Essays on Microeconomics of the Household

by

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Abstract

These essays are concerned with the problem of cooperation among individuals in a household and among households in a community under lack of commitment. The first chapter provides a theoretical investigation of consumption patterns in a household in which income is stochastic and some expenditures are public to household members. It is shown that in the constrained efficient agreement, private expenditures of household members do not necessarily co-move, as they do in both the first-best agreement and the constrained efficent agreement when all expenditures are private. In particular, the absence of co-movement of private expenditures in the household do not necessarily imply the absence of mutual insurance; and, indeed, negative correlation in private expenditures can be consistent with a cooperative agreement. These results indicate that caution should be used in interpreting the correlation of private expenditures of household members as a measure of cooperation and mutual insurance within the household.

Chapter two investigates the effect of lack of commitment on household savings in a constrained efficient mutual insurance agreement among different households. It is shown that a saving rule is part of the agreement if and only if risk aversion changes with wealth. If not, no gains can be had from contracting on savings. Under reasonable assumptions about risk preferences, the constrained efficient agreement tends to depress savings to a greater extent for poorer individuals than for richer individuals, thus increasing inequality in consumption and wealth over time relative to the case where savings are not contracted.

Chapter three (co-authored with Harounan Kazianga) provides evidence, using a survey of agricultural households in Burkina Faso, that plots owned by the head of the household is farmed much more intensively than plots, with similar characteristics and planted to the same crops, owned by other household members (of both genders). As in previous studies, this evidence is inconsistent with the assumption of Pareto efficiency in household decisions, but additionally suggests that status within the household rather than gender per se may be the most important factor in determining the allocation of productive resources within the household. We argue that the higher yields achieved by the household head may be explained in terms of social norms that require him to spend the earnings from some farms under his control exclusively on household public goods, as has been observed in the anthropological literature on this region.

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Chapter 1

A Theory of Intra-household Bargaining with Limited Commitment and Public Goods

1.1 Introduction

It is well recognised in the economics literature that public goods play an important role within the household[1]. Members of a household are bound together by their emotional attachments to one another and the close physical proximity in which they live. We are hard-pressed to find examples of such cohesiveness in any other social organisation in the modern economy. Within a family, an important example of public goods are child-related expenditures, which would affect the well-being of both parents. And the satisfaction that parents derive from providing for their children arguably has no equivalent in society. Moreover, members of a household, by definition, live and eat together; such that many of the things they use or consume are inevitably shared among them – a roof, a water faucet, a latrine, and so on – to an extent not seen anywhere else in society. Therefore, it seems reasonable to assert that in terms of the share of total expenditures that are 'public' among its members, the household ranks higher than any other social organisation, and that this marks a crucial difference between the household and other types of social groups.

Recent empirical studies provide considerable evidence of inefficiency in intra-household allocation; and this has increasingly brought attention to the possibility of 'limited commitment' within the household[2]. Broadly, the term relates to the idea that if there are limited means to enforce

[1] See, for example, a survey in Bergstrom (1995) and Deaton (1997).

[2] For example, Duflo and Udry (2003), Goldstein (2002) and Dercon and Krishnan (2000) find, for different parts of Africa, that men and women living in the same household do not fully share risk. Duflo and Udry (2003) suggest that their results may be explained by limited commitment. Udry (1996) finds evidence of inefficiency in the allocation fo

a cooperative agreement among a group of individuals, then this also places limits on what individuals can commit to do in such an agreement. For the most part, the literature on limited commitment has worked with the assumption that all consumption is private. This assumption may be reasonably accurate in the context of informal insurance between separate households in a village. However, it becomes highly questionable when the object is to understand behaviour *within* households. Given the importance of public goods in the latter case, it is important to answer the question whether the presence or absence of public goods makes any important qualitative differences in a model of limited commitment, a question that has not been considered in the literature thus far.

This paper demonstrates that the answer is yes for a reason that is simple and intuitive. Co-movement in consumption is a fundamental feature of full insurance, and of partial insurance under limited commitment when all consumption is private[3]. However, we show that if public goods are present in a household characterised by limited commitment, co-movement in the (private) consumption of household members is not a necessary feature of intra-household insurance. In particular, some household members may experience an *increase* in private consumption when the household suffers an adverse income shock (henceforth, we refer to this result as 'perverse' insurance).

To understand the intuition behind this result, consider the following story. In a rural household, a husband and a wife farm on separate plots of land. They face idiosyncratic risks, so that there is scope for mutual insurance between them. However, because of lack of commitment, they cannot insure each other fully. If all consumption within the household is private, we would observe co-movement in the couple's consumption but a shock to one's own income would have a larger effect on one's own consumption than a shock to the spouse's income. This is the result obtained from the standard model of limited commitment (where all consumption is private).

Now suppose that the couple has children, and any child-related expenditures provide utility to both parents. One year, when the wife has had a particularly bad shock, and the husband is unwilling to provide her full insurance, she cannot feed the children as well as usual. The husband is upset at this for he too cares about the well-being of the children. So as to compensate him, the wife accepts that he would not have to spend money to pave the path to the homestead as they had originally agreed. Thus, he would have more to spend at the local bar this year although the wife has just had a bad shock. Co-movement in consumption has broken down, although the couple is still engaged in a cooperative agreement. It is possible to compensate the husband in this manner because, in a pareto efficient allocation, it must be that his marginal utility from expenditure on private goods is always higher than that on public goods.

This result implies that the absence of co-movement in private consumption shoud not be inter-

productive resources across farm plots owned by the same household.

[3]Diamond (1967) and Wilson (1968) derive the result for the full insurance model. Kocherlakota (1996) obtains the result for efficient risk-sharing in an environment with limited commitment and no public goods.

preted as the absence of intra-household insurance, and furthermore that a *negative* correlation in the private consumption of household members can still be consistent with a cooperative agreement. Some evidence of such negative correlation exists in the empirical literature. Goldstein (2002) finds, for agricultural households in southern Ghana, that the private consumption of husbands and wives are negatively correlated. Dercon and Krishnan (2000) finds for agricultural households in Ethiopia that the nutritional status of at least some household members are positively affected by an adverse shock to household income.

In neither of these papers do the authors provide any possible explanation for their counter-intuitive results. Goldstein interprets the absence of co-movement in the consumption of household members as evidence of the absence of intra-household insurance[4]. Dercon and Krishnan uses the effect of an adverse shock to the household on the nutritional status of individuals, averaged across household members, as a measure of the extent of village-level insurance. The main result in this paper indicate that such interpretations are potentially erroneous.

More generally, the result has the following implication for future work on intra-household insurance. The literature on informal insurance has often used co-movement in the consumption of a group of individuals as a test of the presence and extent of insurance within the group[5]. This practice should not be applied wholesale to measure the extent of insurance within a household since, as this paper argues, co-movement in private consumption is not a necessary feature of informal insurance in an environment where public goods and lack of commitment are important factors.

The discussion above assumes that individuals within the household have independent sources of income over which they have full control rights. For many parts of West Africa, this stylization corresponds well with descriptions in the anthropological literature. A husband and a wife may pursue independent economic activities, with each having little claim over the income generated by the other[6]. At the same time, the household head may have nominal control over certain types of income; but is expected, according to social norm, to use the funds to provide for the entire household[7]. Violating the norm may provoke strong punishment from the community. The existence of such social norms thus makes it possible to commit at least some funds to expenditures on household public goods. Duflo and Udry (2003) also find evidence of such norms in Cote d'Ivoire; specifically, that income from the cultivation of yams is associated with education and food expenditures, but not with alcohol, tobacco or adult clothing.

[4]To test whether household public goods serve as a channel for intra-household insurance, Goldstein also estimates the effect of individual shocks on child-related expenditures. The effect of agricultural shocks on these expenditures are close to zero, which provides some further evidence of the absence of intra-household insurance; however, the effect of illness shocks are imprecisely estimated, such that it is not possible to rule out that adults lower child-related expenditures when they suffer from adverse health shocks.

[5]The seminar work in this literature is Townsend (1994), which rejects the full insurance model for households in rural India, but finds that household consumption co-moves with village average consumption.

[6]See, for example, Guyer (1980) and Oppong (1983).

[7]Duflo and Udry (2003) provide a short survey of the anthropological literature that highlights the importance of such norms in African society.

Given the evidence for such social norms, I consider how a model of limited commitment and public goods is affected by the presence of a separate account out of which all public expenditures are made (to be called 'family' income). The analysis reveals two interesting results. First, a negative shock to the income of either spouse will necessarily lead to a decrease in the private consumption by both individuals in the current period (as in the setting where there are no public goods); however, under plausible assumptions about the outside option, a negative shock to the 'family' income can lead to an increase in the private consumption of an individual in the same period.

I provide some empirical evidence of the effect of income shocks on private consumption for households in Côte d'Ivoire using the Côte d'Ivoire Living Standards Measurement Survey. Following Duflo & Udry (2003), I use variation in rainfall and its differential effects on crops cultivated by men versus women, to obtain exogenous shocks to the incomes of each spouse. It is found that a negative shock to income from crops cultivated by women has a positive and significant effect on private expenditures by the male head of the household (measured as transfers made to his relatives) two years later; furthermore, a negative shock to income from yam cultivation (which, according to the social norm, is for public goods expenditures only[8]) has a positive effect on the private expenditures of the household head during the same year. These effects are unorthodox but consistent with the predictions of the model.

The idea of limited commitment has been explored and developed extensively in the literature. It appears in Kimball (1988) as a possible basis of mutual insurance schemes in a rural setting; it is also central to Thomas and Worrall (1988), which examines long-term wage contracts between a risk-neutral firm and a risk-averse worker where, at any point in time, either party can renege and contract at a spot market wage. Coate and Ravallion (1993) characterise the conditions under which the first-best insurance agreement is self-enforcing, for a setting with two risk-averse agents. For the same setting, Kocherlakota (1996) characterise constrained efficient agreements, and examine the long-run dynamics of such agreements; and Gauthier and Poitevin (1997) show that if agents have the ability to make ex-ante payments, this can lead to improved efficiency for self-enforcing agreements. Ligon, Thomas and Worrall (2002) show that the constrained efficient agreements are characterised by a simple updating rule; specifically, that for each state of nature, there is a time-invariant interval for the ratio of marginal utilities; and in each period, the ratio of marginal utilities adjusts by the smallest amount necessary to bring it into the current interval. Ligon (2002) develops an axiomatic approach to household bargaining where the Nash bargaining solution is modified to capture the idea of limited commitment. Unlike the present paper, this literature has focused on a setting where all consumption is private.

The remainder of this paper proceeds as follows. Section 2 presents the formal model, and Section 3 analyses an extension involving 'family' income. The empirical evidence for the theoretical

[8]See Meillasoux (1965), quoted in Duflo and Udry (2003).

results involving the Côte d'Ivoire Living Standards Survey are to be found in Section 4. Section 5 concludes.

1.2 A Model of Limited Commitment

Consider a household consisting of two individuals, A and B. In each period, the household may devote resources to three different types of goods: x^A, x^B, and z ; x^A is composed of private goods consumed by A and x^B those consumed by B. Both individuals derive utility from expenditure on good z ; i.e. the good is public to the household. The per-period utilities of the two individuals are given by $u^A\left(x^A, z\right)$ and $u^B(x^B, z)$ respectively where u^k is increasing, and strictly concave in x^k and z. The individuals each receive stochastic income streams $\left\{y^A\left(t\right)\right\}_{t=0}^{\infty}$ and $\left\{y^B\left(t\right)\right\}_{t=0}^{\infty}$ which depend on the current state of nature, s, drawn from the set $\{1, 2, ..N\}$. The distribution of s is i.i.d. with $Pr[s(t) = i] = \pi_i$. We assume there is no scope for saving.

We assume that the household is characterised by lack of commitment; i.e. there is no external mechanism to enforce an agreement, and either individual can renege in any period if she finds it in her interest to do so. If an individual does renege on her agreement, then she receives her outside option. For the present analysis, the nature of this outside option is not important. Two realistic possibilities in the context of West African households are divorce and non-cooperative behaviour within the marriage. The value of the outside option to person k in a period where the income levels to A and B are y^A and y^B respectively will be written as

$$d^k\left(y^A, y^B\right) + \beta v_{aut}^k$$

where $d^k\left(y^A, y^B\right)$ is the utility derived from the outside option in the current period, a value which ought to depend on current income levels; v_{aut}^k is the future expected utility from pursuing the outside option and does not depend on current income levels as we have assumed that the income process is independent of history; and β is the discount rate.

Let $\mu^k(y^A, y^B, \lambda)$, where $\lambda \in (0, \infty)$, be the current-period utility that person k receives from the allocation given by the solution to the following maximisation problem:

$$\begin{aligned} \max_{(x^A, x^B, z)} &: \lambda u^A\left(x^A, z\right) + u^B\left(x^B, z\right) \\ \text{s.t.} &: x^A + x^B + z \leq y^A + y^B \end{aligned}$$

We make the following assumptions about the functions $d^A\left(.\right)$ and $d^B\left(.\right)$.

Assumption 1: For each $s \in S$, $\exists\lambda$ such that $\mu^k\left(y_s^A, y_s^B, \lambda\right) > d^k\left(y_s^A, y_s^B\right)$, $k \in \{A, B\}$

Assumption 2: $\mu^A(y^A, y^B, \lambda) - d^A(y^A, y^B)$ is decreasing in y^A and increasing in y^B. Similarly, $\mu^B(y^A, y^B, \lambda) - d^B(y^A, y^B)$ is decreasing in y^B and increasing in y^A.

Assumption 1 ensures that there is at least one way to allocate resources across states that leaves both individuals strictly better off than they are from pursuing their outside option. Assumption 2 says that the allocation given by the joint maximisation of current period utilities for a given pair of Pareto weights becomes more attractive relative to the current period outside option as one's own income declines or as the spouse's income increases.

Given this environment, we shall consider feasible 'agreements' within the household. Formally, an 'agreement' is a plan for allocating funds to each type of good in each period, (potentially) contingent on the current state and the history of past states. An agreement is feasible if neither individual has an incentive to opt for the outside option over the agreement after any possible history.

1.2.1 An Example with Memoryless Agreements

In this section, we analyse 'memoryless' agreements; i.e. agreements in which expenditures are contingent upon the current state of nature but independent of the history of past shocks. In particular, these agreements rule out the possibility that household members borrow from one another to cope with adverse shocks. This restriction is very to be likely unrealistic: allowing for borrowing and lending in an environment with limited commitment should make Pareto improving reallocations possible, and the empirical evidence considered later on suggests that such transactions are indeed taking place within the household. However, this restricted setting is ideal for demonstrating the effect, described in the introduction, that the presence of public goods have on the agreement.

Furthermore, I assume that $\frac{\partial d_A}{\partial y_B} = \frac{\partial d_B}{\partial y_A} = 0$; i.e. a change in the income of one's partner has no effect on one's own utility from the current period in autarky. This condition may be unrealistic given the presence of public goods on which one or both partners may spend money in a noncooperative equilibrium. However, the assumption simplifies the exposition. Both restrictions are relaxed in the following section, where the analysis is carried out in a more general setting.

A memoryless agreement can be written as $\left\{x_s^A, x_s^B, z_s\right\}_{s \in S}$, where the expenditure levels x_s^A, x_s^B and z_s are prescribed in each period that the realised state of nature is s. If the environment is characterised by limited commitment, then such an agreement is feasible if and only if, for each state of nature, the continuation value each individual receives from the agreement is at least as large as that obtained from the outside option. We can write the feasibility constraints as follows:

$$\begin{aligned} u^i\left(x_s^i, z_s\right) + \frac{\beta}{1+\beta}\sum_{r\in S}\pi_r u^i\left(x_r^i, z_r\right) &\geq d^i\left(y_s^i, y_s^{-i}\right) + \beta v_{aut}^i \\ \text{for each } s &\in S, i \in \{A, B\} \end{aligned}$$

Let $M(v)$ be the maximum ex-ante utility that B can obtain from a memoryless agreement if the environment is characterised by limited comitment and the ex-ante utility received by A is at least v. Then $M(v)$ is given by

$$\max_{\{x_s^A, x_s^B, z_s\}} : \frac{1}{1+\beta} \sum_{r \in S} \pi_r u^B \left(x_r^B, z_r\right) \tag{1.1}$$

subject to

$$\lambda : \frac{1}{1+\beta} \sum_{r \in S} \pi_r u^A \left(x_r^A, z_r\right) \geq v \tag{1.2}$$

and for each $s \in S$,

$$\theta_s^A \quad : \quad u^A \left(x_s^A, z_s\right) + \frac{\beta}{1+\beta} \sum_{r \in S} \pi_r u^A \left(x_r^A, z_r\right) \geq d^A \left(y_s^A, y_s^B\right) + \beta v_{aut}^A \tag{1.3}$$

$$\theta_s^B \quad : \quad u^B \left(x_s^B, z_s\right) + \frac{\beta}{1+\beta} \sum_{r \in S} \pi_r u^B \left(x_r^B, z_r\right) \geq d^B \left(y_s^A, y_s^B\right) + \beta v_{aut}^B \tag{1.4}$$

$$\tau_s \quad : \quad x_s^A + x_s^B + z_s \leq y_s^A + y_s^B \tag{1.5}$$

The condition in (1.2) ensures that person A receives an ex-ante utility of at least v. The conditions in (1.3) and (1.4) ensure that in each possible state, the utility levels A and B receive from the agreement are at least as large as that obtained from the outside option. The budget constraints, one for each state, are in (1.5). From the first-order conditions, we obtain

$$\begin{aligned} & \frac{1}{1+\beta} u_1^B (..) + \theta_s^B \left(1 + \frac{\beta \pi_s}{1+\beta}\right) u_1^B (..) \\ = & \frac{\lambda}{1+\beta} u_1^A (..) + \theta_s^A \left(1 + \frac{\beta \pi_s}{1+\beta}\right) u_1^A (..) \\ = & \frac{1}{1+\beta} \left[u_2^B (..) + \lambda u_2^A (..)\right] + \theta_s^B \left(1 + \frac{\beta \pi_s}{1+\beta}\right) u_2^B (..) + \theta_s^A \left(1 + \frac{\beta \pi_s}{1+\beta}\right) u_2^A (..) \end{aligned}$$

where $u_1^i (..)$ and $u_2^i (..)$ are the marginal utility to person i from the private and the public good respectively.

If the participation constraints do not bind for some state r, then $\theta_r^A = \theta_r^B = 0$ and

$$u_1^B (..) = \lambda u_1^A (..) = u_2^B (..) + \lambda u_2^B (..)$$

Thus, the first-order conditions are the same in each state that the participation constraint does not bind. It is straightforward to show that, for such states, the larger is aggregate income, the more is spent on each type of good: i.e. if states r and s are such that $y_s^A + y_s^B > y_r^A + y_r^B$, and $\theta_r^i = \theta_s^i = 0$ for $i = A, B$, then $x_s^A > x_r^A, x_s^B > x_r^B$ and $z_s > z_r$.

If person A's participation constraint binds, and that of person B is slack in state s, then $\theta_s^A > 0, \theta_s^B = 0$ and

$$u_1^B (..) = \lambda_s u_1^A (..) = u_2^B (..) + \lambda_s u_2^B (..)$$

where $\lambda_s = \lambda + \theta_s^A (1 + \beta + \beta\pi_s) > \lambda$.

Furthermore, using the fact that the condition in (1.3) is satisfied with equality, and the promised value to A from the agreement equals v in each period, we have

$$u^A(x_s^A, z_s) + \beta v = d^A(y_s^A, y_s^B) + \beta v_{aut}^A$$

Then the allocation of resources in state s prescribed by the agreement must correspond to the solution of the following problem:

$$\begin{aligned} \max_{x^A, x^B, z} &: u^B(x^B, z) \\ \text{subject to} &: \\ &: u^A(x^A, z) \geq d^A(y^A, y^B) - \beta(v - v_{aut}^A) \\ &: x^A + x^B + z \leq y^A + y^B \end{aligned} \tag{1.6}$$

when $y^A = y_s^A, y^B = y_s^B$.

If not, there is a possible reallocation of resources in state s that satisfies person A's promise-keeping and participation constraints and gives person B a strictly higher continuation value, which is ruled out by definition.

The following lemma enables us to compare allocations across states in which person A's participation constraint is binding.

Lemma 1.1 *Let $x^A(y^A, y^B), x^B(y^A, y^B), z(y^A, y^B)$ be the solution to the maximisation problem described in (1.6). Then $x^A(y^A, y^B)$ is decreasing in y^B; and $x^B(y^A, y^B)$ and $z(y^A, y^B)$ are increasing in y^B.*

See Appendix for proof.

Suppose the states r and s are such that $y_r^B < y_s^B, y_r^A = y_s^A$. Then, using Lemma 1.1, $x^A(y_r^A, y_r^B) > x^A(y_s^A, y_s^B)$. If the participation constraint of person A is binding in both states, $x_r^A = x^A(y_r^A, y_r^B)$, $x_s^A = x^A(y_s^A, y_s^B) \implies x_r^A > x_s^A$. We have thus established the following proposition.

Proposition 1.1 *For an agreement that lies on the frontier of the set of feasible and memoryless agreements, if the states r and s are such that $y_r^B < y_s^B, y_r^A = y_s^A$, $\frac{\partial d_A}{\partial y_B} = 0$, and person A's participation constraint binds in both states, then his prescribed private expenditures are such that $x^A(y_r^A, y_r^B) > x^A(y_s^A, y_s^B)$.*

It is straightforward to demonstrate that this result cannot hold if there are no household public goods from which the individuals derive utility. To see this, let $u^A(x^A, z) = \tilde{u}^A(x^A)$, $u^B(x^B, z) = \tilde{u}^B(x^B)$; such that A and B do not derive any utility from the public good. In autarky, each individual spends all of her income on the private good. Therefore, $d^A\left(y^A, y^B\right) = \tilde{u}^A\left(y^A\right)$; $d^B\left(y^A, y^B\right) = \tilde{u}^B\left(y^B\right)$. If person A's participation constraints bind in states r and s as above, then

$$\begin{aligned}\tilde{u}^A\left(x_s^A\right) + \beta v &= \tilde{u}^A\left(y_s^A\right) + \beta v_{aut}^A \\ \tilde{u}^A\left(x_r^A\right) + \beta v &= \tilde{u}^A\left(y_r^A\right) + \beta v_{aut}^A\end{aligned}$$

$$\Longrightarrow \tilde{u}^A\left(x_s^A\right) - \tilde{u}^A\left(x_r^A\right) = \tilde{u}^A\left(y_s^A\right) - \tilde{u}^A\left(y_r^A\right)$$

As $y_r^A = y_s^A$, we obtain $\tilde{u}^A\left(x_s^A\right) = \tilde{u}_A\left(x_r^A\right) \implies x_s^A = x_r^A$.

