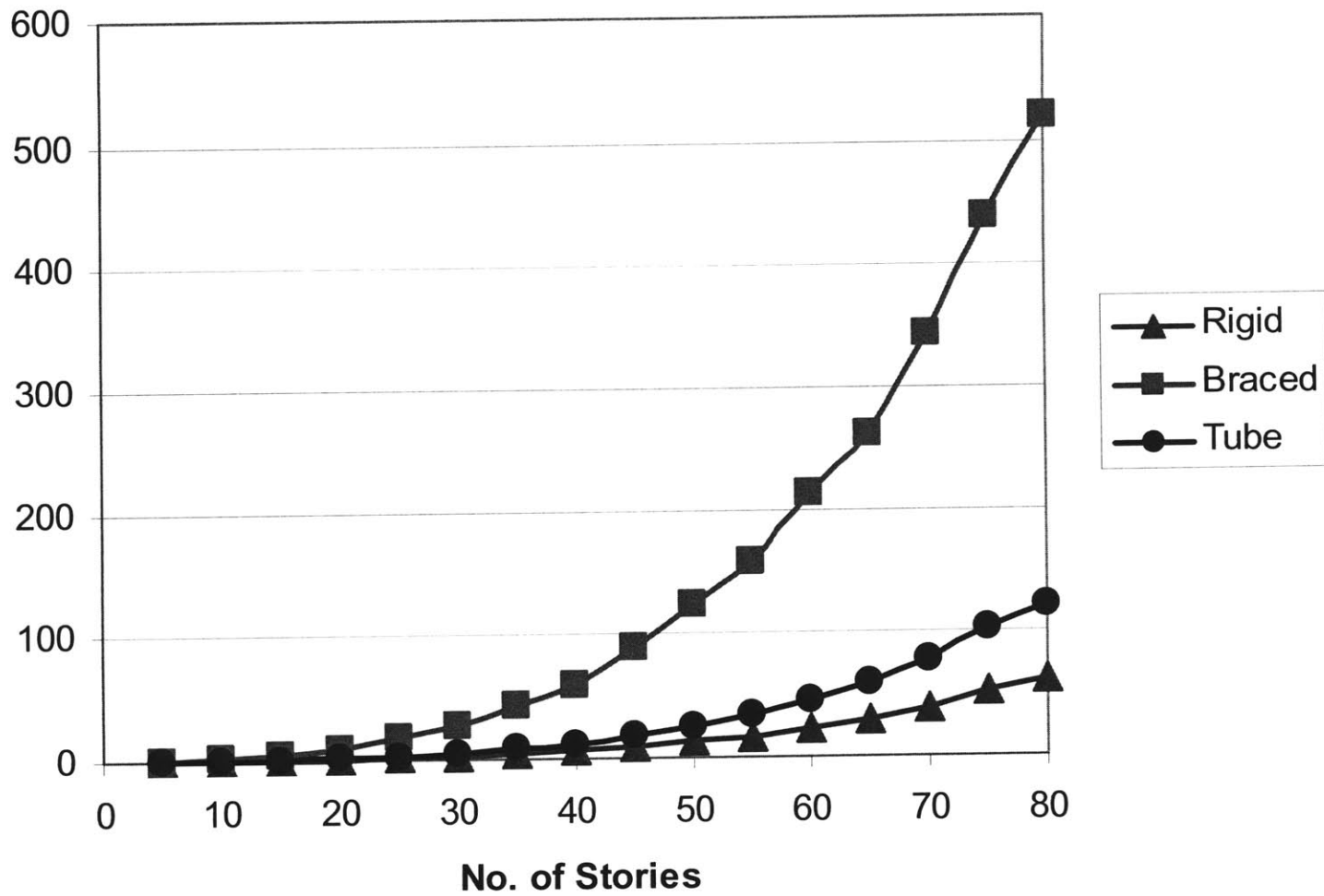








Area of
Column (in²)



“... avoid the temptation of trying spectacular structures where simple structures suffice.”

- Dr. Fazlur Rahman Khan (1929 – 1982)