Thus, in the absence of public goods, if person A's participation constraint binds in states r and s, and he has the income in these two states, then he also consumes the same amount in states r and s, in a memoryless agreement.

1.2.2 History-dependent Agreements

In this section, we analyse agreements where the allocation of expenditures can be contingent on the current state as well as on the past sequence of states. Let H_t be the set of all possible sequence of states $(s_1, s_2.., s_t)$ in period t. Then a history-dependent agreement can be described fully by a sequence of functions $\left\{x_t^A(.), x_t^B(.), z_t(.)\right\}_{t=1..\infty}$ where $x_t^A : H_t \longrightarrow R_+$ specificies the sum of money allocated to person A's private goods for each possible history in period t, and x_t^B and z_t are defined similarly.

For the agreement to be feasible in an environment with limited commitment, it must have the feature that at each date and at every contingency, each member receives as much utility from the agreement as they do from their outside option; i.e.

$$u^k\left(x_t^k(h_t), z_t(h_t)\right) + \beta E_t \sum_{\tau=1}^{\infty} \beta^{\tau-1} u_{t+\tau}^k\left(x^k(h_{t+\tau}), z_{t+\tau}(h_{t+\tau})\right) \geq d^k\left(y_s^A, y_s^B\right) + \beta v_{aut}^k$$

for each $h_t \in H_t$, $k = A, B$, and $t = 0, 1, ..\infty$.

On the left-hand side, the first expression is the utility obtained from the agreement in period t if the realised history is h_t, and the second expression is the discounted expected future utility from the agreement. The right-hand side is the value of the outside option in the current period, assuming that the realised state in period t is s.

In the case of history-dependent agreements, each possible history in each period yields a distinct constraint for each individual. As the set of possible histories grows exponentially with t, it is not possible to adopt the approach used in the previous section to analyse such agreements. Fortunately, the problem is very similar to that considered in Kocherlakota (1996), and the same recursive approach is applicable here. Define $P(v)$ as the maximum ex-ante utility that person B can obtain from a history-dependent agreement if person A must receive a utility of at least v. Then $P(v)$ satisfies the following Bellman equation:

$$P(v) = \max_{\{x_s^A, x_s^B, z_s, w_s\}} \sum_{s=1}^{N} \left[\pi_s u^B\left(x_s^B, z_s\right) + \beta P\left(w_s\right)\right] \tag{1.7}$$

subject to

$$\lambda : \sum_{s=1}^{N} \left[\pi_s u^A\left(x_s^A, z_s\right) + \beta w_s\right] \geq v \tag{1.8}$$

and for each $s \in S$,

$$\theta_s^A \; : \; u^A\left(x_s^A, z_s\right) + \beta w_s \geq d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A \tag{1.9}$$

$$\theta_s^B \; : \; u_B\left(x_s^B, z_s\right) + \beta P\left(w_s\right) \geq d^B\left(y_s^A, y_s^B\right) + \beta v_{aut}^B \tag{1.10}$$

$$\tau_s \; : \; x_s^A + x_s^B + z_s \leq y_s^A + y_s^B \tag{1.11}$$

The formulation above reduces the problem of finding a complete contingency plan which is constrained efficient and awards person A an ex-ante utility of v to the much simpler task of choosing, for each possible state in the current period, an allocation of expenditures x_s^A, x_s^B, z_s and the ex-ante utility (or promised value) w_s with which A would enter the following period. The expression in the maximand is, by definition, the maximum possible utility that B can obtain from such an allocation. The condition in (1.8) ensures that the allocation leaves A with an ex-ante utility of at least v. The conditions in (1.9) and (1.10) ensure that, in each state, A and B receive as much utility from the agreement as they would from their outside options. The budget constraints are in (1.11).

Following the reasoning in Ligon, Thomas and Worrall (2001), it is possible to show that the function $P(.)$ is strictly concave. From the first-order conditions to the problem and the envelope condition, we obtain, for each $s \in S$,

$$P'\left(w_s\right) = \frac{\pi_s P'(v) - \theta_s^A}{\pi_s + \theta_s^B} \tag{1.12}$$

and

$$u_1^B(..) = \lambda_s u_1^A(..) = u_2^B(..) + \lambda_s u_2^A(..) \tag{1.13}$$

where $\lambda_s = -P'\left(w_s\right)$.

Note that the conditions in (1.13) for state s are equivalent to the first-order conditions of the following maximisation problem:

$$\max_{x^A, x^B, z} \lambda u^A\left(x^A, z\right) + u^B\left(x^B, z\right) \tag{1.14}$$
$$\text{subject to:} \quad x^A + x^B + z \leq y^A + y^B$$

when $y^A = y_s^A$, $y^B = y_s^B$, and $\lambda = \lambda_s$. Therefore, λ_s is the weight given to A's preferences in state s in allocating resources among the three types of goods; and the current utility obtained by A and B can be written as $\mu^A\left(y_s^A, y_s^B, \lambda_s\right)$ and $\mu^B\left(y_s^A, y_s^B, \lambda_s\right)$ respectively. Thus, the allocation of resources and the promised future utilities in a state s can be described fully by the pair $(\lambda_s, w_s) = (-P'(w_s), w_s)$. To see how (λ_s, w_s) varies across states for a given v, it is convenient to group the states according to the following three categories:

(i) Let $S^A(v)$ be the set of states for which the pair (λ, v) violates person A's participation constraint; i.e.

$$S^A(v) = \left\{s \in S : \mu^A\left(y_s^A, y_s^B, \lambda\right) + \beta v < d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A\right\}$$

where $\lambda = -P'(v)$.

(ii) Similarly, let $S^B(v)$ be the set of states for which the pair (λ, v) violates B's participation constraint:

$$S^B(v) = \left\{s \in S : \mu^B\left(y_s^A, y_s^B, \lambda\right) + \beta P(v) < d^B\left(y_s^A, y_s^B\right) + \beta v_{aut}^B\right\}$$

(iii) Let $S^0(v)$ be the set of states for which the pair (λ, v) satisfies the constaints of both individuals:

$$S^0(v) = \left\{\begin{array}{c} s \in S : \mu^A\left(y_s^A, y_s^B, \lambda\right) + \beta v \geq d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A \\ \text{and } \mu^B\left(y_s^A, y_s^B, \lambda\right) + \beta P(v) \geq d^B\left(y_s^A, y_s^B\right) + \beta v_{aut}^B \end{array}\right\}$$

The three categories exhaust all possibilities. Further, because of Assumption 1, the first two categories are mutually exclusive. Then, following the reasoning of Proposition 1 in Ligion, Thomas and Worrall (2002), it is straightforward to establish the following result.

Lemma 1.2 *If, in a constrained efficient agreement, person A is awarded a utility of* v*, then (i) for* $s \in S^A(v)$, $\theta_s^A > 0, \theta_s^B = 0$ *and* $\lambda_s > \lambda, w_s > v$*; (ii) for* $s \in S^B(v)$, $\theta_s^A = 0, \theta_s^B > 0$ *and* $\lambda_s < \lambda, w_s < v$*; and (iii) for* $s \in S^0(v)$, $\theta_s^A = \theta_s^B = 0$ *and* $\lambda_s = \lambda, w_s = v$.

See Appendix for Proof.

Thus, in the constrained efficient agreement, the allocation of resources across states follow a simple and intuitive rule. If the pair $(\lambda, v) = (-P'(v), v)$ causes the participation constraint of either individual to be violated in a particular state s, then that individual receives a higher utility both in the current period and in the future so that his constraint is just satisfied. If (λ, v) does not violate either participation constraint in some state s, then the weights and the promised utilities in that state are unchanged from the preceding period.

Comparing expenditures across states which belong to $S^0(v)$ is straightforward. As the weights used for A's and B's preferences to allocate resources are the same in all these states, expenditure on each type of good is increasing in aggregate income: if $r, s \in S^0(v)$, and $y_r^A + y_r^B > y_s^A + y_s^A$ then $x_r^A > x_r^A$, $x_r^B > x_r^B$, and $z_r > z_s$.

To compare expenditures across states which belong to $S^A(v)$ (and, similarly, $S^B(v)$) we proceed as follows. If $s \in S^A(v)$, the allocation in that state is given by the solution to the following maximisation problem:

$$\begin{aligned} \max_{x^A, x^B, z, w} &: u^B(x^B, z) + \beta P(w) \\ \text{s.t.} &: \\ \lambda &: u^A(x^A, z) = d^A(y^A, y^B) - \beta(w - v_{aut}^A) \\ \tau &: x^A + x^B + z \leq y^A + y^B \end{aligned} \tag{1.15}$$

with $y^A = y_s^A$, $y^B = y_s^B$.

To see this, note, first, that the solution to (1.15) yields an allocation that is feasible: by construction, it satisfies person A's participation constraints; and it must also satisfy person B's participation constraints given the assumption that there is at least one allocation in each period that leaves both individuals at least as well off as in autarky. Furthermore, an agreement that is constrained efficient must solve the maximisation problem in (1.15). If not, there is a possible reallocation that leaves person A at least as well-off and provides strictly higher utility to person B.

Note that the solution to (1.15) depends on the current state of nature s but is independant of v, the promised value in the preceding period. This means that when the participation constraint binds for either individual at a point in time, we ignore the past history of states, and the allocation in that period is contingent only on the current state. Therefore, if person A's participation constraint binds in state s after any history, we can denote this allocation by $x_s^A[A], x_s^B[A], z_s[A], w_s[A]$. Similarly, if person B's participation constraint binds in state r after any history, we can denote this allocation by $x_s^A[B], x_s^B[B], z_s[B], w_s[B]$.

Compared to the maximisation problem in (1.6), which related to 'memoryless' agreements, there is an additional choice variable, w, in (1.15). In the former case, the promised value to each individual remained fixed over time, as we were considering a type of agreement that ignored the past sequence of events. In the latter case, the promised values can change over time, and serves as an additional mechanism to ensure that the participation constraints of each individual are satisfied in each period. Intuitively, this additional mechanism should dampen the effect – of raising private expenditures at the expense of public expenditures in order to satisfy participation constraints –

noted in the previous section.

However, the lack of commitment also imposes restrictions on the extent to which individuals can "borrow" from one another. It may not be possible to promise a high level of future consumption to one individual as this makes it tempting for the partner to renege on the agreement. In particular, if the individuals are very impatient, (i.e. β is low), they do not value compensation that is received in the future. In addition, this impatience limits the amount that one can promise to 'pay back'. Thus, when impatience is high, we are more likely to observe situations where compensation takes the form of increased private expenditures in the current period at the expense of public expenditures. This argument is formalised in the following proposition.

Proposition 1.2 *If the states r and s are such that $y_r^A = y_s^A$, $y_r^B < y_s^B$, then $z_r[A] < z_s[A]$, $x_r^B[A] < x_s^B[A]$, and $w_r[A] > w_s[A]$. Furthermore, there exists a $\beta^* > 0$ such that for $\beta < \beta^*$, and $\frac{\partial d^A}{\partial y^B}$ sufficiently small, $x_r^A[A] > x_s^A[A]$.*

See Appendix for proof.

Person B is poorer in state r and would thus like to receive a larger transfer in this state. The proposition states that if β is sufficiently small, person A is compensated for this additional transfer, to a large extent, in the current period (by allowing him to use money, assigned to public expenditures in state s, for his private consumption); thus raising his private expenditures above their level in state s. Note that for this 'perverse' insurance effect to take place, we also require that person A's outside option is not significantly worse in state r compared to state s; i.e. $\frac{\partial d^A}{\partial y^B}$ is required to be small. This is because, if person A's outside option is significantly less attractive in the state that B is poorer, it would not be necessary to 'bribe' A to tempt him to remain in the agreement in this state.

This comparison of allocations across states translates easily into a comparison of allocations over time. Suppose in period t, the realised state is s and this is followed in period $t+1$ by state r. If the participation constraint of person A is binding in period t, it must bind again in period $t+1$ (see Proposition 1.5 in the appendix). Therefore, his private expenditures is $x_s^A[A]$ in period t followed by $x_r^A[A]$ in period $t+1$. Thus, we have the following corollary to Proposition 1.2.

Corollary 1.1 *If the participation constraint is binding for A in state s, period t and the state of nature, r in period $t+1$ is such that $y_r^A = y_s^A$, $y_r^B < y_s^B$, then $x^B(t+1) < x^B(t)$, $z(t+1) < z(t)$ and $\exists \beta^* > 0$ s.t. for $\beta < \beta^*$, $x^A(t+1) > x^A(t)$.*

It is straightforward to show that Proposition 1.2 cannot hold if there are no public goods. As in the previous section, let $u^A(x^A, z) = \tilde{u}^A(x^A)$, $u^B(x^B, z) = \tilde{u}^B(x^B)$; $d^A(y^A, y^B) = \tilde{u}^A(y^A)$; $d^B(y^A, y^B) = \tilde{u}^B(y^B)$. As $y_r^A = y_s^A$, the outside option to person A is the same in states r and s. Therefore, the utility awarded to person A when his participation constraint binds, must also be the same in both states; i.e.

$$\tilde{u}^A(x_r^A[A]) + \beta w_r[A] = \tilde{u}^A(x_s^A[A]) + \beta w_s[A] \tag{1.16}$$

We can show by contradiction that $w_r[A] \geq w_s[A]$. If $w_r[A] < w_s[A]$, then the concavity of $P(.)$ implies that $-P'(w_r) < -P'(w_s)$. Therefore, the pareto weights are less favourable to A in state r; this statement along with the fact that aggregate income is also smaller in state r would imply that $x_r^A[A] < x_s^A[A]$. Then, $\tilde{u}^A\left(x_r^A[A]\right) + \beta w_r[A] < \tilde{u}^A\left(x_s^A[A]\right) + \beta w_s[A]$, which contradicts (1.16). Therefore, $w_r[A] \geq w_s[A]$. From (1.16), we obtain

$$\tilde{u}^A\left(x_r^A[A]\right) - \tilde{u}^A\left(x_s^A[A]\right) = \beta\left(w_s[A] - w_r[A]\right)$$

$$\Longrightarrow x_r^A[A] \leq x_s^A[A]$$

Thus, if no public goods are present, and person A's participation constraint binds in states r and s, with person B poorer in state r, then A's level of private consumption can be no larger in state r than it is in state s.

1.3 A Separate Account for Public Expenditures

Anthropologists writing about African households often emphasize the importance of social norms that restrict the use of certain types of income to household public goods. For example, Meillasoux (1965) – quoted by Duflo and Udry (2003) – observes that among the Gouro in Cote d'Ivoire, income from "'appreciated products' are always under the control of the household head for redistribution to the entire household in the form of food." In this section, I investigate the effect of social norms of this kind on a model of limited commitment with public goods. Specifically, I introduce an additional source of income for the household, to be called 'family' income; and consider agreements whereby this income is used for household public expenditures only.

Denote by $\left\{y^Z(t)\right\}_{t=0}^{\infty}$ an income stream, to be called 'family' income available to the household in addition to the private incomes $\left\{y^A(t)\right\}_{t=0}^{\infty}$ and $\left\{y^B(t)\right\}_{t=0}^{\infty}$. We assume that the social norm is to spend all of the family income on public goods in each period. Furthermore, we assume that the family income is sufficiently large in any state that individuals A and B would not wish to supplement it with their private incomes either in the agreement or in autarky[9]. Let $\tilde{d}^k\left(y^k, y^Z\right)$ be the utility that person k receives from his outside option when his private income and the family income are y^k and y^Z respectively. It is reasonable to suppose that the spouse's income would not enter this function since, in autarky, she would spend all her private income on her private goods. Let $\tilde{\mu}^k(y^A, y^B, y^Z, \lambda)$, where $\lambda \in (0, \infty)$, be the current-period utility that person k receives from

[9] The anthropological descriptions regarding such social norms and also the evidence in Duflo and Udry (2003) are consistent with this assumption.

the allocation given by the solution to the following maximisation problem:

$$\begin{aligned}\max_{(x^A,x^B)} &: \lambda u^A\left(x^A,y^Z\right)+u^B\left(x^B,y^Z\right)\\ \text{s.t.} &: x^A+x^B\le y^A+y^B\end{aligned}$$

We make the following assumptions about the functions $\tilde{d}^A(..)$ and $\tilde{d}^B(..)$.

Assumption 1F: For each $s\in S$, $\exists\lambda$ such that $\tilde{\mu}^k\left(y_s^A,y_s^B,y^Z,\lambda\right)>d^k\left(y_s^k,y_s^Z\right)$, $k\in\{A,B\}$

Assumption 2F: $\tilde{\mu}^A\left(y^A,y^B,y^Z,\lambda\right)-\tilde{d}^A\left(y^A,y^Z\right)$ is decreasing in y^A and increasing in y^Z; and likewise $\tilde{\mu}^B\left(y^A,y^B,y^Z,\lambda\right)-\tilde{d}^B\left(y^A,y^Z\right)$ is decreasing in y^B and increasing in y^Z.

Assumption 1F is equivalent to Assumption 1 in section 1.2. The first part of Assumption 2F is also unchanged from that in Assumption 2. The second part of Assumption 2F says that as the level of family income increases, income pooling for any given set of pareto weights becomes more attractive relative to the outside option for both individuals. The story behind such an assumption is that a fraction of the family income is misallocated or wasted in autarky; therefore, the cost of switching to autarky becomes larger when the family income increases.

Let $\tilde{P}(v)$ be the maximum ex-ante utility that can be awarded to person B from an agreement that satisfies the participation constraints and follows the social norm, when the ex-ante utility to person A is at least as large as v. Then

$$\tilde{P}(v)=\max_{\{x_s^A,x_s^B,w_s\}}\sum_{s\in S}\left[\pi_s u^B\left(x_s^B,y_s^Z\right)+\beta\tilde{P}\left(w_s\right)\right]$$

subject to

$$\lambda:\sum_{s\in S}\left[\pi_s u^A\left(x_s^A,y_s^Z\right)+\beta w_s\right]\ge v$$

and for each $s\in S$,

$$\theta_s^A \;:\; u^A\left(x_s^A,y_s^Z\right)+\beta w_s\ge\hat{d}^A\left(y_s^A,y_s^Z\right)+\beta v_{aut}^A \tag{1.17}$$

$$\theta_s^B \;:\; u^B\left(x_s^B,y_s^Z\right)+\beta\tilde{P}\left(w_s\right)\ge\hat{d}^B\left(y_s^B,y_s^Z\right)+\beta v_{aut}^B \tag{1.18}$$

$$\tau_s \;:\; x_s^A+x_s^B\le y_s^A+y_s^B \tag{1.19}$$

An agreement that follows the social norm will specify only the private consumption of A and B in each period, and as such, is similar to one in a setting where no public goods exist. In fact, if the level of family income is non-varying over time, then the programme above is equivalent to that for constrained efficient agreements in an environment with private goods only. In particular, it is straightforward to establish the following result.

Proposition 1.3 *If the participation constraint for A is binding in state r, period t and the state of nature, s in period $t+1$ is such that $y_s^A=y_r^A$, $y_s^Z=y_r^Z$, $y_s^B<y_r^B$, then $w(t+1)>w(t)$ and*

$x^A(t+1) < x^A(t)$.

The proposition states that if the participation constraint is binding for person A in period t and B suffers an adverse shock in period $t+1$ (and A's private income and the family income remain at their former levels) then, as in the case without public goods, we do not observe 'perverse' insurance: person A has a lower level of private consumption in period $t+1$ but a higher future promised utility as compensation. In section 2, we noted that it is possible to slacken the participation constraint of either individual at a point in time by enabling him to raise private consumption at the expense of public goods. As modelled here, the social norm prescribes a separate account for public expenditures, and thus makes it impossible to use public goods in this manner in an agreement.

Because of Assumption 2F, changes in the level of family income will affect the participation constraints of both individuals and, as such, can also affect the private expenditures prescribed by a constrained efficient agreement. In particular, we can establish the following proposition.

Proposition 1.4 *If the participation constraint is binding for A in state r, period t and the state of nature, s in period t + 1 is such that* $y_s^A = y_r^A$, $y_s^B = y_r^B$, $y_s^Z < y_r^Z$, *then* $w(t+1) > w(t)$ *and* $x^A(t+1) > x^A(t)$.

The proposition states that if the participation constraint is binding for person A in period t and the family income suffers an adverse shock in period $t+1$ (while the private incomes remain at their period t levels) then, we should observe 'perverse' insurance: an increase in private consumption by A. Intuitively, a decline in the family income makes it more attractive for both individuals to quit the agreement because of Assumption 2F. As the participation constraint was initially binding for A, he has to be compensated to induce him to stay in the agreement. Thus, he receives a larger share of private consumption in the current period as well as in the future. As the total amount available for private consumption is the same in period $t+1$ as in period t, private expenditures by A will actually increase after the decline in family income.

1.4 Empirical Evidence

In this section I discuss some empirical evidence for the model of limited commitment presented above using the Côte d'Ivoire Living Standards Survey (CILSS). The empirical analysis draws on the work by Duflo & Udry (2003). Here I shall provide a brief summary of the relevant part of their empirical strategy and refer the reader to the paper in question for details.

The key prediction of the model relate to a change in private expenditures of one spouse in response to an exogenous change in the income of the other spouse. Additionally, the model has predictions about the responsiveness of private expenditures to an exogenous shock to family income, and the extent to which such a shock could lead to borrowing and leading between spouses, in comparison to shocks to private incomes.

We are therefore interested in regressing the change in expenditures that are private to one individual in the household on exogenous changes in the income of the spouse and in some measure of 'family' income. Duflo & Udry (2003) make use of the fact that agricultural income is strongly affected by rainfall patterns, which varies regionally and affects different crops in different ways. In the context of West Africa, certain crops are significantly more likely to be cultivated by one gender than the other, and the social norms about the use of income may relate to the type of crop from which that income is derived (discussed in more detail in the following section). Therefore, income from different groups of crops may correspond roughly to the categories "man's income", "woman's income" and "family income" used in the theoretical discussion above.

Duflo & Udry (2003) groups together crops according to these categories and for each group, regress the change in the logarithm of income from specific crops between two successive years on measures of rainfall for the corresponding years. Specifically, they estimate:

$$\log y_{is2} - \log y_{is1} = (R_{i2} - R_{i1})\,\gamma_{ys} + (X_{i2} - X_{i1})\,\delta_{ys} + (\xi_{i2} - \xi_{i1}) \tag{1.20}$$

where y_{ist} is the income in household i from the crop group s in period t, R_{it} is a vector of measures of rainfall that affects the period t harvest (including rainfall in the preceding year), and X_{it} are year and region interactions. Then the predicted values from each of these regressions provide a measure of exogenous change in the income from the group of crops concerned. The exogenous change in income from crop group s in household i is given by $DR_{is2} = (R_{i2} - R_{i1})\,\hat{\gamma}_{ys}$. Note that the estimate $\hat{\gamma}_{ys}$ enables us to create estimates of exogenous changes in crop income further back in time using historical rainfall data, even if income data is not available for these years. We calculate $DR_{is1} = (R_{i1} - R_{i0})\,\hat{\gamma}_{ys}$ and $DR_{is0} = (R_{i0} - R_{i,-1})\,\hat{\gamma}_{ys}$. If a couple is dealing with the problem of limited commitment by 'borrowing' from and 'lending' to each other, then expenditures in the current period should be affected by these measures of historical shocks.

Finally, following Duflo & Udry (2003), we estimate

$$\log c_{i2} - \log c_{i1} = \sum_{t=0}^{2}\sum_{s\in S} DR_{ist}\pi_{st} + (X_{i2} - X_{i1})\,\delta + (v_{i2} - v_{i1}) \tag{1.21}$$

where c_{it} is total expenditures on good c in household i in period t.

1.4.1 Data

The Côte d'Ivoire Living Standards Measurement Survey (CILSS) was carried out in 4 consecutive years, between 1985 and 1988. The first round was conducted with 1,500 households. In each of the following years, half the sample was replaced with a new set of households; such that each household would be surveyed in two consecutive years (except for the 750 dropped after the first round and the new ones included in the fourth). Thus, it is possible to construct a panel of roughly 2,250 households, each observed twice.

The survey included detailed questions on agricultural income and expenditures. For the present purpose, the data is limiting in that the unit of observation is the household rather than the individual. However, it is possible to get around this problem using two feasures of the data. First, agricultural output and costs are reported for each crop cultivated by the household. Therefore, given the strong association in this region of certain crops with either men or women, it is possible to construct approximate measures of male and female income.

Specifically, guided by the ethnographic literature on the subject, Duflo & Udry (2003) construct three linear combinations of rainfall realisations, each pertaining to the predicted change in the income from a crop or group of crops: one for the cash crops cultivated by men, one for yams (also cultivated by men) and one for crops cultivated by women. Leaving out crops that do not have any strong gender associations, they are able to place roughly 80 percent of the value of agricultural output in one of these three categories. The assumption behind this categorisation is not that certain crops are cultivated exclusively by men and others by women; but rather that there are systemic differences across crops in the likelihood that a man or a woman would exercise nominal control over the income from the crop.

Second, the survey included questions regarding transfers made by members of the household to individuals who do not belong to the household. Although it is not reported which member is making the transfer, the relationship between the recepient and the head of the household is recorded. If we assume that the head would derive more utility from a 'gift' made to his own relatives than other members of the household, these expenditures can be construed as being 'private' for him. We do not construct a parallel measure for the spouse of the head of the household for there seem to be very few transfers being made to the in-laws of the household head. A possible explanation for this is that questions regarding transfers were addressed only to the household head, who may not have been well-informed about transfers being made by other members of his household.

Aggregate rainfall data, collected at local rainfall stations, is available for Côte d'Ivoire for the past 14 years. Duflo & Udry (2003) construct, for each household, a series of aggregate rainfall at the closest rainfall station for each of the eight quarters preceding the most recent harvest. The series are used to construct the instruments for crop income.

The analysis is carried out on a sample of about 850 households that are engaged in agriculture, where there is at least one adult male and one adult female member, and which reports at least one crop classified as 'male only' and another classified as 'female only'.

1.4.2 Results

Table A1, reproduced from Duflo & Udry (2003), presents the F-statistics from estimating equation (1.20) for male non-yam cash crops, yams and female crops. Year and region effects and their interactions are used in each equation. The F tests indicate that the rainfall variables are jointly significant for each equation and the coefficients are significantly different from each other.

The predicted values from these regressions are then used on the right-hand side of an equation with the logarithm of transfers made to relatives of the head of the household as the dependant variable. A small number of observations, where the head of the household is a woman, are excluded from the sample. One-year and two-year lagged values of predicted changes in log income are also included on the right-hand side of the regression for the reason that if the household is characterised by limited commitment, the current allocation of goods in the household could depend on the past history of shocks. A one percent increase in yam income lowers transfers to relatives of the household head by 0.85 percent (significant at the 6 percent level) during the same year. Female income has a positive but insignificant effect on transfers during the first two years. But two years after the shock, transfers decline by 0.57 percent in response to a one percent increase in female income.

These results are inconsistent with a model of the household characterised by Pareto efficiency and the first piece of evidence – the negative relation between yam income and male private expenditures – cannot be explained by means of a standard model of limited commitment[10]. However, the theoretical results obtained in section 1.3, where we assume that public goods expenditures are made out of a separate account, can provide a consistent explanation for the signs of the coefficients observed here.

When the woman suffers an adverse shock, the household head would be unwilling to provide her insurance if his limited commitment constraint is binding. However, he may be willing to 'lend' her money. Then, the shock to female income would cause his private expenditures to decrease in the same period, but increase at a future date when he is 'repaid'. This is the story behind proposition 1.3.

On the other hand, if Assumption 2F in section 1.3 holds, then a decline in yam income would lower the surplus from any agreement to all members of the household. If the limited commitment constraint begins to bind for the household head as a result of this shock, then he would be given a greater share of the surplus so that he does not quit the agreement. In particular, he would receive a greater share of total non-yam income in the current period for his private consumption; then, as the non-yam income is unaffected by the shock, his private consumption would increase contemporaneously with the shock to yam income, as in proposition 1.4.

[10] Duflo and Udry (2003) find suggestive evidence along the same lines. Specifically, they estimate (1.21) without lagged income for various categories of household expenditures and find that an increase in output from yams has a *negative* association with two of the categories: 'adult' goods and 'prestige' goods such as jewelry. Like transfers made to relatives of the household head, these expenditures are likely to be private for adult members of the household; however these effects are not statistically significant.

The fact that, in table 1, the coefficient for a change in female crop income two years earlier is negative indicates that the male household head is required to wait two years after a woman in the household has suffered a negative shock before receiving compensation in the form of increased private expenditures. Two years may be reasonable time-frame for a loan in this context if agricultural income depends on the level of rainfall in both the current and the past year. Table A2, reproduced from Duflo & Udry (2003), shows the regression results of income from female crops, male cash crops and yam on measures of rainfall in the current and preceding years. The coefficients for rainfall measures in the two years are roughly similar; which means, for example, that if the rainfall this year was bad for this year's crops, it would also be bad for next year's crops. Therefore, an individual who borrows during a year that the rainfall has been bad for her crops would be unwilling to repay the following year when this year's rainfall would still have a negative effect on her income.

1.5 Conclusion

Theories of limited commitment were originally developed to provide a better understanding of interactions between households; and in particular to explain why informal arrangements among households may not lead to full insurance. These theories may now serve to develop a better understanding of intra-household allocation; and to account for the growing evidence of inefficiency in the allocation of expenditures within the household.

This paper makes the point that, in importing these theories for developing models of intra-household dynamics, we ought not to ignore the fact that public goods constitute a very significant part of household expenditures. In the standard model of limited commitment (where public goods are absent), individual consumption covaries with aggregate consumption; but this relationship can break down if public goods are present.

Therefore, co-movement in the private consumption of household members should not be regarded as a necessary condition for (partial) insurance within the household. The model used in this paper illustrates that divergence of (private) consumption can take place within a cooperative agreement. In the presence of commitment problems and public goods, such divergence may be the efficient way to deal with adverse shocks.

Additionally, the paper provides analysis for an environment where public goods expenditures are made out of a separate account, a practice that is consistent with evidence from anthropological studies of households in Africa. By keeping such an account, a couple would effectively surrender decisions about public goods expenditures to a social norm, and the relevant dynamic bargaining game would be one where all expenditures are private. We obtain the result that private consumption co-moves in response to shocks that affect private income but not the account out of which public expenditures are made.

However, under plausible assumptions about the outside option, an adverse shock to the account for public expenditures can lead to an increase in the private consumption of an individual in the same period if his participation constraint were initially binding; and thus co-movement in private consumption would break down.

The theoretical results seem to be consistent with evidence from the Côte d'Ivoire Living Standards Measurement Survey. It is found that transfers made to the relatives of the head of the household (which may be construed as expenditures that are private to him) respond negatively to exogenous changes in income from yams. Furthermore, the transfers respond positively to exogenous changes to income from crops farmed by women during the year of the shock but negatively two years after the shock. Thus we observe 'perverse' insurance in the case of yams – a source of income used primarily for household public goods – but partial insurance combined with borrowing and lending between the spouses in the case of a shock to the private income of women.

In the recent past, household surveys in developing countries have begun to collect information on individual well-being; for example, measures of the nutritional status of household members. While this data can provide valuable insights about decision-making and bargaining power within the household, it is evident that measures of individual well-being present only a partial picture of household dynamics. The analysis provided in this paper highlights the importance of analysing this data within a conceptual framework that explicitly accounts for the presence of public goods within the household. In particular, the absence of co-movement in private consumption, by itself, should not be interpreted as the absence of intra-household insurance, as some of the empirical literature has tended to do.

1.6 Appendix

The proofs of Lemmas 1.1 and 1.3 makes use of the following theorem in Topkis (1998).

Theorem 1.1 (2.8.1 in Topkis (1998)) *If X is a lattice, T is a partially ordered set, Δ_t is a subset of X for each t in T, Δ_t is increasing in t in T, $f(x,t)$ is supermodular in x on X for each t in T and $f(x,t)$ has increasing differences in (x,t) on $X \times T$, then $\arg\max_{x\in\Delta_t} f(x,t)$ is increasing in t on $\left\{t : t \in T, \arg\max_{x\in\Delta_t} f(x,t) \text{ is nonempty}\right\}$.*

Proof. **of Lemma 1.1:** Let $g\left(y^A, y^B, z\right)$ be such that $u^A\left(g\left(y^A, y^B, z\right), z\right) = d^A\left(y^A, y^B\right) - \beta\left(v - v^A_{aut}\right)$. Then the maximisation problem in (1.6) is equivalent to

$$\max_{x^B,z} \quad : \quad u^B\left(x^B, z\right) \tag{1.22}$$

$$\text{subject to} \quad : \quad g\left(y^A, y^B, z\right) + x^B + z \leq y^A + y^B \tag{1.23}$$

where the value of x^A is given by $g\left(y^A, y^B, z\right)$. By assumption in section 1.2.1, $\frac{\partial d^A(..)}{\partial y^B} = 0 \implies \frac{\partial g(..)}{\partial y^B} = 0$. Therefore, if $\Delta\left(y^A, y^B\right) = \left\{\left(x^B, z\right) : (1.23) \text{ is satisfied}\right\}$, $\Delta\left(y^A, y^B\right)$ is increasing in y^B. Also, by definition, $\frac{\partial^2 u_B(..)}{\partial x_B \partial z} \geq 0$; i.e. $u^B(..)$ is supermodular in x^B and z. Therefore, the conditions in Theorem 2.8.1. in Topkis (1998) are satisfied, and the theorem applies. Thus, we obtain the result $x^B\left(y^A, y^B\right)$ and $z\left(y^A, y^B\right)$ are increasing in y^B. Furthermore, since $u^A(..)$ is increasing in x^A and z, we have $g\left(y^A, y^B, z\right)$ decreasing in z. Therefore, $x^A\left(y^A, y^B\right)$ is decreasing in y^B. ■

Proof. of Lemma 1.2: (i) For $s \in S^A(v)$, suppose $\theta_s^A = 0$. Then, using equation (1.12), $P'(w_s) \geq P'(v) \iff w_s \leq v$; and since $\lambda_s = -P'(w_s)$, we obtain $\lambda_s \leq \lambda$. Then,

$$\begin{aligned}
\mu^A\left(y_s^A, y_s^B, \lambda_s\right) + \beta w_s &\leq \mu^A\left(y_s^A, y_s^B, \lambda\right) + \beta v \\
&< d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A
\end{aligned}$$

i.e. person A's participation constraint would be violated in state s. Therefore, we must have $\theta_s^A > 0$. This implies that the constraint is binding with equality:

$$\mu^A\left(y_s^A, y_s^B, \lambda_s\right) + \beta w_s = d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A$$

Then, using Assumption 1, we obtain

$$\mu^B\left(y_s^A, y_s^B, \lambda_s\right) + \beta P(w_s) > d^B\left(y_s^A, y_s^B\right) + \beta v_{aut}^B$$

$$\implies \theta_s^B = 0$$

Then, using equation (1.12), we obtain $w_s > v, \lambda_s > \lambda$.

(ii) The argument used for part (i) can be used here as well. If $s \in S^B(v)$ and $\theta_s^B = 0$, then person B's participation constraint would be violated. Therefore, we obtain $\theta_s^B > 0$ and $\theta_s^A = 0$ $\implies w_s < v, \lambda_s < \lambda$.

(iii) For $s \in S^0(v)$, suppose $\theta_s^A > 0$. Then, person A's participation constraint binds with equality in state s:

$$\mu^A\left(y_s^A, y_s^B, \lambda_s\right) + \beta w_s = d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A$$

and using Assumption 1, we obtain

$$\mu^B\left(y_s^A, y_s^B, \lambda_s\right) + \beta P(w_s) > d^B\left(y_s^A, y_s^B\right) + \beta v_{aut}^B$$

$$\implies \theta_s^B = 0$$

Then, using equation (1.12), we obtain $w_s > v$ and $\lambda_s > \lambda$. This implies

$$\mu^A\left(y_s^A, y_s^B, \lambda\right) + \beta v < d^A\left(y_s^A, y_s^B\right) + \beta v_{aut}^A$$

which contradicts the definition of $S^0(v)$. Therefore, we must have $\theta_s^A = 0$. Similarly, we can show that for $s \in S^0(v)$, we must have $\theta_s^B = 0$. Then, using equation (1.12), we obtain $w_s = v \iff \lambda_s = \lambda$. ■

The proof of Proposition 1.2 uses the following lemma.

Lemma 1.3 *Let* $\left\{x^A\left(y^A, y^B\right), x^B\left(y^A, y^B\right), z\left(y^A, y^B\right), w\left(y^A, y^B\right)\right\}$ *denote the solution to the maximisation problem in (1.15). Then (i)* $\frac{\partial z\left(y^A, y^B\right)}{\partial y^B} > 0$, $\frac{\partial x^B\left(y^A, y^B\right)}{\partial y^B} > 0$.

Let $\sigma\left(y^A, y^B\right)$ *be the value of the Lagrange multiplier to the problem in (1.15). Denote by* $\bar{x}^A\left(y^A, y^B, \lambda\right)$, $\bar{x}^B\left(y^A, y^B, \lambda\right)$, $\bar{z}\left(y^A, y^B, \lambda\right)$, $\bar{w}\left(y^A, y^B, \lambda\right)$ *the solution to the following problem.*

$$\max_{x^A, x^B, z, w} \lambda u^A\left(x^A, z\right) + u^B\left(x^B, z\right) + \beta\left(w + P(w)\right) \tag{1.24}$$

$$\textit{subject to:} \quad x^A + x^B + z \leq y^A + y^B$$

Then, (ii) $\frac{\partial x^A\left(y^A, y^B\right)}{\partial y^B} \geq 0$ *iff*

$$u_2^A(..)\left(\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B} - \frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda}\right) - d_2^A(..)\frac{\partial \bar{x}^A}{\partial \lambda} \leq \beta\frac{\partial \bar{w}}{\partial \lambda}\frac{\partial \bar{x}^A}{\partial y^B} \tag{1.25}$$

at $\lambda = \sigma\left(y^A, y^B\right)$.

Proof. of Lemma 1.3:

(i) To prove the first part of the lemma, we proceed as in the proof of Lemma **1.1.** Let $\tilde{g}\left(y^A, y^B, z, w\right)$ be such that $u^A\left(\tilde{g}\left(y^A, y^B, z, w\right), z\right) + \beta w = d^A\left(y^A, y^B\right) + \beta v_{aut}^A$. Then the maximisation problem in (1.15) is equivalent to

$$\max_{x^B, z, w} \ : \ u^B\left(x^B, z\right) + \beta P(w) \tag{1.26}$$

$$\text{subject to} \ : \ \tilde{g}\left(y^A, y^B, z, w\right) + x^B + z \leq y^A + y^B \tag{1.27}$$

where the value of x^A is given by $\tilde{g}\left(y^A, y^B, z, w\right)$. If $\tilde{\Delta}\left(y^A, y^B\right) = \left\{\left(x^B, z, -w\right) : (1.27) \text{ is satisfied}\right\}$, $\tilde{\Delta}\left(y^A, y^B\right)$ is increasing in y^B iff $\frac{\partial \tilde{g}}{\partial y^B}(..) < 1 \iff u_1^A\left(\tilde{g}\left(y^A, y^B, z, w\right), z\right) \geq d_2^A\left(y^A, y^B\right)$. The latter inequality follows from Assumption 2. [11] Also if $\hat{u}\left(x^B, z, w\right) = u^B\left(x^B, z\right) + \beta P(w)$, then

[11] To see this, note that by Assumption 2, $\frac{\partial \mu^A\left(y^A, y^B, \lambda\right)}{\partial y^B} > d_2^A\left(y^A, y^B\right)$.
And $\frac{\partial \mu^A}{\partial y^B} = u_1^A\frac{\partial \bar{x}^A}{\partial y^B} + u_2^A\frac{\partial \bar{z}}{\partial y^B} < u_1^A\left(\frac{\partial \bar{x}^A}{\partial y^B} + \frac{\partial \bar{z}}{\partial y^B}\right) < u_1^A$.

$\tilde{u}\left(x^B, z, w\right)$ is supermodular in x^B, z and $-w$. Therefore, we can apply Theorem 2.8.1. in Topkis (1998). Thus, we obtain the result $\frac{\partial z(y^A,y^B)}{\partial y^B} > 0$, $\frac{\partial x^B(y^A,y^B)}{\partial y^B} > 0$, $\frac{\partial w(y^A,y^B)}{\partial y^B} < 0$.

(ii) It is straightforward to show that $x^A\left(y^A, y^B\right)$ can be written as $\bar{x}^A(y^A, y^B, \sigma\left(y^A, y^B\right))$, and that corresponding expressions can be used for $x^B\left(y^A, y^B\right), z\left(y^A, y^B\right)$ and $w\left(y^A, y^B\right)$. Furthermore, as the allocation $x^A\left(y^A, y^B\right)$, $x^B\left(y^A, y^B\right)$, $z\left(y^A, y^B\right)$, $w\left(y^A, y^B\right)$ by definition satisfies the participation constraint of person A with equality we have

$$u^A\left(\bar{x}^A(y^A, y^B, \sigma\left(y^A, y^B\right)), \bar{z}(y^A, y^B, \sigma\left(y^A, y^B\right))\right) + \beta\bar{w}(y^A, y^B, \sigma\left(y^A, y^B\right)) = d^A\left(y^A, y^B\right) + \beta v^A_{aut}$$

$$\Longrightarrow \mu^A\left(y^A, y^B, \sigma\left(y^A, y^B\right)\right) + \beta\bar{w}(y^A, y^B, \sigma\left(y^A, y^B\right)) = d^A\left(y^A, y^B\right) + \beta v^A_{aut} \tag{1.28}$$

where $\mu^A(..)$ is as defined in section 1.2. Taking the derivative throughout w.r.t. y^B in (1.28) and rearranging, we obtain

$$\Longrightarrow \frac{\partial \sigma}{\partial y^B} = \frac{\left[\frac{\partial d^A}{\partial y^B} - \frac{\partial \mu^A}{\partial y^B}\right]}{\left[\frac{\partial \mu^A}{\partial \sigma} + \beta\frac{\partial \bar{w}}{\partial \sigma}\right]} \tag{1.29}$$

Taking the derivative throughout w.r.t. y^B in the identity $x^A\left(y^A, y^B\right) = \bar{x}^A(y^A, y^B, \sigma\left(y^A, y^B\right))$ and substituting for $\frac{\partial \sigma}{\partial y^B}$ using (1.29), we obtain

$$\frac{\partial x^A}{\partial y^B} = \frac{\partial \bar{x}_A}{\partial y_B} + \frac{\partial \bar{x}^A}{\partial \sigma}\frac{\left[\frac{\partial d^A}{\partial y^B} - \frac{\partial \mu^A}{\partial y^B}\right]}{\left[\frac{\partial \mu^A}{\partial \sigma} + \beta\frac{\partial \bar{w}}{\partial \sigma}\right]} \tag{1.30}$$

Therefore, $\frac{\partial x^A}{\partial y^B} \geq 0$ iff

$$\frac{\partial \bar{x}_A}{\partial y_B} + \frac{\partial \bar{x}^A}{\partial \sigma}\frac{\left[\frac{\partial d^A}{\partial y^B} - \frac{\partial \mu^A}{\partial y^B}\right]}{\left[\frac{\partial \mu^A}{\partial \sigma} + \beta\frac{\partial \bar{w}}{\partial \sigma}\right]} \geq 0 \tag{1.31}$$

From the definition of $\mu^A\left(y^A, y^B, \lambda\right)$, we have $\frac{\partial \mu^A}{\partial y^B} = u^A_1(..)\frac{\partial \bar{x}^A}{\partial y^B} + u^A_2(..)\frac{\partial \bar{z}}{\partial y_B}$ and $\frac{\partial \mu^A}{\partial \lambda} = u^A_1(..)\frac{\partial \bar{x}_A}{\partial \lambda} + u^A_2(..)\frac{\partial \bar{z}}{\partial \lambda}$. Substituting for $\frac{\partial \mu^A}{\partial y^B}$ and $\frac{\partial \mu^A}{\partial \lambda}$ in (1.31) using these expressions and simplifying, we obtain

$$u^A_2(..)\left(\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B} - \frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda}\right) - d^A_2(..)\frac{\partial \bar{x}^A}{\partial \lambda} \leq \beta\frac{\partial \bar{w}}{\partial \lambda}\frac{\partial \bar{x}^A}{\partial y^B}$$

■

The proof of Proposition 1.2 makes use of the following lemma.

Lemma 1.4 : *Let $\bar{x}^A\left(y^A, y^B, \right)$, $\bar{x}^B\left(y^A, y^B, \right)$, and $\bar{z}\left(y^A, y^B, \right)$ be as in the statement of Lemma*

$\Longrightarrow u^A_1\left(\bar{x}^A\left(y^A, y^B, \lambda\right), \bar{z}\left(y^A, y^B, \lambda\right)\right) > d^A_2\left(y^A, y^B\right)$ for any λ.
$\Longrightarrow u^A_1\left(\tilde{g}\left(y^A, y^B, z, w\right), z\right) \geq d^A_2\left(y^A, y^B\right)$

1.3. Then

$$\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B} - \frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda} > 0$$

for all values of y^A, y^B *and* λ.

Proof. of Lemma 1.4: From the first-order conditions to the maximisation problem in (1.24),

$$\lambda u_1^A\left(\bar{x}^A\left(y^A,y^B,\lambda\right),\bar{z}\left(y^A,y^B,\lambda\right)\right) \equiv \lambda u_2^A\left(\bar{x}^A\left(y^A,y^B,\lambda\right),\bar{z}\left(y^A,y^B,\lambda\right)\right)+u_2^B\left(\bar{x}^B\left(y^A,y^B,\lambda\right),\bar{z}\left(y^A,y^B,\lambda\right)\right) \tag{1.32}$$

The terms $\bar{x}^A\left(y^A,y^B,\lambda\right)$, $\bar{x}^B\left(y^A,y^B,\lambda\right)$, and $\bar{z}\left(y^A,y^B,\lambda\right)$ must also satisfy the budget constraint:

$$\bar{x}^A\left(y^A,y^B,\lambda\right)+\bar{x}^B\left(y^A,y^B,\lambda\right)+\bar{z}\left(y^A,y^B,\lambda\right)=y^A+y^B \tag{1.33}$$

Derivating throughout (1.32) w.r.t. y^B and λ and rearranging, we obtain

$$\lambda\left(u_{11}^A-u_{12}^A\right)\frac{\partial \bar{x}^A}{\partial y^B} \equiv \lambda\left(u_{22}^A-u_{12}^A\right)\frac{\partial \bar{z}}{\partial y^B}+u_{12}^B\frac{\partial \bar{x}^B}{\partial y^B}+u_{22}^B\frac{\partial z}{\partial y^B} \tag{1.34}$$

$$\lambda\left(u_{11}^A-u_{12}^A\right)\frac{\partial \bar{x}^A}{\partial \lambda}+u_1^A \equiv \lambda\left(u_{22}^A-u_{12}^A\right)\frac{\partial \bar{z}}{\partial \lambda}+u_{12}^B\frac{\partial \bar{x}^B}{\partial \lambda}+u_{22}^B\frac{\partial z}{\partial \lambda}+u_2^A \tag{1.35}$$

Similarly, derivating throughout (1.33) w.r.t. y^B and λ we obtain

$$\frac{\partial \bar{x}^B}{\partial y^B} = 1-\frac{\partial \bar{x}^A}{\partial y^B}-\frac{\partial \bar{z}}{\partial y^B} \tag{1.36}$$

$$\frac{\partial \bar{x}^B}{\partial \lambda} = -\frac{\partial \bar{x}^A}{\partial \lambda}-\frac{\partial \bar{z}}{\partial \lambda} \tag{1.37}$$

Substituting in (1.34) and (1.35) using (1.36) and (1.37) and rearranging,

$$\left[\lambda\left(u_{11}^A-u_{12}^A\right)+u_{12}^B\right]\frac{\partial \bar{x}^A}{\partial y^B} \equiv \left[\lambda\left(u_{22}^A-u_{12}^A\right)-u_{12}^B+u_{22}^B\right]\frac{\partial \bar{z}}{\partial y^B}+u_{12}^B \tag{1.38}$$

$$\left[\lambda\left(u_{11}^A-u_{12}^A\right)+u_{12}^B\right]\frac{\partial \bar{x}^A}{\partial \lambda} \equiv \left[\lambda\left(u_{22}^A-u_{12}^A\right)-u_{12}^B+u_{22}^B\right]\frac{\partial \bar{z}}{\partial \lambda}+u_2^A-u_1^A \tag{1.39}$$

Dividing (1.38) by (1.39) and rearranging,

$$\left[\lambda\left(u_{22}^A-u_{12}^A\right)-u_{12}^B+u_{22}^B\right]\left(\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B}-\frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda}\right) \equiv \left(\frac{\partial \bar{x}^A}{\partial y^B}\left(u_2^A-u_1^A\right)-\frac{\partial \bar{x}^A}{\partial \lambda}u_{12}^B\right)$$

The first term within square brackets on the left-hand side, and the term on the right-hand side, are both negative. Therefore,

$$\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B} - \frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda} > 0$$

■

Proof. of Proposition 1.2: Consider two states r and s for which $y_r^A = y_s^A$ and $y_r^B < y_s^B$. From the fundamental theorem of calculus,

$$z\left(y_s^A, y_s^B\right) = z\left(y_r^A, y_r^B\right) + \int_{y_r^B}^{y_s^B} \frac{\partial z\left(y^A, y^B\right)}{\partial y^B} dy^B$$

Then, using Lemma 1.3, $z\left(y_s^A, y_s^B\right) > z\left(y_r^A, y_r^B\right) \implies z_s[A] > z_r[A]$. Likewise, we can show that $x_s^B[A] > x_r^B[A]$.

To find sufficient conditions for $x_s^A[A] < x_r^A[A]$, we define the following terms. Denote by $P(.|\beta)$ the pareto frontier of feasible history-dependent agreements, for the discount factor β. Let $\tilde{\lambda}\left(y^A, y^B, \beta\right)$ be the pareto weight for person A which would cause his participation constraint to just bind if the income levels in the current period are y^A and y^B and the discount factor is β. Let

$$\tilde{\gamma}\left(y^A, y^B, \beta\right) = \frac{\left(\frac{\partial \bar{x}^A}{\partial \lambda}\right)}{\left(\frac{\partial \bar{w}}{\partial \lambda}\frac{\partial \bar{x}^A}{\partial y^B}\right)}\left[\frac{u_2^A(..)}{\left(\frac{\partial \bar{x}^A}{\partial \lambda}\right)}\left(\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B} - \frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda}\right) - d_2^A(..)\right]$$

where the expression on the right-hand side is computed for $y^A, y^B, \tilde{\lambda}\left(y^A, y^B, \beta\right)$ and $P(.|\beta)$. (In particular, $\frac{\partial \bar{w}}{\partial \lambda} = -1/P''(w|\beta)$ where w is given by $-P'(w|\beta) = \tilde{\lambda}\left(y^A, y^B, \beta\right)$). As β approaches 0 or 1, $\tilde{\lambda}\left(y^A, y^B, \beta\right)$ approaches $\lambda\left(y^A, y^B\right)$ defined as follows.

$$\mu^A\left(y^A, y^B, \lambda\left(y^A, y^B\right)\right) = d^A\left(y^A, y^B\right)$$

Let

$$\gamma\left(y^A, y^B\right) = \inf\left\{\tilde{\gamma}\left(y^A, y^B, \beta\right)\right\}_{\beta \in (0,1)}$$

It can be shown that the term $\frac{\partial \bar{x}^A}{\partial \lambda}\frac{\partial \bar{z}}{\partial y^B} - \frac{\partial \bar{x}^A}{\partial y^B}\frac{\partial \bar{z}}{\partial \lambda} > 0$ for all values when x^A is a private good and z is a public good (see Lemma 1.4).Therefore, for $d_2^A(..)$ sufficiently small, $\gamma\left(y^A, y^B\right) > 0$. If $\beta < \gamma\left(y^A, y^B\right)$, then using Lemma 1.3, $\frac{\partial x^A\left(y^A, y^B\right)}{\partial y^B} < 0$. Choose

$$\beta^* = \min\left\{\gamma\left(y^A, y^B\right)\right\}_{y^B \in \left(y_r^B, y_s^B\right)}$$

Therefore, if $\beta < \beta^*$, then

$$\begin{aligned} x^B\left(y_s^A, y_s^B\right) &= x^B\left(y_r^A, y_r^B\right) + \int_{y_r^B}^{y_s^B} \frac{\partial x^B\left(y^A, y^B\right)}{\partial y^B} dy^B \\ &< x^B\left(y_r^A, y_r^B\right) \end{aligned}$$

■

Proof. of Proposition 1.3: We show by contradiction that $w(t+1) > w(t)$. If $w(t+1) \leq w(t)$ then we also have $x^A(t+1) < x^A(t)$ (since $\frac{u_1^B(..)}{u_1^A(..)} = -P'(w)$ in each period and $y_r^A + y_r^B > y_s^A + y_s^B$). Now, person A's participation constraint is satisfied with equality in period t and he faces the same outside option in period t and $t+1$. Therefore, if $w(t+1) \leq w(t)$ and $x^A(t+1) < x^A(t)$, his participation constraint cannot be satisfied in period $t+1$. Therefore, we must have $w(t+1) > w(t)$ which can happen only if person A's participation constraint is binding in period $t+1$. Therefore, he receives his outside option from the agreement in this period; and therefore, we must also have $x^A(t+1) < x^A(t)$. ■

Proof. of Proposition 1.4: Let $\lambda(t)$ be the value of the Lagrange multiplier on person A's promise-keeping constraint in period t. Then, since person A's participation constraint is binding in period t, we have

$$\tilde{\mu}^A\left(y_r^A, y_r^B, y_r^Z, \lambda(t)\right) - \tilde{d}^A\left(y_r^A, y_r^Z\right) = \beta\left(v_{aut}^A - w(t)\right)$$

And, by Assumption 2F,

$$\tilde{\mu}^A\left(y_s^A, y_s^B, y_s^Z, \lambda(t)\right) - \tilde{d}^A\left(y_s^A, y_s^Z\right) < \tilde{\mu}^A\left(y_r^A, y_r^B, y_r^Z, \lambda(t)\right) - \tilde{d}^A\left(y_r^A, y_r^Z\right)$$

$$\Longrightarrow \tilde{\mu}^A\left(y_s^A, y_s^B, y_s^Z, \lambda(t)\right) - \tilde{d}^A\left(y_s^A, y_s^Z\right) < \beta\left(v_{aut}^A - w(t)\right)$$

Therefore person A's participation constraint cannot be satisfied in period $t+1$ unless $\lambda(t+1) > \lambda(t)$. Therefore, the constraint must be binding in period $t+1$. Therefore $w(t+1) > w(t)$. Furthermore, total expenditures on private goods are equal in the two periods. Therefore, $x^A(t+1) > x^A(t)$. ■

Proposition 1.5 *If the participation constraint is binding for A in state s, period t and the realised state in period t+1, s^* is such that $y_{s^*}^A = y_s^A$, $y_{s^*}^B < y_s^B$, then the participation constraint is binding for A again in period $t+1$.*

Proof. of Proposition 1.5: Suppose the participation constraint for A does not bind in period $t+1$. Then his promised value in period $t+1$ is no larger than that in period t: $w(t+1) \leq w(t)$. And, by Assumption 2,

$$u^A\left(x^A(t+1), z(t+1)\right) - d^A\left(y_{s^*}^A, y_{s^*}^B\right) < u^A\left(x^A(t), z(t)\right) - d^A\left(y_s^A, y_s^B\right) \tag{1.40}$$

As A's participation constraint binds in period t, we have

$$u^A\left(x^A(t), z(t)\right) - d^A\left(y_s^A, y_s^B\right) = \beta\left(v_{aut}^A - w(t)\right)$$

Then equation (1.40) and $w(t+1) \leq w(t)$ implies

$$u^A\left(x^A(t+1), z(t+1)\right) - d^A\left(y_{s*}^A, y_{s*}^B\right) < \beta\left(v_{aut}^A - w(t+1)\right)$$

which violates A's participation constraint in period $t+1$. Therefore, the participation constraint must bind for A in period $t+1$. ■

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Table 1: Dependant Variable: Change in Log (Transfers to Relatives of Household Head)

	OLS Coefficients
Predicted change in log male non-yam income	
year t	0.685
	(0.564)
t-1	0.239
	(0.253)
t-2	0.056
	(0.253)
Predicted change in log yam income	
year t	-0.842
	(0.441)
t-1	0.205
	(0.123)
t-2	0.112
	(0.095)
Predicted change in log female income	
year t	0.828
	(0.692)
t-1	0.357
	(0.400)
t-2	-0.569
	(0.270)

Standard errors in parentheses

Table A1: First stage summary statistics (F statistics for following null hypotheses)

	Dependent variables		
	Male cash crop	Current Yam Income	Female Income
	(1)	(2)	(3)
All rainfall variables = 0	1.99	3.50	2.53
	(0.014)	(0.000)	(0.000)
Current yr rainfall variables = 0	1.18	3.38	2.43
	(0.315)	(0.000)	(0.005)
Past yr rainfall variables = 0	2.79	4.64	2.64
	(0.005)	(0.000)	(0.001)
Coefficients equal to those for			
Male Cash Crop	NA		
Yam Income	2.1	NA	
	(0.010)		
Female Income	2.1	2.38	NA
	(0.009)	(0.002)	

p-values in parentheses

Full specification includes year dummies, region dummies.

Results reproduced from Duflo & Udry (2003)

Table A2: First stage regression results

	Dependent variables					
	Female crop income		Male cash crop		Yam Income	
	Forest coefficients (1)	Savannah interaction (2)	Forest coefficients (3)	Savannah interaction (4)	Forest coefficients (5)	Savannah interaction (6)
Difference (year 2 – year 1) in:						
Aggregate rainfall current year, season 1	-0.001518	0.0040317	0.0004811		-0.003153	-0.010761
	(0.001)	(0.003)	(0.002)		(0.002)	(0.006)
Aggregate rainfall current year, season 2	0.0007268	0.0013814	-0.001099		0.0015603	0.0015827
	(0.000)	(0.002)	(0.001)		(0.001)	(0.004)
Aggregate rainfall current year, season 3	-0.000613	0.0038313	0.0001552		-0.002321	-0.003099
	(0.001)	(0.001)	(0.001)		(0.001)	(0.002)
Aggregate rainfall current year, season 4	0.0007069	-0.0042	-0.000169		0.0005378	-0.006442
	(0.001)	(0.005)	(0.001)		(0.001)	(0.006)
Aggregate rainfall past year, season 1	-0.000357	0.0068233	-0.004016		-0.00618	-0.010605
	(0.002)	(0.008)	(0.003)		(0.003)	(0.011)
Aggregate rainfall past year, season 2	0.0000808	-0.006707	0.0008669		0.0023795	-0.000265
	(0.000)	(0.005)	(0.001)		(0.001)	(0.006)
Aggregate rainfall past year, season 3	-0.00138	0.0033809	-9.57E-05		-0.00226	0.0027378
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)
Aggregate rainfall past year, season 4	-0.000769	-0.003408	0.0014161		0.0007269	0.0053683
	(0.001)	(0.005)	(0.001)		(0.001)	(0.006)
Dummy for shock, current year, season 1	-0.476518		-0.093278		-0.894238	
	(0.233)		(0.364)		(0.439)	
Dummy for shock, current year, season 2	0.4592265	0.3583756	0.127267		0.4623188	-2.75326
	(0.193)	(0.485)	(0.300)		(0.283)	(0.828)
Dummy for shock, current year, season 3						
Dummy for shock, current year, season 4	-0.411497	-0.023197	0.1107528		-2.134331	
	(0.378)	(0.531)	(0.362)		(0.520)	
Dummy for shock, past year, season 1	0.2208379	0.1403303	-0.037784		0.2016122	3.537023
	(0.208)	(0.429)	(0.183)		(0.262)	(1.107)
Dummy for shock, past year, season 2	-0.0745	0.5705816	-0.037784		-0.133787	-2.962664
	(0.119)	(1.027)	(0.183)		(0.204)	(0.911)
Dummy for shock, past year, season 3	-0.31524	0.4587139	-1.324416		-0.124188	-3.387585
	(0.245)	(1.366)	(0.384)		(0.386)	(1.389)
Dummy for shock, past year, season 4	-0.720612		0.7792504		-1.74257	1.238107
	(0.267)		(0.437)		(0.408)	(1.274)
Number of observations	976		614		607	

Standard errors in parentheses

Full specification includes year dummies, region dummies.

Results reproduced from Duflo & Udry (2003)

Chapter 2

Limited Commitment, Individual Savings and Risk Aversion

2.1 Introduction

The concept of self-enforcing insurance contracts have received considerable attention in the literature in recent years, given their ability to explain important features in the pattern of consumption and transfers in village economies[1].

For the most part, the theoretical literature has adopted a framework where households do not have access to an intertemporal technology, and therefore must consume all of their earnings at each point in time. For many village environments, this assumption is unrealistic. Agricultural households usually have the means to store a part of their produce after each harvest. In addition, some household assets may be used in the production process. For example, seeds may be stored for planting in the following season. If incomes are subject to seasonal variations or uncertainty, households have strong incentives to make use of these intertemporal technologies. Therefore, it is natural to develop the existing theoretical framework along these dimensions.

This paper extends the basic theoretical framework along one dimension: it is assumed that individuals have access to a riskless intertemporal technology; but the production technology does not rely on the level of existing assets. This extension reveals an additional purpose of saving/dissaving which is relevent if households are engaged in a self-enforcing insurance agreement; but not when the first-best agreement is enforceable or when households live in autarky. If risk-aversion decreases with wealth, then having individuals save less today will increase their aversion to autarky,

[1]For example, Ligon, Thomas and Worrall (2002) find, using Indian village data, that a model of limited commitment (i.e. of self-enforcing insurance contracts) can explain the consumption path of households more effectively than either the full insurance or the autarkic model. Fafchamps (1999) argue that the practice of gift-giving and informal credit in rural societies have important similarities with the predictions of the limited commitment model.

and therefore the largest transfer they can be asked to make within a self-enforcing agreement. Thus, dis-saving becomes a devise to increase future commitment to an agreement. The opposite is true if risk-aversion increases with wealth; and the effect disappears altogether in the case of constant absolute risk aversion.

In mathematical terms, limited commitment introduces an additional term to the right-hand side of the standard Euler equation; and the sign and magnitude of the term is directly related to the rate at which risk-aversion changes with the level of wealth.

The corollary of this result is that whenever risk aversion changes with the level of wealth, and the first-best contract is not enforceable, it is possible to achieve a higher level of utility with an agreement on both transfers and saving than with an agreement on transfers only. Equivalently, a group of households can achieve better risk-sharing when they can monitor each others' assets. Furthermore, at each point in time, the constrained efficient agreement would specify savings that are lower (higher) than what individuals with decreasing (increasing) risk aversion would choose on their own, given the future contingent transfers specified by the agrement.

The paper further argues that this effect of limited commitment on saving is especially large for the borderline poor, who face a positive risk of experiencing acute distress – e.g. homelessness or death in the family – without the agreement and therefore are very committed to a mutual insurance agreement which protects them from such distress. If these households or individuals accumulate wealth, then this positive risk of acute distress would diminish rapidly and so would their commitment to an insurance agreement. Therefore, the constrained efficient insurance agreement would force these individuals to save significantly less than what they would choose to do if the decision were left to them. In this manner, the informal insurance agreement presents a formidable obstacle to escaping asset poverty.

The remainder of the paper proceeeds as follows. Section 2.2 discusses the related literature. The model is introduced in Section 2.3. Section 2.4 provides a characterisation of subgame perfect equilibria of the game. Section 2.5 analyses the pareto-optimal equilibria for the case where individuals or households have finite lives. Section 2.6 concludes.

2.2 Related Literature

The idea of limited commitment as a basis of mutual insurance has been explored and developed extensively in the literature; it was first formalised in Kimball (1988) in the context of farm households in a rural community. Coate and Ravallion (1993) characterised the conditions under which the first-best insurance can be implemented under limited commitment. Kocherlakota (1996) provided a characterisation of constrained efficient agreements, and examined their long-run dynamics. Gauthier, Poitevin and Gonzalez (1997) showed that if agents have the ability to make ex-ante payments, this can lead to improved efficiency for self-enforcing agreements. Ligon, Thomas and Worrall (2001) showed that the constrained efficient agreements are characterised by a simple updating rule; specifically, that for each state of nature, there is a time-invariant interval for the ratio of marginal utilities; and in each period, the ratio of marginal utilities adjusts by the smallest amount necessary to bring it into the current interval.

Fafchamps (1999) argued that the theoretical characteristics of informal insurance under limited commitment correspond closely to the empirical evidence on gift-giving and informal credit in rural societies. For example, Udry (1994) finds that the terms of repayment of informal credit in rural Nigeria is affected by both shocks to the creditor and the debtor; which corresponds to the characteristics of informal insurance under limited commitment. Ligon, Thomas and Worrall (2002) test the limited commitment model using Indian village data and find that it can explain the consumption path of households more effectively than either the full insurance or the autarkic model.

The framework for informal insurance under limited commitment has also been discussed in the context of intra-household bargaining. Recent empirical research has shown that consumption and production within the household, in a large number of cases, do not match the implications of pareto efficiency; and the idea of limited commitment is an intuitively appealing constraint to introduce to the pareto efficient model of the household. Duflo and Udry (2004) study the responsiveness of household expenditures on various goods to different types of shocks, using data from agricultural households in Côte d'Ivoire, and find that it fails the test of pareto efficiency, but correspond broadly to the implications of a model of mutual insurance under limited commitment. In theoretical work, Ligon (2003) developed an axiomatic approach to intra-household bargaining where the Nash bargaining solution is modified to capture the idea of limited commitment. Wahhaj (2005) showed that the result of co-movement in the consumption of agents, which characterises the model of informal insurance under limited commitment, breaks down when public goods are introduced to the setting, public goods arguably being important in the context of intra-household bargaining.

The theoretical literature discussed thus far have generally assumed, for simplicity, that no intertemporal technology is available in the economy, such that in each period the output had to be consumed in the same period. Ligon, Thomas and Worrall (2000) relaxed this assumption and considered the welfare effects of an improvement in the intertemporal technology; in addition, they show that, under limited commitment, the intertemporal savings decision satisfies a 'modified' Euler equation; such that the mariginal rate of substitution between any two periods may be larger or smaller than the marginal rate of transformation. In this paper, we study a setting very similar to that in Ligon, Thomas and Worrall (2000) and establish that the gap between the marginal rate of substitution and the marginal rate of transformation under limited commitment depends on the rate at which risk aversion changes with wealth for the agent; in addition, we consider the implications of this result for the consumption path of poor and rich agents under limited commitment.

2.3 The Model

We consider an environment where two individuals A and B, receive an uncertain income in each period, $t = 1, 2...T$ (although A and B will be referred to as individuals throughout the paper, they can also be regarded as households). In each period, there are S possible states of the world; and the probability of each state s equals π_s, with $\sum_{s=1}^{S} \pi_s = 1$; i.e. the distribution of states is independantly and identically distributed over time. The income earned by person i when the realised state is s is denoted y_s^i. In addition, each person has access to an intertemporal technology whereby ρ units of the good stored at the end of period t is transformed into 1 unit in period $t+1$. Individuals also have the means to transfer some of their savings or income to each other in each period. The exact sequence of events within each period is as follows:

(i) the state of nature s is realised and each individual receives her income for the period y_s^i;
(ii) each person chooses an amount $\tau^i \geq 0$ to transfer to the other;
(iii) each person allocates available resources between consumption $c^i \geq 0$ and saving, $k^i \geq 0$.

Assuming that it is not possible to hold negative assets, we must have

$$\frac{1}{\rho} k_{t-1}^i + y_s^i + \tau_t^{-i} \leq k_{t+1}^i + \tau_t^i + c_t^i$$

Person i's preferences over different streams are given by the following expression:

$$E \sum_{t=1}^{T} \beta^{t-1} u^i \left(c_t^i\right)$$

where $u^i()$ is increasing and strictly concave.

Since $u^i()$ is concave, individuals prefer to smooth consumption across time and over different states of the world. They have two means of doing so; by engaging in precautionary saving using the intertemporal technology, and by participating in a mutual insurance agreement. The remaining sections consider these possibilities.

Before considering mutual insurance agreements, we introduce some terms and notation that will be used later in the analysis. Let $h_t = (s_1, s_2, ..., s_t)$ denote a particular history of realised states up to and including period t. Let $\mathcal{H}_t$ denote the set of all possible histories of states in period t.

Definition 2.1 *An allocation is a complete plan for transfers and savings by A and B contingent on the history of states.*

An allocation can be described by $\mathcal{A} = \left\{k_t^A(h_t), k_t^B(h_t), \tau_t^A(h_t), \tau_t^B(h_t)\right\}_{t=1..T}$; such that if the history of realised states in period t is h_t, then person A should transfer $\tau_t^A(h_t)$ to B during period t and have savings of $k_t^A(h_t)$ at the end of the period. Similarly, the level of transfers and saving to be made by B are given by $\tau_t^B(h_t)$ and $k_t^B(h_t)$ respectively. An allocation implicitly defines a consumption stream if we assume that whatever assets are not saved at the end of a period will be consumed. Then the consumption stream $\left\{c_t^A(h_t), c_t^B(h_t)\right\}_{t=1..T}$ is given by

$$c_t^i(h_t) = \frac{1}{\rho}k_{t-1}^i(h_{t-1}) + y_s^i + \tau_t^{-i}(h_t) - \tau_t^i(h_t) - k_t^i(h_t), i = A, B$$

where s is the realised state in period t.

Note that an allocation is not equivalent to a complete strategy profile because the actions that it specifies are not contingent on the history of past actions, which can also be part of the information available to both individuals at any point in time. We shall also sometimes refer to an allocation as an "agreement on transfers and savings", as opposed to an "agreement on transfers" which is defined below.

Definition 2.2 *An agreement on transfers is a plan for transfers to be made by A and B to each other contingent on the history of states.*

An agreement on transfers can be described by $\mathcal{T} = \left\{\tau_t^A(h_t), \tau_t^B(h_t)\right\}_{t=1..T}$. Such an agreement does not specify what portions of their assets individuals should consume and save at the end of the period. In other words, an agreement on transfers is more lax than an agreement on transfers and saving in that it allows individuals to choose the level of saving independently. If it is not possible to monitor the level of saving, then individuals would have to engage in mutual insurance using an agreement on transfers only.

Define $U_T^i\left(z^i, \left\{\tau_t^A, \tau_t^B\right\}_{t=1}^T\right)$ as the maximum utility person i can obtain, given initial assets z^i (after period 1 income has been received) and an agreement on transfers $\left\{\tau_t^A, \tau_t^B\right\}_{t=1}^T$, which is obeyed by both individuals[2]. Then, $U_T^i\left(z^i, \left\{\tau_t^A, \tau_t^B\right\}_{t=1}^T\right)$ is given by the following programme:

$$U_T^i\left(z^i, \left\{\tau_t^A, \tau_t^B\right\}_{t=1}^T\right) = \max_{\{c^i,k^i\}} u^i\left(c_1^i\right) + \beta E \sum_{t=2..T} \beta^t u^i\left(c^i(h_t)\right)$$

[2]For legibility, we will often use a more abbreviated notation for such an agreement on transfers: $\left\{\tau_t^j\right\}$.

subject to

$$\begin{aligned} c_1^i + \rho k_2^i + \tau_1^i &\leq z^i + \tau_1^{-i} \\ c_t^i(h_t) + k_t^i(h_t) + \tau_t^i(h_t) &\leq \frac{1}{\rho} k_{t-1}^i(h_{t-1}) + y_s^i + \tau_t^{-i}(h_t) \\ \text{for } t &= 2..T, h_t = H_t \end{aligned}$$

Using ϕ to denote the absence of an agreement, $U_T^i\left(z^i, \phi\right)$ denotes the maximum expected utility person i can achieve in autarky given initial resources z^i.

We define $I_T^i\left(z^i, \left\{\tau_t^j\right\}\right)$ as the amount of money person i would be willing to forego in the current period, given assets z^i, to participate in a scheme which prescribes transfers $\left\{\tau_t^j\right\}_{t=2}^T$ in future periods. Thus if the transfers provide insurance (i.e. dictates a higher level of net transfer for higher income realisations), then $I_T^i\left(z^i, \left\{\tau_t^j\right\}\right)$ is a measure of the risk-aversion of person i at wealth level z^i. If a transfers scheme $\left\{\tau_t^j\right\}$ specifies net transfers $I_T^i\left(z^i, \left\{\tau_t^j\right\}\right)$ from person i in period 1 followed by $\left\{\tau_t^j\right\}_{t=2}^T$, we have, by definition,

$$U_T^i\left(z^i, \left\{\tau_t^j\right\}\right) - U_T^i\left(z^i, \phi\right) \equiv 0$$

This equation will frequently appear in our analysis since it will hold whenever the participation constraint is binding for person i for a particular allocation of resources (and the future allocation of resources can be represented by an agreement on transfers). We will, in particular, be interested in the effect of additional saving by person i on a binding participation constraint, which is the subject of the following lemma.

Lemma 2.1 *:Given a transfers scheme* $\left\{\tau_t^j\right\}$, *if* $\tau_1^i(s) - \tau_1^{-i}(s) = I_T^i\left(z^i, \left\{\tau_t^j\right\}_{t=2}^T\right)$ *for some* s, *then*

$$\frac{\partial U_T^i\left(z^i, \left\{\tau_t^j\right\}\right)}{\partial z^i} - \frac{\partial U_T^i\left(z^i, \phi\right)}{\partial z^i} = -\frac{\partial U_T^i\left(z^i, \left\{\tau_t^j\right\}\right)}{\partial \tau_1^i(s)} \frac{\partial I_T^i\left(z^i, \left\{\tau_t^j\right\}\right)}{\partial z^i}$$

The first-term on the right-side is the effect on person i's utility of making an additional unit of transfer in period 1, state s and is always negative. The second term is the effect of additional wealth on the 'premium' that individual i is willing to pay in period t to participate in a future transfers scheme $\left\{\tau_t^j\right\}_{t=2}^T$. If the transfers scheme provides insurance, then the premium depends on the degree of risk aversion of individual i; and the sign of the second term depends on how risk aversion changes with the level of wealth.

2.4 Characterisation of Subgame-perfect Allocations

In this section, we characterise allocations and agreements on transfers that can be supported in a subgame perfect equilibrium. These characterisations will be used later to analyse constrained efficient agreements. The results in this section are based on the techniques developed in Abreu (1988) and closely follows the reasoning behind a similar proposition in Kocherlakota (1996).

Proposition 2.1 *: An allocation* $\mathcal{A} = \{k_t^i(h_t), \tau_t^i(h_t)\}_{i=A,B;t=1..T}$ *and the associated consumption stream* $\{c_t^i(h_t)\}_{i,t,h_t}$ *can be obtained in a subgame perfect equilibrium if and only if it satisfies the following conditions:*

$$\begin{aligned} u^i\left(c_t^i(h_t)\right) + \beta E_t \sum_{\varepsilon=t+1}^{\infty} u^i\left(c_\varepsilon^i(h_\varepsilon)\right) &\geq U^i\left(k_{t-1}^i(h_{t-1}) + \tau_t^{-i}(h_t) + y_s^i, \phi\right) \qquad (2.1) \\ i &\in \{A,B\}, \forall h_t \in \mathcal{H}_t, t = 1..\infty \end{aligned}$$

where $c_t^i(h_t) = \frac{1}{\rho}k_{t-1}^i(h_{t-1}) + y_s^i + \tau_t^{-i}(h_t) - \tau_t^i(h_t) - k_t^i(h_t)$.

The conditions ensure that for each person i, after each possible history, the expected utility obtained from the consumption path specified by the allocation is at least as large as the maximum expected utility that can be obtained under autarky (after person i has received the transfers due to him in that period). The reasoning behind the proposition is briefly sketched here. The proof can be found in the Appendix. If a particular allocation is obtained in a subgame perfect equilibrium, then it must satisfy the conditions above; if it does not hold for some individual after some history, she could improve her expected utility by deviating to the autarkic strategy in that subgame. Conversely, if an allocation satisfies the conditions specified above, then a strategy profile along the following lines would be subgame perfect: each individual, after each possible history, chooses transfers and savings as specified by the allocation; if any individual deviates at a point in time, then he receives no transfers thereafter. As the cost of deviation is autarky, which cannot be utility-improving by construction, this strategy profile is subgame perfect.

We also consider allocations that can be obtained in a subgame perfect equilibrium when the strategy space is restricted to 'asset-blind' strategies; i.e. strategies which may be contingent on the history of shocks and transfers but not the level of saving made by other individuals. When individuals are using 'asset-blind' strategies, one may choose to save as much or as little as one wishes in any period. Therefore, an allocation obtained in a subgame perfect equilibrium using 'asset-blind' strategies will be called an 'agreement on transfers' as opposed to an 'agreement on transfers and saving'. In the next section, we provide a characterisation of constrained efficient agreements on transfers, which will be used later to analyse constrained efficient agreements on transfers and savings. However, agreements on transfers can be of interest in their own right; for, under many circumstances, the level of individual savings may not be publicly observable.

It is possible to characterise the set of allocations that can be obtained in subgame perfect equilibria using asset-blind strategies by adding a set of 'Euler' conditions to those specified in Proposition 2.1. As strategies do not depend on saving decisions, individuals can choose to save as they wish without being penalised:

Proposition 2.2 *: An allocation $\mathcal{A} = \left\{k_t^i(h_t), \tau_t^i(h_t)\right\}_{i,t,h_t}$ and the associated consumption stream $\left\{c_t^i(h_t)\right\}_{i,t,h_t}$ can be obtained in a subgame perfect equilibrium with asset-blind strategies if and only if it satisfies the following conditions:*

$$
\begin{aligned}
u^i\left(c_t^i(h_t)\right) + \beta E_t \sum_{\varepsilon=t+1}^{\infty} u^i\left(c_\varepsilon^i(h_\varepsilon)\right) &\geq U_{T-t}^i\left(k_{t-1}^i(h_{t-1}) + \tau_t^{-i}(h_t) + y_s^i, \phi\right) && (2.2)\\
\rho u^{i\prime}\left(c_t^i(h_t)\right) &\geq \beta E\left[u^{i\prime}\left(c_t^i(h_{t+1})\right)|h_t\right] && (2.3)\\
i &\in \{A, B\}, \forall h_t \in \mathcal{H}_t, t = 1..\infty
\end{aligned}
$$

where $c_t^i(h_t) = \frac{1}{\rho}k_{t-1}^i(h_{t-1}) + y_s^i + \tau_t^{-i}(h_t) - \tau_t^i(h_t) - k_t^i(h_t)$ and (2.3) is satisfied with strict equality if $k_t^i(h_t) > 0$.

The intuition behind this result is as follows. If an allocation specifies a consumption stream that does not satisfy the 'Euler' condition for some individual, after some history (or in other words, the marginal rate of intertemporal substitution is not equal to the marginal rate of transformation), this individual could improve expected utility by choosing a different level of saving, without changing the prescribed transfers. As the strategies are asset-blind, such a deviation cannot carry a penalty. Therefore, if an allocation is to be obtained in a subgame perfect equilibrium using asset-blind strategies, it must satisfy the Euler conditions. Conversely, if the consumption stream specified by an allocation satisfies the 'Euler' condition for each individual, in each period, it is not possible for an individual to improve expected utility through a change in the savings decisions alone; as the savings, by construction, are optimal given the income streams and conditional transfers. In addition, using an argument similar to that made in the case of the previous proposition, it is not possible to improve utility through a deviation involving transfers alone. Using the Single-Deviation rule, we can then argue that the allocation can be obtained in a subgame perfect equilibrium using asset-blind strategies only.

2.5 Constrained Efficient Agreements

In this section, we consider constrained efficient agreements between A and B, for a game that lasts for a finite number of periods. As the game lasts a finite number of periods, it is necessary to assume the existence of an exogenous mechanism that ensures some level of commitment with an agreement. Without such a mechanism, individuals would have no incentive to make transfers in the final period; and consequently, it would not be possible obtain insurance in the earlier periods eithers. For analytical simplicity, it is assumed for this example that individuals who violate an agreement during the final period of their lives can be punished severely; such that there is full commitment in this period. No such mechanism is available during the earlier periods, such that compliance with an agreement can be obtained only with the threat of autarky in subsequent periods.

The main insights in this paper can be captured in a three-period repetition of the stage game. Therefore, we first analyse and provide results for constrained efficient agreements for such a game. Section 2.5.4 extends the analysis to a finite horizon game of length $T > 3$.

2.5.1 Two-Period Continuation Game

In any constrained efficient agreement for the three-period game, the continuation agreements beginning in period 2 must also be constrained efficient. As argued in Ligon, Thomas and Worrall (2001), if any continuation agreement is not constrained efficient, it is possible to replace this continuation agreement by another which is Pareto dominating; and thus weakly relax all participation constraints. Then if the original agreement were feasible, so would be the new one and it would pareto-dominate the first. Therefore, we first characterise constrained efficient agreements for the continuation game beginning in period 2, and use this characterisation to analyse constrained efficient agreements for the three-period game.

Define $V_2\left(k^A, k^B, \bar{U}\right)$ as the maximum utility that person B can obtain in an equilibrium of this subgame, if person A must receive a utility of at least U; and the level of savings for A and B at the beginning of the period are given by k^A and k^B. Then $V_2\left(k^A, k^B, U\right)$ is given by the following programme:

$$
\begin{aligned}
V_2\left(k^A, k^B, U\right) &= \\
\max_{\left\{k_s^j, \tau_s^j\right\}_{s \in S, j \in \{A,B\}}} &: \quad E\left[u^B\left(c_s^B\right) + \beta u^B\left(c_{sr}^B\right)\right] \\
\text{subject to} &: \\
\lambda &: \quad E\left[u^A\left(c_s^A\right) + \beta u^A\left(c_{sr}^A\right)\right] \geq U \\
\pi_s \theta_s^i &: \quad u^i\left(c_s^i\right) + \beta E u^i\left(c_{sr}^i\right) \geq U_2^i\left(k^i + y_s^i + \tau_s^{-i}, \phi\right), \forall s \in S, i \in \{A, B\}
\end{aligned}
$$

where $c_s^i = k^i + y_s^i - \tau_s^i + \tau_s^{-i} - \rho k_s^i$, $c_{sr}^i = k_s^i + y_r^i - \tau_{sr}^i + \tau_{sr}^{-i}$, $i \in \{A, B\}$.

The first constraint ensures that person A receives an expected utility of at least U. The second set of constraints correspond to the conditions stated in Proposition 2.1 for subgame perfect equilibria[3]. From the first-order conditions of the programme we obtain the following equations.

$$\frac{u^{B\prime}\left(c_s^B\right)}{u^{A\prime}\left(c_s^A\right)}=\frac{u^{B\prime}\left(c_{sr}^B\right)}{u^{A\prime}\left(c_{sr}^A\right)}=\lambda_s=\frac{\lambda+\theta_s^A}{1+\theta_s^B}$$

$$\rho=\beta\frac{Eu^{i\prime}\left(c_{sr}^i\right)}{u^{i\prime}\left(c_s^i\right)}$$

The first equation says that the ratio of marginal utilities in this equilibrium will be fixed across periods 2 and 3. The second equation states that the expected marginal rate of substitution will equal the marginal rate of transformation, ρ for each person between periods 2 and 3. These results are the same as those obtained for the full-insurance model and hold here because we have effectively assumed full insurance in period 3.

On the other hand, in period 2, the ratio of marginal utilities can vary across states. In states where both participation constraints are slack, i.e. $\theta_s^A=\theta_s^B=0$, we obtain $\lambda_s=\lambda$. If the constraint binds for B but is slack for A, i.e. $\theta_s^B>0$, $\theta_s^A=0$ then $\lambda_s=\frac{\lambda}{1+\theta_s^B}<\lambda$; and similarly, if the constraint binds for A but is slack for B, then $\lambda_s=\lambda+\theta_s^A>\lambda$. The participation constraints of both individuals will not bind in any state as long as some allocation other than autarky is subgame perfect. This parallels the well-known result in the literature on informal insurance under limited commitment, (discussed in Kocherlaka 1996 and Ligon, Thomas and Worrall 2001 among others) that the ratio of marginal utilities is more favourable to an individual in the states where his participation constraint is binding.

Using the Envelope Theorem, we obtain the following conditions:

$$\begin{aligned}\frac{\partial V_2}{\partial k^i}&=\lambda^i Eu^{i\prime}\left(c_s^i\right)+\sum_s\pi_s\theta_s^i\left[u^{i\prime}\left(c_s^i\right)-\frac{\partial U_2^i}{\partial k^i}\left(k^i+y_s^i+\tau_s^{-i}\right)\right],i=A,B\\ \frac{\partial V_2}{\partial U}&=-\lambda\end{aligned}$$

where $\lambda^i=\lambda$ for $i=A$ and $\lambda^i=1$ for $i=B$.

Note that we can write the continuation utility to person i from a two-period agreement after state s has been realised in the first period as $U_2^i\left(k^i+y_s^i,\left\{\tau_t^j\right\}_{t=2}^3\right)$. Then, using Lemma 2.1 obtained in section 2.3, we can rewrite the Envelope conditions as follows:

[3] We may ask whether we can innocuously drop the term τ_s^{-i} from the expression on the right-hand side of person i's particpation constraint by assuming that in the transfers scheme only one person is making (non-zero) transfers to the other at any point in time. The answer is no. The reason is that a person who is required to make zero transfers after some history may still pocket the transfers that are due to him and then exit the agreement at the savings stage. He will then receive a utility of $U_2^i\left(k^i+y_s^i+\tau_s^{-i},\phi\right)$ as in the programme.

$$\frac{\partial V_2}{\partial k^i} = \lambda^i E u^{i\prime}\left(c_s^i\right) - \sum_s \theta_s^i \frac{\partial U^i}{\partial \tau_s^i}\frac{\partial I^i}{\partial z^i}\left(k^i + y_s^i, \left\{\tau_t^j\right\}_{t=3}\right), i = A, B \tag{2.4}$$

The main insight in this paper is captured in this equation. The equation represents the increase in person B's expected utility from an additional unit of saving at the start of period 2 by person i, keeping A's expected utility constant at U. For the first-best agreement, this would simply be equal to the expected marginal utility of B in period 2. However, if person i's participation constraints are binding for some states in period 2, then raising her initial level of saving will also affect the maximum transfers that she is willing to pay out in those states; this effect depends on the magnitude of the term $\frac{\partial I^i}{\partial z^i}\left(k^i + y_s^i, \left\{\tau_t^j\right\}_{t=3}\right)$, the change in person B's insurance premium for the insurance provided by the period 3 transfers $\left\{\tau_t^j\right\}_{t=3}$, which is precisely the maximum amount she is willing to pay in period 2 in state s.

It is straightforward to show that the sign and magnitude of this expression depends on whether the Bernoulli utility function, $u^i(.)$ exhibits constant, decreasing or increasing absolute risk aversion. For constant absolute risk aversion, $\frac{\partial I^i}{\partial z^i}\left(k^i + y_s^i, \left\{\tau_t^j\right\}_{t=3}\right) = 0$. And, similarly, for decreasing and increasing absolute risk aversion, we obtain $\frac{\partial I^i}{\partial z^i}\left(k^i + y_s^i, \left\{\tau_t^j\right\}_{t=2}\right) < 0$ and $\frac{\partial I^i}{\partial z^i}\left(k^i + y_s^i, \left\{\tau_t^j\right\}_{t=2}\right) > 0$ respectively. These results are derivated in Lemma 2.2 in the appendix.

2.5.2 Constrained Efficient Agreements for T=3

Define $V_3\left(k^A, k^B, U\right)$ as the maximum utility that person B can obtain in an equilibrium of the three-period game, if person A must receive a utility of at least U; and the level of savings for A and B at the beginning of the period are given by k^A and k^B. Then $V_3\left(k^A, k^B, U\right)$ is given by the following programme.

$$\begin{aligned}
V_3\left(k^A, k^B, U\right) &= && (2.5)\\
\max_{\left\{k_s^A, k_s^B, \tau_s^A, \tau_s^B, U_s\right\}_{s\in S}} &: E\left[u^B\left(c_s^B\right) + \beta V_2\left(k_s^A, k_s^B, U_s\right)\right] \\
s.t &: \\
\lambda &: E\left[u^A\left(c_s^A\right) + \beta U_s\right] \geq U \\
\theta_s^B &: u^B\left(c_s^B\right) + \beta V_2\left(k_s^A, k_s^B, U_s\right) \geq U_3^B\left(k^B + y_s^B + \tau_s^A, \phi\right), \forall s \in S && (2.6)\\
\theta_s^A &: u^A\left(c_s^A\right) + \beta U_s \geq U_3^A\left(k^A + y_s^A + \tau_s^B, \phi\right), \forall s \in S && (2.7)
\end{aligned}$$

where $c_s^i = k^i + y_s^i + \tau_s^{-i} - \left(\rho k_s^i + \tau_s^i\right)$; the variables $\left\{k_s^A, k_s^B, \tau_s^A, \tau_s^B\right\}_{s\in S}$ describe, for each possible state of thr world in period 1, the prescribed savings and transfers by each individual, and U_s is the promised utility to person A in the continuation game. Then, the solution to the programme for $V_2\left(k_s^A, k_s^B, U_s\right)$, as analysed previously, would determine the state contingent savings and transfers during the second and third periods of the game. The conditions in (2.6) and (2.7) correspond to the conditions in Proposition 2.1 at $t = 1$.

From the first-order conditions for the constrained maximisation problem in (2.5), we obtain the following conditions, for each $s \in \mathcal{S}$,

$$\rho u^{B\prime}\left(c_s^B\right) = \beta \frac{\partial V_2}{\partial k_s^B} \tag{2.8}$$

$$\rho \lambda_s u^{A\prime}\left(c_s^A\right) = \beta \frac{\partial V_2}{\partial k_s^A} \tag{2.9}$$

$$\frac{\partial V_2}{\partial U_s} = \frac{u^{B\prime}\left(c_s^B\right)}{u^{A\prime}\left(c_s^A\right)} = -\lambda_s \tag{2.10}$$

where $\lambda_s = \frac{\lambda \pi_s + \theta_s^A}{\pi_s + \theta_s^B}$.

Substituting for $\frac{\partial V_2}{\partial k_s^i}$ in (2.8) and (2.9), using the equations in (2.4) derived above, we obtain

$$\rho u^{i\prime}\left(c_s^i\right) = \beta E u^{i\prime}\left(c_s^i\right) - \beta \frac{1}{\lambda_s^i} \sum_s \theta_s^i \frac{\partial U^i}{\partial \tau_s^i} \frac{\partial I^i}{\partial z^i}\left(k^A + y_s^A, \left\{\tau^j\right\}_{t=3}\right), i = A, B \tag{2.11}$$

where $\lambda_s^i = \lambda_s$ for $i = A$ and $\lambda_s^i = 1$ for $i = B$.

These equations are equivalent to the standard Euler condition except for the additional term on the right-hand side. This term is non-zero if the participation constraint is binding for person i in period 2 for some state r and $\frac{\partial I^i}{\partial z^i}\left(k^i + y_r^i, \left\{\tau^j\right\}_{t=3}\right) \neq 0$; furthermore, since the insurance premium $I^i\left(k^i + y_r^i, \left\{\tau^j\right\}_{t=3}\right)$ depends on person i's aversion to risk, the size and the magnitude of the additional term in the Euler condition depends on how absolute risk aversion changes with wealth for person i. In particular, we can establish the following result.

Proposition 2.3 *: Given a constrained efficient allocation $\mathcal{A} = \left\{k_t^i\left(h_t\right), \tau_t^i\left(h_t\right)\right\}_{i=A,B;t=1..T}$ and the associated consumption stream $\left\{c_t^i\left(h_t\right)\right\}_{i=A,B,t=1..T}$, if the participation constraint binds in some state in period 2 for individual i, then the marginal utilities satisfy the following conditions for each $s \in \mathcal{S}$:*

(i) $\rho u^{B\prime}\left(c_1^B(s)\right) < \beta E u^{B\prime}\left(c_2^B(s,r)\,|s\right)$ if i has decreasing absolute risk aversion

(ii) $\rho u^{B\prime}\left(c_1^B(s)\right) > \beta E u^{B\prime}\left(c_2^B(s,r)\,|s\right)$ if i has increasing absolute risk aversion

If i has constant absolute risk aversion or his participation constraint is slack in all states in period 2, then $\rho u^{B\prime}\left(c_1^B(s)\right) = \beta E u^{B\prime}\left(c_2^B(s,r)\,|s\right)$.

An immediate collorary of Proposition 2.3 is that for an agent with decreasing (increasing) absolute risk aversion, in period 1 the constrained efficient agreement prescribes a level of saving that is less (more) than what is optimal under a full commitment agreement that prescribes the same contingent transfers. A formal statement and proof of this collorary can be found in the appendix.

According to Proposition 2.3, if an individual's aversion to risk does not change with the level of wealth, then in the constrained efficient agreement his savings decision is determined by the standard Euler equation. We know from Proposition 2.2 that if an allocation satisfies the participation constraints and the Euler condition for each individual after each possible history, then it can be obtained in a subgame perfect equilibrium using asset-blind strategies. Therefore, a constrained efficient allocation can be obtained using asset-blind strategies if and only if all individuals exhibit constant absolute risk aversion. On the other hand, if risk aversion changes with the level of wealth for any individual, then his consumption path in the constrained efficient agreement will not satisfy the Euler condition; and consequently the agreement cannot be obtained using asset-blind strategies. We have thus established the following proposition.

Proposition 2.4 : *A constrained efficient allocation can be replicated using an agreement on transfers if and only if A and B both have constant absolute risk aversion.*

Proposition 2.4 addresses the question whether, in the context of an informal insurance agreement among a group of individuals or households, there are any benefits from being able to monitor the savings decisions by agents. If there are no means to monitor these decisions, then we are able to use only asset-blind strategies to implement an agreement. Proposition 2.4 states that if agents have constant absolute risk aversion, then we can use asset-blind strategies to implement any constrained efficient agreement; and therefore there are no additional benefits from being able to monitor agents' savings. On the other hand, if agents' risk aversion changes with wealth, a constrained efficient agreement cannot be attained using asset-blind strategies and therefore there are possible gains from being able to monitor savings by agents.

This result has some parallels with the finding in Rogerson (1985), for a model of repeated moral hazard, that if an agent's saving are unobservable/noncontractable, this is welfare-reducing. Here, we establish that the same is true under limited commitment if risk aversion changes with wealth; but that if preferences exhibit constant absolute risk aversion, an agent's incentives to save are aligned with those of a social planner.

2.5.3 Decreasing Risk-Aversion and Poverty

In this section, we consider a particular setting where the results obtained in the previous section, relating risk-aversion and the intertemporal saving decision under limited commitment, are likely to be important.

We assume that there is a discontinuity in the utility function, such that individuals experience a large disutility when consumption falls below a particular threshold. This threshold can be interpreted as the level of expenditures necessary to provide for one's basic necessities such as food, shelter, heating, etc.; and expenditures below this level would adversely affect one's health, perhaps even posing a threat to one's own life or that of a family member. Therefore, if an individual faces a positive risk of experiencing consumption below the threshold, he or she would be willing to pay a large premium to participate in an agreement that insures him against his income shocks.

Formally, we assume that the Bernoulli utility function takes the following form:

$$u(c) = -e^{-rc}\left[1 + X\mathbf{I}(c < \bar{c})\right]$$

where $\mathbf{I}(c < \bar{c})$ is an indicator function, which takes a value of 1 if $c < \bar{c}$ and 0 otherwise. For small risks, the utility function exhibits constant absolute risk aversion but there is a discontinuity of size X at $\bar{c}$. Suppose the individual has wealth w and, as before, faces stochastic income y_r which depends on the state of nature $r \in S$, and the probability that state r is realised is π_r. Define $\bar{I}(w)$ as the maximum insurance premium that the individual is willing to pay for a scheme that replaces his stochastic income with a certain income of $\bar{y} = Ey_r$. Therefore, $\bar{I}(w)$ is given by

$$\arg\max_{I} \left\{I : u\left(w + \bar{y} - \bar{I}(w)\right) \geq Eu\left(w + y_r\right)\right\}$$

To simplify the analysis, we restrict attention to wealth levels, w which satisfy

$$-e^{-r\bar{c}} \leq -\sum_{r \in S(w)} \pi_r e^{-r(w+y_r)} - \sum_{r \notin S(w)} \pi_r (1 + X) e^{-r(w+y_r)}$$

where $S(w) = \{s \in S : w + y_s \geq \bar{c}\}$. Then, there is some $I \leq w + \bar{y} - \bar{c}$ which solves

$$\implies -e^{-r\left(w+\bar{y}-\bar{I}(w)\right)} = -\sum_{r \in S(w)} \pi_r e^{-r(w+y_r)} - \sum_{r \notin S(w)} \pi_r (1 + X) e^{-r(w+y_r)}$$

That is, the maximum insurance premium $\bar{I}(w)$ leaves the individual with above-the-threshold consumption. Multiplying both sides by e^{rw}, and rearranging, we obtain

$$-e^{-r\left(\bar{y}-\bar{I}(w)\right)} = -\sum_{r \in S} \pi_r e^{-ry_r} - \sum_{r \notin S(w)} \pi_r X e^{-ry_r}$$

Then, the only term on the right-hand side that changes with w is $\sum_{r \notin S(w)} \pi_r e^{-ry_r}$. Since the probability that consumption will fall below the critical level in autarky decreases with wealth, we obtain the result that $\bar{I}(w)$ is (weakly) decreasing in w if $\min_r (w + y_r) < \bar{c}$ and $\bar{I}(w)$ is unchanging in w if not.

Intuitively, if the individual has a low level of wealth, he faces a positive risk of experiencing consumption below the threshold $\bar{c}$ without insurance. Given the disutility associated with consumption below the threshold, he is willing to pay a large amount of money to avoid the risk. As his wealth increases, the probability of falling below the threshold declines and so does his willingness to pay for insurance. When he is sufficiently wealthy, he no longer faces a risk of experiencing consumption below the threshold; and therefore the insurance premium is independent of this risk. Consequently, the insurance premium is less sensitive to changes in wealth for rich individuals than for poor individuals. In our particular example, the insurance premium does not change at all as the Bernoulli utility function exhibits constant absolute risk aversion for small risks.

For these preferences, the results in the previous section imply increasing inequality in consumption in the precise sense to be defined below. Suppose individual i's savings at the end of period 1 is sufficiently low that his consumption in some states in the subsequent periods would fall below the threshold $\bar{c}$ in autarky. Then, given the analysis in this section, individual i exhibits decreasing absolute risk aversion. Therefore, according to Proposition 2.3, the consumption path for person i in the subsequent periods satisfies the inequality

$$\rho u^{i\prime}\left(c_s^i\right) < \beta E_s u^{i\prime}\left(c_{sr}^i\right) \tag{2.12}$$

On the other hand, an individual j whose wealth at the end of period 1 is sufficiently high that his consumption in autarky would remain above the threshold in autarky exhibits constant absolute risk aversion. Therefore his consumption path satisfies the standard Euler condition:

$$\rho u^{j\prime}\left(c_s^j\right) = \beta E_s u^{j\prime}\left(c_{sr}^j\right) \tag{2.13}$$

Dividing (2.12) by (??), we obtain

$$\frac{u^{i\prime}\left(c_s^i\right)}{u^{j\prime}\left(c_s^j\right)} < \frac{E_s u^{i\prime}\left(c_{sr}^i\right)}{E_s u^{j\prime}\left(c_{sr}^j\right)} \tag{2.14}$$

In words, the ratio of marginal utilities in period 1 is smaller than the ratio of expected marginal utilities in period 2. In this sense the allocation of resources for consumption in the constrained efficient agreement increasingly favours the wealthier individual over time.

Although the individual savings at the end of period 1 are endogenous to the agreement, it is straightforward to show that they are increasing in the level of initial assets: a higher level of assets at the beginning of period 1 makes autarky more attractive; participation constraints will be tighter and consequently the individual will have to be promised a higher level of continuation utility when the state of nature is realised in period 1. Now, a higher level of promised utility going into period 2 means that the individual is more committed to the continuation agreement. Therefore, he is also given a greater share of total assets at the end of period 1.

2.5.4 Constrained Efficient Agreements for T>3

It is straightforward to extend the analysis to the case where $T > 3$. If $V_n\left(k^A, k^B, U\right)$ defines the Pareto frontier of continuation agreements in period $T - n$ (i.e. when n periods remain in the game) that can be supported in a subgame perfect equilibrium, then the sequence of functions for $n = 3, ...T$ satisfy the following relationship:

$$
\begin{aligned}
V_n\left(k^A, k^B, U\right) &= && (2.15)\\
\max_{\{k_s^A, k_s^B, \tau_s^A, \tau_s^B, U_s\}_{s\in S}} &: && E\left[u^B\left(c_s^B\right) + \beta V_{n-1}\left(k_s^A, k_s^B, U_s\right)\right]\\
s.t &: && \\
\lambda &: && E\left[u^A\left(c_s^A\right) + \beta U_s\right] \geq U\\
\theta_s^B &: && u^B\left(c_s^B\right) + \beta V_{n-1}\left(k_s^A, k_s^B, U_s\right) \geq U_n^B\left(\frac{1}{\rho}k^B + y_s^B + \tau_s^A, \phi\right), \forall s \in S\\
\theta_s^A &: && u^A\left(c_s^A\right) + \beta U_s \geq U_n^A\left(\frac{1}{\rho}k^A + y_s^A + \tau_s^B, \phi\right), \forall s \in S
\end{aligned}
$$

where $c_s^i = \frac{1}{\rho}k^i + y_s^i + \tau_s^{-i} - \left(k_s^i + \tau_s^i\right)$.

We obtain the following equations using the Envelope theorem:

$$
\frac{\partial V_n}{\partial k^B} = Eu^{B\prime}\left(c_s^B\right) + \sum_s \pi_s \theta_s^B\left[u^{B\prime}\left(c_s^B\right) - \frac{\partial U_n^B}{\partial k^B}\left(k^B + y_s^B + \tau_s^A, \phi\right)\right] \tag{2.16}
$$

$$
\frac{\partial V_n}{\partial k^A} = \lambda Eu^{A\prime}\left(c_s^A\right) + \sum_s \pi_s \theta_s^A\left[u^{A\prime}\left(c_s^A\right) - \frac{\partial U_n^A}{\partial k^A}\left(k^A + y_s^A + \tau_s^B, \phi\right)\right] \tag{2.17}
$$

We cannot proceed as in the previous section to determine the sign of the last term of the right-hand side of equations (2.16) and (2.17), because in period $T - n$, where $n > 1$, the continuation agreement differs from autarky not only in that the agreement prescribes contingent transfers but also because it may require individuals to save more or less than what is individually optimal.

To compare continuation values from an agreement and from autarky, we define $\tilde{U}_n^i\left(z^i, \left\{k_t^j\right\}_{t=1}^n, \left\{\tau_t^j\right\}_{t=1}^T\right)$ as the maximum expected utility to person i from an agreement that specifies transfers and savings during the first n periods, followed by a continuation agreement on transfers only; i.e. the agreement prescribes transfers $\left\{\tau_t^j\right\}_{t=1}^T$, and savings $\left\{k_t^j\right\}_{t=1}^n$ during the first n periods, which are potentially different from the saving path that is optimal given $\left\{\tau_t^j\right\}_{t=1}^T$. Then, if the solution to the programme above is given by $\left(\left\{k_t^j\right\}_{t=1}^T, \left\{\tau_t^j\right\}_{t=1}^T\right)$, then the continuation utility to person i, after state s is realised in period $T - n$, can be written as

$$
\tilde{U}_n^i\left(k^i + y_s^i, \left\{k_t^j\right\}_{t=T-n+1}^{T-2}, \left\{\tau_t^j\right\}_{t=T-n}^T\right)
$$

since the level of saving in the last two periods are always individually optimal. It is straightforward to show, using the Envelope theorem, that

$$\frac{d\tilde{U}_n^i}{dk^i}\left(k^i + y_s^i, \left\{k_t^j\right\}_{t=T-n+1}^{T-2}, \left\{\tau_t^j\right\}_{t=T-n}^{T}\right) = u^{i\prime}\left(c_s^i\right) \tag{2.18}$$

where c_s^i is the level of consumption allocated to person i in the agreement after state s is realised in period $T-n$. Then, substituting for $u^{i\prime}\left(c_s^i\right)$ in (2.16) and (2.17) using (2.18), we obtain

$$\begin{aligned}
\frac{dV_n}{dk^B}\left(k^A, k^B, U\right) &= Eu^{B\prime}\left(c_s^B\right) + \sum_{s\in S}\theta_s^B\left[\frac{d\tilde{U}_n^B}{dk^B}\left(k^B + y_s^B, \left\{k_t^j\right\}_{t=T-n+1}^{T-2}, \left\{\tau_t^j\right\}_{t=T-n}^{T}\right) - \frac{\partial U_n^B}{\partial k^B}\left(k^B + y_s^B + \tau_s^A\right.\right. \\
\frac{dV_n}{dk^A}\left(k^A, k^B, U\right) &= \lambda Eu^{A\prime}\left(c_s^A\right) + \sum_{s\in S}\theta_s^A\left[\frac{d\tilde{U}_n^A}{dk^A}\left(k^A + y_s^A, \left\{k_t^j\right\}_{t=T-n+1}^{T-2}, \left\{\tau_t^j\right\}_{t=T-n}^{T}\right) - \frac{\partial U_n^A}{\partial k^A}\left(k^A + y_s^A + \tau_s^B\right.\right.
\end{aligned}$$

For $n = 3$, we can rewrite equation (2.19) as follows:

$$\begin{aligned}
\frac{dV_3}{dk^B}\left(k^A, k^B, U\right) &= Eu^{B\prime}\left(c_s^B\right) \\
&+\sum_{s\in S}\theta_s^B\left[\frac{d\tilde{U}_3^B}{dk^B}\left(k^B + y_s^B, k_{T-2}^B, \left\{\tau_t^j\right\}_{t=T-2}^{T}\right) - \frac{dU_3^B}{dk^B}\left(k^B + y_s^B, \left\{\tau_t^j\right\}_{t=T-1}^{T}\right)\right] \\
&+\sum_{s\in S}\theta_s^B\left[\frac{dU_3^B}{dk^B}\left(k^B + y_s^B, \left\{\tau_t^j\right\}_{t=T-1}^{T}\right) - \frac{\partial U_n^B}{\partial k^B}\left(k^B + y_s^B + \tau_s^A, \phi\right)\right]
\end{aligned} \tag{2.21}$$

The last term on the right-hand side is the same as that obtained in Section 2.5 for the two-period continuation agreement, and we can substitute it, using Lemma 2.1, with an expression involving the insurance premium. The second-last term on the right-hand side can also be expressed in terms of the insurance premium using the following equation, which is derived in the appendix.

$$\frac{d\tilde{U}_3^B}{dk^B}\left(k^B + y_s^B, k_{T-2}^B, \left\{\tau_t^j\right\}_{t=T-2}^{T}\right) - \frac{dU_3^B}{dk^B}\left(k^B + y_s^B, \left\{\tau_t^j\right\}_{t=T-1}^{T}\right) = -\beta\sum_s\theta_{T-1,s}^B\frac{\partial U_2^B}{\partial\tau_{T-2}^B}\frac{\partial I^B}{\partial z^B}\left(k^B + y_s^B, \left\{\tau_T^j\right\}\right. \tag{2.22}$$

Thus, substituting for the last two terms on the right-hand side of equation (2.21) using Lemma

2.1 and equation (2.22), we obtain

$$\begin{aligned}
\frac{dV_3}{dk^B}\left(k^A,k^B,U\right) &= Eu^{B\prime}\left(c_s^B\right) \\
&\quad -\beta\sum_{s\in\mathcal{S}}\theta_{T-2}^B(s)\sum_{r\in\mathcal{S}}\theta_{T-1}^B(s,r)\frac{\partial U_{T-1}^B}{\partial\tau_{T-1}^B}\frac{\partial I^B}{\partial k^B}\left(k_{T-1,s}^B+y_r^B,\left\{\tau_T^j\right\}\right) \\
&\quad -\sum_{s\in\mathcal{S}}\theta_{T-2}^B(s)\frac{\partial U_{T-2}^B}{\partial\tau_{T-2}^B}\frac{\partial I^B}{\partial k^B}\left(k^B+y_s^B,\left\{\tau_t^j\right\}_{t=T-1}^T\right)
\end{aligned}$$

where $\theta_t^i(h_t)$ is the value of the Kuhn-Tucker multiplier for person i's participation constraint in period t following history h_t.

This equation can be interpreted in the same manner as the Envelope condition for the continuation agreement in the penultimate period, derived in the preceding section. It represents the increase in person B's expected utility from an additional unit of saving at the start of period $T-2$ by B, keeping A's expected utility constant at U. For the first-best agreement, this would be equal to the increase in person B's expected marginal utility in period $T-2$, given by $Eu^{B\prime}\left(c_s^B\right)$. However, in the constrained efficient agreement, raising person B's initial wealth will also affect her future participation constraints, if the premium she is willing to pay for future insurance changes with wealth; the last term on the right-hand side represents the effect on participation constraints that bind in period $T-2$; and the second term represents the effect on the constraints that bind in period $T-1$.

Applying the procedure used to obtain the intertemporal condition above recursively, we obtain the following general result:

$$\begin{aligned}
\frac{dV_n}{dk^B}\left(k^A,k^B,U\right) &= Eu^{B\prime}\left(c_s^B\right)-\sum_{t=T-n}^{T-1}\beta^{t-1}\sum_{h_t\in\mathcal{H}_t}\hat{\theta}_{T-n,t}^B(h_t)\frac{\partial U_t^B}{\partial\tau_t^B}\frac{\partial I^B}{\partial k_t^B}\left(k_t^B+y_s^B,\left\{\tau_\epsilon^j\right\}\right) \\
\frac{dV_n}{dk^A}\left(k^A,k^B,U\right) &= \lambda Eu^{A\prime}\left(c_s^A\right)-\sum_{t=T-n}^{T-1}\beta^{t-1}\sum_{h_t\in\mathcal{H}_t}\hat{\theta}_{T-n,t}^A(h_t)\frac{\partial U_t^A}{\partial\tau_t^A}\frac{\partial I^A}{\partial k_t^A}\left(k_t^A+y_s^A,\left\{\tau_\epsilon^j\right\}\right)
\end{aligned}$$

where $\hat{\theta}_{T-n,t}^B(h_t)=\prod_{\eta=T-n}^{t}\theta_\eta^B\left(h_\eta|h_{T-n}\right)$, and $h_\eta|h_t$ is the history of the first η periods contained in h_t. This result is similar to that obtained for the three-period model. It says that, for the constrained efficient agreement, an extra unit of saving by person B after any history will affect all future binding constraints for person B, depending on whether the insurance premium increases or decreases with wealth. Then, using the first-order conditions to the maximisation problem in

(2.15), we obtain the equivalent of the Euler equation:

$$u^{i\prime}\left(c^i_{T-n-1}\right) = Eu^{i\prime}\left(c^i_{T-n}\right) - \frac{1}{\lambda^i_s}\sum_{t=T-n}^{T-1}\beta^{t-1}\sum_{h_t\in\mathcal{H}_t}\hat{\theta}^i_{T-n,t,h_t}\frac{\partial U^i_t}{\partial\tau^i_t}\frac{\partial I^i}{\partial k^i_t}\left(k^i_t + y^i_r, \{\tau^j_\epsilon\}\right)$$

where $i = \{A, B\}$, $\lambda^A_s = \lambda_s$, $\lambda^B_s = 1$. The equivalent of propositions 2.3 and 2.4 can then be obtained following the same arguments used for the three-period model.

2.6 Conclusion

In this paper we anaylsed self-enforcing insurance agreements among individuals who have the means to save independently. We showed that if risk aversion changes with the level of wealth, then the constrained optimal agreement would require individuals to save at a rate different from that which is individually optimal given the contingent transfers prescribed by the agreement.

Of particular interest is the case where individuals have decreasing absolute risk aversion; then the constrained optimal agreement would tend to depress saving to increase commitment to the agreement in future periods. Then the levels of saving observed in the economy would be lower than that predicted by the standard model of competitive equilibrium.

The reason is that in this economy saving carries an additional cost in that it reduces commitment and thus the scope for future insurance. We argue that this cost is especially high in the case of individuals or households who face a significant risk of falling into acute poverty. These individuals would be willing to pay a large premium for insurance against income shocks and are therefore very committed to a mutual insurance agreement. Each additional unit of saving diminishes the risk of falling into poverty under autarky and thus the degree of commitment to the agreement; therefore, in the context of an informal insurance agreement, saving by the poor carries large social costs.

This result implies that the consumption stream prescribed by the constrained efficient insurance agreement under limited commitment will be 'front-loaded' for the poor; they will receive higher levels of consumption in the earlier periods. Therefore, the gap in consumption and assets between the rich and the poor will tend to increase over time.

In many environments, it may not be possible to monitor the savings decisions by agents. For such environments, it is possible to approximate the constrained efficient agreement if and only if individual preferences are close to constant absolute risk aversion. Equivalently, there can be large gains to developing a mechanism to monitor saving decisions if risk aversion changes rapidly with wealth.

2.7 Appendix

Proof. of Lemma 2.1: We are given

$$\tau_1^i(s) - \tau_1^{-i}(s) = I_T^i\left(z^i, \left\{\tau_t^j\right\}_{t=2}^T\right)$$

Therefore, by definition of $I_T^i\left(z^i, \left\{\tau_t^j\right\}_{t=2}^T\right)$, we must have

$$U_T^i\left(z^i, \left\{\tau_t^j\right\}_{t=1}^T\right) - U_T^i\left(z^i, \phi\right) \equiv 0$$

Differentiating throughout with respect to z^i, we obtain

$$\frac{dU_T^i}{dz^i}\left(z^i, \left\{\tau_t^j(h_t)\right\}_{t=1}^T\right) - \frac{dU_T^i}{dz^i}\left(z^i, \phi\right) \equiv 0$$

$$\begin{aligned} \Longrightarrow \quad & \frac{\partial U_T^i}{\partial z^i}\left(z^i, \left\{\tau_t^j(h_t)\right\}_{t=1}^T\right) + \frac{\partial U_T^i}{\partial \tau_1^i}\frac{\partial I^i}{\partial z^i}\left(z^i, \left\{\tau_t^j(h_t)\right\}_{t=2}^T\right) - \frac{\partial U_T^i}{\partial z^i}\left(z^i, \phi\right) = 0 \\ \Longrightarrow \quad & \frac{\partial U_T^i}{\partial z^i}\left(z^i, \left\{\tau_t^j(h_t)\right\}_{t=1}^T\right) - \frac{\partial U_T^i}{\partial z^i}\left(z^i, \phi\right) = -\frac{\partial U_T^i}{\partial \tau_1^i}\frac{\partial I_T^i}{\partial z^i}\left(z^i, \left\{\tau_t^j(h_t)\right\}_{t=2}^T\right) \end{aligned}$$

■

Lemma 2.2 : *Let $I(w, \{\tau_s\})$ be defined as in Section x and suppose $\{-\tau_s\}$ is a 'patent increase in risk' as defined in Kimball (1993). Then $I(w, \{\tau_s\})$ is decreasing/constant/increasing in w if and only if the associated Bernoulli utility function $u(.)$ exhibits decreasing/constant/increasing risk aversion.*

Proof. Let $\theta(w, \{\tau_s\})$ be the insurance premium at wealth level w for contingent transfers $\{\tau_s\}$, associated with the Bernoulli utility function $u(.)$; i.e.

$$Eu\left(w + y_s + \tau_s - \theta\left(w, \{\tau_s\}\right)\right) \equiv Eu\left(w + y_s\right) \tag{2.23}$$

Let k' and k'' be the solutions to the maximisation problem in (2.24) and (2.25) respectively:

$$\begin{aligned} U\left(z, \{\tau_s\}\right) &= \max_k u\left(z - I\left(z, \{\tau_s\}\right) - k\right) + \beta Eu\left(k + y_s + \tau_s\right) && (2.24) \\ U\left(z, .\right) &= \max_k u\left(z - k\right) + \beta Eu\left(k + y_s + \tau_s\right) && (2.25) \end{aligned}$$

Taking the derivative w.r.t. w throughout the identity in (2.23), we obtain

$$Eu'(w + y_s + \tau_s - \theta(w, \{\tau_s\}))[1 - \theta_w] \equiv Eu'(w + y_s)$$

where $\theta_w = \frac{d\theta}{dw}$. If $u(.)$ exhibits decreasing absolute risk aversion, and $\{-\tau_s\}$ is a 'patent increase in risk' then, by definition, $\theta_w < 0$. Then

$$Eu'(w + y_s + \tau_s - \theta(w, \{\tau_s\})) < Eu'(w + y_s)$$

This implies that $k\prime < k'' - \theta(k', \{\tau_s\})$. We can show this by contradiction: If $k\prime \geq k'' - \theta(k', \{\tau_s\})$, then, we have

$$Eu'\left(k' + y_s + \tau_s\right) < Eu'\left(k'' + y_s\right)$$

Therefore, using the Euler conditions, we have

$$u'\left(z - k' - I(z, \{\tau_s\})\right) < u'\left(z - k''\right)$$

$$\Longrightarrow u\left(z - k' - I(z, \{\tau_s\})\right) > u\left(z - k''\right) \tag{2.26}$$

Also, by definition of $\theta(.)$, we have

$$Eu\left(k' + y_s + \tau_s\right) > Eu\left(k'' + y_s\right) \tag{2.27}$$

Then, from (2.26) and (??), we obtain

$$u\left(z - k' - I(z, \{\tau_s\})\right) + \beta Eu\left(k' + y_s + \tau_s\right) > u\left(z - k''\right) + \beta Eu\left(k'' + y_s\right)$$

$$\Longrightarrow U(z - I(z, \{\tau_s\}), \{\tau_s\}) > U(z, \phi) \tag{2.28}$$

which contradicts the definition of $I(.)$. Thus we have established that $k\prime < k'' - \theta(k', \{\tau_s\})$; consequently, $Eu(k' + y_s + \tau_s) < Eu(k'' + y_s)$, which implies that $u(z - k' - I(z, \{\tau_s\})) > u(z - k'')$ and $u'(z - k' - \pi(z, \{\tau_s\})) < u'(z - k'')$. Taking the derivative w.r.t. z throughout (2.28), we obtain

$$\frac{dU}{dz}(z - I(z, \{\tau_s\}), \{\tau_s\}) \left[1 - \frac{dI}{dz}\right] \equiv \frac{dU}{dz}(z, .)$$

Then, using the related Envelope conditions, we obtain

$$u'\left(z - k' - I(z, \{\tau_s\})\right) \left[1 - \frac{dI}{dz}\right] \equiv u'\left(z - k''\right)$$

As we have established that $u'(z - k' - I(z, \{\tau_s\})) < u'(z - k'')$, we must have $\frac{dI}{dz} < 0$. Using similar arguments, we can show that $\frac{dI}{dz} = 0$ if $u(.)$ exhibits constant absolute risk aversion and $\frac{dI}{dz} > 0$ if $u(.)$ exhibits increasing absolute risk aversion. ■

Proof. of Proposition 2.3: If the participation constraint of person i is slack in all states in period 2, then, in equation e1, $\theta^i_{sr} = 0$ for each $r \in \mathcal{S}$. Therefore, $\rho u^{i\prime}\left(c^i_{1,s}\right) = \beta E u^{i\prime}\left(c^i_{2,sr}|s\right)$. If person i has decreasing absolute risk aversion, then, by Lemma 2.2, $\frac{\partial I^i_2}{\partial k^i_{1,s}}\left(k^i_{1,s} + y^i_r, \left\{\tau^j_t\right\}_{t=3}\right) < 0$. Then, if $\theta^i_{sr} > 0$ for some $r \in \mathcal{S}$, then according to equation e1, $\rho u^{i\prime}\left(c^i_{1,s}\right) > \beta E u^{i\prime}\left(c^i_{2,sr}|s\right)$. With similar reasoning, if person i has increasing absolute risk aversion, then $\rho u^{i\prime}\left(c^i_{1,s}\right) < \beta E u^{i\prime}\left(c^i_{2,sr}|s\right)$, and if person i has constant absolute risk aversion, then $\rho u^{i\prime}\left(c^i_{1,s}\right) = \beta E u^{i\prime}\left(c^i_{2,sr}|s\right)$. ■

Corollary 2.1 *to Proposition 2.3:Let $\tilde{k}^i_s$ be the level of savings that is individually optimal for person i after state s is realised in the first period, given initial assets k^i and contingent transfers $\{\tau^j\}$; i.e. $\tilde{k}^i_s$ solves*

$$\max_k u^i\left(k^i + y^i_s - \tau_s - k\right) + \beta E U^i\left(k + y^i_r, \{\tau^j\}\right)$$

If person i exhibits decreasing absolute risk-aversion and his participation constraint binds for some history in period 2, then $k^i_s < \tilde{k}^i_s$.

Proof. of Collorary to Proposition 2.3: $\tilde{k}^i_s$ is given by the first-order condition to the maximisation problem (2.5):

$$\rho u^{i\prime}\left(z^i_s - \tilde{k}^i_s\right) = \beta E \frac{\partial U^i}{\partial \tilde{k}^i_s}\left(\tilde{k}^i_s + y^i_r, \{\tau^j\}^3_{t=2}\right)$$

whereas k^i_s, person i's level of saving in state s, period 1, prescribed in the agreement is given by the following 'modified Euler equation' derived in section

$$x : \rho u^{i\prime}\left(z^i_s - k^i_s\right) = \beta E \frac{\partial U^i}{\partial k^i_s}\left(k^i_s + y^i_r, \{\tau^j\}^3_{t=2}\right) - \beta \sum_r \theta^i_{sr} \frac{\partial U^i}{\partial \tau^i_{sr}} \frac{\partial I^i}{\partial z^i}\left(k^i + y^i_r, \{\tau^j\}^3_{t=2}\right) \qquad (2.29)$$

If person i exhibits decreasing absolute risk-aversion, then $\frac{\partial I^i}{\partial z^i}\left(k^i + y^i_r, \{\tau^j\}^3_{t=2}\right) < 0$. Furthermore, $\frac{\partial U^i}{\partial \tau^i_{sr}} < 0$. Therefore, the last term on the right-hand side of (2.29) is negative. Because $u^i(.)$ is concave and $U^i(.)$ is concave in its first term, we must have $\tilde{k}^i_s < k^i_s$. ■

Proof. of Proposition 2.1: Given an allocation $\mathcal{A}$, if it is obtained in a subgame perfect equilibrium, the conditions in (2.1) must be satisfied. If it is not satisfied in some period t, after history h_t for person i, then person i would obtain a higher utility by deviating to an autarkic strategy for the continuation game, which contradicts the definition of subgame perfection.

Conversely, if an allocation $\mathcal{A}$ satisfies the conditions in (2.1), then we can construct a strategy profile as follows. Each individual, after each possible history chooose transfers and savings as specified in the allocation, if all previous actions in the game correspond to the allocation (the cooperation phase); after any deviation, each individual adopt autarkic strategies for the continuation game (the punishment phase). Then, the conditions in (2.1) ensure that, in the cooperation phase, a deviation in phase (ii) of the stage game (the transfers stage) cannot improve welfare. It follows that a deviation in phase (iii) of the stage game (the consumption/saving stage) also cannot improve welfare because an individual can do no better by deviating in phase (iii) than in phase (ii), as

$$U^i\left(k^i_{t-1}\left(h_{t-1}\right)+\tau^{-i}_t\left(h_t\right)+y^i_s\right)\geq U^i\left(k^i_{t-1}\left(h_{t-1}\right)+\tau^{-i}_t\left(h_t\right)-\tau^i_t\left(h_t\right)+y^i_s\right)$$

Furthermore, in the punishment phase, a deviation cannot improve welfare because the autarkic strategies are subgame perfect. Therefore, the outlined strategy profile is also subgame perfect. ■

Proof. of Proposition (2.2): Given an allocation $\mathcal{A}$, if it is obtained in a subgame perfect equilibrium with 'asset-blind' strategies, then, by Proposition (2.1), the conditions in (2.2) must be satisfied. For 'asset-blind' strategies, a deviation in savings by an individual after some history should not affect the continuation strategies of the other individuals. Therefore, if the conditions in (2.3) are *not* satisfied for some individual after some history, he can improve his expected utility by choosing a different level of saving at that node. Therefore, the conditions in (2.3) are necessary for an allocation to be obtained in a subgame perfect equilibrium using asset-blind strategies.

Conversely, suppose $\mathcal{A}$ satisfies conditions (2.2) and (2.18). We construct an 'asset-blind' strategy profile as follows. Each individual, after each possible history choooses transfers and savings as specified in the allocation, if all previous actions in the game correspond to the allocation (the cooperation phase); after any deviation involving transfers of the stage game, each individual adopts the autarkic strategy for the continuation game (the punishment phase). Note that this strategy profile does not prescribe any punishment after any deviation involving savings. Then, (2.2) ensures that it is not possible for any individual i to improve utility through a single-deviation involving transfers. In addition, (2.18) ensures that it is not possible to improve utility through a single-deviation involving savings after any history. Then using the single-deviation principle, we assert that for the given strategy profile, no deviation by any individual can be welfare-improving. Therefore, the allocation $\mathcal{A}$ can be obtained in a subgame perfect equilibrium using 'asset-blind' strategies. ■

Proof. of Proposition 2.4: For a constrained efficient allocation, any continuation agreement beginning in period 2 satisfies the 'Euler' equation for both individuals, as shown in section x, and satisfies the participation constraints by assumption. Therefore, the conditions in Proposition 2.2, for agreements on transfers that can be supported in a subgame perfect equilibrium, are satisfied. Furthermore, for constant absolute risk aversion, the consumption stream in a constrained efficient allocation also satisfies the Euler equation in period 1. Thus we obtain the result that for constant absolute risk averson utility, a constrained efficient allocation can be obtained using asset-blind strategies, and consequently in an agreement on transfers. However, for increasing or decreasing absolute risk aversion, the Euler equation is not satisfied and therefore, by Proposition 2.2, the allocation cannot be obtained using an agreement on transfers only. ■

Derivation of Equation 2.22 in Section 2.5.4

Equivalent to intertemporal conditions in (??) for period 1in the three-period game, we can write the conditions in period $T-2$ in the T - period game as follows:

$$\rho u^{B\prime}\left(c_{T-2}^{B}\right)=\beta E u^{B\prime}\left(c_{T-1}^{B}\right)-\beta\sum_{r}\theta_{T-1}^{B}\frac{\partial U^{B}}{\partial\tau_{T-1}^{B}}\frac{\partial I^{B}}{\partial z^{B}}\left(k^{B}+y_{r}^{B},\left\{\tau_{T}^{j}\right\}\right)$$

Substituting for $u^{B\prime}\left(c_{T-2}^{B}\right)$ with $\frac{d\tilde{U}_{T-2}^{B}}{dk^{B}}\left(k^{B}+y_{s}^{B},k_{T-1,s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-2}^{T}\right)$ and for $u^{B\prime}\left(c_{T-1}^{B}\right)$ with $\frac{dU_{T-1}^{B}}{dk_{T-1,s}^{B}}\left(k_{T-1,s}^{B}+y_{s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right)$, we obtain

$$\begin{aligned}\rho\frac{d\tilde{U}_{T-2}^{B}}{dk^{B}}\left(k^{B}+y_{s}^{B},k_{T-1,s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-2}^{T}\right) &= \beta E\frac{dU_{T-1}^{B}}{dk_{T-1,s}^{B}}\left(k_{T-1,s}^{B}+y_{s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right)\\ &\quad -\beta\sum_{r}\theta_{T-1}^{B}\frac{\partial U^{B}}{\partial\tau_{T-1}^{B}}\frac{\partial I^{B}}{\partial z^{B}}\left(k^{B}+y_{r}^{B},\left\{\tau_{T}^{j}\right\}\right)\end{aligned} \tag{2.30}$$

Note that, by definition,

$$U_{T-2}^{B}\left(z,\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right)=\max_{k}u\left(z-\rho k\right)+\beta E U_{T-1}^{B}\left(k+y_{s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right)$$

Using the Envelope condition and the first-order condition to this maximisation problem, we obtain

$$\frac{dU_{T-2}^{B}}{dz}\left(z,\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right)=\beta E\frac{dU_{T-1}^{B}}{dk_{T-1,s}^{B}}\left(k_{T-1,s}^{B}+y_{s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right) \tag{2.31}$$

Thus, substituting in (2.30) using (2.31) and rearranging, we obtain

$$\begin{aligned}&\frac{d\tilde{U}_{T-2}^{B}}{dk^{B}}\left(k^{B}+y_{s}^{B},k_{T-1,s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-2}^{T}\right)-\frac{dU_{T-2}^{B}}{dk^{B}}\left(k^{B}+y_{s}^{B},\left\{\tau_{t}^{j}\right\}_{t=T-1}^{T}\right)\\ =\;&-\beta\sum_{s}\theta_{T-1,s}^{B}\frac{\partial U_{T-1}^{B}}{\partial\tau_{T-1,s}^{B}}\frac{\partial I^{B}}{\partial k^{B}}\left(k^{B}+y_{s}^{B},\left\{\tau_{T}^{j}\right\}\right)\end{aligned}$$

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Chapter 3

Gender and Household Production

3.1 Introduction

Recent empirical research on the household have cast serious doubts about the assumption of Pareto efficiency in the allocation of resources within households[1]. These findings pose a challenge to the existing theories of the household; as Pareto efficiency underlies the models of the household that have so far received the most attention in the literature: the unitary, cooperative bargaining and collective models[2]. Consequently, the theoretical discussion has recently turned its attention to possible constraints that can prevent an efficient outcome within the household; such as lack of commitment and informational asymmetries[3]. To date, however, there is little evidence as to the relative importance of each of these in determining household outcomes.

In addition, the empirical literature has shown that gender is an important determinant in the allocation of resources within the household, in that the initial distribution of resources between men and women affect household outcomes, and inequality within the household tend to be correlated with gender[4].

In the theoretical literature why gender matters remains an open question. Theories of intra-household allocation treat the household as a collection of agents, with possibly conflicting preferences, but makes no distinction between the genders.

In this paper, we provide some evidence on the allocation of productive resources within the household that may help to shed some light on these two issues, central to the discussion of intra-

[1] For example, Duflo and Udry (2004), Dubois and Ligon (2002) and Dercon and Krishnan (2000) show that risk-sharing within the household is not Pareto efficient for rural households in Côte d'Ivoire, the Philippines and Ethiopia respectively. Udry (1996) shows, for agricultural households in Burkina Faso, that the allocation of productive resources within the household is inefficient.

[2] See Becker (1980) for the unitary model; Manser and Brown (1980) and McElroy and Horney (1981) for the cooperative bargaining model; and Chiappori (1988, 1992) for the collective bargaining model. Bergstrom (1997) provides an extensive survey of the theoretical literature on the household.

[3] Ligon (2003) develops a model of dynamic bargaining within the household, characterised by limited commitment.

[4] See, for example, Thomas (1990) and (1993), Lundberg, Pollak and Wales (1997), and Duflo (2000).

household allocation. We replicate the test of efficiency in household production implemented by Udry (1996) using a sample of agricultural households in Burkina Faso, in 8 villages around the country, surveyed during the years 1993 and 1994. As in Udry's analysis with the households surveyed by ICRISAT in Burkina Faso ten years earlier, we find strong evidence of inefficiency within the household: some farm plots achieve significantly higher yields than others within the same household, after controlling physical characteristics of the plot and the crops planted.

However, we show that the higher yields are achieved on plots owned by the household head, who is almost invariably male; the yields achieved by other adult men within the same household do not differ significantly from those achieved by women in the household.

While these results are consistent with the findings in Udry (1996), they help to shed further light on the question why gender matters. Within the household, the head occupies a unique position. Traditional institutions bestow upon him certain types of authority over – as well as responsibilities towards – other members of the household. These institutions may enable him to overcome problems of commitment and moral hazard within the household more effectively than other household members. We present a simple model of the household to explain our results.

The remainder of this paper proceeds as follows. Section 3.2 introduces the theoretical framework which provides a test of pareto efficiency in household production. A brief description of the household survey is given in Section 3.3. Section 3.4 discusses our empirical findings and section 3.5 discusses possible explanations of the results. Section 3.6 formally presents the most plausible of these explanations and section 3.7 concludes.

3.2 Theoretical Framework

In this section, we introduce a model of an agricultural household and a test of pareto efficiency in household production based on Udry (1996).

Suppose the household consists of N individuals, indexed $j = 1..N$. There are k different private goods and consumption of these by inidividual j is denoted by the vector $\mathbf{x}^j$. There is also a household public good, denoted by z. Individual j's preferences over consumption are given by the utility function $U^j\left(\mathbf{x}^j, \mathbf{x}^{-j}, z\right)$, increasing in $\mathbf{x}^j$ and z; we allow for the possibility that $U^j\left(.\right)$ is increasing in $\mathbf{x}^{-j}$ to represent altruism in the household.

The household owns a number of farm plots on which it can on which it can engage in the production of any of the k private goods. Let $P^k = \{i : \text{plot } i \text{ is planted to crop } k\}$. Then the total production of good k by the household is given by

$$Y^k = \sum_{i \in P^k} G^k\left(N_F^i, N_M^i, A^i\right)$$

where $G^k\left(.\right)$ is the production function of good k, N_F^i and N_M^i denote the total use of female and male labour on plot i respectively, and A^i is the area of plot i.

Udry (1996) shows that, in this setting, pareto efficiency requires that outputs are equal on plots which have the same characteristics and are planted with the same crop: i.e. $G^k\left(N_F^i, N_M^i, A^i\right) = G^k\left(N_F^j, N_M^j, A^j\right)$ if $A^i = A^j$. Then, adjusting the notation to allow for multiple households and time periods, the following specification enables us to test for efficiency in household production:

$$Q_{htci} = \mathbf{X}_{hci}\beta + \gamma G_{htci} + \lambda_{htc} + \epsilon_{htci} \tag{3.1}$$

where Q_{htci} is the log of yield on plot i in year t, planted to crop c and belonging to household h; X_{hci} is a vecor of physical characteristics of plot i; λ_{htc} is a household-year-crop fixed effect; and S_{htci} are characteristics of the individual in household h who controls plot i in year t. Udry (1996) shows that if productive resources are allocated efficiently within the household, then $S_{htci} = 0$; i.e. yield on a plot should not depend on the characteristics of the individual who controls it.

3.3 Description of the Household Survey

The household survey was conducted by the University of Laval, Quebec and the University of Ouagadougou in Burkina Faso. It was carried out in 4 provinces in different parts of Burkina Faso: the Namentenga province in the Central Plateau, the Soum province in the North, the Kossi province in the West and the Nahouri province in the Southeast. In each province, two villages were chosen, one to represent the wealthier villages and the other to present the poorer ones in that region. A sample of 35 households were randomly selected in each village except in the North where 40 households were chosen in anticipation of a higher dropout rate due to out-migration. Agriculture is the primary source of livelihood in all the villages, but livestock is relatively more important in the North, the region where population pressures and soil degradation have been most acute. Unfortunately, cotton, the most important agricultural export of Burkina Faso is not farmed extensively in any of the sample villages; therefore it is not possible to study the effects of the devaluation on cotton producers using this survey.

The survey was conducted during the 1993 and 1994 agricultural seasons and covered farm characteristics, production technologies, agricultural inputs and outputs, market activities, household expenditures and consumption. Information relating to each farm plot was obtained from the individual in the household who had responsibility for it during that season, while information regardaring the 'common' plots were obtained from the head of the household. Information about expenditures, sales, livestock holding and transfers were also gathered at the individual level; while information about housing and farming equipment were gathered at the household level, with the head of the household usually providing most of the information.

3.4 Results

3.4.1 Yield on Agricultural Plots

The estimated coefficients for equation (2.5) are shown in column 1 of Table 2. The estimated coefficient for the household head dummy is large and significantly different from zero. Therefore, we are able to reject the hypothesis of pareto efficiency in household production. However, the coefficient for the 'junior' male dummy variable (which takes a value of 1 if the plot owner is an adult male who does not head the household and zero otherwise) is close to zero; therefore, for plots owned by same household, have similar characteristics and are planted to the same crops in the same year, there are no significant differences in yields between those controlled by women and those controlled by 'junior' men. However, yields on plots controlled by the household head are significantly higher than those on other plots within the household. Therefore, the results strongly suggest that the source of inefficiency in the allocation of production resources within the household is not gender per se, but factors associated with household headship; or more generally, status within the household.

3.4.2 Labour Allocation

To examine the extent to which the differences in plot yields are driven by variations in the allocation of labour across farm plots owned by the household, we estimate the following equation:

$$L^{f}_{htci} = \mathbf{X}_{hci}\beta + \gamma G_{htci} + \lambda_{htc} + \epsilon_{htci} \tag{3.2}$$

where L^{f}_{htci} is labour of type f per hectare on plot i owned by household h, planted to crop c in year t. Labour type can by family adult male, adult female, child, and hired labour.

The estimated coefficients for equation (3.2) are shown in Table 3. The results show that plots controlled by the household head use all types of family labour – adult male, adult female, and child labour – more intensively than other plots owned by the same household, with similar characteristics and planted to the same crop in the same year. While plots controlled by 'junior' men use male family labour more intensively than those controlled by women, the opposite is true for 'female' family labour, by about the same order of magnitude. There are no significant differences in the use of child labour between the 'junior' men and the women. Therefore, the pattern of labour allocation across agricultural plots within the household correspond closely with the variation in plot yields.

3.4.3 Unobserved Plot Characteristics

It is possible that the information on plot characteristics obtained through the household survey do not adequately capture differences in land quality across farm plots. Returns to labour may be higher on better quality land. If the farm plots controlled by the household head are of better quality than those controlled by the other household members, this may explain why these plots would be cultivated more intensively and achieve higher yields. While it is not possible to rule out this possibility, one can note that the variation in labour allocation across plots can explain all of the variation in plot yields if a doubling of labour intensity leads to a fifty percent increase in yields. In this case, unobserved variations in land quality cannot justify the observed variations in labour intensity across farm plots owned by the same household.

3.4.4 Devaluation of the CFA Franc

In January 1994, the 14 countries of the 'Franc zone' in West and Central Africa, pressured by the IMF, agreed to devalue their common currency, the CFA franc by fifty-percent against the French franc. Although such a devaluation had been debated for a number of years, its timing was unexpected for the population. It was widely suggested in the press that its immediate cause was the death in December 1993 of Houphouët-Boigny, the former president of Côte d'Ivoire, who strongly opposed the devaluation and had close relations with the French Government.

It was anticipated that while the devaluation would lead to a sharp increase in the prices of imported goods, it would provide a boost to the agricultural sector in the CFA zone; by making the local produce more competitive in both regional and export markets. Savadogo and Kazianga (1999) find that urban dwellers in Burkina Faso, especially low-income households lowered their consumption of imported rice and wheat in favour of locally grown cereals following the devaluation.

The CEDRES/Laval survey in Burkina Faso was conducted during the years immediately before and after the devaluation. As such, the timing of the survey is ideal to study the immediate response of household production and consumption to the devaluation. However, such analysis is hindered by the fact that the survey did not collect any information about prices. In addition, the absence of information about consumption of own produce makes it difficult to assess whether agricultural households experienced the devaluation as an income shock or an economic opportunity.

Here, we provide a limited analysis of the devaluation by asking whether the variation in labour intensity and yields across agricultural plots within the household were driven or affected by the devaluation. Specifically, in the plot yield and labour intensity regressions, we interact the 'household head' and 'junior male' variables with a post-devaluation dummy, and replace the household-crop-year fixed effects with household-crop fixed effects and a year dummy. The estimates of the plot yield equation with these interactions, as well as controls for variations in rainfall, are shown in column 4 of Table 2. While yields increase on all plots following the devaluation, the difference in yields across different types of plot owners noted earlier are present in both 1993 and 1994, i.e. both before and after the devaluation.

The estimates of the labour intensity equation with the plot-owner – year interactions are shown in Table 4. The results indicate that while the variation in labour intensity across farm plots noted in the last section were present during 1993, they increased following the devaluation. Plots owned by the household head used adult male and female family labour more intensively following the devaluation. For female-owned plots, the increase was limited to female family labour, whereas for other male-owned plots, the increase was limited to male family labour. This last group actually sees a significant *decrease* in female family labour.

The large increase in labour intensity, and the corresponding increase in yields, on farm plots following the devaluation could have happened because the devaluation presented an economic opportunity or because it impoverished agricultural households. However, Azam and Wane (2001) calculate, using household surveys in Côte d'Ivoire and Mali, that the CFA franc devaluation led to a deepening of poverty in all sections of the population in these countries.

3.5 Possible Explanations

The near absence of land and labour markets in Africa has been frequently noted in the anthropological literature. The literature has pointed to the weakness of property rights in Africa as the reason why land is rarely rented out. And the predominance of family labour in agricultural production has been attributed to the fact that farm plots are usually small and widely dispersed, making it costly to monitor the labour activity of a hired agent.

Udry (1996) argues that the factors that prevent the efficient allocation of productive resources across households should also act as significant constraints within the household. The literature records little evidence of land being leased out to another household member; and although members of a household frequently do work on each other's farm plots, they rarely receive an explicit compensation for such work. The presence of moral hazard wthin the household, and the inability to commit to a wage would prevent the efficient allocation of labour across farm plots within the household.

However, the ties that bind together members of a household, such as their emotional attachments to each other and the presence of public goods, play an important role in overcoming the constraints imposed by moral hazard and lack of commitment.

The empirical evidence in this paper suggests how the household is able to overcome these problems, albeit in a limited manner. The evidence shows that the household head is able to call upon other members of the household to work on his land much more successfully than they are able to call upon each other. Plots which are controlled by the head of the household use both male and female family labour more intensively than plots which are controlled by women or other men within the same household. Therefore, it appears that an individual has much stronger incentives to work on a plot owned by the household head than on a plot belonging to another member of the household. We consider possible reasons why this may be true.

The household head acquires authority, as well as responsibilities within the household on the basis of traditional social institutions. Duflo and Udry (2004) discusses a social norm in West Africa which places certain cash crops, such as rice and yams under the control of the household head; but also requires him to use the revenue from these crops to provide household public goods. Furthermore, they find, using a survey of households in Côte d'Ivoire, that expenditures within the household on items such as children's education are more responsive to shocks to income from yams than other crops.

In Burkina Faso, a similar social norm places certain 'common' plots under the control of the household head; who is expected to use the income from these plots to provide for his household. Such a social norm would ensure that an individual receive some return for labour on a 'common' plot in the form of increased expenditures on public goods. The social norm would ameliorate the problem of moral hazard, as well as the inability to commit to provide compensation for farm-work. Consequently, higher yields would be achieved on 'common' plots than on plots privately owned within the household.

Our findings also have close similarities to an analysis of agricultural production in rural Ghana by Goldstein and Udry (2005). Goldstein and Udry find that, female farmers achieve lower profits than their male counterparts within the same household and in the same year, after controlling for plot characteristics and crops planted. They find that all of this difference can be attributed to the length of time that a parcel of land is left fallow between production cycles. Fallowing improves soil fertility but individuals with little political power in the community risk losing rights over their land if it is left fallow for extended periods. Thus the gender difference in productivity is traced to differences in political power in the community, which is correlated with gender. This explanation is also consistent with our findings that the household head, who is likely to have greater say in the community, achieves higher yields than women and other men in the same household. In the absence of information on land fallowing in the CEDRES/Laval survey, it is not possible to ascertain to what extent this factor is responsible for observed differences in yields.

An alternate model that can account for the cross-sectional variation in labour intensity across plots is one where members of the household have a social obligation to provide labour on plots owned by the head of the household (equivalently, it is socially acceptable for the household head to punish those who do not comply with his wishes) but no such obligation exists for sharing the output from their own plots. Then we would obtain the result that the household head's plots are farmed more intensively than other plots owned by the household. Such obligations would also explain why women provide some labour on farm plots owned by men in the family other than the household head, and vice versa.

However, it is more difficult to explain the change in labour intensity following the devaluation within this framework. According to the evidence presented in the previous section, plots across all types of owners are farmed more intensively during the year after the devaluation. In particular, family members work longer on plots owned by the household head. However, there is no compelling reason to believe that obligations regarding the contibution of labour on a farm plot owned by a family member should increase following the devalution. Indeed, the contribution of family labour on plots owned by family members other than the household head remains unchanged or, in the case of female labour on plots owned by 'junior' men, actually declines.

On the other hand, the increase in labour intensity on the household head's plots following the devaluation is consistent with the view that the cross-sectional variation arise from the presence of household public goods and social norms relating to the use of output from common plots: if the devaluation causes an adverse income shock or leads to an increase in the marginal product of farm labour compared to nonfarm labour, it would correspondingly lead household members to put in more labour on the 'common' plots at the expense of leisure or nonfarm activities.

3.6 An Alternate Equilibrium

In this section we consider the non-cooperative equilibrium for the model introduced in Section 3.2 to explain our empirical findings. We assume that each farm plot is owned by a specific individual within the household and that, because of lack of commitment, the members of the household cannot have a cooperative agreement that would enable them to implement the pareto efficient outcome in household production. However, traditional insitutions require the household head to spend all the income from a 'common' plot, that is under his control, on household public goods, failing which he is subject to social sanctions of some sort.

To simplify the notation and the analysis, we assume here that there is one private good only. The price of the private good is p and the price of the public good is normalised to 1. Each household member owns one farm plot on which the private good may be produced. Each plot is identical and the production function is given by $G\left(N_F, N_M\right)$.

There are two stages in the game. At the first stage, individuals decide how to allocate labour across the farm plots. At the second stage, they choose how to divide expenditures between the private good and the public good. Then, in the non-cooperative equilibrium, we obtain

$$\frac{1}{p}\frac{\partial U^j\left(x^j, z\right)}{\partial x^j} \geq \frac{\partial U^j\left(x^j, z\right)}{\partial z}$$

for each j, and the condition is satisfied with equality if the individual's own expenditure on the public good is greater than zero. In the absence of any social sanctions, the household head uses the income from his farm plots for his private expenditures; and each household member allocates all of his available labour to his own plot. Thus, the allocation of productive resources within the household is autarkic. However, if the household head is required to use the income from the common plots for expenditures on the household public good, then each individual's labour allocation satisfies the following condition:

$$\frac{1}{p}\frac{\partial U^j\left(x^j,z\right)}{\partial x^j}\frac{\partial I\left(N_p^j,0\right)}{\partial N^j} \geq \frac{\partial U^j\left(x^j,z\right)}{\partial z}\frac{\partial I\left(N_c^j,N_c^{-j}\right)}{\partial N^j}$$

for each j, and the condition is satisfied with equality if $N_c^j > 0$. The private plots receive only the plot owner's labour for an individual receives no benefit from labour on the the private plot of another member of the household.

However, the common plot receives labour from all household members who sufficiently value the household public good. In particular, we can show that, given z^{-j}, public goods expenditures by all individuals other than j in the non-cooperative equilibrium, if $\frac{1}{p}\frac{\partial U^j\left(I\left(N_p^j,0\right),z^{-j}\right)}{\partial x^j} < \frac{\partial U^j\left(I\left(N_p^j,0\right),z^{-j}\right)}{\partial z}$ then $N_c^j > 0$. That is, any individual j who does not spend all his resources on private consumption will choose to contribute a positive amount of labour to the common plot.

3.7 Conclusion

In this paper we presented results which indicate that, for agricultural households in Burkina Faso, farm plots controlled by the head of the household are farmed more intenstively and achieve higher yields than other plots owned by the household with similar characteristics and planted to the same crop. These findings can help to illuminate the discussion on intra-household allocation along two dimensions.

First, the recent empirical literature has provided convincing evidence of instances where the allocation of resources within the household is not Pareto efficient. These findings have highlighted the importance of identifying constraints that can prevent the household from reaching the Pareto frontier. Constraints that have been discussed in this regard include informational asymmetries and lack of commitment within the household. The empirical findings in this paper suggest that members of a household have much stronger incentives to work on a plot belonging to the head of the household than they have to work on some other household plot other than their own. In other words, lack of commitment and informational asymmetries are less of a problem in relations involving the household head.

Second, empirical studies of households in developing countries have repeatedly found that the allocation of resources within the household is correlated with gender. By contrast, theories of intra-household allocation, for the most part, have remained gender-blind. The household is treated as a collection of agents with distinct preferences, but with no a priori difference between a man and a woman.

In this paper we suggest that some of the gender-related patterns observed in the empirical investigation of households may be explained by the fact that the head of the household, who is bestowed with authority over as well as responsibilities towards the other members of the household by existing social norms, is in most instances male. We suggest that gender-related patterns in the allocation of resources within the household can be explained without resorting to any assumption of *innate* differences in preferences or power between men and women; but rather by differences in their positions created by social norms. The key piece of evidence we provide for this argument is that, for agricultural households in Burkina Faso, yields on plots owned by men who are not household heads are similar to those achieved on plots owned by women in the same household (controlling for plot characteristics and the crops planted), while yields achieved by male household heads are significantly higher.

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Table 1: Mean Yield, Area and Labour Inputs by Different Types of Cultivator

	Yield (FCFA/Hectare)	Area (Hectare)	Child Labour (Labour Days/Hectare)	Male Labour (Labour Days/Hectare)	Female Labour (Labour Days/Hectare)
Male Household Head	76,811	1.206	11.43	48.49	33.92
	(305,997)	(1.32)	(21.4)	(58.3)	(50.66)
Junior Men	110,101	0.67	10.57	42.81	24.36
	(412,363)	(0.749)	(21.8)	(52.7)	(41.95)
Women	90,512	0.496	6.56	20.07	50.4
	(392,120)	(0.498)	(17.22)	(38.33)	(77.42)

Standard Deviation in Parantheses.

Table 2: Distribution of Primary Crops Across Plots by Type of Owner

	Male Head	Male Non-Head	Female
Millet	33.0	28.0	36.0
White Sorghum	24.0	46.0	26.0
Red Sorghum	11.0	6.0	2.0
Maize	5.9	1.5	0.2
Other Cereals	8.3	3.4	1.1
Earth Peas	0.4	0.0	9.1
Groundnuts	8.6	11.0	17.0
Other	8.8	4.1	8.6

Table 3: Plot Yield

	Dependent Variable: Ln(Plot Yield in CFA Franc)		
	Household-Crop-Year Fixed Effects	Household-Crop Fixed Effects	
	(1)	(2)	(3)
Junior Male Dummy	0.0067	-0.0969	-0.1663
	(0.122)	(0.168)	(0.167)
Male HHead Dummy	0.7542	0.6668	0.7274
	(0.105)	(0.128)	(0.129)
Year 1994		0.1672	1.247
		(0.134)	(0.323)
1994 * Junior Male		0.2864	0.4417
		(0.231)	(0.233)
1994 * Male HHead		0.0294	-0.0408
		(0.15)	(0.159)
Plot Charac. * Rainfall	No	No	Yes
Plot Size (1st decile omitted):			
2nd decile	-0.7742	-1.154	-1.171
	(0.156)	(0.162)	(0.174)
3rd decile	-0.5572	-0.9053	-1.121
	(0.165)	(0.173)	(0.181)
4th decile	-1.046	-1.432	-1.582
	(0.167)	(0.17)	(0.186)
5th decile	-1.276	-1.616	-1.724
	(0.166)	(0.168)	(0.184)
6th decile	-1.505	-1.823	-1.904
	(0.184)	(0.182)	(0.194)
7th decile	-1.314	-1.705	-1.801
	(0.175)	(0.175)	(0.184)
8th decile	-1.62	-1.981	-2.179
	(0.184)	(0.187)	(0.195)
9th decile	-1.903	-2.176	-2.276
	(0.184)	(0.183)	(0.196)
10th decile	-2.082	-2.399	-2.543
	(0.179)	(0.181)	(0.189)
Toposequence ('mid-slope' omitted):			
Near Bottom	-0.474	-0.4434	-0.2183
	(0.267)	(0.244)	(0.251)
Plain	-0.4707	-0.2868	-0.1422
	(0.19)	(0.225)	(0.225)
Soil Type ('clay' omitted):			
Sand	0.0463	-0.1699	-0.0933
	(0.155)	(0.139)	(0.147)

Laterite	-0.3106	-0.3774	-0.2636
	(0.191)	(0.18)	(0.184)
Gravel	-0.1416	-0.7196	-0.6427
	(0.323)	(0.336)	(0.342)
Location (village 'excluded)			
Outside village	0.0049	0.0197	-0.0667
	(0.104)	(0.102)	(0.106)
Observations	1871	1871	1871

Table 4: Labour Intensity

	Dependent Variable: Ln (Labour Days Per Unit Area on Plot of Land)		
	Male Labour (1)	Female Labour (2)	Child Labour (3)
Junior Male Dummy	1.211	-1.163	0.2114
	(0.137)	(0.126)	(0.131)
Male HHead Dummy	2.162	0.5196	0.8146
	(0.114)	(0.104)	(0.108)
Plot Size (1st decile omitted):			
2nd decile	-0.6086	-0.5258	-0.7544
	(0.178)	(0.164)	(0.170)
3rd decile	-0.9657	-0.8277	-0.7673
	(0.188)	(0.173)	(0.180)
4th decile	-1.312	-1.389	-0.8364
	(0.195)	(0.179)	(0.186)
5th decile	-1.474	-1.317	-1.046
	(0.188)	(0.173)	(0.179)
6th decile	-1.723	-1.71	-1.133
	(0.198)	(0.182)	(0.189)
7th decile	-1.757	-1.814	-1.083
	-0.201	(0.185)	(0.192)
8th decile	-1.643	-1.881	-1.012
	(0.203)	(0.187)	(0.194)
9th decile	-1.872	-2.267	-1.035
	(0.210)	(0.193)	(0.201)
10th decile	-2.204	-2.577	-1.21
	(0.217)	(0.199)	(0.207)
Toposequence ('near bottom' omitted):			
Mid-slope	0.3143	0.3994	0.2027
	(0.304)	(0.279)	(0.290)
Plain	-0.1746	-0.0093	-0.3498
	(0.192)	(0.177)	(0.184)
Soil Type ('clay' omitted):			
Sand	-0.2028	-0.0766	0.2597
	(0.152)	(0.140)	(0.146)
Laterite	0.0724	-0.1066	0.4236
	(0.177)	(0.163)	(0.169)
Gravel	0.3381	0.0471	0.8797
	(0.308)	(0.284)	(0.295)
Location ('village' excluded)			
Outside village	0.2246	-0.0013	-0.0065
	(0.112)	(0.103)	(0.107)
Observations	1979	1979	1979

Household-crop-year fixed effects included in each regression. Standard Errors in Parantheses.

Table 5: Change in Labour Intensity between 1993 and 1994

	Dependent Variable: Ln (Labour Days Per Unit Area on Plot of Land)		
	Male Labour	Female Labour	Child Labour
	(1)	(2)	(3)
Junior Male Owner	0.8258	-0.1279	0.0331
	(0.174)	(0.169)	(0.173)
Male HHead Owner	1.6	0.6643	0.388
	(0.13)	(0.126)	(0.129)
Year 1994	0.09	0.9289	0.0451
	(0.12)	(0.117)	(0.12)
1994 * Junior Male	0.69	-1.707	0.3023
	(0.217)	(0.211)	(0.216)
1994 * Male HHead	0.625	-0.4576	0.3564
	(0.139)	(0.135)	(0.138)
Observations	1979	1979	1979

Household-crop fixed effects and controls for plot characteristics included in all regressions. Standard Errors in Parentheses.